Corrections to "On the Quasi-Orthogonality of LoRa Modulation"

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THE PURPOSE of this correspondence is to provide corrections to typos appeared in [1] and to report that [1] contains a missing citation of the reference [2].

First, in [1, eq. (4)], the expressions of $x_2(t-\tau)$ for $\tau \leq t < t_2+\tau$ and $t_2+\tau \leq t < T_2+\tau$ contain a missing term $-\frac{\tau}{T_2}$ in the exponent of the complex exponential and these should be corrected as follows:

$$x_2(t-\tau) = \begin{cases} Ae^{j2\pi\left[\left(\frac{s_2}{M_2}-\frac{1}{2}-\frac{\tau}{T_2}\right)Bt+\frac{B}{2T_2}t^2\right]}, & \tau \leq t < t_2+\tau \\ \mathcal{A}e^{j2\pi\left[\left(\frac{s_2}{M_2}-\frac{3}{2}-\frac{\tau}{T_2}\right)Bt+\frac{B}{2T_2}t^2\right]}, & t_2+\tau \leq t < T_2+\tau \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

Note that these typos do not affect in any way the results, analysis, and conclusions presented in [1].

Meanwhile, in the literature, [2] first presented the mathematical expressions of the cross-correlation functions between two LoRa waveforms in both the continuous- and discrete-time domains considering (possibly) different spreading factors (SFs) and bandwidths. Accordingly, the merit of the pioneering and first investigation on the research topic in [2] should be acknowledged and appropriate corrections or updates need to be made to parts of [1] as follows.

1) In Abstract, the sentence "This article, for the first time, theoretically tackles the quasi-orthogonality of the LoRa modulation." should be corrected as "This article theoretically tackles the quasi-orthogonality of the LoRa modulation." Also, in the fourth paragraph of Section I, the first sentence "In this article, we for the first time theoretically tackle the quasi-orthogonality of the LoRa modulation in both continuous- and discrete-time domains." should be replaced by "In this article, we theoretically tackle the quasi-orthogonality of the LoRa modulation in both continuous- and discrete-time domains."
2) In the second paragraph of Section I, the last sentence "However, in the literature, there has been no analytical effort to establish theoretical validity of the quasi-orthogonality of the LoRa modulation." should be changed with "However, in the literature, there has been rare analytical effort to establish theoretical validity of the quasi-orthogonality of the LoRa modulation: to the best of the authors' knowledge, the issue was touched only in [2]."

REFERENCES

[1] J.-M. Kang and D.-W. Lim, "On the quasi-orthogonality of LoRa modulation," *IEEE Internet Things J.*, vol. 10, no. 14, pp. 12366–12378, Jul. 2023.
[2] F. Benkhelifa, Y. Bouazizi, and J. A. McCann, "How orthogonal is LoRa modulation?" *IEEE Internet Things J.*, vol. 9, no. 20, pp. 19928–19944, Oct. 2022.

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