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RESEARCH ARTICLE

Hidden Mode Based Controller Design for a Class of Hybrid Singular Markovian Jump Delay Systems

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ABSTRACT In this paper, the finite-time H_∞ control problem for a class of hybrid singular Markovian jump systems with time delay and actuator saturation is studied. Considering the discontinuities caused by Markovian jump switching behavior, a hidden mode based controller is designed to ensure the mean-square locally finite-time H_∞ stability of the considered system. Secondly, the parameter solving method of the designed controller is derived based on LMI method. Finally, a numerical example is given to verify the correctness of the method.

INDEX TERMS Finite-time H_∞ control, actuator saturation, time delay, stochastic singular systems.

I. INTRODUCTION

During the past decades, Markov jump mode has been widely used in many practical fields, such as economic system, chemical system and aerospace system. As a research hotspot in the field of control theory, many innovative results have been derived on Markov jump systems [1], [2], [3]. Markov jump systems have brand applications such as T-S fuzzy control problem [4], [5], filtering design problem [6], [7], network control problem [8], [9]. Meanwhile, compared with the above results, when the considered system is singular Markov jump system, the range of application of the results obtained will be wider [10], [14]. For instance, the resilient filter was designed for a class of singular Markov jump systems to deal with deception attacks in [15]. Furthermore, considering dual deception attacks, a dissipative asynchronous controller was designed for a class of T-S fuzzy singular Markov jump systems in [16]. It is worth noting that, the inconsistency between controller mode and real system mode may occur due to the physical limitation or network transmission [17], [18]. In [19], a hidden Markov mode based filter was designed to deal with this problem caused by network transmission and the physical limitation. When the modes of the original system cannot be obtained directly, the hidden Markov mode is used to detect the modal information, and the asynchronous fuzzy integral sliding mode control problem based on dissipation is studied in [20]. In [21], the H_∞ tracking controller was designed when the controller cannot accurately obtain the mode information of the system. Up to now, the hidden Markov model has been widely studied and many results have been obtained.

Moreover, due to the wide application of stochastic systems in science and engineering, scholars are increasingly interested in the research of stochastic systems and lots of results have been obtained in [22], [23], [24], [25], and [26]. To mention a few, by proposing a hybrid model that combines the advantages of KDJ and grey Markov chain, it provides a useful decision support tool for investors participating in the digital currency market in [27]. In [28], an extended stochastic gradient Markov chain Monte Carlo algorithm was proposed to complete the controller design. In [29], a hybrid-triggered controller was presented for class of hybrid-triggered fuzzy Markov jump system subject to input saturation.

On the other hand, in some practical engineering systems, such as aerospace system, robot control system and other short-time working systems, finite time control has a very important application, Therefore, the finite-time control

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problem has become a research hotspot in the field of control, and many important results have been presented in [30], [31], [32], [33], [34], and [35]. It is worth mentioning that, the discontinuities occurring at the moment of switching may lead to system instability or other failures. However, according to the literature review there are only a few results considered about such discontinuities in [36], [37], and [38]. The H_∞ finite-time control problem of the singular hybrid Markov jump delay system has not been fully investigated till now.

Based on the above discussions, the finite-time H_∞ control issue has been investigated for a class of singular Markov jump systems with time delay, input saturation and discontinuity. In this paper, LMI method is used to design the controller parameters, when the above nonlinear factors are combined. The main work of this paper is to obtain strict LMI by applying mathematical technique and reduce the conservatism of the obtained results. There are two main contributions in this work.

1) Compared with [3], various practical factors such as the discontinuities and input saturation are considered in this paper and the proposed method is more general;

2) Compared with [3], by design appropriate Lyapunov-Krasovskii function, the delay-depended result is derived to reduce conservatism in this paper.

II. MODEL DESCRIPTIONS AND PRELIMINARIES

Consider the following singular hybrid Markov jump systems (Σ):

$$\begin{aligned} dE(r(t))x(t) = {} & (A(r(t))x(t) + A_1(r(t))x(t-h) \\ & + A_2(r(t))\int_{t-\tau}^{t} x(s)ds + B(r(t))sat(u(t)) \\ & + D(r(t))D(t))dt + (W(r(t)x(t) \\ & + W_1(r(t))x(t-h) \\ & + W_2(r(t))\int_{t-\tau}^{t} x(s)ds \\ & + B_1(r(t))sat(u(t)))dw(t) \end{aligned} \tag{1}$$

$$\begin{aligned} Z(t) = {} & Z(r(t))x(t) + Z_1(r(t))x(t-h) \\ & + Z_2(r(t))\int_{t-\tau}^{t} x(s)ds + B_2(r(t))sat(u(t)) \end{aligned} \tag{2}$$

$$x(t) = \eta(t), \quad t \in [-max(\tau,\ h),\ 0],\ r_t = r_0, \tag{3}$$

where $x(t) \in \mathbb{R}^n$ is the state of system, $Z(t) \in \mathbb{R}^q$ represents the controlled output of system, the input $u(t) \in \mathbb{R}^m$. $D(t)$ represents the disturbances, the Brownian motion $w(t) \in \mathbb{R}^m$, $\eta(t)$ is the function initial state. The positive constants $\tau > 0$ and $h > 0$ which represent time-delays. $\{r(t)\}$ denotes a Markovian process and the values are taken in a finite set $S = \{1, 2, \ldots, \mathcal{N}\}$ with the following transition probabilities:

$$\Pr\{r(t+\Delta t) = j\,|\,r(t) = i\} = \begin{cases} \lambda_{ij}\Delta t + o(\Delta t) \\ \qquad i \neq j, \\ 1 + \lambda_{ii}\Delta t + o(\Delta t) \\ \qquad i = j, \end{cases}$$

For $j \neq i$, the transition rate $\lambda_{ij} \geq 0$, and

$$\lambda_{ii} = -\sum_{j \in S, j \neq i} \lambda_{ij}. \tag{4}$$

This paper considers the partly known transition rates as the follows:

$$Pr = \begin{bmatrix} \lambda_{11} & ? & \lambda_{13} & \cdots & \lambda_{1n} \\ \lambda_{21} & ? & \lambda_{23} & \cdots & ? \\ \vdots & \vdots & ? & \ddots & \vdots \\ \lambda_{n1} & ? & \lambda_{n3} & \ldots & ? \end{bmatrix}$$

where "?" represents the unknown transition rates. For $\forall i \in S$, the set S^i denotes:

$$S^i = S^i_k \bigcup S^i_{uk}$$

where S^i_k and S^i_{uk} represent the set of unknown parts and known parts of transition rates respectively.

The inputs $u(t)$ are bounded as follows:

$$-u_{0(i)} \leq u_{(i)} \leq u_{0(i)}, \quad u_{0(i)} > 0,\ i = 1, \cdots, m. \tag{5}$$

For $\forall i \in S$, this paper denotes $A_i = A(r(t))$, $A_{1i} = A_1(r(t))$ for the system (Σ). This paper assumes that the controller can not exactly received the value of system mode $r(t)$ in the hidden Markov model. The hidden markov mode based controller is designed as the follows:

$$u(t) = k_{1\sigma(t)}x(t) + k_{2\sigma(t)}\int_{t-\tau}^{t} x(s)ds, \tag{6}$$

where $k_{1\sigma(t)} \in R^{m\times n}$ and $k_{2\sigma(t)} \in R^{m\times n}$. The probability is estimated as the follows:

$$P(\sigma(t) = p|r(t) = i) = \pi_{ip}, \quad \sum_{p=1}^{M}\pi_{ip} = 1. \tag{7}$$

If $\sigma(t) = p,\ r(t) = i$, taking Eq.(6) to Eq.(1) and defining $\psi(u(t)) = sat(u(t)) - u(t)$, one can derive

$$\begin{aligned} dE_i x(t) = {} & ((A_i + B_iK_{1p})x(t) + A_{1i}x(t-h) \\ & + (A_{2i} + B_iK_{2p})\int_{t-\tau}^{t} x(s)ds + B_i\psi(u(t)) + D_iD(t))dt \\ & + ((W_i + B_{1i}K_{1p})x(t) + W_{1i}x(t-h) \\ & + (W_{2i} + B_{1i}K_{2p})\int_{t-\tau}^{t} x(s)ds + B_{1i}\psi(u(t)))dw(t) \end{aligned} \tag{8}$$

For the SJMS (Σ), some definitions and lemmas are given as the follow:

Assumption 1 [11]: The external disturbance $D(t)$ is varying and satisfies the following constraint condition:

$$\int_0^T D(t)^T D(t)dt \leq d,\ d \geq 0. \tag{9}$$

Definition 1 [13]: Regular and impulse-free.

(i) System (1)-(3) with D(t) = 0 is said to be regular, if $det(sE_i - A_i) \neq 0$ for all $t \in [0,\ T]$.

(ii) System (1)-(3) with D(t) = 0 is said to be impulse-free, if $deg(det(sE_i - A_i)) = rank(E_i)$ for all $t \in [0,\ T]$.

Lemma 1 [36]: For$\forall i \in N$, if and only if there exist invertible matrices $\tilde{M}_i$ and $\tilde{N}_i$ such that

$$\tilde{M}_i A_i \tilde{N}_i = \begin{bmatrix} \hat{A}_i & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{M}_i E_i \tilde{N}_i = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \tag{10}$$

the pair $(E_i,\ A_i)$ is said to be regular and impulse-free.
On the otherhand, there exist matrices M_i and N_i such that

$$\begin{aligned} \bar{E} &= M_i E_i N_i = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \\ \bar{A}_i &= M_i A_i N_i = \begin{bmatrix} \bar{A}_{1i} & \bar{A}_{2i} \\ \bar{A}_{3i} & \bar{A}_{4i} \end{bmatrix}. \end{aligned} \tag{11}$$

then the pair $(E_i,\ A_i)$ is impulse-free and regular if and only if $\bar{A}_{4i}$ is nonsingular and the above decomposition is satisfied. Make $\bar{x}(t) = N_i^{-1} x(t)$, the system (8) can be rewritten as the follows:

$$\begin{aligned} d\bar{E}\bar{x}(t) &= f(t)dt + g(t)dw(t), \\ f(t) &= (\bar{A}_i + \bar{B}_i \tilde{K}_{1p})\bar{x}(t) + \bar{A}_{1i}\bar{x}(t-h) \\ &\quad + (\bar{A}_{2i} + \bar{B}_i \tilde{K}_{2p}) \int_{t-\tau}^{t} \bar{x}(s)ds \\ &\quad + \bar{B}_i \psi(u(t)) + \bar{D}_i D(t), \\ g(t) &= (\bar{W}_i + \bar{B}_{1i}\tilde{K}_{1p})\bar{x}(t) + \bar{W}_{1i}\bar{x}(t-h) \\ &\quad + (\bar{W}_{2i} + \bar{B}_{1i}\tilde{K}_{2p}) \int_{t-\tau}^{t} \bar{x}(s)ds + \bar{B}_{1i}\psi(u(t)), \end{aligned} \tag{12}$$

where

$$\bar{A}_{1i} = M_i A_{1i} N_i, \quad \bar{A}_{2i} = M_i A_{2i} N_i,\ \bar{B}_i = M_i B_i,$$

and

$$\begin{aligned} \bar{B}_{1i} &= M_i B_{1i}, \quad \bar{W}_i = M_i W_i N_i,\ \bar{W}_{1i} = M_i W_{1i} N_i, \\ \bar{W}_{2i} &= M_i W_{2i} N_i, \quad \tilde{K}_{1p} = K_{1p} N_i,\ \tilde{K}_{2p} = K_{2p} N_i, \end{aligned}$$

and

$$\bar{A}_i = \bar{A}_i + \bar{B}_i \tilde{K}_p = \begin{bmatrix} \bar{A}_{ip}^1 & \bar{A}_{ip}^2 \\ \bar{A}_{ip}^3 & \bar{A}_{ip}^4 \end{bmatrix}.$$

This paper assumes that the system state before and after the switching moment may not be consistent which is caused by the switching behavior and the mode-dependent singular matrix E_i. Denote $\bar{x}(t_{jq})^-$ and $\bar{x}(t_{jq})^+$ as the state immediately before and after the switching moment t_{jq}, respectively. If the considered system is impulse-free and regular, then we have

$$\begin{aligned} \bar{x}(t_{jq})^+ &= \Gamma_{ij}^q \bar{x}(t_{jq})^-,\ with \\ \Gamma_{ij}^q &= \begin{bmatrix} I & 0 \\ -(\bar{A}_{jq}^4)^{-1}\bar{A}_{jq}^3 & 0 \end{bmatrix} N_j^{-1} N_i. \end{aligned} \tag{13}$$

Before giving the main results of this paper, some definitions and lemmas should be given as the follows.

Lemma 2 [29]: For the controller gain $\tilde{K}_{1p}$, the given appropriate matrix $L_i \in \mathbb{R}^{m\times n}$, if $\bar{x}(t)$ is in the set $D(u_o)$ which is defined as follows:

$$\begin{aligned} D(u_o) = \{&\bar{x}(t) \in \mathbb{R}^n;\ -u_{0(k)} \le (\tilde{K}_{1p(k)} + L_{i(k)})\bar{x}(t) \\ &\le u_{0(k)}, \quad u_{0(k)} > 0,\ k = 1, \ldots, m\}, \end{aligned}$$

then there exist any diagonal matrix $T_i > 0$, such that:

$$\psi(u(t))^T T_i(\psi(u(t)) - L_i\bar{x}(t)) \le 0.$$

Lemma 3 [29]: For the following symmetric matrix $F \in \mathbb{R}^{(n+m)\times(n+m)}$

$$F = \begin{bmatrix} F_{11} & F_{12} \\ F_{12}^T & F_{22} \end{bmatrix},$$

where $F_{11} \in \mathbb{R}^{n\times n}$, $F_{12} \in \mathbb{R}^{n\times m}$, $F_{22} \in \mathbb{R}^{m\times m}$, the following conditions are equivalent:

1) $F < 0$.
2) $F_{11} < 0, \quad F_{22} - F_{12}^T F_{11}^{-1} F_{12} < 0$.
3) $F_{22} < 0, \quad F_{11} - F_{12} F_{22}^{-1} F_{12}^T < 0$.

Definition 2 [33]: For the given scalars $c_1 > 0,\ c_2 > 0$, with $c_1 < c_2$, the constant time $T > 0$, and mode-dependent matrix $\hat{R}_i > 0$, if there exist state feedback controller in form (6) such that

$$\begin{aligned} &E\{\bar{x}^T(t_1)E^T\hat{R}_iE\bar{x}(t_1)\} \le c_1 \\ &\quad \Rightarrow\ E\{\bar{x}^T(t_2)E^T\hat{R}_iE\bar{x}(t_2)\} < c_2, \\ &\quad\quad t_1 \in [-\tau\ 0], \quad t_2 \in [0\ T], \end{aligned} \tag{14}$$

then the systems (12) with $d(t) \neq 0$ is said to be stochastically finite-time bounded stable with respect to $(c_1,\ c_2,\ T,\ \hat{R}_i,\ d)$.

Definition 3 [18]: Make $V(x(t),\ r_t,\ \sigma_t)$ be an $IT\check{o}$ process with the stochastic differential given by

$$\begin{aligned} dV(x(t),\ r_t,\ \sigma_t) = (&V_t(x(t),\ r_t,\ \sigma_t) \\ &+ V_x(x(t),\ r_t,\ \sigma_t)f(t) \\ &+ \frac{1}{2} trace\ g(t)^T V_{xx}(x(t),\ r_t,\ \sigma_t)g(t))dt \\ &+ V_x(x(t),\ r_t,\ \sigma_t)g(t)dwt. \end{aligned} \tag{15}$$

III. MAIN RESULTS

In this section, some sufficient conditions will be firstly proposed to ensure the finite-time H_∞ stability of the systems (12).

Theorem 1: For some given scalars, $\tau > 0,\ h > 0,\ 0 < \upsilon < 0.5,\ \bar{\upsilon} > 0$ and given matrices $L_{1i},\ L_{2i},\ U_{i1},\ U_{i2}$, if there exists some positive scalars $\alpha,\ \lambda_0,\ \lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4,\ \bar{\lambda}_0,\ \bar{\lambda}_1,\ \bar{\lambda}_2,\ \bar{\lambda}_3$ and mode-dependent matrix $\bar{\Omega}_{ip}$, symmetric positive-define matrices $\bar{Q}_1,\ \bar{Q}_2,\ \bar{Q}_3,\ \bar{Q}_4$ and diagonal positive definite matrices

$\hat{\Omega}_{ip}$, G_i, such that

$$\begin{bmatrix} \bar{\Phi} & h\bar{\Pi}_{1i}^T & \sqrt{\frac{1}{2}}\bar{\Pi}_{2i}^T & \bar{\Pi}_{2i}^T & \bar{\Pi}_{3i}^T & \Phi_{ip} \\ * & -\bar{Q}_3 & 0 & 0 & 0 & 0 \\ * & * & -\tilde{\Xi}_{ip} & 0 & 0 & 0 \\ * & * & * & -\frac{1}{h}\bar{Q}_4 & 0 & 0 \\ * & * & * & * & -\bar{Q}_4 & 0 \\ * & * & * & * & * & -\Theta_i \end{bmatrix} < 0, \quad (16)$$

$$\begin{bmatrix} \digamma_{11} & \digamma_{12} \end{bmatrix} < 0, \quad (17)$$

$$\begin{bmatrix} \digamma_{21} & \digamma_{22} \end{bmatrix} \geq 0, \quad (18)$$

$$\begin{bmatrix} \digamma_{31} & \digamma_{32} \end{bmatrix} < 0, \quad (19)$$

$$\begin{bmatrix} \digamma_{41} & \digamma_{42} \end{bmatrix} < 0, \quad (20)$$

$$\hat{\Omega}_{ip} > \upsilon(\bar{E}^T\bar{\Omega}_{ip} + \bar{\Omega}_{ip}^T\bar{E}) + \bar{\upsilon}\check{I} \quad (21)$$

$$\frac{e^{\alpha T}(\Sigma_{i=1}^5 \bar{V}_i(0) + \alpha d\lambda_S(1 - e^{-\alpha T}))}{\lambda_p} \leq C_2 \quad (22)$$

$$\digamma_{11} = \begin{bmatrix} \bar{\Phi} & h\bar{\Pi}_{1i}^T & \sqrt{\frac{1}{2}}\bar{\Pi}_{2i}^T \\ * & -\bar{Q}_3 & 0 \\ * & * & -\tilde{\Xi}_{ip} \\ * & * & * \\ * & * & * \\ * & * & * \end{bmatrix},$$

$$\digamma_{12} = \begin{bmatrix} \bar{\Pi}_{2i}^T & \bar{\Pi}_{3i}^T & \Phi_{ip} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{1}{h}\bar{Q}_4 & 0 & 0 \\ * & -\bar{Q}_4 & 0 \\ * & * & -\Theta_i \end{bmatrix},$$

$$\digamma_{21} = \begin{bmatrix} \bar{\Omega}_{ip}^T\bar{E}^T \\ 0 \\ \bar{K}_{1p} + U_{i1}\bar{\Omega}_{ip} \end{bmatrix}$$

$$\digamma_{22} = \begin{bmatrix} * & * \\ \bar{\Omega}_{ip}^T\bar{E}^T & * \\ \bar{K}_{2p} + U_{i1}\bar{\Omega}_{ip} & u_0(k)^2 \end{bmatrix}$$

$$\digamma_{31} = \begin{bmatrix} -\lambda_0 R_i & 0 & 0 \\ 0 & -\lambda_1 R_i & 0 \\ 0 & 0 & -\lambda_2 R_i \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ * & * & * \end{bmatrix}$$

$$\digamma_{32} = \begin{bmatrix} 0 & 0 & \tilde{A}_i^T \\ 0 & 0 & \bar{A}_{1i}^T \\ 0 & 0 & \tilde{A}_{2i}^T \\ -\lambda_3 R_i & 0 & \bar{B}_i^T \\ 0 & -\lambda_4 R_i & \bar{D}_i^T \\ * & * & -\bar{Q}_3 \end{bmatrix}$$

$$\digamma_{41} = \begin{bmatrix} -\bar{\lambda}_0 R_i & 0 & 0 \\ 0 & -\bar{\lambda}_1 R_i & 0 \\ 0 & 0 & -\bar{\lambda}_2 R_i \\ 0 & 0 & 0 \\ * & * & * \end{bmatrix}$$

$$\digamma_{42} = \begin{bmatrix} 0 & \tilde{W}_i^T \\ 0 & \bar{W}_{1i}^T \\ 0 & \tilde{W}_{2i}^T \\ -\bar{\lambda}_3 R_i & \bar{B}_{1i}^T \\ * & -\bar{Q}_4 \end{bmatrix}$$

$$\tilde{A}_i = \bar{A}_i\bar{\Omega}_{ip} + \bar{B}_i\bar{K}_{1p},$$
$$\tilde{A}_{2i} = \bar{A}_{2i}\bar{\Omega}_{ip} + \bar{B}_i\bar{K}_{2p}$$
$$\tilde{W}_i = \bar{W}_i\bar{\Omega}_{ip} + \bar{B}_{1i}\bar{K}_{1p},$$
$$\tilde{W}_{2i} = \bar{W}_{2i}\bar{\Omega}_{ip} + \bar{B}_{1i}\bar{K}_{2p}$$
$$\bar{J}_i = Sym(\bar{A}_i\bar{\Omega}_{ip} + \bar{B}_i\bar{K}_{1p} + L_{1i}\bar{\Omega}_{ip}) + (\lambda_{ii} - \alpha)\bar{\Omega}_{ip}^T\bar{E} + \bar{Q}_1 + \tau^2\bar{Q}_2,$$
$$\Phi_{ijp} = (\sqrt{\lambda_{ij}\pi_{j1}}\bar{\Omega}_{ip}^T\Upsilon_{ij}^T\bar{E}^T, \sqrt{\lambda_{ij}\pi_{j2}}\bar{\Omega}_{ip}^T\Upsilon_{ij}^T\bar{E}^T, \cdots, \sqrt{\lambda_{ij}\pi_{jM}}\bar{\Omega}_{ip}^T\Upsilon_{ij}^T\bar{E}^T),$$
$$\tilde{\Xi}_{jq} = \upsilon(\bar{E}^T\bar{\Omega}_{jq} + \bar{\Omega}_{jq}^T\bar{E}) + \bar{\upsilon}\check{I}$$
$$\Xi_j = diag(\tilde{\Xi}_{j1}, \ \tilde{\Xi}_{j2}, \ \cdots \ \tilde{\Xi}_{jM})$$
$$\Phi_{ip} = (\Phi_{i1p}, \ \Phi_{i2p}, \ \cdots \ \Phi_{iNp})$$
$$\tilde{\Omega}_{ip} = \lambda_{ii}\bar{\Omega}_{ip}^T\bar{E}^T, \quad \Theta_i = diag(\Xi_1, \ \Xi_2, \ \cdots, \ \Xi_N)$$

with

$$\bar{\Pi}_{1i}{}^T = \begin{bmatrix} \bar{K}_{1p}^T\bar{B}_i^T + \bar{\Omega}_{ip}^T\bar{A}_i^T \\ \bar{\Omega}_{ip}^T\bar{A}_{1i}^T \\ \bar{K}_{2p}^T\bar{B}_i^T + \bar{\Omega}_{ip}^T\bar{A}_{2i}^T \\ 0 \\ \bar{B}_i^T G_i \\ \bar{D}_i^T \end{bmatrix},$$

$$\bar{\Pi}_{2i}{}^T = \begin{bmatrix} \bar{K}_{1p}^T\bar{B}_{1i}^T + \bar{\Omega}_{ip}^T\bar{W}_i^T \\ \bar{\Omega}_{ip}^T\bar{W}_{1i}^T \\ \bar{K}_{2p}^T\bar{B}_{1i}^T + \bar{\Omega}_{ip}^T\bar{W}_{2i}^T \\ 0 \\ \bar{B}_{1i}^T G_i \\ 0 \end{bmatrix},$$

$$\bar{\Pi}_{3i}{}^T = \begin{bmatrix} L_{1i}\bar{Q}_4 \\ L_{2i}\bar{Q}_4 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$\bar{\Phi} = [\bar{\Phi}^1 \ \bar{\Phi}^2 \ \bar{\Phi}^3]$$

$$\bar{\Phi}^1 = \begin{bmatrix} \bar{J}_i & \bar{A}_{1i}\bar{\Omega}_{ip} + L_{1i}\bar{\Omega}_{ip} - \bar{\Omega}_{ip}^T L_{2i}^T \\ * & -\bar{Q}_1 - Sym(L_{1i}\bar{\Omega}_{ip}) \\ * & * \\ * & * \\ * & * \\ * & * \end{bmatrix},$$

$$\bar{\Phi}^2 = \begin{bmatrix} \bar{A}_{2i}\bar{\Omega}_{ip} + \bar{B}_i\bar{K}_{2p} & -L_{1i}\bar{Q}_3 \\ 0 & -L_{2i}\bar{Q}_3 \\ -\bar{Q}_2 & 0 \\ * & -\bar{Q}_3 \\ * & * \\ * & * \end{bmatrix},$$

$$\bar{\Phi}^3 = \begin{bmatrix} \bar{B}_i G_i + \bar{\Omega}_{ip}^T U_{i1}^T & \bar{\Omega}_{ip}^T \bar{D}_i \\ 0 & 0 \\ 0 & 0 \\ \bar{Q}_3 U_{i2}^T & 0 \\ -2G_i & 0 \\ * & -\alpha S \end{bmatrix},$$

$$\bar{I} = \begin{bmatrix} 0 & I \end{bmatrix}, \quad \tilde{I} = \begin{bmatrix} I \\ 0 \end{bmatrix},$$

$$\Lambda = \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix}, \quad \check{I} = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix},$$

$$\hat{\Omega}_{ip} = \bar{\Omega}_{ip} - \bar{I}\bar{\Omega}_{ip}\tilde{I}\Lambda, \quad \bar{Q}_1 = \bar{\Omega}_{ip}^T Q_1 \bar{\Omega}_{ip},$$

$$\bar{Q}_2 = \bar{\Omega}_{ip}^T Q_2 \bar{\Omega}_{ip}$$

$$\bar{Q}_3 = Q_3^{-1}, \quad \bar{Q}_4 = Q_4^{-1}, \ \bar{\Omega}_{ip} = \begin{bmatrix} \bar{\Omega}_{ip}^1 & 0 \\ \bar{\Omega}_{ip}^2 & \bar{\Omega}_{ip}^3 \end{bmatrix},$$

$$\bar{V}_1(0) = \lambda_P C_1, \quad \bar{V}_2(0) = \lambda_{Q_1} h\sigma_1 C_1,$$

$$\bar{V}_3(0) = \lambda_{Q_2}\tau^2\sigma_2 C_1,$$

$$\bar{V}_4(0) = h^2(\lambda_0 C_1 + \lambda_1\sigma_1 C_1 + \lambda_2\sigma_2 C_1 + \lambda_3\sigma_3 C_1 + \lambda_4\sigma_4 C_1),$$

$$\bar{V}_5(0) = h(\bar{\lambda}_0 C_1 + \bar{\lambda}_1\sigma_1 C_1 + \bar{\lambda}_2\sigma_2 C_1 + \bar{\lambda}_3\sigma_3 C_1).$$

and $\lambda_P = max_{i\in S}\sigma_{max}(\bar{P}_i)$, $\lambda_p = min_{i\in S}\sigma_{min}(\bar{P}_i)$, $\lambda_{Q_i} = \sigma_{max}(\hat{Q}_i)$, $\lambda_S = \sigma_{max}(S)$, $\hat{Q}_i = R_i^{-1/2}Q_iR_i^{-1/2}$, $\bar{P}_i = R_i^{-1/2}P_iR_i^{-1/2}$, the systems(12) with initial conditions belonging to $\varepsilon(\bar{E}^T\Omega_{ip},\ 1)$ is said to be locally stochastically finite-time bounded stabilizable with respect to $(C_1,\ C_2,\ T,\ R_i,\ \alpha,\ d)$ with controller gain $K_{1p} = \bar{K}_{1p}\bar{\Omega}_{ip}^{-1}N_i^{-1}$, $K_{2p} = \bar{K}_{2p}\bar{\Omega}_{ip}^{-1}N_i^{-1}$.

Proof: For $\forall\ r(t) = i \in S$ of system (12), define the following Lyapuonv-Krasovkii functional:

$$\begin{aligned} V(\bar{x}(t), r(t), \sigma(t)) &= \bar{x}(t)^T\bar{E}^T P_{r(t)\sigma(t)}\bar{E}\bar{x}(t) \\ &\quad + \int_{t-h}^t \bar{x}^T(s)Q_1\bar{x}(s)ds, \\ &\quad + \tau\int_{-\tau}^0\int_{t+\theta}^t \bar{x}^T(s)Q_2\bar{x}(s)dsd\theta \\ &\quad + h\int_{-h}^0\int_{t+\theta}^t f^T(s)Q_3f(s)dsd\theta \\ &\quad + \int_{-h}^0\int_{t+\theta}^t g^T(s)Q_4g(s)dsd\theta \end{aligned}$$

set $r(t) = i,\ \sigma(t) = p$, and denote $\Omega_{ip} = P_{ip}\bar{E} + SW$, with $\bar{E}^TS = 0$. Using the well-known Taylor expansion formula, we get one part of $V(\bar{x}(t), t) - V(\bar{x}(0), 0)$ as the follows:

$$\begin{aligned} &\sum_{i=0}^{k-1} V_x(\bar{x}(t_i),\quad t_i)\Delta\bar{x}_i \\ &\quad = 2\sum_{i=0}^{k-1}(\bar{x}^T(t_i)\Omega_{ip}^T\bar{E})\Delta\bar{x}_i \\ &\quad = 2\sum_{i=0}^{k-1}(\bar{x}^T(t_i)\Omega_{ip}^T)\Delta\bar{E}\bar{x}_i \\ &\quad \to 2\int_0^t \bar{x}^T(s)\Omega_{ip}^Tf(s)ds + 2\int_0^t \bar{x}^T(s)\Omega_{ip}^Tg(s)dw_s \end{aligned}$$

and we can also rewrite another part of $V(\bar{x}(t), t) - V(\bar{x}(0), 0)$ as the follows

$$\begin{aligned} &\frac{1}{2}\sum_{i=0}^{k-1} V_{xx}(\bar{x}(t_i),\ t_i)(\Delta\bar{x}_i)^2 \\ &\quad = \frac{1}{2}\sum_{i=0}^{k-1}(\bar{E}^TP_{ip}\bar{E})(\Delta\bar{x}_i)^2 = \frac{1}{2}\sum_{i=0}^{k-1}(\bar{E}^TP_{ip}\bar{E})(\Delta\bar{E}\bar{x}_i)^2 \\ &\quad = \frac{1}{2}\sum_{i=0}^{k-1}(\bar{E}^T\Omega_{ip})(\Delta\bar{x}_i)^2 = \frac{1}{2}\sum_{i=0}^{k-1}(\bar{E}^T\Omega_{ip})g_i^2(\Delta\bar{w}_i)^2 \\ &\quad \to \frac{1}{2}\int_0^t(\bar{E}^T\Omega_{ip})g(s)ds \end{aligned}$$

Then we achieve

$$dV(\bar{x}_t,\ i,\ p,\ t) = LV(\bar{x}_t,\ i,\ p,\ t)dt + 2\bar{x}^T(t)\Omega_{ip}^Tg(t)dw(t)$$

Then we have

$$\begin{aligned} &LV(\bar{x}_t,\ i,\ p,\ t) \\ &\quad = 2\bar{x}^T(t)\Omega_{ip}^Tf(t) \\ &\qquad + \frac{1}{2}g^T(t)\bar{E}^T\Omega_{ip}g(t) + \lambda_{ii}\bar{x}^T(t)\bar{E}^T\Omega_{ip}\bar{x}(t) \\ &\qquad + \Sigma_{j=1,\ j\neq i}^N\Sigma_{q=1}^M\lambda_{ij}\pi_{jq}\bar{x}^T(t)(\Gamma_{ij}^q)^T\bar{E}^T\Omega_{ip}\bar{E}\Gamma_{ij}^q\bar{x}(t) \\ &\qquad + \bar{x}^T(t)Q_1\bar{x}(t) + \bar{x}^T(t-h)Q_1\bar{x}(t-h) \\ &\qquad + \tau^2\bar{x}^T(t)Q_2\bar{x}(t) - \tau\int_0^t \bar{x}^T(s)Q_2\bar{x}(s)ds \\ &\qquad + h^2f^T(t)Q_3f(t) - h\int_{t-h}^t f^T(s)Q_3f(s)ds \\ &\qquad + hg^T(t)Q_4g(t) - \int_{t-h}^t g^T(s)Q_4g(s)ds. \end{aligned} \tag{23}$$

then we have

$$-\tau\int_0^t \bar{x}^T(s)Q_2\bar{x}(s)ds$$

$$\leq -(\int_{t-\tau}^{t} \bar{x}(s)ds)^T Q_2 (\int_{t-\tau}^{t} \bar{x}(s)ds), \tag{24}$$

$$-h\int_{t-h}^{t} f^T(s) Q_3 f(s)ds$$
$$\leq -(\int_{t-h}^{t} f(s)ds)^T Q_2 (\int_{t-h}^{t} f(s)ds). \tag{25}$$

From system (12), one can obtained:

$$2[\bar{x}^T(t)\Omega_{ip}^T L_{1i} + \bar{x}^T(t-h)\Omega_{ip}^T L_{2i}] \times [\bar{x}^T(t) - \bar{x}^T(t-h) - \int_{t-h}^{t} f(s)ds - \int_{t-h}^{t} g(s)dw(s)] = 0, \tag{26}$$

where L_{1i}, L_{2i} are matrices with appropriate dimensions and for each $i \in S$, we have

$$\begin{aligned}&-2(\bar{x}^T(t)\Omega_{ip}^T L_{1i} + \bar{x}^T(t-h)\Omega_{ip}^T L_{2i}) \int_{t-h}^{t} g(s)dw(s)\\ &\leq (\bar{x}^T(t)\Omega_{ip}^T L_{1i} + \bar{x}^T(t-h)\Omega_{ip}^T L_{2i})^T Q_4^{-1} (\bar{x}^T(t)\Omega_{ip}^T L_{1i}\\ &+ (\int_{t-h}^{t} g(s)dw(s))^T Q_4 (\int_{t-h}^{t} g(s)dw(s))\\ &+ \bar{x}^T(t-h)\Omega_{ip}^T L_{2i}).\end{aligned} \tag{27}$$

By employing Ito formula, we derive

$$\varepsilon[(\int_{t-h}^{t} g(s)dw(s))^T Q_4 (\int_{t-h}^{t} g(s)dw(s))] = \varepsilon[\int_{t-h}^{t} g^T(s) Q_4 g(s)ds]. \tag{28}$$

Note that $u(t) = \tilde{k}_{1p}\bar{x}(t) + \tilde{k}_{2p}\int_{t-\tau}^{t} \bar{x}(s)ds$, make $L_i = [U_{i1}\ U_{i2}]$, from Lemma 2 we have

$$2\psi^T(u)T_i[\psi(u) - U_{i1}\bar{x}(t) - U_{i2}\int_{t-\tau}^{t} x(s)ds] \leq 0. \tag{29}$$

Denoting

$$\Upsilon_{ij} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} N_j^{-1} N_i,$$

we have $\bar{E}^T\Gamma_{ij}^q = \bar{E}^T\Upsilon_{ij}$. From $\Omega_{ip}^T\bar{E} = \bar{E}^T\Omega_{ip}$, one have $\bar{\Omega}_{ip}^T\bar{E} = \bar{E}^T\bar{\Omega}_{ip}$, thus the matrix $\bar{\Omega}_{ip}^T\bar{E}$ is symmetric, then we can rewrite condition (18) as the follows

$$\begin{aligned}&\hat{\Omega}_{jq} > \upsilon(\bar{E}^T\bar{\Omega}_{jq} + \bar{\Omega}_{jq}^T\bar{E}) + \bar{\upsilon}\check{I},\\ &(\upsilon(\bar{E}^T\bar{\Omega}_{jq} + \bar{\Omega}_{jq}^T\bar{E}) + \bar{\upsilon}\check{I})^{-1} > \bar{E}^T\hat{\Omega}_{jq}^{-1} = \bar{E}^T\Omega_{jq}\end{aligned} \tag{30}$$

From (22)-(29), one has $\xi[LV(\bar{x}_t, i, p, t) - \alpha V(\bar{x}_t, i, p, t)] - \alpha D^T(t)SD(t) \leq \xi(t)\Phi_i\xi^T(t)$, where

$$\xi(t) = [\bar{x}^T(t)\ \bar{x}^T(t-h)\ \int_{t-\tau}^{t} \bar{x}^T(s)ds \times \int_{t-h}^{t} f^T(s)ds\ \psi^T(u)\ D^T(t)]$$

$$\Phi_i = \Phi + h^2\Pi_{1i}^T Q_3 \Pi_{1i} + \Pi_{2i}^T(hQ_4 + \bar{E}^T\Omega_{ip})\Pi_{2i} + \Pi_{3i}^T Q_4^{-1}\Pi_{3i}$$

$$\Phi = \begin{bmatrix} \Phi^1 & \Phi^2 & \Phi^3 \end{bmatrix},$$

where

$$\Phi^1 = \begin{bmatrix} J_i & \Omega_{ip}^T\bar{A}_{1i} + \Omega_{ip}^T L_{1i} - L_{2i}^T\Omega_{ip} \\ * & -Q_1 - Sym(\Omega_{ip}^T L_{1i}) \\ * & * \\ * & * \\ * & * \\ * & * \end{bmatrix},$$

$$\Phi^2 = \begin{bmatrix} \Omega_{ip}^T\bar{A}_{2i} + \Omega_{ip}^T\bar{B}_i\tilde{K}_{2p} & -\Omega_{ip}^T L_{1i} \\ 0 & -\Omega_{ip}^T L_{2i} \\ -Q_2 & 0 \\ * & -Q_3 \\ * & * \\ * & * \end{bmatrix},$$

$$\Phi^3 = \begin{bmatrix} \Omega_{ip}^T\bar{B}_i + U_{i1}^T T_i & \bar{D}_i \\ 0 & 0 \\ 0 & 0 \\ U_{i2}^T T_i & 0 \\ -2T_i & 0 \\ * & -\alpha S \end{bmatrix},$$

$$\begin{aligned}J_i &= Sym(\Omega_{ip}^T\bar{A}_i + \Omega_{ip}^T\bar{B}_i\tilde{K}_{1p} + \Omega_{ip}^T L_{1i})\\ &+ (\lambda_{ii} - \alpha)\bar{E}^T\Omega_{ip} + Q_1 + \tau^2 Q_2\\ &+ \sum_{j=1,\, j\neq i}^{N}\sum_{q=1}^{M}\lambda_{ij}\pi_{jq}\bar{x}^T(t)(\Upsilon_{ij})^T\bar{E}^T\Omega_{ip}\bar{E}\Upsilon_{ij}\bar{x}(t),\end{aligned}$$

$$\Pi_{1i}{}^T = \begin{bmatrix} \tilde{K}_{1p}^T\bar{B}_i^T + \bar{A}_i^T \\ \bar{A}_{1i}^T \\ \tilde{K}_{2p}^T\bar{B}_i^T + \bar{A}_{2i}^T \\ 0 \\ \bar{B}_i^T \\ \bar{D}_i^T \end{bmatrix},$$

$$\Pi_{2i}{}^T = \begin{bmatrix} \tilde{K}_{1p}^T\bar{B}_{1i}^T + \bar{W}_i^T \\ \bar{W}_{1i}^T \\ \tilde{K}_{2p}^T\bar{B}_{1i}^T + \bar{W}_{2i}^T \\ 0 \\ \bar{B}_{1i}^T \\ 0 \end{bmatrix},$$

$$\Pi_{3i}{}^T = \begin{bmatrix} \Omega_{ip}^T L_{1i} \\ \Omega_{ip}^T L_{2i} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Then pre- and post multiplying matrix inequality (16) with $\bigcap$ and $\bigcap^T$, where

$$\bigcap = [\Omega_{ip}^T\ \Omega_{ip}^T\ \Omega_{ip}^T\ Q_3\ T_i\ I\ I\ I\ Q_4\ I]$$

Then we have

$$LV(\bar{x}(t),\ i,\ p) < \alpha D(t)^T SD(t) + \alpha V(\bar{x}(t),\ i,\ p).$$

Multiplying the above inequality by $e^{-\alpha t}$, we get

$$L[e^{-\alpha t} V(\bar{x}(t),\ i,\ p)] < \alpha e^{-\alpha t} D(t)^T SD(t).$$

By integrating the above inequality between 0 and t, it follows that

$$\begin{aligned} &e^{-\alpha t} V(\bar{x}(t),\ i,\ p) - V(\bar{x}(0),\ r_0,\ \sigma_0) \\ &\quad < \alpha \int_0^t e^{-\alpha s} D(s)^T SD(s)ds. \end{aligned}$$

Then, we have the follows

$$\begin{aligned} &E\{\bar{x}(t)^T \bar{E}^T \bar{P}_i \bar{E}\bar{x}(t)\} \\ &\quad \le V(\bar{x}(t),\ i,\ p) \\ &\quad < e^{\alpha t} V(\bar{x}(0),\ r_0,\ \sigma_0) + \alpha d\sigma_S e^{\alpha t} \int_0^t e^{-\alpha s} ds \\ &\quad < e^{\alpha t}[\alpha d\sigma_S(1-e^{\alpha t}) + V(\bar{x}(0),\ r_0,\ \sigma_0)] \\ &E\{\bar{x}(t)^T \bar{E}^T R_i \bar{E}\bar{x}(t)\} \\ &\quad \le \frac{e^{\alpha t}[\alpha d\sigma_S(1-e^{\alpha t}) + V(\bar{x}(0),\ r_0,\ \sigma_0)]}{\lambda_p} \end{aligned}$$

Since that $E\{\bar{x}^T(0)\bar{E}^T R_i \bar{E}\bar{x}(0)\} \le c_1$ and denote $\int_{t-h}^t \bar{x}^T(s) R_i \bar{x}(s)ds < \sigma_1 c_1$, $\int_{t-\tau}^t \bar{x}^T(s) R_i \bar{x}(s)ds < \sigma_2 c_1$, $sat^T(u(t)) R_i sat(u(t) < \sigma_3 c_1$, $D^T(t) R_i D(t) < \sigma_4 c_1$ with $t = 0$. Consider conditions (20)-(21), and based on Schur lemma, one can derived $E\{\bar{x}^T(t)\bar{E}^T R_i \bar{E}\bar{x}(t)\} \le c_2$ from condition (19).

Pre- and post- multiplying matrix inequality (17) with $[\Omega_{ip}^T\ \Omega_{ip}^T\ I]$ and $[\Omega_{ip}^T\ \Omega_{ip}^T\ I]^T$, one can derive

$$\begin{bmatrix} \bar{E}^T \Omega_{ip}^T & * & * \\ 0 & \bar{E}^T \Omega_{ip}^T & * \\ \tilde{K}_{1p} + U_{i1} & \tilde{K}_{2p} + U_{i1} & u_0(k)^2 \end{bmatrix} \ge 0, \quad k = 1, \cdots, m,$$

which implies that $\varepsilon(\bar{E}^T P_i \bar{E},\ 1) \in D(u(0))$. Denote

$$\Omega_{ip} = \begin{bmatrix} \Omega^1 & \Omega^2 \\ \Omega^3 & \Omega^4 \end{bmatrix}.$$

From condition (16), we have

$$\lambda \bar{\Omega}_{ip}^T \bar{E}^T + \bar{\Omega}_{ip}^T \bar{A_{ip}}^T + \bar{A_{ip}}^T \bar{\Omega}_{ip} < 0. \tag{31}$$

Consider that $\bar{E}^T \Omega_{ip} = \Omega_{ip}^T \bar{E}$, which implies $\Omega^2 = 0$, from (30) one can derive

$$\begin{bmatrix} \iota^1 & \iota^2 \\ \iota^3 & (\Omega_{ip}^4)^T (\bar{A_{ip}}^4) + (\bar{A_{ip}}^4)^T \Omega_{ip}^4 \end{bmatrix} < 0.$$

we derive $(\Omega_{ip}^4)^T (\bar{A_{ip}}^4) + (\bar{A_{ip}}^4)^T \Omega_{ip}^4 < 0$, and then further get $\bar{A_{ip}}^4 < 0$. which ensures that the considered system (12) is impulse-free and regular.

Theorem 2: For the considered system (12) with initial conditions belonging to $\varepsilon(\bar{E}^T \Omega_{ip},\ 1)$ and the same parameter description as theorem 1, if there exists a constant $\gamma > 0$, such that conditions (17)-(18) and (20)-(21) hold and

$$\begin{bmatrix} (16) & \Gamma \\ * & -I \end{bmatrix} < 0, \tag{32}$$

$$\frac{e^{\alpha T}(\Sigma_{i=1}^5 \bar{V}_i(0) + \gamma^2 d\lambda_S(1-e^{-\alpha T}))}{\lambda_p} \le C_2. \tag{33}$$

with

$$\begin{aligned} \bar{\Phi} &= \begin{bmatrix} \bar{\Phi}^1 & \bar{\Phi}^2 & \bar{\Phi}^3 \end{bmatrix}, \\ \Gamma^T &= [\bar{Z}_i \bar{\Omega}_{ip} + B_{2i} \bar{K}_{1p}\ \bar{Z}_{1i} \bar{\Omega}_{ip} \\ &\quad \bar{Z}_{2i} \bar{\Omega}_{ip} + B_{2i} \bar{K}_{2p}\ B_{2i}\ \mathbf{0}] \end{aligned}$$

and

$$\bar{\Phi}^1 = \begin{bmatrix} \bar{J}_i & \bar{A}_{1i} \bar{\Omega}_{ip} + L_{1i} \bar{\Omega}_{ip} - \bar{\Omega}_{ip}^T L_{2i}^T \\ * & -\bar{Q}_1 - Sym(L_{1i} \bar{\Omega}_{ip}) \\ * & * \\ * & * \\ * & * \\ * & * \end{bmatrix},$$

$$\bar{\Phi}^2 = \begin{bmatrix} \bar{A}_{2i} \bar{\Omega}_{ip} + \bar{B}_i \bar{K}_{2p} & -L_{1i} \bar{Q}_3 \\ 0 & -L_{2i} \bar{Q}_3 \\ -\bar{Q}_2 & 0 \\ * & -\bar{Q}_3 \\ * & * \\ * & * \end{bmatrix},$$

$$\bar{\Phi}^3 = \begin{bmatrix} \bar{B}_i G_i + \bar{\Omega}_{ip}^T U_{i1}^T & \bar{\Omega}_{ip}^T \bar{D}_i \\ 0 & 0 \\ 0 & 0 \\ -\bar{Q}_3 U_{i2}^T & 0 \\ -2G_i & 0 \\ * & -\gamma^2 I \end{bmatrix},$$

where $\bar{Z}_i = Z_i N_i$, $\bar{Z}_{1i} = Z_{1i} N_i$, $\bar{Z}_{2i} = Z_{2i} N_i$, the systems(12) is said to be locally stochastically H_∞ finite-time bounded stabilizable via state feedback with respect to $(C_1,\ C_2,\ T,\ R_i,\ \gamma,\ d)$.

Proof. Select the similar Lyapuonv-Krasovkii functional of theorem 1, one can easily derive based on theorem 2:

$$\begin{aligned} LV(\bar{x}(t),\ i,\ p) &< \gamma^2 D(t)^T D(t) - z(t)^T z(t) \\ &\quad + \alpha V(\bar{x}((t),\ i,\ p). \end{aligned}$$

Since that $-z(t)^T z(t) < 0$, we derive

$$LV(\bar{x}(t),\ i,\ p) < \gamma^2 D(t)^T D(t) + \alpha V(\bar{x}((t),\ i,\ p).$$

Then conditions (32) can be obtained by following the similar proof of Theorem 1. On the other hand, we derive the follows under the assumed zero initial condition

$$e^{-\alpha t} V(\bar{x}(t),\ i,\ p) < \int_0^T (\gamma^2 D(t)^T D(t) - z(t)^T z(t))dt.$$

Further, the follows can be obtained

$$\int_0^T z(t)^T z(t)dt \leq \int_0^T \gamma^2 D(t)^T D(t)dt \leq \gamma^2 \int_0^T D(t)^T D(t)dt$$

Then follow the similar proof of theorem 1.

Theorem 3: For the system (12) with initial conditions belonging to $\varepsilon(\bar{E}^T\Omega_{ip}, 1)$ and the same parameter description as theorem 2, if there exists mode-dependent positive matrix $\bar{H}_i > 0$, such that conditions (17)-(18) and (20)-(21) hold and

$$\bar{Q}_1 + \bar{\Omega}_{ip}^T E^T - \bar{H}_i \geq 0, \quad i = j \in S_{uk}, \tag{34}$$

$$(31) + \begin{bmatrix} \bar{Q}_1 & \mathbf{0} \\ * & \mathbf{0} \end{bmatrix} < 0, \quad i \neq j \in S_k,\ i = j \in S_{uk}, \tag{35}$$

$$\begin{bmatrix} -\bar{H}_i & \Phi_{ijp} \\ * & -\Xi_j \end{bmatrix} < 0, \quad i \neq j \in S_{uk}, \tag{36}$$

$$(31) < 0, \quad i \neq j \in S_k,\ i = j \in S_k, \tag{37}$$

the systems(12) is said to be locally stochastically H_∞ finite-time bounded stabilizable via state feedback with respect to $(C_1, C_2, T, R_i, \gamma, d)$.

Proof: Select the similar Lyapuonv-Krasovkii functional of theorem 1, note that $\Sigma_{j=1}^N \lambda_{ij} = 0$.

$$\begin{aligned} LV(\bar{x}_t, i, p, t) &= H + \lambda_{ii}\bar{x}^T(t)\bar{E}^T\Omega_{ip}\bar{x}(t) \\ &+ \bar{x}^T(t)Q_1\bar{x}(t) + \Sigma_{j=1}^N \lambda_{ij}\bar{x}^T(t)(-H_i)\bar{x}(t) \\ &+ \Sigma_{j=1,\ j\neq i}^N \Sigma_{q=1}^M \lambda_{ij}\pi_{jq}\bar{x}^T(t)(\Gamma_{ij}^q)^T\bar{E}^T\Omega_{ip}\bar{E}\Gamma_{ij}^q\bar{x}(t) \end{aligned}$$

where $H_i > 0$ and

$$\begin{aligned} H &= 2\bar{x}^T(t)\Omega_{ip}^T f(t) + \frac{1}{2}g^T(t)\bar{E}^T\Omega_{ip}g(t) \\ &+ \bar{x}^T(t-h)Q_1\bar{x}(t-h) + \tau^2\bar{x}^T(t)Q_2\bar{x}(t) \\ &- \tau\int_0^t \bar{x}^T(s)Q_2\bar{x}(s)ds \\ &+ h^2 f^T(t)Q_3 f(t) - h\int_{t-h}^t f^T(s)Q_3 f(s)ds \\ &+ hg^T(t)Q_4 g(t) - \int_{t-h}^t g^T(s)Q_4 g(s)ds. \end{aligned}$$

Then we have

$$\begin{aligned} &LV(\bar{x}_t, i, p, t) \\ &= H + \bar{x}^T(t)(\lambda_{ii}(\bar{E}^T\Omega_{ip} - H_i)\bar{x}(t) \\ &+ \bar{x}^T(t)Q_1\bar{x}(t) + [\Sigma_{j=1,i\neq j}^N \lambda_{ij}\bar{x}^T(t)(-H_i)\bar{x}(t) \\ &+ \Sigma_{j=1,\ j\neq i}^N \Sigma_{q=1}^M \lambda_{ij}\pi_{jq}\bar{x}^T(t)(\Gamma_{ij}^q)^T\bar{E}^T\Omega_{ip}\bar{E}\Gamma_{ij}^q\bar{x}(t)]_{j\in S_k} \\ &+ [\Sigma_{j=1,i\neq j}^N \lambda_{ij}\bar{x}^T(t)(-H_i)\bar{x}(t) \\ &+ \Sigma_{j=1,\ j\neq i}^N \Sigma_{q=1}^M \lambda_{ij}\pi_{jq}\bar{x}^T(t)(\Gamma_{ij}^q)^T\bar{E}^T\Omega_{ip}\bar{E}\Gamma_{ij}^q\bar{x}(t)]_{j\in S_{uk}} \end{aligned} \tag{38}$$

If λ_{ii} is unknown, denote $-\lambda_{ii} < 1$, now we rewrite the first part of (37) as the follows:

$$\begin{aligned} &H + \bar{x}^T(t)(\lambda_{ii}(\bar{E}^T\Omega_{ip} - H_i + Q_1)\bar{x}(t) \\ &+ \bar{x}^T(t)(1-\lambda_{ii})Q_1\bar{x}(t) \\ &< H + \bar{x}^T(t)(\lambda_{ii}(\bar{E}^T\Omega_{ip} - H_i + Q_1)\bar{x}(t) \\ &+ \bar{x}^T(t)2Q_1\bar{x}(t) \end{aligned} \tag{39}$$

Then one can easily derive based on theorem 3:

$$LV(\bar{x}(t), i, p) < \gamma^2 D(t)^T D(t) - z(t)^T z(t) + \alpha V(\bar{x}((t), i, p).$$

Then follow the proof of theorems 1 and 2. Proof is completed.

IV. SIMULATION EXAMPLE

In this section, a numerical example is provided to demonstrate the effectiveness of the proposed method.

Example 1: Consider the system (1)-(3) with the following modes:

Subsystem 1

$$A_1 = \begin{bmatrix} 8 & 2 \\ 3 & -2 \end{bmatrix},\quad A_{11} = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 3 \\ -1 \end{bmatrix},\quad A_{21} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix},\quad W_1 = \begin{bmatrix} 0.6 & 0.5 \\ 0.3 & -0.3 \end{bmatrix},$$

$$W_{11} = \begin{bmatrix} -0.2 & 0.2 \\ -0.1 & 0.2 \end{bmatrix},\quad B_{11} = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix},$$

$$W_{21} = \begin{bmatrix} 0.1 & 0 \\ 0.5 & 0.8 \end{bmatrix}.$$

Subsystem 2

$$A_2 = \begin{bmatrix} 7 & -3 \\ 2 & -1 \end{bmatrix},\quad A_{12} = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\quad A_{22} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\quad W_2 = \begin{bmatrix} 0.5 & 0.6 \\ 0.4 & -0.4 \end{bmatrix},$$

$$W_{12} = \begin{bmatrix} -0.3 & 0.2 \\ -0.2 & 0.1 \end{bmatrix},\quad B_{12} = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix},$$

$$W_{22} = \begin{bmatrix} 0.2 & 0 \\ 0.6 & 0.8 \end{bmatrix}.$$

Subsystem 3

$$A_3 = \begin{bmatrix} 6 & -2 \\ 1 & -1 \end{bmatrix},\quad A_{13} = \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\quad A_{23} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix},$$

$$E_3 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\quad W_3 = \begin{bmatrix} 0.6 & -0.2 \\ 0.4 & -0.3 \end{bmatrix},$$

$$W_{13} = \begin{bmatrix} -0.3 & 0.2 \\ -0.1 & 0.1 \end{bmatrix},\quad B_{13} = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix},$$

$$W_{23} = \begin{bmatrix} 0.2 & 0 \\ 0.5 & 0.7 \end{bmatrix}.$$

Subsystem 4

$$A_4 = \begin{bmatrix} 9 & -2 \\ 3 & -1 \end{bmatrix}, \quad A_{14} = \begin{bmatrix} 6 & 1 \\ -1 & 3 \end{bmatrix},$$
$$B_4 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad A_{24} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix},$$
$$E_4 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad W_4 = \begin{bmatrix} 0.6 & 0.5 \\ 0.33 & -0.3 \end{bmatrix},$$
$$W_{14} = \begin{bmatrix} -0.2 & 0.2 \\ -0.1 & 0.2 \end{bmatrix}, \quad B_{14} = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix},$$
$$W_{24} = \begin{bmatrix} 0.1 & 0 \\ 0.5 & 0.8 \end{bmatrix}.$$

Choose

$$M_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{i=1,\ 2,\ 3,\ 4},$$
$$N_1 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \quad N_i = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}_{i=2,\ 3,\ 4}.$$

Then one can obtained

$$\bar{E} = M_i E_i N_i = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}_{i=1,\ 2,\ 3,\ 4}$$

In this example, we give the following initial parameters: $h = 0.5,\ \tau = 0.2,\ c_1 = 0.5,\ c_2 = 0.8,\ \gamma = 0.8,\ T = 10,\ R_i = I_2,\ D(t) \le d = 1, \bar{\upsilon} = 0.2,\ \upsilon = 0.25$, the input is bounded as $|u_t| \le 0.5$. The initial state function is defined as the follows,

$$\eta(t) = \begin{bmatrix} 0.2t + 0.8 \\ 0.2t - 0.7 \end{bmatrix}, \quad t \in [-max(h,\ \tau),\ 0].$$

The follows are the transition rate matrix of system and controller mode:

$$\lambda_{ij} = \begin{bmatrix} -0.5 & ? & 0.1 & ? \\ 0.2 & ? & ? & 0.3 \\ ? & ? & -0.6 & 0.3 \\ 0.2 & ? & ? & -0.2 \end{bmatrix},$$
$$\pi_{ij} = \begin{bmatrix} 0.2 & 0.3 & 0.2 & 0.3 \\ 0.3 & 0.3 & 0.2 & 0.2 \\ 0.1 & 0.2 & 0.4 & 0.3 \\ 0.3 & 0.2 & 0.3 & 0.2 \end{bmatrix}.$$

Based on theorem 3, the controller parameters can be derived

$$K_{21} = \begin{bmatrix} -7.3921 & -1.0976 \end{bmatrix},$$
$$K_{22} = \begin{bmatrix} -8.4726 & 0.8531 \end{bmatrix},$$
$$K_{23} = \begin{bmatrix} -13.1476 & -1.0561 \end{bmatrix},$$
$$K_{24} = \begin{bmatrix} -11.1721 & 0.9768 \end{bmatrix},$$
$$K_{21} = \begin{bmatrix} -7.3921 & -1.0976 \end{bmatrix},$$
$$K_{22} = \begin{bmatrix} -8.4726 & 0.8531 \end{bmatrix},$$
$$K_{23} = \begin{bmatrix} -13.1476 & -1.0561 \end{bmatrix},$$
$$K_{24} = \begin{bmatrix} -11.1721 & 0.9768 \end{bmatrix}.$$

Remark: Figs. $1 - 2$ are system and controller mode, *Fig*. 3 are state response of Example 1. From the provided figures, the designed controllers ensure that the considered system (12) is mean-square locally finite-time bounded stable, and the H_∞ performance $\gamma = 0.8$ is less than the results of literature [3].

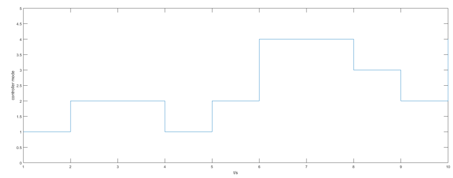

FIGURE 1. Controller mode.

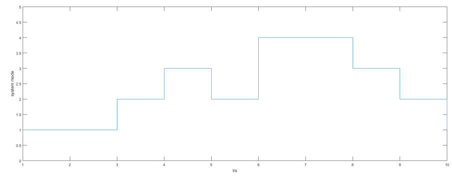

FIGURE 2. System mode.

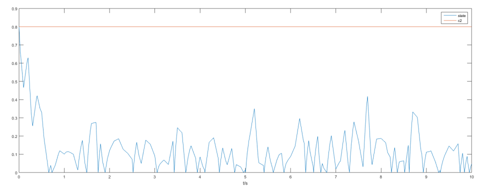

FIGURE 3. The state response of the closed-loop system (12).

V. CONCLUSION

This paper designs the hidden Markov model based memory feedback controller to ensure the finite-time H_∞ stability of the considered singular stochastic Markovian jump systems. Some sufficient conditions to the solution for this problem are given in terms of linear matrix inequalities which considering the discontinues caused by singular system matrix and Markovian jump behavior. Considering that using LMI method to deal with saturation and partially known transition rates will increase conservatism, more advanced theoretical methods will be used to reduce conservatism in the future work.

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STATEMENTS AND DECLARATIONS

Conflict of Interest: The authors declare that we have no conflicts of interests about the publication of this paper.

Availability of Data and Material: The authors declare that all data generated or analyzed during this study are included in this article.

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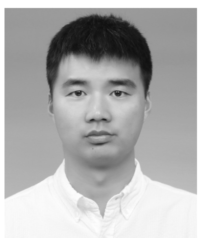

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