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RESEARCH ARTICLE

A Construction Method of Phase Sequences for the Selected Mapping Scheme

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ABSTRACT In this paper, a construction method of phase sequences is proposed for the selected mapping (SLM) scheme. Three principles are applied to the construction process: periodicity, binary (± 1), and maximization of the minimum Hamming distance between different sequences. Using the first and second principle, except the original frequency signal, the calculation process of other candidate time signals can be simplified as the linear addition of some intermediate signals, which are obtained in the inverse fast Fourier transform (IFFT) operation of the original frequency signal. Using the third principle, greater differences can be generated between different candidate signals, which is helpful to improve the peak-to-average power ratio (PAPR) reduction performance. Theoretical analysis and simulation results show that, with the help of new phase sequences, the SLM scheme can effectively reduce the computational complexity, while it shows almost the same performance of PAPR reduction as that of the traditional SLM scheme, whose phase sequences are from the odd row vectors in the Hadamard matrix.

INDEX TERMS Peak-to-average power ratio (PAPR), a construction method of phase sequences, selected mapping scheme (SLM), low complexity, Hamming distance.

I. INTRODUCTION

The Orthogonal frequency division multiplexing (OFDM) technology was widely used in many communication applications [1], [2], [3]. The major limitation of OFDM was the high peak-to-average power ratio (PAPR) problem. Several techniques were applied to mitigate PAPR in OFDM systems [4], [5]. The simple and widely used method is clipping the signal to limit the PAPR below a threshold level [6], but it causes both in-band distortion and out-of-band radiation. Block coding is another technique to reduce PAPR, but it incurs the loss of transmission data rate, and has high computation complexity problem [7]. The other method called the partial transmission sequence (PTS) modifies signals by increasing peak cancellation carrier or constellation graph changes, and chooses the minimum PAPR signal through optimization of parameters [8], [9].

The selected mapping (SLM) scheme is also a well-known method to lower the PAPR [10], [11], [12]. This method aims to generate a certain amount of candidate signals and then transmits the signal with minimum PAPR. In the traditional SLM scheme, the generation process of each candidate time signal requires one inverse fast Fourier transform (IFFT) operation, so the computational complexity is increased in proportion to the number of candidate signals. From different perspectives, researchers had proposed many different modified SLM schemes to decrease the computation complexity [13], [14], [15], [16], [17]. However, these schemes can only decrease the computation complexity to a limited extent.

To effectively reduce the computational complexity, a construction method of phase sequences is proposed for the SLM scheme. All the phase sequences are periodic, and are composed of -1 and 1. This construction method can simplify the calculation process of candidate time signals as the linear addition of some intermediate signals, which are generated during the IFFT operation of the original frequency signal, and can significantly reduce the number of complex multiplication and addition. Moreover, another principle of maximization of the minimum Hamming

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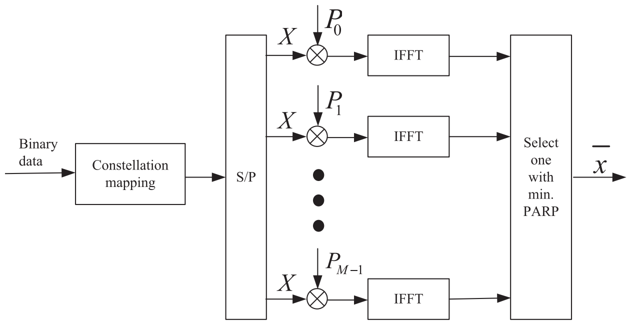

FIGURE 1. Diagram of the traditional SLM scheme.

distance between different sequences is also used to search the phase sequences, which makes the candidate signals more different and improves the performance of PAPR reduction.

The paper is organized as follows: Section II reviews the basic principle of traditional SLM scheme and the recursive computation process of the IFFT operation. Section III introduces specific contents of the new SLM scheme, including three basic lemmas about the phase sequences design, the construction method of the phase sequences, the calculation process of candidate time signals, and the computational complexity analysis. Section IV describes the simulation results of PAPR reduction performances of the proposed scheme under some parameters. Section V draws conclusions.

II. THE TRADITIONAL SLM SCHEME AND IFFT RECURSIVE CALCULATION PROCESS

A. TRADITIONAL SLM SCHEME

The PAPR value of a discrete-time signal $x = [x(0), x(1), \ldots, x(N-1)]$ can be expressed as

$$PAPR(x) = \max_{0\leq n\leq N-1} \frac{|x(n)|^2}{E[|x(n)|^2]}, \quad (1)$$

where $E[.]$ denotes the expected value [5].

The SLM scheme is a common technique for reducing PAPR of OFDM signal, and the signal processing flow chart of traditional SLM scheme is shown in Fig.1. The dot multiplication operation $\otimes$ is performed between the original frequency signal $X = [X(0), X(1), \ldots, X(N-1)]$ and the phase sequences $P = \{P_0, P_1, \ldots, P_{M-1}\}$ to generate candidate signals in frequency domain. Then the IFFT operations are performed on these signals respectively. Finally, the signal $\bar{x}$ is selected for transmission, where

$$\bar{x} = \arg\min_{0\leq m\leq M-1} [PAPR(IFFT(X \otimes P_m))]. \quad (2)$$

To obtain better PAPR reduction performance, the traditional SLM scheme needs to increase the number of phase sequence, which causes the number of the IFFT operation in transmitter to increase linearly.

B. IFFT RECURSIVE CALCULATION PROCESS

In this paper, the number of subcarriers $N = 2^{\mu}$ is the default, where μ is a positive integer. The relationship between the time signal $x = [x(0), x(1), \ldots, x(N-1)]$ and the frequency signal $X = [X(0), X(1), \ldots, X(N-1)]$ can be expressed as [18]

$$x(k) = IDFT(X) = \sum_{n=0}^{N-1} X(n)e^{j\frac{2\pi k}{N}n}, 0 \leq k \leq N-1. \quad (3)$$

Based on the IFFT algorithm, the inverse discrete Fourier transform (IDFT) operation of one long sequence can be continuously decomposed into two IDFT operations of two shorter sequences, and the computation complexity of Eq.(3) can be reduced from $O(N^2)$ to $O(N \log_2 N)$. For $0 \leq k \leq N-1$, the decomposition process can be expressed as

$$x(k) = \sum_{n=0}^{N/2-1} X_0(n)e^{j\frac{2\pi k}{2N}n} + e^{j\frac{2\pi k}{N}} \sum_{n=0}^{N/2-1} X_1(n)e^{j\frac{2\pi k}{2N}}. \quad (4)$$

Therefore,

$$IDFT(X) = \begin{bmatrix} IDFT(X_0) \\ IDFT(X_0) \end{bmatrix} + \phi_N \otimes \begin{bmatrix} IDFT(X_1) \\ IDFT(X_1) \end{bmatrix}, \quad (5)$$

where

$$\begin{cases} \phi_N &= [e^{j\frac{2\pi}{N}0}, e^{j\frac{2\pi}{N}1}, \ldots, e^{j\frac{2\pi}{N}(N-1)}] \\ X_0 &= [X(0), X(2), \ldots, X(N-2)] \\ X_1 &= [X(1), X(3), \ldots, X(N-1)] \end{cases}. \quad (6)$$

The decomposition process will continue until the length of the sequences is two.

III. CONSTRUCTION METHOD OF PHASE SEQUENCES

Although it has been reduced, the computation complexity of IFFT operation is still at a higher level. Therefore, decreasing the number of IFFT operation is the major direction to reduce the computational complexity of the improved SLM scheme. And further consideration shall be provided to optimize the performance in PAPR reduction with the same number of candidate signals.

A. THREE BASIC LEMMAS

In this paper, let $Q = 2^{\omega} \leq N/4$, where ω is a positive integer. The new phase sequence construction scheme is mainly based on the following three conclusions.

Lemma 1: Let $k = 0, 1, \ldots, N/Q$, decompose the original frequency signal X into the following Q signals

$$\underline{X}_q(n) = \begin{cases} X(n), & if\, n = q + kQ, k = 0, 1, \ldots, N/Q \\ 0, & else \end{cases}, \quad (7)$$

then during the calculation of $IDFT(X)$ using IFFT algorithm, the values of the following Q intermediate signals will be obtained

$$IDFT(\underline{X}_0), IDFT(\underline{X}_1), \ldots, IDFT(\underline{X}_{Q-1}), \quad (8)$$

and these signals satisfy

$$IDFT(X) = \sum_{q=0}^{Q-1} IDFT(\underline{X}_q). \quad (9)$$

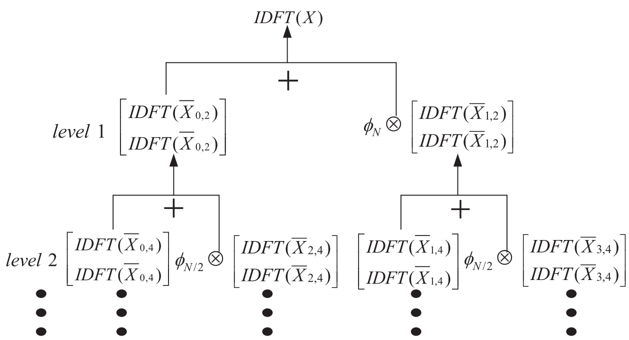

FIGURE 2. Tree diagram of IFFT recursive calculation.

Proof: According to Eq.(7), we have $X = \sum_{q=0}^{Q-1} \underline{X}_q$. Therefore, $IDFT(X) = IDFT(\sum_{q=0}^{Q-1} \underline{X}_q) = \sum_{q=0}^{Q-1} IDFT(\underline{X}_q)$, which proves Eq.(9).

Let $\overline{X}_{k,n} = [X(k), X(k+n), \ldots, X(N+k-n)]$, and the operation of $IDFT(X)$ can be divided into two operations of $IDFT(\overline{X}_{k,2}), k = 0, 1$, and then divided into four operations of $IDFT(\overline{X}_{k,4}), k = 0, 1, 2, 3$. The remainder can be similarly performed, and finally divided into $N/2$ operations of $IDFT(\overline{X}_{k,N/2}), 0 \le k \le N/2-1$. The tree diagram in Fig.2 shows the above recursive calculation process.

According to Fig.2, the value of $IDFT(\overline{X}_{k,n}), n = 2, 4, \ldots, N/2, k = 0, 1, \ldots, n-1$ in all level can be obtained during the calculation of $IDFT(X)$, including $A_q = IDFT(\overline{X}_{q,Q}), 0 \le q \le Q-1$ in level $\log_2 Q$.

Then according to Eq.(5), we have

$$IDFT(X) = \begin{bmatrix} A_0 \\ A_0 \\ \vdots \\ A_0 \end{bmatrix} + \sum_{q=1}^{Q-1} \Delta_q \otimes \begin{bmatrix} A_q \\ A_q \\ \vdots \\ A_q \end{bmatrix} = \sum_{q=0}^{Q-1} IDFT(\underline{X}_q), \tag{10}$$

where $\Delta_q, q = 1, 2, \ldots, Q-1$ are some known complex constant vectors with a length of N.

Therefore, the values of $IDFT(\underline{X}_q), 0 \le q \le Q-1$ can be obtained during the calculation of $IDFT(X)$ using IFFT algorithm.

End of the proof.

Lemma 2: When the phase sequences $P_m = [p_{m,0}, p_{m,1}, \ldots, p_{m,N-1}], m = 0, 1, \ldots, M-1$ are periodic sequences and the periodic value is Q, except the original frequency signal X, the transmitter can use the following equation to produce other $M-1$ candidate time signals

$$IDFT(P_m \otimes X) = \sum_{q=0}^{Q-1} p_{m,q} IDFT(\underline{X}_q), m \ge 1. \tag{11}$$

Proof: We have $P_m \otimes X = \sum_{q=0}^{Q-1} p_{m,q}\underline{X}_q$, thus,

$$\begin{aligned} IDFT(P_m \otimes X) &= IDFT(\sum_{q=0}^{Q-1} p_{m,q}\underline{X}_q) \\ &= \sum_{q=0}^{Q-1} p_{m,q} IDFT(\underline{X}_q). \end{aligned} \tag{12}$$

End of the proof.

Lemma 3: For the frequency signal X, the PAPR value of the following four signals are equal

$$PAPR(IDFT(X)) = PAPR(IDFT(X \otimes S_i)), i = 1, 2, 3, \tag{13}$$

where

$$\begin{cases} S_1 &= [-1, -1, -1, -1, \ldots, -1, -1] \\ S_2 &= [-1, 1, -1, 1, \ldots, -1, 1] \\ S_3 &= [1, -1, 1, -1, \ldots, 1, -1] \end{cases}. \tag{14}$$

Proof: Because

$$IDFT(X \otimes S_1) = IDFT(-X) = -IDFT(X), \tag{15}$$

we have

$$PAPR(IDFT(X)) = PAPR(IDFT(X \otimes S_1)). \tag{16}$$

Let $x_{S_2} = IDFT(X \otimes S_2)$, for $0 \le k \le N/2-1$, according to Eq.(4), we have

$$\begin{cases} x_{S_2}(k) &= -x(k+\frac{N}{2}) \\ x_{S_2}(k+\frac{N}{2}) &= -x(k) \end{cases}, \tag{17}$$

therefore

$$PAPR(IDFT(X)) = PAPR(IDFT(X \otimes S_2)). \tag{18}$$

Let $x_{S_3} = IDFT(X \otimes S_3)$, for $0 \le k \le N/2-1$, according to Eq.(4), we have

$$\begin{cases} x_{S_3}(k) &= x(k+\frac{N}{2}) \\ x_{S_3}(k+\frac{N}{2}) &= x(k) \end{cases}, \tag{19}$$

therefore

$$PAPR(IDFT(X)) = PAPR(IDFT(X \otimes S_3)). \tag{20}$$

End of the proof.

The contents of Lemma 1 and 2 reveal that, using the new phase sequences with the periodicity (Q) and duality (± 1), except the original frequency signal X, the IFFT operation is no longer required for the calculation process of other candidate signals, instead, the linear addition operations of Q intermediate signals.

Fig.3 shows the amplitude of the four related signals mentioned in Lemma 3, where X is an QPSK modulated frequency signal with 64 subcarriers. Obviously, the maximum amplitude values of the four signals are the same. Therefore, the content of Lemma 3 reveals that, in order to prevent useless effort on the performance of PAPR reduction, if a

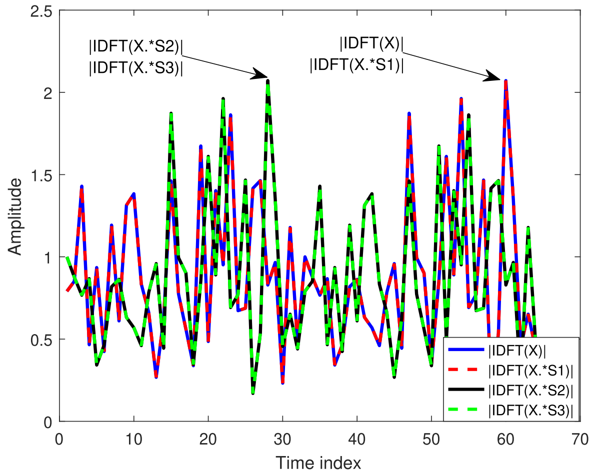

FIGURE 3. Amplitude of four signals in Lemma 3.

sequence P_m is selected to be a phase sequence, then it is necessary to avoid selecting other three sequences of $P_m \otimes S_1$, $P_m \otimes S_2$, and $P_m \otimes S_3$ to be the phase sequence.

B. A CONSTRUCTION METHOD OF PHASE SEQUENCES

The basic idea of the SLM scheme is to generate a set of different time signals, each of length N, all representing the same information as the original frequency signal, then transmit the one with least PAPR. If the length of sideband information is m bits, so wo need to construct a phase sequence set, which contains $M = 2^m$ sequences.

According to the contents of Lemma 1-3, a new construction method of phase sequences is provided for the SLM scheme, its details are as follows.

a) All the phase sequences are composed of 1 and -1, i.e.,

$$p_{m,n} \in \{-1, 1\}, 0 \le m \le M-1, 0 \le n \le N-1 \quad (21)$$

b) All the phase sequences are periodic sequences and the periodic value is Q, i.e.,

$$p_{m,q} = p_{m,q+kQ}, 0 \le q \le Q-1, 1 \le k \le N/Q-1, \quad (22)$$

where $2^{(Q-2)} \ge M$.

c) Let $\Omega \in \{\Omega_0, \Omega_1, \ldots, \Omega_{2^Q-1}\}$ be a set of binary (± 1) sequences with length Q, and M sequences $\overline{P} = [\overline{P}_0, \overline{P}_1, \ldots, \overline{P}_{M-1}]$ are selected from the set Ω, then repeat each sequence N/Q times to get the required phase sequences. The principle used in the selection process is

$$\overline{P} = \underset{0 \le m_1 \ne m_2 \le M-1}{\arg\max} \quad min(\left\| \Theta(\overline{P}_{m_1}) - \Theta(\overline{P}_{m_2}) \right\|_H), \quad (23)$$

where $\Theta(\overline{P}_{m_1}) = \left\{\overline{P}_{m_1}, \overline{P}_{m_1} \otimes S_1, \overline{P}_{m_1} \otimes S_2, \overline{P}_{m_1} \otimes S_3\right\}$, $\left\| \Theta(\overline{P}_{m_1}) - \Theta(\overline{P}_{m_2}) \right\|_H$ represents the minimum Hamming distance between any two sequences in set $\Theta(\overline{P}_{m_1})$ and set $\Theta(\overline{P}_{m_2})$, and the Hamming distance represents the sum of the number of different numerical elements in the same position of two equal length sequences.

The requirements of items a) and b) can help the SLM scheme to reduce the computational complexity, and the requirement of item c) makes the candidate time signals more different, which is helpful to improve the PAPR reduction performance of the phase sequences. At present, no mathematical method has been found to achieve the principle of Eq.(23). However, considering the limited search range in the sequence set Ω, the search of phase sequence with better PAPR reduction performance can be carried out by the computer, and the search process is as follows.

Step 1) Set an expected value C for the search process, and select the full 1 sequence with length Q from Ω as the first sequence, i.e., $\overline{P}_0 = [1, 1, \ldots, 1]$.

Step 2) Randomly select a sequence $\overline{P}_1$ from Ω as the second sequence, where

$$\left\| \Theta(\overline{P}_1) - \Theta(\overline{P}_0) \right\|_H = C. \quad (24)$$

Step 3) Randomly select a sequence $\overline{P}_2$ from Ω as the third sequence, where

$$\left\| \Theta(\overline{P}_2) - \Theta(\overline{P}_i) \right\|_H = C, i = 0, 1. \quad (25)$$

...

Step M) Randomly select a sequence $\overline{P}_{M-1}$ from Ω as the M-1th sequence, where

$$\left\| \Theta(\overline{P}_{M-1}) - \Theta(\overline{P}_i) \right\|_H = C, i = 0, 1, \ldots, M-2. \quad (26)$$

Step M+1) Finally, repeat each sequence in set $\{\overline{P}_0, \overline{P}_1, \ldots, \overline{P}_{M-1}\}$ N/Q times to get the required phase sequence set.

C. CALCULATION PROCESS OF THE CANDIDATE TIME SIGNALS

Based on the new construction method of the phase sequences, the candidate time signals can be generated in the following steps.

Step 1) Let $m = 0$ ($P_0 = [1, 1, \ldots, 1]$), then calculate the value of $IDFT(X)$ using the IFFT operation. Meanwhile, the values of Q intermediate signals in Eq.(8) are obtained in the calculation process of the IFFT operation, and stored as $\underline{A}_q = IDFT(\underline{X}_q), 0 \le q \le Q-1$.

Step 2) Use the following formula to calculate other $M-1$ candidate time signals

$$IDFT(P_m \otimes X) = \sum_{q=0}^{Q-1} \pm \underline{A}_q, m = 1, 2, \ldots, M-1. \quad (27)$$

The signal processing flow chart of the SLM scheme with new phase sequences is shown in Fig.4.

D. COMPUTATION COMPLEXITY ANALYSIS

The computation complexity of the traditional SLM scheme and the new scheme is compared in this section.

$N/2 \log_2 N$ complex multiplications and $N \log_2 N$ complex additions are needed to complete one IFFT operation with N points [19]. Therefore, plus point multiplication calculation process, $MN/2 \log_2 N + MN$ complex multiplications and $MN \log_2 N$ complex additions are needed to generate M candidate signals for the traditional SLM scheme.

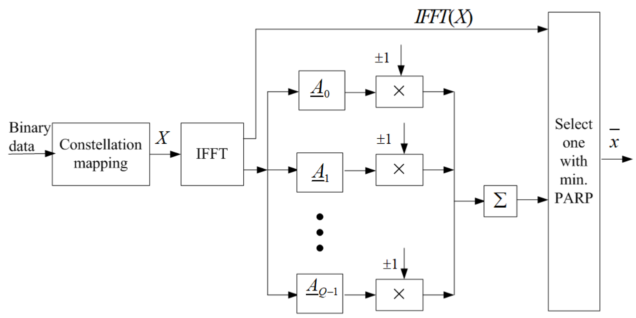

FIGURE 4. The signal processing chart of the SLM scheme with new phase sequences.

In the SLM scheme with new phase sequences, only one IFFT operation is necessary to generate the first candidate time signal, then store the following signal sets

$$\eta_q = \{\underline{A}_q, -\underline{A}_q\}, q = 0, 1, \ldots, Q-1. \tag{28}$$

So the total number of complex multiplications to generate M candidate signals for the new SLM scheme is only

$$num_{\times} = N/2 \log_2 N \tag{29}$$

To avoid unnecessary repetitive calculation, a tree-shaped calculation method is proposed to minimize the number of complex additions in the calculation process of Eq.(27). Fig.5 shows the relevant calculation process as a reference, where $Q = 8$, and the sign $\oplus$ refers to the combinatorial addition between two sets $\Delta = \{\Delta_1, \Delta_2, \ldots \Delta_K\}$ and $\Lambda = \{\Lambda_1, \Lambda_2, \ldots \Lambda_K\}$, i.e.,

$$\Delta \oplus \Lambda = \{\lambda | \lambda = \Delta_{k_1} + \Lambda_{k_2}\}, 1 \leq k_1, k_2 \leq K. \tag{30}$$

The tree-shaped calculation process is as follows.

Step 1) Calculate the following $Q/2$ sets

$$\eta_{q,q+1} = \eta_q \oplus \eta_{q+1}, q = 0, 2, \ldots, Q-2 \tag{31}$$

Step 2) Calculate the following $Q/4$ sets

$$\eta_{q,q+1,q+2,q+3} = \eta_{q,q+1} \oplus \eta_{q+2,q+3}, q = 0, 4, \ldots, Q-4 \tag{32}$$

...

Step $\log_2 Q - 1$) Calculate the following two sets

$$\eta_{0,1,\ldots Q/2-1} = \eta_{0,1,\ldots,Q/4-1} \oplus \eta_{Q/4,Q/4+1,\ldots,Q/2-1} \tag{33}$$

and

$$\eta_{Q/2,Q/2+1,\ldots Q-1} = \eta_{Q/2,Q/2+1,\ldots,3Q/4-1} \oplus \eta_{3Q/4,3Q/4+1,\ldots,Q-1}. \tag{34}$$

Step $\log_2 Q$) According to the value of phase sequence P_m, $1 \leq m \leq M-1$, select the corresponding sequence from set $\eta_{0,1,\ldots Q/2-1}$ and set $\eta_{Q/2,Q/2+1,\ldots Q-1}$ to add or subtract, and get the value of candidate time signal $IDFT(P_m \otimes X)$.

The number of complex additions in Step $q(1 \leq q \leq \log_2 Q - 1)$ is $\frac{Q}{2^q} N 2^{2^{q-1}} 2^{2^{q-2}}$, so the total number of complex

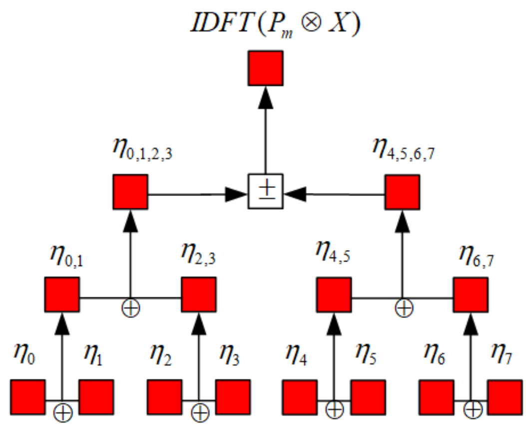

FIGURE 5. Tree-shaped calculation diagram ($Q = 8$).

TABLE 1. The complexity of traditional SLM and New SLM.

Scheme	Complex
traditional SLM	$MN/2 \log_2 N + MN$ multiplications $MN \log_2 N$ additions
new SLM	$N/2 \log_2 N$ multiplications $(M + 2^{3Q/8+1})N$ additions

additions to complete the previous $\log_2 Q - 1$ steps is

$$num1_{+} = \sum_{q=1}^{\log_2 Q-1} \frac{Q}{2^q} N 2^{2^{q-1}} 2^{2^{q-2}} \approx N 2^{3Q/8+1}, \tag{35}$$

and the total number of complex additions to complete the last step is

$$num2_{+} = MN. \tag{36}$$

Therefore, the total number of complex additions to complete the entire tree calculation process is as follows:

$$num_{+} = num1_{+} + num2_{+} \approx (M + 2^{3Q/8+1})N \tag{37}$$

Table 1 summarizes the computational complexity for the traditional SLM scheme and the SLM scheme with new phase sequences.

IV. SIMULATION RESULTS OF THE PERFORMANCE OF PAPR REDUCTION

In practice, the empirical complementary cumulative distribution function (CCDF) is the most informative metric used for evaluating the PAPR of OFDM time signals [5]

$$CCDF(PAPR(x)) = prob(PAPR(x) > \delta), \tag{38}$$

where δ is some threshold.

The Hadamard matrix is a square matrix whose elements are all composed of $+1$ and -1, and any two different rows are orthogonal to each other. The Hadamard matrix is generated in the following way [20]

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, H_4 = \begin{bmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{bmatrix}, \ldots$$
$$H_N = \begin{bmatrix} H_{N/2} & H_{N/2} \\ H_{N/2} & -H_{N/2} \end{bmatrix}. \tag{39}$$

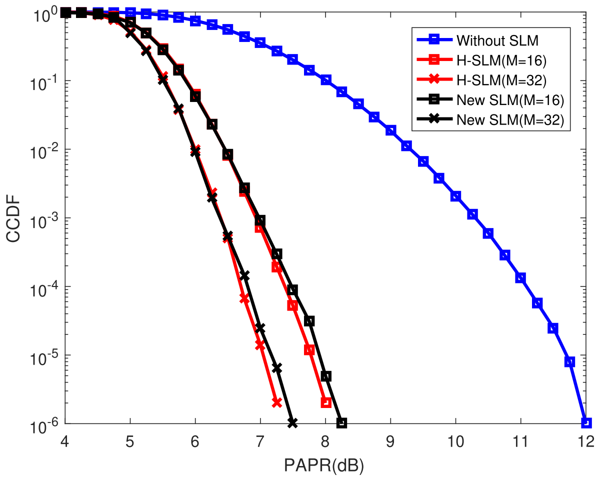

FIGURE 6. **The CCDF performance of the original OFDM without SLM scheme, the traditional SLM scheme based on Hadamard matrix H_{64}, and the SLM scheme with new phase sequence sets ($N = 64$ and $M = 16, 32$).**

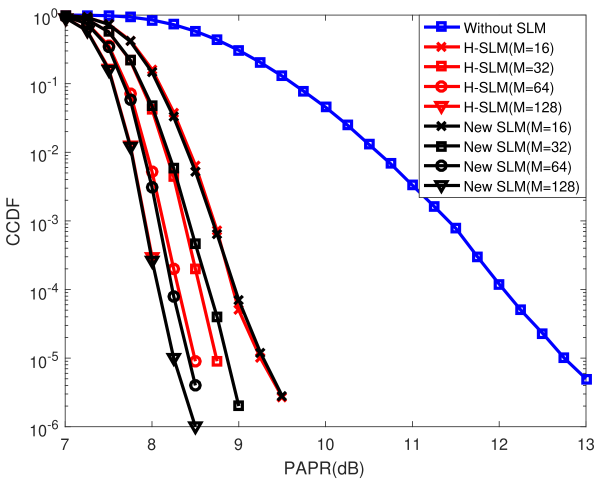

FIGURE 7. **The CCDF performance of the original OFDM without SLM scheme, the traditional SLM scheme based on Hadamard matrix H_{1024}, and the SLM scheme with new phase sequence sets ($N = 1024$ and $M = 16, 32, 64, 128$).**

According to the construction way of Hadamard matrix in Eq.(39), half of the element values of any two vectors in the Hadamard matrix are the same, and the other half of the element values are opposite (i.e.,180 degree phase rotation) to each other. So when using the vectors in Hadamard matrix as the phase sequences in the SLM scheme, this characteristic can help to generate greater differences between different candidate signals, and get excellent PAPR reduction performance [21]. Therefore, the SLM scheme based on Hadamard matrix is often used as a comparison object for the PAPR reduction performance of other similar schemes.

Here, a supplementary explanation is proposed for the traditional SLM scheme based on Hadamard matrix. According to Eq.(39), the 2n-1th row vector h_{2n-1} and the 2nth row vector h_{2n} in the Hadamard matrix have the following association

$$h_{2n-1} = h_{2n} \otimes S_3, n = 1, 2, \ldots, N/2. \tag{40}$$

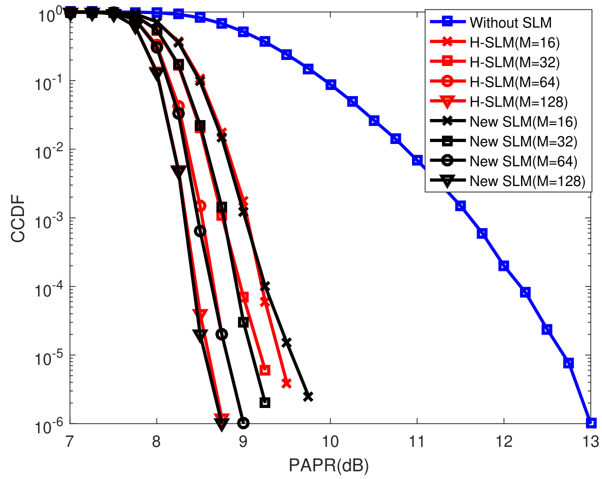

FIGURE 8. **The CCDF performance of the original OFDM without SLM scheme, the traditional SLM scheme based on Hadamard matrix H_{2048}, and the SLM scheme with new phase sequence sets ($N = 2048$ and $M = 16, 32, 64, 128$).**

TABLE 2. **The CCRR of traditional SLM and New SLM.**

Parameter	H-SLM	New SLM	CCRR(%)
N=64,M=16(QPSK)	4096(×)	192(×)	95.31
	6144(+)	2048(+)	66.67
N=64,M=32(QPSK)	8192(×)	192(×)	97.66
	12288(+)	3072(+)	75.00
N=1024,M=16(16QAM)	98304(×)	5120(×)	94.79
	163840(+)	32768(+)	80.00
N=1024,M=32(16QAM)	196608(×)	5120(×)	97.40
	327680(+)	49152(+)	85.00
N=1024,M=64(16QAM)	393216(×)	5120(×)	98.70
	655360(+)	196608(+)	70.00
N=1024,M=256(16QAM)	786432(×)	5120(×)	99.35
	1310720(+)	262144(+)	80.00
N=2048,M=128(256QAM)	212992(×)	11264(×)	94.71
	360448(+)	65536(+)	81.82
N=2048,M=32(256QAM)	425984(×)	11264(×)	97.36
	720896(+)	98304(+)	86.36
N=2048,M=64(256QAM)	851968(×)	11264(×)	98.68
	1441792(+)	393216(+)	72.73
N=2048,M=128(256QAM)	1703936(×)	11264(×)	99.34
	2883584(+)	524688(+)	81.80

Basing on the content of Lemma 3, we have that half of the vectors in the Hadamard matrix are useless for improving the PAPR reduction performance. Therefore, only the odd row vectors in the Hadamard matrix are used to be the phase sequences in this paper.

Let $Q = 8, M = 16, C = 2$, based on the construction scheme of phase sequences, we use the computer to search and get a phase sequence set $P = \{P_0, P_1, \ldots, P_{15}\}$. To facilitate content display, hexadecimal is used to represent its value, where

$$P^1_{0x} = \{FF, F3, C9, CF, E7, C6, FC, C3, C5, D4, CC, E1, F5, DD, D7, F6\}_{16}. \tag{41}$$

Then convert the data format of Eq.(41) from hexadecimal to binary (0 and 1), i.e.,

$$P^1_B = P^1_{0x}, \tag{42}$$

and let

$$\overline{P}^1 = \{\overline{P}_0, \overline{P}_1, \ldots, \overline{P}_{15}\} = 2 \times P_B^1 - 1. \quad (43)$$

Let $Q = 8, M = 32, C = 2$, and $Q = 16, M = 64, C = 4$ and $Q = 16, M = 128, C = 4$ respectively, we use the similar method to get another three sets and their hexadecimal values are

$$P_{0x}^2 = \{FF, F6, D1, E2, EE, DD, FA, C9, D8, F9, F3, D4, C0, CC, DE, ED, E1, EB, CF, CA, C5, C3, E7, F0, E4, D2, F5, C6, D7, E8, DB, FC\}_{16}, \quad (44)$$

and

$$P_{0x}^3 = \{FFFF, CABE, CF4B, F1FB, CFB4, CEFD, DF37, E0CE, F45E, DCB6, F6CC, E6F8, CAA5, EEA0, E4E2, EB95, EF51, F215, FCD1, C54E, E089, E5EC, EA01, EC45, E71B, E85F, DC98, C17C, F33C, E307, F1B0, C087, E586, E0D0, E2ED, C0B8, FDA7, F0F6, DB24, E293, CB92, D386, C8A2, EB6F, CC7E, D403, D2D1, D860, D68F, E073, F8A8, C6B7, FADB, CDF2, DA2F, C328, F971, C665, FBF8, D144, D263, C3AF, C97B, C9E4\}_{16}, \quad (45)$$

and (46), as shown at the bottom of the page.

Then use a similar method in Eq.(42) and Eq.(43) to get the sets $\overline{P}^2$, $\overline{P}^3$ and $\overline{P}^4$ respectively. In order to obtain the PAPR reduction performance of these sequences under different simulation parameters, 10^7 randomly generated OFDM symbols are used in the simulation of each scheme.

For an OFDM system using QPSK modulation and $N = 64$, repeat the sequences of set $\overline{P}^1$ and set $\overline{P}^2$ 8 times respectively, and get two phase sequence set P^1 and P^2 for the new SLM scheme, whose length is all 64. Fig.6 shows the CCDF performance of three different scheme: the original OFDM without any SLM scheme, the traditional scheme which uses the first $M = 16$ and $M = 32$ odd row vectors in Hadamard matrix H_{64} as the phase sequences (H-SLM), and the new SLM scheme with phase sequence set P^1 and P^2.

In subsequent simulations, by repetition, the length of sequences $\overline{P}^i, i = 1, 2, 3, 4$ is extended to the number of subcarriers to generate the phase sequence sets of new SLM scheme. Then when using 16QAM modulation and $N = 1024$, Fig.7 shows the CCDF performance of three different scheme: the original OFDM without any SLM scheme, the traditional scheme which uses the first $M = 16, 32, 64, 128$ row vectors in Hadamard matrix H_{1024} as the phase sequences, and the new SLM scheme with four phase sequence sets. And Fig. 8 shows the similar simulation results when using 256QAM modulation and $N = 2048$.

The results in Fig.6, Fig.7 and Fig.8 all reveal that, in OFDM systems with different parameters, as long as the number of phase sequences is the same, the phase sequence set based on the new construction method can reach almost the same level as that of the traditional SLM scheme, whose phase sequences are from the odd row vectors in the Hadamard matrix H_N.

The computational complexity reduction ratio (CCRR) of the new SLM scheme over the traditional H-SLM scheme is defined as [3]

$$CCRR = \left(1 - \frac{complexity \quad of \quad new \quad SLM}{complexity \quad of \quad H-SLM}\right) \times 100\% \quad (47)$$

$$P_{0x}^4 = \{FFFF, F677, C674, D616, FF71, D8CF, C846, C4F8, FDD0, C13D, DF4A, ED6F, C381, F5F3, D625, E0D3, FF24, ED41, E925, DB66, D680, F4C7, EEBD, DEDE, CA6F, E174, E6CB, C542, C18A, F0B1, DC37, FD1A, E608, F426, D4AA, E319, E235, E6A4, C57E, CA95, E73C, C873, CF50, E3B0, DDAC, F1B6, F0FF, D8D8, F78A, C7D6, CBB6, CF87, DD95, F977, D68F, C1A4, C74C, CD1F, D549, C012, E558, C95C, F9A3, CEBA, F17A, C713, C1F1, F8E5, C5A9, DAC1, D2DB, E94A, CB31, D5A7, F797, DB8D, FACC, ED8B, EA9B, E63B, EC13, D71D, D9BF, F20B, D570, C82C, F93C, EE14, DB82, E241, DE99, FC4B, F4E9, FCE2, CEE0, F3BD, F333, D3AB, CD2A, C485, EBF3, F088, D2E7, D7F9, FA18, D72E, D1E8, FAB7, E169, CCE7, D893, ECF1, C298, D951, F4DA, C107, FC56, FA63, FE7A, D00E, E455, CCD2, E8FA, EC3E, CAA3, EEC5, F25E, E8D4\}_{16}. \quad (46)$$

The CCRR of the proposed scheme over the traditional SLM scheme with the parameters of Fig.6, Fig.7, and Fig.8 are given in Table 2, where the symbols of (×) and (+) represent complex multiplication and complex addition, respectively. And Table 2 tells us that the proposed scheme can effectively reduce the computational complexity, for example, when $Q = 8, M = 32, N = 1024$, over the traditional SLM scheme, the number of complex multiplication and complex addition of new scheme can separately be decreased by about 97.40% and 85.00%. And when $Q = 16, M = 128, N = 2048$, the reduction can reach about 99.34% and 81.80%.

V. CONCLUSION

A construction method of phase sequences is proposed for the SLM scheme. Its core idea is to make the phase sequence have periodicity and duality (±1), meanwhile, the principle of maximization of the minimum Hamming distance between different sequences is introduced to search the phase sequences, which can help the SLM scheme to improve the performance of PAPR reduction. Theoretical analysis and simulation results show that, the SLM scheme with new phase sequences can have the advantages of low computational complexity and excellent PAPR reduction performance.

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