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RESEARCH ARTICLE

Model of a Queuing System With BPP Elastic and Adaptive Traffic

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ABSTRACT This paper presents an analytical model of a multiservice queueing system that services elastic and adaptive BPP traffic. The model presented in the paper was developed as an extension of earlier works published by the authors. The model is based on several concepts specific to modelling multi-service systems. In order to derive the model, the idea of equivalent bandwidth and the discretisation process were used first. Subsequently, the MIM-BPP method was generalised in relation to a system with traffic that undergoes the compression mechanism. Since the proposed model is an approximate one, the results of the calculation are compared with the simulation results. The comparison of the results confirms the accuracy of the proposed model.

INDEX TERMS Adaptive and elastic traffic, BPP traffic, compression mechanism, two-dimensional Markov process, traffic modeling, queuing systems, multirate systems.

I. INTRODUCTION

In modern multiservice networks that are based on the IP protocol, data streams are typically generated by Internet applications with different modes of operation. This, in turn, translates into the need to transmit streams with different requirements guaranteeing at the same time the required quality parameters (QoS - Quality of Service, GoS - Grade of Service) of the conforming sources. Good examples of such streams are, among others, video on demand streams, Internet telephony, events transmitted in real time, e-mail sending and receiving process, etc. Each of these services can undergo different regulations depending on the expectations of the end user. The issue of providing adequate and appropriate quality parameters for serviced streams is further amplified by the sharp increase in the amount of data transferred (this phenomenon of increasing traffic intensity in the Internet network is thoroughly discussed in reports [1], [2]). Additionally, the Covid-19 pandemic has caused an increase in network traffic. At the beginning of the pandemic, the network traffic increased by approximately 40% in Poland [3]. This trend was observed practically all over the world. A vast increase was observed mainly in the transmission related to videoconferences, multimedia transmission, and solutions used for remote work, such as OpenVPN and IPSec1 [3], [4], [5]. Video transmission traffic in countries such as the United States, France, Great Britain, Spain, and Italy increased by about 20 to 30 % [4], [5]. On the other hand, the number of users of videoconferencing solutions increased even more significantly, where the number of users of the ZOOM application in April 2020 was 300 million, and just a few months earlier, in December 2019, only 10 million [5].

In these circumstances, the modelling of telecommunications systems seems to be a particularly important issue. Modelling makes it possible to dimension and optimise the existing infrastructure in direct relation to offered services. In the literature on the subject, the analysis of multiservice systems is typically carried out at the calls level, in which each call is defined as a packet stream with specific requirements pertaining to the resources. These requirements define constant bitrates (CBR) to be applied that are most frequently determined on the basis of the maximum bitrates (speeds) of real packet streams with variable bitrate (VBR) or on the basis of the so-called equivalent bandwidth (EB) [6], [7], [8]. The equivalent bitrate is a certain value of the bandwidth that can be determined on the basis of the adopted assumptions, e.g., as the mean

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bitrate of VBR streams or other parameters indicated by the service provider. The choice of the equivalent bandwidth itself does not translate, however, into the way a system is modelled, but only into its parametrisation. Therefore, this issue is not the subject of considerations addressed in this article. References [9] and [10] proves that traffic at the call level can be approximated by Poisson streams. This can be translated into the possibility of analysing the multiservice system on the basis of multi-dimensional Markov processes. In this process, we can distinguish micro- and macro-states. The former are defined by the number of demands from individual call classes that are in the system, whereas the latter are defined as the total number of busy resources in the occupancy state of the system under consideration [11].

In the modelling process, the traffic management mechanisms that are used by operators must be taken into consideration. The application of these mechanisms mainly results from the attempt to limit the ongoing expansion of the network in order to utilise the existing resources in the best possible way. These mechanisms include, among others:

- call queuing [12], [13], [14], [15], [16], which allows calls to wait until access to the system is possible in the case of a temporary lack of resources;
- dynamic resource reservation [17], [18], [19], [20], in which certain load levels (thresholds) are introduced into the system that, when exceeded, allow the system to service calls of strictly defined call classes only;
- priorities [21], [22], which allow the operator to determine the significance of individual call classes and to assign preferences in allocated resources;
- traffic compression [9], [11], [13], [23], [24], [25], which makes it possible to change the bitrates of serviced calls depending on the load of the system;
- traffic overflow [26], [27], [28], which is based on redirecting the call that cannot be serviced in one system to another system that has a sufficient number of resources to service this call.

In mobile and Internet networks that are based on the TCP/IP protocol, the most frequently used mechanisms are queueing of calls and thresholdless compression. The thresholdless compression mechanism allows the transmission speed of serviced streams to be either increased or decreased. In the literature, the traffic flow that is subject to such a mechanism is called elastic traffic or adaptive traffic [9], [13], [20], [25]. If the decrease in bitrate is accompanied by the extension of service time necessary for all data to be transferred, then we speak about elastic traffic. A good example of the applications related to this type of traffic can be all operations that involve data downloading, email sending, and the like. In turn, if the decrease in bitrate is not accompanied by any change in call service time, then we can speak about adaptive traffic. No change in service time can be linked to, for example, a change in the quality of the transmitted signal and often occurs at the time of an on-line transmission overload. A change in bitrate in systems with threshold-less compression occurs when all resources of the system are busy. In these circumstances, a new call that arrives at the input of the system causes all currently serviced streams to decrease their bitrates, and consequently this new call can be admitted for service.

To perform an accurate analysis of network systems that are based on the TCP/IP protocol stack properly, it is necessary to develop queuing models that would take into account the service of elastic and adaptive traffic. Regrettably, the literature does not provide many analytical models of multiservice queuing systems. The vast majority of the proposals presented are based on the M/G/R PS (M/G/R Processor Sharing) models [29], [30], in which the dimensioning process consists in determining the minimum compression level. The minimum compression level is understood as the definition of a bitrate for which the delay will not exceed the assumed value (calculated based on Erlang's "C" formula) [31]. The Erlang "C" formula is also used to develop models of packet streams in the nodes of an Internet network [32]. Another approach to modelling queueing systems involves the recursive method for full-availability multiservice systems with elastic traffic [33]. However, none of these solutions allows the delay parameters for the individual traffic classes to be determined. In turn, the models presented in [34] make it possible to determine individual characteristics of delays for all classes of connections offered to the system within the so-called state-dependent FIFO service (SD FIFO). In [14], the authors propose a multiservice queuing model with the state-dependent FIFO discipline of calls. This discipline, based on the balanced fairness algorithm, allocates resources in the multiservice server. Therefore, a queuing system with SD FIFO discipline can be imaged as a sequence of virtual queuing systems in which a single virtual queue is dedicated to a single traffic class. Moreover, virtual servers in this system have a variable capacity, which depends on two parameters: the number of serviced packet streams and the number of calls waiting in virtual queues. Due to these assumptions, this model enables the determination of the queue length for particular call classes. This model does not take into account the service of elastic traffic; therefore, the concept of a generalised SD FIFO model on the macrostate level was proposed [12], [35]. Then it was expanded to develop a comprehensive model of the SD FIFO queue system with elastic and adaptive Erlang traffic at the micro and macrostate level [36].

The author of this article discusses the model [36] that has been expanded and generalised to include Erlang, Engset, and Pascal traffic. As a result, a queueing model with elastic and adaptive traffic is considered, with the assumption that the offered traffic creates a mixture of Erlang, Engset, and Pascal traffic. The proposed model allows all important characteristics of the occupancy of the system with the FIFO discipline to be determined.

A. RESEARCH MOTIVATION

In this paper, the authors decided to address the topic of modelling a queuing system with elastic and adaptive

BPP traffic. To the best of the authors' knowledge, this is the first model of a system of this type in the world. The implementation of the research was an extension of previous research carried out in the research team, resulting in publications [12], [13], [24], [25], [36], [37], [38], [39], [40]. In these publications, various ICT systems with BPP traffic were considered, but none of them used queuing, which is widely used in today's systems. In the literature, queuing systems with adaptive [13] and elastic [35] traffic can be found independently, as well as model with a mixture of elastic and adaptive [36] traffic, but let us emphasise that in these models considering only Erlang-type traffic is considered, which is subject to a compression mechanism. Therefore, the realisation of the assembly of these systems became a natural step in the research.

B. RESEARCH CONTRIBUTION

The main achievements of this article are as follows:

- The development of a new analytical model based on the reversible Markov process for a queuing system with a mixture of Erlang, Engset, and Pascal elastic and adaptive traffic (compression-sensitive traffic). In the system under consideration, both the process of incoming calls (Engset's and Pascal's traffic classes) and the servicing of calls (extending the handling time of flexible requests) are state-dependent. Incorporating both of these dependencies in the analytical model was the main obstacle and required the use of a double approximation, which allowed the average number of calls serviced in a given state to be determined while taking into account the level of call compression. To the best of authors' best, this is the first model to take into account the aforementioned types of traffic found in modern data communication networks.
- In the proposed model, the authors introduced a mechanism to determine the approximate occupancy distribution of the different states of the system, which made it possible to determine qualitative parameters such as blockage and system loss coefficients, as well as the average queue length and average system occupancy. As demonstrated by the simulation studies carried out, the model has a high level of accuracy.

C. ARTICLE STRUCTURE

The article is structured as follows. Chapter 2 describes and defines the basic parameters of the offered traffic and the queuing system. Then, a model of a multiservice with elastic and adaptive Erlang, Engset, and Pascal traffic is presented. This model was developed on the basis of a multi-dimensional Markov process. In Chapter 3, appropriate formulae are derived and discussed. These formulae allow the average number of occupied resources in the queue and all of the system to be determined. Chapter 5 presents a comparison of the results of analytical calculations with the results of digital simulation for a number of selected traffic management scenarios. Chapter 6 provides the conclusions that can be drawn from this study.

II. ANALYTICAL MODEL OF THE QUEUING SYSTEM WITH BPP TRAFFIC WITH COMPRESSION MECHANISM

The assumption in the analytical modelling of the traffic characteristics of multiservice systems is that the resources demanded by calls for individual traffic classes are the multiple number of the so-called allocation unit (AU) that can be defined as the greatest common divisor of the equivalent bandwidths of all traffic classes offered to the system:

$$c_{AU} = GCD(t_{u,1,d}, t_{u,2,d}, \ldots), \tag{1}$$

where $t_{u,i,d}$ denotes the bitrate that corresponds to the equivalent bandwidth to a call of u (Erlang, Engset, Pascal) traffic of class i that undergoes compression of type d (adaptive, elastic). Thus, determined bitrate c_{AU} makes it possible to express the capacity of the system C_r and the number of demanded resources necessary to set up a connection $c_{d,i,u}$ in AUs:

$$C_r = \left\lfloor \frac{V_r}{c_{AU}} \right\rfloor, \tag{2}$$

$$c_{d,i,u} = \left\lceil \frac{t_{u,i,d}}{c_{AU}} \right\rceil, \tag{3}$$

where V_r is the total capacity of the server expressed in Bps.

Consider then a queuing system with real capacity C_r AUs and the buffer capacity C_q AUs, to which a mixture of Erlang, Engset, and Pascal elastic and adaptive traffic classes is offered. The call stream of each Erlang traffic class is a Poisson stream whose intensity does not depend on the occupancy state of the system (i.e., on the number of serviced calls). The call stream of each Engset traffic class is a Bernoulli stream, whose intensity decreases with increasing occupancy state of the system. In turn, in the case of Pascal traffic classes, the call stream is a Pascal stream whose intensity increases with the increasing occupancy state of the system. This mixture of traffic streams is called in the literature BPP traffic [38], [41]. Each of the traffic streams offered to the system is either elastic or adaptive, which means that the call service process is also state-dependent. This dependence can be described in the following way: a lack of free resources necessary to service a new call triggers compression (a decrease in bitrate) of all currently serviced calls. If admission of a new call would cause a given compression level (called compression boundary) to be exceeded, then this call is transferred to the queue in its uncompressed form. If there is no free space in the queue, this call will be rejected. To determine the compression boundary, the notion of virtual capacity of the system is introduced and denoted by the parameter C_v ($C_v > C_r$). The introduction of the parameter C_v makes it possible to assume the following interpretation of the compression mechanism: calls can undergo the compression mechanism until the number of busy (occupied) AUs in the server, defined as the sum of uncompressed demands of calls of all classes,

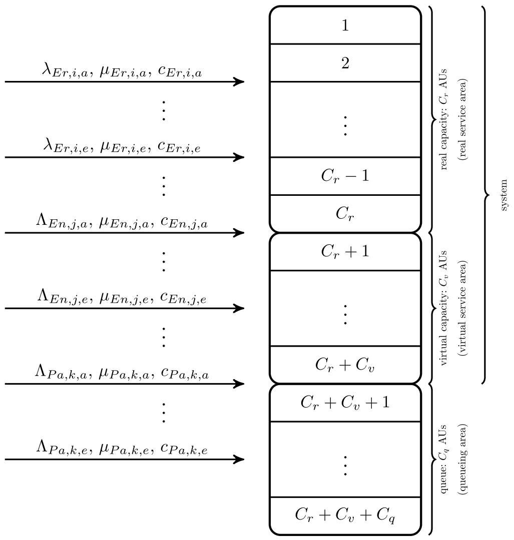

FIGURE 1. Multiservice queuing system with elastic and adaptive BPP traffic.

exceeds the virtual capacity of the system. This means that the choice of the virtual capacity of the system C_v is equivalent to the maximum compression level ratio on the server, which can again be defined as the ratio between the real capacity of the system and the virtual capacity $\frac{Cr}{Cv}$. Figure 1 shows an illustrative (diagram showing a multiservice system with BPP traffic, compression, and queue. The service process in a multiservice system can be considered on the basis of the so-called microstates and macrostates. The microstate X is defined as a sequence (string) of natural numbers $\mathbb{N}_+$ that can be determined by the number of serviced calls of individual classes:

$$X = \Big(x^*_{Er,1,a}, \ldots, x^*_{Er,M_{Er,a},a}, x^*_{Er,1,e}, \ldots, x^*_{Er,M_{Er,e},e}, x^*_{En,1,a}, \ldots, x^*_{En,M_{En,a},a}, x^*_{En,1,e}, \ldots, x^*_{En,M_{En,e},e}, x^*_{Pa,1,a}, \ldots, x^*_{Pa,M_{Pa,a},a}, x^*_{Pa,1,e}, \ldots, x^*_{Pa,M_{Pa,e},e}\Big), \quad (4)$$

where $x^*_{d,i,u}(X)$ is the number of serviced calls of class i of traffic d compression type u, ($d \in \{Er -$ Erlang traffic streams, $En -$ Engset traffic streams , $Pa -$ Pascal traffic streams$\}$ i $u \in \{e -$ elastic traffic, $a -$ adaptive traffic$\}$) in the microstate X. The total number of AUs serviced in the microstate X, (denoted by n_X), is then equal to

$$n_X = \sum_{u\in\{a,e\}} \sum_{i=1}^{M_{Er,u}} x^*_{Er,i,u} c_{Er,i,u} + \sum_{u\in\{a,e\}} \sum_{i=1}^{M_{En,u}} x^*_{En,i,u} c_{En,i,u} + \sum_{u\in\{a,e\}} \sum_{i=1}^{M_{Pa,u}} x^*_{Pa,i,u} c_{Pa,i,u}. \quad (5)$$

The macrostate, in turn, defines the total number of AUs that are serviced in the system, without the share (division) of AUs among individual call classes taken into consideration. This means that the macrostate $\Omega(n)$ can be formally defined as the set of such microstates in which the total number of busy AUs is equal to n:

$$\Omega(n) = \{X : n_X = n\}. \quad (6)$$

Let u denote the parameter that determines the type of compression of calls of individual classes. This parameter can take on two values, $u = e$ for elastic traffic and $u = a$ for adaptive traffic, respectively.

In this article, the following formal notation of the call stream parameters is adopted:

- Erlang traffic:
  - $M_{Er,u}$ - the number of Erlang call classes, compression type u
  - $\lambda_{Er,i,u}$ - intensity of Poisson stream of class i compression type u
  - $\mu_{Er,i,u}$ - intensity of the service stream of calls of class i compression type u (exponentially distributed)
  - $c_{Er,i,u}$ - the number of resources necessary to set up a connection of calls of class i compression type u (expressed in AUs)
- Engset traffic:
  - $M_{En,u}$ - the number of Engset call classes, compression type u offered to the system
  - $N_{En,i,u}$ - the number of traffic sources of class i compression type u
  - $\Lambda_{En,i,u}$ - intensity of the Bernoulli stream of class i compression type u generated by a single free traffic source
  - $\lambda_{En,i,u}\left(x^*_{En,i,u}\right)$ denotes the intensity of call arrival of calls of Engset class i compression type u; this parameter is the function of the number of serviced calls and can be expressed by the following formula:

$$\lambda_{En,j,u}\left(x^*_{En,j,u}\right) = \left[N_{En,j,u} - x^*_{En,j,u}\right] \Lambda_{En,j,u} \quad (7)$$

  - $\mu_{En,i,u}$ - intensity of the service stream of calls of class i compression type u (exponentially distributed)
  - $c_{En,i,u}$ - the number of resources necessary to establish the connection of calls of class i compression type u (expressed in AUs)
- Pascal traffic:
  - $M_{Pa,u}$ - the number of Pascal call classes, compression type u offered to the system
  - $S_{Pa,i,u}$ - the number of traffic sources of class i compression type u
  - $\Lambda_{Pa,i,u}$ - intensity of the Pascal stream of class i compression type u generated by a single free traffic source
  - $\lambda_{Pa,i,u}\left(x^*_{Pa,i,u}\right)$ denotes the intensity of the arrival of Pascal calls of class i compression type u;

this parameter is the function of the number of serviced calls and can be expressed by the following formula:

$$\lambda_{Pa,k,u}\left(x^{*}_{Pa,k,u}\right) = \left[S_{Pa,k,u} + x^{*}_{Pa,k,u}\right]\Lambda_{Pa,k,u} \quad (8)$$

- $\mu_{Pa,i,u}$ - intensity of the call service stream of class i compression type u (which undergo exponential distribution)
- $c_{Pa,i,u}$ - the number of resources necessary to establish a connection of calls of class i compression type u (expressed in AUs).

Equations (7) and (8) indicate a state-dependent call admission process. In the case of Engset traffic classes, the call arrival intensity decreases along with the increase in the occupancy state of the system, whereas the call admission intensity of Pascal traffic classes increases with the increase of the occupancy of the system.

A. ANALYSIS OF CALL SERVICE PROCESS AT THE MICROSTATE LEVEL

The compression of elastic traffic classes is followed by a concurrent extension of call service time and the decrease in the number of demanded resources. Therefore, the service intensity $\mu_{d,i,u}$ and the number of demanded resources $c_{d,i,u}$ can be written in the following way:

$$\mu_{d,i,e}(X) = \begin{cases} \mu_{d,i,e}, & 0 \le n_X \le C_r, \\ \frac{C_r}{n_X}\mu_{d,i,e}, & C_r < n_X \le C_r + C_v, \\ \frac{C_r}{C_v}\mu_{d,i,e}, & n_X > C_r + C_v \text{ and} \\ & n_X \le C_r + C_v + C_q, \end{cases} \quad (9)$$

and

$$c_{d,i,e}(X) = \begin{cases} c_{d,i,e}, & 0 \le n_X \le C_r, \\ \frac{C_r}{n_X}c_{d,i,e}, & C_r < n_X \le C_r + C_v, \\ \frac{C_r}{C_v}c_{d,i,e}, & n_X > C_r + C_v \text{ and} \\ & n_X \le C_r + C_v + C_q, \end{cases} \quad (10)$$

where $d \in \{Er, En, Pa\}$ i $u \in \{e, a\}$.

In the case of adaptive traffic classes, call compression is followed by the decrease in the number of demanded AUs only, therefore we can write:

$$\mu_{d,i,a}(X) = \mu_{d,i,a}, \quad \text{for} \quad 0 \le n_X \le C_r + C_v + C_q, \quad (11)$$

and

$$c_{d,i,a}(X) = \begin{cases} c_{d,i,a}, & 0 \le n_X \le C_r, \\ \frac{C_r}{n_X}c_{d,i,a}, & C_r < n_X \le C_r + C_v, \\ \frac{C_r}{C_v}c_{d,i,a}, & n_X > C_r + C_v \text{ and} \\ & n_X \le C_r + C_v + C_q, \end{cases} \quad (12)$$

where $d \in \{Er, En, Pa\}$ and $u \in \{e, a\}$.

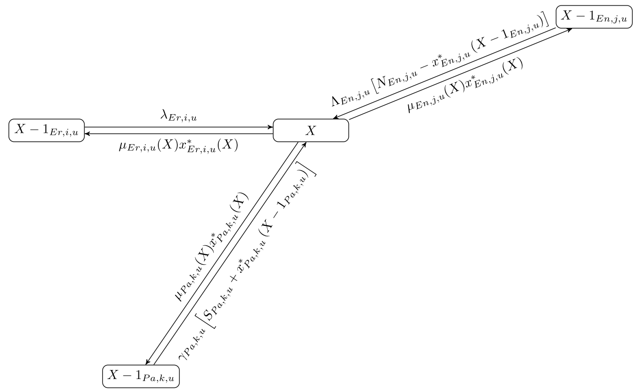

FIGURE 2. A fragment of a diagram of a multidimentional Markov process in the system with BPP elastic and adaptive traffic.

The coefficient $\frac{C_r}{C_v}$ in the queuing area (in the macrostates for which the inequality $C_v < n_X \le C_q$ is satisfied) does not necessarily mean that calls admitted to the queue undergo a compression mechanism. This means that the admission of a call from the queuing area to the service area triggers (implies) the maximum compression of all serviced elastic and adaptive calls. A similar interpretation also applies to the service time of elastic calls. The service time of this call will be extended to the maximum, and thus its service intensity will decrease by $\frac{C_r}{C_v}$ times.

In [36], an analysis of the service process that takes place in systems with adaptive and elastic traffic was carried out, while the results of the analysis show that in such systems the property of reversibility of the service process is not met.

Let us assume, however, that in the multiservice system with BBP traffic that is subject to compression (of elastic and adaptive traffic) under consideration, the service process is a reversible process. The property of reversibility of the process implies local balance equations between two neighbouring microstates of the process. Consider then a fragment of a multi-dimensional Markov process that occurs in the considered system (Fig. 2).

On the basis of Figure 2, the equations for the Erlang traffic class of class i compression type u, Engset of class j compression type u, and Pascal of class k compression type u can be written as follows:

$$\lambda_{Er,i,u}P\left(X - 1_{Er,i,u}\right) = \mu_{Er,i,u}(X)\,x^{*}_{Er,i,u}(X)\,P(X), \quad (13)$$

$$\Lambda_{En,j,u}[N_{En,j,u} - x^{*}_{En,j,u}\left(X - 1_{En,j,u}\right)] \cdot P\left(X - 1_{En,j,u}\right) = \mu_{En,j,u}(X)\,x^{*}_{En,j,u}(X)\,P(X), \quad (14)$$

$$\Lambda_{Pa,k,u}[S_{Pa,k,u} + x^{*}_{Pa,k,u}\left(X - 1_{Pa,k,u}\right)] \cdot P\left(X - 1_{Pa,k,u}\right) = \mu_{Pa,k,u}(X)\,x^{*}_{Pa,k,u}(X)\,P(X). \quad (15)$$

After simple asymmetric transformations, Equations (13)-(15) can be written in the following way:

$$\frac{\lambda_{Er,i,u}}{\mu_{Er,i,u}(X)}c_{Er,i,u}(X)\,P\left(X - 1_{Er,i,u}\right) = c_{Er,i,u}(X)\,x^{*}_{Er,i,u}(X)\,P(X), \quad (16)$$

$$\frac{\Lambda_{En,j,u}[N_{En,j,u} - x^*_{En,j,u}\left(X - 1_{En,j,u}\right)]}{\mu_{En,j,u}\left(X\right)} \cdot c_{En,j,u}\left(X\right) P\left(X - 1_{En,j,u}\right) = c_{En,j,u}\left(X\right) x^*_{En,j,u}\left(X\right) P\left(X\right), \quad (17)$$

$$\frac{\Lambda_{Pa,k,u}[S_{Pa,k,u} + x^*_{Pa,k,u}\left(X - 1_{Pa,k,u}\right)]}{\mu_{Pa,k,u}\left(X\right)} \cdot c_{Pa,k,u}\left(X\right) P\left(X - 1_{Pa,k,u}\right) = c_{Pa,k,u}\left(X\right) x^*_{Pa,k,u}\left(X\right) P\left(X\right). \quad (18)$$

The fractional expressions determine the intensity of Erlang, Engset, and Pascal offered traffic, respectively. Therefore, Equations (16)-(18) can be rewritten in the following form:

$$A_{Er,i,u}\left(X\right) c_{Er,i,u}\left(X\right) P\left(X - 1_{Er,i,u}\right) = c_{Er,i,u}\left(X\right) x^*_{Er,i,u}\left(X\right) P\left(X\right), \quad (19)$$

$$\alpha_{En,j,u}\left(X\right)[N_{En,j,u} - x^*_{En,j,u}\left(X - 1_{En,j,u}\right)] \cdot c_{En,j,u}\left(X\right) P\left(X - 1_{En,j,u}\right) = c_{En,j,u}\left(X\right) x^*_{En,j,u}\left(X\right) P\left(X\right), \quad (20)$$

$$\beta_{Pa,k,u}\left(X\right)[S_{Pa,k,u} + x^*_{Pa,k,u}\left(X - 1_{Pa,k,u}\right)] \cdot c_{Pa,k,u}\left(X\right) P\left(X - 1_{Pa,k,u}\right) = c_{Pa,k,u}\left(X\right) x^*_{Pa,k,u}\left(X\right) P\left(X\right), \quad (21)$$

where

$$A_{Er,i,u}\left(X\right) = \frac{\lambda_{Er,i,u}}{\mu_{Er,i,u}\left(X\right)} = \begin{cases} A_{Er,i,u}, & (u = a \wedge 0 \le n_X \le C_s + C_v + C_q) \vee (u = e \wedge 0 \le n_X \le C_r), \\ \frac{n_X}{C_r} A_{Er,i,u}, & u = e \wedge C_r < n_X \le C_r + C_v, \\ \frac{C_v}{C_r} A_{Er,i,u}, & u = e \wedge C_r + C_v < n_X \le C_r + C_v + C_q \end{cases} \quad (22)$$

determines the intensity of Erlang offered traffic of class i type u, whereas

$$\alpha_{En,j,u}\left(X\right) = \frac{\Lambda_{En,j,u}}{\mu_{En,j,u}\left(X\right)} = \begin{cases} \alpha_{En,j,u}, & (u = a \wedge n_X \ge 0 \wedge n_X \le C_r + C_v + C_q) \vee (u = e \wedge 0 \le n_X \le C_r), \\ \frac{n_X}{C_r} \alpha_{En,j,u}, & u = e \wedge C_r < n_X \le C_r + C_v, \\ \frac{C_v}{C_r} \alpha_{En,j,u}, & u = e \wedge n_X > C_r + C_v \wedge n_X \le C_r + C_v + C_q \end{cases} \quad (23)$$

and

$$\beta_{Pa,k,u}\left(X\right) = \frac{\Lambda_{Pa,k,u}}{\mu_{Pa,k,u}\left(X\right)} = \begin{cases} \beta_{Pa,k,u}, & (u = a \wedge 0 \le n_X \le C_q) \vee (u = e \wedge 0 \le n_X \le C_r), \\ \frac{n_X}{C_r} \beta_{Pa,k,u}, & u = e \wedge C_r < n_X \le C_v, \\ \frac{C_v}{C_r} \beta_{Pa,k,u}, & u = e \wedge C_v < n_X \le C_q \end{cases} \quad (24)$$

determine the intensity of traffic offered by a single free Engset traffic source of class j compression type u and Pascal k compression type u, respectively.

Since Equations (19)-(21) are not satisfied for elastic traffic classes [36], the expression $c_{d,j,u}\left(X\right) x^*_{d,j,u}\left(X\right)$ will be replaced by a value such as $r_{d,j,u}\left(X\right)$ expressed in AUs, which would satisfy the reversibility condition of the service process. By taking into consideration Equations (19)-(21) we can write:

$$r_{Er,i,u}\left(X\right) = \frac{A_{Er,i,u}\left(X\right) c_{Er,i,u}\left(X\right) P\left(X - 1_{Er,i,u}\right)}{P\left(X\right)}, \quad (25)$$

$$r_{En,j,u}\left(X\right) = \frac{\alpha_{En,j,u}\left(X\right)[N_{En,j,u} - x^*_{En,j,u}\left(X - 1_{En,j,u}\right)]}{P\left(X\right)} \cdot c_{En,j,u}\left(X\right) P\left(X - 1_{En,j,u}\right), \quad (26)$$

$$r_{Pa,k,u}\left(X\right) = \frac{\beta_{Pa,k,u}\left(X\right)[S_{Pa,k,u} + x^*_{Pa,k,u}\left(X - 1_{Pa,k,u}\right)]}{P\left(X\right)} \cdot c_{Pa,k,u}\left(X\right) P\left(X - 1_{Pa,k,u}\right). \quad (27)$$

The independence of call streams offered to the system allows us to add up all $M_{Er,a}$ and $M_{Er,e}$ equations of type (19) for elastic and adaptive Erlang streams, $M_{En,a}$ and $M_{En,e}$ equations of type (20) for adaptive and elastic Engseta streams, and $M_{Pa,a}$ and $M_{Pa,e}$ equations of type (21) elastic streams and adaptive Pascal for microstate X. Thus, we can get:

$$P\left(X\right) \cdot \sum_{u\in\{e,a\}} \sum_{o\in\{Er,En,Pa\}} \sum_{i=1}^{M_{o,u}} r_{o,i,u}\left(X\right) = \sum_{u\in\{e,a\}} \sum_{i=1}^{M_{Er,u}} A_{Er,i,u}\left(X\right) c_{Er,i,u}\left(X\right) P\left(X - 1_{Er,i,u}\right) + \sum_{u\in\{e,a\}} \sum_{j=1}^{M_{En,u}} \{\alpha_{En,j,u}\left(X\right) \cdot \left[N_{En,j,u} - x^*_{En,j,u}\left(X - 1_{En,j,u}\right)\right] \cdot c_{En,j,u}\left(X\right) P\left(X - 1_{En,j,u}\right)\}$$

$$+ \sum_{u\in\{e,a\}} \sum_{k=1}^{M_{Pa,u}} \left\{ \beta_{Pa,k,u}(X) \cdot \left[S_{Pa,k,u} + x^*_{Pa,k,u}\left(X - 1_{Pa,k,u}\right)\right] \cdot c_{Pa,k,u}(X)\, P\left(X - 1_{Pa,k,u}\right)\right\}. \quad (28)$$

The expression in square parenthesis on the left-hand side of Equation (28) determines the total number of serviced calls in microstate X. Therefore, and on the basis of the adopted assumptions, we can write:

$$\sum_{u\in\{e,a\}} \sum_{o\in\{Er,En,Pa\}} \sum_{i=1}^{M_{o,u}} r_{o,i,u}(X) = \begin{cases} n_X, & 0 \le n_X \le C_r, \\ C_r, & C_r < n_X \le C_r + C_v + C_q. \end{cases} \quad (29)$$

By taking into account Equations (28)-(29), the occupancy distribution in the system under investigation at the microstate level will eventually take on the following form:

$$\begin{aligned} P(X) \min\{n_X, C_r\} &= \sum_{u\in\{e,a\}} \sum_{i=1}^{M_{Er,u}} \left\{ A_{Er,i,u} \cdot (X)\, c_{Er,i,u}(X)\, P\left(X - 1_{Er,i,u}\right)\right\} \\ &+ \sum_{u\in\{e,a\}} \sum_{j=1}^{M_{En,u}} \left\{ \alpha_{En,j,u}(X) \cdot \left[N_{En,j,u} - x^*_{En,j,u}\left(X - 1_{En,j,u}\right)\right] \cdot c_{En,j,u}(X)\, P\left(X - 1_{En,j,u}\right)\right\} \\ &+ \sum_{u\in\{e,a\}} \sum_{k=1}^{M_{Pa,u}} \left\{ \beta_{Pa,k,u}(X) \cdot \left[S_{Pa,k,u} + x^*_{Pa,k,u}\left(X - 1_{Pa,k,u}\right)\right] \cdot c_{Pa,k,u}(X)\, P\left(X - 1_{Pa,k,u}\right)\right\}, \end{aligned} \quad (30)$$

for $0 \le n_X \le C_r + C_v + C_q$, for other cases $P(X) = 0$, while the value of the probability $P(0)$ results from the normative condition.

B. ANALYSIS OF SERVICE PROCESS AT THE MACROSTATE LEVEL

The probability of macrostate $P(n)$ determines the occupancy probability of n AUs and can be expressed as the sum of the probabilities of particular (corresponding) microstates:

$$P(n) = \sum_{\Omega(n)} P(X), \quad (31)$$

where $\Omega(n)$ is a macrostate, that is, a set of all microstates X that satisfy the following condition (5).

Note that for all microstates X that belong to the macrostate $\Omega(n)$, the traffic characteristics of offered traffic take on identical values. This means that for each macrostate $\Omega(n)$, where $0 \le n \le C_r + C_v + C_q$, Equations (9)-(12) take on the following form:

$$\mu_{d,i,e}(X) = \mu_{d,i,e}(n) = \begin{cases} \mu_{d,i,e}, & 0 \le n \le C_r, \\ \frac{C_r}{n}\mu_{d,i,e}, & C_r < n \le C_r + C_v, \\ \frac{C_r}{C_v}\mu_{d,i,e}, & C_r + C_v < n \le C_r + C_v + C_q, \end{cases} \quad (32)$$

$$c_{d,i,e}(X) = c_{d,i,e}(n) = \begin{cases} c_{d,i,e}, & 0 \le n \le C_r, \\ \frac{C_r}{n}c_{d,i,e}, & C_r < n \le C_r + C_v, \\ \frac{C_r}{C_v}c_{d,i,e}, & C_r + C_v < n \le C_r + C_v + C_q, \end{cases} \quad (33)$$

and

$$\mu_{d,i,a}(n) = \mu_{d,i,e}, \quad 0 \le n \le C_q, \quad (34)$$

$$c_{d,i,a}(n) = \begin{cases} c_{d,i,a}, & 0 \le n \le C_r, \\ \frac{C_r}{n}c_{d,i,a}, & C_r < n \le C_r + C_v, \\ \frac{C_r}{C_v}c_{d,i,a}, & C_r + C_v < n \le C_r + C_v + C_q, \end{cases} \quad (35)$$

where $d \in \{Er, En.Pa\}$.

Hence, and on the basis of Formulae (22)-(24), we can write:

$$A_{Er,i,u}(X) = A_{Er,i,u}(n) = \begin{cases} A_{Er,i,u}, & (u = a \wedge 0 \le n \le C_r + C_v + C_q) \vee (u = e \wedge 0 \le n \le C_r), \\ \frac{n}{C_r}A_{Er,i,u}, & u = e \wedge C_r < n \le C_r + C_v, \\ \frac{C_v}{C_r}A_{Er,i,u}, & (u = e \wedge n > C_r + C_v \wedge n \le C_r + C_v + C_q), \end{cases} \quad (36)$$

$$\alpha_{En,j,u}(X) = \alpha_{En,j,u}(n) = \begin{cases} \alpha_{En,j,u}, & (u = a \wedge 0 \le n \le C_r + C_v + C_q) \vee (u = e \wedge 0 \le n \le C_r), \\ \frac{n}{C_r}\alpha_{En,j,u}, & u = e \wedge C_r < n \le C_r + C_v, \\ \frac{C_v}{C_r}\alpha_{En,j,u}, & (u = e \wedge C_r + C_v < n \le C_r + C_v + C_q, \end{cases} \quad (37)$$

and

$$\beta_{Pa,k,u}(X) = \beta_{Pa,k,u}(n) = \begin{cases} \beta_{Pa,k,u}, & (u = a \wedge 0 \leq n \leq C_r + C_v + C_q) \vee (u = e \wedge 0 \leq n \leq C_r), \\ \frac{n}{C_r}\beta_{Pa,k,u}, & u = e \wedge C_r < n \leq C_r + C_v, \\ \frac{C_v}{C_r}\beta_{Pa,k,u}, & (u = e) \wedge C_r + C_v < n \leq C_r + C_v + C_q). \end{cases} \tag{38}$$

The assumption of reversibility of the service process at the microstate level adopted in the study allows us to sum up on both sides the Equations (30) for all microstates that belong to the macrostate $\Omega(n)$. Taking therefore the definition of macrostate probability (31) and Formula (30) into consideration, the probability of an event in which the system is in the occupancy state n AUs can be written as follows:

$$\begin{aligned} \min\{n, C_r\} \sum_{\Omega(n)} P(X) &= \sum_{\Omega(n)} \Bigg(\sum_{u\in\{e,a\}} \sum_{i=1}^{M_{Er,u}} A_{Er,i,u}(X)\, c_{Er,i,u}(X) \cdot P(X - 1_{Er,i,u}) \\ &+ \sum_{u\in\{e,a\}} \sum_{j=1}^{M_{En,u}} \{\alpha_{En,j,u}(X) \cdot [N_{En,j,u} - x^*_{En,j,u}(X - 1_{En,j,u})] \cdot c_{En,j,u}(X)\, P(X - 1_{En,j,u})\} \\ &+ \sum_{u\in\{e,a\}} \sum_{k=1}^{M_{Pa,u}} \{\beta_{Pa,k,u}(X) \cdot [S_{Pa,k,u} + x^*_{Pa,k,u}(X - 1_{Pa,k,u})] \cdot c_{Pa,k,u}(X)\, P(X - 1_{Pa,k,u})\} \Bigg), \end{aligned} \tag{39}$$

The average number of serviced calls of traffic $d \in \{Er, En, Pa\}$ of class j compression type $u \in \{e, a\}$ in the occupancy state n AUs can be defined as follows:

$$y_{d,j,u}(n) = \sum_{X\in\Omega(n)} \frac{x^*_{d,j,u}(X)\, P(X)}{P(n)}. \tag{40}$$

To simplify the notation, we will introduce the following notation for the intensity of traffic offered by the Engset and Pascal elastic and adaptive call classes:

$$A_{En,j,u}(n) = \alpha_{En,j,u}(n) \cdot [N_{En,j,u} - y_{En,j,u}(n - c_{En,j,u})], \tag{41}$$

$$A_{Pa,k,u}(n) = \beta_{Pa,k,u}(n) \cdot [S_{Pa,k,u} + y_{Pa,k,u}(n - c_{Pa,k,u})]. \tag{42}$$

At this point, we need another approximation. This time the numbers of serviced calls of Engset and Pascal traffic taken into account in (39) will be replaced by their average values described by Formula (40). For this purpose, in Equation (39) we multiply and divide the components of the sums that correspond to the Engset and Pascal traffic classes by the corresponding macrostate probabilities ($\sum_{X\in\Omega(n)} P(X - 1_{d,j,u})$). Then, sequentially and by taking into consideration the definition of the average number of serviced calls in the occupancy state n AUs (Formula (40)) as well as the introduced notation (Formulae (41)-(42)), the occupancy distribution in the multiservice with BPP elastic and adaptive traffic (39) will eventually be rewritten in the following form:

if $0 \leq n \leq C_r$ then

$$\begin{aligned} nP(n) &= \sum_{u\in\{e,a\}} \sum_{i=1}^{M_{Er,u}} A_{Er,i,u} c_{Er,i,u} P(n - c_{Er,i,u}) \\ &+ \sum_{u\in\{e,a\}} \sum_{j=1}^{M_{En,u}} A_{En,j,u}(n)\, c_{En,j,u} \cdot P(n - c_{En,j,u}) \\ &+ \sum_{u\in\{e,a\}} \sum_{k=1}^{M_{Pa,u}} A_{Pa,k,u}(n)\, c_{Pa,k,u} \cdot P(n - c_{Pa,k,u}) \end{aligned} \tag{43}$$

if $C_r < n \leq C_r + C_v$ then

$$\begin{aligned} C_r P(n) &= \sum_{i=1}^{M_{Er,e}} A_{Er,i,e} c_{Er,i,e} P(n - c_{Er,i,e}) \\ &+ \sum_{i=1}^{M_{Er,a}} \frac{C_r}{n} A_{Er,i,a} c_{Er,i,a} P(n - c_{Er,i,a}) \\ &+ \sum_{u\in\{e,a\}} \sum_{j=1}^{M_{En,u}} \frac{C_r}{n} A_{En,j,u}(n)\, c_{En,j,u} \cdot P(n - c_{En,j,u}) \\ &+ \sum_{u\in\{e,a\}} \sum_{k=1}^{M_{Pa,u}} \frac{C_r}{n} A_{Pa,k,u}(n)\, c_{Pa,k,u} \cdot P(n - c_{Pa,k,u}) \end{aligned} \tag{44}$$

finally, if $C_r + C_v < n \leq C_r + C_v + C_q$ then

$$\begin{aligned} C_r P(n) &= \sum_{i=1}^{M_{Er,e}} A_{Er,i,e} c_{Er,i,e} P(n - c_{Er,i,e}) \\ &+ \sum_{i=1}^{M_{Er,a}} \frac{C_r}{C_v} A_{Er,i,a} c_{Er,i,a} P(n - c_{Er,i,a}) \end{aligned}$$

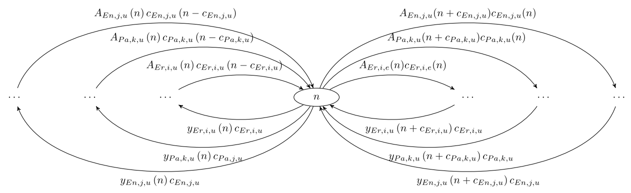

FIGURE 3. A fragment of one-dimensional Markov process in the multiservice system with BPP elastic and adaptive traffic.

$$+\sum_{u\in\{e,a\}}\sum_{j=1}^{M_{En,u}}\frac{C_r}{C_v}A_{En,j,u}(n)\,c_{En,j,u}\cdot P\left(n-c_{En,j,u}\right)+\sum_{u\in\{e,a\}}\sum_{k=1}^{M_{Pa,u}}\frac{C_r}{C_v}A_{Pa,k,u}(n)\,c_{Pa,k,u}\cdot P\left(n-c_{Pa,k,u}\right) \quad (45)$$

The average number of serviced calls in the individual occupancy states of the system can be determined using the call service process that takes place in the system at the macrostate level. Let us consider then the fragment of the Markov chain shown in Figure 3, which is a graphic interpretation of the service process in the system under consideration. Similarly as at the microstate level, the assumption at the macrostate level is that the local balance equations for all classes of elastic and adaptive calls offered to the system are satisfied. This means that the following equation is true

$$y_{d,i,u}(n)\,P(n)=A_{d,i,u}(n)\,P\left(n-c_{d,i,u}\right), \quad (46)$$

for $d\in\{Er, En, Pa\}$, $u\in\{e,a\}$. Hence

$$y_{d,i,u}(n)=\frac{A_{d,i,u}(n)\,P\left(n-c_{d,i,u}\right)}{P(n)}, \quad (47)$$

where $d\in\{Er, En, Pa\}$, $u\in\{e,a\}$, for $n-c_{d,i,u}\geq 0$. In the remaining cases, the assumption is $y_{d,i,u}(n)=0$.

Note that to determine the occupancy distribution it is necessary to determine the average number of serviced calls in the system. In turn, to determine the average number of serviced calls, it is necessary to first determine the occupancy state. Therefore, to determine the numerical values for the state probabilities, an appropriate modification of the iterative MIM-BPP method, presented in [37], is used. The change, as opposed to the MIM-BPP method, is based on the fact that now the call compression mechanism is taken into consideration. The values of the probabilities $P(n)$ and the average number of serviced calls of traffic of type d class w compression type u in the macrostate n are the values determined in the last step of the iteration of the MIM-BPP method [37]. These values are used to determine the queuing characteristics of the system. The block diagram of the modification of the MIM-BPP method adapted to determine the occupancy distribution in the considered system is presented in Figure 4.

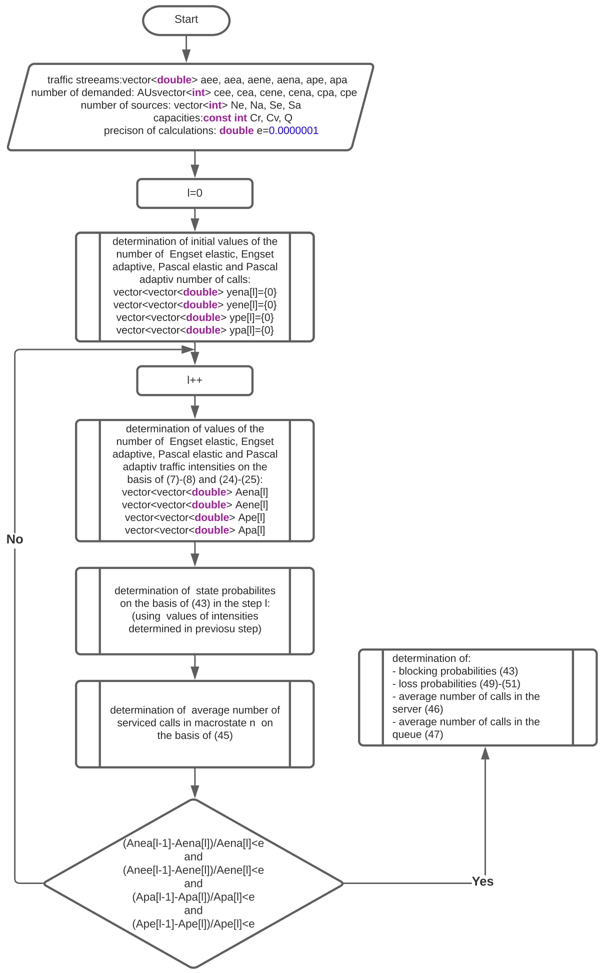

FIGURE 4. Block diagram of modification of MIM-BPP method with stream, elastic and adaptive traffic.

III. CHARACTERISTIC PARAMETERS OF THE SYSTEM

With the knowledge of the occupancy distribution of the system, we can determine the queueing characteristics of the system.

A. THE AVERAGE NUMBER OF BUSY AUs IN THE SYSTEM

The average number of busy E_C AUs can be determined as follows:

$$E_C=\sum_{n=0}^{C_r+C_v} nP(n)+\sum_{n=C_r+C_v+1}^{C_r+C_v+C_q}(C_r+C_v)P(n). \quad (48)$$

The second sum in Formula (48) describes the states in which the resources of the system are completely occupied and part of calls are waiting in the queue.

B. THE AVERAGE QUEUE LENGTH

The random variable that describes the queue length takes on the value 0 for all states n such that $0\leq n\leq C_r+C_v$, and respectively the values $n-C_r-C_v$ for $n=C_r+C_v+1,\ldots,C_r+C_v+C_q$ with the probabilities resulting from the distribution allow us to determine the average queue length without any division into call classes and in accordance with the following formula:

$$Q = \sum_{n=C_r+C_v+1}^{C_r+C_v+C_q} (n - C_r - C_v)P(n). \tag{49}$$

C. BLOCKING AND LOSS PROBABILITY

The blocking probability $E_{d,i,u}$ for calls of type $d \in \{Er, En, Pa\}$ class i compression type $u \in \{e, a\}$ in the considered system can be determined by blocking states, i.e., those states in which admission of a new call is not possible due to the lack of free AUs in the queue:

$$E_{d,i,u} = \sum_{n=C_r+C_v+C_q-c_{d,i,u}+1}^{C_r+C_v+C_q} P(n). \tag{50}$$

The loss probability is defined as the ratio between the average number of lost calls and the offered calls.

In the case of Erlanga calls, the loss probability is equal to the blocking probability:

$$B_{Er,d,u} = E_{Er,d,u}. \tag{51}$$

The number of Engset and Pascal traffic calls offered in macrostate n depends directly on the number of serviced calls in this macrostate. Hence the loss probability for elastic and adaptive calls of Engset and Pascal calls can be written in the following way:

$$E_{En,i,u} = \frac{\sum_{n=C-c_{En,i,u}+1}^{C} \Lambda_{En,i,u}[N_{En,i,u} - y_{En,i,u}(n)]P(n)}{\sum_{n=0}^{C} \Lambda_{En,i,u}[N_{En,i,u} - y_{En,i,u}(n)]P(n)} \tag{52}$$

$$E_{Pa,i,u} = \frac{\sum_{n=C-c_{Pa,i,u}+1}^{C} \Lambda_{Pa,w,u}[N_{Pa,i,u} - y_{Pa,i,u}(n)]P(n)}{\sum_{n=0}^{C} \Lambda_{Pa,i,u}[N_{Pa,i,u} - y_{Pa,i,u}(n)]P(n)}, \tag{53}$$

where $C = C_r + C_v + C_q$.

IV. NUMERICAL RESULTS

To verify the accuracy and correctness of the proposed model of a queuing system with Erlang, Engset, and Pascal elastic and adaptive traffic, the results of modelling were compared with the results of simulation experiments. For this particular purpose, a dedicated simulator of the considered system developed in the C++ language was prepared. The simulator used the event scheduling method. Each experiment consisted of 8 series of simulations, each with at least 500,000 calls of this class that required the highest number of AUs. The results of the simulation are presented in the graphs in the form of plotted points with a confidence interval of 95%, calculated after the t-Student distribution. The results obtained are presented in the function of offered traffic per a single AU of the system:

$$a = \frac{\sum_{d\in\{Er,En,Pa\}} \sum_{u\in\{a,e\}} \sum_{i=1}^{M_{d,u}} A_{d,i,u}}{C_r}. \tag{54}$$

Yet another assumption was that the system was offered a mixture of different traffic classes in the following proportion:

$$a_{d,1,u}c_{d,1,u} : a_{d,2,u}c_{d,2,u} : \cdots : a_{d,M,u}c_{d,M,u} = 1 : 1 : \cdots : 1, \tag{55}$$

where $M_{d,u}$ denotes the number of traffic classes offered to the system.

Figures 5, 8, 11 show the sequence of the results of the analytical and simulation modelling of the loss probability in the three systems considered for each of the traffic classes. The parameters of the individual systems are presented in Table 1-3. Figures 6, 9, 12 show the results of the modelling of the average number of busy resources, while Figures 7, 10, 13 show the results of the modelling of the average queue length.

The analytical model proposed in this article is an approximate model, which results from the assumed reversibility of the call service process. However, the presented results

TABLE 1. Parameters of the system I.

Capacity	Real	Virtual	Queue	Total
-	10	10	5	25
Number of class (i)	Demands (c)	Traffic Type	Kind of traffic	Number of sources (N)
1	1	elastic	Erlang	-
2	2	adaptive	Erlang	-
3	2	elastic	Engset	40
4	3	adaptive	Engset	40
5	2	elastic	Pascal	70
6	1	adaptive	Pascal	70

TABLE 2. Parameters of the system II.

Capacity	Real	Virtual	Queue	Total
-	15	8	4	27
Number of class (i)	Demands (c)	Traffic Type	Kind of traffic	Number of sources (N)
1	2	adaptive	Erlang	-
2	2	elastic	Engset	90
3	3	elastic	Pascal	25
4	1	adaptive	Pascal	25

TABLE 3. Parameters of the system III.

Capacity	Real	Virtual	Queue	Total
-	10	6	5	21
Number of class (i)	Demands (c)	Traffic Type	Kind of traffic	Number of sources (N)
1	2	elastic	Erlang	-
2	1	adaptive	Engset	50
3	3	adaptive	Pascal	20

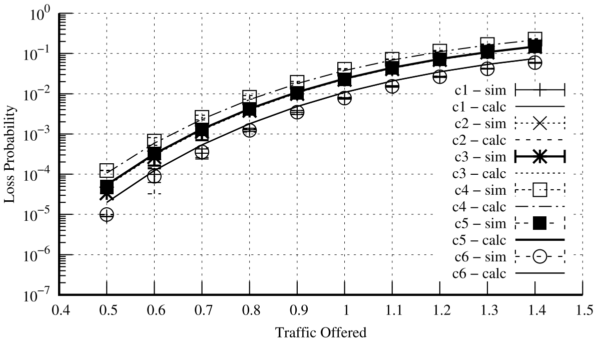

FIGURE 5. Loss probability - System 1.

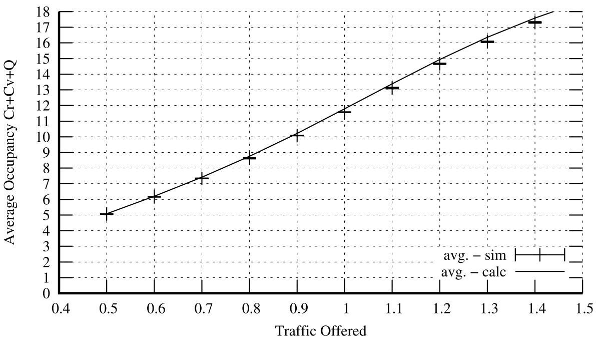

FIGURE 6. Average occupancy of system - System 1.

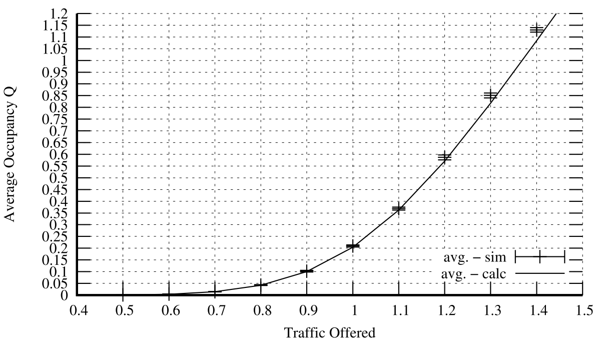

FIGURE 7. Average occupancy of queue - System 1.

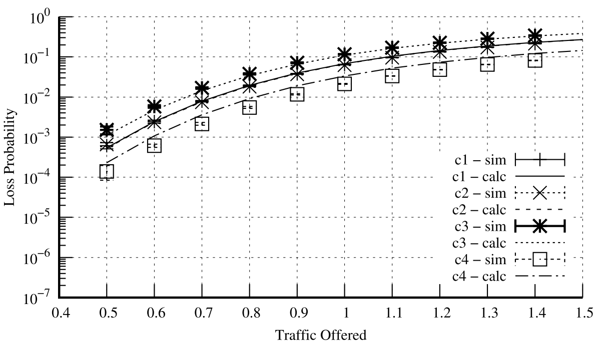

FIGURE 8. Loss probability - System 2.

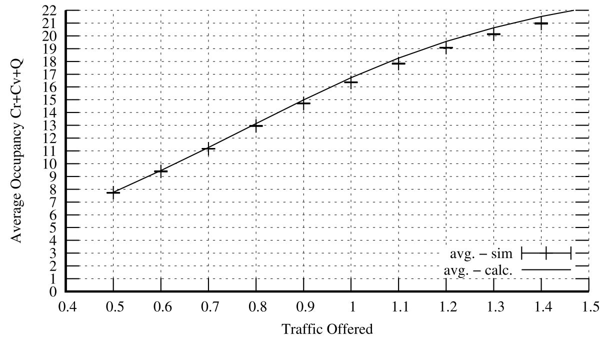

FIGURE 9. Average occupancy of system - System 2.

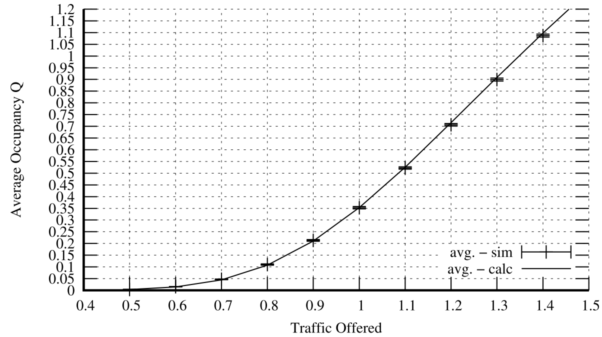

FIGURE 10. Average occupancy of queue - System 2.

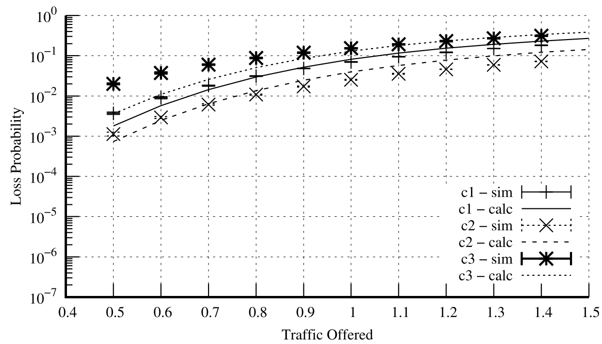

FIGURE 11. Loss probability - System 3.

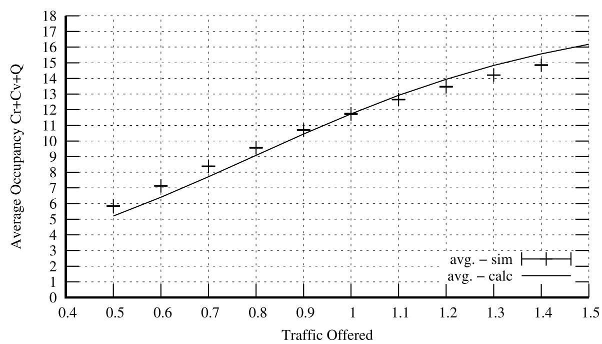

FIGURE 12. Average occupancy of system - System 3.

clearly indicate that the model approximates queuing systems with a mixture of Erlang, Engset, Engset, and Pascal elastic and adaptive traffic very well. The accuracy of the loss coefficient for individual call classes determined using the

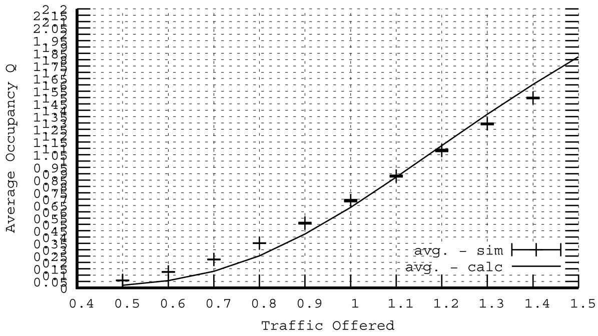

FIGURE 13. **Average occupancy of queue - System 3.**

model depends neither on their demands nor on the traffic offered to the system. In addition, the accuracy of the model is not influenced by offered mixtures of traffic, i.e., the number of Erlang, Engset, and Pascal traffic classes, regardless of their type - elastic or adaptive (Fig. 5). On the basis of the occupancy distribution, analytically determined, it is possible to determine the average number of busy AUs in both the system and the queue (Figs. 5, 6, 8, 9, 11, and 12). The results obtained in this way are also characterised by fair accuracy.

V. SUMMARY

In this article, an analytical model of a queueing FIFO system that services multiservice Erlang, Engset, and Pascal elastic and adaptive traffic is presented. The model allows the quality parameters of the system, such as the loss probability for individual call classes and the average number of busy AUs in the queue and in all of the system, to be determined. The accuracy of the model is verified on the basis of a simulation study that additionally shows that the accuracy of the model is independent of both the type of offered traffic (mixture of Erlang, Engset, and Pascal elastic and adaptive traffic) and its intensity. This model can in the future be applied to analyse and optimise telecommunications and computer systems that would offer modern services that involve dynamic changes in bitrates.

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