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RESEARCH ARTICLE

Robust Reset Controller Design for Switched Nonlinear Uncertain Systems

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ABSTRACT This paper presents a design of a robust switching reset controller for a class of nonlinear uncertain switched systems. We consider the norm-bounded time-varying parameter uncertainties in switched nonlinear systems obeyed by the average dwell-time switching signal. The proposed switching reset controller uses the measured output in resetting the controller's states, whereas the previous studies did not. A weighted mixed $\mathcal{L}_2/l_2$-gain is introduced to take into account the discrete disturbances induced by the measured output when resetting the controller's states. The proposed reset controller and switched nonlinear uncertain plant form a closed-loop system that is a class of nonlinear impulsive switched uncertain systems. Hence, we first provide sufficient conditions for the $\mathcal{L}_2$ stability of the nonlinear impulsive switched uncertain systems. Based on the conditions, we propose linear matrix inequality (LMI)-based design conditions to choose the dynamic output feedback control and output feedback reset laws guaranteeing the weighted mixed $\mathcal{L}_2/l_2$-gain performance of the controlled systems with continuous and discrete disturbances. Numerical examples demonstrate the effectiveness of the proposed method.

INDEX TERMS Impulsive switched systems, $\mathcal{L}_2$-stability, linear matrix inequality, nonlinear uncertain systems, reset control systems.

I. INTRODUCTION

Switched systems, which are typically described by a family of subsystems and a switching signal governing the switching logic, have attracted much attention over the past decades due to their wide applications in communication networks, electrical systems, machine power management, and aircraft control, see [1], [2], [3], [4], [5], and [6]. Several interesting research topics are represented in the switched systems such as stability analysis and the design of controller and switching signal. For example, in [7], a class of switching signals based on the state estimates was composed by using multiple Lyapunov functions, and then the design of feedback nonlinear controllers was proposed. In [8], the Karush–Kuhn–Tucker condition was used to present a necessary and sufficient quadratic stability condition of switched nonlinear systems. [9] proposed a stabilization method of switched systems under bounded additive disturbance using a quasi-time-varying Lyapunov functional. In [10], the robust $\mathcal{L}_2$-stability and stabilization of discrete-time switched linear systems were presented using a set of convex lifted conditions with minimum dwell-time switching property. In [11], the adaptive finite-time tracking control problem was investigated for a class of switched nonlinear uncertain systems based on the backstepping control.

Many studies have been conducted to consider practical systems' properties. For example, output feedback control approaches have been presented for the switched systems as only partial state is measurable in many real-world control applications due to various constraints such as cost of sensor installation and collection, hardware limitations, lack of measurement, and so on. In [12], network-delay-dependent switching controllers are designed for a class of systems over asymmetric path delay on arbitrary

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communication networks. [13] addressed the stability analysis and control design of switched linear systems based on Lyapunov–Metzler inequalities. [14] addressed the output tracking control for switched linear systems with time delay using dynamic output feedback control and state-dependent switching. In [15], the observer-based controller design problem for a class of switched systems with a semi-Markov random switching signal was investigated. However, many studies on switched systems mentioned above are limited to linear systems or a class of systems that do not take into account uncertainty. To overcome these limitations, in [16], nonlinear switched systems were described using the T-S fuzzy approach and proposed observer-based nonlinear output feedback controllers. [17] proposed a fault detection observer for switched systems with the Lipschitz nonlinearity and average dwell time (ADT) switching. In [18], an adaptive dynamic programming approach was proposed for the linear quadratic optimal control of discrete-time switched nonlinear systems under arbitrary switching laws. Considering uncertainties, in [19], an adaptive control approach was proposed for switched uncertain nonlinear systems. [20] proposed a linear matrix inequality (LMI)-based state-feedback controller synthesis for switched uncertain systems under asynchronous switching.

Meanwhile, the reset controller, which is a type of hybrid dynamical system that its states jump to specified values based on the predetermined reset laws, was firstly proposed by Clegg [21]. Since the introduction of the control methodology, many studies have been presented for transient performance improvement with its impulsive behaviors [22], [23], [24]. [25] provided stability analysis of time-delay reset systems. In [26], a generalized first-order reset element was utilized to cross the theoretical limitation of improving overshoot and reaching-time performances. In [27] and [28], the reset strategy was applied to unknown input observers to estimate the states and faults for linear systems or in a class of nonlinear uncertain systems. [29] proposed an adaptive reset observer design scheme for states and actuator fault estimation in a class of nonlinear time-varying delayed models. In [30], the reset control approach which consists of a proportional-integral (PI) controller with a Clegg integrator (CI) was utilized to synchronize multi-agent systems.

However, few studies have investigated the reset control for the switched systems. As the switched systems suffer sudden changes in system parameters, the controlled output can undergo impulsive behaviors and poor transient performance after switching instants. Adopting the reset control strategy can be a good choice to overcome these undesirable transient behaviors. In [31], the design methodology of state-feedback reset control law was proposed for switched linear parameter varying systems. In [32], LMI-based reset controller design method was proposed for switched linear systems under the ADT. [33] proposed a fault detection observer-based reset control strategy for switched linear systems under sensor and actuator faults. Recently, [34] addressed a reset control-based stabilization for fractional order switched linear systems under the ADT. However, the previous studies on reset control assumed that the system parameters were perfectly known. Furthermore, in many previous studies, linear switched systems were taken into account without consideration of the nonlinearity of the plant model. Nevertheless, as we mentioned above, many practical systems are described as nonlinear systems. In addition, parameter uncertainties in the system model can deteriorate the static and dynamic performances, if not considered in the controller design, see [35], [36], [37], and [38]. Finally, the feedback on the reset law has not been applied in the previous studies. Namely, the dynamic controller's states were only used for resetting the controller's states without any information on the measured outputs.

Motivated by the above discussions, this study presents a robust switching reset controller design strategy for uncertain nonlinear switched systems with the ADT switching property. We define the weighted mixed $\mathcal{L}_2/l_2$-gain to consider the influence of discrete disturbances at impulse instants. Considering nonlinear impulsive switched systems with norm-bounded time-varying parameter uncertainties, the LMI-based stability analysis guaranteeing the weighted $\mathcal{L}_2/l_2$-gain performance was presented. Moreover, we propose an LMI-based design conditions for the dynamic output feedback control and output feedback reset laws. It is worth mentioning that it is difficult to design controller parameters for complex reset-controlled switched systems. Furthermore, there is no analytical methodology to design a robust reset controller for nonlinear switched systems with time-varying uncertainty due to the high system complexity. Therefore, the use of LMI inequalities is a reasonable approach for designing a robust reset controller for nonlinear uncertain switched systems as LMI constraints are convex constraints, meaning that the controller parameters can be effectively obtained in polynomial time using numerical optimization methods such as primal-dual interior point methods. Finally, a convex optimization problem is presented using the proposed LMI-based design condition, enabling the $\mathcal{L}_2$ gain to be minimized. Hence, the reset controller can be designed to reduce influence of the exogenous disturbances. We present numerical examples to show the effectiveness of the proposed design methodology. The main contributions of this study are summarized as follows:

1) This study addresses the simultaneous design methodology of the switched dynamic output feedback control and output feedback reset law for nonlinear uncertain switched systems.
2) The weighted mixed $\mathcal{L}_2/l_2$-gain is addressed to consider not only the continuous-time disturbance but also discrete-time disturbances induced by output feedback reset laws when switching occurs.
3) We provide LMI-based controller design conditions for the switched nonlinear systems in the presence of time-varying norm-bounded uncertainties.
4) An optimization problem using the proposed LMI conditions is presented to optimize the proposed controller, reducing the influence of the exogenous disturbances.

TABLE 1. Qualitative comparison with the existing studies.

Ref	System type	Model Uncertainty	Reset method	Controller design
Proposed	NS	NU	OFR	LMI
[13], [14]	LS	NC	NC	NC
[28]	NS	CU	NC	NC
[31]	LPV	NC	CSF	LMI
[32]	LS	NC	CSF	LMI
[34]	LS	NU	CSF	LMI

* NS: nonlinear system, LS: linear system, LPV: linear parameter varying system, CSF: controller state feedback, NU, norm-bounded uncertainty, CU: constant uncertainty, OFR: output feedback resetting, LMI, linear matrix inequality, NC: not considered.

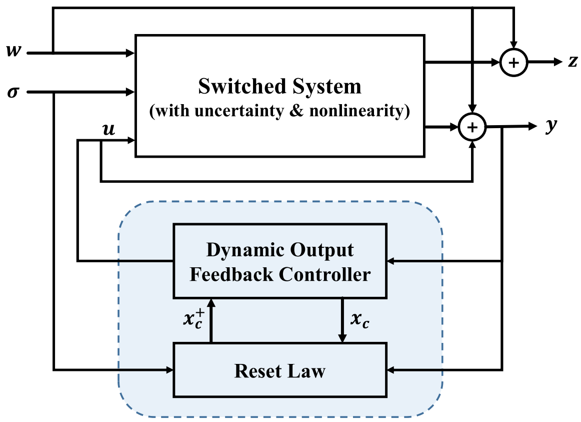

FIGURE 1. Schematic diagram of the switched system with the proposed controller.

The numerical examples show the effectiveness of the proposed controller in the ADT switching scenario.

Finally, based on the above-mentioned discussions, Table 1 presents the comparison of characteristics between our study and the existing ones.

The rest of this paper is organized as follows. Section II presents the representation of the nonlinear uncertain switched plants and switching reset controllers. Section III provides the LMI-based $\mathcal{L}_2$ stability analysis for switched impulsive nonlinear uncertain systems. Subsequently, we present the LMI-based design methodology of the switching reset controller for switched nonlinear uncertain systems. The numerical simulations are presented in Section IV. Finally, the conclusions are made in Section V.

Notation: Throughout this paper, $\mathbb{R}$ denotes for the set of real numbers and $\mathbb{Z}^+$ for the positive integer numbers. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ stand for the set of real n-dimensional vectors and real $n\times m$ matrices, respectively. $\mathcal{L}_2\,[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$, while $l_2\,[0,\infty)$ is the space of square-summable vector sequences over $[0,\infty)$. $\|\cdot\|_{\mathcal{L}_2}$ stands for the $\mathcal{L}_2\,[0,\infty)$ norm over $[0,\infty)$, $\|\cdot\|_{l_2}$ denotes the $l_2\,[0,\infty)$ norm over $[0,\infty)$. The Hermitian operator $He\{\cdot\}$ represents that $He\{A\} = A + A^T$ for real matrices. $diag\{\cdots\}$ stands for a block-diagonal matrix, the symbol $\star$ in LMIs is used to denote a term that is induced by symmetry.

II. PROBLEM STATEMENT

We consider the following uncertain nonlinear switched system:

$$\dot{x}_p = A_\sigma x_p + B_{f,\sigma} f_\sigma(x_p) + B_{w,\sigma} w + B_{u,\sigma} u \tag{1a}$$

$$y = C_{y,\sigma} x_p + D_{yw,\sigma} w + D_{yu,\sigma} u, \tag{1b}$$

$$z = C_{z,\sigma} x_p + D_{zw,\sigma} w + D_{zu,\sigma} u \tag{1c}$$

where $x_p \in \mathbb{R}^{n_x}$ is the plant state, $u \in \mathbb{R}^{n_u}$ is the control input, $y \in \mathbb{R}^{n_y}$ is the measurement output, $z \in \mathbb{R}^{n_z}$ is the controlled output, and $w \in \mathbb{R}^{n_w}$ is the disturbance. We consider a plant that includes model uncertainty. A_σ is represented by the sum of the nominal plant matrix $A_{0,\sigma}$ and the time-varying parametric uncertainty $\Delta A_\sigma(t)$ that is represented as $E_{\Delta,\sigma}\Delta_\sigma(t)F_{\Delta,\sigma}$, i.e., $A_\sigma = A_{0,\sigma} + E_{\Delta,\sigma}\Delta F_{\Delta,\sigma}$ with $\Delta_\sigma^T\Delta_\sigma \leq I$. $f_\sigma(x_p) \in \mathbb{R}^{n_f}$ is a nonlinear function where $f_\sigma(0) = 0$ and $\sigma(t) : \mathbb{R}^+ \rightarrow \Sigma$, $\Sigma := \{1,\ldots,M\}$ is a switching signal that is a piecewise constant function of time and $M \in \mathbb{Z}^+$ is the number of subsystems. Given a switching time sequence $0 < t_1 < t_2 < \cdots$, σ is continuous from the right everywhere and obeys the ADT switching logic [39]. When $t \in [t_k, t_{k+1})$, the $\sigma(t_k)$th subsystem is activated and thus the trajectory $x_p(t)$ of the switched system (1) is the trajectory of the $\sigma(t_k)$th subsystem. We present the following assumptions for the uncertain nonlinear switched system.

Assumption 1: The system (1) is stabilizable and detectable for all $i \in \Sigma$.

Assumption 2: $D_{yu,i} = 0$ in the system (1) for all $i \in \Sigma$.

Assumption 3: The smooth nonlinear function $f_\sigma(x_p)$ in the system (1) satisfied the following Lipschitz condition

$$\|f_\sigma(x) - f_\sigma(y)\| \leq \beta\|x - y\|, \quad \forall x, y \in \mathbb{R}^n$$

where $\beta > 0$ is the Lipschitz constant.

Remark 1: It should be noted that the first assumption guarantees the existence of a dynamic output-feedback controller to stabilize each subsystem of the switched system and the second one can be relaxed by loop transformation, see [40]. The Lipschitz condition described in Assumption 3 is often used because of the advantages of solving problems using LMI. In addition, the Lipschitz condition for nonlinearity has been applied to various systems such as inverted pendulums, chaotic systems, and power systems. It is also reasonable to assume the local Lipschitz condition in the specified interval that a plant operates.

The proposed switching reset control scheme, which consists of a dynamic output-feedback switching controller and a reset law, is represented as follows:

$$\begin{bmatrix} \dot{x}_c \\ u \end{bmatrix} = \begin{bmatrix} A_{c,\sigma} & B_{c,\sigma} \\ C_{c,\sigma} & D_{c,\sigma} \end{bmatrix} \begin{bmatrix} x_c \\ y \end{bmatrix}, \quad t \neq t_k \tag{2a}$$

$$x_c^+ = R_{1,ij}y + R_{2,ij}x_c, \qquad t = t_k \tag{2b}$$

where $x_c \in \mathbb{R}^{n_c}$ is the controller state, the subscript of the reset matrices $R_{1,ij}$ and $R_{2,ij}$ are used to denote the indices of the pre-switching subsystem i and the post-switching subsystem j, i.e., at the switching instant, we have $\sigma = i$, $\sigma^+ = j$.

From the nonlinear uncertain system (1) and controller (2), we obtain the following closed-loop system formed as a class of switched nonlinear impulsive systems:

$$\begin{bmatrix}\dot{\bar{x}}\\ z\end{bmatrix} = \begin{bmatrix}\bar{A}_\sigma & \bar{B}_\sigma\\ \bar{C}_\sigma & \bar{D}_\sigma\end{bmatrix}\begin{bmatrix}\bar{x}\\ w\end{bmatrix} + \begin{bmatrix}\bar{B}_{f,\sigma}\\ 0\end{bmatrix} f_\sigma(x_p), \quad t \neq t_k \tag{3a}$$

$$\bar{x}^+ = \bar{R}_{1,ij}\bar{x} + \bar{R}_{2,ij}w, \quad t = t_k \tag{3b}$$

where $\bar{x} = [x_p^T, x_c^T]^T$ and

$$\begin{aligned}
\bar{A}_\sigma &= \bar{A}_{0,\sigma} + \bar{E}_{\Delta,\sigma}\Delta_\sigma \bar{F}_{\Delta,\sigma},\\
\bar{A}_{0,\sigma} &= \begin{bmatrix}A_{0,\sigma} + B_{u,\sigma}D_{c,\sigma}C_{y,\sigma} & B_{u,\sigma}C_{c,\sigma}\\ B_{c,\sigma}C_{y,\sigma} & A_{c,\sigma}\end{bmatrix},\\
\bar{E}_{\Delta,\sigma} &= \begin{bmatrix}E_{\Delta,\sigma}\\ 0\end{bmatrix}, \quad \bar{F}_{\Delta,\sigma} = \begin{bmatrix}F_{\Delta,\sigma} & 0\end{bmatrix},\\
\bar{B}_{f,\sigma} &= \begin{bmatrix}B_{f,\sigma}\\ 0\end{bmatrix}, \quad \bar{B}_\sigma = \begin{bmatrix}B_{w,\sigma} + B_{u,\sigma}D_{c,\sigma}D_{yw,\sigma}\\ B_{c,\sigma}D_{yw,\sigma}\end{bmatrix},\\
\bar{C}_\sigma &= \begin{bmatrix}C_{z,\sigma} + D_{zu,\sigma}D_{c,\sigma}C_{y,\sigma} & D_{zu,\sigma}C_{c,\sigma}\end{bmatrix},\\
\bar{D}_\sigma &= \begin{bmatrix}D_{zw,\sigma} + D_{zu,\sigma}D_{c,\sigma}D_{yw,\sigma}\end{bmatrix},\\
\bar{R}_{1,ij} &= \begin{bmatrix}I & 0\\ R_{1,ij}C_{y,i} & R_{2,ij}\end{bmatrix}, \bar{R}_{2,ij} = \begin{bmatrix}0\\ R_{1,ij}D_{yw,i}\end{bmatrix}.
\end{aligned}$$

Note that there exist both continuous and discrete disturbances in the closed-loop system (3). To take into consideration of the influence of the continuous and discrete disturbances together with the ADT, for a given scalar $0 < \lambda < 1$, we define the following weighted mixed $\mathcal{L}_2/l_2$-gain:

$$\int_0^\infty e^{-\lambda t} z^T(t)z(t)dt \leq \gamma^2 \left\{\|w(t)\|^2_{\mathcal{L}_2} + \|w(t_k)\|^2_{l_2}\right\} \tag{4}$$

where the disturbance attenuation level $\gamma > 0$.

Remark 2: We introduced a weighted mixed $\mathcal{L}_2/l_2$-gain condition to consider the influence of discrete disturbances at impulse times. The conventional weighted $\mathcal{L}_2$-gain does not consider the l_2 norm of discrete disturbance sequences because the discrete disturbances do not exist in the conventional switched systems. However, as seen from (3b), disturbances are applied to the controller's states at reset instants, making the closed-loop impulsive switched system (3) have discrete disturbances at switching instants.

We give the following definitions used in the next section.

Definition 1: [39] Given a switching signal $\sigma(t)$ and each $t_2 \geq t_1 \geq 0$, let $N_\sigma(t_1, t_2)$ denote the number of discontinuities of $\sigma(t)$ in the open interval (t_1, t_2). We denote by $\mathcal{S}_{ave}[\tau_a, N_0]$ the set of all switching signals for which

$$N_\sigma(t_1, t_2) \leq N_0 + \frac{t_2 - t_1}{\tau_a}, \quad \forall t_2 \geq t_1 \geq 0$$

for the given constants $\tau_a, N_0 > 0$, which are called the ADT and the chatter bound, respectively.

Definition 2: [39] Given a set of piecewise constant switching signals $\mathcal{S}_{ave}[\tau_a, N_0]$, we say that (1) is globally uniformly asymptotically stable over $\mathcal{S}_{ave}[\tau_a, N_0]$ if there exists a function α of class $\mathcal{KL}$ such that, for each $\sigma \in \mathcal{S}_{ave}$,

$$\|x_p(t)\| \leq \alpha(\|x_p(t)\|, t - \tau), \quad \forall t \geq \tau \geq 0,$$

along solutions to (1).

Here, we are interested in providing the $\mathcal{L}_2$ stability analysis of the closed-loop system (3) and deriving the design condition of the controller's parameters $A_{c,\sigma}$, $B_{c,\sigma}$, $C_{c,\sigma}$, $D_{c,\sigma}$, $R_{1,ij}$, and $R_{2,ij}$ in (2) for the switched system under the ADT switching.

III. MAIN RESULTS

In this section, we provide the $\mathcal{L}_2$-gain analysis of the closed-loop switched nonlinear uncertain systems (3). Subsequently, the LMI-based design methodology of switching reset controller for switched nonlinear uncertain systems.

A. WEIGHTED MIXED $\mathcal{L}_2/l_2$ ANALYSIS FOR IMPULSIVE SWITCHED SYSTEMS WITH THE ADT

Before we present our main results, we introduce the following lemmas that are necessary for the proof of the main results.

Lemma 1: [41] Let $V_0(\zeta)$ and $V_1(\zeta)$ be two arbitrary quadratic forms over $\mathbb{R}^n$. Then $V_0(\zeta) < 0$ is a consequence of $V_1(\zeta) \leq 0$ if and only if there exists $\rho \geq 0$ such that

$$V_0(\zeta) < \rho V_1(\zeta), \quad \forall \zeta \in \mathbb{R}^n - \{0\}.$$

Lemma 2: [42] Let $x \in \mathbb{R}^n$, $\mathcal{Q} = \mathcal{Q}^T \in \mathbb{R}^{n\times n}$ and $\mathcal{B} \in \mathbb{R}^{m\times n}$ such that rank$(\mathcal{B}) < n$. The following statements are equivalent:

1) $x^T\mathcal{Q}x < 0$ for all $\mathcal{B}x = 0, x \neq 0$;
2) $\mathcal{B}^{\perp T}\mathcal{Q}\mathcal{B}^\perp < 0$ where $\mathcal{B}^\perp$ is the kernel of $\mathcal{B}$, i.e., $\mathcal{B}\mathcal{B}^\perp = 0$;
3) $\exists \mathcal{V} \in \mathbb{R}^{n\times m}$: $\mathcal{Q} + \mathcal{V}\mathcal{B} + \mathcal{B}^\perp\mathcal{V}^\perp < 0$.

The following theorem provides sufficient conditions for the $\mathcal{L}_2$ stability of the closed-loop system (3).

Theorem 1: For given scalars $\gamma > 0$, $\beta > 0$, $\mu \geq 1$, $\nu \geq 1$, $1 > \lambda > 0$, ε_1, and ε_2, the impulsive switched system (3) is globally uniformly asymptotically stable (GUAS) for every switching signal σ with the ADT $\tau_a \geq \ln(\mu)/\lambda$ and achieves the weighted mixed $\mathcal{L}_2/l_2$-gain (4) under zero initial condition if there exist matrices $P_i > 0$, S_i, $T_{2,i}$ with appropriate dimensions satisfying the following matrix inequalities $\forall i, j \in \Sigma \times \Sigma, i \neq j$:

$$\Psi < 0, \tag{5}$$

$$\begin{bmatrix} P_j - S_j - S_j^T & \star & \star\\ \bar{R}_{1,ij}^T S_j & -\mu P_i & \star\\ \bar{R}_{2,ij}^T S_j & 0 & -\gamma^2 I\end{bmatrix} \leq 0, \tag{6}$$

where $\Psi = \{\Psi_{kl}\}$, $k, l \in \{1, 2, \ldots, 6\}$ is the symmetric matrix whose components are given as following matrices:

$$\begin{aligned}
\Psi_{11} &= S_i^T\bar{A}_i + \bar{A}_i^T S_i + \lambda P_i,\\
\Psi_{21} &= P_i - S_i + \varepsilon_1 S_i^T\bar{A}_i, \quad \Psi_{22} = -\varepsilon_1(S_i^T + S_i),\\
\Psi_{31} &= \bar{B}_i^T S_i, \quad \Psi_{32} = \varepsilon_1\bar{B}_i^T S_i, \quad \Psi_{33} = -\gamma^2 I,\\
\Psi_{41} &= \varepsilon_2 U_f T_{2,i}^T\bar{A}_i + \bar{B}_i^T S_i,\\
\Psi_{42} &= -\varepsilon_2 U_f T_{2,i}^T + \varepsilon_1\bar{B}_i^T S_i, \quad \Psi_{43} = \varepsilon_2 U_f T_{2,i}^T\bar{B}_i,\\
\Psi_{44} &= -\nu_i I + \varepsilon_2(U_f T_{2,i}^T\bar{B}_i + \bar{B}_i^T T_{2,i}U_f^T),
\end{aligned}$$

$$\Psi_{51} = \bar{C}_i,\quad \Psi_{53} = \bar{D}_i,\quad \Psi_{55} = -I,$$
$$\Psi_{61} = U_p,\quad \Psi_{66} = -(\nu\beta)^{-1}I,$$
$$U_f = \begin{bmatrix} I & 0 \end{bmatrix} \in \mathbb{R}^{n_f \times n_{\bar{x}}},\quad U_p = \begin{bmatrix} I & 0 \end{bmatrix} \in \mathbb{R}^{n_x \times n_{\bar{x}}}.$$

Proof Consider the Lyapunov-like functions as

$$V_i = \bar{x}^T P_i \bar{x},\quad \forall i \in \Sigma. \tag{7}$$

By applying the Schur complements [41] to (5), we have

$$\begin{bmatrix} \Upsilon_1 & \star & \star & \star \\ \Upsilon_2 & \Upsilon_3 & \star & \star \\ \Upsilon_4 & \varepsilon_1\Upsilon_4 & -\gamma^2 I & \star \\ \Upsilon_4+\Upsilon_5 & \varepsilon_1\Upsilon_4+\Upsilon_6 & \Upsilon_7 & \Upsilon_8 \end{bmatrix}$$
$$+\bar{G}^T\bar{G} + \nu\beta\bar{H}^T\bar{H} < 0, \tag{8}$$

where

$$\Upsilon_1 = S_i^T\bar{A}_i + \bar{A}_i^T S_i + \lambda P_i,\ \Upsilon_2 = P_i - S_i + \varepsilon_1 S_i^T\bar{A}_i,$$
$$\Upsilon_3 = -\varepsilon_1(S_i + S_i^T),\ \Upsilon_4 = \bar{B}_i^T S_i,\ \Upsilon_5 = \varepsilon_2 U_f T_{2,i}^T\bar{A}_i,$$
$$\Upsilon_6 = -\varepsilon_2 U_f T_{2,i}^T,\ \Upsilon_7 = \varepsilon_2 U_f T_{2,i}^T\bar{B}_i,$$
$$\Upsilon_8 = -\nu I + \Upsilon_7 + \Upsilon_7^T,\ \bar{G} = \begin{bmatrix} \bar{C}_i & 0 & \bar{D}_i^T & 0 \end{bmatrix},$$
$$\bar{H} = \begin{bmatrix} U_p & 0 & 0 & 0 \end{bmatrix}.$$

Let us define the state vector $\zeta := (\bar{x}, \dot{\bar{x}}, w, f_i(x_p)) \in \mathbb{R}^{n_\zeta}$ and $\mathcal{Z} := \mathrm{diag}\{-\beta U_p^T U_p, 0, 0, I\} \in \mathbb{R}^{n_\zeta \times n_\zeta}$ where $n_\zeta = 2n_{\bar{x}} + n_w + n_f$. Because $f_i(x_p)$ is β-Lipschitz, we have

$$f_i(x_p)^T f_i(x_p) \le \beta x_p^T x_p \iff \zeta^T\mathcal{Z}\zeta \le 0. \tag{9}$$

Then, from (8) with Lemma 1, we have (10).

$$\begin{bmatrix} \Upsilon_1 + \bar{C}_i^T\bar{C}_i & \star & \star & \star \\ \Upsilon_2 & \Upsilon_3 & \star & \star \\ \Upsilon_4 & \varepsilon_1\Upsilon_4 & \Upsilon_9 & \star \\ \Upsilon_4+\Upsilon_5 & \varepsilon_1\Upsilon_4+\Upsilon_6 & \Upsilon_7 & He\{\Upsilon_7\} \end{bmatrix} < 0, \tag{10}$$

where $\Upsilon_9 = -\gamma^2 I + \bar{D}_i^T\bar{D}_i$.

From (3a), we have

$$\begin{bmatrix} \bar{A}_i & -I & \bar{B}_i & \bar{B}_{f,i} \end{bmatrix}\begin{bmatrix} \bar{x} \\ \dot{\bar{x}} \\ w \\ f_i(x_p) \end{bmatrix} = 0,\quad \forall i \in \Sigma. \tag{11}$$

We obtain the following inequality by (10) and (11):

$$\begin{bmatrix} \bar{C}_i^T\bar{C}_i + \lambda P_i & \star & \star & \star \\ P_i & 0 & \star & \star \\ \bar{D}_i^T\bar{C}_i & 0 & -\gamma^2 I + \bar{D}_i^T\bar{D}_i & \star \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$+He\left\{\begin{bmatrix} S_i^T \\ \varepsilon_1 S_i^T \\ 0 \\ \varepsilon_2 U_f T_{2,i}^T \end{bmatrix}\begin{bmatrix} \bar{A}_i & -I & \bar{B}_i & \bar{B}_{f,i} \end{bmatrix}\right\} < 0. \tag{12}$$

Then, we apply Lemma 2 to (12) to get

$$\dot{\bar{x}}^T P_i\bar{x} + \bar{x}^T P_i\dot{\bar{x}} < -\lambda\bar{x}^T P_i\bar{x} - z^T z + \gamma^2 w^T w$$

and thus,

$$\dot{V}_i(t) < -\lambda V_i(t) - z^T(t)z(t) + \gamma^2 w^T(t)w(t). \tag{13}$$

Moreover, we can derive the following equation from (3b) and

$$\begin{bmatrix} -I & \bar{R}_{1,ij} & \bar{R}_{2,ij} \end{bmatrix}\begin{bmatrix} \bar{x}(t_k^+) \\ \bar{x}(t_k) \\ w(t_k) \end{bmatrix} = 0,\quad \forall i \in \Sigma. \tag{14}$$

From (6) and (14) with Lemma 2, we have

$$V_j(t_k^+) - \mu V_i(t_k) \le \gamma^2 w^T(t_k)w(t_k). \tag{15}$$

at every impulse time t_k.

According to [39], [43], (13) and (15) for all $i, j \in \Sigma$ are sufficient to guarantee the globally uniformly asymptotic stability of the closed-loop system (3) with $w = 0$. From now on, we will show the closed-loop system (3) satisfies the weighted mixed $\mathcal{L}_2/l_2$-gain condition (4) under zero initial condition. Given the switching signal σ, $t_1 < \cdots < t_k$ ($k > 1$) represent the switching instant of σ over the interval $(0, t)$. Integrating (13) for $t \in [t_k^+, t)$ yields

$$V_\sigma(t) \le V_\sigma(t_k^+)e^{-\lambda(t-t_k^+)} - \int_{t_k}^{t} e^{-\lambda(t-\tau)}\chi(\tau)d\tau, \tag{16}$$

where $\chi(\tau) = h_z(\tau) - \gamma^2 h_w(\tau)$, $h_x(\cdot) := x(\cdot)^T x(\cdot)$. According to (15) and (16), we have

$$\begin{aligned} V_\sigma(t) \le\ & \left(\mu V_\sigma(t_k) + \gamma^2 h_w(t_k)\right)e^{-\lambda(t-t_k)} \\ & - \int_{t_k}^{t} e^{-\lambda(t-\tau)}\chi(\tau)d\tau \\ \le\ & \mu\Big((V_\sigma(t_{k-1})e^{-\lambda(t_k-t_{k-1})} \\ & - \int_{t_{k-1}}^{t_k} e^{-\lambda(t_k-\tau)}\chi(\tau)d\tau\Big)e^{-\lambda(t-t_k)} \\ & - \int_{t_k}^{t} e^{-\lambda(t-\tau)}\chi(\tau)d\tau + \gamma^2 h_w(t_k)e^{-\lambda(t-t_k)} \\ & \vdots \\ \le\ & \mu^k e^{-\lambda t}V_\sigma(0) - \mu^k\int_0^{t_1} e^{-\lambda(t-\tau)}\chi(\tau)d\tau \\ & - \cdots - \mu^0\int_{t_k}^{t} e^{-\lambda(t-\tau)}\chi(\tau)d\tau \\ & +\gamma^2\left(\mu^{k-1}h_w(t_1)e^{-\lambda(t-t_1)} + \cdots\right. \\ & \left. +h_w(t_k)e^{-\lambda(t-t_k)}\right) \\ =\ & e^{N_\sigma(0,t)\ln\mu}e^{-\lambda t}V_\sigma(0) \\ & - \int_0^{t} e^{N_\sigma(\tau,t)\ln\mu}e^{-\lambda(t-\tau)}\chi(\tau)d\tau \\ & +\gamma^2\sum_{t_k\in(0,t)} e^{N_\sigma(t_k^+,t)\ln\mu}e^{-\lambda(t-t_k)}h_w(t_k). \end{aligned} \tag{17}$$

By using the fact $V_\sigma(0) = 0$ and multiplying both side of (17) by $e^{-N_\sigma(0,t)\ln\mu}$, we have

$$\begin{aligned} e^{-N_\sigma(0,t)\ln\mu}V_\sigma(t) \le & -\int_0^t e^{-\lambda(t-\tau)-N_\sigma(0,\tau)\ln\mu}\chi(\tau)d\tau \\ & +\gamma^2\sum_{t_k\in(0,t)} e^{-\lambda(t-t_k)-N_\sigma(0,t_k)\ln\mu}h_w(t_k). \end{aligned}$$

As $V(t) \geq 0 \, \forall \, t > 0$, we have the following from the above inequality:

$$\int_0^t e^{-\lambda(t-\tau)-N_\sigma(0,\tau)\ln\mu}\chi(\tau)d\tau \leq \gamma^2 \sum_{t_k\in(0,t)} e^{-\lambda(t-t_k)-N_\sigma(0,t_k)\ln\mu} h_w(t_k). \quad (18)$$

Because $N_\sigma\,(0,\tau) \leq \tau/\tau_a^*$ and $\tau_a > \tau_a^* = \ln\mu/\lambda$, we have $N_\sigma\,(0,\tau)\ln\mu \leq \lambda\tau$ [43], [44]. Thus, it follows from (18) that

$$\int_0^t e^{-\lambda(t-\tau)-\lambda\tau}h_z(\tau)d\tau \leq \gamma^2\Big\{\int_0^t e^{-\lambda(t-\tau)}h_w(\tau)d\tau + \sum_{t_k\in(0,t)} e^{-\lambda(t-t_k)}h_w(t_k)\Big\}. \quad (19)$$

By integrating (19) from $t = 0$ to ∞, one can have

$$\begin{aligned}
&\int_0^\infty \int_0^t e^{-\lambda(t-\tau)-\lambda\tau}h_z(\tau)d\tau dt \\
&\quad \leq \int_0^\infty \gamma^2\Big\{\int_0^t e^{-\lambda(t-\tau)}h_w(\tau)d\tau + \sum_{t_k\in(0,t)} e^{-\lambda(t-t_k)}h_w(t_k)\Big\}dt \\
&\Leftrightarrow \int_0^\infty e^{-\lambda\tau}h_z(\tau)\left(\int_\tau^\infty e^{-\lambda(t-\tau)}dt\right)d\tau \\
&\quad \leq \gamma^2\Big\{\int_0^\infty h_w(\tau)\left(\int_\tau^\infty e^{-\lambda(t-\tau)}dt\right)d\tau + \sum_{t_k\in(0,\infty)} h_w(t_k)\left(\int_{t_k}^\infty e^{-\lambda(t-t_k)}dt\right)\Big\} \\
&\Leftrightarrow \frac{1}{\lambda}\int_0^\infty e^{-\lambda\tau}h_z(\tau)d\tau \\
&\quad \leq \frac{\gamma^2}{\lambda}\Big\{\int_0^\infty h_w(\tau)d\tau + \sum_{t_k\in(0,\infty)} h_w(t_k)\Big\}.
\end{aligned}$$

Finally, we have

$$\int_0^\infty e^{-\lambda\tau}h_z(\tau)d\tau \leq \gamma^2\Big\{\int_0^\infty h_w(\tau)d\tau + \sum_{t_k\in(0,\infty)} h_w(t_k)\Big\},$$

implying that the inequality (4) holds. This completes the proof. □

Theorem 1 presents the LMI-based $\mathcal{L}_2$-gain analysis for impulsive switched nonlinear uncertain systems for the first time. The introduction of the additional matrix variable S_i based on Lemma 2 enables the LMI-based design of dynamic output feedback controller parameters $A_{c,\sigma}$, $B_{c,\sigma}$, $C_{c,\sigma}$, $_{c,\sigma}$, and reset control parameters $R_{1,ij}$, $R_{2,ij}$. In other words, the use of the variable allows the multiple Lyapunov matrices P_σ to be multiplied by the control parameters to avoid the occurrence of BMI terms, and the LMI-based sufficient conditions for the controller design scheme can be induced. Finally, we present the controller synthesis to be achieved as shown in the next subsection.

B. ROBUST SWITCHING RESET CONTROLLER DESIGN

In this subsection, a robust reset controller design scheme is proposed for switched nonlinear uncertain systems. The LMI-based design conditions and procedures to obtain the controller parameters are provided through the following theorem.

Theorem 2: For given scalars $\gamma > 0$, $\beta > 0$, $\rho > 0$, $\mu \geq 1$, $\nu \geq 1$, $1 > \lambda > 0$, ε_1, and ε_2, the closed-loop system (3) is GUAS for every switching signal σ with the ADT $\tau_a \geq \ln(\mu)/\lambda$ and achieves a weighted mixed $\mathcal{L}_2/l_2$-gain (4) under zero initial condition if there exist matrices $\hat{P}_i > 0$, X_i, Y_i, Z_i, $\hat{A}_{c,i}$, $\hat{B}_{c,i}$, $\hat{C}_{c,i}$, $\hat{D}_{c,i}$, $\hat{R}_{1,ij}$, $\hat{R}_{2,ij}$ with appropriate dimensions such that $\forall i, j \in \Sigma\times\Sigma$, $i \neq j$:

$$\Omega < 0, \quad (20)$$

$$\begin{bmatrix} \hat{P}_j - \Theta_{2,j}^T - \Theta_{2,j} & \star & \star \\ \Xi_{1,ij}^T & -\mu\hat{P}_i & \star \\ \Xi_{2,ij}^T & 0 & -\gamma^2 I \end{bmatrix} \leq 0 \quad (21)$$

where $\Omega = \{\Omega_{kl}\}$, $k, l \in \{1, 2, \ldots, 8\}$ is the symmetric matrix whose components are given as follows:

$$\begin{aligned}
\Omega_{11} &= \Theta_{1,i} + \Theta_{1,i}^T + \lambda\hat{P}_i, \quad \Omega_{21} = \hat{P}_i - \Theta_{2,i}^T + \varepsilon_1\Theta_{1,i}, \\
\Omega_{22} &= -\varepsilon_1(\Theta_{2,i} + \Theta_{2,i}^T), \quad \Omega_{31} = \Theta_{3,i}^T, \quad \Omega_{32} = \varepsilon_1\Theta_{3,i}^T, \\
\Omega_{33} &= -\gamma^2 I, \quad \Omega_{41} = \varepsilon_2 U_f\Theta_{1,i} + \Theta_{4,i}, \\
\Omega_{42} &= -\varepsilon_2 U_f\Theta_{2,i} + \varepsilon_1\Theta_{4,i}, \quad \Omega_{43} = \varepsilon_2 U_f\Theta_{3,i}^T, \\
\Omega_{44} &= \nu_i I + \varepsilon_2(U_f\Theta_{4,i}^T + \Theta_{4,i}U_f^T), \quad \Omega_{51} = \Theta_{5,i}, \\
\Omega_{53} &= \Theta_{6,i}, \quad \Omega_{55} = -I, \quad \Omega_{61} = \Theta_{7,i}, \\
\Omega_{66} &= -(\nu\beta)^{-1}I, \quad \Omega_{71} = \Theta_{8,i}, \quad \Omega_{72} = \varepsilon_1\Theta_{8,i}, \\
\Omega_{74} &= \varepsilon_2\Theta_{8,i}U_f^T, \quad \Omega_{77} = -\rho^{-1}I, \\
\Omega_{81} &= \Theta_{9,i}, \quad \Omega_{88} = -\rho I, \\
\Theta_{1,i} &= \begin{bmatrix} A_{0,i}X_i + B_{u,i}\hat{C}_{c,i} & A_{0,i} + B_{u,i}\hat{D}_{c,i}C_{y,i} \\ \hat{A}_{c,i} & Y_iA_{0,i} + \hat{B}_{c,i}C_{y,i} \end{bmatrix}, \\
\Theta_{2,i} &= \begin{bmatrix} X_i & I \\ Z_i & Y_i \end{bmatrix}, \quad \Theta_{3,i} = \begin{bmatrix} B_{w,i} + B_{u,i}\hat{D}_{c,i}D_{yw,i} \\ Y_iB_{w,i} + \hat{B}_{c,i}D_{yw,i} \end{bmatrix}, \\
\Theta_{4,i} &= \begin{bmatrix} B_{f,i}^T & B_{f,i}^TY_i^T \end{bmatrix}, \\
\Theta_{5,i} &= \begin{bmatrix} C_{z,i}X_i + D_{zu,i}\hat{C}_{c,i} & C_{z,i} + D_{zu,i}\hat{D}_{c,i}C_{y,i} \end{bmatrix}, \\
\Theta_{6,i} &= \begin{bmatrix} D_{zw,i} + D_{zu,i}\hat{D}_{c,i}D_{yw,i} \end{bmatrix}, \quad \Theta_{7,i} = \begin{bmatrix} X_i & I \end{bmatrix}, \\
\Theta_{8,i} &= \begin{bmatrix} E_{\Delta,i}^T & E_{\Delta,i}^TY^T \end{bmatrix}, \quad \Theta_{9,i} = \begin{bmatrix} F_{\Delta,i}X_i & F_{\Delta,i} \end{bmatrix}, \\
\Xi_{1,ij} &= \begin{bmatrix} X_i & I \\ \hat{R}_{2,ij} & Y_j + \hat{R}_{1,ij}C_{y,i} \end{bmatrix}, \quad \Xi_{2,ij} = \begin{bmatrix} 0 \\ \hat{R}_{2,ij}D_{yw,i} \end{bmatrix}.
\end{aligned}$$

Furthermore, the controller's parameters can be obtained using the following steps $\forall i, j \in \Sigma\times\Sigma$, $i \neq j$:

Step 1: Find $\hat{A}_{c,i}$, $\hat{B}_{c,i}$, $\hat{C}_{c,i}$, $\hat{D}_{c,i}$, X_i, Y_i and Z_i such that (20) and (21).

Step 2: Obtain M_i and N_i from the relation that $M_iN_i^T = Z_i - X_iY_i$.

Step 3: Calculate the controller's parameters as follows:

$$\begin{bmatrix} A_{c,i} & B_{c,i} \\ C_{c,i} & D_{c,i} \end{bmatrix} = \begin{bmatrix} N_i & Y_iB_{u,i} \\ 0 & I \end{bmatrix}^{-1} \begin{bmatrix} \hat{A}_{c,i} - Y_iA_{c,i}X_i & \hat{B}_{c,i} \\ \hat{C}_{c,i} & \hat{D}_{c,i} \end{bmatrix}$$

$$\times \begin{bmatrix} M_i^T & 0 \\ C_{y,i}X_i & I \end{bmatrix}^{-1}, \tag{22}$$
$$R_{1,ij} = N_j^{-1}\hat{R}_{1,ij},$$
$$R_{2,ij} = N_j^{-1}\left(\hat{R}_{2,ij} - Y_jX_i - \hat{R}_{1,ij}C_{y,i}X_i\right)M_i^{-T}.$$

Proof: Please see the Appendix for the proof. □

The above LMI conditions were able to be obtained by means of the introduction of additional matrix variables S_i in Theorem 1 and the newly proposed change of variables in reset law $R_{2,ij}$. For the details on the derivation of the LMI-based design condition of Theorem 2, see the proof in Appendix.

Remark 3: Since, for given β, ρ, μ, ν, ε_1, ε_2 and λ, the conditions (20) and (21) are formulated in terms of a set of LMIs, which are not only over the matrix variables but also over the scalar γ^2, letting $\bar{\gamma} = \gamma^2$, γ can be minimized by the following optimization problem:

$$\begin{aligned} &\text{minimize } \bar{\gamma} \\ &\text{subject to } (20)-(21). \end{aligned} \tag{23}$$

IV. NUMERICAL EXAMPLES

We demonstrate the effectiveness of the controller proposed in this paper through two examples in this section. In example 1, we show the stability of the switched system by the switching reset controller and discuss the effect according to uncertainty. In example 2 and example 3, we present the effectiveness of the proposed method by comparing the results when the reset control scheme is applied and it is not and by applying it to a nonlinear switched DC motor system as a practical application, respectively.

A. EXAMPLE 1

We consider the following uncertain nonlinear switched system (1), modified from the example in [32] and [45]:

$$A_{0,1} = \begin{bmatrix} 0.5108 & -0.9147 & -0.2 \\ -0.6563 & 0.1798 & 0.113 \\ 0.881 & -0.7841 & 0.1 \end{bmatrix}, \ E_{\Delta,1} = \begin{bmatrix} 0 \\ 0 \\ \theta_1 \end{bmatrix}$$
$$F_{\Delta,1} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \ \Delta_1 = \sin(5t), \ B_{f,1} = \begin{bmatrix} 0 \\ 0 \\ 0.05 \end{bmatrix},$$
$$B_{w,1} = \begin{bmatrix} 0.1056 \\ 0.1284 \\ 0.1 \end{bmatrix}, \ B_{u,1} = \begin{bmatrix} 0.3257 \\ 1.2963 \\ 2.43 \end{bmatrix},$$
$$C_{z,1} = \begin{bmatrix} 0.01 & 0.06 & 0.03 \end{bmatrix}, \ C_{y,1} = \begin{bmatrix} -5 & 0.2 & 0.5 \end{bmatrix},$$
$$D_{zw,1} = D_{zu,1} = 0, \ D_{yw,1} = 0.1,$$
$$A_{0,2} = \begin{bmatrix} -0.125 & -0.9833 & -0.34 \\ -0.5305 & 0.3848 & 0.58 \\ 1.0306 & 0.6521 & 0.1 \end{bmatrix}, \ E_{\Delta,2} = \begin{bmatrix} 0 \\ 0 \\ \theta_2 \end{bmatrix}$$
$$F_{\Delta,2} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \ \Delta_2 = \sin(4t), \ B_{f,2} = \begin{bmatrix} 0 \\ 0 \\ 0.04 \end{bmatrix},$$

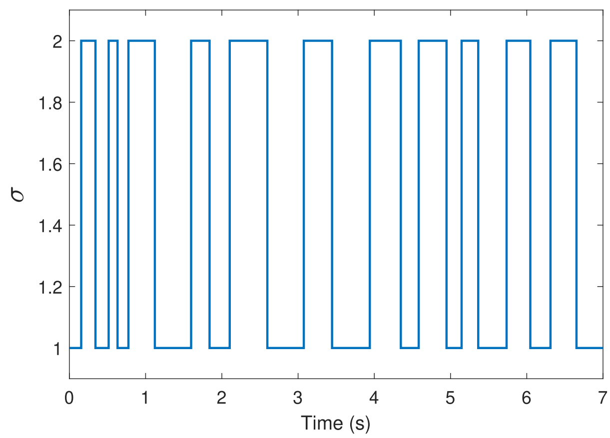

FIGURE 2. Switching signal $\sigma(t)$.

$$B_{w,2} = \begin{bmatrix} 0.7425 \\ 0.1436 \\ 0.1 \end{bmatrix}, \ B_{u,2} = \begin{bmatrix} 1.0992 \\ 0.6532 \\ 3.5 \end{bmatrix},$$
$$C_{z,2} = \begin{bmatrix} 0.01 & 0.02 & 0.05 \end{bmatrix}, \ C_{y,2} = \begin{bmatrix} -6 & 6 & -1 \end{bmatrix},$$
$$D_{zw,2} = D_{zu,2} = 0, \ D_{yw,2} = 0.1$$

where θ_i for $i \in \{1,2\}$ are unknown constants used for discussions and $f_i(x_p) = \sin(3x_{p1})$ is the nonlinear function with $\beta = 1$.

We select the following parameters $\lambda = 0.1$, $\varepsilon_1 = 0.1$, $\varepsilon_2 = -0.5$, $\nu = 0.05$, $\rho = 5$, and $\mu = 1.5$. Then, given $\theta_1 = 0.05$ and $\theta_2 = 0.04$, the reset controller (2) is designed based on Theorem 2 as follows:

$$A_{c,1} = \begin{bmatrix} -17.8577 & 1.9928 & 3.4230 \times 10^{-4} \\ 36.8266 & -5.0967 & 4.9544 \times 10^{-4} \\ -446.8741 & 24.1520 & -18.5694 \end{bmatrix},$$
$$B_{c,1} = \begin{bmatrix} 59.6176 & -119.4748 & 1.4951 \times 10^3 \end{bmatrix}^T,$$
$$C_{c,1} = \begin{bmatrix} 1.2491 & -0.1482 & -2.5136 \times 10^{-5} \end{bmatrix},$$
$$D_{c,1} = -4.2254,$$
$$R_{1,12} = \begin{bmatrix} -0.2859 & 5.7040 & 240.4930 \end{bmatrix}^T,$$
$$R_{2,12} = \begin{bmatrix} 1.7170 & -0.0196 & 8.6202 \times 10^{-6} \\ -1.1190 & -0.4621 & -4.3634 \times 10^{-5} \\ -80.0655 & 9.9483 & -0.0014 \end{bmatrix},$$
$$A_{c,2} = \begin{bmatrix} -5.834 & 51.8977 & 0.0440 \\ 2.4474 & -26.0976 & -0.0088 \\ -41.8892 & 420.4544 & -15.6989 \end{bmatrix},$$
$$B_{c,2} = \begin{bmatrix} 120.8324 & -59.2046 & 1.2263 \times 10^3 \end{bmatrix}^T,$$
$$C_{c,2} = \begin{bmatrix} 0.1627 & -1.6522 & -0.0014 \end{bmatrix},$$
$$D_{c,2} = -3.8194,$$
$$R_{1,21} = \begin{bmatrix} 0.2883 & 10.4825 & -20.4461 \end{bmatrix}^T,$$
$$R_{2,21} = \begin{bmatrix} 0.6048 & 0.1089 & 5.5572 \times 10^{-5} \\ 0.1462 & 2.7527 & 0.0024 \\ -0.4840 & 5.9095 & 0.0329 \end{bmatrix}.$$

We give a time-domain simulation to verify the controller parameters obtained by the LMI conditions and procedure in Theorem 2. The switching signals were randomly generated

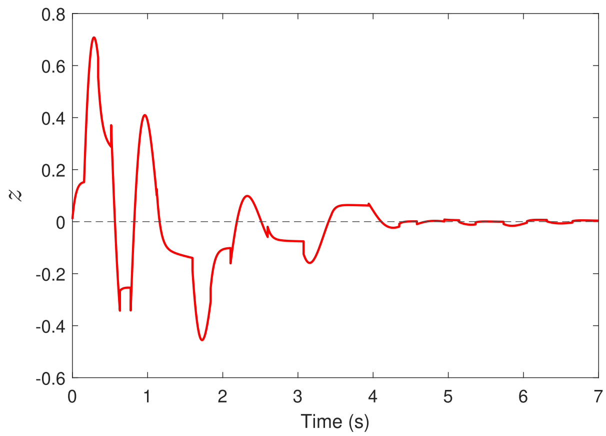

FIGURE 3. Trajectory of the controlled output $z(t)$ in Example 1.

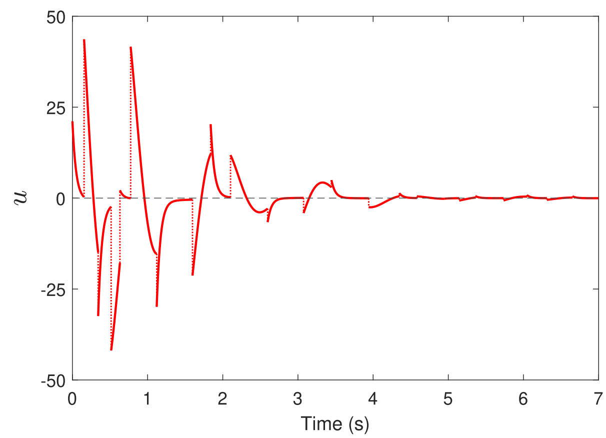

FIGURE 4. Trajectory of the control input $u(t)$ in Example 1.

TABLE 2. $\gamma_{\min}$ according to various μ in Example 1.

μ	1.4384	2.0	4.0	7.0
Theorem 2	198.9846	30.9410	10.8366	7.1263

and switching occurs twenty-two times for seven seconds (Fig. 2). The ADT of the generated switching signals was calculated by $\tau_a = 7/22 = 0.3181$. Therefore, one can see that the ADT condition in [44] $\tau_a \geq \tau_a^\star = \frac{\ln \mu}{\lambda} = 4.0547$ is satisfied. For the simulation, the disturbance $w(t) = 0.1\sin(0.3t)$ was applied and the initial condition was set as $x_p(0) = [1, 0, 0]^T$ and $x_c(0) = [0, 0, 0]^T$. Fig. 3 and 4 show the simulation results of the controlled output and control input trajectories, respectively. It shows that the proposed controller designed based on the LMI conditions and procedure in Theorem 2 effectively stabilizes the nonlinear switched system with uncertainty.

Moreover, the minimum disturbance attenuation level $\gamma_{\min}$ from various μ can be obtained based on the optimization problem (23) using Theorem 2. (Table 2) It can be seen that as the value μ increases, the attenuation level $\gamma_{\min}$ decreases. In other words, by allowing a large rate of change at the impulsive instants, it provides better performance against disturbance. From Fig. 5, we can see that the attenuation level gradually increases as the magnitude of the uncertainty

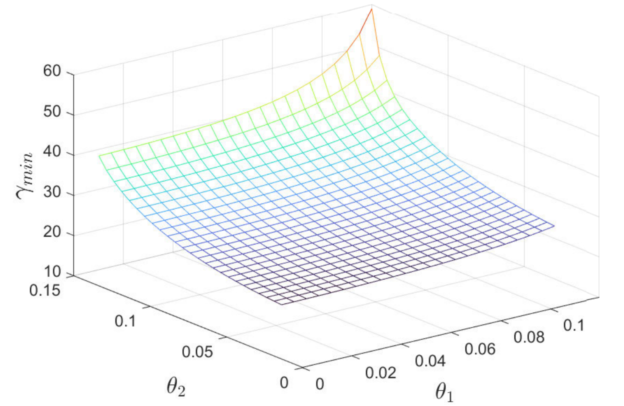

FIGURE 5. $\gamma_{\min}$ according to various θ_1 and θ_2 with $\mu = 2.5$ in Example 1.

increases. It means that the uncertainty degrades the robustness of the controller against disturbance.

B. EXAMPLE 2

Consider the following uncertain nonlinear switched system (1):

$$\begin{aligned}
A_{0,1} &= \begin{bmatrix} 1 & 1 \\ 0 & 0.5 \end{bmatrix},\ E_{\Delta,1} = \begin{bmatrix} 0 \\ \theta_1 \end{bmatrix},\ F_{\Delta,1} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\
\Delta_1 &= \sin(5t),\ B_{f,1} = \begin{bmatrix} 0 \\ -0.6 \end{bmatrix},\ B_{w,1} = \begin{bmatrix} 2.1 \\ -1.1 \end{bmatrix}, \\
B_{u,1} &= \begin{bmatrix} 1.6 \\ 2 \end{bmatrix},\ C_{z,1} = \begin{bmatrix} 0.5 & 0.2 \end{bmatrix},\ C_{y,1} = \begin{bmatrix} -3 & 0.2 \end{bmatrix}, \\
D_{zw,1} &= D_{zu,1} = 0,\ D_{yw,1} = 0.1, \\
A_{0,2} &= \begin{bmatrix} 0.2 & 1 \\ 0.5 & 0.2 \end{bmatrix},\ E_{\Delta,2} = \begin{bmatrix} 0 \\ \theta_2 \end{bmatrix},\ F_{\Delta,2} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\
\Delta_2 &= \sin(4t),\ B_{f,2} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix},\ B_{w,2} = \begin{bmatrix} 1.5 \\ 1.2 \end{bmatrix}, \\
B_{u,2} &= \begin{bmatrix} 1.8 \\ 1 \end{bmatrix},\ C_{z,2} = \begin{bmatrix} -0.1 & 0.1 \end{bmatrix},\ C_{y,2} = \begin{bmatrix} -2 & 0.7 \end{bmatrix}, \\
D_{zw,2} &= D_{zu,2} = 0,\ D_{yw,2} = 0.1
\end{aligned}$$

where the unknown system uncertainty parameters are $\theta_1 = 0.1$ and $\theta_2 = 0.08$. The nonlinear term is defined as $f_i(x_p) = \sin(3x_{p1})$ with $\beta = 1$.

We chose the following parameters $\lambda = 0.02$, $\varepsilon_1 = 0.1$, $\varepsilon_2 = -0.01$, $\nu = 0.1$, $\rho = 5$ and $\mu = 2.15$ for obtaining controller's parameters using (23). Based on Theorem 2, the reset controller (2) is obtained as

$$\begin{aligned}
A_{c,1} &= \begin{bmatrix} -12.3626 & -0.1065 \\ 488.2267 & -15.9618 \end{bmatrix},\ B_{c,1} = \begin{bmatrix} 1.1579 \\ -56.2608 \end{bmatrix}, \\
C_{c,1} &= \begin{bmatrix} 12.1417 & 0.2024 \end{bmatrix},\ D_{c,1} = -0.0571, \\
R_{1,12} &= \begin{bmatrix} 0.2647 \\ 0.0327 \end{bmatrix},\ R_{2,12} = \begin{bmatrix} -0.8384 & -0.0075 \\ 1.2914 & 0.0075 \end{bmatrix}, \\
A_{c,2} &= \begin{bmatrix} 1.0068 & 0.4631 \\ 0.1198 & -15.7937 \end{bmatrix},\ B_{c,2} = \begin{bmatrix} 0.7099 \\ 7.0875 \end{bmatrix}, \\
C_{c,2} &= \begin{bmatrix} 4.4337 & 0.2830 \end{bmatrix},\ D_{c,2} = 1.5753,
\end{aligned}$$

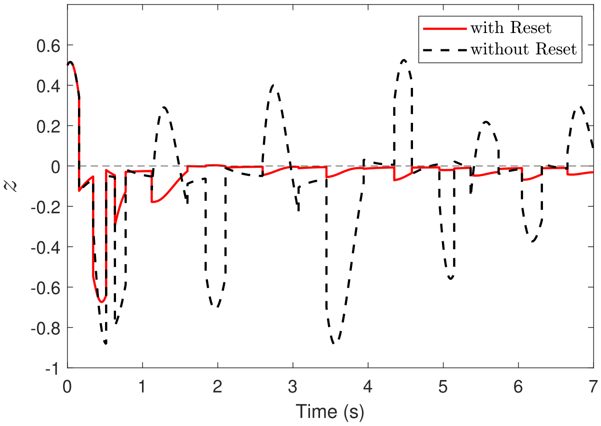

FIGURE 6. Trajectories of the controlled output $z(t)$ when the reset control is applied and when it is not applied.

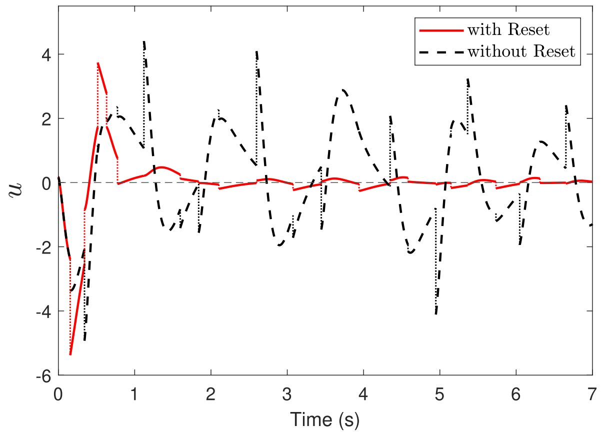

FIGURE 7. Trajectories of the control input $u(t)$ when the reset control is applied and when it is not applied.

$$R_{1,21} = \begin{bmatrix} 0.1950 \\ 1.7727 \end{bmatrix}, \; R_{2,21} = \begin{bmatrix} 0.2346 & 0.0065 \\ -0.5697 & -0.0178 \end{bmatrix}.$$

To show the effectiveness of the proposed controller, we compare the case where the reset control is applied with the case where it is not applied using a time-domain simulation. For the simulation, the switching signal $\sigma(t)$ is applied in the same way as in Example 1, which satisfies the ADT condition for the selected parameters. As in Example 1, the disturbance $w(t) = 0.1\sin(0.3t)$ was applied during the simulation. The initial condition was set as $x_p(0) = [1, 0]^T$ and $x_c(0) = [0, 0]^T$. Fig. 6 and Fig. 7 show the controlled output and control input trajectories for the two cases, respectively. The simulation results show that the proposed reset controller effectively reduced the transient behavior of the controlled outputs compared to the conventional switching controller when the switched system experience impulsive behavior in its controlled output by switching. In addition, the $\mathcal{L}_2$ gains in the presence or absence of a reset action are analytically calculated as 1.7086 and 4.3374, respectively. That is, the $\mathcal{L}_2$ gain in the case where the controller is applied is only 39.3934% of that in the case where it is not applied. This shows that the proposed controller effectively handles the disturbance.

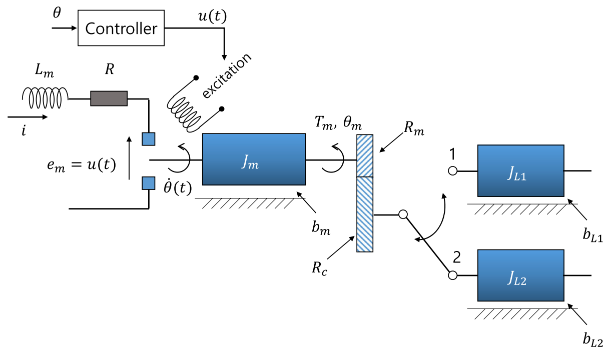

FIGURE 8. Nonlinear switched motor model.

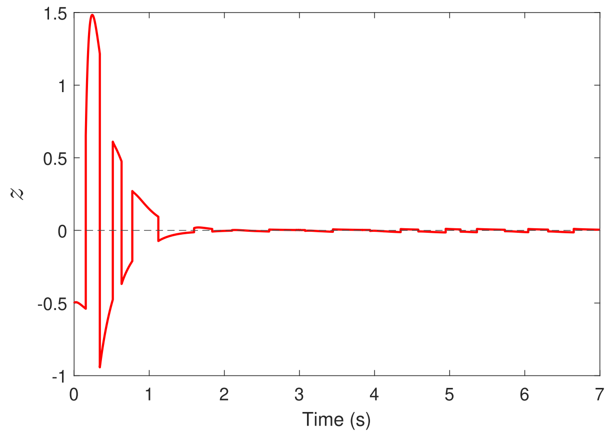

FIGURE 9. Trajectory of the controlled output $z(t)$ in Example 3.

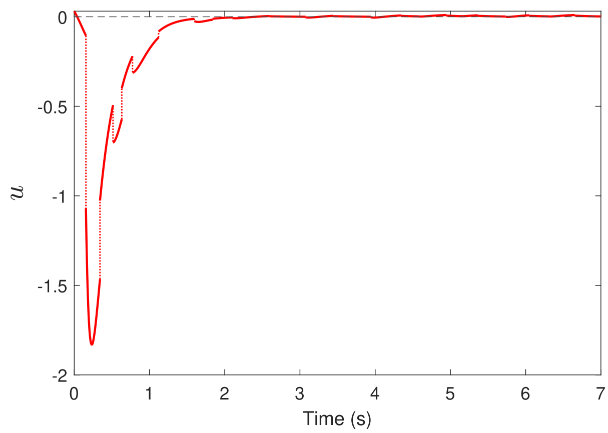

FIGURE 10. Trajectory of the control input $u(t)$ in Example 3.

C. EXAMPLE 3

Let us consider the following nonlinear switched DC motor system (Fig. 8) [46], [47]:

$$\dot{x}_p = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B_{e,\sigma}}{J_{e,\sigma}} \end{bmatrix} x_p + \begin{bmatrix} 0 \\ -1 \end{bmatrix} f_\sigma(x_p) + \begin{bmatrix} 0 \\ \frac{A_m}{J_{e,\sigma}} \end{bmatrix} u$$

where $x_p = [\theta^T, \; \dot{\theta}^T]^T$ is the plant state and A_m is the actuator gain. $J_{e,\sigma}$ and $B_{e,\sigma}$ are the total moment of inertia and equivalent damping term, respectively, which are switched by the switching sigmal σ. The friction parameter $f_\sigma(x_p)$ is expressed as the following nonlinear function:

$$f_\sigma(x_p) = 0.0174\mathrm{sgn}(x_{p2}) + 0.0087e^{-\frac{|x_{p2}|}{0.064}}\mathrm{sgn}(x_{p2}).$$

Then, the system can be expressed in the uncertain nonlinear switched system (1) as follows:

$$\begin{aligned}
A_{0,1} &= \begin{bmatrix} 0 & 1 \\ 0 & -34.3333 \end{bmatrix},\; E_{\Delta,1} = \begin{bmatrix} 0 \\ \theta_1 \end{bmatrix},\; F_{\Delta,1} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\
\Delta_1 &= \sin(5t),\; B_{f,1} = \begin{bmatrix} 0 \\ -1 \end{bmatrix},\; B_{w,1} = \begin{bmatrix} 0.1 \\ -0.3 \end{bmatrix}, \\
B_{u,1} &= \begin{bmatrix} 0 \\ 61.3038 \end{bmatrix},\; C_{z,1} = \begin{bmatrix} -0.5 & 0.3 \end{bmatrix},\; C_{y,1} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\
D_{zw,1} &= D_{zu,1} = 0,\; D_{yw,1} = 0.1, \\
A_{0,2} &= \begin{bmatrix} 0 & 1 \\ 0 & -51.9444 \end{bmatrix},\; E_{\Delta,2} = \begin{bmatrix} 0 \\ \theta_2 \end{bmatrix},\; F_{\Delta,2} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\
\Delta_2 &= \sin(4t),\; B_{f,2} = \begin{bmatrix} 0 \\ -1 \end{bmatrix},\; B_{w,2} = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}, \\
B_{u,2} &= \begin{bmatrix} 0 \\ 71.5212 \end{bmatrix},\; C_{z,2} = \begin{bmatrix} 0.6 & -0.4 \end{bmatrix},\; C_{y,2} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\
D_{zw,2} &= D_{zu,2} = 0,\; D_{yw,2} = 0.1
\end{aligned}$$

where the switching parameters of the motor system are $J_{e,1} = 0.0021$, $B_{e,1} = 0.0721$, $J_{e,2} = 0.0018$, and $B_{e,2} = 0.0935$. The unknown system uncertainty parameters are $\theta_1 = 0.1$ and $\theta_2 = 0.08$. We chose the parameters $\lambda = 0.02$, $\varepsilon_1 = 0.1$, $\varepsilon_2 = -0.01$, $\beta = 3$, $\nu = 0.1$, $\rho = 5$ and $\mu = 2$ to design the controller.

Through the scheme in Theorem 2, the controller (2) is designed as

$$\begin{aligned}
A_{c,1} &= \begin{bmatrix} -8.4414 & -3.3876 \\ 6.9309 & 0.3367 \end{bmatrix},\; B_{c,1} = \begin{bmatrix} -0.7432 \\ 1.3880 \end{bmatrix}, \\
C_{c,1} &= \begin{bmatrix} 6.3852 & 2.8766 \end{bmatrix},\; D_{c,1} = 0.0312, \\
R_{1,12} &= \begin{bmatrix} -0.0029 \\ 6.1547 \end{bmatrix},\; R_{2,12} = \begin{bmatrix} 0.1724 & 0.0722 \\ 290.1550 & 163.7795 \end{bmatrix}, \\
A_{c,2} &= \begin{bmatrix} -22.8664 & -0.0043 \\ 150.1907 & -19.8642 \end{bmatrix},\; B_{c,2} = \begin{bmatrix} -1.6969 \\ 51.7697 \end{bmatrix}, \\
C_{c,2} &= \begin{bmatrix} 17.0229 & 0.0056 \end{bmatrix},\; D_{c,2} = -0.9759, \\
R_{1,21} &= \begin{bmatrix} -0.3172 \\ 0.6477 \end{bmatrix},\; R_{2,21} = \begin{bmatrix} 0.5708 & 2.0744 \times 10^{-4} \\ 5.1779 & 0.0023 \end{bmatrix}.
\end{aligned}$$

For the time-domain simulation with the obtained controller, the switching signal $\sigma(t)$ and the disturbance $w(t)$ are applied in the same way as in example 1. The initial condition was set as $x_p(0) = [1, 0]^T$ and $x_c(0) = [0, 0]^T$. Fig. 9 and 10 show the simulation results of the controlled output and control input trajectories, respectively. It shows that the proposed controller designed based on the LMI conditions and procedure in Theorem 2 effectively stabilizes the nonlinear switched motor system with uncertainty.

V. CONCLUSION

This paper has proposed a design methodology of the output feedback robust switching reset controller for switched nonlinear uncertain systems. The proposed controller has a reset action every switching instant, and the measured output has

$$\begin{bmatrix}
S_i^T\bar{A}_{0,i} + \bar{A}_{0,i}^T S_i + \lambda P_i & \star & \star & \star & \star & \star \\
P_i - S_i + \varepsilon_1 S_i^T \bar{A}_{0,i} & -\varepsilon_1(S_i + S_i^T) & \star & \star & \star & \star \\
\bar{B}_i^T S_i & \varepsilon_1 \bar{B}_i^T S_i & -\gamma^2 I & \star & \star & \star \\
\varepsilon_2 U_f T_{2,i}^T \bar{A}_{0,1} + \bar{B}_i^T S_i & -\varepsilon_2 U_f T_{2,i}^T + \varepsilon_1 \bar{B}_i^T S_i & \varepsilon_2 U_f T_{2,i}^T \bar{B}_i & -\nu I + \varepsilon_2 \Phi & \star & \star \\
\bar{C}_i & 0 & \bar{D}_i & 0 & -I & \star \\
U_p & 0 & 0 & 0 & 0 & -(\nu\beta)^{-1} I
\end{bmatrix}
+ He\left\{ \begin{bmatrix} S_i^T \bar{E}_{\Delta,i} \\ \varepsilon_1 S_i^T \bar{E}_{\Delta,i} \\ 0 \\ \varepsilon_2 U_f T_{2,i}^T \bar{E}_{\Delta,i} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \bar{F}_{\Delta,i} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \right\} < 0, \tag{24}$$

$$\Phi = U_f T_{2,i}^T \bar{B}_i + \bar{B}_i^T T_{2,i} U_f^T \tag{25}$$

$$\begin{bmatrix}
S_i^T\bar{A}_{0,i} + \bar{A}_{0,i}^T S_i + \lambda P_i & \star & \star & \star & \star & \star & \star & \star \\
P_i - S_i + \varepsilon_1 S_i^T \bar{A}_{0,i} & -\varepsilon_1(S_i + S_i^T) & \star & \star & \star & \star & \star & \star \\
\bar{B}_i^T S_i & \varepsilon_1 \bar{B}_i^T S_i & -\gamma^2 I & \star & \star & \star & \star & \star \\
\varepsilon_2 U_f T_{2,i}^T \bar{A}_{0,1} + \bar{B}_i^T S_i & -\varepsilon_2 U_f T_{2,i}^T + \varepsilon_1 \bar{B}_i^T S_i & \varepsilon_2 U_f T_{2,i}^T \bar{B}_i & -\nu I + \varepsilon_2 \Phi & \star & \star & \star & \star \\
\bar{C}_i & 0 & \bar{D}_i & 0 & -I & \star & \star & \star \\
U_p & 0 & 0 & 0 & 0 & -(\nu\beta)^{-1} I & \star & \star \\
\bar{E}_{\Delta,i}^T S_i & \varepsilon_1 \bar{E}_{\Delta,i}^T S_i & 0 & \varepsilon_2 \bar{E}_{\Delta,i}^T T_{2,i} U_f^T & 0 & 0 & -\rho^{-1} I & \\
\bar{F}_{\Delta,i} & 0 & 0 & 0 & 0 & 0 & 0 & \rho I
\end{bmatrix} < 0, \tag{26}$$

been used for resetting the controller state, whereas the previous studies did not use the measured output in the reset law. Because the measured output has been affected by the disturbance, the controller's reset action has been also affected by disturbance at switching instants. To take into account the discrete disturbance, the weighted mixed $\mathcal{L}_2/l_2$-gain has been introduced. Due to the controller's reset action, the closed-loop system was formulated as the nonlinear uncertain impulsive switched system. Thus, we first provide the $\mathcal{L}_2$-gain analysis for the nonlinear uncertain impulsive switched systems using the weighted mixed $\mathcal{L}_2/l_2$-gain. Then, based on the analysis, an LMI-based controller design methodology was proposed, enabling the controller's parameters to be optimized using convex optimization methods. The two numerical examples demonstrated the feasibility of the proposed design method and the fact that the proposed switching reset controller outperforms the conventional switching controller.

In real applications, delays inevitably occur and become one of the factors that degrade the stability and performance of the system. Furthermore, the complexity and interconnections of controlled systems continue to increase. Therefore, research for the controller design scheme for multi-agent switched systems having system or measurement delays is still an open problem and would be an interesting research direction.

APPENDIX

Before providing the proof of Theorem 2, we present the following lemma that are necessary for the proof.

Lemma 3: [48] Given constant matrices $\mathcal{M}, \mathcal{N}, \mathcal{Y}$; positive semi-definite matrix R with appropriate dimensions and $\mathcal{Y} = \mathcal{Y}^T$, then for any Δ satisfying $\Delta^T\Delta \leq \mathcal{R}$, the following inequality holds:

$$\mathcal{Y} + \mathcal{M}\Delta\mathcal{N} + \mathcal{N}^T\Delta^T\mathcal{M}^T < 0$$

if and only if there exists a constant $\rho > 0$ such that:

$$\mathcal{Y} + \nu\mathcal{M}\mathcal{M}^T + \rho^{-1}\mathcal{N}^T\mathcal{R}\mathcal{N} < 0.$$

Proof of Theorem 2: Considering $A_i = A_{0,i} + E_{\Delta,i}\Delta F_{\Delta,i}$, (5) can be rewritten as (24), shown at the bottom of the previous page. By applying Lemma 3 and Schur complements [41], we can obtain (26), as shown at the bottom of the previous page.

Let us define auxiliary matrix $S_i \in \mathbb{R}^{(n+n_c)\times(n+n_c)}$ and

$$T_{1,i} = \begin{bmatrix} X_i & I \\ M_i^T & 0 \end{bmatrix}, \quad T_{2,i} = \begin{bmatrix} I & Y_i^T \\ 0 & N_i^T \end{bmatrix}, \quad \forall i \in \Sigma \tag{27}$$

such that $S_iT_{1,i} = T_{2,i}$ and $M_iN_i^T = Z_i^T - X_i^TY_i^T$. Then, pre- and post-multiplying (5) by a matrix $diag\{T_{1,i}^T, T_{1,i}^T, I, I, I, I, I, I\}$ and its transpose, one has by congruent transformation

$$\begin{aligned} T_{1,i}^TS_i^T\bar{A}_{0,i}T_{1,i} &= T_{2,i}^T\bar{A}_{0,i}T_{1,i} \\ &= \begin{bmatrix} A_{0,i}X_i + B_{u,i}\hat{C}_{c,i} & A_{0,i} + B_{u,i}\hat{D}_{c,i}C_{y,i} \\ \hat{A}_{c,i} & Y_iA_{0,i} + \hat{B}_{c,i}C_{y,i} \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} T_{1,i}^TP_iT_{1,i} &= \hat{P}_i, \quad T_{1,i}^TS_i^TT_{1,i} = T_{2,i}^TT_{1,i} = \begin{bmatrix} X_i & I \\ Z_i & Y_i \end{bmatrix}, \\ B_{cl,i}^TS_iT_{1,i} &= B_{cl,i}^TT_{2,i} \\ &= \begin{bmatrix} B_{w,i}^T + D_{yw,i}^T\hat{D}_{c,i}^TB_{u,i}^T & B_{w,i}^TY_i^T + D_{yw,i}^T\hat{B}_{c,i}^T \end{bmatrix}, \\ \bar{B}_{f,i}^TS_iT_{1,i} &= \bar{B}_{f,i}^TT_{2,i} = \begin{bmatrix} B_{f,i}^T & B_{f,i}^TY_i^T \end{bmatrix}, \\ \bar{C}_iT_{1,i} &= \begin{bmatrix} C_{z,i}X_i + D_{zu,i}\hat{C}_{c,i} & C_{z,i} + D_{zu,i}\hat{D}_{c,i}C_{y,i} \end{bmatrix}, \\ U_pT_{1,i} &= \begin{bmatrix} X_i & I \end{bmatrix}, \\ \bar{E}_{\Delta,i}^TS_iT_{1,i}^T &= \bar{E}_{\Delta,i}^TT_{2,i}^T = \begin{bmatrix} E_{\Delta,i}^T & E_{\Delta,i}^TY_i^T \end{bmatrix}, \\ \bar{F}_{\Delta,i}T_{1,i} &= \begin{bmatrix} F_{\Delta,i}X_i & F_{\Delta,i} \end{bmatrix}, \end{aligned}$$

where

$$\begin{aligned} \hat{A}_{c,i} &= Y_i\left(A_i + B_{u,i}D_{c,i}C_{y,i}\right)X_i + N_iB_{c,i}C_{y,i}X_i \\ &\quad + Y_iB_{u,i}C_{c,i}M_i^T + N_iA_{c,i}M_i^T, &\text{(28a)} \\ \hat{B}_{c,i} &= Y_iB_{u,i}D_{c,i} + N_iB_{c,i}, &\text{(28b)} \\ \hat{C}_{c,i} &= D_{c,i}C_{y,i}X_i + C_{c,i}M_i^T, &\text{(28c)} \\ \hat{D}_{c,i} &= D_{c,i}. &\text{(28d)} \end{aligned}$$

Thus, we obtain (20) and $\hat{P}_i > 0$ for all $i \in \Sigma$. We multiply (6) to the right by a matrix $diag\{T_{1,j}, T_{1,i}, I\}$, to the left by its transpose. Then, the following equations can be obtained as

$$\begin{aligned} T_{1,j}^TS_j^T\bar{R}_{1,ij}T_{1,i} &= T_{2,j}^T\bar{R}_{1,ij}T_{1,i} \\ &= \begin{bmatrix} X_i & I \\ \hat{R}_{2,ij} & Y_j + \hat{R}_{1,ij}C_{y,i} \end{bmatrix}, \\ T_{1,j}^TS_j^T\bar{R}_{2,ij} &= \begin{bmatrix} 0 \\ \hat{R}_{1,ij}D_{yw,i} \end{bmatrix}, \end{aligned}$$

where

$$\begin{aligned} \hat{R}_{1,ij} &= N_jR_{1,ij}, &\text{(29a)} \\ \hat{R}_{2,ij} &= Y_jX_i + \hat{R}_{1,ij}C_{y,i}X_i + N_jR_{2,ij}M_i^T. &\text{(29b)} \end{aligned}$$

Then, the condition (21) is deduced. Finally, the reset control law (22) can be obtained by using relations in (28) and (29). □

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