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RESEARCH ARTICLE

Bipartite Containment Disturbance Rejection Using Disturbance Observers

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ABSTRACT This paper mainly addresses bipartite containment control of multi-agent systems (MASs) subject to exogenous disturbances. With dynamic gain technique independent of any global information, the disturbance observer method is applied to estimate disturbances generated by heterogeneous nonlinear exosystems. Two disturbance observer-based controllers are accordingly presented via state feedback approach and output feedback approach. By means of appropriate Lyapunov method, it is shown that the bipartite containment control is realized under sufficient criteria. Finally, simulations are employed to validate the effectiveness and correctness of our proposed controllers.

INDEX TERMS Bipartite containment control, disturbance observer, heterogeneous nonlinear exosystems, feedback control.

I. INTRODUCTION

Over the past decades, the substantial attention has been paid to control systems owing to its broad applications, especially for networked systems [1], [2], [3], [4] and multi-agent systems [5], [6], [7], [8], [9], [10]. Compared with networked systems, the MASs has the advantage of greater efficiency, lower cost, less communication requirement. So far, there have been considerable researches about MASs. Among them, containment control is a research hotspot currently, which renders each follower gradually convergent to the convex hull composed of all leaders. Yu et al. [11] took into account a group of auxiliary systems to handle finite-time containment control for the unknown internal nonlinearity. Li et al. [12] investigated the containment problem by the observer-based output feedback controller and anti-windup compensator to enhance the performance for MASs.

Remarkably, above-mentioned results mainly consider that the agents interact collaboratively. However, cooperation and competition coexist in some practical networks. Therefore, the communication topology of agents can be represented as signed weighted digraphs with positive/negative weights corresponding to cooperation/competition [13], which extends the synchronization to bipartite synchronization. The bipartite containment control [14] was first introduced by utilizing signed digraphs [15], [16]. It was found that the followers cooperating with leaders asymptotically entered into dynamic convex hull spanned by leaders, while the followers competing with leaders asymptotically entered into the virtual convex hull opposite to the dynamic convex hull. Up to now, some works about bipartite containment have been done. Zuo et al. [17] figured out the bipartite output containment problem for linear heterogeneous multi-agent systems based on state feedback control and output feedback control. Zhou et al. [19] designed adaptive bipartite containment control protocols via state-observer-based approach and event-triggered control to effectively save limited resource. Yang et al. [18] designed delayed control protocols to solve the fractional bipartite containment control under in fixed and time-varying signed networks. However, external disturbances are few discussed in the above literatures.

As is known to all, disturbances may cause the systems with bad performance or even instability. Therefore, it is of great significance to study the disturbance

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rejection [20], [21], [22]. Li et al. [24] used an novel fixed-time control method to facilitate the disturbance-based controller design in improving the transient response and the steady performance. Yan et al. [25] presented a distributed disturbance-based controller for MASs with disturbances, where disturbances were imposed on all leaders and followers. Li et al. [23] designed a sliding mode controller to achieve finite-time containment control for nonlinear MASs by the pinning control, but it was easy to cause the harmful chattering phenomenon. But, as far as we know, exogenous disturbances of MASs with multiple leaders have not been studied in the cooperation-competition network. For MASs with multiple leaders in the sign network, the challenging of the control design includes two aspects: firstly, it will consider the communication connections between the follower with several leaders, rather than with one/no leader; secondly, the nonlinear disturbance compensator in controller can be designed to estimate the exogenous disturbances, which needs the dynamic gain approach to solve the nonlinearity. As a result, the follower does not require to track the leader, but to enter the convex hull formed by leaders.

Motivated by foregoing discussion, we handle the bipartite containment for MASs with disturbances generated from heterogeneous nonlinear exosystems. In the paper, the major contributions include three aspects. (i) Unlike bipartite containment control with bounded disturbances [26], disturbance observers are designed via adaptive control in the paper, which are irrespective of the whole spectrum information of the interaction topology. (ii) Compared with the disturbance rejection problem [27], it extends the containment on nonnegative communication digraphs to signed communication digraphs with antagonistic interactions from the reality aspect. Moreover, exogenous disturbances generated from nonlinear exosystems are investigated, which generate much more exogenous signals and have wider application than linear systems do. (iii) The proposed state feedback disturbance-observer-based control algorithm guarantees agents realize the bipartite containment control if the state is obtainable. Moreover, an output feedback disturbance-observer-based controller is constructed to complete the bipartite containment if the state is unobtainable. To sum up, it extends the linear exosystems model to the nonlinear exosystems model and extends the cooperative-cooperative communication connections to cooperative-competitive connections. Both the state feedback controller and the output feedback controller are considered while disturbance observers are designed via adaptive control to reduce the dependence of communication information. The problems are generally less restrictive in this article.

The outline of this paper is organized as follows. Notations, graph theory and problem formulation are introduced briefly in Section 2. In Section 3, main results about bipartite containment control are respectively shown on the basis of state feedback approach and output feedback approach. Numerical simulations are given in Section 4. Section 5 reports the final conclusion.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. NOTATIONS

Throughout this paper, $R, R^{l\times l}$ are respectively denoted as the real number and the $l \times l$ real matrix. I refers to the identity matrix while $1 = [1, 1, \cdots, 1]^T$ with the compatible dimensions. The Kronecker product of C and D is represented by $C \otimes D$. Let $\lambda(E)$, E^T be respectively the eigenvalue and the transpose of E. We define $\|x\|_p = (\sum_{i=1}^{l} |x_i|^p)^{1/p}$ for $p > 0$ and for a vector $x = [x_1, x_2, \cdots, x_l]^T$.

B. GRAPH THEORY

The adjacency matrix $A = (a_{ij})_{l\times l}$ describes the connections of followers, where $a_{ij} > 0$ if the follower i can get the cooperative interaction from the follower j, $a_{ij} < 0$ if the follower i can get the the antagonistic interaction from the follower j, otherwise $a_{ij} = 0$. The Laplacian matrix is defined as $L = D - A$, where $D = diag(d_1, d_2, \cdots, d_l)$ is a diagonal matrix with diagonal elements $d_1, d_2, \cdots, d_l (d_i = \sum_{j=1}^{l} a_{ij})$.

What is more, $G_k = diag(g_i^k)$ represents the communication connection between the $th - k$ leader and the $th - i$ follower. $g_i^k > 0$ if the interaction is cooperation, $g_i^k < 0$ if the interaction is competition, otherwise $g_i^k = 0$. Moreover, $\tilde{G}_k = diag(|g_i^k|)$, $\varphi_k = \frac{1}{m}L + \tilde{G}_k$.

Assumption 1: There is at least one leader which has a directed path to every follower in the communication topology.

C. PROBLEM FORMULATION

Consider a group of agents with m leaders and l followers in this section. Let's begin with the following kinematics

Followers:

$$\begin{aligned} \dot{r}_i(t) &= Ar_i(t) + Bu_i(t) + Bd_i(t), \\ y_i(t) &= Dr_i(t), \end{aligned} \tag{1}$$

Leaders:

$$\begin{aligned} \dot{r}_k(t) &= Ar_k(t), \\ y_k(t) &= Dr_k(t), \end{aligned} \tag{2}$$

where $r_i(t), d_i(t), u_i(t), y_i(t)$ are respectively the state vector, exogenous disturbance, control input and output of the follower i, meanwhile, $r_k(t), y_k(t)$ respectively represent the state vector and output of the leader k. A, B, D are constant system matrices.

For system (1), disturbances $d_i(t)$ are generated by heterogeneous nonlinear exosystems:

$$\begin{aligned} d_i(t) &= H_i\eta_i(t), \\ \dot{\eta}_i(t) &= F_i\eta_i(t) + \psi_i(\eta_i(t)), \end{aligned} \tag{3}$$

where $\eta_i(t)$ denotes the internal state of the nonlinear exogenous system. H_i, F_i are constant matrices with appropriate dimensions. Nonlinear function $\psi_i(\eta_i(t))$ is continuous and differential in t. Define the bipartite containment error as

$$e_i(t) = \sum_{j=1}^{l} \left(\left| a_{ij} \right| r_i(t) - a_{ij} r_j(t) \right) + \sum_{k=l+1}^{l+m} \left(\left| g_i^k \right| r_i(t) - g_i^k r_k(t) \right). \quad (4)$$

Setting $e = [{e_1}^T, {e_2}^T, \cdots, {e_l}^T]^T$, $\bar{r}_k = 1 \otimes r_k$ and $r = [{r_1}^T, {r_2}^T, \cdots, {r_l}^T]^T$ for convenience, the bipartite containment error can be equally rewritten as

$$\begin{aligned} e(t) &= ((D-A) \otimes I)\, r(t) + \sum_{k=l+1}^{l+m} \left(\tilde{G}_k \otimes I \right) r(t) \\ &\quad - \sum_{k=l+1}^{l+m} (G_k \otimes I)\,(1 \otimes r_k(t)) \\ &= \sum_{k=l+1}^{l+m} (\varphi_k \otimes I)\, r(t) - \sum_{k=l+1}^{l+m} (G_k \otimes I)\, \bar{r}_k(t). \end{aligned}$$

Assumption 2: There exists a non-negative constant C such that

$$\left\| \psi_i(\eta_{i1}(t)) - \psi_i(\eta_{i2}(t)) \right\| \leq C \left\| \eta_{i1}(t) - \eta_{i2}(t) \right\|.$$

Lemma 1: Under Assumptions 1, the bipartite containment problem of system (1) and (2) is realized if $\lim_{t \to \infty} e_i(t) = 0$.

Proof: It can be testified in the similar way as the proof process of lemma 3 in [28].

Remark 1: The linear exosystems $d_i(t) = H_i \eta_i(t)$, $\dot{\eta}_i(t) = F_i \eta_i(t)$ can be seen in [29]. In addition, equation $d_i(t) = H_i \eta_i(t)$, $\dot{\eta}_i(t) = F_i \eta_i(t)$ is equivalent to equation $\dot{d}_i(t) = S_i d_i(t)$, where it is common in mass-damper-spring systems [30], unmanned aerial vehicles (UAVs) [31] with disturbances as the vibration frequency in radians per second. The system is generally nonlinear with the improvement of industrial automation degree, the increasing diversity of production processes and increasing complexity of controlled object. Therefore, the equation(3) is reasonable.

III. MAIN RESULTS

In this section, two disturbance-observer controllers are correspondingly constructed based on state feedback control and output feedback control to solve bipartite containment control. The design procedures are detailed below.

A. BIPARTITE CONTAINMENT VIA STATE FEEDBACK CONTROL

The following distributed controller, using state feedback approach, is designed to handle out the bipartite containment problem

$$u_i(t) = K_1 e_i(t) - H_i \hat{\eta}_i(t), \quad (5)$$

where the estimates $\hat{\eta}_i(t)$ of $\eta_i(t)$ is generated by

$$\hat{\eta}_i(t) = \xi_i(t) + M_i r_i(t),$$

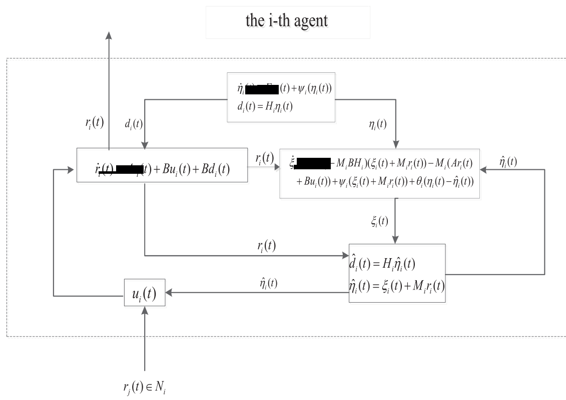

FIGURE 1. **The block diagram of the proposed control strategy of the follower i.**

$$\begin{aligned} \dot{\xi}_i(t) &= (F_i - M_i B H_i)(\xi_i(t) + M_i r_i(t)) - M_i(A r_i(t) \\ &\quad + B u_i(t)) + \psi_i(\xi_i(t) + M_i r_i(t)) + P_i^{-1} \theta_i(t)(\eta_i(t) \\ \dot{\theta}_i(t) &= (\eta_i(t) - \hat{\eta}_i(t))^T (\eta_i(t) - \hat{\eta}_i(t)), -\hat{\eta}_i(t)), \\ \hat{d}_i(t) &= H_i \hat{\eta}_i(t), \end{aligned} \quad (6)$$

where $\xi_i(t)$ is the internal variable of the observer, $\hat{d}_i(t)$ is the disturbance observer and P_i^{-1} is the inverse matrix of the P_i which will be determined later. The feedback gain K_1 and observer gain M_i are designed later.

Assumption 3: Positive real constant $\bar{\theta}$ is selected to assure $\bar{\theta} \geq \frac{1}{2}\lambda + C \left\| P_i \right\|$ with $\lambda \geq 2 \left\| Q_e((\sum_{k=l+1}^{l+m} \varphi_k) \otimes B) H \right\|^2 + 1$, where $H = diag(H_1, \cdots, H_l)$.

Theorem 1: Under Assumptions 1, 2 and 3, the control input (5) addresses the bipartite containment control problem if the following conditions hold: (i) M_i and K_1 are designed such that $F_i - M_i B H_i$ and $(I \otimes A) + ((\sum_{k=l+1}^{l+m} \varphi_k) \otimes B K_1)$ are Hurwitz matrices;
(ii) positive definite matrix P_i is designed to satisfy $P_i(F_i - M_i B H_i) + (F_i - M_i B H_i)^T P_i = -I$.

Proof: Let $\sigma_i(t) = d_i(t) - \hat{d}_i(t)$, $z_i(t) = \eta_i(t) - \hat{\eta}_i(t)$. According to (3) and (6), one can easily conclude that

$$\begin{aligned} \dot{z}_i(t) &= \dot{\eta}_i(t) - \dot{\hat{\eta}}_i(t) \\ &= F_i \eta_i(t) + \psi_i(\eta_i(t)) - [(F_i - M_i B H_i)\hat{\eta}_i(t) \\ &\quad - M_i(A r_i(t) + B u_i(t)) + \psi_i(\hat{\eta}_i(t)) \\ &\quad + {P_i}^{-1} \theta_i(t)(\eta_i(t) - \hat{\eta}_i(t)) + M_i \dot{r}_i(t)] \\ &= F_i \eta_i(t) + \psi_i(\eta_i(t)) - (F_i - M_i B H_i)\hat{\eta}_i(t) \\ &\quad - \psi_i(\hat{\eta}_i(t)) - {P_i}^{-1} \theta_i(t) z(t) - M_i B d_i(t) \\ &= (F_i - M_i B H_i) z_i(t) + \psi_i(\eta_i(t)) - \psi_i(\hat{\eta}_i(t)) \\ &\quad - {P_i}^{-1} \theta_i(t) z_i(t). \end{aligned} \quad (7)$$

Define the Lyapunov function as $V_i(t) = {z_i}^T(t) P_i z_i(t) + (\theta_i(t) - \bar{\theta})^2$ under (ii). Differentiate $V_i(t)$ as follows

$$\dot{V}_i(t) = {z_i}^T(t)[P_i(F_i - M_i B H_i) + (F_i - M_i B H_i)^T P_i] z_i(t)$$

$$
\begin{aligned}
&+2z_i^T(t)P_i[\psi_i(\eta_i(t))-\psi_i(\hat{\eta}_i(t))]\\
&-2z_i^T(t)\theta_i(t)z_i(t)+2(\theta_i(t)-\bar{\theta})z_i^T(t)z_i(t)\\
\le& -\|z_i(t)\|^2+2C\|P_i\|\|z_i(t)\|^2-2\bar{\theta}\|z_i(t)\|^2\\
\le& -2(\bar{\theta}-C\|P_i\|)\|z_i(t)\|^2\\
\le& -\lambda\|z_i(t)\|^2.
\end{aligned}
$$

Under Assumption 3 and stability theory, we have $\lim_{t\to\infty} z_i(t) = 0$. Moreover, the dynamics of agents can be expressed in a matrix form

$$
\begin{aligned}
\dot{r}(t) &= (I\otimes A)\,r(t) + (I\otimes BK_1)\,e(t)+(I\otimes B)\,Hz(t),\\
\dot{\bar{r}}_k(t) &= (I\otimes A)\,\bar{r}_k(t),
\end{aligned}
$$

where $z=[z_1{}^T, z_2{}^T, \cdots, z_l{}^T]^T$. Therefore, the derivative of $e(t)$ obtains

$$
\begin{aligned}
\dot{e}(t) =& \sum_{k=l+1}^{l+m}(\varphi_k\otimes I)((I\otimes A)r(t)+(I\otimes BK_1)e(t)\\
&+(I\otimes B)Hz(t))-\sum_{k=l+1}^{l+m}(G_k\otimes I)(I\otimes A)\bar{r}_k(t)\\
=& (I\otimes A)e(t)+((\sum_{k=l+1}^{l+m}\varphi_k)\otimes BK_1)e(t)\\
&+((\sum_{k=l+1}^{l+m}\varphi_k)\otimes B)Hz(t)\\
=& A_f e(t)+((\sum_{k=l+1}^{l+m}\varphi_k)\otimes B)Hz(t).
\end{aligned}
$$

Because A_f is a Hurwitz matrix, there must be a positive definite matrix Q_e satisfying $A_f Q_e+Q_e{}^T A_f=-I$. Construct the Lyapunov function as $V(t)=e^T(t)Q_e e(t)+\sum_{i=1}^{l}V_i(t)$. The derivative of $V(t)$ obtains

$$
\begin{aligned}
\dot{V}(t) =& e^T(t)[A_f Q_e+Q_e{}^T A_f]e(t)+2e^T(t)Q_e\\
&((\sum_{k=l+1}^{l+m}\varphi_k)\otimes B)Hz(t)+\sum_{i=1}^{l}\dot{V}_i(t)\\
\le& -\|e(t)\|^2+\frac{1}{2}\|e(t)\|^2-\lambda\|z(t)\|^2\\
&+2\left\|Q_e((\sum_{k=l+1}^{l+m}\varphi_k)\otimes B)H\right\|^2\|z(t)\|^2\\
\le& -\frac{1}{2}\|e(t)\|^2-\|z(t)\|^2.
\end{aligned}
$$

Under Assumption 3, $\lim_{t\to\infty} z_i(t)=0$ and Lyapunov stability theory, we have $\lim_{t\to\infty} e_i(t)=0$, $\lim_{t\to\infty}\sigma_i(t)=0$. The proof is completed.

B. BIPARTITE CONTAINMENT VIA OUTPUT FEEDBACK CONTROL

The following distributed controller, using output feedback approach, is designed to handle out the bipartite containment

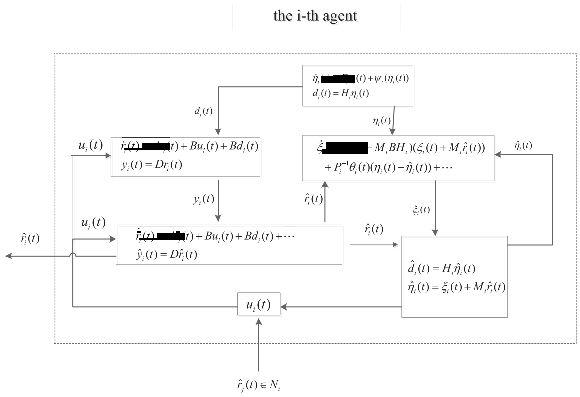

FIGURE 2. The block diagram of the proposed control strategy of the follower i.

problem

$$
u_i(t)=K\hat{e}_i(t)-H_i\hat{\eta}_i(t), \tag{8}
$$

where the estimates $\hat{\eta}_i(t)$ of $\eta_i(t)$ is generated by

$$
\begin{aligned}
\hat{\eta}_i(t) =& \xi_i(t)+M_i\hat{r}_i(t),\\
\dot{\xi}_i(t) =& (F_i-M_iBH_i)(\xi_i(t)+M_i\hat{r}_i(t))-M_i(A\hat{r}_i(t)\\
&+Bu_i(t))+P_i^{-1}\theta_i(t)(\eta_i(t)-\hat{\eta}_i(t))\\
&+\psi_i(\xi_i(t)+M_i\hat{r}_i(t)),\\
\dot{\theta}_i(t) =& (\eta_i(t)-\hat{\eta}_i(t))^T(\eta_i(t)-\hat{\eta}_i(t)),\\
\dot{\hat{r}}_i(t) =& A\hat{r}_i(t)+Bu_i(t)\\
&+B\hat{d}_i(t)-S(\sum_{j=1}^{l}(\left|a_{ij}\right|(y_i(t)\\
&-\hat{y}_i(t))-a_{ij}(y_j(t)-\hat{y}_j(t)))+\sum_{k=l+1}^{l+m}(\left|g_i^k\right|(y_i(t)\\
&-\hat{y}_i(t))-g_i^k(y_k(t)-y_k(t)))),\\
\hat{d}_i(t) =& H_i\hat{\eta}_i(t),
\end{aligned} \tag{9}
$$

where $\hat{y}_i(t)=D\hat{r}_i(t)$, $\hat{r}_i(t)$ is state observer, $\xi_i(t)$ is the internal variable of the observer $\hat{\eta}_i(t)$, $\hat{d}_i(t)$ is constructed to estimate $d_i(t)$, K, P_i and S are designed later. Moreover, the bipartite containment observer errors

$$
\begin{aligned}
\hat{e}_i(t) =& \sum_{j=1}^{l}(\left|a_{ij}\right|\hat{r}_i(t)-a_{ij}\hat{r}_j(t))\\
&+\sum_{k=l+1}^{l+m}(\left|g_i^k\right|\hat{r}_i(t)-g_i^k r_k(t)).
\end{aligned} \tag{10}
$$

Assumption 4: Positive real constant $\bar{\theta}$ is selected to assure $\bar{\theta}\ge\frac{1}{2}\lambda_{\max}(B^TQQB)\|H\|^2+(C\|P_i\|)_{\max}+\frac{1}{2}\|P_iM_iSD\|^2{}_{\max}$, where $H=diag(H_1,\cdots H_l)$.

Theorem 2: Under Assumptions 1, 2 and 4, the control input (8) addresses the bipartite containment if the following conditions hold: (i) F_i is a Hurwitz matrix and positive definite matrix P_i is designed to satisfy $P_iF_i+F_i{}^TP_i=-I$;

(ii) the state observer gain and the control gain are respectively designed as $S = -Q^{-1}D^T$, $K = -B^T Q$, where the positive definite matrix Q meets

$$QA + A^T Q + (\lambda_{\max}(D^T DD^T D) + \lambda_{\max}^2(\sum_{k=l+1}^{l+m} \varphi_k) + 1)I < 0. \quad (11)$$

Proof: Let $\sigma_i(t) = d_i(t) - \hat{d}_i(t)$, $z_i(t) = \eta_i(t) - \hat{\eta}_i(t)$ and $\tilde{e}_i(t) = e_i(t) - \widehat{e}_i(t)$. According to (2) and (9), one can easily conclude that

$$\begin{aligned}\dot{z}_i(t) &= \dot{\eta}_i(t) - \dot{\hat{\eta}}_i(t)\\ &= F_i\eta_i(t) + \psi_i(\eta_i(t)) - [(F_i - M_iBH_i)\hat{\eta}_i(t) + M_i\dot{\hat{r}}_i(t)\\ &\quad -M_i(A\hat{r}_i(t) + Bu_i(t)) + {P_i}^{-1}\theta_i(t)(\eta_i(t) - \hat{\eta}_i(t))\\ &\quad +\psi_i(\hat{\eta}_i(t)))\\ &= F_iz_i(t) + \psi_i(\eta_i(t)) - \psi_i(\hat{\eta}_i(t)) - {P_i}^{-1}\theta_i(t)z_i(t)\\ &\quad +M_iSD\tilde{e}_i(t). \end{aligned} \quad (12)$$

Define the Lyapunov function as $V_i(t) = {z_i}^T(t)P_iz_i(t) + (\theta_i(t) - \bar{\theta})^2$ under (i). The time derivative of $V_i(t)$ is

$$\begin{aligned}\dot{V}_i(t) &= {z_i}^T(t)[P_iF_i + {F_i}^T P_i]z_i(t) + 2(\theta_i(t) - \bar{\theta}){z_i}^T(t)z_i(t)\\ &\quad +2{z_i}^T(t)P_i[\psi_i(\eta_i(t)) - \psi_i(\hat{\eta}_i(t))] - 2{z_i}^T(t)\theta_i(t)z_i(t)\\ &\quad +2{z_i}^T(t)P_iM_iSD\tilde{e}_i(t)\\ &\leq -\|z_i(t)\|^2 + 2C\|P_i\|\|z_i(t)\|^2 - 2\bar{\theta}\|z_i(t)\|^2\\ &\quad +2{z_i}^T(t)P_iM_iSD\tilde{e}_i(t).\end{aligned}$$

Setting $\hat{e} = [\hat{e}_1^T, \hat{e}_2^T, \cdots, \hat{e}_l^T]^T$, $\hat{r} = [\hat{r}_1^T, \hat{r}_2^T, \cdots, \hat{r}_l^T]^T$, the observer tracking errors are equally expressed

$$\hat{e}(t) = \sum_{k=l+1}^{l+m} (\varphi_k \otimes I)\hat{r}(t) - \sum_{k=l+1}^{l+m} (G_k \otimes I)\bar{r}_k(t).$$

Based on $K = -B^T Q$, the dynamics of agents under the distributed controller (8) are expressed in a matrix form

$$\begin{aligned}\dot{r}(t) &= (I \otimes A)\, r(t) - \left(I \otimes BB^T Q\right)\hat{e}(t) + (I \otimes B)\, Hz(t),\\ \dot{\hat{r}}(t) &= (I \otimes A)\,\hat{r}(t) - \left(I \otimes BB^T Q\right)\hat{e}(t) - (I \otimes SD)\,\tilde{e}(t),\\ \dot{\bar{r}}_k(t) &= (I \otimes A)\,\bar{r}_k(t),\end{aligned}$$

where $z = [{z_1}^T, {z_2}^T, \cdots, {z_l}^T]^T$. Therefore, the derivative of $e(t)$ obtains

$$\begin{aligned}\dot{e}(t) &= \sum_{k=l+1}^{l+m} (\varphi_k \otimes I)((I \otimes A)\, r(t) - (I \otimes BB^T Q)\hat{e}(t)\\ &\quad + (I \otimes B)\, Hz(t)) - \sum_{k=l+1}^{l+m} (G_k \otimes I)((I \otimes A)\bar{r}_k(t))\\ &= (I \otimes A)\, e(t) - ((\sum_{k=l+1}^{l+m} \varphi_k) \otimes BB^T Q)\hat{e}(t)\\ &\quad +((\sum_{k=l+1}^{l+m} \varphi_k) \otimes B)Hz(t).\end{aligned}$$

Similarly, one also gets the time derivative of $\hat{e}(t)$

$$\begin{aligned}\dot{\hat{e}}(t) &= ((I \otimes A) - ((\sum_{k=l+1}^{l+m} \varphi_k) \otimes BB^T Q)\hat{e}(t)\\ &\quad -((\sum_{k=l+1}^{l+m} \varphi_k) \otimes SD)\tilde{e}(t).\end{aligned}$$

Therefore, it is not hard to derive that

$$\begin{aligned}\dot{\tilde{e}}(t) &= ((I \otimes A) + ((\sum_{k=l+1}^{l+m} \varphi_k) \otimes SD)\tilde{e}(t)\\ &\quad +((\sum_{k=l+1}^{l+m} \varphi_k) \otimes B)Hz(t).\end{aligned}$$

Considering the Lyapunov function as $V(t) = \hat{e}^T(t)(I \otimes Q)\hat{e}(t) + \tilde{e}^T(t)(I \otimes Q)\tilde{e}(t) + \sum_{i=1}^{l} V_i(t)$ and designing $S = -Q^{-1}D^T$, it can be derived that

$$\begin{aligned}\dot{V}(t) &= \tilde{e}^T(t)[I \otimes (QA + A^T Q) - 2((\sum_{k=l+1}^{l+m} \varphi_k) \otimes D^T D)]\tilde{e}(t)\\ &\quad +\hat{e}^T(t)[I \otimes (QA + A^T Q) - 2((\sum_{k=l+1}^{l+m} \varphi_k)\\ &\quad \otimes QBB^T Q)]\hat{e}(t) + 2\tilde{e}^T(t)((\sum_{k=l+1}^{l+m} \varphi_k) \otimes QB)Hz(t)\\ &\quad +\sum_{i=1}^{l} \dot{V}_i(t) + 2\hat{e}^T(t)((\sum_{k=l+1}^{l+m} \varphi_k) \otimes D^T D)\tilde{e}(t).\end{aligned}$$

It follows from the Young's inequality that

$$\begin{aligned}&2\hat{e}^T(t)((\sum_{k=l+1}^{l+m} \varphi_k) \otimes D^T D)\tilde{e}(t) \leq \lambda_{\max}^2(\sum_{k=l+1}^{l+m} \varphi_k)\|\hat{e}(t)\|^2\\ &\quad +\lambda_{\max}(D^T DD^T D)\|\tilde{e}(t)\|^2,\\ &2\tilde{e}^T(t)((\sum_{k=l+1}^{l+m} \varphi_k) \otimes QB)Hz(t) \leq \lambda_{\max}^2(\sum_{k=l+1}^{l+m} \varphi_k)\|\tilde{e}(t)\|^2\\ &\quad +\lambda_{\max}(B^T QQB)\|H\|^2\|z(t)\|^2.\end{aligned}$$

On the basis of above analysis, it yields

$$\begin{aligned}\dot{V}(t) &\leq \lambda_{\max}^2(\sum_{k=l+1}^{l+m} \varphi_k)\|\hat{e}(t)\|^2 + \lambda_{\max}(D^T DD^T D)\|\tilde{e}(t)\|^2\\ &\quad +\hat{e}^T(t)[-2((\sum_{k=l+1}^{l+m} \varphi_k) \otimes QBB^T Q) + I \otimes (QA\\ &\quad +A^T Q)]\hat{e}(t) + \tilde{e}^T(t)[-2((\sum_{k=l+1}^{l+m} \varphi_k) \otimes D^T D)\\ &\quad +I \otimes (QA + A^T Q)]\tilde{e}(t) + \lambda_{\max}^2(\sum_{k=l+1}^{l+m} \varphi_k)\|\tilde{e}(t)\|^2\end{aligned}$$

$$
\begin{aligned}
&+\sum_{i=1}^{l}(2C\,\|P_i\|\,\|z_i(t)\|^{2} - 2\bar{\theta}\,\|z_i(t)\|^{2}\\
&+\|P_iM_iSD\|^2\,\|z_i(t)\|^{2} + \|\tilde{e}_i(t)\|^2)\\
&+\lambda_{\max}(B^TQQB)\|H\|^2\|z(t)\|^2\\
\le &\sum_{i=1}^{l}\bar{\hat{e}}_i^{T}(t)[QA+A^TQ-2\lambda_i(\sum_{k=l+1}^{l+m}\varphi_k)QBB^TQ\\
&+\lambda^2_{\max}(\sum_{k=l+1}^{l+m}\varphi_k)I]\bar{\hat{e}}_i(t)+\sum_{i=1}^{l}\bar{\tilde{e}}_i^{T}(t)[QA+A^TQ\\
&-2\lambda_i(\sum_{k=l+1}^{l+m}\varphi_k)D^TD+(\lambda_{\max}(D^TDD^TD)\\
&+\lambda^2_{\max}(\sum_{k=l+1}^{l+m}\varphi_k)+1)I]\bar{\tilde{e}}_i(t)+(\|P_iM_iSD\|^2{}_{\max}\\
&+\lambda_{\max}(B^TQQB)\|H\|^2+(2C\,\|P_i\|)_{\max}-2\bar{\theta})\,\|z(t)\|^{2},
\end{aligned}
$$

where $\bar{\hat{e}}^T(t) = (U\otimes I)\hat{e}^T(t), \bar{\tilde{e}}^T(t) = (U\otimes I)\tilde{e}^T(t)$, U is an orthogonal constant matrix satisfying $U^T(\sum_{k=l+1}^{l+m}\varphi_k)U = diag(\lambda_1,\lambda_2,\cdots,\lambda_l)$, $0<\lambda_1\le\lambda_2\le\cdots\le\lambda_l$ are the eigenvalues of $\sum_{k=l+1}^{l+m}\varphi_k$. $\bar{\hat{e}}(t), \bar{\tilde{e}}(t)$ are respectively the column stack vectors of $\bar{\hat{e}}_i(t), \bar{\tilde{e}}_i(t)$. In addition, it is easy to yield the following inequalities

$$
\begin{aligned}
&QA+A^TQ-2\lambda_i(\sum_{k=l+1}^{l+m}\varphi_k)QBB^TQ+\lambda^2_{\max}(\sum_{k=l+1}^{l+m}\varphi_k)I\\
&\quad\le QA+A^TQ+\lambda^2_{\max}(\sum_{k=l+1}^{l+m}\varphi_k)I,\\
&\quad QA+A^TQ-2\lambda_i(\sum_{k=l+1}^{l+m}\varphi_k)D^TD+(\lambda_{\max}(D^TDD^TD)\\
&\quad+\lambda^2_{\max}(\sum_{k=l+1}^{l+m}\varphi_k)+1)I\\
&\quad\le QA+A^TQ+(\lambda_{\max}(D^TDD^TD)\\
&\quad+\lambda^2_{\max}(\sum_{k=l+1}^{l+m}\varphi_k)+1)I.
\end{aligned}
$$

Therefore, when (11) and Assumption 4 are satisfied, it is obviously that $\dot{V}(t)<0$. Based on Lyapunov stability theory, we obtain $\lim_{t\to\infty} V(t)\to 0$. Thus, $\hat{e}(t)\to 0, \tilde{e}(t)\to 0, z(t)\to 0$ as $t\to\infty$, that is to imply, $e_i(t)\to 0, \sigma_i(t)\to 0$. The rigorous proof is completed.

Remark 2: Due to communication constraints, packet dropout and other reasons, the state is always not easy to obtain or unobtainable while output information is most available. As a result, bipartite containment problem via output feedback approach is more realistic than state feedback approach in most practical engineering. However, the sate feedback approach has better system performance than the output feedback approach because it uses more signal. Therefore, different control methods can be selected for different situations.

Remark 3: The approach to handle disturbances includes sliding mode control [32], robust control [33], and disturbance-observer-based control [34]. The main idea of SMC is to drive and obtain a system state to the suitably designed sliding mode, which is usually independent of disturbances. However, the control law is a discontinuous switching signal, and it easily generates chattering phenomenon. The core idea of robust control is to consider the control effect in the worst case, it results in the conservative outcome. The above two methods utilize the feedback control structure to reduce the influence of unknown disturbance on system performance. However, the disturbance observer is to use feedforward structure to estimate and compensate for the disturbances so that the systems can have faster response speed to disturbances and can have better system performance. Nevertheless, the disturbance-observer method mainly focuses on the disturbances generated by the linear exosystem. To overcome this limitation, the nonlinear disturbances is proposed in the paper and the adaptive parameter is employed to solve the nonlinearity.

Remark 4: Compared with the issue in [27], it has two advantages. Firstly, the disturbances are generated by nonlinear exosystems, not just linear exosystems. Obviously, it extends the range of disturbance types. Secondly, considering the relations between agents are not only cooperative but also competitive in reality, the issue for containment control under disturbances is extended to the issue for bipartite containment control. Based on the above two aspects, the problem for multiagent systems with disturbances is more general from systems model and communication reality. Furthermore, Let $\psi_i(\eta_i(t)) = 0$ and $a_{ij}>0, a_{ik}>0$, the methods in this paper can solve the problem in [27].

Remark 5: Through the comparison with [37], [38], and [39], we find disturbance estimator in the three articles estimates a limited disturbance. However, estimator in this article can estimate high-order disturbance.

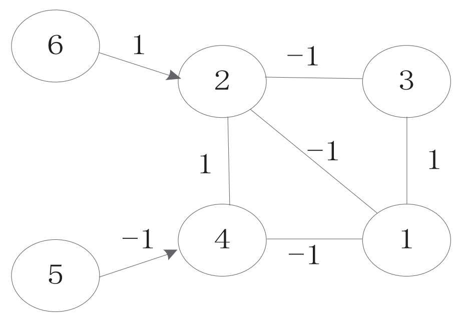

FIGURE 3. The interaction topology.

IV. NUMERICAL SIMULATIONS

Suppose that the relations of of agents are shown in Fig.3 with followers labeled $1,2,3,4$ and leaders labeled $5,6$.

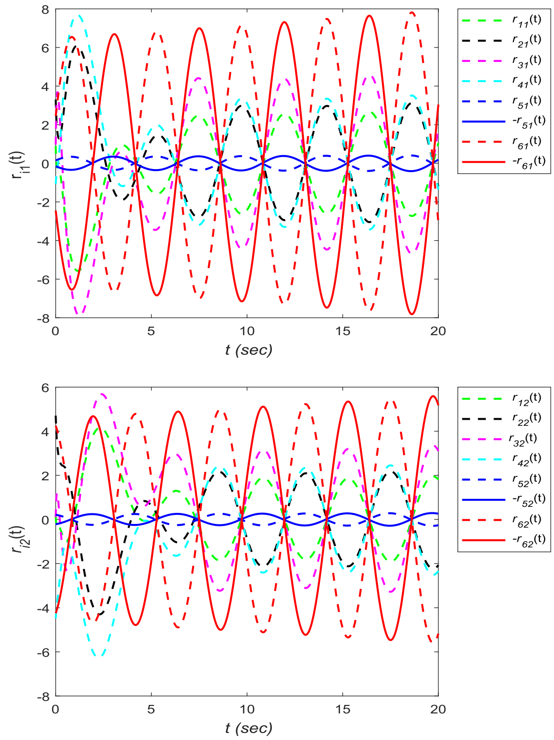

FIGURE 4. The state paths of agents for example 1.

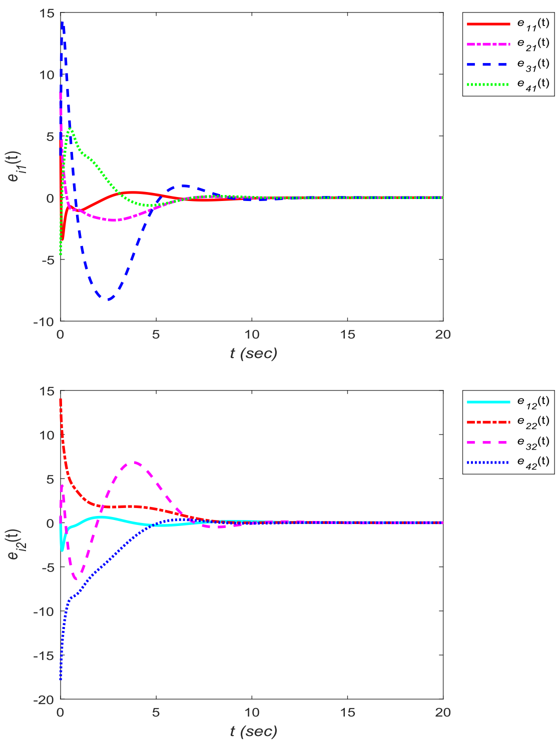

FIGURE 5. The bipartite containment errors of the MASs for example 1.

Numerical examples show the effectiveness of theoretical results. For simplicity, the original values of each agent are chosen randomly within $[-10, 10]$.

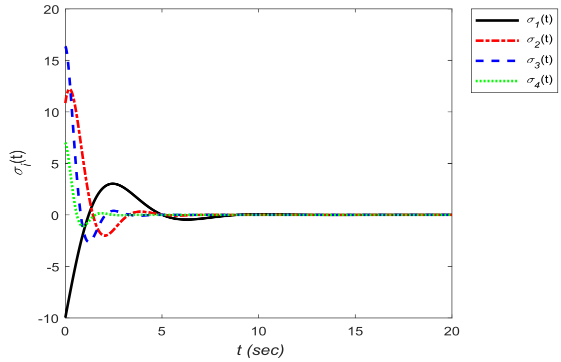

FIGURE 6. The disturbance observer errors of agents for example 1.

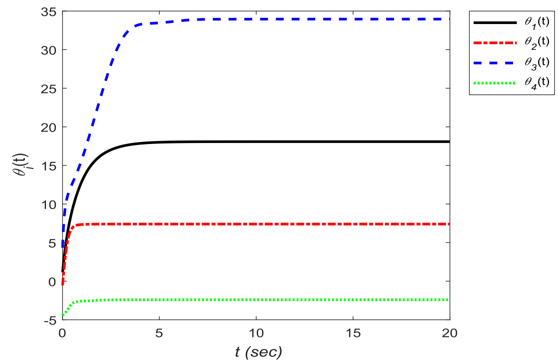

FIGURE 7. The coupling weights for example 1.

Example 1: In this example, the validity of Theorem 1 is tested. The dynamics of agents are given by (1) and (2) with a network of mass-spring systems in [35] $A = \begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \end{bmatrix}$. In addition, the disturbances $d_i(t)$ are generated by the following systems

$$\begin{bmatrix} \dot{\eta}_{i1}(t) \\ \dot{\eta}_{i2}(t) \end{bmatrix} = \begin{bmatrix} -i & i \\ -i & 0 \end{bmatrix} \begin{bmatrix} \eta_{i1}(t) \\ \eta_{i2}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0.1i\sin(\eta_{i2}(t)) \end{bmatrix},$$
$$d_i(t) = \begin{bmatrix} 2i & 0 \end{bmatrix} \begin{bmatrix} \eta_{i1}(t) \\ \eta_{i2}(t) \end{bmatrix}, i = 1, 2, 3, 4.$$

Assuming $K_1 = \begin{bmatrix} -3 & -1 \end{bmatrix}$ and $M_i = \begin{bmatrix} 0 & 0.1i \\ 0 & 0.1i \end{bmatrix}$ such that $F_i - M_iBH_i(i = 1, 2, 3, 4)$ are Hurwitz matrices, which is the premise to reach the bipartite containment control. Fig.4 and Fig.5 respectively show the state paths and state errors of agents while the disturbance observer errors are represented in Fig.6. From the Fig.4, the followers cooperating with the leaders will go to the dynamic convex hull spanned by the leaders, while the followers competing with the leaders will asymptotically go to the opposite one. And the bipartite containment error gradually tends to zero in Fig.7. Obviously, the bipartite containment control has been realised under the state-feedback controller (5) according to Fig.4 and Fig.5. The disturbance observer errors is shown in Fig.6, which means the disturbance observer is valid. The adaptive

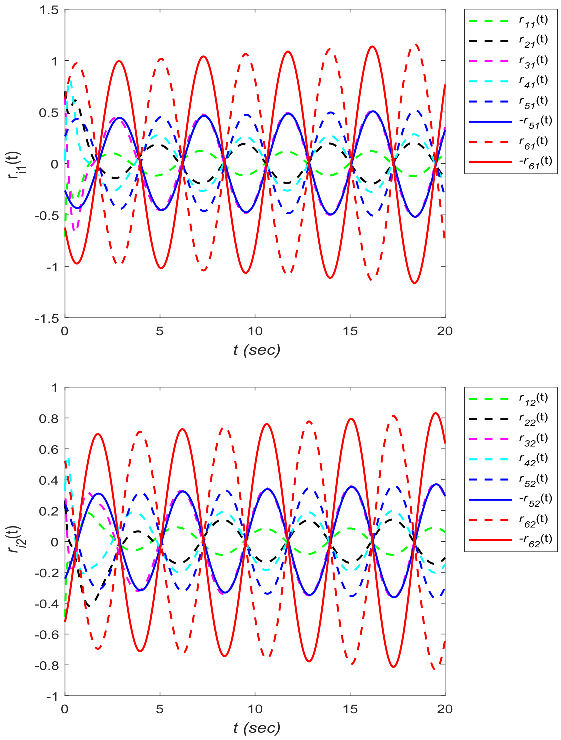

FIGURE 8. The state paths of agents for example 2.

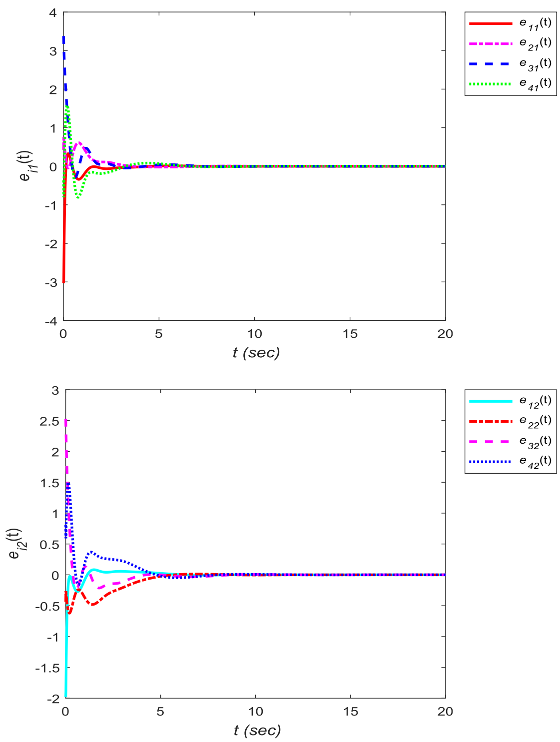

FIGURE 9. The bipartite containment errors of the MASs for example 2.

coupling weights are represented to be convergent to their own constants in Fig.7.

Example 2: The validity of Theorem 2 is verified in this example. The dynamics of agents and disturbances are the

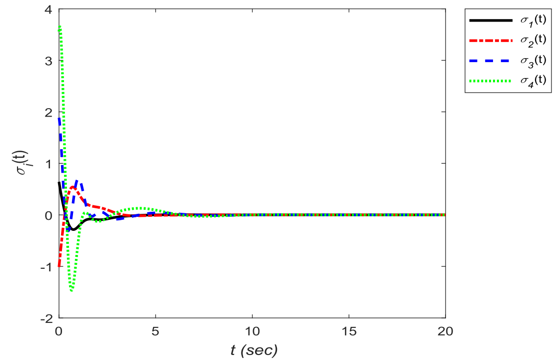

FIGURE 10. The disturbance observer errors of agents for example 2.

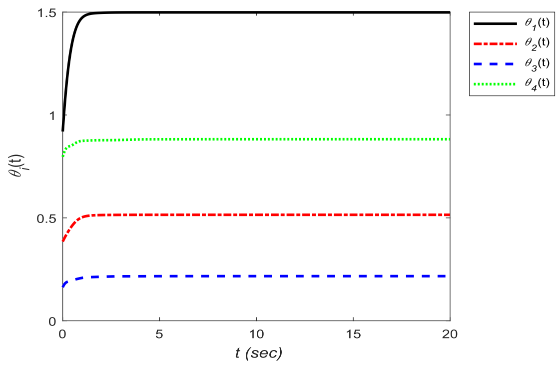

FIGURE 11. The coupling weights for example 2.

same as in Example 1. By solving the matrix inequalities (11), the feasible solutions of S, K can be elected as $S = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$, $K = \begin{bmatrix} -1.25 & -0.5 \end{bmatrix}$. Moreover, $M_i = \begin{bmatrix} 0 & 0.05i \\ 1 & -2 \end{bmatrix}$ $(i = 1, 2, 3, 4)$. The state trajectories of agents in Fig.8 and the state errors in Fig.9 illustrate that the bipartite containment control is realized for systems (1) and (2). Moreover, the state observer is valid from the fact that disturbance observer errors gradually enter into zero in Fig.9 and Fig.10. The adaptive coupling weights are no doubt to convergence to constants in Fig.11.

Example 3: In this example, the method in this paper is compared with the method in [27]. The dynamics of agents are given by (1) and (2) as the same dynamics in [35]. Considering communication connections of agents are all cooperative and the disturbances are generated by the linear exo-systems in the paper [35], the parameters $a_{ij} > 0$, $g_i^k > 0$ and $\psi_i(\eta_i(t)) = 0$ is selected in this paper. Based on this, the control gain K is selected as $K = [-3 - 1]$. Simulation results are presented in the Fig.12 and Fig.13. Evidently, the bipartite containment state errors tend to zeros and the trajectories of followers is surrounded by the trajectories of leaders in Fig.12. Therefore, it is enough to explain the method in the paper is also applied in the problem of the paper [27].

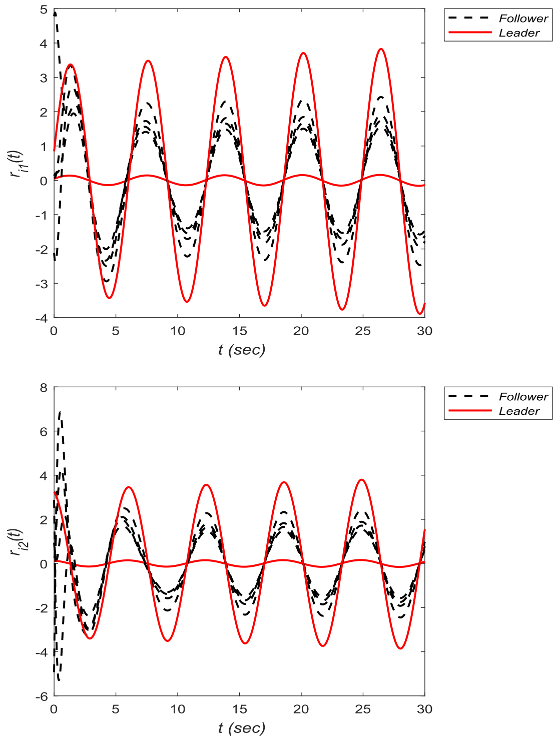

FIGURE 12. The state paths of agents for example 3.

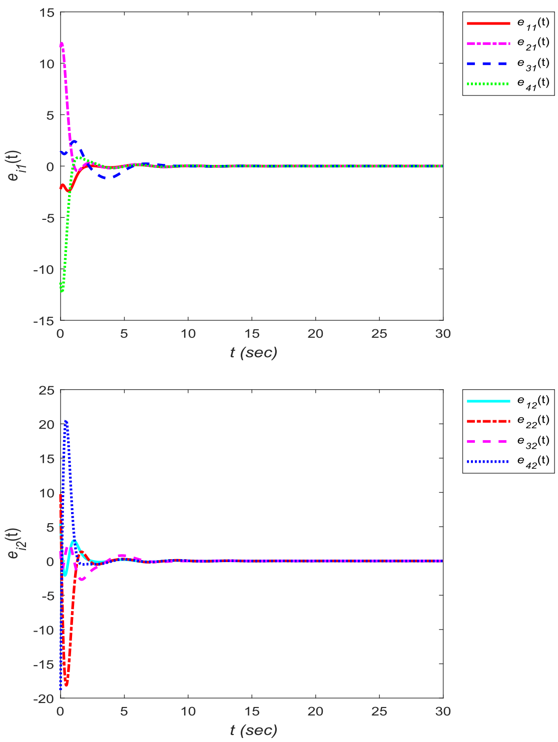

FIGURE 13. The bipartite containment errors of the MASs for example 3.

V. CONCLUSION

The bipartite containment control for MASs subject to disturbances generated by heterogeneous nonlinear exosystems is investigated in this paper. The disturbance observers with coupling gains are provided to compensate for the exogenous disturbances of MASs via adaptive control. Two different control laws are respectively provided by state-feedback method and output-feedback method. Sufficient conditions are presented to realize the bipartite containment control by Lyapunov stability theory and other mathematical analysis. Finally, numerical simulations are carried out to validate theoretical results. Future work will be on the bipartite containment for MASs with more complex disturbances, such as more general disturbances, such as stochastic disturbances [36] and completely unknown disturbances.

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