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RESEARCH ARTICLE

Robust H_∞ Fuzzy Observer-Based Fault-Tolerant Tracking Control for Nonlinear Stochastic System: A Sum of Square Approach

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ABSTRACT In this study, the robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy is proposed for the stochastic polynomial fuzzy system (SPFS) under the effect of external disturbance, measurement noise, system/sensor fault signals and continuous/discontinuous internal random fluctuations. At first, the smoothed models of fault signals are constructed to describe their dynamic behavior. Then, by integrating the SPFS with the smoothed models of fault signals as one augmented system, the state/fault signal estimation problem can be transformed to a state estimation problem of augmented system by the proposed polynomial fuzzy observer. With the utilization of estimated state/fault signals and reference trajectory, a fuzzy polynomial fault-tolerant tracking controller can be implemented. To attenuate the effect of undesired external disturbance and measurement noise on the state/fault signal estimation and tracking control performance, the robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy is proposed in this study. By utilizing the homogeneous Lyapunov function and Lipschiz condition, the Itô-Lévy formula is reformulated to relax the compensation terms of stochastic processes during the design. Then, the design conditions are derived in terms of interpolation function-dependent matrix inequality and consequently transformed to a two-step sum of square (SOS) condition design problem for the robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy. A simulation example of double lane maneuvering task for autonomous ground vehicle (AGV) is provided to illustrate the effectiveness of proposed method. **Index terms:** Observer-based tracking control, polynomial fuzzy system, stochastic control, fault-tolerant control, sum of square (SOS).

INDEX TERMS Network control system, actuator/sensor attack signals, AGV, robust observer-based tracking control, T-S fuzzy interpolation method, linear matrix inequalities.

I. INTRODUCTION

Due to the fact that most of physical plants are nonlinear, the nonlinear system theory and the nonlinear control strategies have been widely investigated in the past three decades [1]. Among various kinds of nonlinear control techniques, Takagi-Sugeno (T–S) fuzzy control approach is a popular nonlinear control technique due to its simple design procedure [2]. In the T-S fuzzy control scheme, the nonlinear system is interpolated by a set of local linearized systems with suitable interpolation functions. Then, based on the concept of parallel distributed compensation (PDC) [3], a single linear controller is constructed for each local linearized system and the fuzzy controller can be implemented by the interpolation of these local linear controllers with T-S fuzzy interpolation functions. In this situation, the design conditions can be transformed to a set of Riccati-like inequalities or linear matrix inequalities which can be easily solved via current convex optimization techniques, e.g., interior point method [4]. By applying T-S fuzzy control approach, there are a lot of fruitful results for nonlinear control design with wide applications such as the flight vehicles tracking problem [5],

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the stabilization problem of time-delay nonlinear systems [6], the sample-data control for chaotic systems [7] and fuzzy control with online membership function learning policy [8].

Recently, the polynomial fuzzy system for nonlinear system modeling approach becomes a hot topic and it can be regarded as an extension of the conventional T–S fuzzy system approach [9]. Instead of using constant local matrices in the T–S fuzzy model, the polynomial fuzzy system enables the local systems to include polynomial terms to significantly improve the accuracy of nonlinear system modeling. Consequently, the polynomial fuzzy controller or polynomial fuzzy observer can be applied for various control issues (i.e., stabilization problem [10] and estimation problem [11]). Therein, the design conditions are derived in terms of sum-of-square (SOS) conditions which could be solved via SOSTOOLS [12]. Several control design problems of fuzzy polynomial system have been investigated including the sliding mode control design [13], the observer-based control design [14] and the output–feedback control design [15]. However, by applying the polynomial matrix $P(x)$ in the Lyapunov function during the design, the partial derivative of $P(x)$ (i.e., $\frac{\partial P(x)}{\partial x_k}$ with the kth component x_k of x) induced from the gradient function $\frac{\partial x P(x) x}{\partial x}$ in the derivation process will increase the analysis difficulty. In this case, it leads to the fact that the quadratic Lyapunov function is commonly adopted in most of studies especially for complicated control problems, e.g., the output–feedback control design in [15]. Recently, to relax the using of quadratic Lyapunov function during the design, the homogeneous polynomial Lyapunov function (HPLF) has been put forward as an alternative solution [16]. By using the Euler homogeneity of HPLF, the gradient function of HPLF can be replaced by the multiplication of its Hessian matrix and state with some coefficient scaling. Consequently, the partial derivative w.r.t. the polynomial Lyapunov matrix (i.e., $\frac{\partial P(x)}{\partial x_k}$) will vanish and the design variable becomes the corresponding Hessian matrix. By using HPLF, the separation principle is successfully developed to resolve the observer-based control of polynomial fuzzy system in [17].

In general, even the polynomial fuzzy system is applicable to describe the ideal nonlinear model, some external disturbances from the environment and modeling uncertainty, which can not be well modeled, will deteriorate the control performance and have to be considered during the design. Conventionally, the robust control approach is a powerful tool to passively attenuate the effect of external disturbance and model uncertainty on the control performance [18]. However, it is not easy to solve the robust control design problem for polynomial system with the adoption of polynomial Lyapunov function. For example, in [19], the design condition is based on perturbation theory to iteratively find the polynomial Lyapunov function for the robust control design and it is hard to be implemented for practical applications. Recently, by the well properties of HPLF, the problems of robust control design for the deterministic ordinary differential system and deterministic partial differential system are investigated and the design conditions are derived in terms of solvable SOS conditions [20], [21].

Despite the effect of external disturbance on system, there may exists fault signals on system/sensor because of system component damage. Hence, the fault-tolerant control (FTC) becomes a popular field and it aims to estimate these faults for fault signal compensation [22], [24], [23], [25]. With the utilization of conventional descriptor system for fault estimation, there are some results of FTC on the polynomial fuzzy system. The descriptor-based FTC design for the polynomial fuzzy system is proposed in [26] to deal with the stabilization problem and fault compensation. Beside, in [27], the descriptor-based FTC design is proposed for the optimal tracking control problem of wind energy conversion systems. However, due to the characteristics of descriptor system and polynomial fuzzy system, the design conditions involve the polynomial equality constraints, which are not easy to be solved.

Recently, the stochastic control design is a crucial field to be addressed in modern control field. In fact, for the most of physical systems in various fields, the system characteristic randomly varies and this random fluctuation can be formulated as the random process in system modeling [28], [29], e.g., the Wiener process is used to describe the random price fluctuation in financial system [30]. Compared with the large amount of researches of T–S fuzzy stochastic control [31], [32], [33], there have very few issues about the stochastic polynomial fuzzy control [13], [34]. Therein, due to the compensation terms of stochastic process during the derivation, it makes the analysis become more difficult than the cases of deterministic polynomial system and T–S fuzzy system.

To the best of authors' knowledge, even the reference tracking problem and robust control problem have been investigated [15], [20], [34], there is no research to address the observer-based reference tracking control design for stochastic polynomial fuzzy system (SPFS) with the influence of external disturbance, measurement noise and system/sensor fault signal. Hence, an observer-based fault-tolerant tracking control strategy should be further investigated for SPFS. On the other hand, to estimate the fault signal for fault signal compensation, the conventional descriptor estimation scheme in SPFS will lead to complicated polynomial matrix equality constraints which are hard to be solved [26], [27]. Thus, it is more appealing to seek another fault estimation scheme to avoid solving complicated design conditions. Furthermore, due to the compensation terms in Itô-Lévy formula, it is difficult to use polynomial Lyapunov matrix for the design in SPFS and thus it will decrease the design flexibility for practical applications [13], [34].

Motivated by the above discussion, the robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy design is addressed for the SPFS under the effect of external disturbance, measurement noise and system/sensor fault signals. At first, by using the smoothed model in [36] to describe the behavior of system/sensor fault signals, these smoothed models can be embedded in SPFS as one augmented system for simultaneous estimation of state and system/sensor fault signals by the polynomial fuzzy observer. Then, the polynomial fuzzy tracking controller can be implemented by using the information of estimated state/fault signals and reference trajectory to achieve the desired reference trajectory tracking control and fault signal compensation. Moreover, the robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy is considered in the design of nonlinear stochastic system to attenuate the effect of external disturbance and measurement noise on the tracking/estimation performance. With the utilization of HPLF and Lipchitz condition of nonlinear system matrices, the gradient of HPLF and the compensation terms of stochastic processes can be bounded by the Hessian matrix of HPLF with some scaling. Thus, the Itô-Lévy formula can be further reformulated to a compact form containing the system matrices and Hessian matrix of HPLF. By using the reformulated Itô-Lévy formula and HPLF, the design condition of the robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy is derived as an interpolation function-dependent matrix inequality problem. To relieve the design difficulty, a two-step design procedure is developed to transform the interpolation function-based matrix inequality problem into a two-step SOS condition problem, which can be solved via SOSTOOLS in [12] to obtain the fuzzy polynomial observer gains and the fuzzy polynomial tracking controller gains. A simulation example of double-lane maneuvering task for autonomous ground vehicle (AGV) is provided to illustrate the design procedure and verify the tracking/estimation performance of proposed robust H_∞ fuzzy observer-based fault tolerant tracking control strategy.

This work is an extension of authors' previous work in [34] and the main contributions as well as the improvements of this study are summarized as follows: (i) With the consideration of robust H_∞ polynomail fuzzy observer-based fault-tolerant tracking control strategy, the designed fuzzy polynomial observer and fuzzy polynomial tracking controller are proposed for SPFS to simultaneously achieve the robust state/fault signal estimation and reference tracking control with a desired H_∞ attenuation level of external disturbance as well as the compensation of the effect of fault signals during the reference tracking control process. The proposed method provides an efficient way to estimate the fault signal in nonlinear stochastic system for the fault signal compensation purpose to achieve robust H_∞ observer-based fault-tolerant reference tracking control of nonlinear system. In this case, it can ensure a system to maintain its normal operation for the desired target tracking even the system is influenced by system/sensor fault signals. On the other hand, by utilizing the polynomial fuzzy control method, the derived fault-tolerant control/estimation scheme of nonlinear stochastic system can save more computational time for the practical industrial applications; (ii) By using the smoothed models to describe the fault signals, the design condition can be formulated in terms of polynomial inequalities which can be solved easier than the polynomial equality constraints in the descriptor design method [26], [27]; (iii) By using the HPLF and Lipchitz condition of nonlinear system matrices, the compensation terms of stochastic processes in Itô-Lévy formula are replaced by the Hessian matrix of HPLF and then a reformulated Itô -Lévy formula can be derived. In this case, the design condition can be transformed to a set of solvable two-step SOS conditions. The reformulated Itô-Lévy formula derived in this study can be regarded as a powerful analysis tool to deal with various control design issues and more relaxed design conditions for SPFS, e.g., the reference tracking control in [34] can be extended to the SOS design conditions with polynomial Lyapunov matrix.

The organization of this study is given as follows: In Section II, the preliminary of SPFS is given and the smoothed models are introduced to describe the fault signals. Also, the robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy is introduced. With the help of HPLF, the design condition is derived as a interpolation function-based matrix inequality in Section III and consequently a two-step design procedure is developed to transform the polynomial matrix inequalities into a two-step SOS condition problem. In Section IV, a simulation example of the maneuvering task for AGV system is given for the performance validation. Conclusion is made in Section V.

Notation: $P > 0$ ($P \geq 0$) : The positive definite (semi-definite) matrix P; $E\{\cdot\}$: The expectation operator; $(\Omega, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{F}, \mathcal{P})$: The complete probability space with sample space Ω, non-decreasing set of $\sigma-$algebras $\{\mathcal{F}_t\}_{t\geq 0}$, $\mathcal{F} = \cup_{t\geq 0}\mathcal{F}_t$ and probability measure $\mathcal{P}$; $\|x(t)\|_{2,\mathcal{F}} = E\{[\int_0^\infty x^T(t)x(t))dt]^{\frac{1}{2}}\}$; $\mathcal{L}_2^{\mathcal{F}}(\mathbb{R}^+, \mathbb{R}^n) = \{x(t)|\ \|x(t)\|_{2,\mathcal{F}} < \infty\}$; $\|x(t)\|_2^D = \sqrt{x^T(t)x(t)}$; $C_{n,m}^k$: The function space which collects the differentiable functions $f(x) : \mathbb{R}^n \rightarrow \mathbb{R}^m$ of order k; $\frac{\partial V(x)}{\partial x}$:The gradient column vector of differentiable function $V(x)$ w.r.t. x; $\frac{\partial^2 V(x)}{\partial^2 x}$: The Hessian matrix of twice differentiable function $V(x)$ w.r.t. x; I_n : n-dimension identity matrix; $0_{a\times b}$: $a \times b$ zero matrix; $[A^T B + (*)] + C$: The abbreviation of matrix operation $A^T B + B^T A + C$; A polynomial function $f(x)$ is an SOS if there exist polynomials $\{f_i(x)\}_{i=1}^m$ such that $f(x) = \sum_{i=1}^m f_i^2(x)$.

II. PRELIMINARY

In this section, the SPFS will be introduced to model the nonlinear stochastic system under the effect of external disturbance, measurement noise and system/sensor fault signals. Then, for the state/fault estimation purpose and the fault-tolerant tracking purposes, the polynomial fuzzy observer, polynomial reference model and the polynomial observer-based controller will be discussed. To attenuate the effect of unavailable external disturbance and measurement noise on the tracking/estimation performance, a robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy of SPFS is introduced.

A. STOCHASTIC POLYNOMIAL FUZZY SYSTEMS AND SMOOTHED SIGNAL MODEL

Consider the following nonlinear stochastic system with external disturbance, measurement noise and system/sensor fault signals:

$$\begin{aligned} dx(t) &= [f(x(t)) + g(x(t))u(t) + d_a(x(t))f_a(t) + v(t)]dt \\ &\quad + h(x(t))dw(t) + o(x(t))dp(t) \\ x(0) &= x_0 \\ y(t) &= c(x(t)) + d_s(x(t))f_s(t) + n(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state with initial state x_0, $u(t) \in \mathbb{R}^{n_u}$ denotes the control input, $v(t) \in \mathcal{L}_2^{\mathcal{F}}(\mathbb{R}^+, \mathbb{R}^n)$ represents the external disturbance, $f_a(t) \in C^1_{1,n_a}$ is the system fault signal, $c(x(t)) \in \mathbb{R}^m$ is the nonlinear measurement output, $f_s(t) \in C^1_{1,m_s}$ is the sensor fault signal, $n(t) \in \mathcal{L}_2^{\mathcal{F}}(\mathbb{R}^+, \mathbb{R}^m)$ represents the measurement noise, $w(t)$ is the 1-D Wiener process and $p(t)$ is the Poisson counting process with jump intensity $\lambda > 0$. These two processes $\{w(t), p(t)\}$ are defined on a complete probability space $(\Omega, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{F}, \mathcal{P})$ in which $\sigma-$algebra $\mathcal{F}_t$ is generated by $\{w(s), p(s)\}_{0\leq s\leq t}$ with $w(s)$ and $p(s)$ being assumed to be independent. The nonlinear functional matrices $\{f(x(t)), g(x(t)), d_a(x(t)), h(x(t)), o(x(t)), c(x(t)), d_s(x(t))\}$ are satisfied with (i) Lipschitz condition and (ii) growth condition (Section 6.2 in [28]). In (1), $h(x(t))dw(t)$ is used to describe the state-dependent continuous fluctuations (e.g., the thermal noise of resistance unit in circuit system) and $o(x(t))dp(t)$ can be regarded as the state-dependent discontinuous jump behavior (e.g., the short-circuit phenomenon).

Remark 1: [29] *The stochastic processes in (1) satisfy with the following properties: (i) $E\{w(t)\} = E\{dw(t)\} = 0$, (ii) $E\{dw^2(t)\} = dt$, (iii) $E\{dp(t)\} = \lambda dt$ with jump intensity $\lambda > 0$.*

Under the concept of sector nonlinearity [9], the following plant rules of SPFS is presented to approximate the nonlinear stochastic system in (1)

The i*th Plant Ruleif* $\omega_1(t)$ is $\varpi_{i,1}, \cdots, \omega_g(t)$ is $\varpi_{i,g}$

$$\begin{aligned} dx(t) &= [A_i(x(t))x(t) + B_i(x(t))u(t) + G_i(x(t))f_a(t) \\ &\quad + v(t)]dt + H_i(x(t))x(t)dw(t) \\ &\quad + O_i(x(t))x(t)dp(t) \\ y(t) &= C_i(x(t))x(t) + S_i(x(t))f_s(t) + n(t), \\ &\forall i = 1, \cdots, l \end{aligned} \tag{2}$$

where $\{\omega_i(t)\}_{i=1}^g$ are premise variables, g is the number of premise variables, $\varpi_{i,j}$ is the membership function of the ith rule associated with the jth premise variables, l is the number of plant rules and $\{A_i(x(t)), B_i(x(t)), G_i(x(t)), H_i(x(t)), O_i(x(t)), C_i(x(t)), S_i(x(t))\}_{i=1}^l$ are the polynomial matrices with appropriate dimensions.

Then, the following SPFS can be inferred to represent the nonlinear stochastic system in (1) [9]

$$\begin{aligned} dx(t) &= \sum_{i=1}^l h_i(\omega(t))\{[A_i(x(t))x(t) + B_i(x(t))u(t) \\ &\quad + G_i(x(t))f_a(t) + v(t)]dt \\ &\quad + H_i(x(t))x(t)dw(t) + O_i(x(t))x(t)dp(t)\} \\ y(t) &= \sum_{i=1}^l h_i(\omega(t))[C_i(x(t))x(t) \\ &\quad + S_i(x(t))f_s(t) + n(t)] \end{aligned} \tag{3}$$

with

$$h_i(\omega(t)) = \frac{\prod_{j=1}^g \varpi_{i,j}(\omega_j(t))}{\sum_{i=1}^l \prod_{j=1}^g \varpi_{i,j}(\omega_j(t))}$$

where $\omega(t) = [\omega_1(t), \cdots, \omega_g(t)]$, $\varpi_{i,j}(\omega_j(t))$ is the grade of membership function of the ith plant rule w.r.t. the jth premise variable and $\{h_i(\omega(t))\}_{i=1}^l$ are the fuzzy interpolation functions. To ensure the completeness of SPFS in (3), the following assumption is provided

Assumption 1: [9] *The fuzzy interpolation functions $\{h_i(\omega(t))\}_{i=1}^l$ satisfy with the following conditions: (i) $h_i(\omega(t)) \geq 0, \forall i = 1, \cdots, l$ and (ii) $\sum_{i=1}^l h_i(\omega(t)) = 1$.*

In general, to estimate the fault signals $\{f_a(t), f_s(t)\}$ for the fault signal compensation, these fault signals should be modeled and augmented with SPFS for the estimation purpose. Without using the conventional descriptor model [35], the following smoothed models in [36] are applied to describe the dynamic behavior of fault signals $\{f_a(t), f_s(t)\}$. To begin with, by the right derivative of system fault signal $f_a(t)$ (i.e., $\dot{f}_a(t) = \lim_{h\to 0} \frac{f_a(t+h)-f_a(t)}{h}$), we can construct the following relations of system fault signal

$$\begin{aligned} \dot{f}_a(t) &= \frac{f_a(t+h) - f_a(t)}{h} + \delta_{a,0}(t) \\ &\vdots \\ \dot{f}_a(t-kh) &= \frac{f_a(t-(k-1)h) - f_a(t-kh)}{h} + \delta_{a,k}(t) \end{aligned} \tag{4}$$

where $h > 0$ denotes small time interval, $k \in \mathbb{N}$ is the number of delay sample and $\{\delta_{a,i}(t)\}_{i=0}^k$ are the derivative approximation errors of $\{f_a(t-ih)\}_{i=0}^k$. Moreover, based on linear extrapolation method, the future system fault signal $f_a(t+h)$ can be extrapolated as follows

$$f_a(t+h) = \sum_{i=0}^k \alpha_i f_a(t-ih) + \gamma_a(t) \tag{5}$$

where $\{\alpha_i\}_{i=0}^k$ are the extrapolation coefficients of system fault signal $f_a(t+h)$ and $\gamma_a(t)$ is the extrapolation error of system fault signal $f_a(t+h)$.

By combining (4) and (5), the smoothed model of system fault signal $f_a(t)$ can be constructed as

$$dF_a(t) = [A_a F_a(t) + \bar{\delta}_a(t)]dt \tag{6}$$

where $F_a(t) = [f_a^T(t) \cdots f_a^T(t-kh)]^T$, $\bar{\delta}_a(t) = [(\gamma_a(t)/h + \delta_{a,0}(t))^T \ \delta_{a,1}^T(t) \cdots \delta_{a,k}^T(t)]^T$ and

$$A_a = \begin{bmatrix} \frac{-1+\alpha_0}{h}I & \frac{\alpha_1}{h}I & \cdots\cdots & \frac{\alpha_k}{h}I \\ \frac{1}{h}I & -\frac{1}{h}I & \cdots\cdots & 0 \\ 0 & \frac{1}{h}I & \ddots \cdots & 0 \\ \vdots & \vdots & \vdots \ \ddots & 0 \\ 0 & 0 & \cdots \ \frac{1}{h}I & -\frac{1}{h}I \end{bmatrix}$$

Remark 2: To construct the dynamic behavior of system fault signal $f_a(t)$, the right derivative technique is adopted in (4) with the consideration of derivative error $\{\delta_{a,i}(t)\}_{i=0}^k$. Even the dynamics of $f_a(t), \cdots, f_a(t-kh)$ are constructed, the term $f_a(t+h)$ can not be regarded as state variable in (4). To deal with this problem, the extrapolation method (i.e., Richardson extrapolation [37]) is applied in (5) and $f_a(t+h)$ can be represented as the combination of weighting sum of $f_a(t), \cdots, f_a(t-kh)$ with an extrapolation error $\gamma_a(t)$. In general, it is impossible to select a set of fixed extrapolation coefficients $\{\alpha_i\}_{i=0}^k$ to make $\gamma_a(t) = 0$ for arbitrary fault signal $f_a(t)$. Thus, a possible selection of extrapolation coefficients is given as (i) $\alpha_{i-1} \geq \alpha_i, \forall i = 1, \cdots, k$ and (ii) $\sum_{i=0}^k \alpha_i = 1$. For the first property, due to the continuity of $f_a(t)$, the future sample signal $f_a(t+kh)$ is more related to the current sample signal $f_a(t)$. Also, to avoid over-extrapolation, the sum of these coefficients should be normalized with one [37].

Similar to the above procedure, the smoothed model of sensor fault signal $f_s(t)$ can be constructed as

$$dF_s(t) = [A_s F_s(t) + \bar{\delta}_s(t)]dt \tag{7}$$

where $F_s(t) = [f_s^T(t) \cdots f_s^T(t-kh)]^T$, $\bar{\delta}_s(t) = [(\gamma_s(t)/h + \delta_{s,0}(t))^T \ \delta_{s,1}^T(t) \cdots \delta_{s,k}^T(t)]^T$ with the derivative approximation errors $\{\delta_{s,i}(t)\}_{i=0}^k$ and time-varying extrapolation error of sensor fault signal $\gamma_s(t)$. The system matrix in (7) is defined as

$$A_s = \begin{bmatrix} \frac{-1+\beta_0}{h}I & \frac{\beta_1}{h}I & \cdots\cdots & \frac{\beta_k}{h}I \\ \frac{1}{h}I & -\frac{1}{h}I & \cdots\cdots & 0 \\ 0 & \frac{1}{h}I & \ddots \cdots & 0 \\ \vdots & \vdots & \vdots \ \ddots & 0 \\ 0 & 0 & \cdots \ \frac{1}{h}I & -\frac{1}{h}I \end{bmatrix}$$

with the extrapolation coefficients of sensor fault signal $\{\beta_i\}_{i=0}^k$. In the following discussion, the notation t is dropped and the membership function $h_i(\omega)$ is abbreviated as h_i to lighten the notation, e.g., $x(t) \to x$ and $h_i(\omega(t)) \to h_i$.

By augmenting the SPFS in (3), the smoothed model of system fault in (6) and smoothed model of sensor fault in (7), we have the following augmented stochastic system

$$\begin{aligned} d\bar{x} &= \sum_{i=1}^l h_i\{[\bar{A}_i(\bar{x})\bar{x} + \bar{B}_i(\bar{x})u + D_1\bar{v}]dt \\ &\quad + \bar{H}_i(\bar{x})\bar{x}dw + \bar{O}_i(\bar{x})\bar{x}(t)dp\} \\ y &= \sum_{i=1}^l h_i[\bar{C}_i(\bar{x})\bar{x} + D_2\bar{v}] \end{aligned} \tag{8}$$

where $\bar{x} = [x^T \ F_a^T \ F_s^T]^T$ and $\bar{v} = [v^T \ \bar{\delta}_a^T \ \bar{\delta}_s^T \ n^T]^T$. The detailed polynomial system matrices in (8) are given as

$$\bar{A}_i(\bar{x}) = \begin{bmatrix} A_i(x) & G_i(x)T_a & 0 \\ 0 & A_a & 0 \\ 0 & 0 & A_s \end{bmatrix}, \bar{B}_i(\bar{x}) = \begin{bmatrix} B_i(x) \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{H}_i(\bar{x}) = diag\{H_i(x), 0, 0\}, \bar{O}_i(\bar{x}) = diag\{O_i(x), 0, 0\}$$

$$D_1 = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix}, D_2 = \begin{bmatrix} 0 & 0 & 0 & I \end{bmatrix}$$

$$\bar{C}_i(\bar{x}) = [C_i(x) \ 0 \ S_i(x)T_s],$$

$$T_a = [I \ 0 \ \cdots \ 0], T_s = [I \ 0 \ \cdots \ 0]$$

Remark 3: In this study, the system/sensor fault signals are modeled in (6), (7) based on extrapolation method and smoothed delay methods for the convenience of their estimations. Then, the system state is augmented with system/sensor fault signal as one augmented system in (8) for joint state/fault signal estimation, i.e., the estimation of augmented system in (8) is equivalent to the estimation of system state x and system/sensor fault signals $\{f_a, f_s\}$ in Eq. (3). This method is not only to estimate system state and fault signal simultaneously, but also to avoid the corruption of fault signals on the state estimation by direct estimation from nonlinear stochastic system in (1). Besides, from the system structure of augmented system in (8), the augmented external disturbance $\bar{v}$ involves external disturbance, measurement noise and extrapolation errors of system/sensor fault signal. This fact directly implies the estimation of augmented system will be influenced by $\bar{v}$ and thus the robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy in (15) is considered to passively attenuate the effect of $\bar{v}$ on the state/fault signal estimation and reference tracking simultaneously.

By constructing the augmented stochastic system in (8), the state/fault signal estimation is equivalent to the state estimation of the augmented system in (8). In general, due to the nonliearity of augmented polynomial system matrices $\bar{A}_i(\bar{x})$ and $\bar{C}_i(\bar{x})$, the observability of augmented system in (8) can not be easily ensured by the conventional rank test condition. To address this issue, the following assumption is made.

Assumption 2: The augmeneted system in (8) is observable.

Then, the following observer rules of polynomial fuzzy observer for the augmented system in (8) can be given as

$$\begin{aligned} &\textit{The } j\text{th } \textit{Observer Rule} \\ &\textit{if } \omega_1(t) \text{ is } \varpi_{i,1}, \cdots, \omega_g(t) \text{ is } \varpi_{i,g} \\ &d\hat{\bar{x}} = [\bar{A}_j(\hat{\bar{x}})\hat{\bar{x}} + \bar{B}_j(\hat{\bar{x}})u + L_j(\hat{\bar{x}})(y - \hat{y})]dt \\ &\hat{y} = \bar{C}_j(\hat{\bar{x}})\hat{\bar{x}}, \ \textit{for } j = 1, \cdots, l \end{aligned} \tag{9}$$

where $\hat{\bar{x}}$ is the estimation of $\bar{x}$, $\hat{y}$ is the estimated measurement output of y and $\{L_j(\hat{\bar{x}})\}_{j=1}^l$ are the polynomial fuzzy observer gains, and then the polynomial fuzzy observer can be inferred as follows

$$d\hat{\bar{x}} = \sum_i^l h_i[\bar{A}_i(\hat{\bar{x}})\hat{\bar{x}} + \bar{B}_i(\hat{\bar{x}})u + L_i(\hat{\bar{x}})(y - \hat{y})]dt$$

$$\hat{\bar{x}}(0) = \hat{\bar{x}}_0$$
$$\hat{y}(t) = \sum_{i=1}^{l} h_i \bar{C}_i(\hat{\bar{x}})\hat{\bar{x}} \tag{10}$$

where $\hat{x}_0$ is the initial condition of polynomial fuzzy observer in (10).

Remark 4: Due to the sensor fault signal f_s and measurement noise n in measurement output y in (3), the exact state variables are almost impossible to be obtained by the output sensor. Thus, if the premise variables are associated with state variable, the polynomial matrices of SPFS in (3) and fuzzy polynomial observer in (10) are dependent of state $\bar{x}$ and estimation $\hat{\bar{x}}$, respectively.

Remark 5: Since the fault signals $\{f_a, f_s\}$ are embedded in the state $\bar{x}$ of augmented system in (8), the corruption of $\{f_a, f_s\}$ on the state estimation in (1) or (3) can be avoided in the augmented state estimation in (8). Moreover, the state/fault signal estimation problem of nonlinear system (3) can be transformed to the state estimation problem of the augmented system in (8) by the polynomial fuzzy observer in (10).

Remark 6: In (10), the polynomial observer estimates the augmented state (i.e., system state, system fault signal and sensor fault signal). Indeed, if the observer gains and other system matrices are degenerated to constant matrices, the observer in (10) becomes conventional fuzzy observer. Compared with ordinary fuzzy observers, the H_∞ polynomial fuzzy observer including polynomial terms in observer gain will provide more design flexibility and less conservative ability by conventional H_∞ observer-based reference tracking control design for practical industrial applications.

To generate the reference tracking trajectory of nonlinear stochastic system in (1), the following reference model is adopted

$$dx_r = [A_r(x_r)x_r + B_r(x_r)r]dt$$
$$x_r(0) = x_{r,0} \tag{11}$$

where x_r is the reference trajectory to be tracked by x with the initial condition $x_{r,0}$ in (1), $r \in \mathcal{L}_2^{\mathcal{F}}(\mathbb{R}^+, \mathbb{R}^{n_r})$ is the reference input, $A_r(x_r)$ is the stable polynomial system matrix in (11) and $B_r(x_r)$ denotes the polynomial input matrix.

Remark 7: For the reference model in (11), the desired tracking trajectory x_r is generated by the external reference input r, which is specified by the designer beforehand. Especially, the matrices $\{A_r(x_r), B_r(x_r)\}$ will influence on the transient-response of (11) and it should be carefully chosen to meet the design condition. In the steady state, the relationship between reference tracking trajectory and reference input can be derived as $x_r = -A_r^{-1}(x_r)B_r(x_r)r$. If the designer wants the reference tracking trajectory x_r to approach the reference input r at steady state, one possible selection of system matrices is $A_r(x_r) = -B_r(x_r)$.

For the design purpose of observer-based fault-tolerant tracking control for the augmented stochastic system in (8), the reference model in (11) is extended as follows

$$d\bar{x}_r = [\bar{A}_r(\bar{x}_r)\bar{x}_r + \bar{B}_r(\bar{x}_r)r]dt \tag{12}$$

where $\bar{x}_r = [x_r^T \ x_{r,v_1}^T \ x_{r,v_2}^T]^T$ with the virtual states $\{x_{r,v_1} \in \mathbb{R}^{n_a(k+1)}, x_{r,v_2} \in \mathbb{R}^{m_s(k+1)}\}$, $\bar{A}_r(\bar{x}_r) = diag\{A_r(x_r), A_{r,v_1}, A_{r,v_2}\}$ with two Hurwiz matrices $\{A_{r,v_1}, A_{r,v_2}\}$ and $\bar{B}_r(\bar{x}_r) = [B_r^T(x_r) \ 0 \ 0]^T$.

Remark 8: By using the extended reference model in (12), it can avoid the complicated coordinate transformation during the control strategy design. Also, from the system structure in (12), it can be noticed that the last two augmented states x_{r,v_1} and x_{r,v_2} in $\bar{x}_r$ are zero for anytime. To ensure the stability property of $\bar{A}_r(\bar{x}_r)$, the system matrices A_{r,v_1} and A_{r,v_2} associated in x_{r,v_1} and x_{r,v_2} are selected as Hurwitz matrices during the design, respectively.

By the estimated state/fault signal in (10) and reference trajectory in (12), the following control rules of polynomial observer-based fault-tolerant tracking controller are given

The j*th Controller Rule*

if $\omega_1(t)$ is $\varpi_{i,1}, \cdots, \omega_g(t)$ is $\varpi_{i,g}$

$$u = K_{j,1}(\hat{\bar{x}}, \bar{x}_r)(\hat{\bar{x}} - \bar{x}_r) + K_{j,2}(\hat{\bar{x}}, \bar{x}_r)\bar{x}_r,$$
$$\text{for } j = 1, \cdots, l \tag{13}$$

with fuzzy polynomial controller gains $\{K_{j,1}(\hat{\bar{x}}, \bar{x}_r), K_{j,2}(\hat{\bar{x}}, \bar{x}_r)\}_{j=1}^{l}$, and then the overall polynomial fuzzy observer-based fault-tolerant tracking controller can be inferred as follows

$$u = \sum_{j=1}^{l} h_j[K_{j,1}(\hat{\bar{x}}, \bar{x}_r)(\hat{\bar{x}} - \bar{x}_r) + K_{j,2}(\hat{\bar{x}}, \bar{x}_r)\bar{x}_r] \tag{14}$$

B. ROBUST H_∞ OBSERVER-BASED FAULT-TOLERANT TRACKING CONTROL AND USEFUL LEMMAS

In general, due to the unavailable external disturbance v and measurement noise n in (1) with the unpredictable approximation errors $\{\bar{\delta}_a, \bar{\delta}_s\}$ in the smooth models in (6), (7), the estimation/tracking performance for polynomial observer in (10) and polynomial observer-based tracking controller in (14) of SPFS in (3) will be deteriorated simultaneously. To have a careful consideration of these undesirable effects, the following robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy is proposed with a prescribed attenuation level $\rho > 0$

$$J_\infty(\{K_{i,1}(\hat{\bar{x}}, \bar{x}_r), K_{i,2}(\hat{\bar{x}}, \bar{x}_r), L_i(\hat{\bar{x}})\}_{i=1}^{l})$$
$$= \sup_{\substack{\bar{v} \in \mathcal{L}_2^{\mathcal{F}}(\mathbb{R}^+, \mathbb{R}^{n}) \\ r \in \mathcal{L}_2^{\mathcal{F}}(\mathbb{R}^+, \mathbb{R}^{n_r})}} \frac{\begin{array}{l} E\{\int_0^{t_f}[(\bar{x} - \hat{\bar{x}})^T Q_E(\bar{x} - \hat{\bar{x}}) \\ +(x - x_r)^T Q_T(x - x_r) + u^T R u]dt \\ -V_1(x_0, x_{r,0}, \hat{x}_0)\} \end{array}}{E\{\int_0^{t_f} \bar{v}^T\bar{v} + r^T r dt\}} \le \rho \tag{15}$$

where $Q_E \ge 0$ and $Q_T \ge 0$ are the weighting matrix w.r.t. the estimation error $\bar{x} - \hat{\bar{x}}$ and the tracking error $x - x_r$, respectively, $R > 0$ is the control weighing matrix, $V_1(x_0, x_{r,0}, \hat{x}_0)$ is the effect of initial condition to be deducted and $t_f > 0$ is the terminal time. For the robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy in (15), if one could specify a set of fuzzy polynomial control gains and polynomial observer gains $\{K_{i,1}^*(\hat{\bar{x}}, \bar{x}_r), K_{i,2}^*(\hat{\bar{x}}, \bar{x}_r), L_i^*(\hat{\bar{x}})\}_{i=1}^l$ such that $J_\infty(\{K_{i,1}^*(\hat{\bar{x}}, \bar{x}_r), K_{i,2}^*(\hat{\bar{x}}, \bar{x}_r), L_i^*(\hat{\bar{x}})\}_{i=1}^l) \leq \rho$, for a prescribed atteuation level $\rho > 0$, then the effect of augmented noise $\bar{v}$ and arbitrary reference input r on the estimation/tracking performance in (15) can be attenuated under a prescribed disturbance attenuation level ρ from the view point of energy.

The following lemmas are proposed for the derivation in the sequel

Lemma 1: [4] Given two square matrices $\{X, Y\}$ and a positive definite matrix P, the following inequality holds

$$X^TY + Y^TX \leq X^TPX + Y^TP^{-1}Y \tag{16}$$

Lemma 2: [4] Given a set of matrices $\{X_i\}_{i=1}^l$, a positive definite matrix P and a non-negative series $\{\alpha_i\}_{i=1}^l$ with $\sum_{i=1}^l \alpha_i = 1$, the following matrix inequality holds

$$\sum_{i,j=1}^l \alpha_i\alpha_j X_i^TPX_j \leq \sum_{i=1}^l \alpha_i X_i^TPX_i \tag{17}$$

III. ROBUST H$_\infty$ FUZZY OBSERVER-BASED FAULT-TOLERANT TRACKING CONTROL DESIGN OF SPFS

In this section, the design of robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy for SPFS will be addressed. To simplify the design procedure, the estimation/tracking problem for SPFS in (3) and fuzzy polynomial observer in (10) can be transformed to simple equivalent stabilization problem of an augmented system. Then, based on the Itô-Lévy formula and polynomial Lyapunov function, the design condition of robust H_∞ fuzzy observer-based fault-tolerant tracking control can be derived as a set of coupled polynomial matrix inequalities. Further, by utilizing the homogeneous polynomial Lyapunov function and two–step design procedure, the design condition can be transformed to solvable SOS conditions.

Define estimation vector as $e = \bar{x} - \hat{\bar{x}}$ then the corresponding tracking error dynamic can be obtained by subtracting the polynomial observer in (10) from the augmented SPFS in (8)

$$\begin{aligned} de = \sum_{j,i=1}^l h_ih_j\{[\bar{A}_i(\bar{x})\bar{x} - \bar{A}_j(\hat{\bar{x}})\hat{\bar{x}} - L_j(\hat{\bar{x}})(\bar{C}_i(\bar{x})\bar{x} \\ + D_2\bar{v} - \bar{C}_j(\hat{\bar{x}})\hat{\bar{x}})] + (\bar{B}_i(\bar{x}) - \bar{B}_j(\hat{\bar{x}}))u \\ + D_1\bar{v}]dt + \bar{H}_i(\bar{x})\bar{x}dw + \bar{O}_i(\bar{x})\bar{x}(t)dp\} \end{aligned} \tag{18}$$

Then, by letting $\tilde{x} = [\bar{x}_r^T \ \bar{x}^T \ e^T]^T$, the augmented SPFS can be inferred by (8), (12) and (18)

$$\begin{aligned} d\tilde{x} = \sum_{j,i=1}^l h_ih_j\{[\tilde{A}_{ij}(\tilde{x})\tilde{x} + \tilde{D}_i(\tilde{x})\tilde{v}]dt \\ + \tilde{H}_i(\tilde{x})\tilde{x}dw + \tilde{O}_i(\tilde{x})\tilde{x}dp\} \end{aligned} \tag{19}$$

where $\tilde{v} = [\bar{v}^T \ r^T]^T$. The detailed system matrices in (19) are given as

$$\tilde{A}_{ij}(\tilde{x}) = \begin{bmatrix} \bar{A}_r(\bar{x}_r) & 0 & 0 \\ \tilde{A}_{ij}^{(1)}(\tilde{x}) & \tilde{A}_{ij}^{(2)}(\tilde{x}) & \tilde{A}_{ij}^{(3)}(\tilde{x}) \\ \tilde{A}_{ij}^{(4)}(\tilde{x}) & \tilde{A}_{ij}^{(5)}(\tilde{x}) & \tilde{A}_{ij}^{(6)}(\tilde{x}) \end{bmatrix}$$

$$\tilde{D}_i(\tilde{x}) = \begin{bmatrix} 0 & \bar{B}_r(\bar{x}_r) \\ D_1 & 0 \\ D_1 - L_j(\hat{\bar{x}})D_2 & 0 \end{bmatrix}$$

$$\tilde{H}_i(\tilde{x}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \bar{H}_i(\bar{x}) & 0 \\ 0 & \bar{H}_i(\bar{x}) & 0 \end{bmatrix}, \tilde{O}_i(\tilde{x}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \bar{O}_i(\bar{x}) & 0 \\ 0 & \bar{O}_i(\bar{x}) & 0 \end{bmatrix}$$

where $\tilde{A}_{ij}^{(1)}(\tilde{x}) = \bar{B}_i(\bar{x})(K_{j,2}(\hat{\bar{x}}, \bar{x}_r) - K_{j,1}(\hat{\bar{x}}, \bar{x}_r))$, $\tilde{A}_{ij}^{(2)}(\tilde{x}) = \bar{A}_i(\bar{x}) + \bar{B}_i(\bar{x})K_{j,1}(\hat{\bar{x}}, \bar{x}_r)$, $\tilde{A}_{ij}^{(3)}(\tilde{x}) = -\bar{B}_i(\bar{x})K_{j,1}(\hat{\bar{x}}, \bar{x}_r)$, $\tilde{A}_{ij}^{(4)}(\tilde{x}) = (\bar{B}_i(\bar{x}) - \bar{B}_j(\hat{\bar{x}}))(K_{j,2}(\hat{\bar{x}}, \bar{x}_r) - K_{j,1}(\hat{\bar{x}}, \bar{x}_r))$, $\tilde{A}_{ij}^{(5)}(\tilde{x}) = \bar{A}_i(\bar{x}) - \bar{A}_j(\hat{\bar{x}}) - L_j(\hat{\bar{x}})(\bar{C}_i(\bar{x}) - \bar{C}_j(\hat{\bar{x}})) + (\bar{B}_i(\bar{x}) - \bar{B}_j(\hat{\bar{x}}))K_{j,1}(\hat{\bar{x}}, \bar{x}_r)$, $\tilde{A}_{ij}^{(6)}(\tilde{x}) = \bar{A}_j(\hat{\bar{x}}) - L_j(\hat{\bar{x}})\bar{C}_j(\hat{\bar{x}}) - (\bar{B}_i(\bar{x}) - \bar{B}_j(\hat{\bar{x}}))K_{j,1}(\hat{\bar{x}}, \bar{x}_r)$.

With the augmented SPFS in (19), the robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy in (15) can be rewritten as

$$\begin{aligned} & J_\infty(\{K_{i,1}(\hat{\bar{x}}, \bar{x}_r), K_{i,2}(\hat{\bar{x}}, \bar{x}_r), L_i(\hat{\bar{x}})\}_{i=1}^l) \\ & = \sup_{\tilde{v} \in \mathcal{L}_2^{\mathcal{F}}(\mathbb{R}^+, \mathbb{R}^{n_{\tilde{v}}})} \frac{E\{\int_0^{t_f} [\tilde{x}^T\tilde{Q}\tilde{x} + u^TRu]dt - V(\tilde{x}(0))\}}{E\{\int_0^{t_f} \tilde{v}^T\tilde{v}dt\}} \\ & \leq \rho \\ \bar{Q}_T & = diag\{I_n, 0_{n_a(k+1)\times n_a(k+1)}, 0_{m_s(k+1)\times m_s(k+1)}\} \\ \tilde{Q} & = diag\{[I, -I]^T\bar{Q}_T[I, -I], Q_E\} \end{aligned} \tag{20}$$

where $n_{\tilde{v}}$ is the dimension of $\tilde{v}$ and $V(\tilde{x}(0)) = V_1(x_0, x_{r,0}, \hat{\bar{x}}_0)$. With the augmented SPFS in (19), the robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy in (15) can be transformed to the robust H_∞ fuzzy stabilization strategy in (20) of the augmented SPFS in (19).

In general, due to the stochastic processes (i.e., Wiener process w and Poisson counting process p) of SPFS in (3), the dynamic behavior analysis of augmented SPFS in (19) can not be done by the conventional calculus technique. Thus, the following Itô-Lévy formula is proposed

Lemma 3: (Theorem 1.16 in [29]) Given a Lyapunov function $V(\tilde{x}) \in C_{n_{aug},1}^2$ of SPFS in (19), which satisfies (i) $V(\tilde{x}) \geq 0$, (ii) $V(0) = 0$, then the increment of $V(\tilde{x})$ w.r.t. the augmented SPFS in (19) can be derived as follows:

$$\begin{aligned} dV(\tilde{x}) = \sum_{i,j=1}^l h_ih_j\{[(\frac{\partial V(\tilde{x})}{\partial \tilde{x}})^T(\tilde{A}_{ij}(\tilde{x})\tilde{x} + \tilde{D}_i(\tilde{x})\tilde{v}) \\ + \frac{1}{2}(\tilde{H}_j(\tilde{x})\tilde{x})^T(\frac{\partial^2 V(\tilde{x})}{\partial^2 \tilde{x}})(\tilde{H}_i(\tilde{x})\tilde{x})]dt + \tilde{H}_i(\tilde{x})\tilde{x}dw \\ + (V(\tilde{x} + \gamma(\tilde{x})) - V(\tilde{x}))dp\} \end{aligned} \tag{21}$$

where n_{aug} is the dimension of $\tilde{x}$ and $\gamma(\tilde{x}) = \sum_{a=1}^l h_a\tilde{O}_a(\tilde{x})\tilde{x}$ is the nonlinear system function w.r.t. the Poisson process of augmented SPFS in (19).

Then, the main result is given as follows

Theorem 1: If there exists a polynomial positive-definite matrix $P(\tilde{x})$ and a set of fuzzy polynomial controller gains and fuzzy polynomial observer gains $\{K_{i,1}(\hat{\bar{x}}, \bar{x}_r), K_{i,2}(\hat{\bar{x}}, \bar{x}_r), L_i(\hat{\bar{x}})\}_{i=1}^l$ such that the following polynomial matrix inequalities hold

$$\begin{aligned} & \Phi_{ii}(\tilde{x}) < 0, \ i = 1, \cdots, l \\ & \Phi_{ij}(\tilde{x}) + \Phi_{ji}(\tilde{x}) < 0, \ 1 \leq j < i \leq l \end{aligned} \tag{22}$$

where $\Phi_{ij}(\tilde{x}) = \tilde{x}^T(\tilde{Q} + M_j^T(\tilde{x})RM_j(\tilde{x}) + P(\tilde{x})\tilde{A}_{ij}(\tilde{x}) + \tilde{A}_{ij}^T(\tilde{x})P(\tilde{x}) + \frac{1}{2}\tilde{H}_i^T(\tilde{x})\frac{\partial^2\tilde{x}^T P(\tilde{x})\tilde{x}}{\partial^2\tilde{x}}\tilde{H}_i(\tilde{x}) + \sum_{k=1}^{n_{aug}}\frac{\partial P(\tilde{x})}{\partial\tilde{x}_k}\times\tilde{A}_{ij,k}(\tilde{x})\tilde{x} + \frac{2}{\rho}P(\tilde{x})\tilde{D}_i\tilde{D}_i^T(\tilde{x})P(\tilde{x}) + \lambda(P(\tilde{x} + \gamma(\tilde{x}))\tilde{O}_j(\tilde{x}) + P(\tilde{x} + \gamma(\tilde{x})) + \tilde{O}_i^T(\tilde{x})P(\tilde{x} + \gamma(\tilde{x})) + \tilde{O}_i^T(\tilde{x})P(\tilde{x} + \gamma(\tilde{x}))\ \tilde{O}_i(\tilde{x}) - P(\tilde{x})) + \sum_{k=1}^{n_{aug}}\frac{n_{aug}}{2\rho}\frac{\partial P(\tilde{x})}{\partial\tilde{x}_k}\tilde{x}\tilde{D}_{i,k}(\tilde{x})\tilde{D}_{i,k}^T(\tilde{x})\tilde{x}^T\times\frac{\partial P(\tilde{x})}{\partial\tilde{x}_k})\tilde{x}$, $M_j(\tilde{x}) = [K_{j,2}(\hat{\bar{x}}, \bar{x}_r) - K_{j,1}(\hat{\bar{x}}, \bar{x}_r), K_{j,1}(\hat{\bar{x}}, \bar{x}_r), -K_{j,1}(\hat{\bar{x}}, \bar{x}_r)]$, $\tilde{x}_k$ *is the kth component of* $\tilde{x}$, $\tilde{A}_{ij,k}(\tilde{x})$ *is the kth row vector of* $\tilde{A}_{ij}(\tilde{x})$ *and* $\tilde{D}_{i,k}(\tilde{x})$ *is the kth row vector of* $\tilde{D}_i(\tilde{x})$*, then the robust* H_∞ *fuzzy observer-based fault-tolerant tracking control strategy in (20) is achieved with a prescribed disturbance attenuation level* ρ*. Also, due to the fact that* $\tilde{v}$ *is finite energy (i.e.,* $\tilde{v} \in \mathcal{L}_2^{\mathcal{F}}(\mathbb{R}^+, \mathbb{R}^{n_{\tilde{v}}})$*), the augmented SPFS in (19) is mean square stable, i.e.,* $E\{\tilde{x}^T\tilde{x}\} \to 0$*, as* $t \to \infty$*.*

Proof: Please refer to Appendix A. □

By choosing the polynomial Lyapunov function $\tilde{x}^T P(\tilde{x})\tilde{x}$, the design of robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy is derived in terms of a set of polynomial matrix inequalities in (22). Compared with the deterministic case, the terms $\tilde{H}_i^T(\tilde{x})\frac{\partial^2\tilde{x}^T P(\tilde{x})\tilde{x}}{\partial^2\tilde{x}}\tilde{H}_i(\tilde{x})$ and $\tilde{O}_i^T(\tilde{x})P(\tilde{x} + \gamma(\tilde{x}))\tilde{O}_i(\tilde{x})$ due to the compensation of Wiener process and Poisson counting process in the It ô-Lévy formulat in (21) make the design conditions in (22) become intractable. More specifically, it is impossible to apply any matrix transformation technique to deal with these terms.

To relieve the design difficulty in Theorem 1, the following homogeneous polynomial Lyapunov function (HPLF) is introduced

Definition 1: [17] A function $V(\tilde{x}) : \mathbb{R}^{n_{aug}} \to \mathbb{R}^1$ *is called HPLF with degree* $s \in \mathbb{N}$ *if (i)* $V(\tilde{x}) \in C^2_{n_{aug},1}$*, (ii)* $V(\tilde{x}) \geq 0$ *with* $V(0) = 0$ *and (iii)* $V(a\tilde{x}) = a^s V(\tilde{x})$*,* $\forall a \geq 0$*.*

By the favorable functional characteristic of HPLF, the following lemma is provided to address the relation between HPLF and its gradient vector as well as the relation between HPLF and its Hessian matrix, repectively

Lemma 4: [17] Given HPLF $V(\tilde{x}) : \mathbb{R}^{n_{aug}} \to \mathbb{R}^1$ *with degree* $s \in \mathbb{N} - \{1\}$*, then the following equations hold*

$$\begin{aligned} sV(\tilde{x}) &= \tilde{x}^T\frac{\partial V(\tilde{x})}{\partial\tilde{x}} = (\frac{\partial V(\tilde{x})}{\partial\tilde{x}})^T\tilde{x} \\ V(\tilde{x}) &= \frac{1}{s(s-1)}\tilde{x}^T\frac{\partial^2 V(\tilde{x})}{\partial^2\tilde{x}}\tilde{x} \\ \frac{\partial V(\tilde{x})}{\partial\tilde{x}} &= \frac{1}{(s-1)}\frac{\partial^2 V(\tilde{x})}{\partial^2\tilde{x}}\tilde{x} \end{aligned} \tag{23}$$

Remark 9: Based on the above mentioned lemma, it is obvious that the HPLF and the corresponding gradient function can be represented with its Hessian matrix via some coefficient scaling. Thus, by adopting the corresponding Hessian matrix as new design variables, it can relieve the compensation terms caused by stochastic processes in the Itô-Lévy formula.

Remark 10: If the HPLF with degree $s \in \mathbb{N} - \{1\}$ *is specified as* $V(\tilde{x}) = \tilde{x}^T P(\tilde{x})\tilde{x}$ *for some positive definite homogeneous polynomial matrix* $P(\tilde{x})$*, it is clear that* $V(\tilde{x}) = \tilde{x}^T P(\tilde{x})\tilde{x} = \frac{1}{s(s-1)}\tilde{x}^T\frac{\partial^2 V(\tilde{x})}{\partial^2\tilde{x}}\tilde{x}$ *from Lemma 4. However, in the most of cases, the above relation does not imply* $P(\tilde{x}) = \frac{1}{s(s-1)}\frac{\partial^2 V(\tilde{x})}{\partial^2\tilde{x}}$*. In fact, this relation holds if* $P(\tilde{x})$ *is reduced to constant matrix, i.e., HPLF with degree 2.*

With the merit of HPLF in the above discussion, the compensation terms of the stochastic processes with the Itô-Lévy formula in (21) can be reformulated in the following theorem

Theorem 2: Given a HPLF $V(\tilde{x}) = \tilde{x}^T P(\tilde{x})\tilde{x} \in C^2_{n_{aug},1}$ *of SPFS in (19) with a positive definite homogeneous polynomial matrix* $P(\tilde{x})$ *and degree* $s \in \mathbb{N} - \{1\}$*, the Itô-Lévy formula in (21) can be bounded as follows*

$$\begin{aligned} &E\{dV(\tilde{x})\} \\ &\leq E\{\sum_{i,j=1}^{l} h_i h_j[\frac{1}{2(s-1)}\tilde{x}^T\bar{P}(\tilde{x})(\tilde{A}_{ij}(\tilde{x})\tilde{x} + \tilde{D}_i(\tilde{x})\tilde{v}) \\ &\quad + \frac{1}{2(s-1)}(\tilde{A}_{ij}(\tilde{x})\tilde{x} + \tilde{D}_i(\tilde{x})\tilde{v})^T\bar{P}(\tilde{x})\tilde{x}^T + \frac{1}{2}\tilde{x}^T\tilde{H}_j^T(\tilde{x}) \\ &\quad \times\bar{P}(\tilde{x})\tilde{H}_i(\tilde{x})\tilde{x} + \frac{\lambda((1+L_{poi})^s - 1)}{s(s-1)}\tilde{x}^T\bar{P}(\tilde{x})\tilde{x}]dt\} \end{aligned} \tag{24}$$

where $\bar{P}(\tilde{x}) = \frac{\partial^2 V(\tilde{x})}{\partial^2\tilde{x}}$ *and* L_{poi} *is the Lipschiz constant associated with system matrix* $o(x(t))$ *of Poisson counting process in (1).*

Proof: Please refer to Appendix B. □

By utilizing the relaxed Itô-Lévy formula in (24), the following theorem is proposed

Theorem 3: If there exists a positive definite polynomial matrix $\bar{P}(\tilde{x})$ *and a set of fuzzy polynomial controller gains and fuzzy polynomial observer gains* $\{K_{i,1}(\hat{\bar{x}}, \bar{x}_r), K_{i,2}(\hat{\bar{x}}, \bar{x}_r), L_i(\hat{\bar{x}})\}_{i=1}^l$ *such that the following interpolation function-dependent matrix inequality holds*

$$\begin{aligned} &\sum_{j,i=1}^{l} h_i h_j E\{\tilde{x}^T[(\tilde{Q} + M^T(\tilde{x})RM_j(\tilde{x}) \\ &\quad + \frac{1}{2(s-1)}\bar{P}(\tilde{x})\tilde{A}_{ij}(\tilde{x}) + \frac{1}{2(s-1)}\tilde{A}_{ij}^T(\tilde{x})\bar{P} \\ &\quad + \frac{1}{2}\tilde{H}_i^T(\tilde{x})\bar{P}(\tilde{x})\tilde{H}_i(\tilde{x}) + \frac{\lambda((1+L_{poi})^s - 1)}{s(s-1)}\bar{P}(\tilde{x}) \\ &\quad + \frac{1}{4\rho(s-1)^2}\bar{P}(\tilde{x})\tilde{D}_i(\tilde{x})\tilde{D}_i^T(\tilde{x})\bar{P}(\tilde{x})]\tilde{x}\} \leq 0 \end{aligned} \tag{25}$$

where $s \in \mathbb{N} - \{1\}$ *and* L_{poi} *is Lipschiz constant associated with system matrix of Poisson counting process* $o(x(t))$ *in (1), then the robust* H_∞ *fuzzy observer-based fault-tolerant tracking control strategy in (20) for SPFS in (19) is achieved with a prescribed disturbance attenuation level* $\rho > 0$*. Also, due the fact that* $\tilde{v}$ *is finite energy, the augmented SPFS in (19) is mean square stable, i.e.,* $E\{\tilde{x}^T\tilde{x}\} \to 0$*, as* $t \to \infty$*.*

Proof: Please refer to Appendix C. □

In Theorem 3, by utilizing the property of HPFS and the relaxation of Itô-Lévy formula in Theorem 2, the design condition is successfully transformed to the matrix inequality in (25), i.e., an interpolation function-dependent matrix inequality. However, due to the complicated structure of system matrices $\{\tilde{A}_{ij}(\tilde{x})\}_{i,j=1}^l$, the design variables of controller/observer gains and polynomial positive-definite matrix $\{K_{i,1}(\hat{\bar{x}}, \bar{x}_r), K_{i,2}(\hat{\bar{x}}, \bar{x}_r), L_i(\hat{\bar{x}}), \bar{P}(\tilde{x})\}_{i=1}^l$ are coupled with each others and it can not be directly solved. As a result, the following two-step design procedure is developed to solve the matrix inequality in (25).

Before the discussion of two-step design procedure, some relaxation techniques are utilized to reduce the coupling in (25). To begin with, the HPLF with degree s of augmented system in (19) is chosen as the sum of HPLFs associated with three fuzzy subsystems of augmented system in (19) as follows

$$V(\tilde{x}) = \bar{x}_r^T P_1(\bar{x}_r)\bar{x}_r + \bar{x}^T P_2(\bar{x})\bar{x} + e^T P_3(e)e \tag{26}$$

where $P_1(\bar{x}_r)$, $P_2(\bar{x})$, $P_3(e)$ are of positive definite homogeneous polynomial matrix. Then, there exist positive definite polynomial matrices $\{\bar{P}_1(\bar{x}_r), \bar{P}_2(\bar{x}), \bar{P}_3(e)\}$ such that the following equation holds

$$\begin{aligned} V(\tilde{x}) &= \bar{x}_r^T P_1(\bar{x}_r)\bar{x}_r + \bar{x}^T P_2(\bar{x})\bar{x} + e^T P_3(e)e \\ &= \frac{1}{s(s-1)}\tilde{x}^T \frac{\partial^2 V(\tilde{x})}{\partial^2 \tilde{x}}\tilde{x} = \frac{1}{s(s-1)}\tilde{x}^T \bar{P}(\tilde{x})\tilde{x} \end{aligned} \tag{27}$$

where $\bar{P}(\tilde{x}) = diag\{\bar{P}_1(\bar{x}_r), \det(\bar{P}_2(\bar{x}))\bar{P}_2^{-1}(\bar{x}), \bar{P}_3(e)\}$.

Remark 11: In the conventional T-S fuzzy observer-based tracking control design with the quadratic Lyapunov function, the resulting Lyapunov matrix can be chosen as $P = diag\{P_1, P_2^{-1}, P_3\}$ with $\{P_i > 0\}_{i=1}^3$ in (26). To simplify the design in the sequel, the second term of the Hessian matrix $\bar{P}(\tilde{x})$ in $V(\tilde{x})$ is chosen as specified structure $\det(\bar{P}_2(\bar{x}))\bar{P}_2^{-1}(\bar{x})$ instead of $\bar{P}_2^{-1}(\bar{x})$ and we have $\bar{x}^T P_2(\bar{x})\bar{x} = \frac{1}{s(s-1)}\bar{x}^T \det(\bar{P}_2(\bar{x}))\bar{P}_2^{-1}(\bar{x})\bar{x}$. By the fact that $\det(\bar{P}_2(\bar{x}))\bar{P}_2^{-1}(\bar{x})$ is equal to the adjugate of $\bar{P}_2(\bar{x})$, the increasing property of HPLF $V(\tilde{x})$ in (26) is guaranteed [16], [17].

Besides, by using Lemma 1, the following inequalities can be constructed to relax the term $\bar{P}_3(e)(\bar{B}_i(\bar{x}) - \bar{B}_j(\hat{x}))K_{j,2}(\hat{x}, \bar{x}_r)$ in the non-diagonal position of $\bar{P}(\tilde{x})\tilde{A}_{ij}(\tilde{x})$ in (25)

$$\begin{aligned} &[y^T \bar{P}_3(e)(\bar{B}_i(\bar{x}) - \bar{B}_j(\hat{\bar{x}}))K_{j,2}(\hat{\bar{x}}, \bar{x}_r)x + (*)] \\ &\quad \le x^T K_{j,2}^T(\hat{\bar{x}}, \bar{x}_r)K_{j,2}(\hat{\bar{x}}, \bar{x}_r)x \\ &\qquad + y^T \bar{P}_3(e)(\bar{B}_i(\bar{x}) - \bar{B}_j(\hat{\bar{x}}))(\bar{B}_i(\bar{x}) - \bar{B}_j(\hat{\bar{x}}))^T \bar{P}_3(e)y \\ &\qquad \forall i, j = 1, \cdots, l \end{aligned} \tag{28}$$

where x, y are arbitrary vectors with appropriate dimension.

With the relation of (28), the left hand side (LHS) in (25) can be further relaxed as:

$$\begin{aligned} &\sum_{j,i=1}^{l} h_i h_j E\{\tilde{x}^T[(\tilde{Q} + M^T(\tilde{x})RM_j(\tilde{x}) \\ &\quad + \frac{1}{2(s-1)}\bar{P}(\tilde{x})\tilde{A}_{ij}(\tilde{x}) + \frac{1}{2(s-1)}\tilde{A}_{ij}^T(\tilde{x})\bar{P} \\ &\quad + \frac{1}{2}\tilde{H}_i^T(\tilde{x})\bar{P}(\tilde{x})\tilde{H}_i(\tilde{x}) + \frac{\lambda((1+L_{poi})^s - 1)}{s(s-1)}\bar{P}(\tilde{x}) \\ &\quad + \frac{1}{4\rho(s-1)^2}\bar{P}(\tilde{x})\tilde{D}_i(\tilde{x})\tilde{D}_i^T(\tilde{x})\bar{P}(\tilde{x})]\tilde{x}\} \\ &\le \sum_{j,i=1}^{l} h_i h_j E\{[\tilde{x}^T[\tilde{Q} \\ &\quad + M^T(\tilde{x})RM_j(\tilde{x}) + \Theta_{ij}(\tilde{x}) \\ &\quad + \frac{1}{2}\tilde{H}_i^T(\tilde{x})\bar{P}(\tilde{x})\tilde{H}_i(\tilde{x}) + \frac{1}{4\rho(s-1)^2}\bar{P}(\tilde{x})\tilde{D}_i(\tilde{x}) \\ &\quad \times \tilde{D}_i^T(\tilde{x})\bar{P}(\tilde{x})]\tilde{x}\} \end{aligned} \tag{29}$$

where

$$\Theta_{ij}(\tilde{x}) = \begin{bmatrix} \Theta_{ij}^1(\tilde{x}) & \Theta_{ij}^4(\tilde{x}) & \Theta_{ij}^6(\tilde{x}) \\ * & \Theta_{ij}^2(\tilde{x}) & \Theta_{ij}^5(\tilde{x}) \\ * & * & \Theta_{ij}^3(\tilde{x}) \end{bmatrix}$$

with $\Theta_{ij}^1(\tilde{x}) = \frac{1}{2(s-1)}([\bar{P}_1(\bar{x}_r)\bar{A}_r(\bar{x}_r) + (*)] + K_{j,2}^T(\hat{\bar{x}}, \bar{x}_r) K_{j,2}(\hat{\bar{x}}, \bar{x}_r)) + \frac{\lambda((1+L_{poi})^s-1)}{s(s-1)}\bar{P}_1(\bar{x}_r)$, $\Theta_{ij}^2(\tilde{x}) = \frac{1}{2(s-1)}([\det(\bar{P}_2(\bar{x}))\bar{P}_2^{-1}(\bar{x})(\bar{A}_i(\bar{x}) + \bar{B}_i(\bar{x})K_{j,1}(\hat{\bar{x}}, \bar{x}_r)) + (*)]) + \frac{\lambda((1+L_{poi})^s-1)}{s(s-1)} \det(\bar{P}_2(\bar{x}))\bar{P}_2^{-1}(\bar{x})$, $\Theta_{ij}^3(\tilde{x}) = \frac{1}{2(s-1)}([\bar{P}_3(e)(\bar{A}_j(\hat{\bar{x}}) - L_j(\hat{\bar{x}}) \bar{C}_j(\hat{\bar{x}}) - (\bar{B}_i(\bar{x}) - \bar{B}_j(\hat{\bar{x}}))K_{j,1}(\hat{\bar{x}}, \bar{x}_r)) + (*)] + \bar{P}_3(e)(\bar{B}_i(\bar{x}) - \bar{B}_j(\hat{\bar{x}}))(\bar{B}_i(\bar{x}) - \bar{B}_j(\hat{\bar{x}}))^T \bar{P}_3) + \frac{\lambda((1+L_{poi})^s-1)}{s(s-1)}\bar{P}_3(e)$, $\Theta_{ij}^4(\tilde{x}) = \frac{1}{2(s-1)}(K_{j,2}(\hat{\bar{x}}, \bar{x}_r) - K_{j,1}(\hat{\bar{x}}, \bar{x}_r))^T \bar{B}_i^T(\bar{x}) \det(\bar{P}_2(\bar{x}))\bar{P}_2^{-1}(\bar{x})$, $\Theta_{ij}^5(\tilde{x}) = \frac{1}{2(s-1)}(-\det(\bar{P}_2(\bar{x}))\bar{P}_2^{-1}(\bar{x})\bar{B}_i(\bar{x})K_{j,1}(\hat{\bar{x}}, \bar{x}_r) + (\bar{A}_i(\bar{x}) - \bar{A}_j(\hat{\bar{x}}) - L_j(\hat{\bar{x}})(\bar{C}_i(\bar{x}) - \bar{C}_j(\hat{\bar{x}})) + (\bar{B}_i(\bar{x}) - \bar{B}_j(\hat{\bar{x}}))K_{j,1}(\hat{\bar{x}}, \bar{x}_r))^T \bar{P}_3(e))$, $\Theta_{ij}^6(\tilde{x}) = -\frac{1}{2(s-1)}K_{j,1}^T(\hat{\bar{x}}, \bar{x}_r)(\bar{B}_i(\bar{x}) - \bar{B}_j(\hat{\bar{x}}))^T \bar{P}_3(e)$.

Clearly, the polynomial constraint in (25) holds if the right hand side (RHS) in (29) is negative semi-definite. Moreover, by utilizing Schur complement, the RHS in (29) is negative semi-definite if the following polynomial inequalities hold:

$$\begin{aligned} &\sum_{j,i=1}^{l} h_i h_j \begin{bmatrix} \Theta_{ij}(\tilde{x}) & \Pi_i(\tilde{x}) \\ * & -\Psi_i(\tilde{x}) \end{bmatrix} \le 0 \\ &\Pi_i(\tilde{x}) = [M^T(\tilde{x})\ \tilde{H}_i^T(\tilde{x})\bar{P}(\tilde{x})\ \bar{P}(\tilde{x})\tilde{D}_i(\tilde{x}),\ \tilde{Q}^{\frac{1}{2}}] \\ &\Psi_i(\tilde{x}) = diag\{R^{-1}, 2\bar{P}(\tilde{x}), 4(s-1)^2\rho I, I\}, \end{aligned} \tag{30}$$

i.e., the H_∞ polynomial fuzzy controller gains in (14) and polynomial fuzzy observer gains in (10) (i.e., $\{K_{i,1}(\hat{\bar{x}}, \bar{x}_r), K_{i,2}(\hat{\bar{x}}, \bar{x}_r), L_i(\hat{\bar{x}})\}_{i=1}^l$) need to solve the above interpolation function dependent polynomial inequality.

Based on the above discussion, the following two-step design procedure is developed to solve $\{K_{i,1}(\hat{\bar{x}}, \bar{x}_r), K_{i,2}(\hat{\bar{x}}, \bar{x}_r), L_i(\hat{\bar{x}})\}_{i=1}^l$ from the polynomial inequality in (30).

(STEP I) At first, if the polynomial constraint in (30) holds, then the diagonal terms in the LHS of (30) must be negative-semi-definite. As a result, the term $\Theta_{ij}^2(\tilde{x})$ in $\Theta_{ij}(\tilde{x})$ in (30) is solved in the following at first to obtain the design parameters $K_{j,1}(\hat{\bar{x}}, \bar{x}_r)$ and $\bar{P}_2(\bar{x})$. By pre-multiplying $\bar{P}_2(\bar{x})$ and post-multiplying $\bar{P}_2(\bar{x})$ to $\Theta_{ij}^2(\tilde{x}) \le 0$ with $\{N_{j,1}(\tilde{x}) = K_{j,1}(\hat{\bar{x}}, \bar{x}_r)\bar{P}_2(\bar{x})\}_{j=1}^l$, we have

$$\begin{aligned} &\sum_{j,i=1}^{l} h_i h_j\{(\det(\bar{P}_2(\bar{x}))(\frac{1}{2(s-1)}[(\bar{A}_i(\bar{x})\bar{P}_2(\bar{x}) \\ &\quad + \bar{B}_i \\ &\quad (\bar{x})N_{j,1}(\tilde{x}) + (*)]) + \frac{\lambda((1+L_{poi})^s - 1)}{s(s-1)}\bar{P}_2(\bar{x}))\} \le 0 \end{aligned} \tag{31}$$

and it can be further transformed to the following solvable SOS conditions [17]

$$\begin{aligned} &\varphi_1^T(\bar{P}_2(\bar{x}) - \epsilon_1(\bar{x})I)\varphi_1 && \text{is SOS} \\ &-\varphi_2^T(\Upsilon_{kk}(\tilde{x}) - \epsilon_2(\tilde{x})I)\varphi_2 && \text{is SOS} \\ &-\varphi_3^T(\Upsilon_{ij}(\tilde{x}) + \Upsilon_{ji}(\tilde{x}) - \epsilon_3(\tilde{x})I)\varphi_3 && \text{is SOS} \\ &for\ k = 1, \cdots, l,\ 1 \le i < j \le l \end{aligned} \tag{32}$$

where $\Upsilon_{ij}(\tilde{x}) = \frac{1}{(s-1)}[(\bar{A}_i(\tilde{x})\bar{P}_2(\tilde{x}) + \bar{B}_i(\tilde{x})N_{j,1}(\tilde{x}) + (*)] + \frac{\lambda((1+L_{poi})^s-1)}{s(s-1)}\bar{P}_2(\tilde{x}))$, $\{\epsilon_i(\tilde{x})\}_{i=1}^3$ are non-negative polynomials of $\tilde{x}$ and $\{\varphi_i\}_{i=1}^3$ are vectors independent of $\tilde{x}$. Due to the fact that $\det(\bar{P}_2(\tilde{x})) > 0$, this term can be dropped in SOS conditions in (32).

By solving SOS conditions in (32) via SOSTOOLS in [12] and semi-definite programming, we can obtain the design variables $\{\bar{P}_2(\tilde{x}), N_{j,1}(\tilde{x})\}_{j=1}^l$ and the polynomial fuzzy controller gains can be implemented as $\{K_{j,1}(\hat{\tilde{x}}, \bar{x}_r) = N_{j,1}(\tilde{x})\bar{P}_2^{-1}(\tilde{x})\}_{j=1}^l$. Also, by the fact that $\det(\bar{P}_2(\tilde{x}))\bar{P}_2^{-1}(\tilde{x}) = adj(\bar{P}_2(\tilde{x}))$, where $adj(\bar{P}_2(\tilde{x}))$ is the adjugate of $\bar{P}_2(\tilde{x})$, $\det(\bar{P}_2(\tilde{x}))\bar{P}_2^{-1}(\tilde{x})$ is of polynomial matrix at the next step.

(STEP II) To decouple the bilinear terms $K_{j,2}^T(\hat{\tilde{x}}, \bar{x}_r)$ $K_{j,2}(\hat{\tilde{x}}, \bar{x}_r)$ and $\bar{P}_3(e)(\bar{B}_i(\tilde{x}) - \bar{B}_j(\hat{\tilde{x}}))$ $(\bar{B}_i(\tilde{x}) - \bar{B}_j(\hat{\tilde{x}}))^T\bar{P}_3$ in $\Theta_{ij}^1(\tilde{x})$ and $\Theta_{ij}^3(\tilde{x})$ in (30), respectively, the following slack variables are introduced

$$\begin{aligned} &K_{j,2}^T(\hat{x}, \bar{x}_r)K_{j,2}(\hat{x}, \bar{x}_r) \le N_{j,2}(\tilde{x}) \\ &\bar{P}_3(e)(\bar{B}_i(\tilde{x}) - \bar{B}_j(\hat{x}))(\bar{B}_i(\tilde{x}) - \bar{B}_j(\hat{x}))^T\bar{P}_3(e) \\ &\le N_{i,j,3}(\tilde{x}) \\ &\textit{for } i, j = 1, \cdots, L \end{aligned} \tag{33}$$

where $\{N_{j,2}(\tilde{x}), N_{i,j,3}(\tilde{x})\}_{i,j=1}^l$ are positive-definite homogeneous polynomial matrices to be designed. Then, by letting $\{N_{j,4}(\tilde{x}) = \bar{P}_3(e)L_j(\hat{x})\}_{j=1}^l$, the polynomial constraints in (30) can be relaxed as

$$\textstyle\sum_{j,i=1}^l h_i h_j \begin{bmatrix} \bar{\Theta}_{ij}(\tilde{x}) & \bar{\Pi}_i(\tilde{x}) \\ * & -\Psi_i(\tilde{x}) \end{bmatrix} \le 0 \tag{34}$$

where

$$\bar{\Theta}_{ij}(\tilde{x}) = \begin{bmatrix} \bar{\Theta}_{ij}^1(\tilde{x}) & \Theta_{ij}^4(\tilde{x}) & \Theta_{ij}^6(\tilde{x}) \\ * & \Theta_{ij}^2(\tilde{x}) & \bar{\Theta}_{ij}^5(\tilde{x}) \\ * & * & \bar{\Theta}_{ij}^3(\tilde{x}) \end{bmatrix}$$

$$\bar{\Pi}_i(\tilde{x}) = [M^T(\tilde{x})\ \tilde{H}_i^T(\tilde{x})\bar{P}(\tilde{x})\ \breve{D}_j(\tilde{x})\ \tilde{Q}^{\frac{1}{2}}]$$

$$\breve{D}_j(\tilde{x}) = \begin{bmatrix} 0 & \bar{P}_1(\bar{x}_r)\bar{B}_r(\bar{x}_r) \\ \det(\bar{P}_2(\tilde{x}))\bar{P}_2^{-1}(\tilde{x})D_1 & 0 \\ \bar{P}_3(e)D_1 - N_{j,4}(\tilde{x})D_2 & 0 \end{bmatrix}$$

with $\bar{\Theta}_{ij}^1(\tilde{x}) = \frac{1}{2(s-1)}([\bar{P}_1(\bar{x}_r)\bar{A}_r(\bar{x}_r) + (*)] + N_{j,2}(\tilde{x}) + \frac{\lambda((1+L_{poi})^s-1)}{s(s-1)}\bar{P}_1(\bar{x}_r)$ $\bar{\Theta}_{ij}^3(\tilde{x}) = \frac{1}{2(s-1)}([\bar{P}_3(e)\ \bar{A}_j(\hat{x}) + N_{j,4}(\tilde{x})$ $\bar{C}_j(\hat{x}) + \bar{P}_3(e)(\bar{B}_i(\tilde{x}) - \bar{B}_j(\hat{\tilde{x}}))K_{j,1}(\hat{\tilde{x}}, \bar{x}_r)) + (*)] + N_{i,j,3}(\tilde{x})) + \frac{\lambda((1+L_{poi})^s-1)}{s(s-1)}\bar{P}_3(e)$, , $\bar{\Theta}_{ij}^5(\tilde{x}) = \frac{1}{2(s-1)}(\det(\bar{P}_2(\tilde{x}))\bar{P}_2^{-1}(\tilde{x})\bar{B}_i(\tilde{x})$ $K_{m,1}(\hat{\tilde{x}}, \bar{x}_r) + (\bar{A}_i(\tilde{x}) - \bar{A}_j(\hat{\tilde{x}}))^T\bar{P}_3(e) - (\bar{C}_i(\tilde{x}) - \bar{C}_j(\hat{\tilde{x}}))^T N_{j,4}(\tilde{x}) + ((\bar{B}_i(\tilde{x}) - \bar{B}_j(\hat{\tilde{x}}))\ K_{j,1}(\hat{\tilde{x}}, \bar{x}_r))^T\bar{P}_3(e))$.

By applying Schur complement to (33) with the fixed design variables $\{\bar{P}_2(\tilde{x}), K_{j,1}(\hat{\tilde{x}}, \bar{x}_r)\}_{j=1}^l$ being obtained from STEP 1, the polynomial constraint problem in (34) can be transformed to the following SOS conditions [17]

$$\begin{aligned} &\varphi_4^T(\bar{P}_1(\bar{x}_r) - \epsilon_4(\bar{x}_r)I)\varphi_4 && \text{is SOS} \\ &\varphi_5^T(\bar{P}_3(e) - \epsilon_5(e)I)\varphi_5 && \text{is SOS} \\ &-\varphi_6^T(\Gamma_{i_1i_1}^1(\tilde{x}) - \epsilon_6(\tilde{x}))\varphi_6 && \text{is SOS} \\ &-\varphi_7^T(\Gamma_{ij}^1(\tilde{x}) + \Gamma_{ji}^1(\tilde{x}) - \epsilon_7(\tilde{x}))\varphi_7 && \text{is SOS} \\ &-\varphi_8^T(\Gamma_{i_1j_1}^2(\tilde{x}) - \epsilon_8(\tilde{x}))\varphi_8 && \text{is SOS} \\ &-\varphi_9^T(\Gamma_{i_1j_1}^3(\tilde{x}) - \epsilon_9(\tilde{x}))\varphi_9 && \text{is SOS} \\ &\textit{for } 1 \le i < j \le l,\ i_1, j_1 = 1, \cdots, l \end{aligned} \tag{35}$$

where $\{\epsilon_i(\tilde{x})\}_{i=4}^9$ are non-negative polynomials of $\tilde{x}$ and $\{\varphi_i\}_{i=4}^9$ are vectors independent of $\tilde{x}$ and

$$\Gamma_{ij}^1(\tilde{x}) = \begin{bmatrix} \bar{\Theta}_{ij}(\tilde{x}) & \bar{\Pi}_i(\tilde{x}) \\ * & -\Psi_i(\tilde{x}) \end{bmatrix}$$

$$\Gamma_{ij}^2(\tilde{x}) = \begin{bmatrix} -N_{j,2}(\tilde{x}) & K_{j,2}^T(\hat{x}, \bar{x}_r) \\ * & -I \end{bmatrix}$$

$$\Gamma_{ij}^3(\tilde{x}) = \begin{bmatrix} -N_{i,j,3}(\tilde{x}) & \bar{P}_3(e)(\bar{B}_i(\tilde{x}) - \bar{B}_j(\hat{x})) \\ * & -I \end{bmatrix}$$

By solving SOS conditions in (35), we can get the design variables $\{\bar{P}_1(\bar{x}_r), \bar{P}_3(e), K_{j,2}(\hat{\tilde{x}}, \bar{x}_r),\ N_{j,2}(\tilde{x}), N_{i,j,3}(\tilde{x})\}_{i,j=1}^l$ and the polynomial fuzzy polynomial observer gains $\{L_j(\hat{x}) = \bar{P}_3^{-1}(e)N_{j,4}(\tilde{x})\}_{j=1}^l$. As a result, the design of robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy can be efficiently solved by the developed two-step design procedure.

Remark 12: In Theorem 3, the sufficient condition can be reformulated as interpolation function-independent matrix inequalities. Based on the developed two-step design procedure, $N_h \times l^2$ polynomial constraints have to be solved to get the polynomial fuzzy observer/controller gains where N_h denotes the polynomial constraints w.r.t. one fuzzy rule. On the other hand, even the matrix inequality in (25) is depending on the interpolation functions and it can be further transformed to polynomial matrix inequalities with number $N_h \times l^2/2$ to save the computational time.

Remark 13: Conventionally, due to the coupling of design variables, it is not easy to simultaneously solve the controller gains and observer gains in (25). More specifically, there exists no efficient way to decouple controller/observer gains in (25), especially in the case of polynomial fuzzy system. As a result, the two-step design procedure is developed. Therein, the controller gains can be obtained by solving (32) at the first step, then the observer gains can be obtained by solving (32), too.

Remark 14: Once the effect of stochastic fluctuations, fault signal and external disturbance are simultaneously considered, it will make the derived design conditions become more conservative. For example, to eliminate the effect of system fault signals on observer-based reference tracking control design of the system, the dimension of polynomial controller gains includes the total dimensions of system state, augmented system fault signal and augmented sensor fault signal. Consequently, it will directly enlarge the dimension of derived SOS conditions and thus make the derived conditions more conservative (e.g., $\{K_{j,1}(\hat{\tilde{x}}, \bar{x}_r)\}_{j=1}^l$ in (35)). Similarly, while considering the stochastic effects on the observer-based reference tracking control of nonlinear stochastic system, the derived condition also involve system matrices w.r.t. stochastic effect, which makes the design conditions more difficult to be solved (e.g., $\{\tilde{H}_i(\tilde{x})\}_{i=1}^l$ and $\lambda\frac{((1+L_{poi})^s-1)}{s(s-1)}I$ in (35)).

IV. SIMULATION EXAMPLE

A. MODIFIED AGV SYSTEM WITH GPS SENSOR NETWORK

In this simulation, a maneuvering task for AGV system is provided as an example to validate the effectiveness of the proposed strategy. Generally speaking, during the maneuvering process, there may exist system fault signals caused by the system component damage and the sensor fault signals on the GPS sensor network of AGV system due to the uncertain communication channel or malicious attack signals. On the other hand, caused by the internal system fluctuation and unpredictable environment condition, the AGV system will be influenced by intrinsic continuous/discontinuous fluctuation. As a result, the following modified AGV system is given [38]

$$\begin{aligned} dX &= (V_x \cos\theta - V_y \sin\theta)dt \\ dY &= (V_x \sin\theta + V_y \cos\theta)dt \\ d\theta &= V_\theta dt \\ dV_x &= (V_y V_\theta + \frac{2C_{af}(l_f V_\theta + V_y)}{mV_x(t)}(\delta_f + f_a) - \frac{2(C_{af}+C_{ar})}{m} \\ &\quad \times \delta_b(t) + v_1)dt + 0.01V_X dw + 0.1V_X dp \\ dV_y &= (-V_x V_\theta + \frac{2}{m}(-C_{af}\frac{l_f V_\theta + V_y}{V_x} + C_{ar}\frac{l_r V_\theta - V_y}{V_x}) \\ &\quad + \frac{2C_{af}}{m}(\delta_f + f_a) + v_2)dt + 0.01V_Y dw + 0.01V_y dp \\ dV_\theta &= (-\frac{2}{I_z}(l_f C_{af}\frac{l_f V_\theta + V_y}{V_x} + l_r C_{ar}\frac{l_r V_\theta - V_y}{V_x}) \\ &\quad + \frac{2l_f C_{af}}{I_z}(\delta_f + f_a) + v_3)dt + 0.01V_\theta dw + 0.01V_\theta dp \\ y &= \begin{bmatrix} X \\ Y \\ \theta \end{bmatrix} + \begin{bmatrix} 1 \\ 0.05 \\ 0.05 \end{bmatrix} f_s + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} \end{aligned} \tag{36}$$

where X is the longitudinal displacement with longitudinal velocity V_x, Y denotes the lateral displacement with lateral velocity V_y, θ represents the yaw angle with yaw rate V_θ, δ_f denotes the front wheels' steering angle, δ_b denotes the brake/accelerator, $\{v_i\}_{i=1}^3$ are the external disturbance and are set as the Gaussian distribution with zero mean and 0.1 variance, $\{n_i\}_{i=1}^3$are the measurement noise and are set as the Gaussian distribution with zero mean and 0.1 variance, f_s is the sensor fault signal, f_a is the system fault signal, w is the 1-D Wiener process, p is the Poisson counting process with jump intensity $\lambda = 0.1$, $m = 1280(kg)$ is the mass, $I_z = 2500(kg/m^2)$ denotes the moment of inertia, $C_{af} = 3 \times 10^4(N/rad)$ and $C_{ar} = 3 \times 10^4(N/rad)$ are the stiffness coefficients w.r.t. the front wheel and the rear wheel, respectively, $l_f = 1.22m$ is the distance between the front tire and center of gravity (CG) and $l_r = 1.2m$ is the distance between the rear tire and CG.

Conventionally, for the maneuvering task of AGV system, the first control input δ_f is used to generate suitable control command on longitudinal velocity V_x, lateral velocity V_y and yaw rate V_θ to make AGV turn around based on the reference path trajectory. On the other hand, the second control input δ_b is applied for the control of longitudinal velocity V_x. For the modified AGV system in (36), the Wiener process w can be regarded as the continuous fluctuations of three velocities induced by modeling uncertainties. On the other hand, the Poisson counting process is used to describe the discontinuous reaction forces on the three velocities which is caused by the unknown road conditions. Despite these intrinsic random fluctuations, due to the system component damage with uncertain communication channel of GPS sensor network or malicious attack signals, the following system fault signal and sensor fault signal are given as

$$\begin{aligned} f_s &= \sin 0.2\pi t \\ f_a &= \begin{cases} 0.15, 0s \le t \le 24s \\ 0.1, 26s \le t \le 50s \end{cases} \end{aligned} \tag{37}$$

For the system fault signal f_a in (37), it will cause a bias to the front wheels' steering angle δ_f. In this case, if this effect is not well eliminated, the AGV system will have a constant deviation between the real trajectory and desired trajectory. To ensure that the system fault signal f_a is differentiable functions, f_a is fitted by a smooth function within $24s < t < 26s$. On the other hand, for the sensor fault signal in (37), a single-tone signal is applied to describe the interference from the GPS sensor network.

B. POLYNOMIAL FUZZY MODELING APPROACH AND PARAMETER SETTING

With the merit of polynomial fuzzy modeling approach, the premise variables are chosen as V_x and θ with the following operation points

$$\begin{aligned} V_{x,1} &= 10, V_{x,2} = 20 \\ \theta_1 &= -\frac{1}{4}\pi, \theta_2 = \frac{1}{4}\pi \end{aligned}$$

By adopting the above operation points with trapezoidal interpolation functions $\{h_i(\omega)\}_{i=1}^4$ and $\omega = [V_x\ \theta]$, the following polynomial fuzzy system can be infered to approximate the nonlinear AGV system in (36)

$$\begin{aligned} dx &= \sum\nolimits_{i=1}^4 h_i(\omega)\{[A_i(x)x + B_i(x)u + G_i(x)f_a \\ &\quad + v(t)]dt + Hxdw + Oxdp\} \\ y &= \sum\nolimits_{i=1}^4 h_i(\omega)[Cx + Sf_s + n(t)] \end{aligned} \tag{38}$$

where $x = [X\ Y\ \theta\ V_x\ V_y\ V_\theta]^T$, $u = [\delta_f\ \delta_b]^T$, $v = [0\ 0\ 0\ v_1\ v_2\ v_3]^T$, $n = [n_1\ n_2\ 0]^T$ and $\{A_i(x), B_i(x), G_i(x)\}_{i=1}^4$ and $\{H, O, C, S\}$ are local linearized polynomial matrices and linear matrices obtained from (36), respectively.

To construct the 2nd order smoothed models in (6) and (7) for the estimation of fault signals $\{f_a, f_s\}$, the following extrapolation parameters are selected

$$\begin{aligned} \alpha_0 &= 0.8, \alpha_1 = 0.16, \alpha_2 = 0.04 \\ \beta_0 &= 0.7, \beta_1 = 0.2, \beta_2 = 0.1 \end{aligned} \tag{39}$$

For the construction of the augmented reference model in (12), the following system matrices and input matrix

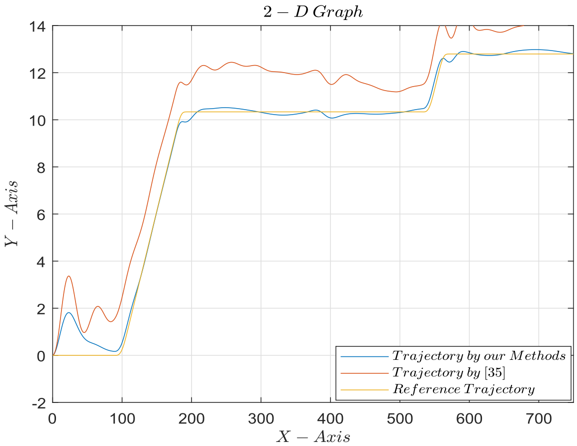

FIGURE 1. The 2–D reference tracking trajectory and the 2–D graph of AGV system controlled by the proposed method and the method in [39].

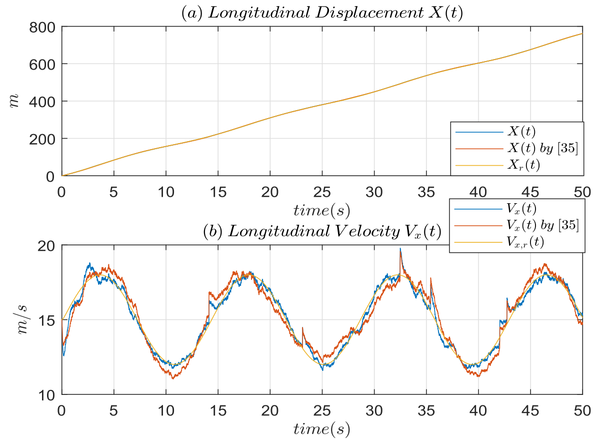

FIGURE 2. The controlled longitudinal displacement X in (a) and controlled longitudinal velocity V_x in (b) of AGV system by the proposed method and the method in [39].

are specified

$$A_r(x_r) = -2I_6, B_r(x_r) = 2I_6$$
$$A_{r,v_1} = A_{r,v2} = -I_3 \tag{40}$$

Also, for the robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy in (15) the following weighting matrices are chosen

$$Q_E = 2I_{12}, Q_T = diag\{2I_3, I_3\}$$
$$R = 0.01I_2 \tag{41}$$

For the choice of weighting matrix Q_T in (41), the designer considers the tracking of position/angle is more important than the tracking of three velocities. Besides, for the choice of Q_E, the estimation of state and the estimation of fault signal are equivalently important. On the other hand, with a small weighting matrix on control input, it enables the control input to use a large control effort during the observer-based tracking control process. Then, with the developed two-step design procedure with disturbance attneuation level $\rho = 1.2$, the SOS conditions in (32) and (35) are efficiently solved to obtain the design variables $\{\bar{P}_1(\bar{x}_r), \bar{P}_3(e), K_{j,2}(\hat{\bar{x}}, \bar{x}_r), N_{j,2}(\tilde{x}), N_{i,j,3}(\tilde{x}), \bar{P}_2(\bar{x}), N_{j,1}(\tilde{x})\}_{i,j=1}^2$. Then the fuzzy polynomial controller gains and fuzzy polynomial observer gains can be obtained as $\{K_{j,1}(\hat{\bar{x}}, \bar{x}_r) = N_{j,1}(\tilde{x})\bar{P}_2^{-1}(\bar{x}), K_{j,2}(\hat{\bar{x}}, \bar{x}_r)\}_{j=1}^4$ and $\{L_j(\hat{\bar{x}}) = \bar{P}_3^{-1}(e)\, N_{j,4}(\tilde{x})\}_{j=1}^4$, respectively. The detailed fuzzy polynomial controller gains and fuzzy polynomial observer gains can be refered to Appendix D.

C. PERFORMANCE EVALUATION

In this section, the AGV system is asked to achieve a double lane maneuvering task as shown in Fig. 1 and the desired tracking trajectory is generated by the reference model in (11). To illustrate the effectiveness of proposed method, the sample-data output reference tracking control scheme for deterministic polynomial fuzzy system in [39] is also carried out. For equality, the sample interval in [39] is chosen as same as the step size for the calculation of state response in the simulation. In the scenario of output feedback

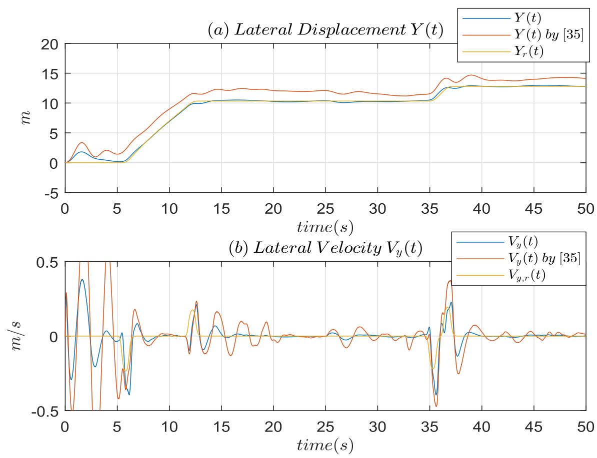

FIGURE 3. The controlled lateral displacement Y in (a) and controlled longitudinal velocity V_Y in (b) of AGV system by the proposed method and the method in [39].

control, the measurement output is directly used for controller synthesis without signal data processing, e.g., noise reduction or compensation of sensor fault signal.

The six state variables by the proposed method and the method in [39] are presented in Figs. 2–4. At first, due to the state dependent continuous/discontinuous stochastic processes, the fluctuation of the longitudinal velocity V_x in Fig. 2 is much higher than the lateral velocity V_y and yaw rate V_θ in Figs. 3–4. However, since the effect of these intrinsic fluctuations is considered during the design, it can be efficiently attenuated during the maneuvering process and the AGV system can be controlled to the desired reference path. For example, once the jump phenomenon occurs on longitudinal velocity V_x in Fig. 2, the longitudinal velocity V_x can be efficiently controlled to the desired trajectory with a less control period than the control scheme in [39]. For the lateral velocity V_y and yaw rate V_θ, there exist small tracking errors due to the estimation errors of AGV state and fault signal at the initial, which cause the AGV system to slightly turn around within X=0m–100m in Fig. 1. On the other hand,

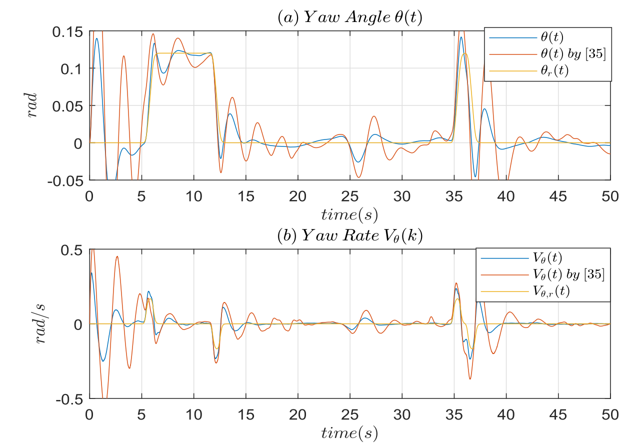

FIGURE 4. The controlled yaw angle θ in (a) and controlled yaw rate V_θ in (b) of AGV system by the proposed method and the method in [39].

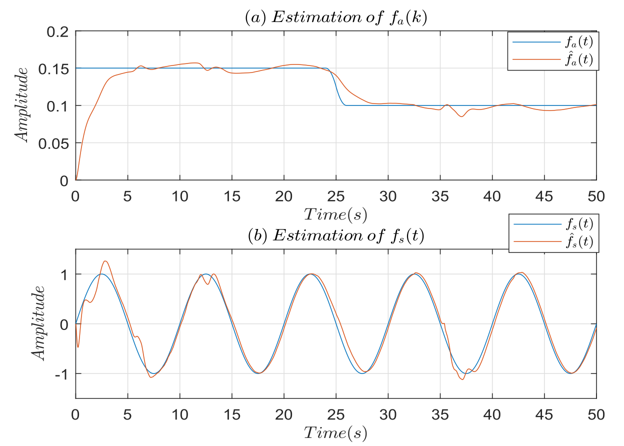

FIGURE 5. The system fault signal f_a with it's esitmaion in (a) and the sensor fault signal f_s with its estimation in (b).

due to the amplitude change of system fault signal f_a within 24s–26s, the lateral velocity V_y and yaw rate V_θ are slightly influenced, which cause a slight turn around behavior within X=380m–400m in Fig. 1. Because of the efficient estimation of fault signals, both the lateral velocity V_y and yaw rate V_θ are very close to the reference signal. Thus, from the 2–D trajectory in Fig. 1, the maximum offset is 2 at the initial by the proposed method and the remaining tracking error is relatively minor.

For the tracking scheme in [39], the high frequency oscillation over the entire tracking process is caused by the fault signals and high gain controller property. Further, without the consideration of fault signals, the lateral velocity V_y and yaw rate V_θ are more hard to be regulated. For example, even the reference signal of yaw rate is 0 within 13s–35s, the yaw rate V_θ by the tracking method in [39] approaches to zero at 20s while the yaw rate V_θ by the proposed method approaches to zero at 15s. Also, due to the effect of constant system fault signal f_a, the 2–D trajectory shows the AGV system controlled by the tracking scheme in [39] has constant deviations over the entire tracking process.

The system fault signal f_a and sensor fault signal f_s with the corresponding estimations are shown in Fig. 5. These two fault signals can be almost estimated only with some small fluctuations. In fact, due to the structure of Luenberger observer in (10), one estimation of state x, system fault signal f_a or sensor fault signal f_s will be influenced by the variations of other estimations. For example, once the magnitude of system fault signal f_a change from 0.15 to 0.1 within 24s–26s, the estimation of sensor fault signal $\hat{f}_s$ slightly deviates from the sensor fault signal f_s. On the other hand, the small osciallation in the estimation of system fault signal $\hat{f}_a$ is caused by the sinusoidal sensor fault signal f_s. However, it can be seen from Fig. 5 that these interactions for fault signal estimation are greatly reduced. Even there exist some fluctuations on the estimation of these fault signals, the estimated faults signals can be utilized by the robust proposed H_∞ observer-based fault-tolerant tracking control

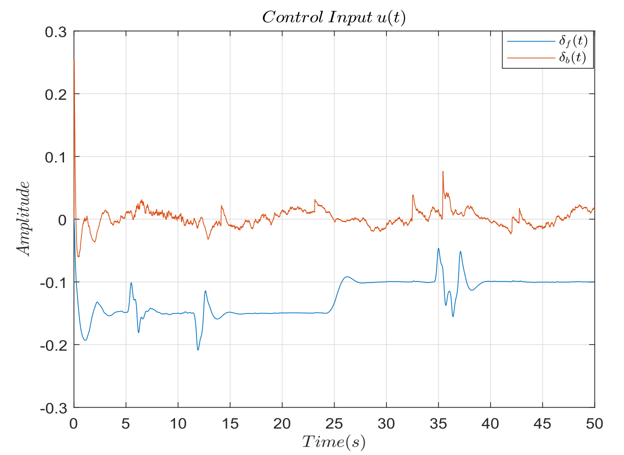

FIGURE 6. The control input $u = [\delta_f\ \delta_b]^T$. Once the system fault signal f_a changes it's magnitude at 25s, the estimated $\hat{f}_a$ is utilized to compensate its effect and δ_f is shifted.

strategy to compensate the effect of real fault signals during the maneuvering process of AGV system as shown in Fig. 1.

The control input is shown in Fig. 6. Due to the influence of stochastic process, there exist some random fluctuations and jump on brake/acceleration δ_b. Since the magnitudes of lateral velocity V_y, lateral distance Y, yaw rate V_θ and yaw angle θ of AGV system are relatively small during the entire tracking process, the fluctuation on the synthesis of steering angle is relatively minor. Besides, with the utilization of system fault estimation, the steering angle δ_f will automatically change it's amplitude to achieve active fault compensation during the entire tracking control process of AGV system.

The estimation errors of six AGV states are shown in Figs. 7–9. Different than the small values of lateral velocity V_y and yaw rate V_θ over the entire tracking control process, the longitudinal velocity V_x is always large and the corresponding stochastic effect on V_x is large too. As a result, for the estimation of longitudinal velocity in Fig. 7, the estimation error is fluctuated due to Wiener process and it has suddenly jump due to Poisson counting process. However, from

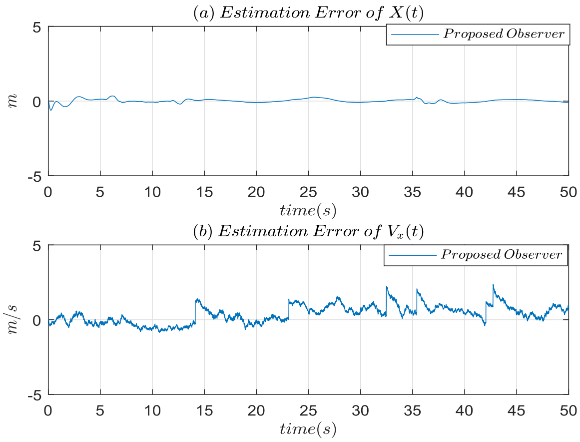

FIGURE 7. The estimation errors of longitudinal displacement X and longitudinal velocity V_x of AGV system by the proposed method.

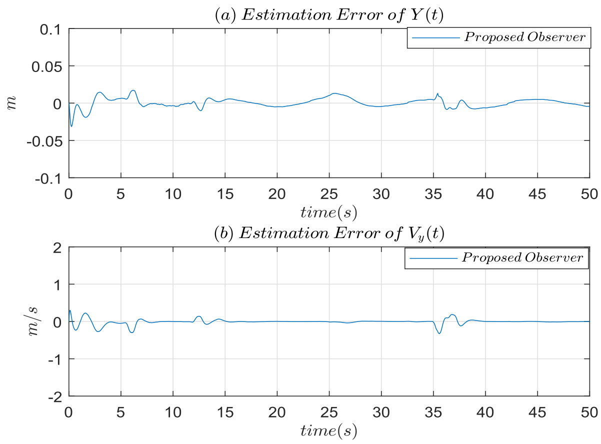

FIGURE 8. The estimation errors of lateral displacement Y and lateral velocity V_y of AGV system by the proposed method.

Fig. 7, it can be seen the stochastic effect of Wiener process on the estimation is efficiently reduced with an acceptable estimation error. Also, once a jump phenomenon occurs, the proposed fuzzy polynomial observer can quickly estimate the real state with a short response time. For example, for the jump phenomenon at 14s, the estimation error converges to zero in probability at 18s. On the other hand, for the efficient estimation of sensor fault signal in Fig. 5, the effect of sensor fault signal is effectively reduced and thus the estimation of longitudinal distance X is very small. For the estimations of lateral velocity V_y and yaw rate V_θ in Figs. 8–9, the fluctuations of estimation errors are mainly caused by the additional reference control command in control input δ_f. For example, once the turn around command is imposed in δ_f at 5s in Fig. 6, there exists a small fluctuation within a very short period. After that, the estimation error quickly approaches to zero. Similar to the estimation of longitudinal distance X, the estimation errors of lateral distance Y and yaw angle θ are very minor due to the efficient compensation of sensor fault signal f_s.

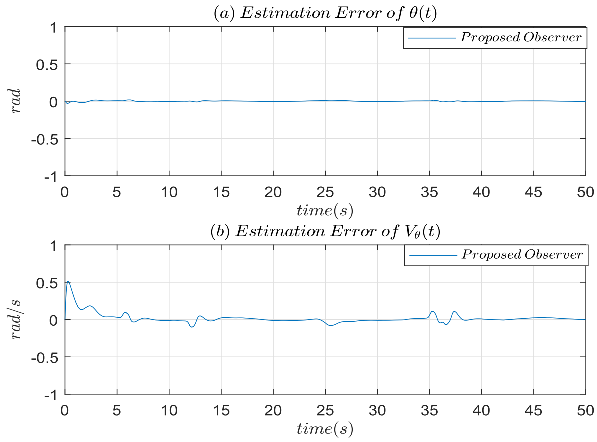

FIGURE 9. The estimation errors of yaw angle θ and yaw rate V_θ of AGV system by the proposed method.

V. CONCLUSION

In this study, a robust H_∞ observer-based fault-tolerant tracking strategy is proposed for SPFS with the consideration of external disturbance, measurement noise and system/sensor fault signals. By using two smoothed models to describe the behaviors of system/sensor fault signals and to embed two fault signals in the nonlinear stochastic system to avoid their corruption on state estimation and control, a polynomial fuzzy observer can be simply constructed for state/fault estimation. With the utilization of estimated information and reference trajectory, a polynomial fuzzy reference tracking controller is implemented to achieve the desired tracking performance and active fault signal compensation. Further, to attenuate the undesired effect of external disturbance and measurement noise during the tracking process, the robust H_∞ fuzzy observer-based fault-tolerant reference tracking control strategy is considered during the design procedure. By using the merit of HPLF, the Itó-Lévy formula can be reformulated and the compensation terms of stochastic processes can be replaced by the Hessian matrix of HPLF with some scaling. Then, the design condition can be formulated in terms of a two–step SOS condition problem which can be easily solved via SOSTOOLs. A simulation example of maneuvering task for AGV system is given to validate the effectiveness of proposed method in comparison with conventional polynomail fuzzy output feedback reference tracking control scheme. In future, the relaxed Itô-Lévy formula can be applied to various control issues with practical applications (e.g., unmanned aerial vehicle tracking design) on SPFS since it provides more design flexibility than the conventional control method for SPFS which has the limitation on the use of quadratic Lyapunov function.

APPENDIX A: PROOF OF THEOREM 1

At first, choose the polynomial Lyapunov function as

$$V(\tilde{x}) = \tilde{x}^T P(\tilde{x})\tilde{x} \tag{42}$$

where $P(\tilde{x})$ is polynomial matrix with $P(\tilde{x}) > 0$. Then, based on the Itô-Lévy formula in (21), the numerator of robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy in (20) can be written as

$$E\{\int_0^{t_f}[\tilde{x}^T\tilde{Q}\tilde{x}+u^TRu]dt\}$$
$$=E\{\int_0^{t_f}[\tilde{x}^T\tilde{Q}\tilde{x}+u^TRu]dt+\int_0^{\infty}dV(\tilde{x})+V(\tilde{x}(0)) -V(\tilde{x}(t_f))\}$$
$$\leq E\{\int_0^{t_f}\sum_{j,i=1}^{l}h_ih_j[\tilde{x}^T\tilde{Q}\tilde{x}+\tilde{x}^TM_j^T(\tilde{x})RM_j(\tilde{x})\tilde{x} +\tilde{x}^TP(\tilde{x})(\tilde{A}_{ij}(\tilde{x})\tilde{x}+\tilde{D}_i(\tilde{x})\tilde{v})+(\tilde{A}_{ij}(\tilde{x})\tilde{x}+\tilde{D}_i(\tilde{x})\tilde{v})^T \times P(\tilde{x})\tilde{x}+\sum_{k=1}^{n_{aug}}\tilde{x}^T\frac{\partial P(\tilde{x})}{\partial\tilde{x}_k}(\tilde{A}_{ij,k}(\tilde{x})\tilde{x}+\tilde{D}_{i,k}(\tilde{x})\tilde{v}))\tilde{x} +\frac{1}{2}\tilde{x}^T\tilde{H}_i^T(\tilde{x})\frac{\partial^2\tilde{x}^TP(\tilde{x})\tilde{x}}{\partial^2\tilde{x}}\tilde{H}_i(\tilde{x})\tilde{x}+\lambda((\tilde{x}+\gamma(\tilde{x}))^T \times P(\tilde{x}+\gamma(\tilde{x}))(\tilde{x}+\gamma(\tilde{x}))-\tilde{x}^TP(\tilde{x})\tilde{x})]dt+V(\tilde{x}(0))\} \tag{43}$$

where $\tilde{x}_k$ is the kth component of $\tilde{x}$, $\tilde{A}_{ij,k}(\tilde{x})$ is the kth row vector of $\tilde{A}_{ij}(\tilde{x})$, $\tilde{D}_{i,k}(\tilde{x})$ is the kth row vector of $\tilde{D}_i(\tilde{x})$ and $M_j(\tilde{x}) = [K_{m,2}(\hat{x},\bar{x}_r) - K_{m,1}(\hat{x},\bar{x}_r), K_{m,1}(\hat{x},\bar{x}_r), -K_{m,1}(\hat{x},\bar{x}_r)]$.

By using Lemma 2, the terms associated with disturbance can be relaxed as

$$\tilde{x}^T\frac{\partial P(\tilde{x})}{\partial\tilde{x}_k}\tilde{x}\tilde{D}_{i,k}(\tilde{x})\tilde{v}$$
$$=\frac{1}{2}\tilde{x}^T\frac{\partial P(\tilde{x})}{\partial\tilde{x}_k}\tilde{x}\tilde{D}_{i,k}(\tilde{x})\tilde{v}+\frac{1}{2}\tilde{v}^T\tilde{D}_{i,k}^T(\tilde{x})\tilde{x}^T\frac{\partial P(\tilde{x})}{\partial\tilde{x}_k}\tilde{x}$$
$$\leq\frac{\rho}{2n_{aug}}\tilde{v}^T\tilde{v}+\frac{n_{aug}}{2\rho}\tilde{x}^T\frac{\partial P(\tilde{x})}{\partial\tilde{x}_k}\tilde{x}\tilde{D}_{i,k}(\tilde{x})\tilde{D}_{i,k}^T(\tilde{x})\tilde{x}^T\frac{\partial P(\tilde{x})}{\partial\tilde{x}_k}\tilde{x}$$
$$\tilde{x}^TP(\tilde{x})\tilde{D}_i(\tilde{x})\tilde{v}+\tilde{v}^T\tilde{D}_i^T(\tilde{x})P(\tilde{x})\tilde{x}$$
$$\leq\frac{2}{\rho}\tilde{x}^TP(\tilde{x})\tilde{D}_i\tilde{D}_i^T(\tilde{x})P(\tilde{x})\tilde{x}+\frac{\rho}{2}\tilde{v}^T\tilde{v}$$
$$for\ k=1,\cdots,n_{aug} \tag{44}$$

with some $\rho > 0$. By substituting (44) into (43), we have

$$E\{\int_0^{t_f}[\tilde{x}^T\tilde{Q}\tilde{x}+u^TRu]dt\}$$
$$\leq E\{\int_0^{t_f}\sum_{j,i=1}^{l}h_ih_j[\tilde{x}^T(\tilde{Q}+M_j^T(\tilde{x})RM_j+P(\tilde{x}) \times\tilde{A}_{ij}(\tilde{x})+\tilde{A}_{ij}^T(\tilde{x})P(\tilde{x})+\frac{1}{2}\tilde{H}_i^T(\tilde{x})\frac{\partial^2\tilde{x}^TP(\tilde{x})\tilde{x}}{\partial^2\tilde{x}}\tilde{H}_i(\tilde{x}) +\sum_{k=1}^{n_{aug}}\frac{\partial P(\tilde{x})}{\partial\tilde{x}_k}\tilde{A}_{ij,k}(\tilde{x})\tilde{x}+\frac{2}{\rho}P(\tilde{x})\tilde{D}_i\tilde{D}_i^T(\tilde{x})P(\tilde{x}) +\lambda(P(\tilde{x}+\gamma(\tilde{x}))+\tilde{O}_i^T(\tilde{x})P(\tilde{x}+\gamma(\tilde{x})) +P(\tilde{x}+\gamma(\tilde{x}))\tilde{O}_j(\tilde{x})+\tilde{O}_i^T(\tilde{x})P(\tilde{x}+\gamma(\tilde{x}))\tilde{O}_i(\tilde{x}) -P(\tilde{x})))\tilde{x}+\sum_{k=1}^{n_{aug}}\frac{n_{aug}}{2\rho}\tilde{x}^T\frac{\partial P(\tilde{x})}{\partial\tilde{x}_k}\tilde{x}\tilde{D}_{i,k}(\tilde{x}) \times\tilde{D}_{i,k}^T(\tilde{x})\tilde{x}^T(\frac{\partial P(\tilde{x})}{\partial\tilde{x}_k})^T\tilde{x}+\rho\tilde{v}^T\tilde{v}]dt+V(\tilde{x}(0))\} \tag{45}$$

If the following polynomial constraints are satisfied

$$\Phi_{ii}(\tilde{x})<0,\ i=1,\cdots,l$$
$$\Phi_{ij}(\tilde{x})+\Phi_{ji}(\tilde{x})<0,\ 1\leq j<i\leq l \tag{46}$$

with

$$\Phi_{ij}(\tilde{x})=\tilde{x}^T(\tilde{Q}+M_j^T(\tilde{x})RM_j(\tilde{x})+P(\tilde{x})\tilde{A}_{ij}(\tilde{x}) +\tilde{A}_{ij}^T(\tilde{x})P(\tilde{x})+\frac{1}{2}\tilde{H}_i^T(\tilde{x})\frac{\partial^2\tilde{x}^TP(\tilde{x})\tilde{x}}{\partial^2\tilde{x}}\tilde{H}_i(\tilde{x}) +\sum_{k=1}^{n_{aug}}\frac{\partial P(\tilde{x})}{\partial\tilde{x}_k}\tilde{A}_{ij,k}(\tilde{x})\tilde{x} +\frac{2}{\rho}P(\tilde{x})\tilde{D}_i\tilde{D}_i^T(\tilde{x})P(\tilde{x})+\lambda(P(\tilde{x}+\gamma(\tilde{x}))\tilde{O}_j(\tilde{x}) +P(\tilde{x}+\gamma(\tilde{x}))+\tilde{O}_i^T(\tilde{x})P(\tilde{x}+\gamma(\tilde{x})) +\tilde{O}_i^T(\tilde{x})P(\tilde{x}+\gamma(\tilde{x}))\tilde{O}_i(\tilde{x})-P(\tilde{x}))+\sum_{k=1}^{n_{aug}}\frac{n_{aug}}{2\rho} \times\frac{\partial P(\tilde{x})}{\partial\tilde{x}_k}\tilde{x}\tilde{D}_{i,k}(\tilde{x})\tilde{D}_{i,k}^T(\tilde{x})\tilde{x}^T\frac{\partial P(\tilde{x})}{\partial\tilde{x}_k})\tilde{x}$$

, then (45) can be written as

$$E\{\int_0^{t_f}[\tilde{x}^T\tilde{Q}\tilde{x}+u^TRu]dt\}$$
$$\leq E\{\int_0^{t_f}\rho[\tilde{v}^T\tilde{v}]dt+V(\tilde{x}(0))\}$$
$$\tilde{v}\in\mathcal{L}_2^{\mathcal{F}}(\mathbb{R}^+,\mathbb{R}^{n_{\tilde{v}}}), \tag{47}$$

where $n_{\tilde{v}}$ is dimension of $\tilde{v}$, and it shows that the robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy in (20) is achieved with a prescibed disturbance attenuation level $\rho > 0$. Moreover, since the augmented noise $\tilde{v}$ is finite energy, the RHS in (47) is finite for any $t_f \in \mathbb{R}^+$, we immediately have

$$E\{\int_0^{t_f}[\tilde{x}^T\tilde{Q}\tilde{x}+u^TRu]dt\}$$
$$\leq E\{\int_0^{t_f}\rho[\tilde{v}^T\tilde{v}]dt+V(\tilde{x}(0))\}<\infty$$
$$\forall t_f\in\mathbb{R}^+,\tilde{v}\in\mathcal{L}_2^{\mathcal{F}}(\mathbb{R}^+,\mathbb{R}^{n_{\tilde{v}}}) \tag{48}$$

,i.e., the augmented system in (19) is mean square stable, i.e., $E\{\tilde{x}^T\tilde{x}\}\rightarrow 0$, as $t\rightarrow\infty$.

APPENDIX B: PROOF OF THEOREM 2

Consider a HPLF $V(\tilde{x}) = \tilde{x}^TP(\tilde{x})\tilde{x}$ of SPFS in (19) with a positive definite homogeneous polynomial matrix $P(\tilde{x})$ and degree $s \in \mathbb{N} - \{1\}$, then there exists a positive definite homogeneous polynomial matrix $\bar{P}(\tilde{x})$ such that

$$V(\tilde{x})=\tilde{x}^TP(\tilde{x})\tilde{x}$$
$$=\frac{1}{s(s-1)}\tilde{x}^T\frac{\partial^2V(\tilde{x})}{\partial^2\tilde{x}}(\tilde{x})\tilde{x}$$
$$=\frac{1}{s(s-1)}\tilde{x}^T\bar{P}(\tilde{x})\tilde{x} \tag{49}$$

where $\bar{P}(\tilde{x})=\frac{\partial^2V(\tilde{x})}{\partial^2\tilde{x}}$.

By selecting the HPFS $V(\tilde{x})$ in (49) with Lemma 4, the Itô-Lévy formula of $V(\tilde{x})$ w.r.t. the augmented system in (19) can be written as

$$\begin{aligned} &E\{dV(\tilde{x})\} \\ &= E\{\sum_{i,j=1}^{l} h_i h_j (\frac{\partial V(\tilde{x})}{\partial \tilde{x}})^T [\tilde{A}_{ij}(\tilde{x})\tilde{x} + \tilde{D}_i(\tilde{x})\tilde{v} \\ &\quad + \frac{1}{2}(\tilde{H}_j(\tilde{x})\tilde{x})^T (\frac{\partial^2 V(\tilde{x})}{\partial \tilde{x}^2})(\tilde{H}_i(\tilde{x})\tilde{x})dt \\ &\quad + \lambda(V(\tilde{x}+\gamma(\tilde{x})) - V(\tilde{x}))]dt\} \\ &= E\{\sum_{j,i=1}^{l} h_i h_j [\frac{1}{2(s-1)}\tilde{x}^T \bar{P}(\tilde{x})(\tilde{A}_{ij}(\tilde{x})\tilde{x} + \tilde{D}_i(\tilde{x})\tilde{v}) \\ &\quad + \frac{1}{2(s-1)}(\tilde{A}_{ij}(\tilde{x})\tilde{x} + \tilde{D}_i(\tilde{x})\tilde{v})^T \bar{P}(\tilde{x})\tilde{x}^T + \frac{1}{2}(\tilde{H}_j(\tilde{x})\tilde{x})^T \\ &\quad \times \bar{P}(\tilde{x})(\tilde{H}_i(\tilde{x})\tilde{x}) + \lambda(V(\tilde{x}+\gamma(\tilde{x})) - V(\tilde{x}))]dt\} \end{aligned} \tag{50}$$

Furthermore, based on Lipschiz condition of system matrix w.r.t. the Poisson counting process $o(x)$ in (1), we have

$$\|o(x)\|_2^D \le L_{poi}\|x\|_2^D \tag{51}$$

where L_{poi} is the Lipschiz constant associated with $o(x)$. Then, to relax the compensation term of Poisson process, we notice that

$$\begin{aligned} &\|\gamma(\tilde{x})\|_2^D \\ &= \left\| \sum_{a=1}^{l} h_a \tilde{O}_a(\tilde{x})\tilde{x} \right\|_2^D \\ &= \left\| [0_{1\times n_{aug}}, o^T(x), 0_{1\times(n_{aug}-n)}, o^T(x), 0_{1\times(n_{aug}-n)}]^T \right\|_2^D \\ &\le L_{poi}\|\tilde{x}\|_2^D \end{aligned} \tag{52}$$

By the triangular inequality with (52), the following relation holds

$$\begin{aligned} \|\tilde{x}+\gamma(\tilde{x})\|_2^D &\le \|\tilde{x}\|_2^D + \|\gamma(\tilde{x})\|_2^D \\ &\le (1+L_{poi})\|\tilde{x}\|_2^D \end{aligned} \tag{53}$$

By (53) with the increasing property of $V(\tilde{x})$, the following inequality holds

$$V(\tilde{x}+\gamma(\tilde{x})) \le V((1+L_{poi})\tilde{x}) \tag{54}$$

Furthermore, by the definition of HPFS, we have

$$\begin{aligned} &E\{(V(\tilde{x}+\gamma(\tilde{x})) - V(\tilde{x}))dp\} \\ &\quad \le \lambda E\{V((1+L_{poi})\tilde{x}) - V(\tilde{x})\} \\ &\quad \le \lambda E\{((1+L_{poi})^s - 1)V(\tilde{x})\} \end{aligned} \tag{55}$$

Then, by using (55) and Lemma 4, the Itô-Lévy formula of $V(\tilde{x})$ in (50) can be bounded as

$$\begin{aligned} E\{dV(\tilde{x})\} &\le E\{\sum_{i,j=1}^{l} h_i h_j [\frac{1}{2(s-1)}\tilde{x}^T \bar{P}(\tilde{x})(\tilde{A}_{ij}(\tilde{x})\tilde{x} \\ &\quad + \tilde{D}_i(\tilde{x})\tilde{v}) + \frac{1}{2(s-1)}(\tilde{A}_{ij}(\tilde{x})\tilde{x} + \tilde{D}_i(\tilde{x})\tilde{v})^T \bar{P}(\tilde{x})\tilde{x} \\ &\quad + \frac{1}{2}(\tilde{H}_j(\tilde{x})\tilde{x})^T \bar{P}(\tilde{x})(\tilde{H}_i(\tilde{x})\tilde{x})dt \\ &\quad + \frac{\lambda((1+L_{poi})^s - 1)}{s(s-1)}\tilde{x}^T \bar{P}(\tilde{x})\tilde{x}]dt\} \end{aligned} \tag{56}$$

APPENDIX C: PROOF OF THEOREM 3

To begin with, consider an HPLS $V(\tilde{x}) = \tilde{x}^T P(\tilde{x})\tilde{x}$ with the positive definite homogeneous polynomial matrix $P(\tilde{x})$ and degree $s \in \mathbb{N} - \{1\}$. Then, with the utilization of the reformulated Itô-Lévy formula in (24) and the similar derivation in (43), the numerator of robust H_∞ observer-based fault-tolerant tracking control strategy in (20) can be written as

$$\begin{aligned} &E\{\int_0^{t_f} [\tilde{x}^T \tilde{Q}\tilde{x} + u^T R u]dt\} \\ &\le E\{\int_0^{t_f} \sum_{j,i=1}^{l} h_i h_j [\tilde{x}^T \tilde{Q}\tilde{x} + \tilde{x}^T M_i^T(\tilde{x}) R M_j(\tilde{x})\tilde{x} \\ &\quad + \frac{1}{2(s-1)}\tilde{x}^T \bar{P}(\tilde{x})(\tilde{A}_{ij}(\tilde{x})\tilde{x} + \tilde{D}_i(\tilde{x})\tilde{v}) \\ &\quad + \frac{1}{2(s-1)}(\tilde{A}_{ij}(\tilde{x})\tilde{x} + \tilde{D}_i(\tilde{x})\tilde{v})^T \bar{P}(\tilde{x})\tilde{x} \\ &\quad + \frac{1}{2}()\tilde{x}^T \tilde{H}_j^T(\tilde{x})\bar{P}(\tilde{x})\tilde{H}_i(\tilde{x})\tilde{x} \\ &\quad + \frac{\lambda((1+L_{poi})^s - 1)}{s(s-1)}\tilde{x}^T \bar{P}(\tilde{x})\tilde{x}]dt + V(\tilde{x}(0))\} \end{aligned} \tag{57}$$

where $\bar{P}(\tilde{x}) = \frac{\partial^2 V(\tilde{x})}{\partial^2 \tilde{x}}$. By using Lemma 2, the terms associated with disturbance can be relaxed as

$$\begin{aligned} &\frac{1}{2(s-1)}(\tilde{x}^T \bar{P}(\tilde{x})\tilde{D}_i(\tilde{x})\tilde{v} + \tilde{v}^T \tilde{D}_i^T(\tilde{x})\bar{P}(\tilde{x})\tilde{x}) \\ &\quad \le \frac{1}{4\rho(s-1)^2}\tilde{x}^T \bar{P}(\tilde{x})\tilde{D}_i(\tilde{x})\tilde{D}_i^T(\tilde{x})\bar{P}(\tilde{x})\tilde{x}^T + \rho\tilde{v}^T\tilde{v}, \\ &\quad for\ i = 1, \cdots, L \end{aligned} \tag{58}$$

with some $\rho > 0$. By substituting (58) into (57), we have

$$\begin{aligned} &E\{\int_0^{t_f} [\tilde{x}^T \tilde{Q}\tilde{x} + u^T R u]dt\} \\ &\le E\{\int_0^{t_f} \sum_{j,i=1}^{l} h_i h_j [\tilde{x}^T (\tilde{Q} + M_j^T(\tilde{x}) R M_j(\tilde{x}) \\ &\quad + \frac{1}{2(s-1)}\bar{P}(\tilde{x})\tilde{A}_{ij}(\tilde{x}) + \frac{1}{2(s-1)}\tilde{A}_{ij}^T(\tilde{x})\bar{P} \\ &\quad + \frac{1}{2}\tilde{H}_j^T(\tilde{x})\bar{P}(\tilde{x})\tilde{H}_i(\tilde{x}) + \frac{\lambda((1+L_{poi})^s - 1)}{s(s-1)}\bar{P}(\tilde{x}) + \frac{1}{4\rho(s-1)^2} \\ &\quad \times \bar{P}(\tilde{x})\tilde{D}_i(\tilde{x})\tilde{D}_i^T(\tilde{x})\bar{P}(\tilde{x}))\tilde{x} + \rho\tilde{v}^T\tilde{v}]dt + V(\tilde{x}(0))\} \end{aligned} \tag{59}$$

Clearly, if following matrix constraint is satisfied

$$\begin{aligned} &\sum_{j,i=1}^{l} h_i h_j E\{[\tilde{x}^T (\tilde{Q} + M_j^T(\tilde{x}) R M_j(\tilde{x}) \\ &\quad + \frac{1}{2(s-1)}\bar{P}(\tilde{x})\tilde{A}_{ij}(\tilde{x}) + \frac{1}{2(s-1)}\tilde{A}_{ij}^T(\tilde{x})\bar{P} \\ &\quad + \frac{1}{2}\tilde{H}_j^T(\tilde{x})\bar{P}(\tilde{x})\tilde{H}_i(\tilde{x}) + \frac{\lambda((1+L_{poi})^s - 1)}{s(s-1)}\bar{P}(\tilde{x}) \\ &\quad + \frac{1}{4\rho(s-1)^2}\bar{P}(\tilde{x})\tilde{D}_i(\tilde{x})\tilde{D}_i^T(\tilde{x})\bar{P}(\tilde{x}))\tilde{x}\} \le 0 \end{aligned} \tag{60}$$

then (59) can be relaxed as

$$E\{\int_0^{t_f} [\tilde{x}^T \tilde{Q}\tilde{x} + u^T R u]dt\} \le E\{\rho\int_0^{t_f} [\tilde{v}^T\tilde{v}]dt + V(\tilde{x}(0))\} \tag{61}$$

which immediately shows the robust H_∞ fuzzy observer-based fault-tolerant tracking control strategy in (20) is achieved with a prescribed disturbance attenuation level $\rho > 0$. Besides, similar to the derivation of Theorem 1, the mean square stability of the augmented system in (19) can be infered from (61).

APPENDIX D: SIMULATION PARAMETERS

The polynomial fuzzy observer gains and polynomial fuzzy controller gains for simulation are given as

$$K_{1,1}(\hat{x},\bar{x}_r)=\begin{bmatrix}0.1 & -0.2 & -1.3 & 0.1\\ 2.1 & 0.2 & 0.1 & 3.9\end{bmatrix}\cdots \cdots\begin{bmatrix}0.5 & -1 & -1 & 0_{1\times5}\\ 0.1 & 0.1 & 0 & 0_{1\times5}\end{bmatrix}$$

$$K_{2,1}(\hat{x},\bar{x}_r)=\begin{bmatrix}-0.1 & -0.1 & -1.3 & 0.1\\ 1.7 & 0.2 & -0.3 & 0.4\end{bmatrix}\cdots \cdots\begin{bmatrix}0.6 & -1.1 & -1 & 0_{1\times5}\\ 0.1 & 0.1 & 0 & 0_{1\times5}\end{bmatrix}$$

$$K_{3,1}(\hat{x},\bar{x}_r)=\begin{bmatrix}-0.1 & -0.1 & -0.7 & 0\\ 1.9 & 0.1 & 0.2 & 0.1\end{bmatrix}\cdots \cdots\begin{bmatrix}0.1 & 0.4 & -1 & 0_{1\times5}\\ 0.1 & 0.1 & 0 & 0_{1\times5}\end{bmatrix}$$

$$K_{4,1}(\hat{x},\bar{x}_r)=\begin{bmatrix}-0.1 & -0.1 & -0.7 & 0\\ 1.9 & 0.1 & 0.1 & -0.3\end{bmatrix}\cdots \cdots\begin{bmatrix}0.1 & 0.4 & -1 & 0_{1\times5}\\ 0.2 & 0.1 & 0 & 0_{1\times5}\end{bmatrix}$$

$$\begin{aligned}K_{1,2}(\hat{x},\bar{x}_r)=&\begin{bmatrix}0.1(\hat{\hat{x}}_1-\bar{x}_{r,1}) & 0.5(\hat{\hat{x}}_2-\bar{x}_{r,2})\\ 1.1(\hat{\hat{x}}_1-\bar{x}_{r,1}) & -0.1(\hat{\hat{x}}_2-\bar{x}_{r,2})\end{bmatrix}\cdots\\ &\cdots\begin{matrix}0.3(\hat{\hat{x}}_3-\bar{x}_{r,3}) & 0.1(\hat{\hat{x}}_4-\bar{x}_{r,4})\\ 0.1(\hat{\hat{x}}_3-\bar{x}_{r,3}) & 1.3(\hat{\hat{x}}_4-\bar{x}_{r,4})\end{matrix}\cdots\\ &\cdots\begin{bmatrix}(\hat{\hat{x}}_5-\bar{x}_{r,5}) & 2.1(\hat{\hat{x}}_6-\bar{x}_{r,6}) & 0_{1\times6}\\ 0.1(\hat{\hat{x}}_5-\bar{x}_{r,5}) & (\hat{\hat{x}}_6-\bar{x}_{r,6}) & 0_{1\times6}\end{bmatrix}\end{aligned}$$

$$\begin{aligned}K_{2,2}(\hat{x},\bar{x}_r)=&\begin{bmatrix}0.1(\hat{\hat{x}}_1-\bar{x}_{r,1}) & 0.2(\hat{\hat{x}}_2-\bar{x}_{r,2})\\ 1.5(\hat{\hat{x}}_1-\bar{x}_{r,1}) & 0.1(\hat{\hat{x}}_2-\bar{x}_{r,2})\end{bmatrix}\cdots\\ &\cdots\begin{matrix}0.7(\hat{\hat{x}}_3-\bar{x}_{r,3}) & 0.1(\hat{\hat{x}}_4-\bar{x}_{r,4})\\ 0.2(\hat{\hat{x}}_3-\bar{x}_{r,3}) & 1.3(\hat{\hat{x}}_4-\bar{x}_{r,4})\end{matrix}\cdots\\ &\cdots\begin{bmatrix}1(\hat{\hat{x}}_5-\bar{x}_{r,5}) & 2.1(\hat{\hat{x}}_6-\bar{x}_{r,6}) & 0_{1\times6}\\ -0.1(\hat{\hat{x}}_5-\bar{x}_{r,5}) & -0.1(\hat{\hat{x}}_6-\bar{x}_{r,6}) & 0_{1\times6}\end{bmatrix}\\ &\cdots\begin{bmatrix}1(\hat{\hat{x}}_5-\bar{x}_{r,5}) & 2.1(\hat{\hat{x}}_6-\bar{x}_{r,6}) & 0_{1\times6}\\ -0.1(\hat{\hat{x}}_5-\bar{x}_{r,5}) & -0.1(\hat{\hat{x}}_6-\bar{x}_{r,6}) & 0_{1\times6}\end{bmatrix}\end{aligned}$$

$$\begin{aligned}K_{3,2}(\hat{x},\bar{x}_r)=&\begin{bmatrix}0.1(\hat{\hat{x}}_1-\bar{x}_{r,1}) & -1.6(\hat{\hat{x}}_2-\bar{x}_{r,2})\\ 0.8(\hat{\hat{x}}_1-\bar{x}_{r,1}) & 0.1(\hat{\hat{x}}_2-\bar{x}_{r,2})\end{bmatrix}\cdots\\ &\cdots\begin{matrix}-1.1(\hat{\hat{x}}_3-\bar{x}_{r,3}) & 0.1(\hat{\hat{x}}_4-\bar{x}_{r,4})\\ 0.1(\hat{\hat{x}}_3-\bar{x}_{r,3}) & 1.3(\hat{\hat{x}}_4-\bar{x}_{r,4})\end{matrix}\cdots\\ &\cdots\begin{bmatrix}1(\hat{\hat{x}}_5-\bar{x}_{r,5}) & 2.1(\hat{\hat{x}}_6-\bar{x}_{r,6}) & 0_{1\times6}\\ -0.1(\hat{\hat{x}}_5-\bar{x}_{r,5}) & -0.1(\hat{\hat{x}}_6-\bar{x}_{r,6}) & 0_{1\times6}\end{bmatrix}\end{aligned}$$

$$\begin{aligned}K_{4,2}(\hat{x},\bar{x}_r)=&\begin{bmatrix}0.1(\hat{\hat{x}}_1-\bar{x}_{r,1}) & 1.6(\hat{\hat{x}}_2-\bar{x}_{r,2})\\ -1.2(\hat{\hat{x}}_1-\bar{x}_{r,1}) & 0.1(\hat{\hat{x}}_2-\bar{x}_{r,2})\end{bmatrix}\cdots\\ &\cdots\begin{matrix}1.3(\hat{\hat{x}}_3-\bar{x}_{r,3}) & 0.2(\hat{\hat{x}}_4-\bar{x}_{r,4})\\ 0.1(\hat{\hat{x}}_3-\bar{x}_{r,3}) & 1.4(\hat{\hat{x}}_4-\bar{x}_{r,4})\end{matrix}\cdots\\ &\cdots\begin{bmatrix}-1.5(\hat{\hat{x}}_5-\bar{x}_{r,5}) & -2.1(\hat{\hat{x}}_6-\bar{x}_{r,6}) & 0_{1\times6}\\ -0.1(\hat{\hat{x}}_5-\bar{x}_{r,5}) & -0.1(\hat{\hat{x}}_6-\bar{x}_{r,6}) & 0_{1\times6}\end{bmatrix}\end{aligned}$$

$$\begin{aligned}L_1(\hat{x})=&\begin{bmatrix}59.2 & 14 & 11 & 30.6\\ 0 & 51.1 & 0 & -9.3\\ 5 & 0 & 56 & -1\end{bmatrix}\cdots\\ &\cdots\begin{matrix}-35 & -20 & 14 & 24\\ 23.5 & 9 & 31 & -70\\ 5 & 12 & 32 & 31\end{matrix}\cdots\\ &\cdots\begin{bmatrix}23 & 16 & 24 & 1.1\\ -27 & -5 & -90 & -1\\ 21 & 16 & 18 & -50\end{bmatrix}^T\end{aligned}$$

$$\begin{aligned}L_2(\hat{x})=&\begin{bmatrix}51.2 & 6 & 6 & 19.6\\ 9 & 51.1 & 0.2 & -0.6\\ 8 & -0.3 & 65 & -1\end{bmatrix}\cdots\\ &\cdots\begin{matrix}-87 & -20 & -35 & 14\\ 26.5 & 7 & 8 & -75\\ 5 & 6 & 32 & 7\end{matrix}\cdots\\ &\cdots\begin{bmatrix}-16 & 16 & -31 & -9\\ -35 & 2 & -8 & 3\\ 31 & 2 & -3 & -13\end{bmatrix}^T\end{aligned}$$

$$\begin{aligned}L_3(\hat{x})=&\begin{bmatrix}53 & 0.7 & -3.8 & 31\\ 11 & 51.1 & 0 & 8\\ 20 & 0 & 53 & 15\end{bmatrix}\cdots\\ &\cdots\begin{matrix}24 & 30 & -11 & -13\\ 26.5 & 13 & 5 & -82\\ 24 & 30 & 4 & -6\end{matrix}\cdots\\ &\cdots\begin{bmatrix}-2 & 16 & -24 & -15\\ -3 & 2 & -9 & -5\\ -3 & 2 & -18 & -7\end{bmatrix}^T\end{aligned}$$

$$\begin{aligned}L_4(\hat{x})=&\begin{bmatrix}51 & 3 & 9 & 59\\ 9 & 51.9 & 0.2 & 1\\ 9 & -0.4 & 53 & 1\end{bmatrix}\cdots\\ &\cdots\begin{matrix}27 & -95 & -4 & 5\\ 27 & 8 & 9 & -17\\ 8 & 8 & 3 & 8\end{matrix}\cdots\\ &\cdots\begin{bmatrix}-7 & 16 & -3 & -9\\ -37 & 2 & -9 & 2\\ -35 & 2 & -2 & -20\end{bmatrix}^T\end{aligned}$$

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