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RESEARCH ARTICLE

Guaranteed-Cost Consensus Control for High-Dimensional Multi-Agent Systems With Input Delays and Parameter Uncertainties

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ABSTRACT In this paper, the problem of consensus control is investigated for a kind of multi-agent systems (MAS) in high-dimensional form. Both the nonlinear dynamics and norm-bound uncertainties are taken into account, which makes the model more comprehensive than those in existing literature. By employing the related error (between each agent and its neighbors) and considering the time-varying input delay, the consensus protocol is constructed to force agents to close to the consensus dynamical function. An augmented closed-loop system is established by taking use of properties of Laplacian matrix and the state space decomposition method. A few of sufficient criteria are gained with the help of feasible solutions for linear matrix inequalities. The asymptotical stability of error system is realized and the upper-bound is obtained for the cost function consisting of both the consensus regulation performance and the control energy consumption. An illustrative example and the corresponding simulations are given for verifying the validity of our results.

INDEX TERMS Consensus control, guaranteed-cost function, input delay, multi-agent system, uncertainty.

I. INTRODUCTION

In last decades, the coordination problem of multi-agent system (MAS) has attracted a large amount of research attention owing to the wide practical utilizations in distributed cloud computing [1], spacecraft formation flying [2], adaptive dynamic programming [3], manipulators [4], and cyber-physical energy networks [5]. Consensus control, whose aim is to propose an appropriate control strategy to render the position and velocity of all agents gradually converge to a common value, has been considered as one of the essential coordination behaviors for MAS [6], [7], [8], [9], [10], [11], [12], [13], [14], [15]. For example, the finite-time consensus tracking control problem has been considered in [12] for nonlinear multi-agent systems in which state

variables are unmeasured and nonlinear functions are totally unknown. In [13], an optimal consensus problem has been studied for a set of integrator systems with dynamic uncertainties. In [14], authors have investigated the observer-based fully distributed containment control for multi-agent systems subject to denial-of-service attacks. In practical application, since the limited speed of information transmission and processing among agents, MASs will inevitably encounter time delay, which may dramatically influence the performance of consensus [16], [17], [18]. Until now, a large number of research results have already been reported to address the issue of consensus control for MASs with various time delays [19], [20], [21], [22], [23]. For instance, in [22], authors have concerned with the problem of consensus of multiple agents with intrinsic nonlinear dynamics and sampled-data information on the basis of a delayed-input approach.

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To solve problems of control synthesis, it is often necessary to give a control strategy which can not only realize the required performance of closed-loop system but also achieve a given level of cost performance. As such, the guaranteed cost control plays an important role in controller design [24], [25]. Recent years, a few of interesting works have been reported to address the guaranteed cost consensus of MASs (see [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37], and references therein). For instance, scholars have investigated the guaranteed cost consensus for a class of discrete-time MAS with singularity and switching topologies via event-triggered consensus protocol [30]. Taking use of the graph theory and the equivalent transformation method for singular systems, some interesting conditions have been derived in the form of the linear matrix inequalities (LMIs) to realize guaranteed cost consensus of discrete-time singular multi-agent systems. By changing variables, the problem of guaranteed-cost consensus has been considered in [31]. By constructing the consensus control protocol with the related cost index, the guaranteed cost consensus has ultimately realized for MAS with some constraints of input.

Most of results mentioned above have been obtained by supposing that all parameters of MASs are accurately known in advance. In fact, the presence of uncertainties in most control plants is unavoidable because of sensor errors, actuator failure, modeling inaccuracy, and so on [38]. For example, air pollution measurement instruments in a sensor network may give imperfect readings, or gas detection sensors may fail to detect gases with a certain probability. That is to say, the system uncertainties are ubiquitous when we take various unmodeled systems dynamics or parameter drifting into consideration. Therefore, it is more reliable to consider the influence of uncertainties on the consensus performance of MASs. Currently, considerable research attention have been drawn on to the consensus control for MASs with various uncertainties [39], [40], [41], [42], [43]. Specifically, the sliding mode control has been exploited to deal with uncertainties incurred by unknown time varying communication delay and disturbance [40]. Based on the control technique and Lyapunov stability theory on finite-time horizon, an adaptive fast finite-time consensus issue has been addressed for second-order uncertain nonlinear MASs with external disturbances and unknown nonsymmetric dead-zone [42]. A fuzzy adaptive control within event-triggered mechanism has been designed for the control of a kind of nonlinear MASs with input dead-zone and uncertainties [43].

On the other research horizon, a physical system in real world might be governed by some nonlinear terms (such as Lipschitz nonlinearity, stochastic nonlinearity, polynomial nonlinearity and so on) due probably to the high maneuverability of the moving target, sudden parameter switching, and environmental fluctuation. For example, the well-known UAVs, which have been widely used for surveillance and reconnaissance missions, is a complex nonlinear dynamic model owing to the coupling of 6-DOF motion and the

nonlinear changes of aerodynamic force and torque with flight. In this case, the state of systems are not only affected by the interaction among neighboring agents, but also by its own intrinsic nonlinear dynamics. It is worth noting that Lipschitz nonlinearity, which is restricted by a linear form of constraint, has aroused the most extensive research attention in analysis for nonlinear systems. Particularly, in recent years, an ever increasing number of studies have been devoted to the filtering and control problems for dynamical systems with Lipschitz nonlinearity. However, few results have emerged to address the problem of guaranteed cost consensus for MASs with both Lipschitz nonlinear dynamics and parameter uncertainties. Thus, the first aim of this paper is to shorten such a gap.

It is to be noted that most of above-mentioned results have only considered the consensus regulation performance for MASs. Generally speaking, the consumption of energy is also fundamental for the control synthesis in practical applications due mainly to the fact that the energy for control input is always limited. Hence, in analyzing and implementing the control of MAS, it is significantly important to design a consensus protocol while considering both the control performance and energy consumption. Recently, some preliminary research work has been done to consider the guaranteed cost consensus problem for MASs with input delays. However, little effort has been devoted to dealing with the guaranteed cost consensus problem for high-dimensional MASs with both time delays and uncertainties, which forms the other motivation of the current research.

Based on the above discussion, this paper is prepared to study the issue of guaranteed cost consensus for a category of high-dimensional MASs, which takes both the nonlinear dynamics and norm-bound uncertainties into account. Two constant matrices and a unsuspected matrix are introduced to characterize the influence resulting from parameter uncertainties. With the help of a Lipschitz constant, the constraint condition with linear form is adopted for the Lipschitz nonlinear term such that the consensus performance is ultimately investigated. The principle novelties of this paper are summarized as follows: (1) The guaranteed cost consensus problem is originally investigated for high-dimensional MASs with uncertain parameter via a distributed control with time-varying input delays. (2) The cost function is constructed to simultaneously evaluate a certain level of both control performance and energy consumption. (3) The control gain matrix are calculated by relying on the feasible solution of LMIs so as to ensure all agents approach a consensus function.

The rest of this paper is structured as follows: Section II provides the graph theory and the problem formulation. In Section III, sufficient conditions are derived to ensure the consensus of MASs and determine the upper bound for the guaranteed cost. In Section IV, the validity of theoretical results is illustrated by calculating a numerical example and simulating the dynamics. Finally, conclusions of this paper are shown in Section V.

Notation: The notations in this paper are standard. R^d is for the d -dimensional real column vector space, and $R^{d \times m}$ stands for the set of $d \times m$ dimensional real matrices. $1_N \in R^N$ denotes an N -dimensional column vector with all components 1. Let $\mathbf{0}$ be the matrix or vector of compatible size with all components 0. $I_N \in R^{N \times N}$ represents the n -order unit matrix. The Kronecker product of matrices is denoted by \otimes . The symbol $*$ stands for the symmetric elements in a symmetric matrix. $P > 0$ (or $P < 0$) means that the symmetric matrix P is positive definite (or negative definite). For $x \in R^N$, $\|x\|$ represents the Euclidean norm of x . $\lambda_{max}(A)$ (or $\lambda_{min}(A)$) represents the maximum (or minimum) eigenvalue of matrix A .

II. PRELIMINARIES AND PROBLEM FORMULATION

A. GRAPH TOPOLOGY

Let $G = G(\mathbb{V}, \mathbb{E}, \mathbb{W})$ be an undirected graph consisting of node set $\mathbb{V} = \{v_1, v_2, \dots, v_N\}$, edge set $\mathbb{E} = \{(v_i, v_j) : v_i, v_j \in \mathbb{V}\}$, and adjacency matrix $\mathbb{W} = [\omega_{ij}] \in R^{N \times N}$ with $\omega_{ij} \geq 0$ for $i \neq j$ and $\omega_{ii} = 0$, where $\omega_{ij} > 0$ if and only if $(v_i, v_j) \in \mathbb{E}$. The node index belongs to a finite index set $\mathbb{I}_N = \{1, 2, \dots, N\}$. The neighboring set of node v_i is denoted by $\mathbb{N}_i = \{v_j \in \mathbb{V} : (v_i, v_j) \in \mathbb{E}\}$. For graph G , the degree matrix is defined as $D = \text{diag}\{\text{deg}_{in}(v_1), \text{deg}_{in}(v_2), \dots, \text{deg}_{in}(v_N)\}$ in which $\text{deg}_{in}(v_i) = \sum_{j \in \mathbb{N}_i} \omega_{ij}$. The Laplacian matrix associated with G is $L = D - \mathbb{W}$. In this paper, we assume G is connected, namely, there is no isolated node in G .

The following properties of the Laplacian matrix are helpful to derive our result.

Lemma 1 [44]: If G is an undirected and connected graph with Laplacian matrix $L \in R^{N \times N}$, then (i) all eigenvalues λ_i ($i \in \mathbb{I}_N$) of L are nonnegative scalars and satisfy $0 = \lambda_1 < \lambda_2 \leq \dots \leq \lambda_N$; (ii) $L1_N = 0$.

B. PROBLEM FORMULATION

Consider a MAS consisting of N identical agents with parameter uncertainties and nonlinear dynamics, which are modeled as follows:

$$\dot{x}_i(t) = (A + \Delta A)x_i(t) + f(x_i(t)) + Bu_i(t) \quad (1)$$

where $i \in \mathbb{I}_N$, $A \in R^{d \times d}$ and $B \in R^{d \times m}$ are known matrices. $x_i(t) \in R^d$ and $u_i(t) \in R^m$ denote the state and input of agent i , respectively. ΔA is for the uncertainties in parameter matrices and satisfies

$$\Delta A = DF(t)E \quad (2)$$

with D and E are known matrices with appropriate dimensions. $F(t)$ is an unsuspected matrix function with condition

$$F^T(t)F(t) \leq I. \quad (3)$$

The uncertainties ΔA is considered to be acceptable provided that (2) and (3) are satisfied.

Remark 1: The dynamical system (1) is an uncertain model described in state space. The constant matrices D and E indicate the range of disturbance from the model uncertainties which would be determined by engineering experience or

expert analysis in practice. This structure of the model uncertainties has been widely employed in existing literature [39], [40], [41], [42].

Assumption 1: The function $f : R^d \rightarrow R^d$ is Lipschitz nonlinearity, namely, there is a positive constant γ such that f fulfills

$$\|f(y) - f(z)\| \leq \gamma \|y - z\|,$$

for any $y, z \in R^d$.

For notation simplicity, let $x(t)$, $u(t)$ and $F(x(t))$ be

$$x(t) = [x_1^T(t), x_2^T(t), \dots, x_N^T(t)]^T,$$

$$u(t) = [u_1^T(t), u_2^T(t), \dots, u_N^T(t)]^T,$$

$$F(x(t)) = [f^T(x_1(t)), f^T(x_2(t)), \dots, f^T(x_N(t))]^T.$$

Then, the model (1) is rewritten to be a compact vector form

$$\begin{aligned} \dot{x}(t) &= (I_N \otimes (A + \Delta A))x(t) \\ &\quad + (I_N \otimes B)u(t) + F(x(t)). \end{aligned} \quad (4)$$

Consider a consensus strategy with time-varying input delay as follows

$$u_i(t) = K \sum_{j \in \mathbb{N}_i} \omega_{ij}(x_j(t - \tau(t)) - x_i(t - \tau(t))),$$

in which $i, j \in \mathbb{I}_N$, $K \in R^{m \times d}$ is the gain matrix for controller. \mathbb{N}_i stands for the set of neighbors of agent i . ω_{ij} denotes the weight strength of the edge between agent j and agent i . Let $\tau(t)$ be the time-varying delay when the control information is transmitted to the actuator, which satisfies $0 \leq \tau \leq \tau_{max}$ and $|\dot{\tau}(t)| \leq l < 1$ with $\tau_{max} > 0$ and $l > 0$. By following from properties of Kronecker product and the definition of Laplacian matrix, the control input is transformed to be

$$u(t) = -(L \otimes K)x(t - \tau(t)). \quad (5)$$

Substituting (5) into (4) gives that

$$\begin{aligned} \dot{x}(t) &= (I_N \otimes (A + \Delta A))x(t) \\ &\quad - (L \otimes BK)x(t - \tau(t)) + F(x(t)). \end{aligned} \quad (6)$$

Without losing generality, we give the initial value to be $x(t) = x_0 \in R^d$, $t \in [-\tau_{max}, 0]$.

In this paper, we employ the following cost function

$$J_C = J_{Cx} + J_{Cu} \quad (7)$$

in which

$$J_{Cx} = \int_0^\infty \sum_{i=1}^N \sum_{j=1}^N \omega_{ij}(x_j(t) - x_i(t))^T Q(x_j(t) - x_i(t))dt,$$

$$J_{Cu} = \int_0^\infty \sum_{i=1}^N u_i^T(t)Ru_i(t)dt$$

with $Q > 0$ and $R > 0$. In general, J_{Cx} denotes the control regulation performance of consensus and J_{Cu} stands for the control energy consumption for MAS.

Remark 2: Recently, the problem of guarantee cost consensus control for multi-agent systems has been considered

in many papers [19], [20], [21], [26], [27]. It is worth noting that in these papers the time delay and parameter uncertainty have not been considered simultaneously. Furthermore, less research effort has been devoted to the consensus control of high-dimensional multi-agent systems with taking both control regulation performance and the control energy consumption into consideration. Based on these reasons, system prepared in this paper is more comprehensive than those in existing literature.

Next, we provide the following several definitions and lemmas which will play a significant role in obtaining the main results.

Definition 1: MAS (6) is said to realize the guaranteed cost consensus provided that there exist a function $c(t)$ and a scalar $J_C^* > 0$ such that

$$\lim_{t \rightarrow \infty} (x(t) - 1_N \otimes c(t)) = 0 \text{ and } J_C \leq J_C^*$$

in which $c(t)$ is the function of consensus and J_C^* is the guaranteed cost.

Definition 2: MAS (4) is said to be guaranteed cost consensusable by control strategy (5) if there is a matrix K render MAS (4) realize the guaranteed cost consensus.

Lemma 2 [45]: For vectors $x, y \in R^d$ and matrices $D, S \in R^{d \times d}$, we have

$$2x^T D S y \leq x^T D D^T x + y^T S^T S y.$$

Lemma 3 [38]: Assume D, E are scalar matrices with appropriate dimensions, and $F(t)$ is an unsuspected matrix function satisfying $F^T(t)F(t) \leq I$. For any scalar $\varepsilon_1 > 0$, one has

$$D F(t) E + E^T F^T(t) D^T \leq \varepsilon_1^{-1} D D^T + \varepsilon_1 E^T E.$$

Lemma 4 [23]: Let $k(t) \in R^d$ be a vector with first-order continuous-derivative entries. For any matrices $\Xi_1, \Xi_2 \in R^{d \times d}$, d -dimensional matrix $H = H^T > 0$, and a nonnegative function $\tau(t) \geq 0$, the following integral inequality holds

$$-\int_{t-\tau(t)}^t \dot{k}^T(s) H \dot{k}(s) ds \leq \chi^T(t) \Xi_a \chi(t) + \tau_{max} \chi^T(t) \Xi_b^T H^{-1} \Xi_b \chi(t),$$

where

$$\chi(t) = \begin{bmatrix} k(t) \\ k(t - \tau(t)) \end{bmatrix}, \quad \Xi_a = \begin{bmatrix} \Xi_1^T + \Xi_1 & -\Xi_1^T + \Xi_2 \\ * & -\Xi_2^T - \Xi_2 \end{bmatrix},$$

$$\Xi_b = \begin{bmatrix} \Xi_1 & \Xi_2 \end{bmatrix}.$$

Lemma 5 Schur Complement Lemma: Let P and Z be real symmetric matrices and Z be invertible. Then,

$$\begin{bmatrix} P & Y \\ * & Z \end{bmatrix} < 0$$

if and only if

$$Z < 0, P - Y Z^{-1} Y^T < 0.$$

III. MAIN RESULTS

For a Laplacian matrix L , it follows from Lemma 1 that the eigenvalue $\lambda_1 = 0$ possesses an associated eigenvector 1_N and $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_N$. Hence, there exists an orthogonal matrix $U = [u_1, u_2, \dots, u_N]$ with $u_1 = 1_N/\sqrt{N}$ such that

$$U^T L U = \Lambda = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_N\}.$$

Let

$$k(t) = (U^T \otimes I_d)x(t) = [k_c^T(t), k_r^T(t)]^T \quad (8)$$

with $k_r(t) = [k_{r2}^T(t), k_{r3}^T(t), \dots, k_{rN}^T(t)]^T \in R^{(N-1)d}$ and $k_c(t) \in R^d$.

Remark 3: According to the property that Laplacian matrix has a zero eigenvalue, we derive the new virtual node state $k_c(t)$ and $k_r(t)$ by using orthogonal linear transformation. Specifically, $k_c(t)$ can not be influenced or controlled by coupling input $u(t)$ because of the zero eigenvalue of Laplacian matrix. Thus, $k_c(t)$ is usually employed to represent the consistent performance with which other nodes need to follow. Meanwhile, $k_r(t)$ stand for the state deviations between all agent nodes and the consistent performance. Within the consensus protocol $u(t)$, $k_r(t)$ will finally tend to zero.

For any $i \in \mathbb{I}_N$, we denote by e_i an N -dimensional column vector with the i -th element 1 and 0 elsewhere. By rewriting

$$x(t) = (U \otimes I_d)k(t),$$

$$k_c(t) = (e_1^T \otimes I_d)(U^T \otimes I_d)x(t),$$

$$k_{ri}(t) = (e_i^T \otimes I_d)(U^T \otimes I_d)x(t),$$

we conclude that MAS (6) is rewritten as

$$\dot{k}_c(t) = (A + \Delta A)k_c(t) + \left(\frac{1_N}{\sqrt{N}} \otimes I_d\right)F(x(t)), \quad (9)$$

$$\dot{k}_{ri}(t) = (A + \Delta A)k_{ri}(t) - \lambda_i B K k_{ri}(t - \tau(t)) + (e_i^T \otimes I_d)(U^T \otimes I_d)F(x(t)). \quad (10)$$

By following the properties of Laplacian matrix L , we have

$$x^T(t) L x(t) = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N w_{ij} (x_i(t) - x_j(t))^T (x_i(t) - x_j(t)),$$

which, together with the form of cost function in (7), indicates that

$$J_{Cx} = \int_0^\infty x^T(t) (2L \otimes Q) x(t) dt,$$

$$J_{Cu} = \int_0^\infty x^T(t - \tau(t)) (L^2 \otimes K^T R K) x(t - \tau(t)) dt.$$

Recalling $\lambda_1 = 0$ and $k(t) = (U^T \otimes I_d)x(t)$, we get

$$J_{Cx} = \int_0^\infty \sum_{i=2}^N 2\lambda_i k_{ri}^T(t) Q k_{ri}(t) dt,$$

$$J_{Cu} = \int_0^\infty \sum_{i=2}^N \lambda_i^2 k_{ri}^T(t - \tau(t)) K^T R K k_{ri}(t - \tau(t)) dt.$$

Thus, the cost function (7) is changed to be

$$J_C = \sum_{i=2}^N \int_0^\infty 2\lambda_i k_{ri}^T(t) Q k_{ri}(t) dt + \sum_{i=2}^N \int_0^\infty \lambda_i^2 k_{ri}^T(t - \tau(t)) K^T R K k_{ri}(t - \tau(t)) dt.$$

We are now in the position to present the main conclusion by which the guaranteed cost consensus will be finally realized for MAS (7).

Theorem 1: Given two positive numbers $h_1 < h_2$, the MAS (6) achieves the guaranteed cost consensus if there exist matrices Ξ_1, Ξ_2 , positive definite matrices $P > 0, W > 0$, diagonal matrix $H > 0$, and matrix K such that

$$h_1 I_d < H < h_2 I_d, \tag{11}$$

$$\Omega_i = \begin{bmatrix} \Omega_{11} & \Omega_{12} & PD & P & \Xi_1^T & 0 & A^T & 0 \\ * & \Omega_{22} & 0 & 0 & \Xi_2^T & \Omega_{26} & \Omega_{27} & \Omega_{28} \\ * & * & \Omega_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Omega_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Omega_{55} & 0 & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & 0 & 0 \\ * & * & * & * & * & * & \Omega_{77} & 0 \\ * & * & * & * & * & * & * & \Omega_{88} \end{bmatrix} < 0, \tag{12}$$

for $i = 2$ and N , in which

$$\begin{aligned} \Omega_{11} &= A^T P + PA + \varepsilon E^T E + \gamma^2 I_d + W \\ &+ 2h_2 \tau_{max} \varepsilon^{-1} A^T D D^T A \\ &+ 2h_2 \tau_{max} \varepsilon E^T E + 2h_2 \tau_{max} \lambda_{max}(D^T D) E^T E \\ &+ 2h_2 \tau_{max} \lambda_i \varepsilon^{-1} E^T E + 2h_2 \tau_{max} \gamma^2 I_d + \Xi_1^T + \Xi_1 \\ &+ 2\lambda_i Q, \end{aligned}$$

$$\Omega_{12} = -\Xi_1^T + \Xi_2 - \lambda_i P B K,$$

$$\Omega_{22} = (l - 1)W - \Xi_2^T - \Xi_2,$$

$$\Omega_{26} = \lambda_i K^T R^T,$$

$$\Omega_{27} = -\lambda_i K^T B^T,$$

$$\Omega_{28} = K^T B^T D,$$

$$\Omega_{33} = -\varepsilon^{-1} I_d,$$

$$\Omega_{44} = -I_d,$$

$$\Omega_{55} = -(h_1 \tau_{max})^{-1} I_d,$$

$$\Omega_{66} = -R,$$

$$\Omega_{77} = -(2h_2 \tau_{max})^{-1} I_d,$$

$$\Omega_{88} = -(2h_2 \tau_{max} \lambda_i \varepsilon)^{-1} I_d.$$

Proof: For convenience, let

$$x_c(t) = (U \otimes I_d)[k_c^T(t), 0]^T, \tag{13}$$

$$x_r(t) = (U \otimes I_d)[0, k_r^T(t)]^T, \tag{14}$$

where $k_c(t)$ and $k_r(t)$ are defined in (8). It is readily seen from (13) and (14) that $x_c(t)$ is linearly independent of $x_r(t)$ because U is nonsingular. According to (8), one deduces

$$x(t) = x_c(t) + x_r(t).$$

By (13), we obtain that

$$x_c(t) = \frac{1_N}{\sqrt{N}} \otimes k_c(t). \tag{15}$$

Considering the structure of $x_c(t)$, we deduce that the consensus of MAS (6) is equivalent to prove $\lim_{t \rightarrow \infty} x_r(t) = 0$. It means that the consensus problem of MAS (6) can be transformed to the asymptotical stability of system (10) via approach of state decomposition. That is to say, we need to verify

$$\lim_{t \rightarrow \infty} k_{ri}(t) = 0, \quad i = 2, 3, \dots, N.$$

Construct a Lyapunov-Krasovskii functional as follows

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$

with

$$V_1(t) = \sum_{i=2}^N k_{ri}^T(t) P k_{ri}(t),$$

$$V_2(t) = \sum_{i=2}^N \int_{t-\tau(t)}^t k_{ri}^T(s) W k_{ri}(s) ds,$$

$$V_3(t) = \sum_{i=2}^N \int_{-\tau_{max}}^0 \int_{t+\theta}^t k_{ri}^T(s) H k_{ri}(s) ds d\theta.$$

A direct calculation for $\dot{V}_1(t)$ along the trajectory of (10) shows

$$\begin{aligned} \dot{V}_1(t) &= \sum_{i=2}^N k_{ri}^T(t) ((A + \Delta A)^T P + P(A + \Delta A)) k_{ri}(t) \\ &- 2 \sum_{i=2}^N \lambda_i k_{ri}^T(t) P B K k_{ri}(t - \tau(t)) \\ &+ 2 \sum_{i=2}^N k_{ri}^T(t) P (u_i^T \otimes I_d) F(x(t)) \\ &\triangleq \dot{V}_{11}(t) + \dot{V}_{12}(t) + \dot{V}_{13}(t), \end{aligned} \tag{16}$$

where

$$\dot{V}_{11}(t) = \sum_{i=2}^N k_{ri}^T(t) ((A + \Delta A)^T P + P(A + \Delta A)) k_{ri}(t),$$

$$\dot{V}_{12}(t) = -2 \sum_{i=2}^N \lambda_i k_{ri}^T(t) P B K k_{ri}(t - \tau(t)),$$

$$\dot{V}_{13}(t) = 2 \sum_{i=2}^N k_{ri}^T(t) P (u_i^T \otimes I_d) F(x(t)).$$

It is obviously inferred from $\dot{V}_{11}(t)$ that

$$\begin{aligned} \dot{V}_{11}(t) &= \sum_{i=2}^N k_{ri}^T(t) (A^T P + PA) k_{ri}(t) \\ &+ \sum_{i=2}^N k_{ri}^T(t) (\Delta A^T P + P \Delta A) k_{ri}(t). \end{aligned}$$

By recalling Lemma 3, one gets

$$\begin{aligned} & \sum_{i=2}^N k_{ri}^T(t)(\Delta A^T P + P \Delta A)k_{ri}(t) \\ &= 2 \sum_{i=2}^N k_{ri}^T(t)P \Delta A k_{ri}(t) \\ &= 2 \sum_{i=2}^N k_{ri}^T(t)P D F(t)E k_{ri}(t) \\ &\leq \sum_{i=2}^N k_{ri}^T(t)(\varepsilon^{-1}P D D^T P + \varepsilon E^T E)k_{ri}(t). \end{aligned}$$

Thus, it is observed that

$$\begin{aligned} \dot{V}_{11}(t) &\leq \sum_{i=2}^N k_{ri}^T(t)(A^T P + P A)k_{ri}(t) \\ &\quad + \sum_{i=2}^N k_{ri}^T(t)(\varepsilon^{-1}P D D^T P + \varepsilon E^T E)k_{ri}(t). \end{aligned} \quad (17)$$

Taking use of Lemma 2, we calculate $\dot{V}_{13}(t)$ and get

$$\begin{aligned} \dot{V}_{13}(t) &\leq \sum_{i=2}^N k_{ri}^T(t)P P^T k_{ri}(t) \\ &\quad + \sum_{i=2}^N F^T(x(t))(u_i \otimes I_d)(u_i^T \otimes I_d)F(x(t)). \end{aligned} \quad (18)$$

By letting $\bar{U} = [u_2, u_3, \dots, u_N]$ and noting $U U^T = I_N$, one derives

$$\bar{U} \bar{U}^T = \sum_{i=2}^N u_i u_i^T = I_N - \frac{1_N 1_N^T}{N}.$$

As such, we further conclude that

$$\begin{aligned} & \sum_{i=2}^N F^T(x(t))(u_i \otimes I_d)(u_i^T \otimes I_d)F(x(t)) \\ &= F^T(x(t))(\bar{U} \bar{U}^T \otimes I_d)F(x(t)) \\ &= \frac{1}{2N} \sum_{i=1}^N \sum_{j=1}^N \|f(x_i(t)) - f(x_j(t))\|^2. \end{aligned}$$

Bearing the Lipschitz condition of f in mind, one derives that

$$\begin{aligned} & \frac{1}{2N} \sum_{i=1}^N \sum_{j=1}^N \|f(x_i(t)) - f(x_j(t))\|^2 \\ &\leq \frac{\gamma^2}{2N} \sum_{i=1}^N \sum_{j=1}^N \|x_i(t) - x_j(t)\|^2 \\ &= \gamma^2 x^T(t)(\bar{U} \bar{U}^T \otimes I_d)x(t). \end{aligned}$$

Noting that $(e_i^T \otimes I_d)(U^T \otimes I_d) = (u_i^T \otimes I_d)$ and $k_{ri}(t) = (e_i^T \otimes I_d)(U^T \otimes I_d)x(t)$, one has

$$\sum_{i=2}^N F^T(x(t))(u_i \otimes I_d)(u_i^T \otimes I_d)F(x(t))$$

$$\begin{aligned} & \leq \gamma^2 x^T(t)(\bar{U} \bar{U}^T \otimes I_d)x(t) \\ &= \gamma^2 x^T(t)(\bar{U} \otimes I_d)(\bar{U}^T \otimes I_d)x(t) \\ &= \gamma^2 \sum_{i=2}^N x^T(t)(u_i \otimes I_d)(u_i^T \otimes I_d)x(t) \\ &= \gamma^2 \sum_{i=2}^N k_{ri}^T(t)k_{ri}(t). \end{aligned} \quad (19)$$

Substituting (19) into (18) yields that

$$\dot{V}_{13}(t) \leq \sum_{i=2}^N k_{ri}^T(t)P P^T k_{ri}(t) + \gamma^2 \sum_{i=2}^N k_{ri}^T(t)k_{ri}(t). \quad (20)$$

Thus, it is concluded from (16), (17), and (20) that

$$\begin{aligned} \dot{V}_1(t) &\leq \sum_{i=2}^N k_{ri}^T(t)(A^T P + P A)k_{ri}(t) \\ &\quad + \sum_{i=2}^N k_{ri}^T(t)(\varepsilon^{-1}P D D^T P + \varepsilon E^T E)k_{ri}(t) \\ &\quad - 2\lambda_i \sum_{i=2}^N k_{ri}^T(t)P B K k_{ri}(t - \tau(t)) \\ &\quad + \sum_{i=2}^N k_{ri}^T(t)P P k_{ri}(t) + \gamma^2 \sum_{i=2}^N k_{ri}^T(t)k_{ri}(t). \end{aligned} \quad (21)$$

By directly calculating $\dot{V}_2(t)$ along the trajectory of (6), we obtain that

$$\begin{aligned} \dot{V}_2(t) &\leq \sum_{i=2}^N k_{ri}^T(t)W k_{ri}(t) \\ &\quad - (1-l) \sum_{i=2}^N k_{ri}^T(t - \tau(t))W k_{ri}(t - \tau(t)). \end{aligned} \quad (22)$$

It follows from $\dot{V}_3(t)$ along the trajectory of (6) that

$$\begin{aligned} \dot{V}_3(t) &= \sum_{i=2}^N \tau_{\max} \dot{k}_{ri}^T(t)H \dot{k}_{ri}(t) \\ &\quad - \sum_{i=2}^N \left(\int_{t-\tau_{\max}}^t \dot{k}_{ri}^T(s)H \dot{k}_{ri}(s)ds \right) \\ &\leq \sum_{i=2}^N \tau_{\max} \dot{k}_{ri}^T(t)H \dot{k}_{ri}(t) \\ &\quad - \sum_{i=2}^N \left(\int_{t-\tau(t)}^t \dot{k}_{ri}^T(s)H \dot{k}_{ri}(s)ds \right) \\ &\leq h_2 \tau_{\max} \sum_{i=2}^N \dot{k}_{ri}^T(t) \dot{k}_{ri}(t) \\ &\quad - \sum_{i=2}^N \left(\int_{t-\tau(t)}^t \dot{k}_{ri}^T(s)H \dot{k}_{ri}(s)ds \right) \\ &\triangleq \dot{V}_{31}(t) + \dot{V}_{32}(t). \end{aligned}$$

For the simplicity of notations, we denote $\chi_i^T(t) = [k_{ri}^T(t), k_{ri}^T(t - \tau(t))]$, $G_i = [A + \Delta A, -\lambda_i BK]$. It is known from Lemma 2, (10) and (19) that

$$\begin{aligned} \dot{V}_{31}(t) &= h_2 \tau_{max} \sum_{i=2}^N (\chi_i^T(t) G_i^T G_i \chi_i(t) \\ &\quad + 2F^T(x(t))(u_i \otimes I_d) G_i \chi_i(t) \\ &\quad + F^T(x(t))(u_i \otimes I_d)(u_i^T \otimes I_d) F(x(t))) \\ &\leq h_2 \tau_{max} \sum_{i=2}^N (\chi_i^T(t) G_i^T G_i \chi_i(t) \\ &\quad + F^T(x(t))(u_i \otimes I_d)(u_i^T \otimes I_d) F(x(t)) \\ &\quad + \chi_i^T(t) G_i^T G_i \chi_i(t) + F^T(x(t))(u_i \otimes I_d) \\ &\quad \times (u_i^T \otimes I_d) F(x(t))) \\ &= 2h_2 \tau_{max} \sum_{i=2}^N (\chi_i^T(t) G_i^T G_i \chi_i(t) \\ &\quad + F^T(x(t))(u_i \otimes I_d)(u_i^T \otimes I_d) F(x(t))) \\ &\leq 2h_2 \tau_{max} \sum_{i=2}^N \chi_i^T(t) G_i^T G_i \chi_i(t) \\ &\quad + 2h_2 \tau_{max} \sum_{i=2}^N \gamma^2 k_{ri}^T(t) k_{ri}(t). \end{aligned}$$

By combining Lemma 2 with Lemma 3, we obtain

$$\begin{aligned} \chi_i^T(t) G_i^T G_i \chi_i(t) &= k_{ri}^T(t) (A + \Delta A)^T (A + \Delta A) k_{ri}(t) \\ &\quad + \lambda_i^2 k_{ri}^T(t - \tau(t)) K^T B^T B K k_{ri}(t - \tau(t)) \\ &\quad - 2\lambda_i k_{ri}^T(t) (A + \Delta A)^T B K k_{ri}(t - \tau(t)) \\ &\leq k_{ri}^T(t) A^T A k_{ri}(t) + \lambda_i \varepsilon^{-1} k_{ri}^T E^T E k_{ri}(t) \\ &\quad + \lambda_i^2 k_{ri}^T(t - \tau(t)) K^T B^T B K k_{ri}(t - \tau(t)) \\ &\quad - 2\lambda_i k_{ri}^T(t) A^T B K k_{ri}(t - \tau(t)) \\ &\quad + k_{ri}^T(t) (\varepsilon^{-1} A^T D D^T A + \varepsilon E^T E \\ &\quad + \lambda_{max}(D^T D) E^T E) k_{ri}(t) \\ &\quad + \lambda_i \varepsilon k_{ri}^T(t - \tau(t)) K^T B^T D D^T B K \\ &\quad \times k_{ri}(t - \tau(t)), \end{aligned}$$

which further implies that

$$\dot{V}_{31}(t) \leq 2h_2 \tau_{max} \sum_{i=2}^N \chi_i^T(t) M \chi_i(t) \quad (23)$$

where $M = [M_{ij}]_{4 \times 4}$ with

$$\begin{aligned} M_{11} &= A^T A + \varepsilon^{-1} A^T D D^T A + \varepsilon E^T E \\ &\quad + \lambda_{max}(D^T D) E^T E + \lambda_i \varepsilon^{-1} E^T E + \gamma^2 I_d, \\ M_{12} &= M_{21}^T = -\lambda_i A^T B K, \\ M_{22} &= \lambda_i^2 K^T B^T B K + \lambda_i \varepsilon K^T B^T D D^T B K. \end{aligned}$$

According to Lemma 4, one has

$$\dot{V}_{32}(t) \leq - \sum_{i=2}^N \int_{t-\tau(t)}^t \dot{k}_{ri}^T(s) H \dot{k}_{ri}(s) ds$$

$$\leq \sum_{i=2}^N (\chi_i^T(t) \Xi_a \chi_i(t) + h_1 \tau_{max} \chi_i^T(t) \Xi_b^T \Xi_b \chi_i(t)).$$

which, together with (23), yields that

$$\begin{aligned} \dot{V}_3(t) &\leq \sum_{i=2}^N \chi_i^T(t) (2h_2 \tau_{max} M + \Xi_a \\ &\quad + h_1 \tau_{max} \Xi_b^T \Xi_b) \chi_i(t). \end{aligned} \quad (24)$$

By taking (21), (22) and (24) into account, we conclude that

$$\dot{V}(t) \leq \sum_{i=2}^N \chi_i^T(t) \bar{\Phi}_i \chi_i(t) \quad (25)$$

in which

$$\bar{\Phi}_i = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ * & \Phi_{22} \end{bmatrix},$$

$$\begin{aligned} \Phi_{11} &= A^T P + PA + \varepsilon^{-1} P D D^T P + \varepsilon E^T E + PP + \gamma^2 I_d \\ &\quad + W + 2h_2 \tau_{max} A^T A + 2h_2 \tau_{max} \varepsilon^{-1} A^T D D^T A \\ &\quad + 2h_2 \tau_{max} \varepsilon E^T E + 2h_2 \tau_{max} \lambda_{max}(D^T D) E^T E \\ &\quad + 2h_2 \tau_{max} \lambda_i \varepsilon^{-1} E^T E + 2h_2 \tau_{max} \gamma^2 I_d \\ &\quad + \Xi_1^T + \Xi_1 + h_1 \tau_{max} \Xi_1^T \Xi_1, \\ \Phi_{12} &= -\lambda_i P B K - 2h_2 \tau_{max} \lambda_i A^T B K - \Xi_1^T + \Xi_2 \\ &\quad + h_1 \tau_{max} \Xi_1^T \Xi_2, \\ \Phi_{22} &= (I - 1)W + 2h_2 \tau_{max} \lambda_i^2 K^T B^T B K - \Xi_2^T - \Xi_2 \\ &\quad + 2h_2 \tau_{max} \lambda_i \varepsilon K^T B^T D D^T B K + h_1 \tau_{max} \Xi_2^T \Xi_2. \end{aligned}$$

Now, we present the following definition

$$\dot{\zeta}(t) = \dot{V}(t) + \bar{J}_C \zeta(t) \quad (26)$$

where

$$\begin{aligned} \bar{J}_C &= \sum_{i=2}^N (2\lambda_i k_{ri}^T(t) Q k_{ri}(t)) \\ &\quad + \sum_{i=2}^N (\lambda_i^2 k_{ri}^T(t - \tau(t)) K^T R K k_{ri}(t - \tau(t))). \end{aligned}$$

According to (25), one derives that

$$\dot{\zeta}(t) \leq \sum_{i=2}^N \chi_i^T(t) \bar{\Phi}_i \chi_i(t) + \bar{J}_C \zeta(t) = \sum_{i=2}^N \chi_i^T(t) \Phi_i \chi_i(t)$$

in which $\Phi_i = \bar{\Phi}_i + J_{xu}$, $J_{xu} = \text{diag}\{2\lambda_i Q, \lambda_i^2 K^T R K\}$.

Considering LMIs possesses the convex property, it is noted that for $i = 2, 3, \dots, N$, $\Omega_i < 0$ is derived from $\Omega_i < 0$ ($i = 2, N$) in (12). Combining with the Schur complement formula in Lemma 5, we have

$$\Phi_i = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ * & \Omega_{22} \end{bmatrix} - M Z^{-1} M^T < 0$$

in which

$$\begin{aligned} Z &= \text{diag}\{-\varepsilon I_d, -I_d, -(h_1 \tau_{max})^{-1} I_d, -R, \\ &\quad -(2h_2 \tau_{max})^{-1} I_d, -(2h_2 \tau_{max} \lambda_i \varepsilon)^{-1} I_d\}, \end{aligned}$$

$$M = \begin{bmatrix} PD P \Xi_1^T & 0 & A^T & 0 \\ 0 & 0 & \Xi_2^T \lambda_i K^T R^T & -\lambda_i K^T B^T K^T B^T D \end{bmatrix}.$$

Thus, it is observed that $\dot{\zeta}(t) < 0$, which together with $\bar{J}_C \geq 0$, further means that $\dot{V}(t) \leq 0$. Therefore, system (10) is asymptotically stable.

Furthermore, it can also be followed by (26) and $\dot{\zeta}(t) \leq 0$ that

$$\bar{J}_C \leq -\dot{V}(t). \quad (27)$$

It is worth noting that the asymptotic stability of system (10) is followed by $\lim_{t \rightarrow \infty} V(t) = 0$. Let us integrate (27) from 0 to $+\infty$ and we derive that $J_C = \int_0^\infty \bar{J}_C dt \leq V(0)$. Hence, the proof of Theorem 1 is complete. ■

Theorem 2: Assume MAS (6) achieves the guaranteed cost consensus. The consensus function $c(t)$ is determined via

$$\dot{c}(t) = (A + \Delta A)c(t) + f(c(t))$$

with initial value $c(0) = \frac{1}{N} \sum_{i=1}^N x_i(0)$.

Proof: It is known from the proof of Theorem 1 the guaranteed cost consensus for MAS indicates that

$$\lim_{t \rightarrow \infty} (x(t) - \frac{1}{\sqrt{N}} 1_N \otimes k_c(t)) = 0. \quad (28)$$

That is

$$\lim_{t \rightarrow \infty} (x_i(t) - \frac{1}{\sqrt{N}} k_c(t)) = 0. \quad (29)$$

From subsystem (9), one deduces that

$$\frac{1}{\sqrt{N}} \dot{k}_c(t) = \frac{1}{\sqrt{N}} (A + \Delta A)k_c(t) + (\frac{1}{N} 1_N^T \otimes I_d)F(x(t)).$$

It is obvious to derive that

$$(\frac{1}{N} 1_N^T \otimes I_d)F(x(t)) = \frac{1}{N} \sum_{i=1}^N f(x_i(t)). \quad (30)$$

By combining (29) with (30), we have

$$\begin{aligned} & \lim_{t \rightarrow \infty} ((\frac{1}{N} 1_N^T \otimes I_d)F(x(t)) - \frac{1}{N} \sum_{i=1}^N f(x_i(t))) \\ &= \lim_{t \rightarrow \infty} ((\frac{1}{N} 1_N^T \otimes I_d)F(x(t)) - \frac{1}{N} Nf(\frac{1}{\sqrt{N}} k_c(t))) \\ &= \lim_{t \rightarrow \infty} ((\frac{1}{N} 1_N^T \otimes I_d)F(x(t)) - f(\frac{1}{\sqrt{N}} k_c(t))) \\ &= 0. \end{aligned}$$

By recalling Definition 1, one notes that

$$\lim_{t \rightarrow \infty} (x(t) - 1_N \otimes c(t)) = 0$$

which together with (28), yields that

$$\lim_{t \rightarrow \infty} (c(t) - \frac{1}{\sqrt{N}} k_c(t)) = 0.$$

That is to say, $\frac{1}{\sqrt{N}} k_c(t)$ can be the candidate for consensus function introduced in (15). By letting $c(t) = \frac{1}{\sqrt{N}} k_c(t)$ and calculating the derivative of $c(t)$, one has

$$\dot{c}(t) = (A + \Delta A)c(t) + f(c(t)).$$

Furthermore, recalling the definition of $k_c(t)$ gives

$$\begin{aligned} k_c(0) &= (e_i \otimes I_d)(U^T \otimes I_d)x(0) \\ &= (\frac{1}{\sqrt{N}} 1_N^T \otimes I_d)x(0) \\ &= \frac{1}{\sqrt{N}} \sum_{i=1}^N x_i(0), \end{aligned}$$

which further implies

$$c(0) = \frac{1}{N} \sum_{i=1}^N x_i(0).$$

Then, we complete the proof. ■

Remark 4: It is obviously seen that Theorem 2 provides an exact description for the dynamical evolution of the target of consensus $c(t)$ that is mainly governed by the isolate dynamics rather than the distributed control input with delay and the guaranteed cost performance. Moreover, Theorem 2 shows that when $t \rightarrow +\infty$, the state of MAS will converge to the consensus function $c(t)$ within any given initial value $x(0)$ and any upper bound τ_{max} for time-varying input delay.

Theorem 3: For given positive numbers $h_1 < h_2$, if there exist matrices $\Xi_1, \Xi_2, K P > 0, W > 0$, and diagonal matrix $H > 0$ such that inequalities (11) and (12) hold, then MAS (6) achieves the guaranteed cost consensus with cost performance satisfying

$$J_C \leq J_C^* = x^T(0)(\Gamma \otimes (P + \tau_{max} W))x(0)$$

with $\Gamma = I_N - 1_N 1_N^T / N$.

Proof: Based on Theorem 1 and Definition 2, we know that MAS achieves the guaranteed cost consensus with $J_C \leq V(0)$. According to the orthogonal matrix $U = [u_1, u_2, \dots, u_N]$ and $u_1 = 1_N / \sqrt{N}$, it is deduced that U can be formulated as $U = [\frac{1}{\sqrt{N}} \tilde{U}]$ with $\tilde{U} = [u_2, u_3, \dots, u_N]$. Bearing in mind $UU^T = I_N$, one has $\tilde{U}\tilde{U}^T = I_N - \frac{1}{N} 1_N 1_N^T$. By denoting $\Gamma = I_N - 1_N 1_N^T / N$, it is clearly observed that

$$\begin{aligned} k(t) &= (U^T \otimes I_d)x(t) = [k_c^T(t), k_r^T(t)]^T, \\ x(t) &= (U \otimes I_d)k(t), \\ k_{ri}(t) &= (e_i^T \otimes I_d)(U^T \otimes I_d)x(t). \end{aligned}$$

According to $V_1(t)$ defined in Theorem 1, one gets

$$\begin{aligned} V_1(t) &= \sum_{i=2}^N k_{ri}^T(t) P k_{ri}(t) \\ &= \sum_{i=2}^N k^T(t) (e_i \otimes I_d) P (e_i^T \otimes I_d) k(t) \end{aligned}$$

$$\begin{aligned} &= \sum_{i=2}^N x^T(t)(U \otimes I_d)(e_i \otimes I_d)P(e_i^T \otimes I_d)(U^T \otimes I_d)x(t) \\ &= \sum_{i=2}^N x^T(t)(u_i \otimes I_d)P(u_i^T \otimes I_d)x(t), \end{aligned}$$

which, together with $\bar{U}\bar{U}^T = \sum_{i=2}^N u_i u_i^T = I_N - \frac{1_N 1_N^T}{N} = \Gamma$, indicates that

$$V_1(t) = x^T(t)(\Gamma \otimes P)x(t).$$

Based on the same deduction line to $V_1(t)$, we get

$$\begin{aligned} V_2(t) &= \sum_{i=2}^N \int_{t-\tau(t)}^t k_{ri}^T(s)Wk_{ri}(s)ds \\ &= \int_{t-\tau(t)}^t x^T(s)(\Gamma \otimes W)x(s)ds, \\ V_3(t) &= \sum_{i=2}^N \int_{-\tau_{max}}^0 \int_{t+\theta}^t \dot{k}_{ri}^T(s)H\dot{k}_{ri}(s)dsd\theta \\ &= \int_{-\tau_{max}}^0 \int_{t+\theta}^t \dot{x}^T(s)(\Gamma \otimes H)\dot{x}(s)dsd\theta. \end{aligned}$$

Noting that $x(t) = x_0$ for $t \in [-\tau_{max}, 0]$, it is readily concluded that $V_1(0) = x_0^T(\Gamma \otimes P)x_0$, $V_2(0) \leq \tau_{max}x_0^T(\Gamma \otimes W)x_0$, and $V_3(0) = 0$. Therefore, we conclude that

$$V(0) \leq x_0^T((\Gamma \otimes P) + \tau_{max}(\Gamma \otimes W))x_0,$$

which further indicates

$$J_C \leq x_0^T(\Gamma \otimes (P + \tau_{max}W))x_0.$$

The proof is complete. ■

Remark 5: This paper is different from other articles in the model and theorem contents. Reference [26] investigates energy-constraint formation design and analysis problems for multiagent systems with two types of switching interaction topologies. In [26], a formation control protocol with switching interaction topologies is shown. The control protocol and topologies is different with this paper. Minimum-energy formation achievement problems for networked multiagent systems are investigated in [27], where information networks are randomly switching. But the difference between this paper and [27] is information networks switching. And this paper mainly investigates the no leader following multi-agent systems.

Remark 6: For a given initial value $x(0)$ and τ_{max} , Theorem 3 provides an approach by which we can directly conclude the upper boundedness for the performance of cost. It is deduced that the guaranteed cost can be minimized by looking for the optimization solution P and W in inequality (12).

Having dealt with the consensus analysis for MAS (7) and the calculation for guaranteed cost, we are now analyzing for the purpose of solving the design problem for the gain matrix K . It should be noted that Theorem 1 is not convenient to derive the gain K due to the presence of nonlinear terms

$-\lambda_i PBK$ with unknown matrix variables P and K . In the following, an improved sufficient condition is proposed for ensuring the desired consensus performance and designing the gain matrix K .

Theorem 4: Let two positive numbers $h_1 < h_2$ be given. MAS (4) is said to be guaranteed cost consensualizable through consensus strategy (5) provided that there are matrices $\bar{P} > 0$, $\bar{W} > 0$ and \bar{K} such that for $i = 2, N$, the following symmetric block matrix satisfies

$$\Upsilon(i) = [\Upsilon_{ij}]_{10 \times 10} < 0$$

with

$$\begin{aligned} \Upsilon_{11} &= A\bar{P} + \bar{P}A^T - \lambda_i B\bar{K} - \bar{W}, \\ \Upsilon_{12} &= -\lambda_i B\bar{K} + \bar{P} - \bar{W}, \\ \Upsilon_{13} &= D, \Upsilon_{14} = I_d, \Upsilon_{16} = \lambda_i \bar{K}^T R^T, \\ \Upsilon_{17} &= \bar{P}A^T - \lambda_i \bar{K}^T B^T, \Upsilon_{18} = \bar{K}^T B^T D, \\ \Upsilon_{19} &= \bar{P}, \Upsilon_{1,10} = \bar{W}, \Upsilon_{22} = -2\bar{W}, \\ \Upsilon_{25} &= I_d, \Upsilon_{26} = \lambda_i \bar{K}^T R^T, \Upsilon_{27} = -\lambda_i \bar{K}^T B^T \\ \Upsilon_{28} &= \bar{K}^T B^T D, \Upsilon_{2,10} = \bar{W}, \Upsilon_{33} = -\varepsilon I_d \\ \Upsilon_{44} &= -I_d, \Upsilon_{55} = -(h_1 \tau_{max})^{-1} I_d, \Upsilon_{66} = -R \\ \Upsilon_{77} &= -(2h_2 \tau_{max})^{-1} I_d, \Upsilon_{88} = -(2h_2 \tau_{max} \lambda_i \varepsilon)^{-1} I_d \\ \Upsilon_{99} &= -\bar{\Lambda}_{11}^{-1}, \Upsilon_{10,10} = -(l-1)^{-1} \bar{W} \\ \bar{\Lambda}_{11} &= \varepsilon E^T E + \gamma^2 I_d \\ &\quad + 2h_2 \tau_{max} \varepsilon^{-1} A^T D D^T A + 2h_2 \tau_{max} \varepsilon E^T E \\ &\quad + 2h_2 \tau_{max} \lambda_{max}(D^T D) E^T E + 2h_2 \tau_{max} \lambda_i \varepsilon^{-1} E^T E \\ &\quad + 2h_2 \tau_{max} \gamma^2 I_d + 2\lambda_i Q. \end{aligned}$$

Moreover, the gain matrix is designed to be $K = \bar{K}W$ and the upper boundedness for cost performance is obtained as

$$J_C^* = x_0^T(\Gamma \otimes (\bar{P}^{-1} + \tau_{max} \bar{W}^{-1}))x_0$$

with $\Gamma = I_N - 1_N 1_N^T / N$.

Proof: Recalling the proof of Theorem 1, we observe that for $i = 2$ and $i = N$, $\Omega_i < 0$ is equivalent to

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} \\ * & \Omega_{22} \end{bmatrix} - MZ^{-1}M^T < 0$$

in which M and Z are given in Theorem 1.

For convenience of statement, we denote

$$\begin{aligned} \bar{A}_i &= \begin{bmatrix} A & -\lambda_i BK \\ I_d & -I_d \end{bmatrix}, S = \begin{bmatrix} P & 0 \\ \Xi_1 & \Xi_2 \end{bmatrix}, \\ \bar{\Lambda} &= \begin{bmatrix} \bar{\Lambda}_{11} & 0 \\ 0 & (l-1)W \end{bmatrix}. \end{aligned}$$

It is easy to deduce that

$$\begin{aligned} &S^T \bar{A}_i + \bar{A}_i^T S \\ &= \begin{bmatrix} A^T P + PA + \Xi_1 + \Xi_1^T & -\lambda_i PBK - \Xi_1^T + \Xi_2 \\ * & -\Xi_2 - \Xi_2^T \end{bmatrix}, \end{aligned}$$

which implies

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} \\ * & \Omega_{22} \end{bmatrix} = S^T \bar{A}_i + \bar{A}_i^T S + \bar{\Lambda}.$$

By choosing $\Xi_1 = -P$ and $\Xi_2 = W$, we derive that the inverse of S equals

$$S^{-1} = \begin{bmatrix} P^{-1} & 0 \\ W^{-1} & W^{-1} \end{bmatrix}. \quad (31)$$

Letting $\Pi^T = \text{diag}\{(S^{-1})^T, I_d, I_d, I_d, I_m, I_d, I_d\}$ and using a direct calculation, we derive $\Upsilon(i) = \Pi^T \Omega_i \Pi < 0$. Taking use of Schur complement lemma, one concludes that $\Upsilon(i) < 0$ if we set $\bar{W} = W^{-1}$, $\bar{P} = P^{-1}$ as well as $\bar{K} = KW^{-1}$. Then, according to Theorem 1, MAS (6) achieves the required consensus performance with

$$J_C^* = x^T(0)(\Gamma \otimes (\bar{P}^{-1} + \tau_{max} \bar{W}^{-1}))x(0).$$

Moreover, the control gain matrix is designed to be $K = \bar{K} \bar{W}^{-1}$. Therefore, the conclusion is finally verified. ■

Remark 7: Undoubtedly, it is a challenge task to strictly prove the existence of feasible solution for a linear matrix inequality from the theoretical point of view. In application, the approximate solution for linear matrix inequality is usually derived by using the interior-point methods. Generally speaking, if all diagonal blocks of the matrix $\Gamma(i)$ are negative definite, then the feasible solution will be easier to obtained.

Remark 8: In this paper, we consider the consensus control architecture with a centralized control center. Therefore, it is necessary to understand the complete topology of the entire agent network so as to design a suitable controller for each agent. It should be pointed out that such a control architecture has been widely adopted in previous literature [1], [7], [15]. Theorems 1 and 4 indicate that the guaranteed cost consensus can be verified by some sufficient conditions which are only dependent on the second smallest and the maximum eigenvalues of all Laplacian matrices in the topology set.

IV. AN ILLUSTRATIVE EXAMPLE AND SIMULATIONS

In the current section, we consider a MAS consisting of eight agents in which the graph of communication is described in Figure 1. The model of MAS is formulated as

$$\begin{aligned} \dot{x}_i(t) &= (A + \Delta A)x_i(t) + BK \sum_{j \in N_i} \omega_{ij}(x_j(t - \tau(t)) \\ &\quad - x_i(t - \tau(t))) + f(x_i(t)) \end{aligned}$$

in which $i = 1, 2, \dots, 8$, $\gamma = 0.3$, $x_i = [x_{i1}, x_{i2}]^T$, and

$$A = \begin{bmatrix} -0.005 & -0.005 \\ -1 & -0.9 \end{bmatrix}, B = \begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix},$$

$$f(x_i) = \begin{bmatrix} 0 \\ -\gamma \sin(x_{i2}) \end{bmatrix}.$$

For the simulation of uncertainty ΔA , we choose $\Delta A = DF(t)E$ with

$$D = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix}, E = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix},$$

$$F(t) = \begin{bmatrix} \sin t & 0 \\ 0 & \cos t \end{bmatrix}.$$

We select the function of delay to be $\tau(t) = 0.04 + 0.03\sin(t)$ with $\tau_{max} = 0.07$. The matrix parameters for cost

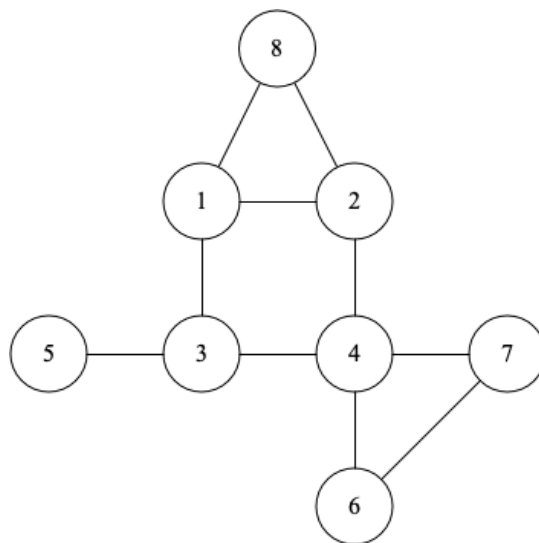


FIGURE 1. Interaction topology G.

function are selected to be $Q = 0.05I_2$, $R = 0.04$. In general, we take all weights of the communication topology as 1. Therefore, it isn't difficult to derive the Laplacian matrix as follows

$$L = \begin{bmatrix} 3 & -1 & -1 & 0 & 0 & 0 & 0 & -1 \\ -1 & 3 & 0 & -1 & 0 & 0 & 0 & -1 \\ -1 & 0 & 3 & -1 & -1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 4 & 0 & -1 & -1 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 & 2 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}.$$

By a direct calculation, one has the eigenvalue $\lambda_2 = 0.5858$ and $\lambda_8 = 5.6017$ for the Laplacian matrix L . For the aim of simulation, let the time step be $T = 0.01s$ and positive numbers $h_1 = 0.1$ and $h_2 = 10$.

In order to show the effect of the controller clearly, we compared the two situation that absence controller and presence controller, respectively.

Case 1: Consider the situation of controller absence, the simulation for dynamical evolution of MAS are presented in Figs. 2 and Fig. 3.

Obviously, the states of the multi-agents didn't achieve the consensus.

Case 2: Let us introduce the consensus control (5). With the help of Matlab toolbox, we obtain a set of feasible solutions for Theorem 4 as follows:

$$P = \begin{bmatrix} 35.1526 & -3.8447 \\ -3.8447 & 3.0037 \end{bmatrix}, W = \begin{bmatrix} 6.1992 & 0.0226 \\ 0.0226 & 6.4020 \end{bmatrix},$$

$$K = [0.3414, 0.0969].$$

According to Theorem 4, MAS achieves the desired guaranteed cost consensus and $J_C^* = 9.0796 \times 10^3$. The simulation for dynamical evolution of MAS with control input are presented in Figure 4-Figure 6. Specifically, it is shown that

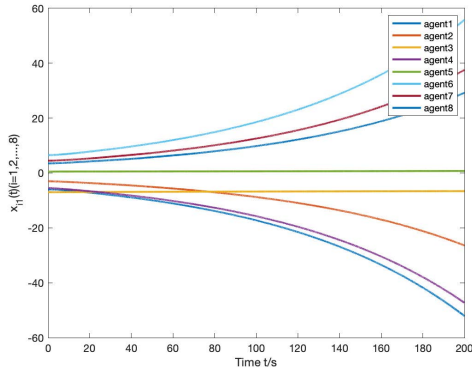


FIGURE 2. State trajectories of $x_{i1}(t) (i = 1, 2, \dots, 8)$.

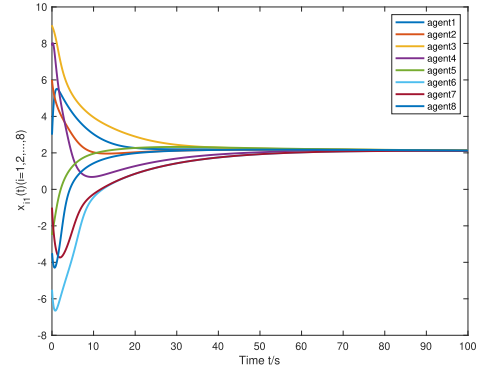


FIGURE 5. State trajectories of $x_{i2}(t) (i = 1, 2, \dots, 8)$.

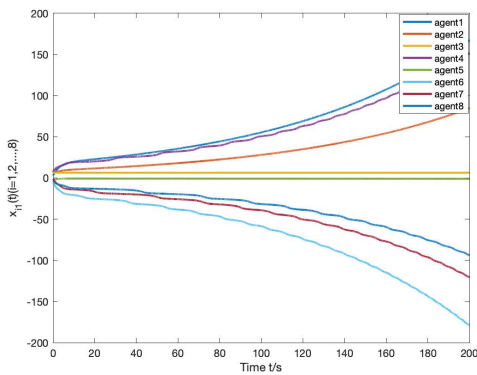


FIGURE 3. State trajectories of $x_{i2}(t) (i = 1, 2, \dots, 8)$.

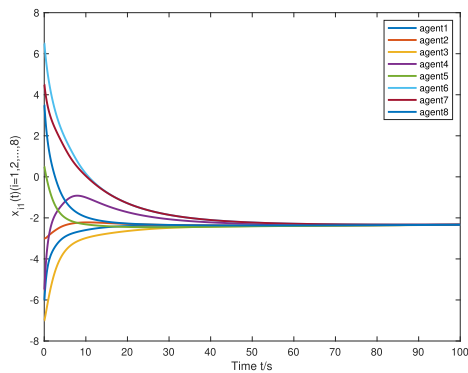


FIGURE 4. State trajectories of $x_{i1}(t) (i = 1, 2, \dots, 8)$.

the state trajectories of MAS are depicted in 4 and Figure 5 from which we see all states of MAS will eventually tend to the consensus function. Figure 6 shows that as time goes to the infinity, the cost function J_C gradually increase from zero while never exceeding a finite upper bound J_C^* .

In order to minimize the upper bound of the cost function, we consider the optimization problem as follows:

$$\begin{aligned} \min \quad & x^T(0)(\Gamma \otimes (P + \tau_{max}W))x(0) \\ \text{s.t.} \quad & \Upsilon(i) < 0. \end{aligned}$$

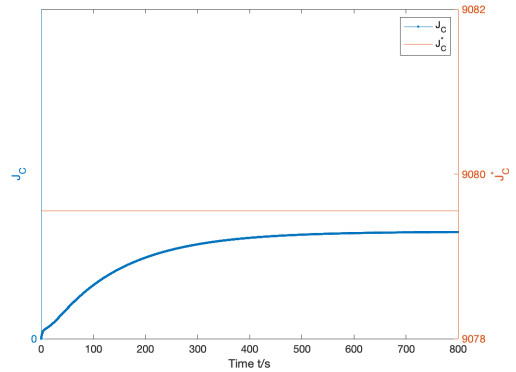


FIGURE 6. Evolution of the cost function.

With the help of LMI toolbox, the optimal solutions for P and W are obtained to be

$$P = \begin{bmatrix} 27.8746 & 1.2994 \\ 1.2994 & 2.6942 \end{bmatrix}, W = \begin{bmatrix} 6.2575 & -0.2825 \\ -0.2825 & 5.2022 \end{bmatrix},$$

In such a case, we deduce that $J_1^* = 5.6799 \times 10^3$. By compared with $J_C^* = 9.0796 \times 10^3$, it is easy to observe that the guaranteed cost performance is further improved by performing this computations tasks.

V. CONCLUSION

Within a certain level of cost performance, this paper has investigated the consensus problem for a type of MASs with nonlinearity dynamics and norm-bound uncertainty. By employing the related error between each agent and its neighbors and introducing the time-varying input delay, a distributed consensus protocol has been constructed in order to force MAS to approach the common consensus function. In addition, an explicit function has been proposed as the consensus function so as to take both the control performance and the energy consumption into account. By employing the properties of Laplacian matrix and Lyapunov-Krasovskii functional, several criteria have been derived for ensuring the expected performance of closed-loop system. Furthermore, the gain matrix for the consensus protocol have been calculated by resorting to the feasible solution of LMIs. Future more, we will consider using only the maximum or minimum

eigenvalues to derive design conditions in Theorem 4 according to [14]. It is worth noting that the methodology used in this paper is applicable to the consensus analysis for MASs subject to some communication constraints, which would be our next research topic. In addition, the heterogeneity and the switching-topology would be another interesting research horizon in the consensus control of MAS with directed topology, which also be our topics in near future.

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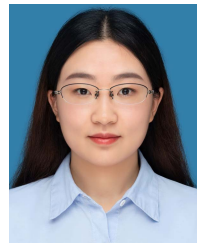
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