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RESEARCH ARTICLE

Distribution Games: A New Class of Games With Application to User Provided Networks

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ABSTRACT User Provided Network (UPN) is a promising solution for sharing the limited network resources by utilizing user capabilities as a part of the communication infrastructure. In UPNs, it is an important problem to decide how to share the resources among multiple clients in decentralized manner. Motivated by this problem, we introduce a new class of games termed distribution games that can be used to distribute efficiently and fairly the bandwidth capacity among users. We show that every distribution game has at least one pure strategy Nash equilibrium (NE) and any best response dynamics always converges to such an equilibrium. We consider social welfare functions that are weighted sums of bandwidths allocated to clients. We present tight upper bounds for the price of anarchy and price of stability of these games provided that they satisfy some reasonable assumptions. We define two specific practical instances of distribution games that fit these assumptions. We conduct experiments on one of these instances and demonstrate that in most of the settings the social welfare obtained by the best response dynamics is very close to the optimum. Simulations show that this game also leads to a fair distribution of the bandwidth.

INDEX TERMS Congestion games, distributed welfare games, user provided networks.

I. INTRODUCTION

A. BACKGROUND AND MOTIVATION

Due to the enormous increase in the mobile data usage, to increase the network capacity provided by network operators in a cost-effective manner is becoming an indispensable necessity. There are several approaches to reduce the load of cellular base stations and to improve the user experience without acquiring more spectrum licences or increasing the number of base stations. One possible solution is to deploy wireless access points and to offload part of mobile data traffic over these access points. This approach is called mobile data offloading and there exist numerous studies on this subject [7]. However, nowadays it is also possible to develop solutions by making use of the users' modern handheld devices, instead of deploying network infrastructure. This gives rise to the concept of User Provided Networks (UPN) that emerged as a novel technique to provide ubiquitous Internet access to subscribers as well as to increase the available bandwidth [1]. In UPNs, mobile subscribers can share their (unused) bandwidth with others through the capabilities of today's devices. Thus, users who need Internet access (due to quota exhaustion, need for higher bandwidth, etc.) can access it by using other users' resources. This kind of access is grouped into two categories: operator-assisted and autonomous. In the former, network operators enforce policies to control subscribers' sharing of their bandwidth, whereas in the second, users can create their own network connections and share their resources independently of the network operator. See [2] for UPN definitions and variations.

UPNs have similarities with Cognitive Radio Networks in the sense that both aim to share the existing spectrum/bandwidth effectively and fairly. Cognitive radio is an optimistic and revolutionary communication concept that tries

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to make a more efficient use of existing wireless spectrum among primary and secondary users. Studies on Cognitive Radio Networks often use game theory (especially potential game theory) as a powerful tool [3], [4]. While cognitive radio is a way to share existing wireless spectrum, the UPN approach aims to utilize unused resources such as bandwidth, energy, quota of other subscribers in a network. In UPNs, the amount of shared bandwidth depends on many factors such as the Internet access quality of the serving user, device-to-device link quality, distance between users, demand of the requesting user, etc.

Figure 1 illustrates a simple UPN scenario. Two suppliers with capacities of κ_1 and κ_2 Mbps are connected to LTE eNodeB. Four clients want to utilize these capacities via device to device (D2D) links (such as Bluetooth, WiFi Direct or LTE Direct) to suppliers. Client i has a demand of d_i Mbps. This is the desirable demand, but not a strict minimum requirement. Rather, the client can make use of the obtained data rate even if it is less than d_i (while this may affect the service quality as in the case of Dynamic Adaptive Streaming over HTTP (DASH) [41]). In Figure 1, red solid lines show that client 1 and 2 are connected to supplier 1, and client 3 and 4 are connected to supplier 2. Blue dashed lines show other possible D2D links. Suppliers assign capacities to the connected clients according to a distribution function. Clients may change their connected supplier to increase the obtained service utility. Strategic decisions of the clients determine the social welfare of the overall UPN system. Due to the nature of the network, it is desirable to adopt a distributed decision making mechanism for both autonomous and operator-assisted UPNs [23]. Therefore, game theory is a powerful tool to model strategies and analyze the outcome of such UPN systems. During the recent years, several game theoretical studies have been performed for both operator-assisted and autonomous UPNs. Most of these studies focus on designing solutions based on the total amount of downloaded data since this quantity is the basis for pricing. In [21] the authors designed a Nash bargaining based solution that considers the amount of offloaded and downloaded data for other participants to compute reimbursements. Despite its theoretical merit, the proposed mechanism necessitates to solve an optimization problem in a distributed fashion and the scheme does not support instant node addition or removal. In [22] an incentive mechanism for operator assisted UPNs in which subscribers are encouraged to be Wi-Fi hotspots for other subscribers (clients) is designed. This scheme provides a quota-based incentive mechanism and a hybrid pricing scheme. There are also several other game theoretical schemes studied in this context [24], [25]. However, most of these studies define the utility as the amount of data transferred, and they do not take the service quality into consideration. In a recent study, Uludag et al. [23] have designed a UPN mechanism where the utility is defined as a function of data rate which is a metric for service quality, but the model is studied only for a single gateway and a single client. In this study, we consider a UPN network with multiple gateways and multiple clients and we focus on accessing and sharing bandwidth, which is the most fundamental problem faced by today's network operators and users.

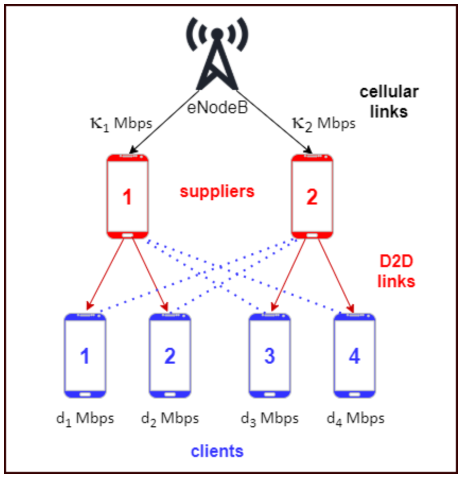

FIGURE 1. A simple UPN scenario.

In a UPN mechanism, the main aim is to share serving subscribers' resources among demanding subscribers in an efficient and fair way both from network operators' and subscribers' perspective. In other words, available resources must be utilized as much as possible and this utilization must be fair enough among participants. Additionally, such a mechanism should be (i) easily implementable on a distributed architecture, (ii) appropriate to networks with frequent changes, i.e., node addition or removal, connection drop, reconnecting or connecting to another node, and (iii) applicable to both operator-assisted and autonomous networks. A two-stage study is required to achieve these objectives. The first stage is the development of a network protocol that enables nodes on the network to communicate with the network operator, discover and communicate with each other. The second stage is the development of a scheme that determines how nodes will share their resources. In this study, we focus on the second stage. In this stage, the designed scheme should not favor some clients over others. The best solution provided by the scheme should perform close to the best results demonstrated. Moreover, the worst solution should not be far from the best. Finally, the scheme should discourage untruthful declarations. First we define a game theoretical sharing mechanism by considering the bandwidth provided by the gateways and data rates demanded by clients. This sharing mechanism is used to serve as the distribution function of our new type of game. We term these games as *distribution games*. We show that every distribution game is a potential game, and discuss its similarities and differences with other games in the literature. Although the distribution games model is inspired by UPNs it can be used in any context where resource sharing is relevant. Smart Grids [46] are new generation energy grids aiming to create a large-scale distributed energy delivery network by using two-way electricity and information flow. Studies on energy consumption scheduling [47] and energy trading [48] mainly aim to shift heavy energy demands to off-peak hours by providing incentives to consumers so that a more reliable and efficient smart grid can be achieved. Distribution games can help solve demand response problems between producers and consumers. Transportation network problems have been investigated for a long time [49], [50] and distribution games can be adopted to solve problems in this area as well. Incentivizing local transportation service providers or travelers across multiple regions or in multi-modal transportation networks [51], [52], [53], [54] is a problem to be tackled. Distribution games can be used as an mechanism for local providers and travelers to agree on a common social welfare as in the case of UPNs.

B. RELATED WORK

Game theory has recently become an indispensable tool to solve decentralized optimization problems in multi-user networks. Congestion Games [5] are one of the most investigated class of games in networking studies. In this type of games, the congestion formed on a congestible element is expressed as a function of the number of players using that network element. A player faces a congestion equal to the sum of the individual congestion on the congestible elements it uses. A simple scenario for the usage of this type of game to network technologies is the following: A person traveling from one city to another faces the congestion created by others along every road that it uses to travel between these cities. This person will choose a route minimizing her congestion. A congestion game is single selection if every player chooses exactly one congestible element. In [11], a specific instance of single selection congestion games is defined. In this type of game, the payoff that each player gets depends only on the number of players playing the same strategy and is expressed as a player-specific function that decreases with the number of those players.

Load Balancing Games are a specific instances of congestion games. In this type of games, each client chooses a server on which it executes her/his own jobs. The aim of a client is to complete her/his job as soon as possible, so they try to choose the server with the minimum load. The load faced by each client is expressed as the load on the server it chooses. The works [17], [18], and [19] present game theoretical solutions to load balancing problems.

A potential game is one in which the increase of the benefit of a user implies an increase in some global potential function. Rosenthal showed that every congestion game is a potential game. It was later shown in [7] that the converse is also true. Namely, for every potential game there exists a congestion game with the same potential function. The potential games model is a key game theoretical tool with a wide range of applications in different areas of networking. For instance, potential games have found many applications in Radio Resource Allocation as explained in [8]. In one of the recent studies on WiFi offloading the authors of [26] designed a user satisfaction aware offloading mechanism in which the problem is defined as a potential game. Access point selection in wireless sensor networks (WSN) is another area of application of potential games. In [27] a game model that is proved to be an exact potential game is proposed for access point (AP) selection in an energy efficient manner. Another access point allocation algorithm for dense wireless LANs backed by software defined wireless networking (SDWN) is proposed in [28]. This potential game based approach dynamically reallocates APs so as to adapt to capacity changes in a WiFi network.

Potential game theory has many other application areas. In [30] spectrum access, power allocation and user scheduling are jointly investigated and an optimization problem is formulated for maximizing the Quality of Experience (QoE) [29] of users in 5G networks. Then, a decentralized solution with local information is provided based on potential game theory. Multi-hop broadcasting is another area of application of potential games. In [31] the authors designed a decentralized, energy efficient, marginal contribution based message broadcasting mechanism in which an optimal broadcast tree is always a Nash equilibrium of the game. A non-cooperative cost sharing game which is shown to be a potential game is proposed in [32]. This game minimizes energy and social cost for data dissemination in wireless networks. Another work on QoE and energy aware resource allocation is [33] in which a potential game is designed to jointly implement power selection, load management and channel allocation for small cell networks. In our previous work [6], we considered a UPN topology and a bandwidth sharing scheme using a potential game where the client utilities are aligned with the global utility of the system. Potential games have also other applications such as video streaming in multi-hop wireless networks [34], caching access point selection in wireless caching networks [35], decentralized resource coordination in coexisting industrial wireless networks [57] and resource allocation in mobile edge computing [55], [56].

The work [9] defined the family of Distributed Welfare Games that are closely related to our study. This family of games is further studied in [10]. In a distributed welfare game, every player chooses to contribute to a subset of the available resources. Some welfare is created at every resource by the set of players contributing to that resource. Every resource has an associated welfare distribution function (protocol) that assigns a utility to every contributing player. Distributed welfare games have a wide range of applications in distributed problems such as sensor network planning [36], content distribution [20], spectrum access in cognitive radio networks [37], interconnection between mobile network operators [40], federated learning [38], and distributed caching in vehicular networks [39].

In this study, without loss of generality, we focus on user provided networks as a potential and coherent application area of our theoretical contributions. UPNs may find different areas of application such as supplying a whole village with internet [43], building a sharing economy in mobile networks [45] or relay the access traffic in 5G Integrated

Access and Backhaul Networks (IAB). During the recent years, a considerably large amount of work has been put in IAB networks [44] which are a type of UPN. In a recent study [42] that is closely related to UPNs and game theory a joint incentive and resource allocation scheme is designed so that the user utility, the sensitivity of battery energy, the incentive compensation and the limitation of network resources are formulated in a Nash Bargaining problem as a cooperative game.

C. OUR CONTRIBUTION

We introduce a family of games termed distribution games in which every player chooses a single supplier from a given set of suppliers. Every player has a demand and it gets a portion of the chosen resource according to its demand and a distribution function governing this supplier. As such, this new family extends the family of single selection congestion games. We prove that every distribution game is a potential game. Thus, any distribution game converges to a pure NE after a finite number of best response moves. We further provide tight bounds on the Price of Anarchy and Price of Stability of games from this new family. We present two practical instances of distribution games and perform a numerical study of one of these games and also of a more general class of distribution games that we analyze only numerically. The numerical study reveals the efficiency of the equilibria in terms of social welfare and fairness. Thus both theoretical and numerical studies show that this new family of games is very appealing for resource sharing applications such as bandwidth distribution in UPNs.

The theoretical results presented in this paper are new, though related to existing results. The result in [7] implies that for every potential game (including distribution games) there exists a congestion game with the same potential function. However, the number of possible strategies of a player in the congestion game implied by that result is quadratic in the number of resources of our game. Therefore, that result does not imply a congestion game with the same set of resources. Distribution games constitute a sub-family of distributed welfare games defined in [9] and further studied in [10]. Though the framework defined therein is very general, most of the positive results pertain to special cases. The results in this work and in [9] and [10] are not comparable in the sense that none implies the other. We elaborate on this in the last part of Section II dedicated to comparisons with existing models and results.

We summarize below the main contributions of this work.

- We introduce a new class of games called distribution games which suits well for various types of resource sharing schemes including user provided networks that could not be properly addressed by the previous game theoretical approaches.
- We prove that every distribution game is a potential game, hence any best response dynamics always converge to a pure strategy NE.
- We provide tight upper bounds for the price of anarchy and the price of stability of distribution games.
- We evaluate the efficiency of equilibria in terms of social welfare and fairness via extensive set of simulations in various UPN setups.
- We provide a characterization of distribution games which make them truthful, such that it is a dominant strategy to declare the true demand for all the clients.

The rest of the paper is organized as follows. In Section II we introduce notation, define distribution games and compare them against congestion games and distributed welfare games. In Section III we prove that every distribution game admits a pure strategy Nash equilibrium. In the same section we introduce two practical instances of distribution games. In Section IV we analyze the quality of the above mentioned Nash equilibria by studying Price of Anarchy and Price of Stability. In Section VI we develop a simple criterion for the truthfulness of a distribution game and apply it to the sample games introduced in earlier sections. In Section V we present a numerical study in a realistic setup and finally, we conclude with some open problems in Section VII.

II. DISTRIBUTION GAMES

In this section, we define a new class of games that are suitable for sharing bandwidth in a distributed UPN setup. To give a basic idea, let us consider an illustrative example based on the simple network shown in Figure 1. Let us define the capacities of two supplier nodes as $\kappa_1 = 30$ Mbps and $\kappa_2 = 20$ Mbps. These two nodes provide service to four client nodes with demands $d_1 = 30$ Mbps, $d_2 = 15$ Mbps, $d_3 = 10$ Mbps and $d_4 = 40$ Mbps. The distribution of the supplier capacities among multiple connected clients can be done in different ways. Figure 2 shows two connection scenarios, where the capacities of the suppliers are distributed equally among the connected clients. In both scenarios, none of the clients can gain more utility by changing their supplier, hence there is an equilibrium. In Figure 3, the capacities of the suppliers are distributed to the clients using a different function, i.e. proportional to their demands. This setting is not in equilibrium because client 3 will have an incentive to switch to supplier 1. We now proceed by giving some preliminaries about game theory, and then define a general class of distribution games to handle strategic decisions in UPN settings using various distribution functions.

A. PRELIMINARIES

We start by giving basic definitions and notation. See Table 1 for frequently used notation. A strategic game is a triple $\mathcal{G} = (n, A, u)$ where $n \in \mathbb{N}$ is the number of players, $A = \times_{i=1}^{n} A_i$, A_i is a finite set of possible strategies of player i, and $u : [n] \times A \rightarrow \mathbb{R}$ is a utility function where $[n]$ denotes the set of positive integers less than or equal to n. A vector $\overrightarrow{a} \in A$ is termed a strategy profile or an outcome of $\mathcal{G}$. When the outcome of the game is $\overrightarrow{a}$, player i receives a utility of $u_i(\overrightarrow{a}) = u(i, \overrightarrow{a})$. For an outcome $\overrightarrow{a}$ and $b \in A_i$, we denote

TABLE 1. Frequently used notation.

$\mathcal{G}$	A strategic game
A_i	Finite set of possible strategies of player i
a_i	Strategy of player i, $a_i \in A_i$
A	Strategy space, $\times_{i=1}^{n} A_i$
$\vec{a}$	A strategy profile or an outcome of $\mathcal{G}$ where $\vec{a} \in A$
$u_i(\vec{a})$	Utility function for player i in outcome $\vec{a}$
Φ	(Ordinal) potential function for $\mathcal{G}$
$W(\vec{a})$	Social welfare of outcome $\vec{a}$
d_i	Demand of client (player) i
$C_j(\vec{a})$	Set of clients connected to supplier j
$\delta_j(\vec{a})$	A monotonically non-increasing distributing function of supplier j in outcome $\vec{a}$
f_i	A monotonically increasing benefit function
$f_i(\delta_j(\vec{a}))$	Utility of client i when connected to supplier j in outcome $\vec{a}$
$D_j(\vec{a})$	Total demand of all clients connected to supplier j in outcome $\vec{a}$
κ_j	Capacity of supplier j

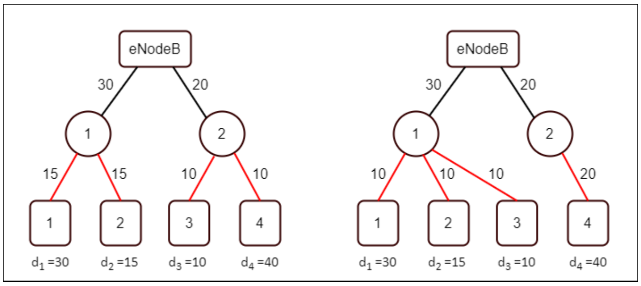

FIGURE 2. An illustrative example for two connection scenarios in a UPN where the supplier capacities are distributed equally among the connected clients.

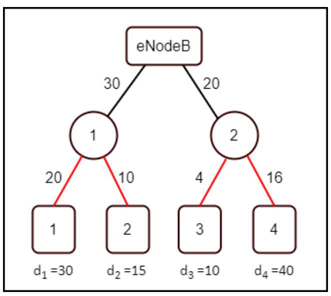

FIGURE 3. An illustrative example for a connection scenario in a UPN where the supplier capacities are distributed proportional to the client demands.

by $(b, \vec{a}_{-i})$ the outcome in which the strategy of player i is b and the strategy of any other player is identical to its strategy in $\vec{a}$. In other words, $(b, \vec{a}_{-i})$ is the outcome obtained from $\vec{a}$ by a move of player i that changes its strategy to b and the strategy of every other player remains intact. A selfish player changes its strategy only if the change will strictly increase its utility assuming that no other player changes its strategy. We denote by $\vec{a} \xrightarrow{i} \vec{a}'$ the fact that $\vec{a}'$ is obtained from $\vec{a}$ by a move of a selfish player i. We use $\vec{a} \rightarrow \vec{a}'$ to denote that $\vec{a} \xrightarrow{i} \vec{a}'$ for some player i.

A Nash equilibrium (NE) of $\mathcal{G}$ is an outcome $\vec{a}$ in which no player can strictly increase its utility by making a move, i.e., there is no outcome $\vec{a}'$ such that $\vec{a} \rightarrow \vec{a}'$.

If there exists a function $\Phi : A \rightarrow \mathbb{R}$ such that $\Phi(\vec{a}) < \Phi(\vec{a}')$ whenever $\vec{a} \rightarrow \vec{a}'$ then $\mathcal{G}$ is an ordinal potential game and Φ is termed an ordinal potential function for $\mathcal{G}$.

Whenever A is finite, it is easy to see that every ordinal potential game has a pure strategy Nash Equilibrium [7]. Furthermore, starting from an arbitrary outcome $\vec{a}$, every finite ordinal potential game converges to a Nash Equilibrium through a finite improvement path. In particular, any best response dynamics, where at each step an arbitrary selfish player plays its best-response strategy, always converges to a Nash Equilibrium after finite number of steps [7].

If a social welfare function $W : A \rightarrow \mathbb{R}$ is defined on the set of outcomes of a game. We compare the social welfare obtained by Nash equilibria to the optimum $W^* = \max\left\{W(\vec{a}) | \vec{a} \in A\right\}$. The price of anarchy (PoA) of such a game is defined as the ratio of the optimum to the performance of a worst NE, i.e, $\text{PoA}(\mathcal{G}) = \frac{W^*}{\min\{W(\vec{a}) | \vec{a} \text{ is a NE}\}}$. Similarly, the price of stability (PoS) is obtained by comparing the optimum to a best NE.

In the following lemma we give a sufficient condition to guarantee that a game is an ordinal potential game. This condition requires a finite family of functions to be defined on the set of outcomes of the game. Specifically, the condition requires that in every selfish move of some player, the minimum of the function values affected by the move strictly decreases.

Lemma 1: Let $\mathcal{G} = (n, A, u)$ be a strategic game, $\mathcal{J}$ be a set of indices with functions $h_j : A \rightarrow \mathbb{R}$ $(j \in \mathcal{J})$. If for every two outcomes $\vec{a}, \vec{a}'$ of $\mathcal{G}$ such that $\vec{a} \rightarrow \vec{a}'$ there exists $\emptyset \subsetneq \bar{\mathcal{J}} \subseteq \mathcal{J}$ such that

1) $\min\left\{h_j(\vec{a}) | j \in \bar{\mathcal{J}}\right\} < \min\left\{h_j(\vec{a}') | j \in \bar{\mathcal{J}}\right\}$, and
2) $h_j(\vec{a}') = h_j(\vec{a})$ for every $j \in \mathcal{J} \setminus \bar{\mathcal{J}}$,

then $\mathcal{G}$ is an ordinal potential game.

Proof: Let $h_{\mathcal{J}}(\vec{a})$ be the multiset $\left\{h_j(\vec{a}) | j \in \mathcal{J}\right\}$, and let $\vec{h}(\vec{a})$ be the vector containing the $|\mathcal{J}|$ values in $h_{\mathcal{J}}(\vec{a})$, sorted in non-decreasing order. Let also $\preceq$ denote the lexicographic order on $\mathbb{R}^{|\mathcal{J}|}$, and $\vec{x} \prec \vec{y}$ denote the fact that $\vec{x} \preceq \vec{y}$ and $\vec{x} \neq \vec{y}$.

Let $\vec{a}$ and $\vec{a}'$ be two outcomes of $\mathcal{G}$ such that $\vec{a} \xrightarrow{i} \vec{a}'$ for some player $i \in [n]$. Then, there is a set $\emptyset \subsetneq \bar{\mathcal{J}} \subseteq \mathcal{J}$ that satisfies the premises of the lemma. Let $x = \min\left\{h_j(\vec{a}) | j \in \bar{\mathcal{J}}\right\}$, $k_<$ (resp. $k_=$) be the number of entries of $\vec{h}(\vec{a})$ that are strictly less than (resp. equal to) x. Then the first $k_<$ entries of $\vec{h}(\vec{a})$ and $\vec{h}(\vec{a}')$ are identical. As of the subsequent $k_=$ entries, the entries of $\vec{h}(\vec{a}')$ are greater or equal than the corresponding entries of $\vec{h}(\vec{a})$ with at least one entry being strictly greater. Therefore, $\vec{h}(\vec{a}) \prec \vec{h}(\vec{a}')$, implying that the rank of $\vec{h}(\vec{a})$ in the lexicographic order is an ordinal potential function for $\mathcal{G}$. □

B. GAME MODEL

We start with an informal description of the model and then proceed with formal definitions. In our model we refer to the players also as clients each of which has a demand of some global resource. In addition, there is a set of suppliers each of which has a limited supply of the resource. Every client connects to a supplier in order to satisfy (possibly partially) its demand.

The utility u_i of a client i when connected to a supplier j depends on the part if its demand satisfied by j and is upper bounded by this demand. Note that in particular, this models the case where the utility u_i is the supply it gets from supplier j. This amount is determined by a) supplier j's current capability to satisfy a clients' demands, and b) the way client i benefits from this capability. These two parameters are determined by two functions, one associated with supplier j, and the other associated with client i. We assume that the capability of a supplier to serve a specific client does not increase when additional clients are connected. This assumption is realistic if the capacity or other characteristics of a supplier do not change very often, i.e., during the game. Therefore, we associate with supplier j a non-increasing real distributing function δ_j that depends solely on the set of clients currently connected to supplier j.

The benefit of client i from the capability of j increases with the capability of j until it reaches the demand of client i. Therefore, we associate with client i a strictly increasing benefit function f_i that depends on the current capability of the supplier. Client i receives more of the resource whenever the capability of supplier j increases unless its demand is already satisfied. Finally, we note that the theoretical model introduced in this section does not require a fixed capacity to be defined for every supplier, despite the fact that capacities are used for the sake of the examples. However, starting from Section IV, we consider special cases of the model in which such a capacity is defined.

The social welfare function of the system is a conical combination of the individual utilities.

We thus have the following formal definition of a distribution game with strategy space A and with utility functions u_i determined by the distributing functions Δ and benefit functions F.

Definition 1: A Distribution game is a tuple $\mathcal{G} = (n, m, d, A, \Delta, F)$ where

- n is the number of players (clients),
- m is the number of suppliers,
- $d : [n] \rightarrow \mathbb{R}^+$ is the demand vector with d_i being the demand of client $i \in [n]$,
- $A = \times_{i=1}^{n} A_i$ is the strategy space with $A_i \subseteq [m]$ for every client $i \in [n]$ (i.e., client i can connect to exactly one supplier from A_i). For an outcome $\overrightarrow{a} \in A$, we denote by $C_j(\overrightarrow{a})$ the set of clients that are connected to supplier j, i.e., $C_j(\overrightarrow{a}) = \{i \in [n] | a_i = j\}$. We denote by $D_j(\overrightarrow{a}) = \sum_{i \in C_j(\overrightarrow{a})} d_i$ the total demand of all clients connected to supplier j.
- $\Delta = \{\delta_j | j \in [m]\}$ is the set of distributing functions with every $\delta_j : 2^{[n]} \rightarrow \mathbb{R}^+$ being a monotonically non-increasing function on sets of clients,
- $F = \{f_i | i \in [n]\}$ is the set of benefit functions where every $f_i : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a monotonically increasing function.
- Moreover, δ_j is strictly decreasing unless all clients connected to supplier j are saturated, i.e., if $\delta_j(\overrightarrow{a}) = \delta_j(\overrightarrow{a}')$ and $C_j(\overrightarrow{a}) \subsetneq C_j(\overrightarrow{a}')$ then $f_i(\delta_j(\overrightarrow{a})) \geq d_i$ for every $i \in C_j(\overrightarrow{a}')$ where $\delta_j(\overrightarrow{a})$ is a shorthand for $\delta_j(C_j(\overrightarrow{a}))$.
- $u_i(\overrightarrow{a}) = \min \{d_i, f_i(\delta_j(\overrightarrow{a}))\}$, i.e., whenever client i is connected to supplier j, its utility u_i is $f_i(\delta_j(\overrightarrow{a}))$ but no more than d_i.

To give an example, let us apply the defined game formulation to the instance given in Figure 3. Using the distribution game notation (and omitting the name $(\overrightarrow{a})$ of the outcome in the figure) we have $C_1 = \{1, 2\}, C_2 = \{3, 4\}, D_1 = d_1 + d_2 = 45$, $D_2 = d_3 + d_4 = 50$. For this instance, δ_j can be defined as the ratio of the capacity of supplier j to the total demand of the clients connected to it, hence $\delta_1 = \frac{30}{45} = \frac{2}{3}$, $\delta_2 = \frac{20}{50} = \frac{2}{5}$. Further, we define $f_i(x) = d_i \cdot x$ for every client i. Then, the utilities of the clients are $u_1 = \min\{d_1, d_1 \cdot \delta_1\} = 20$, $u_2 = \min\{d_2, d_2 \cdot \delta_1\} = 10$, $u_3 = \min\{d_3, d_3 \cdot \delta_2\} = 4$, $u_4 = \min\{d_4, d_4 \cdot \delta_2\} = 16$.

We note that the above formulation of distribution games covers also the case in which the utility of a client is an increasing function g of the resource allocated to it. Consider a distribution game $\mathcal{G} = (n, m, d, A, \Delta, F)$ in which $f_i(\delta_j(\overrightarrow{a}))$ is the resource allocated to client i by the supplier j (where $j = a_i$) Consider the distribution game $\mathcal{G}' = (n, m, d', A, \Delta, F')$ where $d'_i = g(d_i)$ for every $i \in [n]$ and $F' = \{f'_i | i \in [n]\}$ with $f'_i = g \circ f_i$ for every $i \in [n]$. Indeed, $\mathcal{G}'$ is a distribution game since, g and f_i being strictly increasing functions, f'_i is so. Moreover, the utility of client i in an outcome $\overrightarrow{a}$ of $\mathcal{G}'$ is

$$\begin{aligned} u'_i(\overrightarrow{a}) &= \min \{d'_i, f'_i(\delta_j(\overrightarrow{a}))\} \\ &= \min \{g(d_i), g(f_i(\delta_j(\overrightarrow{a})))\} \\ &= g(\min \{d_i, f_i(\delta_j(\overrightarrow{a}))\}) = g(u_i(\overrightarrow{a})) \end{aligned}$$

where $u_i(\overrightarrow{a})$ is the resource allocated to client i in the same outcome of $\mathcal{G}$.

C. RELATION WITH CONGESTION GAMES AND DISTRIBUTED WELFARE GAMES

In a congestion game the strategy of a player is to connect to a subset of given facilities. The cost of a client is the sum of the costs of the facilities it is connected to, and the cost of a facility is a (non-decreasing) function of the number of clients connected to it. In a single selection congestion game a player can be connected to only one facility. We note that when we set $d_i = 1$ and $f_i(x) = x$ for every client i in a distribution game we get a single selection congestion game of which the congestible items are the resources of our game. In the next section we prove that every distribution game is a potential game. A result presented in [7] states that with every potential game there exists a congestion game with the same number of players. However, for our case this result implies a congestion game with $n \cdot m$ possible strategies for every player. Therefore, this result does not imply a congestion game where the set of congestible items is the set of our suppliers.

In a distributed welfare game, players can contribute to any subset of resources. The set S of players that contribute to a resource r produces a welfare $W_r(S)$ in resource r where W_r is a sub-modular function. In other words, the marginal contribution of a player is a non-increasing function of the set of other contributors. Formally, $W_r(S \cup \{i\}) - W_r(S) \geq W_r(T \cup \{i\}) - W_r(T)$ whenever $S \subseteq T$. Contributor $i \in S$ gets a utility of $f_r(i, S)$ where f_r is the distribution function (protocol) at resource r. Such a protocol is termed budget-balanced if the sum of the utilities received by the contributors is equal to the welfare produced in this resource. A set of protocols is scalable if whenever the welfare generation functions of two resources are identical, then their utility distribution functions are identical.

If we consider the total distribution of a supplier as the welfare generated, we get that our utility functions are, by definition, budget-balanced, and scalable. Moreover, our requirements from f and δ imply that the welfare function is sub-modular. Therefore, a distribution game is a single selection distributed welfare game. Moreover, our distribution functions are separable, i.e., the utility received by player i is $\max f_i(\delta_j(S))$ as opposed to a distributed welfare game in which f is not necessarily separable. In this respect the family of distribution games is a sub-family of the family of single selection distributed welfare games. Though, in the next paragraph we argue that our results are incomparable with the results in [9], i.e., none of the results implies the other.

In [9] it is shown that a single selection distributed welfare game admits a NE provided that the game satisfies three sufficient conditions. The analysis of the price of anarchy clearly depends on the same conditions since the very definition of Price of Anarchy requires the existence of a NE. One of these condition is the existence of a total order between the players. More specifically, player i is said to be stronger than player j if the utility that i gets from a resource r with a contributor set S is always greater than or equal to the utility of player j under the same circumstances, for every resource r and every set of players S. Clearly, the "is stronger than" relation is transitive. However, it is also required that for any two players i and j, either i is stronger than j or the other way around, i.e., the mentioned relation is a total order. We finally note that a distribution game does not impose this condition. Such a total order does not necessarily exist since the functions f_i are arbitrary increasing functions, and two such functions are not necessarily comparable.

In [10] it is shown that if a distributed welfare game uses a budget-balanced and scalable protocol, and also guarantees the existence of a Nash equilibrium, then it must be a weighted Shapley value. This makes the implementation of such protocols computationally prohibitive. In the same work the authors also present ways of addressing these problems. On the other hand, the family of distribution games, being more restricted, can use computationally simple, scalable and budget-balanced protocols while admitting Nash equilibria, as we will prove in the next section.

III. EXISTENCE OF NASH EQUILIBRIA

In this section we show that every distribution game has a NE. To this goal, we prove the following lemma that complements Lemma 1 and states that whenever a client moves from one supplier to another it increases the minimum of the δ values of these suppliers.

Lemma 2: Let $\overrightarrow{a} \overset{i}{\rightarrow} \overrightarrow{a}'$ be a move of player i in a distribution game. Then,

$$\min\left\{\delta_{a_i}(\overrightarrow{a}), \delta_{a'_i}(\overrightarrow{a})\right\} < \min\left\{\delta_{a_i}(\overrightarrow{a}'), \delta_{a'_i}(\overrightarrow{a}')\right\}.$$

Proof: Let $j = a_i$ and $j' = a'_i$, i.e., client i moves from supplier j to supplier j'. We have $C_j(\overrightarrow{a}') = C_j(\overrightarrow{a}) \setminus \{i\}$ and $C_{j'}(\overrightarrow{a}') = C_{j'}(\overrightarrow{a}) \cup \{i\}$. By the monotonicity of δ, these imply

$$\delta_{j'}(\overrightarrow{a}') \leq \delta_{j'}(\overrightarrow{a}) \tag{1}$$

and $\delta_j(\overrightarrow{a}) \leq \delta_j(\overrightarrow{a}')$. Suppose that the latter inequality holds with equality, i.e., $\delta_j(\overrightarrow{a}) = \delta_j(\overrightarrow{a}')$. Then we have $f_i(\delta_j(\overrightarrow{a})) \geq d_i$ thus $u_i(\overrightarrow{a}) = d_i$. This contradicts the fact that i moved from supplier j to another supplier. Therefore,

$$\delta_j(\overrightarrow{a}) < \delta_j(\overrightarrow{a}'). \tag{2}$$

Since the utility of i increases after the move, we have

$$\min\left\{d_i, f_i(\delta_j(\overrightarrow{a}))\right\} < \min\left\{d_i, f_i(\delta_{j'}(\overrightarrow{a}'))\right\}.$$

Therefore, $f_i(\delta_j(\overrightarrow{a})) < d_i$ and $f_i(\delta_j(\overrightarrow{a})) < f_i(\delta_{j'}(\overrightarrow{a}'))$. Since f_i is an increasing function, we conclude

$$\delta_j(\overrightarrow{a}) < \delta_{j'}(\overrightarrow{a}'). \tag{3}$$

We combine (1) and (3) to get $\delta_j(\overrightarrow{a}) < \delta_{j'}(\overrightarrow{a}') \leq \delta_{j'}(\overrightarrow{a})$. Thus

$$\min\left\{\delta_j(\overrightarrow{a}), \delta_{j'}(\overrightarrow{a})\right\} = \delta_j(\overrightarrow{a}).$$

We now combine (2) and (3) to get

$$\delta_j(\overrightarrow{a}) < \min\left\{\delta_j(\overrightarrow{a}'), \delta_{j'}(\overrightarrow{a}')\right\}$$

which together with the preceding equality concludes the proof. □

By setting $\bar{J} = \left\{a_i, a'_i\right\}$, since $\delta_j(\overrightarrow{a}) = \delta_j(\overrightarrow{a}')$ for every $j \notin \left\{a_i, a'_i\right\}$, using Lemma 1 and Lemma 2 we get the following corollary.

Corollary 1: Every distribution game is an ordinal potential game.

We now present two families of distribution games by defining the way a supplier j distributes its supply among the set $C_j(\overrightarrow{a})$ of its connected clients, i.e., by defining the functions $\delta_j, j \in [m]$ and $f_i, i \in [n]$.

- **Demand-proportional distribution** ($\mathcal{G}_{DP}$): Supplier j has a supply of κ_j. If $D_j(\overrightarrow{a}) \leq \kappa_j$ then every client $i \in C_j(\overrightarrow{a})$ gets a supply of $u_i(\overrightarrow{a}) = d_i$. Otherwise, client $i \in C_j(\vec{a})$ gets a supply of $d_i \frac{\kappa_j}{D_j(\vec{a})}$. Summarizing, $u_i(\vec{a}) = \min\left\{d_i, d_i \frac{\kappa_j}{D_j(\vec{a})}\right\}$. $\mathcal{G}_{DP}$ is a distribution game since $\delta_j(\vec{a}) = \frac{\kappa_j}{D_j(\vec{a})}$ is monotonically decreasing in $C_j(\vec{a})$, $f_i(x) = d_i \cdot x$ is monotonically increasing in x, and the utility of client i connected to supplier j is $\min\left\{d_i, f_i(\delta_j(\vec{a}))\right\}$.

- **Egalitarian distribution** ($\mathcal{G}_{EG}$): Supplier j has a supply of κ_j which is distributed among the clients in the following iterative procedure. The capacity is distributed equally among the connected clients until the capacity is completely distributed or some client becomes saturated. The remaining capacity is distributed equally among the non-saturated connected clients until the capacity is completely distributed or a client becomes saturated and so on until either the capacity is completely distributed or all connected clients are saturated. Formally,

$$\delta_j(\vec{a}) = \begin{cases} \kappa_j \text{ if } D_j(\vec{a}) \leq \kappa_j \\ \max\left\{ x \middle| \sum_{i \in C_j(\vec{a})} \min\{d_i, x\} \leq \kappa_j \right\} \text{ otherwise.} \end{cases}$$

The utility of client $i \in C_j(\vec{a})$ is $\min\left\{d_i, \delta_j(\vec{a})\right\}$.

Having already observed that $\mathcal{G}_{DP}$ is a distribution game, we proceed to prove the relatively less obvious fact that $\mathcal{G}_{EG}$ is also a distribution game.

Theorem 1: $\mathcal{G}_{EG}$ is a distribution game.

Proof: Clearly, $f_i(x) = x$ is monotonically increasing. It remains to show that δ_j is monotonically non-increasing and if $\delta_j(\vec{a}) = \delta_j(\vec{a}')$ and $C_j(\vec{a}) \subsetneq C_j(\vec{a}')$ then $\delta_j(\vec{a}) \geq d_i$ for every $i \in C_j(\vec{a}')$. Assume that $\vec{a} \subsetneq \vec{a}'$. Clearly, $D_j(\vec{a}) < D_j(\vec{a}')$. We first prove the following claim.

Claim 1: $\delta_j(\vec{a}) < \kappa_j$ if and only if $D_j(\vec{a}) > \kappa_j$.

Proof: If $\delta_j(\vec{a}) < \kappa_j$, we have $D_j(\vec{a}) > \kappa_j$ by the definition of δ_j. To prove the other direction assume for a contradiction that $D_j(\vec{a}) > \kappa_j$ and $\delta_j(\vec{a}) \geq \kappa_j$. Then $\sum_{i \in C_j(\vec{a})} \min\left\{d_i, \delta_j(\vec{a})\right\} \leq \kappa_j$, thus $\min\left\{d_i, \delta_j(\vec{a})\right\} \leq \kappa_j$ for every $i \in C_j(\vec{a})$. Since $\delta_j(\vec{a}) > \kappa_j$, we have $\min\left\{d_i, \delta_j(\vec{a})\right\} = d_i$ for every $i \in C_j(\vec{a})$. Therefore, $D_j(\vec{a}) = \sum_{i \in C_j(\vec{a})} d_i = \sum_{i \in C_j(\vec{a})} \min\left\{d_i, \delta_j(\vec{a})\right\} \leq \kappa_j$, a contradiction. □

We now consider three disjoint and complementing cases:

- $D_j(\vec{a}) < D_j(\vec{a}') \leq \kappa_j$: Then, $d_i \leq \kappa_j$ for every $i \in C_j(\vec{a}')$. Furthermore, by definition of δ_j, we have $\delta_j(\vec{a}) = \delta_j(\vec{a}') = \kappa_j$. We conclude that $f_i(\delta_j(\vec{a}')) = f_i(\kappa_j) = \kappa_j \geq d_i$ for every $i \in C_j(\vec{a}')$.
- $D_j(\vec{a}) \leq \kappa_j < D_j(\vec{a}')$: By Claim 1, we have $\delta_j(\vec{a}') < \kappa_j = \delta_j(\vec{a})$.
- $\kappa_j < D_j(\vec{a}) < D_j(\vec{a}')$: In this case we will show that $\delta_j(\vec{a}') < \delta_j(\vec{a})$. Assume for a contradiction that $\delta_j(\vec{a}) \leq \delta_j(\vec{a}')$. We have

$$\begin{aligned} \sum_{i \in C_j(\vec{a})} \min\left\{d_i, \delta_j(\vec{a})\right\} &\leq \sum_{i \in C_j(\vec{a})} \min\left\{d_i, \delta_j(\vec{a}')\right\} \\ &< \sum_{i \in C_j(\vec{a}')} \min\left\{d_i, \delta_j(\vec{a}')\right\} \\ &\leq \kappa_j \end{aligned}$$

where the last inequality follows from the definition of δ_j. Since the left hand side is continuous in $\delta_j(\vec{a})$, there exists $\epsilon > 0$ such that $\sum_{i \in C_j(\vec{a})} \min\left\{d_i, \delta_j(\vec{a}) + \epsilon\right\} < \kappa_j$. This contradicts the definition of δ_j.

□

The theoretical results presented in this section suggest that in the UPN scenarios where the capacity of suppliers are distributed equally as in Figure 2 or distributed proportional to the client demands as in Figure 3, convergence to a NE is guaranteed by best response dynamics, where the clients update their decisions (on which supplier to connect) via local search. Note that, this result can be extended for other scenarios where the supplier capacity is distributed using a different function, as long as the resulting game is a distribution game.

IV. PRICE OF ANARCHY AND PRICE OF STABILITY

In this section we analyze the efficiency of the equilibria whose existence is proven in the previous section, under some plausible assumptions. We start with definitions regarding these assumptions.

Definition 2: A distribution game $\mathcal{G} = (n, m, d, A, \Delta, F)$ is identity-independent if the function δ_j does not depend on the set of players in $C_j(\vec{a})$ but only on their demands, and the function f_i does not depend on i but only on d_i. We use the term identity-independent to distinguish it from the term anonymous used in [9] for set functions that depend only on the number of players in the set.

The game $\mathcal{G}$ is non-keeper if every supplier j has a supply of κ_j such that $\sum_{i \in C_j(\vec{a})} u_i(\vec{a}) = \min\left\{D_j(\vec{a}), \kappa_j\right\}$.

Clearly, $\mathcal{G}_{EG}$ and $\mathcal{G}_{DP}$ are non-keeper and identity-independent.

We consider social welfare functions that are conical combinations (weighted sums) of the utilities of the players, i.e. $W(\vec{a}) = \sum_{i=1}^{n} \lambda_i u_i(\vec{a})$ where $\lambda_i \geq 0$ for every $i \in [n]$. Such a function is relevant, for instance, in the case where clients pay different prices per unit demand. We assume without loss of generality $\lambda_1 \geq \cdots \geq \lambda_n = 1$. Otherwise we can rename the clients such that the coefficients λ are sorted and then scale them such that the smallest coefficient becomes one. This changes the social welfare function by a constant factor and thus does not affect the price of anarchy and price of stability which are defined as ratios of this function. For a subset $C \subseteq [n]$ of clients we define the social welfare due to C as $W_C(\vec{a}) = \sum_{i \in C} \lambda_i u_i(\vec{a})$.

In order to analyze price of anarchy and price of stability, we have first to provide an upper bound the social welfare. This is done in the following observation.

Observation 1: Let $C = \{i_1, \ldots, i_k\}$ be a set of $k < n$ clients, with $i_1 > \ldots > i_k$. Then, the total utility of C is

- at most d_C, and
- at most $\kappa_{[\min\{k,m\}]}$ (i.e. the sum of the largest $\min\{k, m\}$ capacities). This bound is attained when the clients are matched to the first k suppliers, and the utility of each client is equal to the capacity of its supplier.
- Moreover, the social welfare due to C is at most $\lambda_{i_1}\kappa_1 + \ldots + \lambda_{i_{\min\{k,m\}}}\kappa_{\min\{k,m\}}$.

We have now to provide a way to compare any **NE** against the bounds just introduced. For this purpose we prove that in every **NE**, the excess capacity of every supplier is at most the utility of any non-saturated player. Specifically, denoting by $ex_j(\vec{a}) \stackrel{def}{=} \max\{0, \kappa_j - D_j(\vec{a})\}$ the excess capacity of supplier j in an outcome $\vec{a}$ we prove:

Claim 2: Let $\mathcal{G}$ be an identity-independent, non-keeper distribution game, and let $\vec{a}$ be a **NE** of $\mathcal{G}$. Then $ex_j(\vec{a}) \leq u_i(\vec{a})$ for every client i not saturated in $\vec{a}$ (i.e.,$u_i(\vec{a}) < d_i$) and every supplier j.

Proof: Let i be a client that is not saturated in $\vec{a}$ and a_i be its strategy. Since $\mathcal{G}$ is non-keeper, we have $ex_{a_i}(\vec{a}) = 0 \leq u_i(\vec{a})$. Assume for a contradiction that there is a supplier $j \neq a_i$ such that $ex_j(\vec{a}) > u_i(\vec{a})$. Let $\vec{a}'$ be the outcome obtained from $\vec{a}$ by i changing its strategy to j. If $u_i(\vec{a}') = d_i$ we have $u_i(\vec{a}') > u_i(\vec{a})$ since i is not saturated in $\vec{a}$. This contradicts the fact that $\vec{a}$ is a **NE**, thus $u_i(\vec{a}') < d_i$. Since $\mathcal{G}$ is non-keeper, we have $ex_j(\vec{a}') = 0$. Since $C_j(\vec{a}) \subsetneq C_j(\vec{a}')$, δ_j is non-increasing, and f_i is increasing, we have $u_{i'}(\vec{a}') \leq u_{i'}(\vec{a})$ for every $i' \in C_j(\vec{a})$. Then

$$\begin{aligned} u_i(\vec{a}') &= \kappa_j - \sum_{i' \in C_j(\vec{a})} u_{i'}(\vec{a}') \\ &\geq \kappa_j - \sum_{i' \in C_j(\vec{a})} u_{i'}(\vec{a}) \\ &= ex_j(\vec{a}) > u_i(\vec{a}) \end{aligned} \tag{4}$$

contradicting the fact that $\vec{a}$ is a **NE**. □

In the next Lemma we provide an upper bound to the price of anarchy, by partitioning the clients into two sets, namely the saturated ones and the rest. In this way we can bound the social welfare of the saturated and unsaturated clients separately using different bounds from Observation 1 and then combine the bounds.

Lemma 3: The price of anarchy of every identity-independent non-keeper distribution game is at most $1 + \lambda_1$.

Proof: Let $\mathcal{G}$ be an identity-independent, non-keeper distribution game, and let $\vec{a}$ be a **NE** of $\mathcal{G}$. We first introduce some notation. Denote by $S \stackrel{def}{=} \{i \in [n] | u_i(\vec{a}) = d_i\}$ be the set of clients that are saturated in $\vec{a}$. Let F be the set of suppliers to which the rest of the clients are connected, i.e. $F \stackrel{def}{=} \cup_{i \notin S} \{a_i\}$. For a set C of clients denote by $d_C \stackrel{def}{=} \sum_{i \in C} d_j$ their total demand. Similarly, for a set P of suppliers, we denote by $\kappa_P \stackrel{def}{=} \sum_{j \in P} \kappa_j$ their total capacity, and by $D_P(\vec{a}) = \sum_{j \in P} D_j(\vec{a})$ the total demand of their clients. Without loss of generality we assume $\kappa_1 \geq \ldots \geq \kappa_m$. Note that, since $\mathcal{G}$ is non-keeper, we have

$$\sum_{i \notin S} u_i(\vec{a}) \leq \kappa_F, \tag{5}$$

i.e., the total utility of the non-saturated clients is less than or equal to the total capacity of their suppliers. We now show an important property of a **NE** that we will use to conclude our proof.

Consider an optimal outcome $\vec{a}^*$ of $\mathcal{G}$. Clearly, for every client $i \in S$ we have $u_i(\vec{a}^*) \leq d_i = u_i(\vec{a})$. Therefore,

$$W_S(\vec{a}^*) = \sum_{i \in S} \lambda_i u_i(\vec{a}^*) \leq \sum_{i \in S} \lambda_i u_i(\vec{a}) = W_S(\vec{a}). \tag{6}$$

In the sequel we consider the utilities of the unsaturated clients, i.e., those clients $i \notin S$. Let $\bar{S} = [n] \setminus S = \{i_1, \ldots, i_{n-|S|}\}$ with $i_1 < \cdots < i_{n-|S|}$. Let also $n' = \min\{m, n - |S|\}$ be the smaller among the number of clients that are not saturated in $\vec{a}$ and the number of suppliers. By Observation 1 we have

$$\begin{aligned} W_{\bar{S}}(\vec{a}^*) &\leq \sum_{j=1}^{n'} \lambda_{i_j} \kappa_j \\ &= \sum_{j \in [n'] \setminus F} \lambda_{i_j} \kappa_j + \sum_{j \in [n'] \cap F} \lambda_{i_j} \kappa_j \\ &= \sum_{j \in [n'] \setminus F} \lambda_{i_j} (D_j(\vec{a}) + ex_j(\vec{a})) + \sum_{j \in [n'] \cap F} \lambda_{i_j} \kappa_j \\ &\leq \lambda_1 D_{[n'] \setminus F}(\vec{a}) + \sum_{j \in [n'] \setminus F} \lambda_{i_j} ex_j(\vec{a}) + \lambda_1 \kappa_{[n'] \cap F} \\ &= \lambda_1 \left(D_{[n'] \setminus F}(\vec{a}) + \kappa_{[n'] \cap F}\right) + \sum_{j \in [n'] \setminus F} \lambda_{i_j} ex_j(\vec{a}) \\ &\leq \lambda_1 \sum_{i=1}^{n} u_i(\vec{a}) + \sum_{j \in [n'] \setminus F} \lambda_{i_j} u_{i_j}(\vec{a}) \qquad (7) \\ &\leq \lambda_1 W(\vec{a}) + W_{\bar{S}}(\vec{a}) \qquad (8) \end{aligned}$$

where (7) is obtained by observing that the second term is the total utilization of the suppliers in F, the first term is the total utilization of other suppliers, and using Claim 2 for the third term. Combining inequalities (6) and (8) we obtain

$$W(\vec{a}^*) \leq (1 + \lambda_1) W(\vec{a}) \tag{9}$$

as claimed. □

Note that the game is identity-independent but the social welfare is identity-dependent. In other words, the game is played without identities, thus regardless of the coefficients λ_i, whereas the social welfare is measured using these coefficients. For this reason, a **PoA** of λ_1 seems inevitable. In the rest of this section we show that the above bounds are tight in the sense that there exist distribution games that attain these bounds.

Lemma 4: For every $\epsilon > 0$ there exist a coefficient vector $\vec{\lambda}$ and an identity-independent, non-keeper distribution game $\mathcal{G}$ with $\mathbf{PoA}(\mathcal{G}) \geq \mathbf{PoS}(\mathcal{G}) > 1 + \lambda_1 - \epsilon$.

Proof: Let $n > 2/\epsilon$, $\epsilon' = \frac{\epsilon/2-1/n}{n-1} > 0$ and let $\mathcal{G} = (n, m, d, A, \Delta, F)$ be an identity-independent non-keeper distribution game where every client i has a demand of $d_i = 1$, $\kappa_1 = 1$ and $\kappa_j = 1/n - \epsilon'$ for every $j \in [2, n]$. The social welfare is $\sum_{i=1}^{n} u_i(\vec{a})$, i.e., $\lambda_i = 1$ for every $i \in [n]$. We first prove that the strategy of every player is 1 in every NE, implying that $\mathcal{G}$ has a unique NE. Suppose that the strategy of some player i is $a_i \neq 1$. Then $u_i(\vec{a}) \leq 1/n - \epsilon' < 1/n$. On the other hand for the outcome $\vec{a}'$ obtained from $\vec{a}$ by i changing its strategy to 1, we have $D_1(\vec{a}') \leq n$. Since $\mathcal{G}$ is identity-independent and non-keeper, we have $u_i(\vec{a}') \geq 1/n > u_i(\vec{a})$, contradicting the fact that $\vec{a}$ is a NE. Therefore, the utility of every player is $1/n$ in the unique NE, for a social welfare of 1. An optimal outcome $\vec{a}^*$ is obtained when the strategy of client i is i. In this case $u_1(\vec{a}^*) = 1$ and $u_i(\vec{a}^*) = 1/n - \epsilon'$ for every client $i \in [2, n]$. We have

$$W(\vec{a}^*) = 1 + (n-1)\left(\frac{1}{n} - \epsilon'\right)$$
$$= 2 - \frac{1}{n} - (n-1)\epsilon' = 2 - \frac{\epsilon}{2} > 2 - \epsilon.$$

□

For a UPN where the client utilities are aligned directly with the obtained data rates, the theoretical results presented in this section suggest that the aggregate data rate obtained by the clients in any equilibrium is at least the half of the optimal value. In practice, usually this gap is much lower as will be shown in the next section. Valuation per unit bandwidth may be different for different clients. In this case, the ratio of the optimal social welfare to the attained one does not exceed one plus the ratio of the largest valuation to the smallest one.

V. APPLICATION TO UPN AND NUMERICAL STUDY

In this section we first describe implementation of the proposed scheme in UPN systems. Then we present a numerical study involving simulations in the UPN context that is carried out in order to examine the outcome of the proposed distribution game and also a generalization of it in which the demand of a client may vary from supplier to supplier.

A. UPN IMPLEMENTATION

In order to realize the proposed scheme in a UPN setting, some moderate assumptions should be made. Although the proposed study can be adopted to inband D2D communications where both cellular and D2D links use same licensed spectrum, we assume outband D2D communication such that the suppliers and the clients communicate via unlicensed spectrum such as WiFi or Bluetooth. This approach is used by already deployed UPN implementations such as Karma [59] and Fon [60] which are all based either on a specially designed hardware (WiFi router) or a special firmware (enables the device to emulate a WiFi router) equipped with some capacity sharing features. In such a setting a WiFi router may associate a maximum capacity with each of its connected clients so that a supplier can share its capacity among its connected clients according to our scheme. A full-stack software solution as in the case of M-87 networks [58] is also possible, which leverages smartphones for routing and bridging the data paths, by efficiently and fairly utilizing the outband D2D links.

In a distribution games, every client needs to know the demands of all other users that are connected to its potential suppliers. In an operator-controlled UPN setup, all the clients advertise their demands to the operator via a supplier. Then, demands of the other users could be obtained from the operator. In an autonomous UPN setup, an additional two-step communication protocol has to be implemented between the users. We may describe a node and strategy discovery protocol as follows. In the first step each client advertises its demand to the suppliers in its range. In the second step, i.e., upon collecting all demand declarations, the suppliers share this information with their possible clients in their range. At this point, each client knows others' demands and strategies. Also during the game play whenever a client connects to or drops from a supplier, the relevant supplier informs other possible clients so that clients may try to change their suppliers accordingly. Node discovery and communication between clients and suppliers can be handled via Bluetooth, WiFi direct, or any device-to-device communication technology.

In the rest of this section, we describe the simulation setup details and the performance results obtained by this numerical study. We simulate clients playing best response dynamics and examine the social welfare attained by the resulting equilibria in different network setups. We considered only the unbiased social welfare case, i.e., the case of $\lambda_1 = 1$.

B. SIMULATION SETUP

1) NETWORK AND DEMAND GENERATION

The network generation routine that generates the sample networks is passed the following parameters: a) number of gateways, b) number of clients, c) lower and upper bounds on the bandwidths offered by the gateways, and d) lower and upper bounds on the clients' demands. As for the first two of these parameters, we generate 5 networks comprising of 3 gateways and 6 clients, 5 gateways and 12 clients, 10 gateways and 24 clients, 15 gateways and 36 clients and 20 gateways and 48 clients. These networks are referred as small, medium, large, very large, and huge respectively, in Table 3.

The demands of the clients and gateways are chosen uniformly at random between the respective lower and upper bounds. In the first scenario which is referred as high demand variance in Table 3, clients' demands (d_i) are picked from a wide range (1 Mbps to 65 Mpbs). In the other scenario, referred as low demand variance clients' demands are picked from a narrower range (56 Mbps to 65 Mbps). In both scenarios the capacities (κ_j) of the gateways are chosen uniformly at random from the range 56 Mbps to 65 Mbps. In this way we simulate both random scenarios and scenarios in which the clients have demands close to the gateways' capacities. These data rates reflect the supported capacities and expected

TABLE 2. RSSI and link speed relation in 802.11n [16].

RSSI(dbm)	Link Speed (Mbps)
-82	6.5
-79	13
-77	19.5
-74	26
-70	39
-66	52
-65	58.5
-64	65

demands of modern handheld devices, smart phones, video streaming applications, etc.

We consider two cases in our simulations. In Case 1, the demand of every client is fixed. In this case, all the clients play best response dynamics. In Case 2, we consider a generalization of distribution games in which the demand of a client depends on the gateway to which it is connected. In other words, client i may not be able to get its demand d_i at any gateway j even if that gateway's capacity exceeds d_i. This can be due to distance, obstacles and disruptions in the network. If the maximum data rate that can be transferred from gateway j to client i is R_{ij}, then the maximum satisfiable demand of client i from gateway j is defined as

$$d_{ij} = \min(d_i, R_{ij}). \tag{10}$$

We relate R_{ij} to the distance between client i and gateway j as follows. First we compute received signal strength indicator between client i and gateway j ($RSSI_{ij}$) in dBm by the following formula [15]:

$$RSSI_{ij}(dBm) = P_{ij} - 10\alpha \log_{10}(d_{ij}) - OAF \tag{11}$$

where P_{ij} is the power level to send data from client i to gateway j, d_{ij} is the distance in meters between the client and the gateway, α is the channel attenuation factor and OAF is the obstacle attenuation factor. We used $P_{ij} = -37dBm$, $\alpha = 2$ and $OAF = 1.5$ in all simulations. $RSSI$ levels are then converted to link speed (R_{ij}) according to Table 2.

Throughout all simulations an area of 150 × 150 meters is assumed and all nodes are scattered over that area in a uniformly random fashion. 5 different samples were created from each type of network in Table 3.

The network topologies generated for simulations are summarized in Table 3. Topologies are created for distribution games and generalized distribution games, for networks with different numbers of nodes, and finally for situations where the variance between clients' demands is low and high.

2) SIMULATION

Finding a Nash equilibrium is a hard problem in general [13], [14]. However we do not know the hardness of the problem when restricted to distribution games. Our numerical study aims to determine best and worst Nash equilibria of the proposed scheme for practical instances. For this reason we take two different approaches:

TABLE 3. Simulation scenarios.

Network Size	Demand Variance	Case 1	Case 2
small	high	S-V-1	S-V-2
small	low	S-D-1	S-D-2
medium	high	M-V-1	M-V-2
medium	low	M-D-1	M-D-2
large	high	L-V-1	L-V-2
large	low	L-D-1	L-D-2
very large	high	V-V-1	V-V-2
very large	low	V-D-1	V-D-2
huge	high	H-V-1	H-V-2
huge	low	H-D-1	H-D-2

For networks with a small number of nodes, Nash equilibria are sought in the entire strategy space. This method gives precise results for small networks but its use for networks with a large number of nodes is impractical due to the size of the strategy space.

For large networks, initially clients are connected to gateways randomly and then a best response strategy is played in random order until a Nash equilibrium is reached. This process is repeated 20 times for each network. The best and the worst (in terms of total bandwidth) Nash equilibria among the results of these 20 runs are used for comparison. We computed the averages of the first 5, 10, 15 and 20 runs and observed that the results do not exhibit significant changes. We therefore used the results of 20 runs as a good approximation to the true minimum and maximum.

Optimal solutions are needed as a basis for comparison. In order to find an optimal solution, the integer linear program (12) - (16) is generated for each sample network.

$$\text{maximize} \quad \sum u_{ij} \tag{12}$$

$$\text{s.t.} \quad u_{ij} \leq d_{ij} x_{ij}, \quad \forall i, j \tag{13}$$

$$\sum_i u_{ij} \leq \kappa_j, \quad \forall j \tag{14}$$

$$\sum_j x_{ij} = 1 \quad \forall i \tag{15}$$

$$x_{ij} \in \{0, 1\} \tag{16}$$

where u_{ij} is the utility of client i obtained from gateway j, x_{ij} is an indicator variable that indicates whether client i is connected to gateway j, κ_j is the capacity of gateway j, d_{ij} is the demand of client i from gateway j.

We used the Egalitarian Distribution ($\mathcal{G}_{EG}$) throughout all simulations. This is because the Egalitarian distribution aims to make use of the whole capacity of gateways and assigns this capacity to the clients fairly, whereas the Demand-proportional distribution ($\mathcal{G}_{DP}$) favors the clients with higher declared demands.

3) EVALUATION

The performance evaluation is based on two different criteria: a) the total utility, and b) the Jain index [12]. The total utility is the bandwidth made available for use by the clients. Data rates of each client and the total utility of the sample network

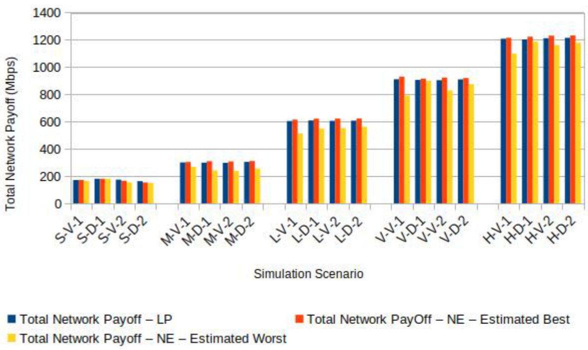

FIGURE 4. Simulation scenario vs total network payoffs (Mbps).

TABLE 4. Simulation scenarios compared.

	PoA	PoS
S-V-1	1.15	1.03
S-D-1	1	1
S-V-2	1.26	1.07
S-D-2	1.12	1.05
M-V-1	1.12	1.03
M-D-1	1.07	1.03
M-V-2	1.06	1.02
M-D-2	1.07	1.01

obtained as the result of the linear program are compared against the results of the proposed scheme. The Jain index is a metric that measures the fairness of the distribution of the available bandwidth among the clients. Specifically,

$$J(u_1, u_2, \ldots, u_n) = \frac{\left(\sum_{i=1}^{n} u_i\right)^2}{n \sum_{i=1}^{n} u_i^2} \tag{17}$$

where u_i is the bandwidth allocated to client i.

Two types of Jain indices are computed for each of the optimal solution and two Nash equilibria. The first Jain index is based on the bandwidth obtained by each client. The second Jain index is based on the ratio of the obtained bandwidth to the demand (of that client). The first Jain index measures the fairness of the bandwidth distribution from the perspective of a network operator, while the second measures the fairness from the clients' perspective.

C. RESULTS

In this section we present the results obtained by the experiments presented in Section V-B.

Price of Anarchy (PoA) and Price of Stability (PoS) are important criteria for the evaluation of the performance of our schemes. For this purpose, an optimal, a best Nash equilibrium and a worst Nash equilibrium (all in terms of total network payoff) are needed. Figure 4 shows total network payoffs of all simulation scenarios (according to the classification in Table 3).

At first glance both distribution games and their generalized version perform almost as good as optimal solutions. In order to better examine the results some metrics need to be considered. Two of them are obviously the PoA and the PoS of the simulation scenarios.

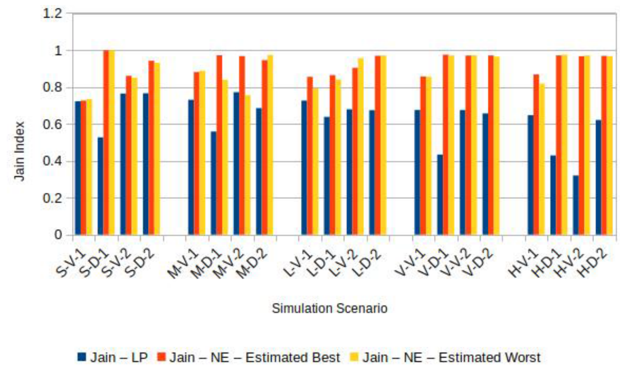

FIGURE 5. Jain index - Network practitioners' perspective.

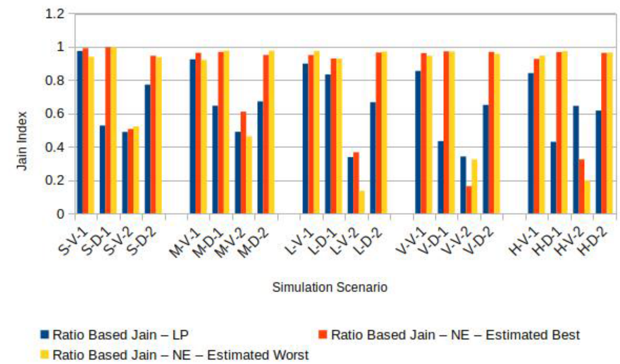

FIGURE 6. Jain index - Subscribers' perspective.

In Table 4 the estimated PoA and PoS of first two scenarios are listed. For larger networks, ILP based solutions take too much time to run and thus do not lead to optimal solutions in reasonable time. For this reason, our solutions could not be compared to optimal ones in large networks. In all scenarios the PoA is close to 1 except one scenario (S-V-2) in which the PoA is 1.26. The PoS is very close to 1 in all scenarios. This indicates that both distribution games and their generalized version introduced in this study converge to near-optimal Nash equilibria despite the lack of centralized control. In other words, when the nodes in the network play selfish best response strategies, the resulting connection graph will yield a good performance and will not cause severe degradation in the network payoff. Simulation results show that the PoA and PoS are much lower than the tight theoretical upper bounds (given in Section IV) in real world UPN deployments. We exclude results for larger networks since problem space for lp solvers are huge and in most of the experiments best Nash equilibria yield better results from the lp solver.

The obtained fairness values, i.e. one from network operators' perspective and another from subscribers' perspective are shown in Figure 5 and Figure 6, respectively, in terms of the Jain index. These indices show that, in this aspect, our scheme outperforms the ILP based optimal solution (that does

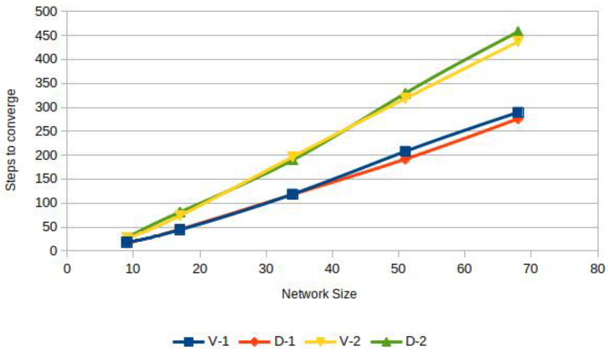

FIGURE 7. Steps to converge - given network size & scenario.

not consider fairness as an optimization criterion) while still maintaining near-optimality.

Since the Egalitarian distribution rule aims both to distribute all available bandwidth and to make it fair among users, the reached Nash equilibria resulted in near-optimal solutions in terms of both fairness and network payoff. When deployed to UPNs, distribution games allow easy adaption to connection changes. Whenever a node (gateway or client) joins the network other nodes may discover the newcomer and clients may change their connections according to the new situation. Thus the connection graph may start to evolve automatically until it reaches an equilibrium after some best/better response steps. The above holds also for the removal of a node from the network. The distribution function included in distribution games prioritizes fair bandwidth sharing since it distributes the bandwidth considering bare minimum values according to clients' demands. While prioritizing fair bandwidth sharing, the distribution function also tries to distribute all available bandwidth, thus trying to maximize the bandwidth usage. Based on the preceding discussion, it can be said that distribution games can be used to establish simple, and self-adapting UPN mechanisms that are efficient and fair from both clients' and operator's perspective. An important metric to consider when using best response dynamics is the time of convergence, i.e., the number of steps it takes the network to converge to an equilibrium. In Figure 7 network sizes versus the number of steps to converge is depicted for different scenarios. V-1 and D-1 represent networks with high and low demand variance clients, respectively, for Case-1. Similarly V-2 and D-2 represent networks with high and low demand variance clients for Case-2. For all scenarios, the number of steps to converge increases almost linearly with the network size, and the maximum number of moves per client is less than 10 in all scenarios, thus demonstrating the effectiveness of our scheme. Our simulations aims to be compatible with real UPNs and cover a wide range of parameters. The data rates used in the numerical results are generated with respect to RSSI which is based on the distance between the gateways and clients. This technique follows a common practice in the literature thus the chosen distances between nodes and the dimensions of the simulation area are not arbitrary. The demands of the clients are picked from data rate ranges that are encountered in daily applications. The 1-65 Mbps range is quite acceptable considering the need for flawless operation of contemporary applications. Also, contemporary devices support these data rates. In the networking context UPNs may consist of various numbers of nodes thus we started from small networks and increased the number of nodes in the simulations in order to cover networks with different sizes to demonstrate the effectiveness of our scheme.

VI. TRUTHFUL DISTRIBUTION GAMES

We note that the game $\mathcal{G}_{DP}$ does not provide a truthful mechanism. Whenever the utility of a client is less than its demand, i.e., when the overall demand of its connected supplier exceeds its capacity, it can get a bigger share of this capacity by declaring a demand that is higher than the true one. On the other hand, $\mathcal{G}_{EG}$ is truthful, since a client cannot increase its share of the capacity by untruthfully declaring a different demand.

In this section we formalize this notion, characterize the functions δ_j and f_j that lead to a truthful distribution games and verify this characterization on these two game families. For this purpose we define a variant of the model in which the demand d_i is not a constant, but part of the strategy of player i. Specifically, the strategy of player i is a supplier j and a demand d_i.

We denote by $\bar{d}_i$ the true demand of player i and by $\overrightarrow{d}$ the vector of demands declared by the clients. The utility of player i is

$$u_i(\overrightarrow{a}, \overrightarrow{d}) = \min\left\{\bar{d}_i, f_i(d_i, \delta_j(\overrightarrow{a}, \overrightarrow{d}))\right\}. \tag{18}$$

Note that this requires

- the functions f_i to possibly depend on d_i, and
- the functions δ_j to possibly depend on $\overrightarrow{d}$.

Note also that the utility of a player is bounded by its true demand as one would expect.

In this discussion we consider Smooth Distribution Games in which the functions f and δ above are continuous and differentiable with respect to every d_i. We assume that δ_j is monotonically non-increasing in d_i for every $i \in [n]$, i.e., if a client connected to supplier j increases its demand then δ_j does not increase. Note that this behaviour is consistent with the case of a connection of an additional client. Thus we have $\frac{\partial \delta_j}{\partial d_i} \leq 0$ for every $i \in [n], j \in [m]$.

We say that a smooth distribution game is truthful if the utility of a client does not increase by playing a value $d_i > \bar{d}_i$. A family of functions f_i and δ_j is truthful if every smooth distribution game using these functions is truthful. Whenever values of the other parameters are fixed, we refer to $f_i(d_i, \delta_j(\overrightarrow{a}, \overrightarrow{d}))$ as a function of d_i only. We now give a necessary and sufficient condition for the truthfulness of a family of functions.

Lemma 5: A smooth distribution game is truthful if and only if $f_i'(d_i) \leq 0$ or $f_i(d_i) \geq d_i$ for every outcome $(\overrightarrow{a}, \overrightarrow{d})$ and $i \in [n], j \in [m]$.

Proof: Suppose that there exists an outcome $(\overrightarrow{a}, \overrightarrow{d})$, a player $i \in [n]$ and a supplier $j \in [m]$ such that $f_i'(d_i) > 0$ and $f_i(d_i) < d_i$. Suppose also that $\bar{d}_i = d_i$. Then player i can increase its utility by setting d_i to $\bar{d}_i + \epsilon$ for a sufficiently small $\epsilon > 0$. The game is thus not truthful.

Conversely, suppose that the the game is not truthful. Then there exist two outcomes $(\overrightarrow{a}, \overrightarrow{d})$ and $(\overrightarrow{a}, \overrightarrow{d}')$ in both of which some client i is connected to the same supplier j, and the vectors $\overrightarrow{d}$ and $\overrightarrow{d}'$ differ only at the i-th entry with $d_i = \bar{d}_i$ and $d_i' > \bar{d}_i$ such that

$$\min\left\{\bar{d}_i, f_i(\bar{d}_i)\right\} < \min\left\{\bar{d}_i, f_i(d_i')\right\}.$$

Then

$$\begin{aligned} f_i(\bar{d}_i) &< f_i(d_i') \\ f_i(\bar{d}_i) &< \bar{d}_i. \end{aligned} \tag{19}$$

By the mean-value theorem, there exists $d'' \in (\bar{d}_i, d_i')$ such that $f_i'(d'') > 0$. If $f_i(d'') < d''$ then the claimed condition holds for d''. Otherwise, let $d''' = \inf\left\{d \in [\bar{d}_i, d'] | f_i(d) = d\right\}$. Since f_i is continuous, $f_i(d'') = d'' > \bar{d}_i$. By the mean value theorem, there exists $d''' \in (\bar{d}_i, d'')$ with $f_i'(d''') > 0$. By the choice of d'' and the continuity of f_i we have $f_i(d''') < d'''$. Thus the claimed condition holds for d'''. □

We now apply Lemma 5 to the smooth variants of $\mathcal{G}_{DP}$ and $\mathcal{G}_{EG}$.

Consider an outcome of $\mathcal{G}_{DP}$ such that at least two clients, one of them being i, are connected to supplier j. Let also choose $\kappa_j < d_i = \bar{d}_i$.

$$\begin{aligned} \frac{df_i}{dd_i} &= \frac{\partial f_i}{\partial d_i} + \frac{\partial f_i}{\partial \delta_j}\frac{\partial \delta_j}{\partial d_i} \\ &= \delta_j(\overrightarrow{a}, \overrightarrow{d}) + d_i\frac{\partial \delta_j}{\partial d_i} = \delta_j(\overrightarrow{a}, \overrightarrow{d}) - d_i\frac{\delta_j(\overrightarrow{a}, \overrightarrow{d})}{D_j(\overrightarrow{a}, \overrightarrow{d})} \\ &= \delta_j(\overrightarrow{a}, \overrightarrow{d})\left(1 - \frac{d_i}{D_j(\overrightarrow{a}, \overrightarrow{d})}\right) > 0 \end{aligned}$$

where the last inequality uses the fact that there are at least two clients connected to j, thus $d_i < D_j(\overrightarrow{a}, \overrightarrow{d})$. Since $\kappa_j < d_i = \bar{d}_j$ we have $f_i(d_i) < d_i$. We conclude that $\mathcal{G}_{DP}$ is not truthful.

For $\mathcal{G}_{EG}$ we have

$$\begin{aligned} \frac{df_i}{dd_i} &= \frac{\partial f_i}{\partial d_i} + \frac{\partial f_i}{\partial \delta_j}\frac{\partial \delta_j}{\partial d_i} \\ &= 0 + 1\frac{\partial \delta_j}{\partial d_i} \leq 0. \end{aligned} \tag{20}$$

for every outcome $(\overrightarrow{a}, \overrightarrow{d})$. Therefore, $\mathcal{G}_{EG}$ is truthful.

VII. CONCLUSION AND OPEN PROBLEMS

In this study we introduced a game theoretical model both keeping total network payoff and fairness in mind. These two concepts are important from both network operators' and subscribers' point of view. Seeking for a Nash equilibrium is conceptually a distributed greedy solution method which may result in non-optimal solutions but this turns out not to be the case in this study.

We now discuss how our model achieves the goals stated in Section I. Since our games do not require any coordination between clients (such as coalitions), they are easily implementable in a distributed environment. We introduced two games that fit into this model, namely the Egalitarian and Demand-proportional distribution games. We note that the Demand-proportional distribution is simpler to implement, since in this case it is sufficient that every supplier advertises its (fixed) capacity and the currently used capacity. On the other hand, Egalitarian distribution requires every supplier to advertise the demands of all its connected clients. The definition of distribution games allows for the restriction of the set of suppliers a client can be connected, thus making our games a good fit to a dynamic environment in which the potential connections between clients and suppliers vary in time, or alternatively when such connections may be restricted by the operator according to various policies. Simple best response dynamics can be played in both operator-assisted and autonomous UPNs in distributed setups. Other solutions such as solving the connection graph either in a distributed or central manner are more costly such that they involve exploring the whole connection graph with all capacities and demands, sharing the connection graph to solve in a distributed or central manner, and assigning the data rates of each client. Also node addition and removal requires re-computation of above steps which is not the case in our scheme.

We conclude with further research directions related to our work. The upper-bound to the number of steps to convergence implied by Lemma 1 is 2^{mn}. This bound is apparently tight for both our distribution schemes, i.e. Egalitarian and Demand-proportional. However, if the set of possible demand values is small, say a constant c, we can lower the number of steps to as low as $\binom{m}{2^c}]$. In this work, we did not consider the time-to-convergence of these games.

We provided a simple characterization of the smooth (continuous and differentiable) functions δ_j and f_j that always lead to truthful distribution games. Using this characterization we have shown that the Egalitarian family of games is truthful whereas Demand-proportional is not. The characterization of non-smooth functions is an open problem. Our functions do not use a payment mechanism. The introduction of such a mechanism will possibly lead to a richer set of truthful functions.

We now mention a few possible generalizations of distribution games: First is the case where the demand of a client depends on the supplier. In this work we studied this generalization only numerically. Our simulations reached a Nash equilibrium in all simulations, by playing random best response. An interesting open problem is to determine whether or not this is always the case. Second, one can study the Bayesian game family by considering the scenarios where users may have incomplete information about other players' demands and only have a prior belief on their types. Another extension is the case when a client is allowed to be connected to more than one supplier and divide its demand among these suppliers. At the first glance, a strategy of this variant seems to be a mixed strategy of the original game. However, this is not the case since the dependence of the utility on the demand is not necessarily linear, as opposed to the expected utility of a mixed strategy whose dependence is linear in the probabilities.

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