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RESEARCH ARTICLE

The l_1 Optimal State Estimator for Load Frequency Control of Power Systems: A Comparative and Extensive Study

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ABSTRACT This paper proposes fully decentralized l_1 optimal dynamic state estimators (DSEs) for load frequency control (LFC) of interconnected power generating systems and provides its comparative and extensive study with respect to three other types of DSEs. To this end, we present the dynamic model of a single area of the power-generating unit occurring from the interconnected power systems. By noting the fact that each area is affected by the frequency deviations in other areas, and it is quite difficult to obtain any property of the load changes in power systems, we characterize the disturbances in the LFC of power systems as bounded persistent signals. As a candidate for DSEs for LFC of power systems, the unknown input observer (UIO), Kalman filter (KF), and H_∞ optimal DSE are considered, and their limitations are analyzed in depth. In connection with this, the l_1 optimal DSE, in which the maximum magnitude of estimation error for the worst bounded persistent disturbances is minimized, is proposed as the most effective state estimator. Finally, the practical validity and effectiveness of the proposed l_1 optimal DSE are demonstrated through some comparative simulations for a three-area power generating system.

INDEX TERMS Dynamics state estimator (DSE), load frequency control (LFC), l_1 optimality.

NOMENCLATURE

\mathbb{N}	The set of positive integers.
\mathbb{N}_0	$\mathbb{N} \cup \{0\}$.
\mathbb{R}^ν	The set of ν -dimensional real vectors.
*	Symmetric terms in a symmetric matrix.
$(\cdot)_i$	i th area index of (\cdot) , $i = 1, 2, \dots, N$.
ΔACE_i	Area control error.
$\Delta P_{s,i}$	Load reference setpoint deviation.
$\Delta P_{v,i}$	Valve position deviation.
$\Delta P_{t,i}$	Turbine power deviation.
$\Delta P_{tie,i}$	Tie-line power deviation.
$\Delta P_{L,i}$	Load change.
Δf_i	Frequency deviation.

ΔR_i	Governor droop characteristic.
ΔB_i	Frequency bias parameter.
$K_{g,i}, T_{g,i}$	Governor gain and time constant.
$K_{t,i}, T_{t,i}$	Turbine gain and time constant.
M_i, D_i, K_i	Inertia constant, load damping coefficient, gain of synchronous generator.
T_{ij}	Synchronous coefficient between i th area and j th area.

I. INTRODUCTION

An imbalance between generated power and load demand might lead to a blackout for the whole power systems. In other words, the power generation amount must coincide with load demand. However, it is a non-trivial task to maintain the balance between them since the load demand is unpredictable and uncertain. In this matter, if we note that such imbalance directly leads to variations in the system frequency,

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then we can conclude that the frequency in power systems can describe the balance (or imbalance) between generated power and load demand. Therefore, one can maintain the balance between the generated power and load demand by regulating the frequency in power systems at prescribed nominal values. In this regard, a control scheme called the load frequency control (LFC) or the automatic generation control (AGC) has been established for regulating the frequency of the system at a nominal value [1], [2], [3].

The LFC has been extensively studied for a few decades [4], [5]. The LFC schemes can be classified into three different frameworks as discussed in [6]: intelligent control framework [7], evolutionary computing framework [8], and state feedback control framework [9]. The state feedback control framework [10], [11], [12], [13] has been intensely studied due to its potential applicability to some robust and optimal properties [14]. However, these schemes are often criticized with respect to the realization of real-time state data acquisitions. To address this problem, dynamic state estimation (DSE) schemes have been suggested to estimate the state variables in real-time [15], [16].

However, it is becoming more challenging to estimate the state variables in modern power systems because they are constructed by many interconnected power generating units. In interconnected power generating systems, not only the load changes in a single area but also the power deviations from the tie-line (so-called tie-line power deviation) affect the frequency deviations. Furthermore, operating these numbers of systems with a single centralized controller requires a massive computation load and communication lines. Thus, it is more efficient to divide the system into interconnected single generating units and operate each system using decentralized controllers based on the effective DSE schemes [17], [18], [19], [20], [21], [22], [23]. In [19], [18], and [17], the decentralized LFC is proposed by using the Kalman filter (KF) based DSEs. However, such KF-based approaches can only handle disturbances with zero means and Gaussian distributions, which is not generally considered the nature of the load changes in power systems. In [21], [20], and [22], novel DSE approaches for the quasi-decentralized LFC are proposed, but they require remote data transmissions such as measurement outputs and control inputs. This solution might be vulnerable with respect to cyber-attacks. Recently, in [23], a fully decentralized DSE scheme was developed by considering the tie-line power deviations as unknown inputs for the first time, which establishes the completely decentralized LFC. However, this approach has limitations on implementation with one of the most practical measurement outputs, the so-called area control error (ACE), due to the limitation of an unknown input observer (UIO). Implementing DSE schemes using only the ACE is practical and efficient since the ACE can represent the frequency deviations together with the tie-line power deviations, and the ACE is commonly used for existing LFC approaches.

Motivated by these limitations of existing studies, this paper proposes a fully decentralized DSE scheme using only

the ACE as the measurement output. Furthermore, we characterize the load changes and the tie-line power deviations as bounded persistent disturbances by noting that it is difficult to characterize such disturbances by specific presumed natures. The proposed approach estimates the state variables by regulating the maximum magnitude of state estimation errors with respect to bounded persistent unknown inputs based on the arguments of the l_∞ -induced norm approach, i.e., the l_1 optimal control theory [24], [25], [26], [27]. The development of the proposed l_1 DSE approach is practically meaningful because the fully decentralized l_1 optimal DSE can handle the real-time DSE problem by using only the ACE.

To put it another way, the main contributions of this paper can be summarized as follows.

- This paper considers a fully decentralized DSE, as discussed in [23]. The tie-line power deviations are modeled as unknown inputs in contrast to the quasi-decentralized LFC schemes [20], [21], [22].
- The disturbances such as the load changes and the tie-line power deviations are considered bounded persistent signals, unlike the KF-based schemes [17], [18], [19].
- This paper suggests three candidates for fully decentralized DSEs for the LFC and extensively discusses their limitations intrinsically occurring from their nature.
- The proposed l_1 DSE uses the measurement output as the ACE, while it cannot be used for the existing fully decentralized DSE scheme [23].
- The proposed scheme can be directly implemented without any construction change from other existing methods using the ACE as an output [4], [5].
- Simulation results with an interconnected three-area power system are given to demonstrate the effectiveness of the proposed l_1 optimal DSE through comparative results with the other candidates.

The organization of this paper is as follows. In Section II, we provide a system description of a power generating unit with the problem definition tackled in this paper. Theoretical analysis for possible DSE candidates is deeply discussed in Section III. An l_1 optimal DSE is proposed in Section IV. Some simulation results are given in Section V to demonstrate the validity of the proposed method compared with other candidates. The conclusions are given in Section VI.

II. LOAD FREQUENCY CONTROL MODEL FOR POWER SYSTEMS AND RELEVANT ISSUES

This section introduces the dynamic model for load frequency control (LFC) of power systems and the relevant problem definition. Modern power systems usually consist of a large number of areas connected by transmission lines so called tie-lines, and thus it is quite difficult to describe the whole dynamics of such power systems in a rigorous fashion. To simplify the dynamics of the power systems without loss of their important properties, we take a decentralized system consisting of power generating units connected by

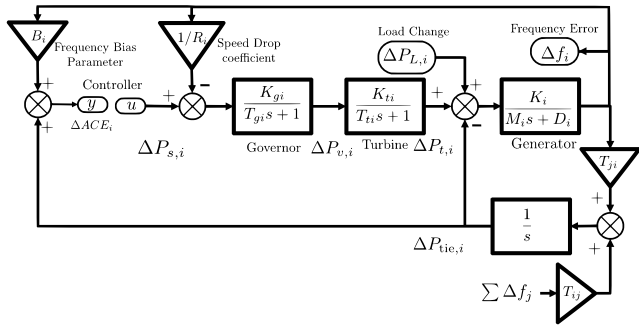


FIGURE 1. The i th area of single power generating unit.

the tie-lines as an effective alternative model of the power systems. In other words, we consider the i th area of the single power generating unit connected with other areas by the tie-lines as shown in Fig 1.

The dynamics of the i th area in this figure can be represented by

$$\Delta \dot{P}_{v,i} = \frac{K_{g,i}}{T_{g,i}} (\Delta P_{s,i} - \frac{1}{R_i} \Delta f_i) - \frac{1}{T_{g,i}} \Delta P_{v,i} \quad (1)$$

$$\Delta \dot{P}_{t,i} = \frac{K_{t,i}}{T_{t,i}} \Delta P_{v,i} - \frac{1}{T_{t,i}} \Delta P_{t,i} \quad (2)$$

$$\Delta \dot{f}_i = \frac{K_i}{M_i} \Delta P_{t,i} + \frac{K_i}{M_i} \Delta P_{L,i} - \frac{K_i}{M_i} \Delta P_{tie,i} - \frac{D_i}{M_i} \Delta f_i \quad (3)$$

$$\Delta ACE_i = B \Delta f_i + \Delta P_{tie,i} \quad (4)$$

$$\Delta P_{tie,i} = \frac{T_{ij}}{s} \left(\sum_i^n \Delta f_j \right) \quad (5)$$

Combining (1)–(5) further admits the representation described by

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_{1,i} w_i(t) + B_{2,i} u(t) \\ y_i(t) = C_i x_i(t) + D_{1,i} w_i(t) \end{cases} \quad (6)$$

where

$$A_i = \begin{bmatrix} -\frac{1}{T_g} & 0 & -\frac{K_g}{RT_g} \\ \frac{K_t}{T_t} & -\frac{1}{T_t} & 0 \\ 0 & \frac{K}{M} & -\frac{D}{M} \end{bmatrix}, \quad B_{1,i} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K}{M} & -\frac{K}{M} \end{bmatrix} \quad (7)$$

$$B_{2,i} = \begin{bmatrix} \frac{K_g}{T_g} \end{bmatrix}, \quad C_i = [0 \ 0 \ B], \quad D_{1,i} = [0 \ 1] \quad (8)$$

with the state variable $x_i := [\Delta P_{v,i} \ \Delta P_{t,i} \ \Delta f_i]^T \in \mathbb{R}^3$, the disturbance $w_i := [\Delta P_{L,i} \ \Delta P_{tie,i}]^T \in \mathbb{R}^2$, the control input $u_i := \Delta P_{s,i} \in \mathbb{R}$, and the measurement output $y_i := \Delta ACE_i$. Here, it should be strongly stressed that describing the tie-line power deviations as disturbances is quite practically meaningful with respect to implementing the completely decentralized LFC [23]. On the other hand, with the fact that dynamic state estimators (DSEs) are usually implemented by digital computing units, we deal with the dynamics of the single

power generating unit as well as DSEs in a discrete-time fashion. To do this, we discretize (6) by taking the sampling period h , i.e.,

$$\begin{cases} x_i[k+1] = A_{d,i} x_i[k] + B_{d1,i} w_i[k] + B_{d2,i} u[k] \\ y_i[k] = C_i x_i[k] + D_{1,i} w_i[k] \end{cases} \quad (9)$$

where

$$A_{d,i} := \exp(A_i h), \quad (10)$$

$$[B_{d1,i} \ B_{d2,i}] := \int_0^h \exp(A\tau) d\tau [B_{1,i} \ B_{2,i}] \quad (11)$$

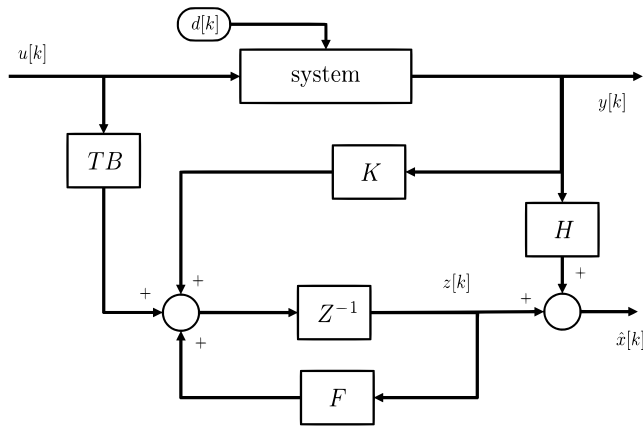
and $x[k] := x(kh)$, $w[k] := w(kh)$, $u[k] := u(kh)$, and $y[k] = y(kh)$ with the time step $k \in \mathbb{N}_0$.

A number of approaches to the LFC of power systems have been introduced in [10], [11], [12], [13] for the state feedback control framework tailored to potential applicabilities to some effective robust and optimal properties [14]. Their effectiveness has been demonstrated extensively through various types of simulations, but it has also been observed that they are quite sensitive to noises or disturbances that affect an exact estimation of the state values. To solve this problem, it should be required to make the DSE more robust for such external signals. In this sense, this paper is concerned with providing effective solutions to the problem of estimating the state x_i against the effects of the disturbance w_i based on (9). This could be conducted by designing an adequate DSE depending on the nature of w_i . However, it is challenging to derive a specific class of signals characterizing w_i , i.e., the load changes because they are often unpredictable. Indeed, this issue becomes more serious when different load changes from other areas affect the considered area through tie-line power fluctuations. In this sense, it cannot be possible to presume several characteristics of w_i but it might be better only to take account of its time-domain boundness as a characteristic.

As a preliminary step to proceed to propose a new state estimation method, characteristics of some conventional methods as well as a possible candidate associated with state estimations for load frequency control of power systems will be discussed in the following section.

III. COMPARATIVE ANALYSIS FOR EXISTING AND POSSIBLE CANDIDATE OF DYNAMIC STATE ESTIMATORS

This section introduces some existing methods as well as a possible candidate used in state estimations for the LFC of power systems. Their intrinsic properties with respect to practical limitations are also discussed comparatively. More precisely, the conventional state estimation schemes of unknown input observer (UIO) [28] and Kalman filter [29] are first introduced, and some difficulties corresponding to achieving high performances through their implementations are deeply discussed. An H_∞ optimal state estimator is also proposed as an alternative for the conventional state estimator, but its limitations relevant to practical effectiveness are given.


FIGURE 2. The unknown input observer.

A. UNKNOWN INPUT OBSERVER

The unknown input observer (UIO) [28] estimates the state variables in linear systems containing additive unknown disturbances. By taking the effectiveness of the UIO relevant to the robustness against the unknown disturbance term, its application to the LFC has been deeply studied in recent years [23], [30], [31], [32].

As a brief sketch of the process for UIOs, we first consider the dynamic plant given by

$$\begin{cases} x[k+1] = Ax[k] + Bu[k] + Ed[k] \\ y[k] = Cx[k] \end{cases} \quad (12)$$

where $x[k] \in \mathbb{R}^{n_x}$ is the state, $u[k] \in \mathbb{R}^{n_u}$ is the known input, $d(t)[k] \in \mathbb{R}^{n_d}$ is the disturbance, and $y[k] \in \mathbb{R}^{n_y}$ is the measurement output. For this plant, we next define a virtual dynamic system given by

$$\begin{cases} z[k+1] = Fz[k] + TBu[k] + Ky[k] \\ \hat{x}[k] = z[k] + Hy[k] \end{cases} \quad (13)$$

where $z[k] \in \mathbb{R}^{n_x}$ and $\hat{x}[k] \in \mathbb{R}^{n_x}$ are the state of the observer and estimated state of (12), respectively, with suitably determined matrix parameters F , T , K and H ; the structure of UIOs is as shown in Fig. 2. Then, defining the state estimation error as $\tilde{x}[k] := x[k] - \hat{x}[k]$ together with considering the additional freedom for taking K as $K = K_1 + K_2$ leads to

$$\begin{aligned} \tilde{x}[k+1] = & (A - HCA - K_1C)\tilde{x}[k] - (F - (A - HCA \\ & - K_1C))z[k] - (K_2 - (A - HCA - K_1C)H)y[k] \\ & - (T - (I - HC))Bu[k] - (HC - I)Ed[k]. \end{aligned} \quad (14)$$

Because the main objective of UIOs is to lead to the asymptotic stability of the system (14) (i.e., $\tilde{x}[k] \rightarrow 0$ as $k \rightarrow \infty$ for an arbitrary $\tilde{x}[0]$), it is obtained in [30] and [31] that

$$\tilde{x}[k+1] = F\tilde{x}[k] \quad (15)$$

with some properties to ensure the following conditions:

- $F = A - HCA - K_1C$.
- $K_2 = FH$.

- $T = I - HC$.
- $(HC - I)E = 0$.
- F is Hurwitz stable.

Such an F is shown in [30] and [31] to be determined if and only if the following conditions hold:

- (a) $\text{rank}(CE) = \text{rank}(E)$.
- (b) For $A_1 := A - E[(CE)^T CE]^{-1}(CE)^T CA$, the pair (C, A_1) is detectable.

Even though the UIO is regarded as an effective DSE for LFC of power systems, as discussed in [30], [31], [23], and [32], it is quite difficult to derive the same conditions as mentioned above for the case with measurement errors. Hence, it is not immediate to establish the asymptotic or bounded-input and bounded-output stability in the UIO-based state estimations for LFC of power systems when there exist measurement noises or their effects are not small as ignored. More importantly, we would like to note that the ACE, which is often taken as an output in LFC of power systems [4], [5], [6], is affected by the tie-line power deviation as can be seen in (4). Indeed, roughly speaking, condition (a) means that the number of output (i.e., n_y) should be larger than that of the disturbance (i.e., n_d), but the former is larger than the latter in this paper. Moreover, with respect to condition (b), the inverse of $(CE)^T CE$ does not always exist. In this sense, the UIO-based schemes cannot be directly applied to the state estimation problem tackled in this paper.

B. KALMAN FILTER

The Kalman filter [29] is used for estimating the state of stochastic systems via the least-squares method [33]. It is generally assumed in the KF-based state estimation methods that the unknown elements are characterized by zero-means and Gaussian distributions.

To introduce the process of KF-based state estimations, we first consider the discrete-time plant given by

$$\begin{cases} x[k+1] = Ax[k] + B_1w[k] + B_2u[k] \\ y[k] = Cx[k] + D_1n[k] + D_2u[k] \end{cases} \quad (16)$$

where $w[k] \in \mathbb{R}^{n_w}$ is the disturbance and $n[k] \in \mathbb{R}^{n_y}$ is the measurement noise.

The state is determined by minimizing the corresponding error covariance in the KF-based schemes, and this process is conducted through recursive updates consisting of the so-called *prediction* and *correction* as follows:

- (i) Prediction: At the time index k , the value of $x[k+1]$ is predicted by using the value of $\hat{x}[k]$ and the information from the plant.
- (ii) Correction: The state value at $k+1$ obtained in the prediction process is modified to be close to the exact value of $x[k+1]$ by using the measurement output. This value is regarded as the estimated state.

The structure consisting of these two update schemes is as shown in Fig. 3, and their mathematical descriptions are given respectively by

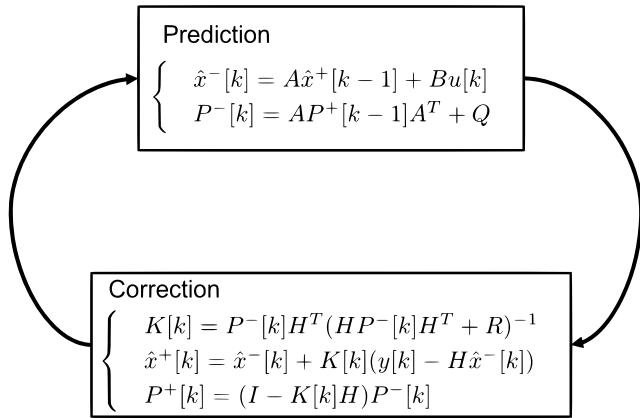


FIGURE 3. The Kalman filter: prediction and correction.

Prediction :

$$\hat{x}^-[k] = A\hat{x}^+[k-1] + B_2u[k] \quad (17)$$

$$P^-[k] = AP^+[k-1]A^T + Q \quad (18)$$

Correction :

$$K[k] = P^-[k]C^T(CP^-[k]C^T + R)^{-1} \quad (19)$$

$$\hat{x}^+[k] = \hat{x}^-[k] + K[k](y[k] - C\hat{x}^-[k] - D_2u[k]) \quad (20)$$

$$P^+[k] = (I - K[k]C)P^-[k] \quad (21)$$

Here, $\hat{x}^-[k]$ and $\hat{x}^+[k]$ are the priori and posteriori estimated states obtained from the prediction and correction procedures, respectively. Finally, $\hat{x}^+[k]$ is regarded as an estimated value for $x[k]$ after both the prediction and correction procedures are completed. The matrix parameters $K[k]$, $P^-[k]$ and $P^+[k]$ are called the Kalman gain, priori and posteriori estimation error covariance matrices, respectively. They are determined by using the covariance matrices $Q \in \mathbb{R}^{n_w \times n_w}$ and $R \in \mathbb{R}^{n_y \times n_y}$ associated with the disturbance and measurement error, respectively.

The basic idea of the KF is as follows. If both n and w are characterized by the zero-means and Gaussian distributions, the estimation error is also described in terms of the zeros means and Gaussian distributions. With this in mind, the estimated state value in the KF-based schemes is obtained by minimizing the covariance of the estimation error, i.e., $P^+ = E[(x - \hat{x})^T(x - \hat{x})]$, where $E[\cdot]$ denotes the mean value of (\cdot) . Thus, the estimation error could be interpreted as located densely around zero [29].

Even though one might argue that the KF could be one of the most effective candidates for a DSE for LFC of power systems and the relevant results are discussed in [17], [18], and [19], its effectiveness cannot be readily achieved since the stochastic properties of the disturbance and measurement error such as the zero means and distributions cannot be obtained in real LFC of a power system.

C. H_∞ OPTIMAL DYNAMIC STATE ESTIMATOR

The arguments of the H_∞ optimality [14] have been widely used for various control applications [34], [35] by noting the fact that they aim at minimizing the l_2 -induced norm from the disturbance to the regulated output. In other words, the H_∞ norm of a system is the maximum energy of the regulated output for the worst disturbance with unit energy. However, there is no study on applying the scheme of the H_∞ optimality to the state estimations for LFC of power systems. In this sense, we propose an H_∞ optimal DSE as an effective candidate with respect to the LFC of power systems and discuss its intrinsic limitations in a qualitative sense (as well as a quantitative sense through simulation results).

As a preliminary step to introduce an H_∞ optimal DSE, let us first consider the discrete-time plant given by

$$\begin{cases} x[k+1] = Ax[k] + B_1w[k] + B_2u[k] \\ y[k] = C_2x[k] + D_{21}w[k] \end{cases} \quad (22)$$

We next consider the virtual dynamic system given by

$$\begin{cases} \hat{x}[k+1] = A\hat{x}[k] + B_2u[k] - \epsilon[k] \\ \hat{y}[k] = C_2\hat{x}[k] \end{cases} \quad (23)$$

where $\epsilon[k] \in \mathbb{R}^{n_x}$ denotes a compensation input for making $\hat{x}(k)$ to be close to $x(k)$ even for the existence of w . With respect to determining $\epsilon[k]$, we regard it as an output of a linear mapping whose input is the measurement estimation error defined as $\tilde{y}[k] := y[k] - \hat{y}[k]$. By combining (22) and (23), we obtain

$$\begin{cases} \tilde{x}[k+1] = A\tilde{x}[k] + B_1w[k] + \epsilon[k] \\ z[k] = C_1\tilde{x}[k] \\ \tilde{y}[k] = C_2\tilde{x}[k] + D_{21}w[k] \end{cases} \quad (24)$$

where $z[k]$ is the regulated output to be minimized and C_1 is a weighting matrix to be selected by the estimation objectives. Based on (24), we propose the dynamic structure for determining $\epsilon[k]$ described by

$$\Psi : \begin{cases} \psi[k+1] = A_\psi\psi[k] + B_\psi\tilde{y}[k] \\ \epsilon[k] = C_\psi\psi[k] + D_\psi\tilde{y}[k] \end{cases} \quad (25)$$

where the matrix-valued parameters A_ψ , B_ψ , C_ψ , and D_ψ are determined to minimize the H_∞ norm of the closed-loop system obtained by connecting (24) and (25). To put it another way, the closed-loop system is described by

$$\Sigma_{cl} : \begin{cases} \xi[k+1] = A_\xi\xi[k] + B_\xi w[k] \\ z[k] = C_\xi\xi[k] \end{cases} \quad (26)$$

where

$$\begin{aligned} A_\xi &= \begin{bmatrix} A + D_\psi C_2 & C_\psi \\ B_\psi C_2 & A_\psi \end{bmatrix}, \\ B_\xi &= \begin{bmatrix} B_1 + D_\psi D_{21} \\ B_\psi D_{21} \end{bmatrix}, \\ C_\xi &= [C_1 \ 0]. \end{aligned} \quad (27)$$

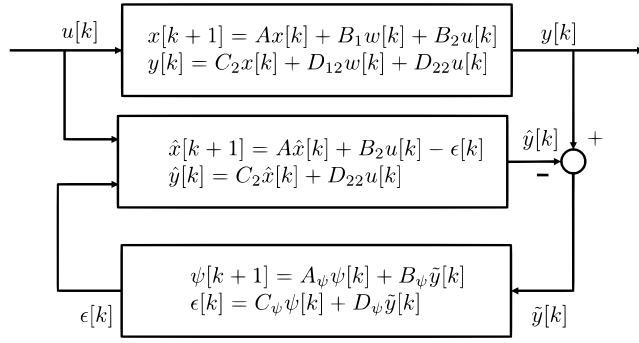


FIGURE 4. Structure of H_∞ optimal dynamic state estimator.

with $\xi[k] := [\tilde{x}^T[k] \ \psi^T[k]]^T$. The H_∞ norm of the closed-loop system Σ_{cl} is defined as

$$\|\Sigma_{cl}\|_{H_\infty} := \sup_{w \neq 0} \frac{\|z\|_2}{\|w\|_2} \quad (28)$$

where $\|\cdot\|_2$ denotes the l_2 norm of (\cdot) . It is shown in [36] that there exists a state estimator Ψ of (26) such that $\|\Sigma_{cl}\|_{H_\infty} < \gamma$ if and only if the following linear matrix inequality (LMI) conditions are feasible:

$$\begin{bmatrix} PJ & AX+L & A+RC_2 & B_1+RD_{21} & 0 \\ *H & Q & YA+FC_2 & Y+FD_{21} & 0 \\ **X+X^T-P & I+S^T-J & 0 & X^T C_1^T & 0 \\ ** & * & Y+Y^T-H & 0 & C_2^T \\ ** & * & * & I & 0 \\ ** & * & * & * & \gamma I \end{bmatrix} < 0 \quad (29)$$

Here, X, L, Y, F, Q, R, S, J and the symmetric matrices P and H are the decision variables, and the matrix-valued parameters A_ψ, B_ψ, C_ψ and D_ψ are given by

$$A_\psi = V^{-1}(Q - YAX)U^{-1} - V^{-1}YLU^{-1} - V^{-1}FC_2XU^{-1} + V^{-1}YRC_2XU^{-1} \quad (30)$$

$$B_\psi = V^{-1}(F - YR) \quad (31)$$

$$C_\psi = (L - RC_2X)U^{-1} \quad (32)$$

$$D_\psi = R \quad (33)$$

Because the H_∞ optimal DSE intrinsically takes signals in the l_2 space, the disturbance is naturally assumed to have bounded energy. In other words, the disturbance $w[k]$ is regarded as converging to 0 as k becomes larger, but this is in contrast to the practical LFC of power systems, in which disturbances should be treated as persistent and bounded signals because the load demands in power systems vary consistently. Thus, direct employment of the H_∞ optimal DSE might not lead to high accuracy in the state estimation for LFC of power systems, and the details will also be discussed in Section V.

Motivated by the limitations of the aforementioned three types of DSEs, we propose another DSE for the LFC of power systems to treat the effects of possible disturbances in a more sophisticated fashion. More precisely, bounded and persistent signals could be regarded as disturbances in the LFC of power systems and their effects will be tackled by developing the l_1 optimal DSE in the following section.

IV. l_1 OPTIMAL DYNAMIC STATE ESTIMATOR

This section introduces the l_1 optimal DSE to take into account the effects of disturbances occurring from real LFC of power systems in a more sophisticated fashion. To put it another way, we take the l_∞ -induced norm from the disturbance to the estimation error as a performance measure with respect to the state estimation for LFC of power systems, and discuss a method for designing the l_1 optimal DSE that minimizes the performance measure. Because bounded and persistent signals are regarded as elements in the l_∞ space, the problem of dealing with the effects of practical disturbances on the LFC of power systems could be tackled by taking the l_1 optimal DSE. Furthermore, if we note that it is often more important to suppress magnitudes of the regulated output rather than its energy in power systems, then taking the l_1 optimal DSE might be interpreted as providing the most effective schemes in the state estimation problem for LFC of power systems, rather than the aforementioned UIO, Kalman and H_∞ -based state estimation methods.

The structure of the l_1 optimal DSE is equivalent to the H_∞ optimal DSE as shown in Fig. 4. In other words, for the plant given by (24), we aim at designing an optimal controller Ψ described by (25), but the control objective is to minimize the l_∞ -induced norm from w to z in (26), i.e.,

$$\|\Sigma_{cl}\|_{l_1} := \inf_{\Psi} \sup_{w \neq 0} \frac{\|z\|_\infty}{\|w\|_\infty} \quad (34)$$

where $\|\cdot\|_\infty$ is the l_∞ norm of (\cdot) . Denote an optimal Ψ leading to the infimum (34) by Ψ_{opt} . Then, the l_1 optimal DSE is described by

$$\begin{cases} \hat{x}[k+1] &= A\hat{x}[k] + B_2u[k] - \epsilon[k] \\ \hat{y}[k] &= C_2\hat{x}[k] \\ \epsilon[k] &= \Psi_{opt}(\tilde{y}[k]) \end{cases} \quad (35)$$

Regarding a synthesis procedure for Ψ_{opt} , we note that the linear programming (LP) is employed [37], unlike the H_∞ -based synthesis procedure with the LMI-based arguments. However, the LP problem tackled in the l_1 synthesis procedure is quite difficult due to its intrinsic properties with respect to the infinite-dimensional cost function. Moreover, the process for deriving Ψ_{opt} is constructed in the frequency domain, not the time domain, in which the overall arguments of this paper are established. In this sense, we would like to briefly sketch the procedure for designing Ψ_{opt} in this section (and see [39], [40] for the details). To address this non-trivial task relevant to the infinite-dimensional properties, the duality theorem [38] is used in [39] and [40] to transform the primal problem with the infinite-dimensional cost function into the dual problem with an infinite number of constraints. Because most of these constraints could be characterized by decaying linear functions, it is shown in [39] and [40] that only a finite number of these constraints is required to be solved. In connection with this, various approximate methods such as truncation idea and delay augmentation are proposed, by which a suboptimal Ψ_{opt} can be obtained within an arbitrary degree of accuracy.

To summarize, the problem of an l_1 optimal DSE synthesis with respect to (34) can be characterized through an LP problem, and such an optimal one can be obtained by taking the truncation idea for infinitely many linear constraints within any degree of accuracy. Its practical effectiveness will also be discussed through some simulation results in the following section.

V. SIMULATION RESULTS

This section demonstrates the practical effectiveness of the proposed l_1 optimal DSE by comparing it to other DSEs discussed in Section III; note that it is impossible to design an UIO-based state estimator with respect to (9) since the feasibility condition (b) in Section III-A does not hold for C_i and $B_{d1,i}$.

A. SIMULATION DESCRIPTION

We first introduce the simulation environments. A three-area power system is adopted; each area is depicted by the power generating unit as shown in Fig. 1. The three areas are interconnected by the tie-lines, and the important model is collected from [1], [2], [22], [23], [32] as follows: $K_{g,i} = K_{t,i} = 1$, $T_{g,i} = 0.83$, $T_{t,i} = 0.3$, $M_i = 0.1667$, $D_i = 0.0083$, $R_i = 2.4$, $B_i = 0.425$, $T_{ij} = T_{ji} = 0.1634$, and $K_i = 0.1$ for $i, j = 1, 2, 3$. The load changes $\Delta P_{L,i}$ are assumed to occur with sudden increases/decreases in each area at arbitrary times and thus they are considered signals with a magnitude of up to 0.02 [p.u].

Since the overall arguments in this paper are initially equipped with a decentralized DSE, we further assume that each area is also operated with pre-designed, fully decentralized LFC schemes. In this assumption, the valve position change, mechanical power error, and area frequency error are estimated for each area through the three types of DSEs (i.e., the l_1 optimal, KF and H_∞ -based estimators). The overall systems are also assumed to be operated with the sampling period $h = 1$ [kHz] in a discrete-time fashion while the power generating systems operate in a continuous-time fashion. Hence, the simulations are conducted in a hybrid continuous/discrete-time fashion. The simulation experiment is performed in the following environments. CPU: Intel Core i7-9700F, RAM: 16.0 GB, and APP: MATLAB 2021b.

B. ANALYSIS OF RESULTS

The simulation results for the valve position deviation, turbine power deviation, and frequency deviation are shown in Figures 5–7, respectively. Furthermore, they are evaluated by taking the root-mean-square (RMS) and maximum magnitude values corresponding to the estimation errors, and the results are given in Table 1 and Table 2, respectively. The average computation times of the DSE algorithms for the three areas is given in Table 3. The computation time is measured for 30 [s], i.e., 30,000 repeated estimations.

It can be observed in Figures 5–7 that the H_∞ optimal DSE presents insufficient estimation accuracies compared to the other two methods. This tendency can be obviously

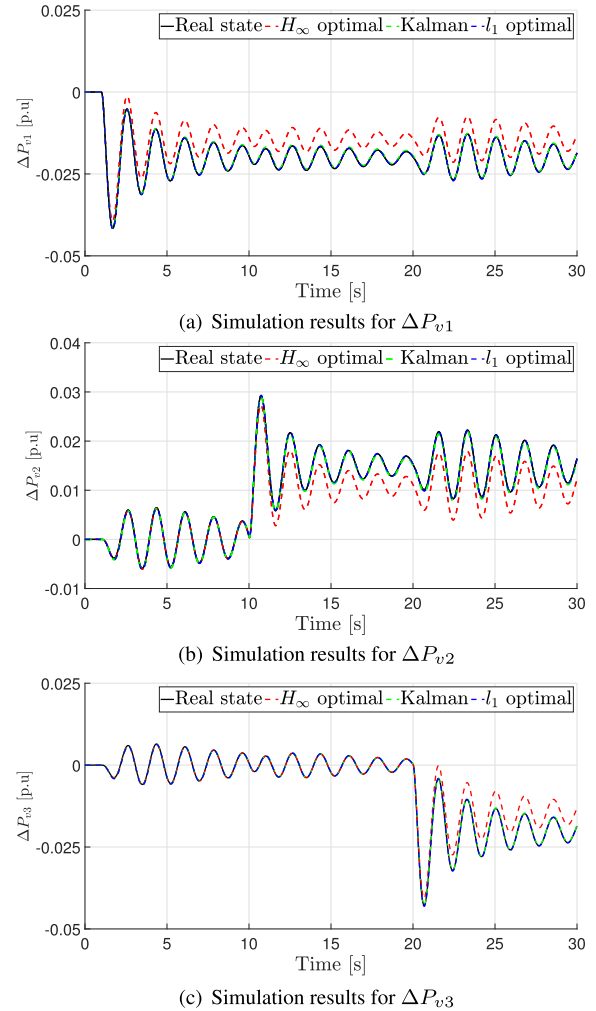


FIGURE 5. Simulation results for $P_{v,i}$ (black: real state, red: H_∞ optimal, green: Kalman filter, blue: l_1 optimal).

TABLE 1. Simulation results for the RMS of estimation errors.

	H_∞ optimal	KF	l_1 optimal
$\Delta P_{v,1}$	5.23×10^{-3}	2.63×10^{-4}	1.55×10^{-4}
$\Delta P_{v,2}$	3.27×10^{-3}	2.59×10^{-4}	1.54×10^{-4}
$\Delta P_{v,3}$	2.84×10^{-3}	1.59×10^{-4}	1.55×10^{-4}
$\Delta P_{t,1}$	3.73×10^{-3}	2.88×10^{-4}	1.45×10^{-4}
$\Delta P_{t,2}$	2.35×10^{-3}	2.67×10^{-4}	1.44×10^{-4}
$\Delta P_{t,3}$	1.98×10^{-3}	1.62×10^{-4}	1.45×10^{-4}
Δf_1	1.35×10^{-2}	1.10×10^{-3}	3.63×10^{-4}
Δf_2	8.46×10^{-3}	8.93×10^{-4}	3.68×10^{-4}
Δf_3	7.38×10^{-3}	6.01×10^{-4}	3.65×10^{-4}

observed in Table 1 and Table 2; the RMS and maximum magnitude values of the estimation errors in the H_∞ optimal DSE are more than an order of magnitude larger than those in the other two methods. Moreover, it can be shown in Figures 5 and 6 that the H_∞ optimal DSE cannot lead to

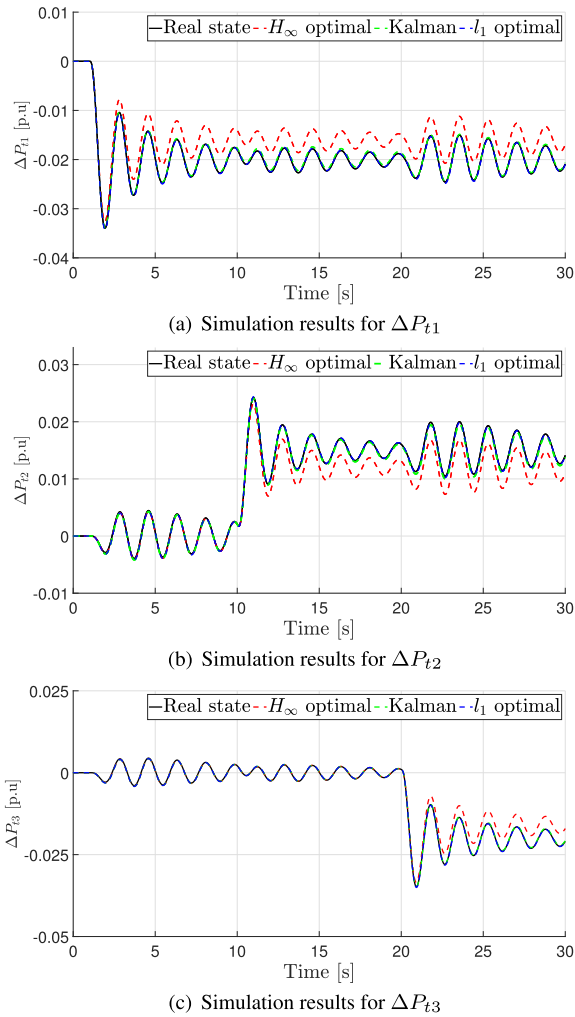


FIGURE 6. Simulation results for $P_{t,i}$ (black: real state, red: H_∞ optimal, green: Kalman filter, blue: l_1 optimal).

	H_∞ optimal	KF	l_1 optimal
$\Delta P_{v,1}$ [p.u.]	5.67×10^{-3}	5.09×10^{-4}	4.21×10^{-4}
$\Delta P_{v,2}$ [p.u.]	4.38×10^{-3}	6.04×10^{-4}	3.97×10^{-4}
$\Delta P_{v,3}$ [p.u.]	5.49×10^{-3}	3.91×10^{-4}	4.10×10^{-4}
$\Delta P_{t,1}$ [p.u.]	4.12×10^{-3}	5.13×10^{-4}	3.55×10^{-4}
$\Delta P_{t,2}$ [p.u.]	3.22×10^{-3}	5.71×10^{-4}	3.40×10^{-4}
$\Delta P_{t,3}$ [p.u.]	3.94×10^{-3}	3.37×10^{-4}	3.40×10^{-4}
Δf_1 [Hz]	1.49×10^{-2}	1.74×10^{-3}	9.39×10^{-4}
Δf_2 [Hz]	1.11×10^{-2}	1.84×10^{-3}	9.50×10^{-4}
Δf_3 [Hz]	1.42×10^{-2}	1.21×10^{-3}	9.50×10^{-4}

TABLE 2. Simulation results for the maximum magnitude of estimation errors.

accurate estimation values, although $\Delta P_{v,i}$ and $\Delta P_{t,i}$ are not directly affected by the disturbances. Indeed, the estimated values for Δf_i with the H_∞ optimal DSE in Figure 7 do not reach the real values. The reason why the H_∞ optimal DSE derives the aforementioned low accuracies might be

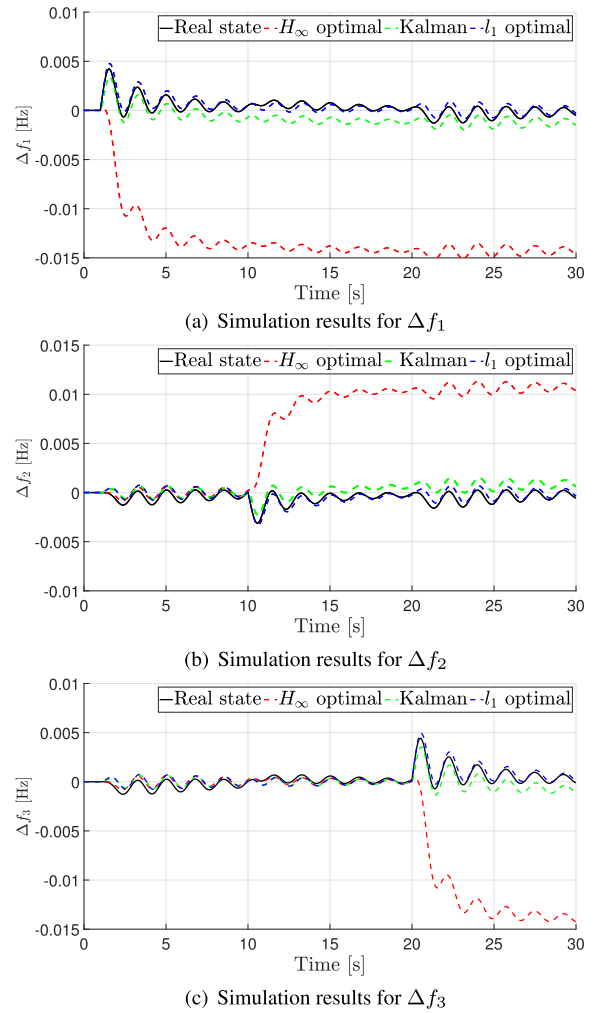


FIGURE 7. Simulation results for f_i (black: real state, red: H_∞ optimal, green: Kalman filter, blue: l_1 optimal).

TABLE 3. The average computation times in simulations for the three areas.

	H_∞ optimal	KF	l_1 optimal
Computation time	0.0123 [s]	0.0983 [s]	0.0136 [s]

interpreted as that it aims at minimizing the estimation errors corresponding to the energy-bounded disturbance, but the load changes in power systems cannot be generally characterized by such energy-bounded signals. Thus, even though one might argue that the H_∞ optimal DSE becomes an effective candidate for a state estimator for LFC of power systems, its practical improvement is still left for an important future topic.

On the other hand, we can observe in Figures 5–7 and Tables 1 and 2 that the KF and l_1 optimal DSE lead to much higher estimation accuracies than the H_∞ DSE. The estimation accuracies of both methods associated with $\Delta P_{v,i}$ and $\Delta P_{t,i}$ are similar to each other, as observed in Figures 5 and 6. However, we can observe from Figure 7

that the KF does not accurately estimate the frequencies as much as the l_1 optimal DSE. This tendency is also obvious in Tables 1 and 2, in which the RMS and maximum magnitude values of the Δf_i estimation errors for the l_1 optimal DSE are reduced up to 67% and 48% from 40% and 21%, respectively compared to the KF.

Such low estimation accuracies of the KF could be interpreted as occurring from the fact that the disturbance $w_i = [\Delta P_{L,i} \Delta P_{tie,i}]^T$ cannot be characterized by the zero means and Gaussian distributions. However, the proposed l_1 optimal DSE presents high performances of estimation accuracies because this approach is based on the arguments of l_1 optimal control, by which the maximum magnitude of estimation errors corresponding to bounded persistent disturbances is minimized. Furthermore, it is also worth noting that the l_1 optimal DSE is performed without any pre-given information such as disturbance and noise covariance matrices, whose explicit values cannot be obtained. In contrast, the KF intrinsically requires them. Moreover, the KF requires more computational cost than the l_1 optimal DSE since the KF consists of two update steps which include an inversion and a multiplication of matrices. Indeed, the computation time of the l_1 optimal DSE is about 13% of that of the KF, as seen in Table 3. These observations indicate that the proposed l_1 optimal DSE can perform as an effective tool for the decentralized DSE for the LFC of power systems.

VI. CONCLUSION

This paper proposed a decentralized l_1 optimal dynamic state estimator (DSE) for the load frequency control (LFC) of power systems and provided its comparative study to other DSEs with the unknown input observer (UIO) [28], Kalman filter (KF) [29] and H_∞ optimal approach [14]. To do this, we first considered decentralized systems consisting of power generating units connected by the tie-lines, and described their dynamic model with respect to the single power generating unit. Noting the fact that it is generally quite difficult to obtain characteristics of the disturbances in the decentralized LFC of power systems, we presumed only the time domain boundness of the disturbances. With respect to state estimations for LFC of power systems even with the existence of the disturbances, we introduced the four candidates for decentralized DSEs: Unknown input observer (UIO) [28], Kalman filter (KF) [29], H_∞ optimal DSE, and l_1 optimal DSE. More precisely, some limitations of the former three decentralized DSEs (i.e., UIO, KF and H_∞ optimal DSE) occurring from their intrinsic nature were deeply discussed, and the l_1 optimal DSE was proposed as the most effective state estimator for LFC of power systems. This conjecture is based on the fact that the maximum magnitude of the estimation errors for bounded persistent disturbances is minimized by using the l_1 optimal DSEs. Finally, we demonstrated the practical validity and effectiveness of the l_1 optimal DSE through comparative simulation results with a three-area power generating system.

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