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RESEARCH ARTICLE

Absolute Exponential Stability for Switching Positive Systems With Time-Delay Based on the Φ-Dependent ADT Strategy

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ABSTRACT The absolute exponential stability problem for the continuous-time and discrete-time, respectively, switching positive nonlinear time-delay system is investigated under the nonlinearity of systems satisfying a certain sector condition in this paper. Two multiple co-positive Lyapunov-Krasovskii functionals (MCLKF) of such systems are constructed for the continuous-time and discrete-time contexts respectively. By using the MCLKF and the Φ-dependent average dwell time switching strategy, more general stability conditions compatible with some existing works are established. Furthermore, the obtained results are generalized to other types of systems. A numerical example, finally, implies the validity and significance of the results proposed.

INDEX TERMS Switching positive systems, absolute exponential stability, average dwell time, Lyapunov-Krasovskii functional.

I. INTRODUCTION

In the last few decades, switching positive systems, as one special kind of switching systems, have attracted considerable attention from investigators due to their importance in both control theory and engineering. Because of the positivity, there are some unusual features for switching positive systems, for instance, the state trajectory is maintained in the positive quadrant and its boundary whenever the initial condition is nonnegative, it has a unique research method: linear co-positive Lyapunov functions method. One can refer to [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], and [11] for more details about switching positive systems. Most of these works are related to switching positive linear systems (SPLSs) and those conditions of the system stability are based on either the common or multiple co-positive Lyapunov functions methods, which are tractable and resolvable in terms of the linear matrix inequalities toolbox. Switching positive nonlinear systems (SPNSs) also have a wide range of applications in chemistry, population dynamics, bioengineering, and other fields [11], [12]. However, there are few results for the stability study of SPNSs, which makes sense to study the stability of SPNSs. The investigation of such systems is full of challenges, it is mainly manifested twofold. First, there is no unified and effective method to deal with the nonlinearity of systems. Second, it is hard to construct a suitable Lyapunov function for SPNSs.

Time-delay phenomena are widespread in practice, and there exist many results to study the stability of switching time-delay systems [2], [8], [9], [10], [11]. What follows is a list of a few. Stability conditions for nonnegative and compartmental dynamical systems with constant delay are obtained in [13]. Recently, in [14], the authors investigate the absolute exponential stability problem of SPNSs with time-delay based on the MCLKF method and ADT switching strategy, then obtain some stability results. At the same time, the concept of absolute exponential stability is usually utilized for some nonlinear systems, such as recurrent neural networks [15], [16], and variable structure systems [17].

On the other hand, most of the stability results for switching systems are based on restricted switching strategies, including ADT [6], [14], [18] switching, mode-dependent ADT (MDADT) [3], [7], [19] switching, Φ-dependent ADT (ΦDADT) [5], [20], [21], [22], [23] switching and so on. And

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it is worth mentioning that the ΦDADT switching provides a unified form of ADT and MDADT switchings [24]. So this paper will investigate the absolute exponential stability of switching positive nonlinear time-delay systems (SPNTSs) under the ΦDADT switching strategy.

Here, the main contributions can be listed as follows:

(i) A MCLKF is constructed for SPNTSs, where the MCLKF is less conservative than the common co-positive Lyapunov-Krasovskii functional;

(ii) The ΦDADT switching strategy is used for the first time to investigate the stability of SPNSs. Under the MCLKF and new strategy, the absolute exponential stability of the considered system is ensured, which is more general than those based on ADT and MDADT ones;

(iii) The obtained results are not only applicable to positive systems but also can be extended to non-positive cases.

The structure of the text is to make the following arrangements. Section 2 provides system descriptions and some preliminaries. In Section 3, some results of absolute exponential stability for SPNTSs in both continuous-time and discrete-time contexts are given, respectively, and they also are generalized to non-positive systems. A typical example is presented in Section 4, which shows the effectiveness of the obtained results. Section 5 summarizes the whole paper.

Notations: $\mathcal{R}$ represents the set of real numbers. $\mathcal{N}$ ($\mathcal{N}_+$) is the set of nonnegative (positive) integers. $\mathcal{R}^{n\times n}$ ($\mathcal{R}^n$ resp.,) is the space of real $n\times n$ matrices (real n-dimensional vectors resp.) $\|\cdot\|$ stands for the Euclidian norm. The transpose of matrix A is marked as $A^\top$, and the element in the ith row and the jth column of matrix A is denoted by a_{ij}. v_i denotes the ith component of $v \in \mathcal{R}^n$ and $\nmid v \nmid \triangleq (|v_1|, |v_2|, \cdots, |v_n|)^\top$. $v \succeq 0$ ($v \succ 0$ resp.) implies that every component of v being nonnegative (positive resp.), i.e., $v_i \geq 0$ ($v_i > 0$ resp.) $D^+f(x)$ represents the right Dini derivatives of function $f(x)$. $A \succeq 0$ denotes the matrix A is nonnegative (that is each element is nonnegative), and a matrix is Metzler means that all off-diagonal elements are non-negative.

II. SYSTEM DESCRIPTIONS AND PRELIMINARIES

Consider the switching positive nonlinear time-delay system (SPNTS)

$$\wedge x(t) = A_{(\varrho(t))}f(x(t)) + B_{(\varrho(t))}f(x(t-h)),\ t \geq 0, \quad (1)$$

where $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^\top \in \mathcal{R}^n$ is the system state, and h denotes the time delay. $\varrho(t)$ represents the switching signal, it takes value from the set $\mathbf{L} = \{1, 2, 3, \cdots, \mathrm{N}\}$, $\mathrm{N} \in \mathcal{N}_+$ and is right continuous for the given switching sequence $t_0 < t_1 < t_2 < \cdots$. The symbol $\wedge x(t)$ means $\dot{x}(t)$ ($x(t+1)$ resp.) for the continuous-time (discrete-time resp.) context. For the continuous-time SPNTS, we assume $A_{(p)}$ is Metzler, $B_{(p)} \succeq 0, p \in \mathbf{L}$, initial condition $x(t_0) = \varphi(t_0)$, $t_0 \in [-h, 0]$, where $\varphi(t_0) \in \mathcal{R}^n$ is continuous on $[-h, 0]$. For the discrete-time SPNTS, we assume $A_{(p)} \succeq 0$ and $B_{(p)} \succeq 0, p \in \mathbf{L}$, initial condition $x(t_0) = \varphi(t_0)$, $t_0 = -h, -h+1, \cdots, 0$. The nonlinear term $f(x(t)) = (f_1(x_1(t)), \cdots, f_n(x_n(t)))^\top \in \mathcal{R}^n$ satisfies $x_i(t)f_i(x_i(t)) > 0$, at the same time, $f(x(t))$ also meets, for given two positive γ, δ and $\gamma < \delta$,

$$\gamma\vartheta^2 \leq \vartheta f_i(\vartheta) \leq \delta\vartheta^2, \forall\vartheta \in \mathcal{R}, f_i(0) = 0, i = 1, 2, \cdots, n, \quad (2)$$

where (2) is called the sector condition. It is assumed that $f_i(x_i(t))$ satisfies some Lipschitz conditions to ensure the existence and uniqueness of the system's solution.

Next, two necessary definitions and a lemma are stated as follows.

Let $\mathbf{G} = \{1,2,\cdots,r\}$ and a surjection operator Φ: $\mathbf{L} \to \mathbf{G}$, where $r \in \mathcal{N}_+$ and $1 \leq r \leq \mathrm{N}$. Denote $\Phi^j = \{q \in \mathbf{L}|\Phi(q) = j, j \in \mathbf{G}\}$.

Definition 1 [20]: For any interval $[a, b)$, $a > b \geq t_0$, let $\mathrm{T}_{\Phi^j}(b, a)$ and $\mathrm{N}_{\varrho\Phi^j}(b, a)$ denote the total activated time and switching numbers of subsystems $\Phi^j \subseteq \mathbf{L}$, if there are two constants $\tau_{a\Phi^j} > 0$ and $\mathrm{N}_{0\Phi^j} > 0$ meeting

$$\mathrm{N}_{\varrho\Phi^j}(b, a) \leq \mathrm{N}_{0\Phi^j} + \frac{\mathrm{T}_{\Phi^j}(b, a)}{\tau_{a\Phi^j}}, \quad (3)$$

then $\varrho(t)$ is saied to have the Φ-dependent average dwell time (ΦDADT) $\tau_{a\Phi^j}$.

Definition 2 [2], [14]: The continuous-time (discrete-time resp.) system (1) with (2) is absolute exponential stable (AES) under given switching signals if, for any admissible nonlinearity $f(x(t))$, there are two positive constants α, β (ν, $\varepsilon < 1$ resp.) satisfying the system's trajectory has, for all $t \geq t_0$,

$$\|x(t)\| \leq \alpha e^{-\beta(t-t_0)}\|\varphi(t_0)\|_{\sup}$$
$$(\|x(t)\| \leq \nu\varepsilon^{t-t_0}\|\varphi(t_0)\|_{\sup} \text{ resp.})$$

where

$$\|\varphi(t_0)\|_{\sup} = \sup_{-h\leq t_0\leq 0}\|x(t_0)\|$$
$$(\|\varphi(t_0)\|_{\sup} = \sup_{t_0\in\{-h,-h+1,\cdots,0\}}\|x(t_0)\| \text{ resp.})$$

The system (1-2) is called to be positive for all switching signals if the state trajectory is nonnegative for any $\varphi(t_0) \succeq 0$.

Lemma 1 [25]: The continuous-time (discrete-time resp.) system $\dot{x}(t) = Ax(t) + r(t)$ ($x(t+1) = Ax(t) + r(t)$ resp.) is positive if A is Metzler ($A \succeq 0$ resp.) and $r(t) \succeq 0$.

III. MAIN RESULTS

This section contains two subsections. One investigates the stability problem of the continuous-time SPNSs and the other studies the corresponding problem of the discrete-time SPNSs.

A. CONTINUOUS-TIME CASE

Theorem 1: Consider the continuous-time SPNTS (1-2) with the nonnegative initial condition. If there are scalars $\psi_{(i)} > 0$, $\xi_{(i)} \geq 1$ and nonnegative vectors $v_{(p)}$ and $\varsigma_{(p)} \in \mathcal{R}^n$ meeting $\forall p \in \mathbf{L}$, $\Phi(p) = i \in \mathbf{G}$,

$$A_{(p)}^\top v_{(p)} + \frac{1}{\gamma}\psi_{(i)}v_{(p)} \preceq -\varsigma_{(p)}, \quad (4)$$

$$B_{(p)}^\top v_{(p)} \preceq e^{-\psi_{(i)}h}\varsigma_{(p)}, \quad (5)$$

$$v_{(p)} \preceq \xi_{(i)} v_{(q)}, \tag{6}$$
$$\varsigma_{(p)} \preceq \xi_{(i)} \varsigma_{(q)}, \tag{7}$$

for $\forall p \neq q$, and $p, q \in \mathbf{L}$. Then the system (1-2) is AES under the ΦDADT

$$\tau_{a\Phi^i} > \tau^*_{a\Phi^i} = \frac{\ln \xi_{(i)}}{\psi_{(i)}}. \tag{8}$$

Proof: We select the MCLKF:

$$V_{(\varrho(t))}(x) = x^\top v_{(\varrho(t))} + \int_{t-h}^{t} e^{\psi_{(\Phi(\varrho(t)))}(-t+s)} f^\top(x(s))\varsigma_{(\varrho(t))} ds, \tag{9}$$

where $\psi_{(\Phi(\varrho(t)))} > 0$, $v_{(\varrho(t))} \succ 0$, $\varsigma_{(\varrho(t))} \succ 0$. Here, $x(t)$ is abbreviated to x.

Suppose the switching points in order are as follows $t_1, t_2, \cdots, t_l, t_{l+1}, \cdots t_{\mathrm{N}_\varrho(T,0)}$ on $[0, T)$ with $T > 0$. $\mathrm{N}_\varrho(T, 0)$ denotes the total switching numbers on $[0, T)$.

For $t \in [t_l, t_{l+1})$, the time derivative of $V_{(\varrho(t))}(x)$ with system (1-2) can be expressed as follows.

$$\begin{aligned}\dot{V}_{(\varrho(t))}(x) &= -\psi_{(\Phi(\varrho(t)))} V_{(\varrho(t))}(x) + \psi_{(\Phi(\varrho(t)))} x^\top v_{(\varrho(t))} \\ &\quad + f^\top(x(t-h))(B^\top_{(\varrho(t))} v_{(\varrho(t))} - e^{-\psi_{(\Phi(\varrho(t)))}h} \varsigma_{(\varrho(t))}) \\ &\quad + f^\top(x)(A^\top_{(\varrho(t))} v_{(\varrho(t))} + \varsigma_{(\varrho(t))}).\end{aligned}$$

By $x \succeq 0$ and inequality (2), it yields that

$$\frac{1}{\delta} f_i(x_i) \leq x_i \leq \frac{1}{\gamma} f_i(x_i).$$

Then,

$$\frac{1}{\delta} f^\top(x) \preceq x^\top \preceq \frac{1}{\gamma} f^\top(x).$$

Thus, we can get

$$\begin{aligned}\dot{V}_{(\varrho(t))}(x) &\leq f^\top(x)(\frac{1}{\gamma}\psi_{(\Phi(\varrho(t)))} v_{(\varrho(t))} + A^\top_{(\varrho(t))} v_{(\varrho(t))} + \varsigma_{(\varrho(t))}) \\ &\quad + f^\top(x(t-h))(B^\top_{(\varrho(t))} v_{(\varrho(t))} - e^{-\psi_{(\Phi(\varrho(t)))}h} \varsigma_{(\varrho(t))}) \\ &\quad - \psi_{(\Phi(\varrho(t)))} V_{(\varrho(t))}(x).\end{aligned}$$

Using (4) and (5),

$$\dot{V}_{(\varrho(t))}(x) \leq -\psi_{(\Phi(\varrho(t)))} V_{(\varrho(t))}(x), \ t \in [t_l, t_{l+1}), \tag{10}$$

i.e.

$$V_{(\varrho(t))}(x) \leq e^{-\psi_{(\Phi(\varrho(t)))}(t-t_l)} V_{(\varrho(t_l))}(x(t_l)). \tag{11}$$

By (6) and (7),

$$V_{(\varrho(t_l))}(x(t_l)) \leq \xi_{(\Phi(\varrho(t_l)))} V_{(\varrho(t_l^-))}(x(t_l^-)). \tag{12}$$

Combining (11) and (12),

$$V_{(\varrho(t))}(x) \leq e^{-\psi_{(\Phi(\varrho(t)))}(t-t_l)} \xi_{(\Phi(\varrho(t_l)))} V_{(\varrho(t_l^-))}(x(t_l^-)). \tag{13}$$

Repeating (11-13),

$$\begin{aligned}V_{(\varrho(t_{\mathrm{N}_\varrho}))}(x(T)) &\leq \xi_{(\Phi(\varrho(t_{\mathrm{N}_\varrho})))} e^{-\psi_{(\Phi(\varrho(t_{\mathrm{N}_\varrho})))}(T-t_{\mathrm{N}_\varrho})} V_{(\varrho(t^-_{\mathrm{N}_\varrho}))}(x(t^-_{\mathrm{N}_\varrho})) \\ &\leq \xi_{(\Phi(\varrho(t_{\mathrm{N}_\varrho})))} \xi_{(\Phi(\varrho(t_{\mathrm{N}_\varrho -1})))} e^{-\psi_{(\Phi(\varrho(t_{\mathrm{N}_\varrho})))}(T-t_{\mathrm{N}_\varrho})} \\ &\quad \times e^{-\psi_{(\Phi(\varrho(t_{\mathrm{N}_\varrho -1})))}(t_{\mathrm{N}_\varrho} - t_{\mathrm{N}_\varrho -1})} V_{(\varrho(t^-_{\mathrm{N}_\varrho -1}))}(x(t^-_{\mathrm{N}_\varrho -1})) \\ &\leq \cdots \leq \xi_{(\Phi(\varrho(t_{\mathrm{N}_\varrho})))} \xi_{(\Phi(\varrho(t_{\mathrm{N}_\varrho -1})))} \cdots \xi_{(\Phi(\varrho(t_1)))} \\ &\quad \times e^{-\psi_{(\Phi(\varrho(t_{\mathrm{N}_\varrho})))}(T-t_{\mathrm{N}_\varrho})} e^{-\psi_{(\Phi(\varrho(t_{\mathrm{N}_\varrho -1})))}(t_{\mathrm{N}_\varrho} - t_{\mathrm{N}_\varrho -1})} \cdots \\ &\quad \times e^{-\psi_{(\Phi(\varrho(0)))}(t_1 - 0)} V_{(\varrho(0))}(x(0)) \\ &= \prod_{i=1}^{r} \xi_{(i)}^{\mathrm{N}_i} e^{-\psi_{(\Phi(\varrho(t_{\mathrm{N}_\varrho})))}(T-t_{\mathrm{N}_\varrho})} e^{-\sum_{m=1}^{\mathrm{N}_\varrho -1} \psi_{(\Phi(\varrho(t_m)))}(t_{m+1}-t_m)} \\ &\quad \times e^{-\psi_{(\Phi(\varrho(0)))}(t_1 - 0)} V_{(\varrho(0))}(x(0)) \\ &= e^{-\sum_{i=1}^{r} \psi_{(i)} (\sum_{\varrho(t_j)\in\Phi^i} (t_{j+1}-t_j))} \prod_{i=1}^{r} \xi_{(i)}^{\mathrm{N}_i} V_{(\varrho(0))}(x(0)) \\ &= \prod_{i=1}^{r} (\xi_{(i)}^{\mathrm{N}_i} e^{-\psi_{(i)}\mathrm{T}_i}) V_{(\varrho(0))}(x(0)),\end{aligned} \tag{14}$$

where $\mathrm{N}_\varrho \triangleq \mathrm{N}_\varrho(T, 0)$, $\mathrm{N}_i \triangleq \mathrm{N}_{\varrho\Phi^i}(T, 0)$, $\mathrm{T}_i \triangleq \mathrm{T}_{\Phi^i}(T, 0)$, $i \in \mathbf{G}$, clearly, $\sum_{i=1}^{r} \mathrm{N}_i = \mathrm{N}_\varrho$, and $\sum_{i=1}^{r} \mathrm{T}_i = T$.
From (14), it follows

$$\begin{aligned}V_{(\varrho(t_{\mathrm{N}_\varrho}))}(x(T)) &\leq \prod_{i=1}^{r} (\xi_{(i)}^{\mathrm{N}_i} e^{-\psi_{(i)}\mathrm{T}_i}) V_{(\varrho(0))}(x(0)) \\ &= \prod_{i=1}^{r} e^{\mathrm{N}_i \ln \xi_{(i)} - \psi_{(i)}\mathrm{T}_i} V_{(\varrho(0))}(x(0)).\end{aligned}$$

By the ΦDADT definition,

$$\begin{aligned}V_{(\varrho(t_{\mathrm{N}_\varrho}))}(x(T)) &\leq \prod_{i=1}^{r} e^{\mathrm{N}_{0\Phi^i} \ln \xi_{(i)} + (\frac{\ln \xi_{(i)}}{\tau_{a\Phi^i}} - \psi_{(i)})\mathrm{T}_i} V_{(\varrho(0))}(x(0)) \\ &\leq c e^{\max_{i\in\mathbf{G}} (\frac{\ln \xi_{(i)}}{\tau_{a\Phi^i}} - \psi_{(i)})T} V_{(\varrho(0))}(x(0)),\end{aligned}$$

where $c = \prod_{i=1}^{r} e^{\mathrm{N}_{0\Phi^i} \ln \xi_{(i)}}$. Let $k_1 = \min_{p\in\mathbf{L}}\{(v_{(p)})_i, i = 1, 2, \cdots, n\}$, $k_2 = \max_{p\in\mathbf{L}}\{(v_{(p)})_i, i = 1, 2, \cdots, n\}$, $k_3 = \max_{p\in\mathbf{L}}\{(\varsigma_{(p)})_i, i = 1, 2, \cdots, n\}$, from the selection of MCLKF, it is easy to know

$$V_{(\varrho(t_{\mathrm{N}_\varrho}))}(x(T)) \geq k_1 \sum_{i=1}^{n} x_i(T),$$
$$V_{(\varrho(0))}(x(0)) \leq k_2 \sum_{i=1}^{n} x_i(0) + k_3 \delta \int_{-h}^{0} \sum_{i=1}^{n} x_i(s) ds.$$

Therefore,

$$\begin{aligned}\sum_{i=1}^{n} x_i(T) &\leq \frac{c}{k_1} e^{\max_{i\in\mathbf{G}} (\frac{\ln \xi_{(i)}}{\tau_{a\Phi^i}} - \psi_{(i)})T} \\ &\quad \times \left(k_2 \sum_{i=1}^{n} x_i(0) + k_3 \delta \int_{-h}^{0} \sum_{i=1}^{n} x_i(s) ds \right).\end{aligned}$$

By $\|x(T)\| \leq \sum_{i=1}^{n} x_i(T) \leq \sqrt{n}\|x(T)\|$, it can be obtained that

$$\|x(T)\| \leq \frac{c}{k_1} e^{\max_{i\in\mathbf{G}}(\frac{\ln \xi_{(i)}}{\tau_{a\Phi^i}} - \psi_{(i)})T} \times \left(k_2\sqrt{n}\|x(0)\| + k_3\sqrt{n}\delta \int_{-h}^{0} \|x(s)\| ds \right).$$

Let $\alpha = \frac{c}{k_1}(k_2+k_3\delta h)\sqrt{n}$, $\beta = \max_{i\in\mathbf{G}}(\frac{\ln \xi_{(i)}}{\tau_{a\Phi^i}} - \psi_{(i)})$. Thus,

$$\|x(T)\| \leq \alpha e^{\beta T} \|\varphi\|_{\sup},$$

where $\|\varphi\|_{\sup} = \sup_{-h\leq t\leq 0} \|x(t)\|$.

By $\beta < 0$ and $\alpha > 0$, the system (1-2) is AES for the ΦDADT (8). □

Remark 1: One can get Theorem 1 in [14] by taking $\mathbf{G} = \{1\}$ in the above theorem. When $\mathbf{G} = \mathbf{L}$ and $\Phi(i) = i$ ($\forall i \in \mathbf{L}$) in Theorem 1, the stability results based on the MDADT strategy can be obtained.

Remark 2: The above theorem is about the stability of positive systems, i.e., system matrices $A_{(p)}$ are Metzler and $B_{(p)} \succeq 0$. While the above approach is general, i.e., it is also valid for non-positive systems. The fact can be seen from the following theorem.

Consider the continuous-time system

$$\begin{aligned} \dot{x}(t) &= A_{(\varrho(t))} f(x(t)) + B_{(\varrho(t))} f(x(t-h)), t \geq 0, \\ x(t_0) &= \varphi(t_0), t_0 \in [-h, 0]. \end{aligned} \tag{15}$$

Here $A_{(p)} \in \mathcal{R}^{n\times n}$ and $B_{(p)} \in \mathcal{R}^{n\times n}$. Other parameters and conditions are the same as the system (1-2). Let $A_{(p)} = [(a_{(p)})_{ij}]$, $B_{(p)} = [(b_{(p)})_{ij}]$, $\overline{A}_{(p)} = [(\overline{a}_{(p)})_{ij}]$, $\overline{B}_{(p)} = [(\overline{b}_{(p)})_{ij}]$ with

$$(\overline{a}_{(p)})_{ij} = \begin{cases} (a_{(p)})_{ij}, & i = j; \\ |(a_{(p)})_{ij}|, & i \neq j; \end{cases} \quad (\overline{b}_{(p)})_{ij} = |(b_{(p)})_{ij}|.$$

Theorem 2: Consider the system (15) with (2). If there exist constants $\psi_{(i)} > 0, \xi_{(i)} \geq 1$ and nonnegative vectors $v_{(p)}, \varsigma_{(p)} \in \mathcal{R}^n$, meeting $\forall p \in \mathbf{L}$, $\Phi(p) = i \in \mathbf{G}$,

$$\overline{A}_{(p)}^{\top} v_{(p)} + \frac{1}{\gamma}\psi_{(i)} v_{(p)} \preceq -\varsigma_{(p)}, \tag{16}$$

$$\overline{B}_{(p)}^{\top} v_{(p)} \preceq e^{-\psi_{(i)}h} \varsigma_{(p)}, \tag{17}$$

$$v_{(p)} \preceq \xi_{(i)} v_{(q)}, \tag{18}$$

$$\varsigma_{(p)} \preceq \xi_{(i)} \varsigma_{(q)}, \tag{19}$$

for $\forall p \neq q$, and $p, q \in \mathbf{L}$. Then the system (15) with (2) is AES for the ΦDADT (8).

Proof: We slightly modify the MCLKF (9):

$$V_{(\varrho(t))}(x) = \sum_{i=1}^{n} \Big\{ |x_i(t)|(v_{(\varrho(t))})_i + \int_{t-h}^{t} e^{\psi_{(\Phi(\varrho(t)))}(-t+s)} |f_i(x_i(s))|(\varsigma_{(\varrho(t))})_i ds \Big\}, \tag{20}$$

where $v_{(\varrho(t))} = ((v_{(\varrho(t))})_1, \cdots (v_{(\varrho(t))})_n)^{\top} \succ 0$, $\varsigma_{(\varrho(t))} = ((\varsigma_{(\varrho(t))})_1, \cdots, (\varsigma_{(\varrho(t))})_n)^{\top} \succ 0$, $\psi_{(\Phi(\varrho(t)))} > 0$. The switching points are given in Theorem 1. For $t \in [t_l, t_{l+1})$, the right Dini derivative of (20) is obtained as follows:

$$\begin{aligned} &D^+V_{(\varrho(t))}(x) \\ &= \sum_{i=1}^{n} \Big\{ \Big(\sum_{j=1}^{n} \big((a_{(\varrho(t))})_{ij} f_j(x_j(t)) + (b_{(\varrho(t))})_{ij} f_j(x_j(t-h))\big)\Big) \\ &\quad \times sign((D^+x_i(t))(v_{(\varrho(t))})_i \\ &\quad - e^{-\psi_{(\Phi(\varrho(t)))}h} |f_i(x_i(t-h))|(\varsigma_{(\varrho(t))})_i \\ &\quad - \psi_{(\Phi(\varrho(t)))} \int_{t-h}^{t} e^{\psi_{(\Phi(\varrho(t)))}(-t+s)} |f_i(x_i(s))|(\varsigma_{(\varrho(t))})_i ds \\ &\quad + |f_i(x_i(t))|(\varsigma_{(\varrho(t))})_i \Big\} \\ &\leq -\psi_{(\Phi(\varrho(t)))} V_{(\varrho(t))}(x) + \sum_{i=1}^{n} \Big\{ \psi_{(\Phi(\varrho(t)))} |x_i(t)|(v_{(\varrho(t))})_i \\ &\quad + \sum_{j=1}^{n} \big((\overline{a}_{(\varrho(t))})_{ij} |f_j(x_j(t))|(v_{(\varrho(t))})_i \big) + |f_i(x_i(t))|(\varsigma_{(\varrho(t))})_i \\ &\quad + \sum_{j=1}^{n} \big((\overline{b}_{(\varrho(t))})_{ij} |f_j(x_j(t-h))|(v_{(\varrho(t))})_i \big) \\ &\quad - e^{-\psi_{(\Phi(\varrho(t)))}h} |f_i(x_i(t-h))|(\varsigma_{(\varrho(t))})_i \Big\}. \end{aligned}$$

By the inequality (2), it follows that

$$\frac{1}{\delta}|f_i(x_i)| \leq |x_i| \leq \frac{1}{\gamma}|f_i(x_i)|.$$

Then,

$$\begin{aligned} &D^+V_{(\varrho(t))}(x) \\ &\leq \dagger f^{\top}(x) \dagger \Big\{ \overline{A}_{(\varrho(t))}^{\top} v_{(\varrho(t))} + \frac{1}{\gamma}\psi_{(\Phi(\varrho(t)))} v_{(\varrho(t))} + \varsigma_{(\varrho(t))} \Big\} \\ &\quad + \dagger f^{\top}(x(t-h)) \dagger \Big\{ \overline{B}_{(\varrho(t))}^{\top} v_{(\varrho(t))} - e^{-\psi_{(\Phi(\varrho(t)))}h} \varsigma_{(\varrho(t))} \Big\} \\ &\quad - \psi_{(\Phi(\varrho(t)))} V_{(\varrho(t))}(x(t)). \end{aligned}$$

Using(16) and (17), one has

$$D^+V_{(\varrho(t))}(x) \leq -\psi_{(\Phi(\varrho(t)))} V_{(\varrho(t))}(x).$$

The rest is similar to that in Theorem 1, so it is omitted. □

B. DISCRETE-TIME CASE

Theorem 3: Consider the discrete-time SPNTS (1-2) with the nonnegative initial condition. If there exist constants $0 < \psi_{(i)} < 1, \xi_{(i)} \geq 1$ and vectors $v_{(p)} \succeq 0, v_{(p)} \in \mathcal{R}^n, \varsigma_{(p)} \succeq 0, \varsigma_{(p)} \in \mathcal{R}^n$ such that $\forall p \in \mathbf{L}$, $\Phi(p) = i \in \mathbf{G}$,

$$A_{(p)}^{\top} v_{(p)} - \frac{\psi_{(i)}}{\delta} v_{(p)} \preceq -\psi_{(i)} \varsigma_{(p)}, \tag{21}$$

$$B_{(p)}^{\top} v_{(p)} \preceq \psi_{(i)}^{1+h} \varsigma_{(p)}, \tag{22}$$

$$v_{(p)} - \xi_{(i)} v_{(q)} \preceq 0, \tag{23}$$

$$\varsigma_{(p)} - \xi_{(i)} \varsigma_{(q)} \preceq 0, \tag{24}$$

for $\forall p \neq q, p, q \in \mathbf{L}$. Then the system (1-2) is AES for the ΦDADT

$$\tau_{a\Phi^i} > \tau_{a\Phi^i}^* = -\frac{\ln \xi_{(i)}}{\ln \psi_{(i)}}. \tag{25}$$

Proof: We select the following MCLKF:

$$V_{(\varrho(k))}(x(k)) = x^{\top}(k)v_{(\varrho(k))} + \sum_{s=k-h}^{k-1} \psi^{k-s}_{(\Phi(\varrho(k)))} f^{\top}(x(s))\varsigma_{(\varrho(k))}, \quad (26)$$

where $0 < \psi_{(\Phi(\varrho(k)))} < 1$, $v_{(\varrho(k))} \succ 0$, $\varsigma_{(\varrho(k))} \succ 0$.

Denote the switching instances in the interval $[0, K)$ are $k_1, k_2, \cdots, k_t, k_{t+1}, \cdots, k_{N_\varrho(K,0)}$, and $0 < k_1 < k_2 < \cdots < k_{N_\varrho(K,0)} < K$, where $N_\varrho(K, 0)$ denotes the total switching numbers in $[0, K)$.

For $k \in [k_t, k_{t+1}]$,

$$\begin{aligned} &V_{(\varrho(k))}(x(k+1)) - \psi_{(\Phi(\varrho(k)))}V_{(\varrho(k))}(x(k)) \\ &= f^{\top}(x(k))A^{\top}_{(\varrho(k))}v_{(\varrho(k))} + f^{\top}(x(k-h))B^{\top}_{(\varrho(k))}v_{(\varrho(k))} \\ &\quad + \psi_{(\Phi(\varrho(k)))}f^{\top}(x(k))\varsigma_{(\varrho(k))} - \psi^{1+h}_{(\Phi(\varrho(k)))}f^{\top}(x(k-h))\varsigma_{(\varrho(k))} \\ &\quad - \psi_{(\Phi(\varrho(k)))}x^{\top}(k)v_{(\varrho(k))}. \end{aligned}$$

Combining (2) and $x(k) \succeq 0$, we have

$$-x_i(k) \le -\frac{1}{\delta}f_i(x_i(k)).$$

Then

$$-x(k) \preceq -\frac{1}{\delta}f(x(k)).$$

Therefore,

$$\begin{aligned} &V_{(\varrho(k))}(x(k+1)) - \psi_{(\Phi(\varrho(k)))}V_{(\varrho(k))}(x(k)) \\ &\quad \le f^{\top}(x(k-h))(B^{\top}_{(\varrho(k))}v_{(\varrho(k))} - \psi^{1+h}_{(\Phi(\varrho(k)))}\varsigma_{(\varrho(k))}) \\ &\quad\quad + f^{\top}(x(k))(-\frac{\psi_{(\Phi(\varrho(k)))}}{\delta}v_{(\varrho(k))} + A^{\top}_{(\varrho(k))}v_{(\varrho(k))} \\ &\quad\quad + \psi_{(\Phi(\varrho(k)))}\varsigma_{(\varrho(k))}). \end{aligned} \quad (27)$$

By (21), (22), and (27),

$$V_{(\varrho(k))}(x(k+1)) \le \psi_{(\Phi(\varrho(k)))}V_{(\varrho(k))}(x(k)),$$

i.e.,

$$\begin{aligned} V_{(\varrho(k_{N_\varrho}))}(x(K)) &\le \psi_{(\Phi(\varrho(k_{N_\varrho})))}V_{(\varrho(k_{N_\varrho}))}(x(K-1)) \le \cdots \\ &\le \psi^{K-k_{N_\varrho}}_{(\Phi(\varrho(k_{N_\varrho})))}V_{(\varrho(k_{N_\varrho}))}(x(k_{N_\varrho})). \end{aligned}$$

By (23) and (24), we obtain

$$V_{(\varrho(k_{N_\varrho}))}(x(K)) \le \xi_{(\Phi(\varrho(k_{N_\varrho})))}\psi^{K-k_{N_\varrho}}_{(\Phi(\varrho(k_{N_\varrho})))}V_{(\varrho(k^-_{N_\varrho}))}(x(k^-_{N_\varrho})).$$

By repeating the process, we can get

$$\begin{aligned} &V_{(\varrho(k_{N_\varrho}))}(x(K)) \\ &\le \xi_{(\Phi(\varrho(k_{N_\varrho})))}\psi^{K-k_{N_\varrho}}_{(\Phi(\varrho(k_{N_\varrho})))}\psi^{k_{N_\varrho}-k_{N_\varrho-1}}_{(\Phi(\varrho(k_{N_\varrho-1})))}\xi_{(\Phi(\varrho(k_{N_\varrho-1})))} \\ &\quad \times V_{(\varrho(k^-_{N_\varrho-1}))}(x(k^-_{N_\varrho-1})) \\ &\le \cdots \le \xi_{(\Phi(\varrho(k_{N_\varrho})))}\xi_{(\Phi(\varrho(k_{N_\varrho-1})))}\cdots\xi_{(\Phi(\varrho(k_1)))} \\ &\quad \times \psi^{K-k_{N_\varrho}}_{(\Phi(\varrho(k_{N_\varrho})))}\psi^{k_{N_\varrho}-k_{N_\varrho-1}}_{(\Phi(\varrho(k_{N_\varrho-1})))}\cdots\psi^{k_1-0}_{(\Phi(\varrho(0)))}V_{(\varrho(0))}(x(0)) \\ &= \prod_{i=1}^{r}\xi^{N_i}_{(i)}\psi_{(i)}^{\sum_{\varrho(k_j)\in\Phi^i}(k_{j+1}-k_j)}V_{(\varrho(0))}(x(0)) \\ &= \prod_{i=1}^{r}(\xi^{N_i}_{(i)}\psi^{T_i}_{(i)})V_{(\varrho(0))}(x(0)) \\ &= \prod_{i=1}^{r}e^{N_i\ln\xi_{(i)}+T_i\ln\psi_{(i)}}V_{(\varrho(0))}(x(0)), \end{aligned}$$

where $N_\varrho \triangleq N_\varrho(K, 0)$, $N_i \triangleq N_{\varrho\Phi^i}(K, 0)$, $T_i \triangleq T_{\Phi^i}(K, 0)$, $i \in \mathbf{G}$, clearly, $\sum_{i=1}^{r} N_i = N_\varrho$, and $\sum_{i=1}^{r} T_i = K$. By Definition 1,

$$\begin{aligned} &V_{(\varrho(k_{N_\varrho}))}(x(K)) \\ &\quad \le \prod_{i=1}^{r}e^{N_{0\Phi^i}\ln\xi_{(i)}+(\frac{\ln\xi_{(i)}}{\tau_{a\Phi^i}}+\ln\psi_{(i)})T_i}V_{(\varrho(0))}(x(0)) \\ &\quad \le ce^{\max_{i\in\mathbf{G}}(\frac{\ln\xi_{(i)}}{\tau_{a\Phi^i}}+\ln\psi_{(i)})K}V_{(\varrho(0))}(x(0)), \end{aligned}$$

where $c = \prod_{i=1}^{r}e^{N_{0\Phi^i}\ln\xi_{(i)}}$.

By (25), $\frac{\ln\xi_{(i)}}{\tau_{a\Phi^i}} + \ln\psi_{(i)} < 0$. The rest of the proof can be obtained by using a similar method in Theorem 1, so it is omitted. □

Similar to the continuous-time one, we also generalize our approach to non-positive systems of the discrete-time one. Now, we aim to address the AES problem of such systems.

Consider the system

$$\begin{aligned} x(k+1) &= A_{(\varrho(k))}f(x(k)) + B_{(\varrho(k))}f(x(k-h)), k \ge 0, \\ x(k_0) &= \varphi(k_0), k_0 = -h, -h+1, \cdots, 0. \end{aligned} \quad (28)$$

Here $A_{(p)} \in \mathcal{R}^{n\times n}$ and $B_{(p)} \in \mathcal{R}^{n\times n}$. Other parameters and conditions are the same as the system (1-2). Let $\hat{A}_{(p)} = [(\hat{a}_{(p)})_{ij}]$, $\hat{B}_{(p)} = [(\hat{b}_{(p)})_{ij}]$, where

$$\begin{cases} (\hat{a}_{(p)})_{ij} = |(a_{(p)})_{ij}|; \\ (\hat{b}_{(p)})_{ij} = |(b_{(p)})_{ij}|. \end{cases}$$

Theorem 4: Consider the system (28) with (2). If there are scalars $0 < \psi_{(i)} < 1$, $\xi_{(i)} \ge 1$ and nonnegative vectors $v_{(p)}, \varsigma_{(p)} \in \mathcal{R}^n$ meeting $\forall p \in \mathbf{L}$, $\Phi(p) = i \in \mathbf{G}$,

$$\hat{A}^{\top}_{(p)}v_{(p)} - \frac{\psi_{(i)}}{\delta}v_{(p)} + \psi_{(i)}\varsigma_{(p)} \preceq 0, \quad (29)$$

$$\hat{B}^{\top}_{(p)}v_{(p)} - \psi^{1+h}_{(i)}\varsigma_{(p)} \preceq 0, \quad (30)$$

$$v_{(p)} - \xi_{(i)}v_{(q)} \preceq 0, \quad (31)$$

$$\varsigma_{(p)} - \xi_{(i)}\varsigma_{(q)} \preceq 0, \quad (32)$$

for $p \ne q$, $\forall p, q \in \mathbf{L}$. Then the system (28) with (2) is AES for the ΦDADT (25).

Proof: We select the following MCLKF:

$$V_{(\varrho(k))}(x(k)) = |x^{\top}(k)|v_{(\varrho(k))} + \sum_{s=k-h}^{k-1}\psi^{k-s}_{(\Phi(\varrho(k)))}|f^{\top}(x(s))|\varsigma_{(\varrho(k))}, \quad (33)$$

where $\psi_{(\Phi(\varrho(k)))}$, $v_{(\varrho(k))}$, $\varsigma_{(\varrho(k))}$, and the switching instances are the same as Theorem 3.

For $k \in [k_t, k_{t+1}]$,

$$V_{(\varrho(k))}(x(k+1)) - \psi_{(\Phi(\varrho(k)))}V_{(\varrho(k))}(x(k))$$

TABLE 1. The different switching designs under (Φ, G).

G	{1}	{1, 2}			{1, 2, 3}
Φ^i	$\Phi^1=\{1,2,3\}$ (ADT switching)	$\Phi^1=\{1,2\},\Phi^2=\{3\}$,	$\Phi^1=\{1,3\},\Phi^2=\{2\}$,	$\Phi^1=\{1\},\Phi^2=\{2,3\}$,	$\Phi^i=\{i\}, i=1,2,3$ (MDADT switching)
$\xi_{(i)}$	$\xi_{(1)}=2$	$\xi_{(1)}=1,\xi_{(2)}=2$	$\xi_{(1)}=1,\xi_{(2)}=2$	$\xi_{(1)}=2,\xi_{(2)}=2$	$\xi_{(1)}=2,\xi_{(2)}=2,\xi_{(3)}=2$
$\psi_{(i)}$	$0.66\le\psi_{(1)}<1$	$0.83\le\psi_{(1)}<1, 0.75\le\psi_{(2)}<1$	$0.84\le\psi_{(1)}<1, 0.81\le\psi_{(2)}<1$	$0.52\le\psi_{(1)}<1, 0.47\le\psi_{(2)}<1$	$0.66\le\psi_{(1)}<1, 0.65\le\psi_{(2)}<1, 0.66\le\psi_{(3)}<1$
$v_{(1)}$	$[556.3513, 340.7297]^\top$	$[180.5086, 124.6652]^\top$	$[135.0893, 94.1105]^\top$	$[99.3425, 51.9686]^\top$	$[665.2943, 406.6888]^\top$
$v_{(2)}$	$[448.2654, 303.2321]^\top$	$[180.5086, 124.6652]^\top$	$[179.1714, 131.9655]^\top$	$[65.6695, 38.5737]^\top$	$[535.2087, 358.9391]^\top$
$v_{(3)}$	$[424.8913, 336.2510]^\top$	$[238.6858, 185.8745]^\top$	$[135.0893, 94.1105]^\top$	$[62.5498, 46.8338]^\top$	$[508.3995, 401.3798]^\top$
$\varsigma_{(1)}$	$[657.8089, 560.3628]^\top$	$[251.4345, 208.5402]^\top$	$[193.4519, 151.9024]^\top$	$[114.8132, 96.0254]^\top$	$[788.1092, 669.4375]^\top$
$\varsigma_{(2)}$	$[822.7246, 589.1552]^\top$	$[251.4345, 208.5402]^\top$	$[294.5025, 226.3035]^\top$	$[169.5663, 106.7669]^\top$	$[994.5263, 706.0514]^\top$
$\varsigma_{(3)}$	$[794.7096, 583.8494]^\top$	$[423.8770, 332.9150]^\top$	$[193.4519, 151.9024]^\top$	$[177.1961, 106.5410]^\top$	$[949.3445, 697.5554]^\top$
Signal design	$\tau^*_{a\Phi^1}=1.668$,	$\tau^*_{a\Phi^1}=0, \tau^*_{a\Phi^2}=2.409$	$\tau^*_{a\Phi^1}=0, \tau^*_{a\Phi^2}=3.289$	$\tau^*_{a\Phi^1}=1.60, \tau^*_{a\Phi^2}=0.918$	$\tau^*_{a\Phi^1}=1.668, \tau^*_{a\Phi^2}=1.609, \tau^*_{a\Phi^3}=1.668$
Signal instance	$\tau_1=1,\tau_2=2,\tau_3=2$	$\tau_1=1,\tau_2=1,\tau_3=3$	$\tau_1=1,\tau_2=1,\tau_3=4$	$\tau_1=2,\tau_2=1,\tau_3=1$	$\tau_1=2,\tau_2=2,\tau_3=2$

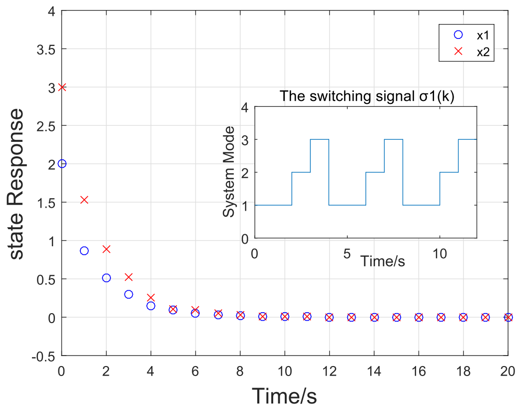

FIGURE 1. The state response of the system under $\sigma 1(k)$.

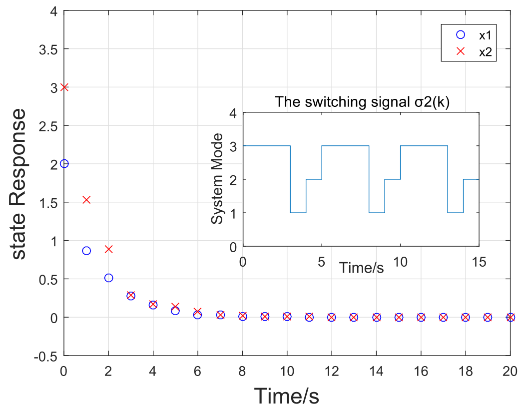

FIGURE 2. The state response of the system under $\sigma 2(k)$.

$$= -\psi_{(\Phi(\varrho(k)))}\|x^\top(k)\|v_{(\varrho(k))}+\|f^\top(x(k))\|\hat{A}^\top_{(\varrho(k))}v_{(\varrho(k))} + \|f^\top(x(k-d))\|\hat{B}^\top_{(\varrho(k))}v_{(\varrho(k))} - \psi^{1+h}_{(i)}\|f^\top(x(k-d))\| \times \varsigma_{(\varrho(k))} + \psi_{(\Phi(\varrho(k)))}\|f^\top(x(k))\|\varsigma_{(\varrho(k))}.$$

By (2), it yields that

$$-\|x_i\| \le -\frac{1}{\delta}\|f_i(x_i)\|,$$

i.e.,

$$-\|x\| \le -\frac{1}{\delta}\|f(x)\|.$$

Therefore,

$$\begin{aligned} &V_{(\varrho(k))}(x(k+1)) - \psi_{(\Phi(\varrho(k)))}V_{(\varrho(k))}(x(k)) \\ &\quad\le \|f^\top(x(k-d))\|(\hat{B}^\top_{(\varrho(k))}v_{(\varrho(k))} - \psi^{1+d}_{(\Phi(\varrho(k)))}\varsigma_{(\varrho(k))}) \\ &\qquad + \|f^\top(x(k))\|(-\frac{\psi_{(i)}}{\delta}v_{(\varrho(k))} + \hat{A}^\top_{(\varrho(k))}v_{(\varrho(k))} \\ &\qquad + \psi_{(\Phi(\varrho(k)))}\varsigma_{(\varrho(k))}). \end{aligned} \tag{34}$$

By (29), (30), and (34),

$$V_{(\varrho(k))}(x(k+1)) \le \psi_{(\Phi(\varrho(k)))}V_{(\varrho(k))}(x(k)).$$

The rest of the proof is omitted since it is similar to that in Theorem 3. □

Remark 3: After we proposed ΦDADT strategy [20], the works [21] and [22] further applied it to switched T-S fuzzy systems and switched singular systems respectively. Different from them, this paper combines this strategy with the MCLKF approach for the first time to deal with switched positive time-delay systems.

IV. A NUMERICAL EXAMPLE

This section gives a numerical example to show the validity of the proposed approach.

Example 1: Consider the discrete-time SPNTS (1) with three modes whose relevant parameters are presented as follows:

$$A_1 = \begin{bmatrix} 0.7 & 0.15 \\ 0.8 & 0.47 \end{bmatrix}, B_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0.2 & 0.3 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.2 & 0.05 \\ 0.26 & 0.18 \end{bmatrix}, B_2 = \begin{bmatrix} 0.1 & 0.3 \\ 0.8 & 0.1 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0.1 & 0.2 \\ 0.08 & 0.15 \end{bmatrix}, B_3 = \begin{bmatrix} 0.3 & 0.2 \\ 0.4 & 0.2 \end{bmatrix},$$

and $h = 1, f_i(x_i(t)) = 0.3x_i(t) + \frac{0.1x_i(t)}{x_i^2(t)+1}$, then $\gamma = 0.2$ and $\delta = 0.3$.

For $\mathbf{L} = \{1, 2, 3\}$, there are five possibilities of Φ^i based on the different $\mathbf{G}$ (see Table 1 for details). Wherein $\Phi^1 = \{1, 2, 3\}$, and $\Phi^i = \{i\}, i = 1, 2, 3$ correspond those results of the classical ADT and MDADT ones respectively.

We present the different designs based on $(\Phi, \mathbf{G})$ to demonstrate their strengths by selecting parameters appropriately in Table 1 (where τ_i means the ith subsystem's ADT).

From the table, one can also get these facts as follows.

When $\mathbf{G} = \{1, 2\}$, we can draw some conclusions for SPNTSs that have not been paid attention to before. Choose a switching signal $\varrho 1(k)$ with $\tau_1 = 2, \tau_2 = 1, \tau_3 = 1$, the system's stability under $\varrho 1(k)$ can be got based on $\Phi^1 = \{1\}, \Phi^2 = \{2, 3\}$ but not for the ADT strategy. Similarly, given the signal $\varrho 2(k)$ with $\tau_1 = 1, \tau_2 = 1, \tau_3 = 3$, we can obtain the system's stability under $\varrho 2(k)$ by $\Phi^1 = \{1, 2\}, \Phi^2 = \{3\}$ but not for the MDADT one. It can be seen in Figures 1 and 2 with $x(0) = (2, 3)^\top$.

V. CONCLUSION

This paper investigates the AES problem of SPNTSs based on the ΦDADT switching strategy. By combining the MCLKF with the ΦDADT switching strategy, some less conservative stability conditions of SPNTSs are obtained in both continuous-time and discrete-time cases. Furthermore, the obtained results are developed into non-positive systems. These sufficient AES criteria are compatible with classic ADT and MDADT ones. An illustrated example is presented to show the merits and features of our results. In future works, we will apply the ΦDADT switching strategy to switching systems with time-varying and multi-delay cases.

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