

Received 1 September 2022, accepted 27 October 2022, date of publication 31 October 2022, date of current version 4 November 2022. *Digital Object Identifier 10.1109/ACCESS.2022.3218317*

RESEARCH ARTICLE

Adaptive Feedback Information Switching for Reliable Wireless Networked Control Systems

KOJI ISHII^{vo}[,](https://orcid.org/0000-0001-5878-7705) (Senior Member, IEEE)
Faculty of Engineering and Design, Kagawa University, Takamatsu, Kagawa 7610396, Japan

e-mail: ishii.koji@kagawa-u.ac.jp

This work was supported by the Japan Society for the Promotion of Science (JSPS) KAKENHI under Grant 20K04485.

ABSTRACT This paper proposes an adaptive switching method of feedback information for a wireless networked control system (WNCS) with unreliable communication links, which is based on a cross-layered design approach between communication and control layers. In the focused WNCS where communication errors can occur, the uncertainty in the estimation of control-state at the controller side consists of the following two factors. The one is the uncertainty as to whether the control-input has been correctly input (received). The other is the disturbance added to the controlled object (plant). This work considers the following two modes which determine the feedback information: (1) the mode in which the plant side transmits the communication result of feedforward channel (from the controller to the plant), which will be referred to as to *ACK* mode, (2) the mode in which the plant transmits the information of control-state observed by the sensor, which is referred to as *STA* mode. From a communication perspective, in the case with ACK mode, the use of ACK/NACK signaling allows a small number of bits (communication rate) to be used, and thus enables reliable feedback communication. As a result, the controller can reliably know the uncertainty of whether the control-input has been successfully received, but the uncertainty due to the disturbance still remains. On the other hand, in the case with STA mode, all the uncertainties can be eliminated only if the feedback communication is successful. However, the increase of information (rate) makes the feedback communication unreliable compared to the case with ACK mode. Furthermore, the control-input should be calculated considering the reliability of feedforward and feedback channels as well as the accuracy of the estimated control-state. As a result, the selection of feedback information deeply affects the quality of WNCS. This paper provides an adaptive switching method of feedback information and corresponding control-input, and shows that the proposed method can efficiently improve the quality of control in the WNCS with unreliable communication links.

INDEX TERMS Wireless networked control system, communication protocol, model predictive control, power allocation.

I. INTRODUCTION

Recent advances in wireless communication technology and control systems have focused attention on networked control systems (NCS), especially wireless networked control systems (WNCSs) [1], [2], [3], [4]. For example, it is indispensable for wireless remote control systems such as factory automation [5], [6], remote maintenance of buildings [7], autonomous car [8], [9], and so on. Recent researches in

The associate editor coordinating the re[view](https://orcid.org/0000-0002-7455-5872) of this manuscript and approving it for publication was Rute C. Sofia[.]

control theory cover in a wide range of areas from the control of conventional equipment and plants to the control of some event. For example, the real-time output feedback dynamic control via membership functions online learning policy has proposed for fuzzy systems in [10]. The efficient control methods for multi-agent systems have been proposed in [11] and [12]. Based on control theory approach, the pandemic mitigation strategies have been proposed in [13] and [14]. Meanwhile, in the field of wireless communication research, the 5th generation mobile communication system (5G) is now in service, and discussions have recently begun regarding

the future of wireless communication technology, including the system using local 5G [15] and other technologies, as well as beyond 5G and 6G [16], [17]. These developments in wireless communication and control technologies have greatly increased expectations for the realization of reliable and efficient WNCSs.

A. RELATED WORKS IN WNCS

A WNCS, which is a joint research area of control theory and communication theory, has attracted much attention in recent years. However, most researches in the last few decades have been tackled by researchers of control theory. To realize a reliable WNCS, the features of wireless communication should be considered and some design criteria of controller and observer have been provided considering the limitations due to data rate [18], communication errors [19], [20], quantization errors [21], [22], communication delay [23]. However, most of them did not provide the design of the wireless communications for the WNCS. More recently, some sophisticated design methods based on cross-layered design of wireless communication and control layers have been proposed. These design methods depend on the focused WNCS as well as the design objectives. The works [19], [24], [25] have focused on the simplest WNCS in which one controller tries to control a single controlled object over the wireless network. The studies [26], [27], [28] have provided an efficient wireless resource management method for the WNCS in which one controller controls multiple controlled objects over the common wireless network. A WNCS with more advanced wireless networks such as multihop network has been studied in [29]. An efficient coding scheme for WNCS has been provided by [30]. The works [31], [32], [33] have focused on a secure WNCS.

B. CONTRIBUTIONS OF THIS PAPER

Similar to [19], [24], and [25], this work focuses on the simplest WNCS in which a single controller controls a single controlled object (plant) and aims to enhance the quality of control by properly designing the control and wireless communication systems. We assume that the observer and controller are embedded in the controller side, while the controlled object and sensor are in plant side, and the controller and plant are connected through *unreliable* wireless networks. The work [19] provided the design criteria at the controller and observer, that is, how to calculate the controller gain and observer gain considering the features of wireless network, but it did not provide the design of wireless communication. The work [24] has provided the joint design of wireless communication and control system, where the transmission powers at the feedforward (controller-to-plant) and feedback (plant-to-controller) channels are adaptively changed and the control-input is calculated considering the reliability of wireless links. However, the communication protocol (the kind of feedback information) is fixed and the disturbance at the controlled object is not considered. This work considers the disturbance. As a result, there are three

uncertainty factors as follows: (1) the disturbance added to the controlled object, (2) the uncertainty as to whether the control-input was successfully input, that is, whether the *feedforward* channel was successfully communicated, (3) the estimation uncertainty at the controller side, which is determined by the communication result of *feedback* channel and the type of feedback information transmitted from the plant side. It is worth noting that the quality of WNCS depends on what kind of information is fed back, since the feedback information affects the accuracy of estimation at the observer and the reliability of wireless communication links.

This work proposes to appropriately switch the feedback information considering the control-state and the reliability of the communication links. We consider two type of feedback information as follows: (1) The plant side transmits the *state-information* observed at the sensor to the controller side, which will be referred to as *STA* mode. (2) The plant side transmits the *communication result* of the feedforward channel, commonly named acknowledgement/ negative-acknowledgement (ACK/NACK) signal in wireless communications [34], which will be referred to as *ACK* mode. In the case when STA mode is applied, if the feedback communication is successful, the controller side can obtain the true control-state including the uncertainties caused by the communication result at the feedforward channel and the additive disturbance. In the case with ACK mode, if the feedback communication is successful, the controller side can only know whether the control-input has been successfully received. Consequently, the estimated control-state at the controller side remains the uncertainty due to the disturbance and the estimation error in the previous estimated controlstate. However, it is worth noting that the feedback communication, when ACK mode is applied, is more reliable than the case with STA mode since the transmission rate of ACK/NACK signal is much lower than the case with STA mode. As a result, there is the trade-off relationship between obtaining the true control-state with low probability and obtaining only the availability of the control-input with high probability. Intuitively, at the beginning of control, it is more important to successfully input the control-input than to obtain the true control-state, i.e., ACK mode should be applied. At the end of control, in order to mitigate the influence of disturbances, it is important to calculate the meticulous control-input from the exact (true) control-state, i.e., STA mode should be applied.

This work provides a cross-layer optimization method for both wireless and control systems. Specifically, in the wireless communication layer, adaptive power-allocation method to feedforward and feedback channels and adaptive switching of feedback information are provided. While, in the control layer, the control-input considering the reliability of feedforward and feedback channels is provided. The main contributions of this work are summarized as follows:

• The cost function based on the quality of control considering the accuracy of estimation depending on the applied mode and transmission powers allocated

into the feedforward and feedback channels is provided.

- This work provides a joint-optimization of the controlinput, the applied mode, and the transmission power at the feedforward and feedback channels.
- Computer simulations show that the proposed optimization can adaptively switch the feedback information, appropriately allocate transmission-power, and provide the optimum control-input corresponding to the quality of control and the accuracy of the estimation of controlstate, which can enhance the quality of control in the WNCS with unreliable communication links.

C. ORGANIZATION OF THIS PAPER

The rest of this paper is organized as follows: In Sec.II, the focused WNCS is presented where the controller and the plant are connected via unreliable wireless links. In Sec.III, the integrated design of communication and control layers is proposed. Specifically, joint optimization of transmission mode, transmission power, and control-input is provided. In Sec.IV, some numerical evaluations are presented to validate the efficiency of the proposed method. Lastly, the conclusions are drawn in Sec.V.

D. NOTATIONAL REMARKS

Let $\mathbb R$ denote the set of real numbers. For the vector v , let v^T denote the transpose of *v*. For brevity, we sometimes use the symbol ''0'' instead of zero matrix with appropriate dimensions and omit the index of vector. Let $Pr(x)$ and $E[X]$ be the probability of the event *x* and the expected value of a random variable X , respectively. Let I_n be an identity matrix of size *n*.

II. SYSTEM MODEL

The focused WNCS is composed of a single controller and a single plant in which they are connected via a wireless communication link as depicted in Fig[.1.](#page-3-0) The observer and controller are embedded in the controller side, while the controlled object and sensor are in the plant side. The channels from controller to controlled object and from sensor to observer are referred to as *feedforward* and *feedback* channels, respectively. This work focuses on a discrete-time control since we consider a digital wireless communication. The wireless communication network and control system can be modeled as follows.

A. WIRELESS COMMUNICATION NETWORK

The controller at the controller side wants to transmit the control-input $u[k]$ to the controlled object at the plant side through the feedforward wireless channel, while the sensor at the plant side wants to transmit the measured control-state $Cx[k]$ or ACK/NACK signal to the observer at the controller side through the feedback wireless channel. We assume that the receiver can ideally detect communication error. The details of control-input and estimated control-state will be described in the next subsection. Let $s_{FF}[k]$ and $s_{FB}[k]$ be

the transmitting signals from the controller and sensor at the *k*th time-slot and with average power of 1 ($\mathbb{E}[|s_{FF}[k]|^2] =$ $\mathbb{E}[|s_{FB}[k]|^2] = 1$). $s_{FF}[k]$ and $s_{FB}[k]$ are composed of the control-input *u*[*k*] and the feedback information corresponding to the selected mode, respectively. The role of feedback communication is to observe the control-state at the controller side. There are three uncertainly factors in the estimation of control-state at the controller side: (1) the disturbance added to the controlled object, (2) the uncertainly as to whether the control-input was successfully received, i.e., whether the feedforward channel was successfully communicated, (3) the residual estimation error in the previous estimation of controlstate. These uncertainties are determined by the results of communication in the previous time-slots and the feedback information. This study considers the following two modes of determining the feedback information. The one is that the observer transmits the communication result of feedforward channel, that is, ACK/NACK signal is transmitted, which will be referred to as *ACK* mode. The other is that the observer transmits the state-information, which will be referred to as *STA* mode. The impacts of selection of feedback information on the control system will be described in Sec. [II-B](#page-3-1) and Sec. [III-B.](#page-5-0) The received signals at the controlled object (plant) and observer are respectively given by

$$
r_{\rm FF}[k] = h_{\rm FF}[k] \sqrt{P_{\rm FF}^{\mathcal{M}}[k]} s_{\rm FF}[k] + n_{\rm FF}[k],\tag{1}
$$

$$
r_{\rm FB}[k] = h_{\rm FB}[k] \sqrt{P_{\rm FB}^{\mathcal{M}}[k]} s_{\rm FB}[k] + n_{\rm FB}[k],\tag{2}
$$

where h_{FF} and h_{FB} are the Rayleigh-fading coefficients in the feedforward and feedback channels and independently drawn from the complex Gaussian distribution $CN(0, 1)$, $P_{\text{FF}}^{\mathcal{M}}[k]$ and $P_{\text{FB}}^{\mathcal{M}}[k]$ are the transmitting power at the controller and plant at the *k*th time-slot corresponding to the selected mode $M \in \{ACK, STA\}, n_{FF}[k]$ and $n_{FB}[k]$ are the noise factors which follow Gaussian distributed random variables with mean zero and unit variance (n_{FF} , $n_{FB} \sim \mathcal{CN}(0, 1)$). The channel estimation and time-frequency synchronization at the receiver side are assumed to be ideal. Let R_{FF} and R_{FB} be the transmission rate at the feedforward and feedback channels, respectively. R_{FF} is a fixed value, while R_{FB} is determined by the selected mode. When ACK or STA mode is selected, R_{FB} is given by R_{ACK} or R_{STA} . Since the amount of information in the ACK/NACK signal is less than that of the control-state, the required transmission rate of ACK mode is obviously smaller than that of STA mode (i.e., R_{ACK} < *R*_{STA}). Different from general wireless data communications, this work does not apply the retransmission techniques such as ARQ (Automatic Repeat Request) when NACK signal is received at the sender, because the retransmissions cause communication delays that can have a significant negative impact on the control system. But, note that the retransmission technique can be expected to be still effective because the communication interval is generally much shorter than control interval.

For the sake of simple discussion, this work uses ideal modulation and error correcting code (random Gaussian

FIGURE 1. Wireless networked control system.

code) that can achieve the Shannon limit. In this case, the outage probabilities of the feedforward and feedback channels are given by

$$
p_{\text{FF}}^{\mathcal{M}}[k] = \Pr\left(\log_2\left(1 + |h_{\text{FF}}|^2 P_{\text{FF}}^{\mathcal{M}}[k]\right) < R_{\text{FF}}\right)
$$
\n
$$
= \Pr\left(|h_{\text{FF}}|^2 < \frac{2^{R_{\text{FF}}}-1}{P_{\text{FF}}^{\mathcal{M}}[k]}\right) = \int_0^{2\frac{R_{\text{FF}}-1}{P_{\text{FF}}[k]}} \exp(-x) \, dx
$$
\n
$$
= 1 - \exp\left(-\frac{2^{R_{\text{FF}}}-1}{2^{M_{\text{FF}}}}\right),\tag{3}
$$

$$
= 1 - \exp\left(-\frac{1}{P_{\text{FF}}^{\mathcal{M}}[k]}\right),
$$
\n
$$
[k] = 1 - \exp\left(-\frac{2^{R_{\text{FF}}^{\mathcal{M}}}-1}{1}\right)
$$
\n(3)

$$
p_{\text{FB}}^{\mathcal{M}}[k] = 1 - \exp\left(-\frac{2^{R_{\text{FB}}^{\mathcal{M}}} - 1}{P_{\text{FB}}^{\mathcal{M}}[k]}\right),
$$
 (4)
where $R_{\text{FB}}^{\mathcal{M}} \in \{R_{\text{ACK}}, R_{\text{STA}}\}$ [34]. To improve the quality

of control, the system can control the quality of wireless channels, i.e., $p_{\text{FF}}^{\mathcal{M}}$ and $p_{\text{FB}}^{\mathcal{M}}$, by selecting the transmission mode M and changing the transmission powers at the feedforward and feedback channel, i.e., $P_{\text{FF}}^{\mathcal{M}}$ and $P_{\text{FB}}^{\mathcal{M}}$. Let P_T be the total transmitting power of the system. The transmit powers of feedforward and feedback channels at the *k*th timeslot are given by $P_{\text{FF}}^{\mathcal{M}}[k] = (1 - \lambda[k])P_T$ and $P_{\text{FB}}^{\mathcal{M}}[k] =$ $\lambda[k]P_T$ with the power allocation factor $\lambda[k]$ ($0 \leq \lambda[k] \leq 1$). Given the transmission rates and total transmission power, the outage probabilities of feedforward and feedback channels become the function of the power allocation factor $\lambda[k]$ and applied mode, that is, $p_{\text{FF}}^{\mathcal{M}}[k] = f_{p_{\text{FF}}}(\lambda[k], \mathcal{M}[k])$, $p_{\text{FB}}^{\mathcal{M}}[k] =$ $f_{p_{\text{FR}}}(\lambda[k],\mathcal{M}[k]).$

B. CONTROL SYSTEM

Consider a linear time-invariant system of the discrete-time form

$$
x[k+1] = Ax[k] + v[k]Bu[k] + Dw[k],
$$
 (5)

$$
y_o[k] = Cx[k],\tag{6}
$$

where $x[k] \in \mathbb{R}^{n_x}$, $v[k] \in \{0, 1\}$, $u[k] \in \mathbb{R}^{n_u}$, $w[k] \in$ \mathbb{R}^{n_w} , and $y_o[k] \in \mathbb{R}^{n_y}$ denote the state of the controlled object (control-state), communication result at the feedforward channel, control-input, disturbance with zero-mean and covariance matrix $\mathbf{W} = \text{diag}(\sigma_w^2, \dots, \sigma_w^2) \in \mathbb{R}^{n_x \times n_x}$, and measured output at sampling instant k , respectively. $A \in$ $\mathbb{R}^{n_x \times n_x}$, $B \in \mathbb{R}^{n_x \times n_u}$, $C \in \mathbb{R}^{n_y \times n_x}$, and $D \in \mathbb{R}^{n_x \times n_w}$ are the coefficient matrices of the control system. For simplicity, this work assumes that C is an identity matrix, i.e., the observer can observe all states without measurement errors including quantization error. The control-state is determined corresponding to the communication result at feedforward channel. $\nu[k] = 0$ when the communication error occurred at the feedforward channel, whereas $v[k] = 1$ when the communication successfully completed. Similarly, $\gamma[k]$ = 1 when the observer can get the feedback signal at the *k*th time-slot, and $\gamma[k] = 0$ when the feedback communication fails. The probabilities of $v[k] = 0$ and $v[k] = 1$, when the communication mode M is selected, are given by $Pr(v[k] = 0) = p_{\text{FF}}^{\mathcal{M}}[k]$ and $Pr(v[k] = 1) = 1 - p_{\text{FF}}^{\mathcal{M}}[k],$ respectively. Similarly, the probabilities of $\gamma[k] = 0$ and $\gamma[k] = 1$ are given by $Pr(\gamma[k] = 0) = p_{FB}^{\mathcal{M}}[k]$ and $Pr(\gamma[k] = 1) = 1 - p_{FB}^{\mathcal{M}}[k]$. It is important to note that the separation principle does not hold in the focused WNCS, since the control-input affects the estimation error and viceversa (details are seen in Sec. [III\)](#page-4-0). By integrating the control system and the wireless communication networks, the WNCS focused on in this study can be represented as shown in Fig. [1.](#page-3-0)

Since this work utilizes model predictive control (MPC) based optimization method, the fundamentals of MPC are introduced from here. Assuming that a disturbance-free and error-free communication at both feedforward and feedback

channels can be achieved, perfect knowledge of the controlstate $x[k]$ and the control-input $u[k]$ is considered available for both sides. The optimal control-input at the *k*th time-slot $u^*[k]$ can be given by

$$
u^*[k] = \left(\mathbf{U}^*[k]\right)_1,\tag{7}
$$

$$
\mathbf{U}^*[k] = \arg \min J_k(\mathbf{U}),\tag{8}
$$

$$
J_k(\mathbf{U}) = \sum_{i=0}^{L-1} \left(x[k+i]^T Q x[k+i] + u[k+i]^T R u[k+i] \right) + x[k+L]^T P x[k+L],
$$
\n(9)

where $(\mathbf{U}^*[k])_1$ represents the extraction of the first n_u elements of $\mathbf{U}^* [k] (\in \mathbb{R}^{L n_u})$ which corresponds to the control-inputs for the *k*th to $(k + L - 1)$ th time-slot, $Q > 0$, $P \geq 0$, and $R > 0$ are weight matrices and *L* is the receding horizon (prediction period). For brevity, we use the matrix form defined as

$$
\begin{bmatrix}\nx[k+1] \\
x[k+2] \\
\vdots \\
x[k+L]\n\end{bmatrix}
$$
\n
$$
= \begin{bmatrix}\nA \\
A^2 \\
\vdots \\
A^L\n\end{bmatrix} x[k]
$$
\n
$$
+ \begin{bmatrix}\nB & 0 & \cdots & 0 \\
AB & B & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
A^{L-1}B & A^{L-2}B & \cdots & B\n\end{bmatrix} \begin{bmatrix}\nu[k] \\
u[k+1] \\
\vdots \\
u[k+L-1]\n\end{bmatrix},
$$
\n
$$
\stackrel{\triangle}{=} \mathbf{B} \in \mathbb{R}^{Ln_x \times Ln_u} \stackrel{\triangle}{=} \mathbf{U} \in \mathbb{R}^{Ln_u} (10)
$$

$$
\mathbf{X} = \mathbf{A}\mathbf{x}[k] + \mathbf{B}\mathbf{U}.\tag{11}
$$

٦ \perp \perp \perp \perp

,

 \tilde{x}

Thus, equation [\(9\)](#page-4-1) can be rewritten as

$$
J_k(\mathbf{U}) = \mathbf{X}^T \bar{Q} \mathbf{X} + \mathbf{U}^T \bar{R} \mathbf{U}
$$

= $(\mathbf{A}x[k] + \mathbf{B} \mathbf{U})^T \bar{Q} (\mathbf{A}x[k] + \mathbf{B} \mathbf{U}) + \mathbf{U}^T \bar{R} \mathbf{U}$
= $\mathbf{U}^T (\bar{R} + \mathbf{B}^T \bar{Q} \mathbf{B}) \mathbf{U} + 2 x[k]^T \mathbf{A}^T \bar{Q} \mathbf{B} \mathbf{U}$
+ $x[k]^T \mathbf{A}^T \bar{Q} \mathbf{A}x[k],$ (12)

where \overline{Q} = diag $(Q, \dots, Q, P) \in \mathbb{R}^{Ln_x \times Ln_x} \geq 0$, \overline{R} = $diag(R, \dots, R) \in \mathbb{R}^{Ln_u \times Ln_u} > 0$. Since the cost function J_k (U) is quadratic, the optimization problem can be solved as a convex quadratic programming (QP) problem. It is wellknown that QP problem has a unique minimum and can be efficiently solved by using standard techniques such as internal point method [35], [36], [37], [38]. Since the calculation of control-input is completed in a receding-horizon manner, only the first element of the calculated control-input

sequence is used for the control-input at the *k*th time-slot. At every time slot, the MPC optimization is solved and only the first element of the calculated control-input sequence is utilized. The above procedure is repeated until the end of control. We assume that the controlled plant applies not a hold-input scheme, but a zero-input scheme in the case where the communication error occurs at the plant side. A detailed suggestion about which scheme is better can be seen in [39].

Next, let us consider the estimation of control-state at the controller side, which is used as the initial control-state $x[k]$ in the calculation of control-input. The estimation result depends on the applied communication protocol and the communication result of feedforward channel. In the case where STA mode is applied, the estimated state-information depending on the communication result of feedback channel can be given by

$$
\tilde{x}[k+1]
$$
\n
$$
= \begin{cases}\n x[k+1] & \text{if successfully received} \\
 \mathbb{E} \left[x[k+1] | \tilde{x}[k], u[k] \right] \\
 = A\tilde{x}[k] + (1 - p_{\text{FF}}^{\text{STA}}[k])Bu[k] & \text{otherwise (failure)}\n \end{cases}
$$
\n(13)

The controller can get true state-information if it can successfully receive the signal from the observer with a probability of $(1 - p_{FB}^{STA}[k])$. If it fails with a probability of $p_{FB}^{STA}[k]$, the control-state at $(k + 1)$ th time-slot needs to be estimated from the control-input and the control-state estimated at the previous (*k*th) time-slot.

In the case where ACK mode is applied, the estimated state-information can be given by

$$
[k+1]
$$

\n
$$
= \begin{cases} A\tilde{x}[k] + Bu[k] & \text{if ACK is received} \\ A\tilde{x}[k] & \text{if NACK is received} \\ A\tilde{x}[k] + (1 - p_{\text{FF}}^{\text{ACK}}[k])Bu[k] & \text{otherwise.} \end{cases}
$$
\n(14)

Even if the controller side can successfully receive the ACK/NACK signal with a probability of $(1 - p_{\text{FB}}^{\text{ACK}}[k])$, only the communication result in the feedforward channel, i.e., the true control-input at the controlled object, can be used for the state-estimation. Note that in the case where ACK mode is applied, even if the communication at the feedback channel is successful, the estimated control-state still contains the uncertainty due to the disturbances and residual error in the previous estimated result. If the communication at the feedback channel fails, the control-state needs to be estimated as in the case of STA mode.

III. INTEGRATED DESIGN OF FEEDBACK INFORMATION, TRANSMISSION POWER, AND CONTROL-INPUT

A. PROBLEM FORMULATION

The aim of this work is to provide an integrated design of communication and control. The communication method

should be designed considering the quality of control, specifically the estimation accuracy at the controller side and the control-state at the plant side, while the control-input should be designed considering the quality of communication at the feedforward and feedback channels. The design for communication consists of two parts: the selection of an appropriate mode (feedback information) and the power allocation of transmission powers to the feedforward and feedback channels. Therefore, the problem focused in this work can be given by

$$
u^*[k] = (\mathbf{U}^*[k])_1, \mathcal{M}^*[k]
$$

= (\mathbf{M}^*[k])_1, \lambda^*[k] = (\Lambda^*[k])_1, (15)
[k], \mathbf{M}^*[k], \Lambda^*[k] = \arg \min J_k (\mathbf{U}, \mathbf{M}, \Lambda, \tilde{\chi}[k]), (16)

$$
J_k(\mathbf{U}, \mathbf{M}, \Lambda, \tilde{x}[k]) = \mathbb{E}\left[\mathbf{X}^T \bar{Q} \mathbf{X} + \mathbf{U}^T \bar{R} \mathbf{U} | \mathbf{M}, \Lambda, \tilde{x}[k]\right],
$$
\n(17)

where **M** is the scheduling vector consisting of the transmission modes to be applied in the prediction period *L*, i.e., $\mathbf{M} = \{ \mathcal{M}[k], \mathcal{M}[k+1], \cdots, \mathcal{M}[k+L-1] \} \in \{\text{ACK}, \text{STA}\}^L,$ Λ is the vector consisting of the power allocation factors, i.e., $\Lambda = {\lambda[k], \lambda[k+1], \cdots, \lambda[k+L-1]}, J_k$ (U, M, $\Lambda, \tilde{x}[k]$) is the expected cost function in the case where the estimated control-state at *k*th time-slot $\tilde{x}[k]$ is given and the transmission modes M and the power-allocation factors Λ are adopted. Based on the cost function given by Eq. [\(17\)](#page-5-1), the control-input, transmission mode, and power allocation factor can be optimized simultaneously. Specifically, the optimization process is to find \mathbf{U}^* [k], \mathbf{M}^* [k], Λ^* [k] that can minimize the cost function J_k .

B. EXPECTED CONTROL-STATE AND ESTIMATION ERROR

To formulate the cost function, we consider the expected control-state and the estimation error. The real control-states that depend on the communication results of the feedforward channel are given by

$$
x[k + 1] = Ax[k] + v[k]Bu[k] + Dw[k]
$$

\n
$$
x[k + 2] = Ax[k + 1] + v[k + 1]Bu[k + 1] + Dw[k + 1]
$$

\n
$$
\vdots
$$

\n
$$
x[k + L] = Ax[k + L - 1] + v[k + L - 1]Bu[k + L - 1]
$$

\n
$$
+Dw[k + L - 1],
$$
\n(18)

while the expected control-states can be given by

$$
\tilde{x}[k+1] = A\tilde{x}[k] + (1 - p_{\text{FF}}[k])Bu[k]
$$

\n
$$
\tilde{x}[k+2] = A\tilde{x}[k+1] + (1 - p_{\text{FF}}[k+1])Bu[k+1]
$$

\n
$$
\vdots
$$

\n
$$
\tilde{x}[k+L] = A\tilde{x}[k+L-1]
$$

\n
$$
+(1 - p_{\text{FF}}[k+L-1])Bu[k+L-1]. \quad (19)
$$

Let us consider the gap (error) between the real control-state and the expected (estimated) control-state, which is defined

- **U** ∗

as $e[k] = x[k] - \tilde{x}[k]$. As seen in Eqs. [\(13\)](#page-4-2) and [\(14\)](#page-4-3), the estimated control-state depends on the communication results of the feedback channel and the applied communication mode, so that the error also depends on these factors. When ACK mode is adopted, if feedback (ACK/NACK) signal is successfully received with a probability of $(1 - p_{FB}^{ACK}[k])$, only the control-input can be known at the controller side. As a result, only the disturbance component is added to the estimation error for the *k*th time-slot, as expressed as

$$
e[k+1] = Ae[k] + Dw[k],
$$
 (20)

where $e[k]$ is the error component in the estimated control-state at the *k*th (previous) time-slot. If the feedback communication fails, the error includes not only the disturbance component but also the uncertainty whether the control-input was successfully input. Specifically, it can be given by

$$
e[k+1] = \begin{cases} Ae[k] - p_{\text{FF}}^{\text{ACK}}[k]Bu[k] + Dw[k] \\ \text{with a prob. of } p_{\text{FB}}^{\text{ACK}}[k] (1 - p_{\text{FF}}^{\text{ACK}}[k]) \\ Ae[k] + (1 - p_{\text{FF}}^{\text{ACK}}[k])Bu[k] + Dw[k] \\ \text{with a prob. of } p_{\text{FB}}^{\text{ACK}}[k] p_{\text{FF}}^{\text{ACK}}[k] \end{cases}
$$
(21)

On the other hand, when STA mode is adopted, if the feedback signal is successfully received with a probability of $(1 - p_{FB}^{STA}[k])$, the controller side can ideally obtain the control-state $(\tilde{x}[k+1] = x[k+1])$ and thus the estimated error becomes zero $(e[k+1] = 0)$. If the feedback communication fails with a probability of $p_{\text{FB}}^{\text{STA}}[k]$, similar to the case with ACK mode, the observer needs to estimate with the previously estimated control-state and control-inputs. As a result, the estimation error can be given by

$$
e[k+1] = \begin{cases} 0 & \text{with a prob. of } 1 - p_{\text{FB}}^{\text{STA}}[k] \\ Ae[k] - p_{\text{FF}}^{\text{STA}}[k]Bu[k] + Dw[k] \\ & \text{with a prob. of } p_{\text{FB}}^{\text{STA}}[k] (1 - p_{\text{FF}}^{\text{STA}}[k]) \\ Ae[k] + (1 - p_{\text{FF}}^{\text{STA}}[k])Bu[k] + Dw[k] \\ & \text{with a prob. of } p_{\text{FB}}^{\text{STA}}[k] p_{\text{FF}}^{\text{STA}}[k] \end{cases}
$$
 (22)

In the following, the error vector defined as $\mathbf{E} = [e[k +]$ 1], $e[k + 2], \cdots, e[k + L]]^T$ is used.

C. COST FUNCTION CORRESPONDING TO THE APPLIED PROTOCOL AND POWER-ALLOCATION FACTOR

The control-state vector for the *L* prediction period **X** can be given by $\mathbf{X} = \tilde{\mathbf{X}} + \mathbf{E}$ where $\tilde{\mathbf{X}} = \left[\tilde{x}[k+1], \cdots, \tilde{x}[k+L] \right]^T$. Substituting X into Eq. (17) , the cost function depending on the scheduling vector M , power allocation vector Λ , and the initial estimated control-state $\tilde{x}[k]$, can be rewritten as

$$
J_k (\mathbf{U}, \mathbf{M}, \Lambda, \tilde{x}[k])
$$

= $\mathbb{E}\left[\left(\tilde{\mathbf{X}} + \mathbf{E} \right)^T \bar{Q} \left(\tilde{\mathbf{X}} + \mathbf{E} \right) + \mathbf{U}^T \bar{R} \mathbf{U} | \mathbf{M}, \Lambda, \tilde{x}[k] \right]$
= $\mathbb{E}\left[\tilde{\mathbf{X}}^T \bar{Q} \tilde{\mathbf{X}} + \mathbf{E}^T \bar{Q} \mathbf{E} | \mathbf{M}, \Lambda, \tilde{x}[k] \right] + \mathbf{U}^T \bar{R} \mathbf{U},$ (24)

where $\mathbb{E}\left[\tilde{\mathbf{X}}^T\bar{Q}\mathbf{E}\right] = 0$ since $\mathbb{E}\left[e[k+l]\right] = 0$, $(l =$ $1, \dots, L$). Let us focus on the first term of Eq. [\(24\)](#page-5-2), i.e., $\mathbb{E} \left[\tilde{\mathbf{X}}^T \bar{Q} \tilde{\mathbf{X}} \, | \, \mathbf{M}, \, \Lambda, \tilde{\mathbf{x}}[k] \right]$. Since $\tilde{\mathbf{X}}$ is the vector composed of the expected control-state for the *L* predictive period, the *l*th component of **X**˜ can be given by

$$
\tilde{x}[k+l] = A\tilde{x}[k+l-1] + (1 - p_{\text{FF}}^{\mathcal{M}}[k+l-1])Bu[k+l-1].
$$
 (25)

And thus, the matrix form can be given by

$$
\tilde{\mathbf{X}} = \mathbf{A}\tilde{x}[k] + \mathbf{B}'\mathbf{U}
$$
 (26)

where B' is the modified B that takes into account the error probability of the feedforward channel, concretely, the row components from $((l - 1) n_x + 1)$ to ln_x of \mathbf{B}' $(l = 1, \dots, L)$ can be given by

$$
\mathbf{B}'_{(l)} = \left(1 - p_{\text{FF}}^{\mathcal{M}}[k+l-1]\right) \mathbf{B}_{(l)}.
$$
 (27)

As described in Sec. [II-A,](#page-2-0) $p_{\text{FF}}^{\mathcal{M}}$ is the function of the applied mode M and the power allocation vector Λ . Substituting Eq. [\(26\)](#page-6-0) into the the first term of Eq. [\(24\)](#page-5-2), it can be expressed as

$$
\mathbb{E}\left[\tilde{\mathbf{X}}^T\bar{Q}\tilde{\mathbf{X}} | \mathbf{M}, \Lambda, \tilde{x}[k]\right] \n= \mathbb{E}\left[\left(\mathbf{A}\tilde{x}[k] + \mathbf{B}'\mathbf{U}\right)^T \bar{Q} \left(\mathbf{A}\tilde{x}[k] + \mathbf{B}'\mathbf{U}\right)\right] \n= \mathbf{U}^T \left(\mathbf{B}'^T \bar{Q} \mathbf{B}'\right) \mathbf{U} + 2\tilde{x}[k]^T \mathbf{A}^T \bar{Q} \mathbf{B}' \mathbf{U} \n+ \mathbb{E}\left[\tilde{x}[k]^T \mathbf{A}^T \bar{Q} \mathbf{A}\tilde{x}[k]\right].
$$
\n(28)

Since B' depends on the error probabilities of the feedforward channel corresponding to the transmission modes **M** and power allocation factors Λ applied in the L prediction periods, the first and second terms in Eq. [\(28\)](#page-6-1) depends on them as well as the control-input **U**. The last term in Eq. [\(28\)](#page-6-1) is independent of the optimization factors and thus can be ignored in the optimization process given by Eq. [\(16\)](#page-5-1).

Next, we focus on the second term of Eq. [\(24\)](#page-5-2), i.e., $\mathbb{E}[\mathbf{E}^T\tilde{Q}\mathbf{E}|\mathbf{M}, \Lambda, \tilde{\chi}[k]]$, which is the expected estimation error at the controller side for the *L* prediction periods. $\mathbb{E}[\mathbf{E}^T \bar{Q} \mathbf{E} | \mathbf{M}, \Lambda, \tilde{\chi}[k]]$ can be decomposed into the following three factors: 1) the uncertainty of whether the control-input can be successfully input, 2) the uncertainty due to the disturbance added into the controlled object, and 3) the uncertainty attributed to the initial estimation error $e[k]$, which can be expressed as

$$
\mathbb{E}\left[\mathbf{E}^T\bar{Q}\mathbf{E} | \mathbf{M}, \Lambda, \tilde{\mathbf{x}}[k]\right] = f_u(\mathbf{U}, \mathbf{M}, \Lambda) + f_w(\mathbf{W}, \mathbf{M}, \Lambda) + f_{e_0}(e[k], \mathbf{M}, \Lambda). \quad (29)
$$

We first consider the first term of Eq. [\(29\)](#page-6-2), which depends on the control-inputs and communication reliability of the feedforward and feedback channels. Given the transmission mode vector M and the power allocation vector Λ applied to the *L* prediction periods, the communication reliability of the feedforward and feedback channels can be determined

as described in Sec[.II-A.](#page-2-0) Let $p_3^{\mathcal{M}}[k+l]$ be the probability that the residual error remains at the *l*th prediction time-slot, which is defined as

$$
p_3^{\mathcal{M}}[k+l] = \begin{cases} 1 & \text{if } \mathcal{M}[k+l] = \text{ACK} \\ p_{\text{FB}}^{\text{STA}}[k+l] & \text{if } \mathcal{M}[k+l] = \text{STA.} \end{cases} (30)
$$

The first term of Eq. [\(29\)](#page-6-2) can be given by

$$
f_u(\mathbf{U}, \mathbf{M}, \Lambda) = \mathbb{E}\left[\mathbf{U}^T \bar{Q}_{uu} \mathbf{U}\right] = \mathbf{U}^T \mathbb{E}\left[\bar{Q}_{uu}\right] \mathbf{U}, \quad (31)
$$

where

$$
\mathbb{E}\left[\bar{Q}_{\text{uu}}\right] = \text{diag}\left(Q_{\text{uu}}[1], \cdots, Q_{\text{uu}}[L]\right),\tag{32}
$$
\n
$$
Q_{\text{uu}}[l] = p_{\text{uu}}'(l) \sum_{n=l}^{L} \left(A^{L-n}B\right)^{T} Q\left(A^{L-n}B\right) \cdot p_{\text{uu}}''[l, n],
$$

$$
p'_{\text{uu}}[l] = p'^{M}_{\text{FB}}[k+l-1]p'^{M}_{\text{FF}}[k+l-1] \times \left(1 - p'^{M}_{\text{FF}}[k+l-1]\right), \tag{34}
$$

(33)

$$
p_{\text{uu}}^{"}[l,n] = \prod_{m=l+1}^{L-n+l} p_3^{\mathcal{M}}[k+m-1].
$$
 (35)

For details, see Appendix. We next consider the second term of Eq. [\(29\)](#page-6-2) which represents the expected cumulative disturbance corresponding to the selected transmission mode and power allocation and can be given by

$$
f_{w}(\mathbf{W}, \mathbf{M}, \Lambda) = \mathbf{W}_{w}^{T} \mathbb{E} \left[\bar{Q}_{ww} \right] \mathbf{W}_{w}, \tag{36}
$$

$$
\mathbf{W}_w = [\sigma_w, \cdots, \sigma_w]^T \in \mathbb{R}^{Ln_x \times 1}, \tag{37}
$$

$$
\mathbb{E}\left[\bar{Q}_{ww}\right] = \text{diag}\left(Q_{ww}[1], \cdots, Q_{ww}[L]\right),\tag{38}
$$

$$
Q_{\text{ww}}[l] = \sum_{n=l}^{L} \left(A^{L-n} D \right)^{T} Q \left(A^{L-n} D \right) \cdot p_{\text{ww}}[l, n],
$$
\n(39)

$$
p_{\text{ww}}[l,n] = \prod_{m=l}^{L-n+l} p_3^{\mathcal{M}}[k+m-1]. \tag{40}
$$

Finally, we consider the third term of Eq. [\(29\)](#page-6-2) which represents the expected cumulative estimation error attributed to the initial estimation error and can be given by

$$
f_{e_0}(e[k], \mathbf{M}, \Lambda) = \mathbb{E}\left[e^T[k]Q_{e_0}e[k]\right],\tag{41}
$$

$$
Q_{\mathbf{e}_0} = \sum_{l=1}^{L} \left(\left(A^l \right)^T Q A^l \right) \cdot p_{\mathbf{e}_0}[l], \quad (42)
$$

$$
p_{e_0}[l] = \prod_{m=1}^{l} p_3^{\mathcal{M}}[k+m-1]. \tag{43}
$$

Only when STA mode was selected at the $(k-1)$ th (previous) time-slot and the feedback communication was successful, $e[k]$ becomes zero so that the third term (Eq. [\(41\)](#page-6-3)) also becomes zero. On the other hand, when ACK mode was selected at the previous time-slot, even if the feedback communication was successful, *e*[*k*] cannot be zero as shown in Eq. [\(20\)](#page-5-3). Thus, since *e*[*k*] depends on the selected mode and communication results of feedback channel from the time when STA mode was selected and feedback communication was successful to the $(k - 1)$ th time-slot, the controller needs to stock them to calculate $f_{e_0}(e[k], \mathbf{M}, \Lambda)$. For detailed proofs of Eqs. [\(29\)](#page-6-2)-[\(41\)](#page-6-3), see Appendix. Substituting from Eqs. [\(28\)](#page-6-1) and [\(29\)](#page-6-2) into Eq. [\(24\)](#page-5-2), the cost function can be given by

$$
J_k (\mathbf{U}, \mathbf{M}, \Lambda, \tilde{x}[k])
$$

= $\mathbf{U}^T (\mathbf{B}^T \bar{Q} \mathbf{B} + \bar{R} + \mathbb{E} [\bar{Q}_{uu}]) \mathbf{U} + 2\tilde{x}[k]^T \mathbf{A}^T \bar{Q} \mathbf{B}^{\prime} \mathbf{U}$
+ $\mathbf{W}_w^T \mathbb{E} [\bar{Q}_{ww}] \mathbf{W}_w + \mathbb{E} [e^T[k] Q_{e_0} e[k]] + \text{Cnt}, \quad (44)$

where Cnt is a constant value and does not affect on the optimization. Given the the transmission modes **M** and the power allocation factors Λ to be applied in the *L* prediction period, the cost function is a quadratic convex function and thus can be solved using quadratic programming. Note that the proposed optimization does not ensure stability, since the optimization is based on MPC and the focused WNCS has stochastic events in communication links.

D. COMPUTATIONAL COMPLEXITY

The computational complexity depends on the number of combinations of transmission protocols and power allocation factors. Since this work considers two type of transmission modes, the total number of transmission mode combinations to be applied during the *L* prediction period is 2^L . On the other hand, since the power allocation is as a continuous value, we convert it to discrete value in order to calculate the optimization based on a quadratic programming. As a result, the computational complexity depends on the number of candidates for λ. Given the number of candidates of power allocation factor as N_{λ} , the total number of power allocation factor combinations is N_{λ}^{L} . Since the power allocation factor and the applied transmission mode are independent, the order of calculation for the optimization is given by $O(2^L N_\lambda^L)$ if we use a brute-force approach. Note that the combination of the applied mode and power allocation factor is given by integer number, the optimization can be solved by a mixed integer programming (MIP), which leads to reduction of computational burden.

IV. NUMERICAL EVALUATION

A. SIMULATION SETTINGS

The parameters used in the numerical evaluations are given by as follows.

$$
A = \begin{bmatrix} 1.19 & 0.08 \\ 0.22 & 0.89 \end{bmatrix}, \quad B = \begin{bmatrix} 1.75 \\ 0.9 \end{bmatrix}, C = D = I_2,
$$

$$
x[0] = \begin{bmatrix} 50 \\ -50 \end{bmatrix}, \quad P = Q = R = I_2
$$
 (45)

The eigenvalues of matrix *A* are eig(A) = (1.2402, 0.8398), which consists of one stable and unstable values. Thus, *A* is unstable but the pair (A, B) is controllable. The coefficient matrices for the observer and the disturbance (i.e., *C* and *D*) are assumed to be identity matrices. Initial and target

FIGURE 2. Behavior of the proposed system in the case where SNR = 15 dB, $\sigma_W^2 = 1$. The upper part shows the control-state (blue and red solid lines) and communication results of feedforward (green dotted line with square mark) and feedback channels (red dotted line with triangle mark). The lower part shows the power allocation factor $\lambda[k]$ (red dotted line with square mark) and the selected transmission mode $\mathcal{M}[k]$ (blue solid line with cross mark).

control-states are set as $[50, -50]^T$ and $[0, 0]^T$, respectively. The weighting matrices for MPC optimization (i.e., *P*, *Q*, and *R*) are assumed to be identity matrices. The mean square error (MSE) defined as $\mathbb{E} \left[|x[k]|_2^2 \right]$ is used for the evaluation of quality of control. The signal-to-noise ratio (SNR) is defined as SNR = $10 \log_{10} P_T/N_0$ in dB, where P_T is the total transmitting power of the feedforward and feedback channels and N_0 is the noise power of each channel (i.e., $N_0 = \mathbb{E}[|n_{\text{FF}}|^2] =$ $\mathbb{E}[|n_{FF}|^2] = 1$. The required transmission rate of feedback channel is determined by the applied transmission mode. The transmission rate of feedforward is set as $R_{FF} = 3$ bps/Hz for both ACK and STA, while that of feedback is set as $R_{\text{FB}}^{\text{ACK}} =$ 0.25 bps/Hz or $R_{\text{FB}}^{\text{STA}}$ = 6 bps/Hz for ACK or STA mode. Note that, even though the AKC/NACK transmission can be accomplished with one bit, the data transmission requires several bits for the header. Thus, given the transmission mode and power allocation factor, the error probabilities of the feedforward and feedback channels are uniquely determined. As described in Sec. [III-D,](#page-7-0) the computational complexity exponentially increases in proportion to the prediction period *L* and the number of candidates of power allocation factor $\lambda[k]$. To make the computational complexity for the optimization feasible, we assume that the power allocation factors for the cases with ACK and STA modes are fixed during the prediction period, that is, $\lambda_{\text{ACK}}[k] = \lambda_{\text{ACK}}[k+1] = \cdots$ $\lambda_{\text{ACK}}[k+L-1]$ and $\lambda_{\text{STA}}[k] = \lambda_{\text{STA}}[k+1] = \cdots = \lambda_{\text{STA}}[k+1]$ *L* − 1]. As a result, the computational complexity can be reduced to $O(2^L \times 2N_\lambda)$. Specifically, we set as $L = 3$, $\lambda_{\text{ACK}} \in$ $\{0, 0.1, 0.2, \cdots, 1\}$, and $\lambda_{\text{STA}} \in \{0, 0.1, 0.2, \cdots, 1\}$ in the following evaluations and thus the computational complexity becomes $O(2^3 \times 22)$.

B. BEHAVIOR OF THE PROPOSED OPTIMIZATION

Fig. [2](#page-7-1) shows the trajectory of control-state and its corresponding power allocation factor in the case where $SNR = 15 dB$

and the variance of disturbance $\sigma_w^2 = 1$. The upper part of Fig. [2](#page-7-1) shows the control-state and communication results, where the green and red dotted lines show the error events at the feedforward and feedback channels, respectively. The lower part of Fig. [2](#page-7-1) shows the power allocation factor and the selected transmission mode $\mathcal{M}[k]$, where the blue solid line and red dotted line show the power allocation factor $\lambda[k]$ and the selected transmission mode $\mathcal{M}[k]$, specifically, if ACK (or STA) mode is selected, the value of red line has "0" (or "1"). For reference, in the case where ACK mode is selected and power allocation factor is 0.1, the error (outage) probabilities of the feedforward and feedback channels are given by $p_{\text{FF}}^{\text{ACK}} = 0.1157$ and $p_{\text{FB}}^{\text{ACK}} = 0.0295$. Whereas, in the case where STA mode is selected and power allocation factor is 0.9, those are given by $p_{\text{FF}}^{\text{STA}} = 0.6694$ and $p_{\text{FB}}^{\text{STA}} =$ 0.6694. Apparently, the SNR is not enough to make a reliable communication in the case with STA mode. If ACK mode is selected, the communication is more reliable than the case with STA mode but the controller side cannot estimate correct control-state. On the other hand, if STA mode is selected, the controller attempts to estimate the true control-state with unreliable probability. The proposed system tends to select ACK mode in the early stages of control in order to prioritize the input of control-input to the controlled object. After the burst communication error at the feedback channel (e.g., time-slot = 3 \sim 4, 9 \sim 15), the proposed system tends to select STA mode in order to eliminate the estimation errors. At the end of the control, the proposed system tends to select the STA mode in order to achieve accurate control (suppress the effects of disturbance).

C. AVERAGE PERFORMANCE OF CONTROL QUALITY Figs. [3](#page-8-0) and [4](#page-8-1) show the control performances of the proposed and the conventional systems averaged over 10000 trials.

FIGURE 4. Control performance comparison between the conventional and proposed systems in the case where SNR = 20 dB and $\sigma_{\bm{w}}^2 = 1$ or 0.01.

As the conventional systems, three types of the conventional systems are used, (1) ACK mode fixed and adaptive power allocation, (2) STA mode fixed and adaptive power allocation, (3) adaptive mode switching but the fixed power allocation where $p_{\text{FF}}^{\mathcal{M}} = p_{\text{FB}}^{\mathcal{M}}$ is achieved. The conventional systems (1) and (2) are the same systems proposed in [24]. Fig. [3](#page-8-0) shows the case where $SNR = 15$ dB. The system with ACK mode and adaptive power allocation can achieve well control up to around the 9th time-slot, because the control-input can be successfully supplied to the controlled object with a high probability. However, after around the 10th time slot, the control-state diverges to infinity due to the accumulated estimation errors caused by disturbances. The system with STA mode and adaptive power allocation cannot achieve stable control due to unreliable communication links. The system with adaptive mode switching and fixed power allocation factor can improve the performance compared to the other conventional systems. For reference, when applying ACK mode and $\lambda = 0.0263$, the outage probabilities of feedforward and feedback channels can be given by $p_{\text{FF}}^{\text{ACK}} = p_{\text{FB}}^{\text{ACK}} =$ 0.1074. Whereas, when applying STA mode and $\lambda = 0.900$, those are given by $p_{\text{FF}}^{\text{STA}} = p_{\text{FB}}^{\text{STA}} = 0.6694$. Since the feedback channel is unreliable in the case of STA mode, the system is still unstable. The proposed system can achieve the best performance since the communication mode and transmission powers can be adaptively selected considering the control-state and estimation accuracy. Therefore, the proposed system can efficiently work even in the WNCS with unreliable communication links.

Figs. [4](#page-8-1) shows the control performances in the case where $SNR = 20$ dB. Similar to the case of $SNR = 15$ dB, the system with ACK-fixed mode cannot achieve stable control due to

the accumulated estimation errors. The other three systems have almost the same performances, but the proposed system can still achieve the best performance. When applying STA mode and $\lambda = 0.900$, the outage probabilities of feedforward and feedback channels can be given by $p_{\text{FF}}^{\text{STA}} = p_{\text{FB}}^{\text{STA}} =$ 0.2953 which is relatively more reliable than the case of $SNR = 15$ dB. As a result, the probability that STA mode is selected becomes higher than the case of $SNR = 15$ dB and thus the performance gaps among three systems become small. From above evaluations, the proposed system always works most effectively at the expense of increased computational complexity.

V. CONCLUSION

This work has provided the design criterion for the WNCS with unreliable communication links based on a cross-layer approach between the communication and control layers. Specifically, the cost function considering the control-state and the reliability of communication links has been provided. By minimizing the cost function, the transmission mode (feedback information), transmission power, and control-input have been jointly optimized based on MPC

optimization method. Simulation results have showed that the proposed scheme can improve the quality of control in the WNCS with unreliable communication links by efficiently selecting feedback information and allocating the transmission power considering the control-state and the accuracy of estimation.

APPENDIX

Here, we provide the details of Eqs. [\(29\)](#page-6-2)-[\(41\)](#page-6-3). Given the transmission modes and power allocation factors to be applied during the prediction period *L*, the event probability, whether the feedforward and/or feedback communication can be successfully done, can be given so that the expected estimation error can be computed. Concretely, the expected estimation error for the prediction period can be sequentially computed based on Eqs.[\(20\)](#page-5-3), [\(21\)](#page-5-4), and [\(22\)](#page-5-5). As an example, consider the case where the prediction period is $L = 3$ and the applied power allocation factor and transmission mode are Λ and $M = \{STA, ACK, STA\}$, respectively. Regardless of the applied transmission mode, there are four types of event: (1) The feedforward and feedback communications fail. (2) The feedforward communication fails and the feedback communication succeeds. (3) The feedforward communication succeeds and the feedback communication fails. (4) The feedforward and feedback communications succeed. Since the transmission mode and power allocation factor are given, the individual probabilities corresponding to each event can be calculated, and the expected value can be calculated with these probabilities. Table [1](#page-9-0) shows the controlstate, the estimated control-state, and the estimation error corresponding to each event. As seen in Table [1](#page-9-0) and Eqs. [\(20\)](#page-5-3), [\(21\)](#page-5-4), and [\(22\)](#page-5-5), when STA mode is applied, if the feedback communication is successful, the estimation errors, including residual error, can be completely eliminated. On the other hand, when ACK mode is applied, even if the feedback communication is successful, the estimation errors still remain and the uncertainty due to disturbance is added. Considering above facts, the expected square estimation errors at the $k+1$, $k + 2$, and $k + 3$ can be calculated as follows;

$$
\mathbb{E}[e[k+1]^{T}Qe[k+1]]
$$
\n
$$
= p_{FB}^{STA}[k]^{T} \mathbb{E}[(Ae[k])^{T}Q(Ae[k])]
$$
\n
$$
+ p_{FF}^{STA}[k](1 - p_{FF}^{STA}[k])p_{FB}^{STA}[k](Bu[k])^{T}Q(Bu[k])
$$
\n
$$
+ p_{FB}^{STA}[k](Dw[k])^{T}QDw[k], \qquad (46)
$$
\n
$$
\mathbb{E}[e[k+2]^{T}Qe[k+2]]
$$
\n
$$
= \mathbb{E}[(Ae[k+1])^{T}Q(Ae[k+1])]
$$
\n
$$
+ p_{FF}^{ACK}[k+1](1 - p_{FF}^{ACK}[k+1])p_{FB}^{ACK}[k+1]
$$
\n
$$
\times (Bu[k+1])^{T}Q(Bu[k+1])
$$
\n
$$
+ (Dw[k+1])^{T}QDw[k+1]
$$

$$
= p_{FB}^{STA}[k] \mathbb{E}[(A^{2}e[k])^{T}Q(A^{2}e[k])]
$$

\n
$$
+ p_{FF}^{STA}[k][(1 - p_{FF}^{STA}[k])p_{FB}^{STA}[k](ABu[k])^{T}Q(ABu[k])
$$

\n
$$
+ p_{FB}^{STA}[k](ADw[k])^{T}QADw[k], \qquad (47)
$$

\n
$$
\mathbb{E}[e[k + 3]^{T}Qe[k + 3]]
$$

\n
$$
= p_{FB}^{STA}[k + 2]
$$

\n
$$
\times \mathbb{E}[(Ae[k + 2])^{T}Q(Ae[k + 2])]
$$

\n
$$
+ p_{FF}^{STA}[k + 2](1 - p_{FF}^{STA}[k + 2])p_{FB}^{STA}[k + 2]
$$

\n
$$
\times (Bu[k + 2])^{T}Q(Bu[k + 2])
$$

\n
$$
+ p_{FB}^{STA}[k + 2](Dw[k + 2])^{T}QDw[k + 2]
$$

\n
$$
= p_{FB}^{STA}[k]p_{FB}^{STA}[k + 2]\mathbb{E}[(A^{3}e[k])^{T}Q(A^{3}e[k])]
$$

\n
$$
+ p_{FB}^{STA}[k]p_{FB}^{STA}[k + 2](A^{2}Dw[k])^{T}QA^{2}Dw[k]
$$

\n
$$
+ p_{FB}^{STA}[k + 2](ADw[k + 1])^{T}QADw[k + 1]
$$

\n
$$
+ p_{FF}^{STA}[k + 2](Dw[k + 2])^{T}QDw[k + 2]
$$

\n
$$
\times (A^{2}Bu[k])^{T}Q(A^{2}Bu[k])
$$

\n
$$
+ p_{FF}^{ACA}[k + 1](1 - p_{FF}^{STA}[k])p_{FB}^{STA}[k + 2]
$$

\n
$$
\times (ABu[k + 1])^{T}Q(ABu[k + 1])
$$

\n
$$
+ p_{FF}^{STA}[k + 2](1 - p_{FF}^{STA}[k + 2])p_{FB}^{STA}[k + 2]
$$

\n
$$
\times (Bu[k + 2])^{T}Q(Bu[k + 2]).
$$

\n(48)

With Eqs. [\(46\)](#page-10-0), [\(47\)](#page-10-0), and [\(48\)](#page-10-0), the components of the estimation error in the evaluation function defined in [\(29\)](#page-6-2), can be calculated as Eq. [\(49\)](#page-10-1), shown at the bottom of the page. The probability coefficients p'_{uu} , p''_{uu} , p_{ww} , p_{e_0} are composed of outage probabilities at the feedforward and feedback channels

as seen in Eqs. (34) , (35) , (40) , and (43) , and are determined by the applied communication protocols and power allocation factors.

REFERENCES

- [1] P. Park, S. Ergen, C. Fischione, C. Lu, and K. Johansson, "Wireless network design for control systems: A survey,'' *IEEE Commun. Surveys Tuts.*, vol. 20, no. 2, pp. 978–1013, Dec. 2017.
- [2] D. Zhang, P. Shi, Q.-G. Wang, and L. Yu, ''Analysis and synthesis of networked control systems: A survey of recent advances and challenges,'' *ISA Trans.*, vol. 66, pp. 376–392, Jan. 2017.
- [3] P. Antsaklis and J. Baillieul, ''Special issue on technology of networked control systems,'' *Proc. IEEE*, vol. 95, no. 1, pp. 5–8, Jan. 2007.
- [4] R. A. Gupta and M.-Y. Chow, ''Networked control system: Overview and research trends,'' *IEEE Trans. Ind. Electron.*, vol. 57, no. 7, pp. 2527–2535, Jul. 2010.
- [5] B. Hu, Y. Wang, P. V. Orlik, T. Koike-Akino, and J. Guo, ''Co-design of safe and efficient networked control systems in factory automation with state-dependent wireless fading channels,'' *Automatica*, vol. 105, pp. 334–346, Jul. 2019.
- [6] P. A. M. Devan, F. A. Hussin, R. Ibrahim, K. Bingi, and F. A. Khanday, "A survey on the application of WirelessHART for industrial process monitoring and control,'' *Sensors*, vol. 21, no. 15, p. 4951, Jul. 2021.
- [7] E. Witrant, P. Di Marco, P. Park, and C. Briat, ''Limitations and performances of robust control over WSN: UFAD control in intelligent buildings,'' *IMA J. Math. Control Inf.*, vol. 27, no. 4, pp. 527–543, Dec. 2010.
- [8] R. Nagel, S. Eichler, and J. Eberspacher, ''Intelligent wireless communication for future autonomous and cognitive automobiles,'' in *Proc. IEEE Intell. Vehicles Symp.*, Jun. 2007, pp. 716–721.
- [9] M. N. Ahangar, Q. Z. Ahmed, F. A. Khan, and M. Hafeez, ''A survey of autonomous vehicles: Enabling communication technologies and challenges,'' *Sensors*, vol. 21, no. 3, p. 706, Jan. 2021.
- [10] Y. Pan, Q. Li, H. Liang, and H.-K. Lam, ''A novel mixed control approach for fuzzy systems via membership functions online learning policy,'' *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 9, pp. 3812–3822, Sep. 2021, doi: [10.1109/TFUZZ.2021.3130201.](http://dx.doi.org/10.1109/TFUZZ.2021.3130201)
- [11] F. Li, F. Yang, and Q. Li, "Event-triggered consensus control for nonlinear multiagent systems with unmeasured state variables,'' *IEEE Access*, vol. 10, pp. 77030–77038, 2022.
- [12] H. Liang, Z. Du, T. Huang, and Y. Pan, "Neuroadaptive performance guaranteed control for multiagent systems with power integrators and unknown measurement sensitivity,'' *IEEE Trans. Neural Netw. Learn. Syst.*, early access, Mar. 29, 2022, doi: [10.1109/TNNLS.2022.3160532.](http://dx.doi.org/10.1109/TNNLS.2022.3160532)
- [13] M. Ogura and V. M. Preciado, "Stability of SIS spreading processes in networks with non-Markovian transmission and recovery,'' *IEEE Trans. Control Netw. Syst.*, vol. 7, no. 1, pp. 349–359, Mar. 2020.
- [14] P. Godara, S. Herminghaus, and K. M. Heidemann, ''A control theory approach to optimal pandemic mitigation,'' *PLoS ONE*, vol. 16, pp. 1–16, Feb. 2021, doi: [10.1371/journal.pone.0247445.g006.](http://dx.doi.org/10.1371/journal.pone.0247445.g006)
- [15] Y. Siriwardhana, P. Porambage, M. Ylianttila, and M. Liyanage, ''Performance analysis of local 5G operator architectures for industrial internet,'' *IEEE Internet Things J.*, vol. 7, no. 12, pp. 11559–11575, Dec. 2020.
- [16] A. Dogra, R. K. Jha, and S. Jain, "A survey on beyond 5G network with the advent of 6G: Architecture and emerging technologies,'' *IEEE Access*, vol. 9, pp. 67512–67547, 2020.
- [17] W. Jiang, B. Han, M. A. Habibi, and H. D. Schotten, ''The road towards 6G: A comprehensive survey,'' *IEEE Open J. Commun. Soc.*, vol. 2, pp. 334–366, 2021.
- [18] S. Tatikonda and S. Mitter, "Control under communication constraints," *IEEE Trans. Autom. Control*, vol. 49, no. 7, pp. 1056–1068, Jul. 2004.
- [19] L. Schenato, B. Sinopoli, M. Franceschetti, K. Poolla, and S. S. Sastry, ''Foundations of control and estimation over lossy networks,'' *Proc. IEEE*, vol. 95, no. 1, pp. 163–187, Jan. 2007.
- [20] K. Okano and H. Ishii, ''Stabilization of uncertain systems with finite data rates and Markovian packet losses,'' *IEEE Trans. Control Netw. Syst.*, vol. 1, no. 4, pp. 298–307, Dec. 2014.
- [21] K. Tsumura, K. Ishii, and H. Hoshina, ''Tradeoffs between quantization and packet loss in networked control of linear systems,'' *Automatica*, vol. 45, no. 12, pp. 2963–2970, Dec. 2009.
- [22] H. Ishii, "Limitations in remote stabilization over unreliable channels without acknowledgements,'' *Automatica*, vol. 45, no. 10, pp. 2278–2285, Oct. 2009.
- [24] K. Ishii, ''MPC based power allocation for reliable wireless networked control systems,'' *IEEE Access*, vol. 9, pp. 60913–60922, 2021.
- [25] K. Gatsis, A. Ribeiro, and G. J. Pappas, "Optimal power management in wireless control systems,'' *IEEE Trans. Autom. Control*, vol. 59, no. 6, pp. 1495–1510, Jun. 2014.
- [26] A. Kundu and D. E. Quevedo, "Stabilizing scheduling policies for networked control systems,'' *IEEE Trans. Control Netw. Syst.*, vol. 7, no. 1, pp. 163–175, Mar. 2020.
- [27] E. G. W. Peters, D. E. Quevedo, and M. Fu, "Controller and scheduler codesign for feedback control over IEEE 802.15.4 networks,'' *IEEE Trans. Control Syst. Technol.*, vol. 24, no. 6, pp. 2016–2030, Nov. 2016.
- [28] Y. Sadi and S. C. Ergen, ''Joint optimization of wireless network energy consumption and control system performance in wireless networked control systems,'' *IEEE Trans. Wireless Commun.*, vol. 16, no. 4, pp. 2235–2248, Apr. 2017.
- [29] R. Alur, A. D'Innocenzo, K. H. Johansson, G. J. Pappas, and G. Weiss, ''Compositional modeling and analysis of multi-hop control networks,'' *IEEE Trans. Autom. Control*, vol. 56, no. 10, pp. 2345–2357, Oct. 2011.
- [30] W. Liu, P. Popovski, Y. Li, and B. Vucetic, ''Wireless networked control systems with coding-free data transmission for industrial IoT,'' *IEEE Internet Things J.*, vol. 7, no. 3, pp. 1788–1801, Dec. 2021.
- [31] A. S. Leong, D. E. Quevedo, D. Dolz, and S. Dey, "Transmission scheduling for remote state estimation over packet dropping links in the presence of an eavesdropper,'' *IEEE Trans. Autom. Control*, vol. 64, no. 9, pp. 3732–3739, Sep. 2019.
- [32] M. S. Chong, H. Sandberg, and A. M. H. Teixeira, ''A tutorial introduction to security and privacy for cyber-physical systems,'' in *Proc. 18th Eur. Control Conf. (ECC)*, Jun. 2019, pp. 968–978.
- [33] Y. Pan, Y. Wu, and H.-K. Lam, "Security-based fuzzy control for nonlinear networked control systems with DoS attacks via a resilient event-triggered scheme,'' *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 10, pp. 4359–4368, Oct. 2022, doi: [10.1109/TFUZZ.2022.3148875.](http://dx.doi.org/10.1109/TFUZZ.2022.3148875)
- [34] J. G. Proakis, *Digital Communications*, 5th ed. New York, NY, USA: McGraw-Hill, 2008.
- [35] J. Nocedai and S. Wright, *Numerical Optimization* (Operations Research and Financial Engineering). New York, NY, USA: Springer, 2006.
- [36] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.
- [37] M. Farina, L. Giulioni, and R. Scattolini, ''Stochastic linear model predictive control with chance constraints—A review,'' *J. Process Control*, vol. 44, pp. 53–67, Aug. 2016.
- [38] T. A. Heifung, J. A. Paulson, J. O'Leary, and A. Mesbah, ''Stochastic model predictive control - how does it work?'' *Comput. Chem. Eng.*, vol. 114, pp. 158–170, Jun. 2018.
- [39] J. Nilsson, Real-time control system with delays, Ph.D. dissertation, Dept. Autom. Control, Lund Inst. Technol., Lund, Sweden, 1998.
- [40] X. Ge, F. Yang, and Q.-L. Han, "Distributed networked control systems: A brief overview,'' *Inf. Sci.*, vol. 380, pp. 117–131, Feb. 2017.

KOJI ISHII (Senior Member, IEEE) received the B.E., M.E., and Ph.D. degrees in electrical and computer engineering from Yokohama National University, Japan, in 2000, 2002, and 2005, respectively. He has been with the Faculty of Engineering, Kagawa University, Japan, since 2005, and currently works as an Associate Professor. From 2012 to 2013, he was a Visiting Scholar at The University of Sydney, Sydney, Australia. He is a member of SCI and IEICE.