IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received 28 August 2022, accepted 3 October 2022, date of publication 28 October 2022, date of current version 4 November 2022.

Digital Object Identifier 10.1109/ACCESS.2022.3217812

RESEARCH ARTICLE

Optimization of the Walking Robot Parameters on the Basis of Isotropy Criteria

SAYAT IBRAYEV[1], ARMAN IBRAYEVA[1,2], NUTPULLA JAMALOV[1,2], AND SAROSH H. PATEL[3]

[1]Institute of Mechanics and Engineering, Almaty 050013, Kazakhstan
[2]Mechanical Engineering Department, Al-Farabi Kazakh National University, Almaty 050040, Kazakhstan
[3]School of Engineering, University of Bridgeport, Bridgeport, CT 06604, USA

Corresponding author: Arman Ibrayeva (sayatqyzy@gmail.com)

This work was supported in part by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan under Grant AP09259589; and in part by the U.A. Joldasbekov Institute of Mechanics and Engineering, Almaty.

ABSTRACT Many authors proposed various criteria and concepts to assess manipulability of the robotic system. The isotropy criterion is used by other authors for synthesis of parallel manipulators, as a criterion of optimal force and transfer in all directions. In this research we applied this method for the first time for investigation and synthesis of turning mechanism of a walking robot. Isotropy conditions were derived for the walking robot with orthogonal-type propellers and on its basis the optimal configurations of the robot in terms of force and motion transmission are defined. Solutions of the isotropy equations were found and on the basis of the analysis of isotropic solutions, optimal metric parameters of the robot were found. An experimental prototype of the legged robot is developed.

INDEX TERMS Walking robot, turning mechanism, isotropy criteria, optimal synthesis.

I. INTRODUCTION

Various "measures" have been proposed to estimate how far the position of the mechanism is "remote" from the nearest singular position of the second kind. In Mechanisms and Machines Theory, for example, there is the concept of the quality of motion transmission (the closest concept in German literature is "Uebertragungsguete"). In the English-language literature, there are a number of concepts that are close in meaning, such as manipulability, the ability of force and motion transfer, the kinematic performance index, etc.

So, in work [1], as such a measure, the "transfer coefficient" is proposed as the product of the sines of the pressure angles related to the output and input links. Takeda et al. [2], [3] propose to use the sine of the pressure angle related to the output link, wherein consider the angle formed by the direction of motion of a rotational or spherical kinematic pair on the platform and the direction of the relative motion of this pair around the joint on the input link. Despite the fact that the equality of the named angle to zero is a sufficient condition of singularity, its magnitude is not linear with respect to the force transmission.

Yoshikava [4] uses the determinant of the Jacobian matrix as a criterion of manipulability, which transforms the generalized velocities at the input into the generalized velocities at the output object, and the determinant is a measure that reflects the transformation of a unit sphere into an ellipsoid, more precisely, the change in its dimensions along the principal axes. In other words, this measure indicates some integrated value of the velocity that is achieved by the generalized output velocities in different directions. Considering special cases of symmetric planar and spatial manipulators, Duffy [5] also introduced a measure based on the determinant of the Jacobian matrix. It is obvious that these measures are local criteria and depend on the configuration of the manipulator. Tsai [6] introduces a global criterion that is the integral of the square of the determinant (of a matrix that is the product of the Jacobian matrix and its transposition) over the entire working volume of the manipulator.

A clearer interpretation of the transfer criterion was described in the works of Angeles [7], [8], who proposed to consider as the measure the condition number k of the Jacobian matrix, which is equal to the ratio of the largest and

The associate editor coordinating the review of this manuscript and approving it for publication was Shuai Liu.

smallest eigenvalues. If we consider the ellipsoids of generalized velocities and generalized forces, then the number k reflects the uniformity of the motions and forces transmission. The maximum reciprocal value *1/k* over the entire working volume (which varies from zero to one), is called the kinematic conditioning index of the manipulator. In addition to the Chebyshev norm, the root mean square criterion is also introduced as an integral of the Euclidean norm over the entire working volume.

The most complete to date is the method of optimal synthesis of parallel manipulators, proposed by representatives of the German school. In the works of Schoenherr [9], [10], [11] a generalized functional (Guetefunktional) is introduced based on the use of the product of the weighted norm of the Jacobian matrix and the weighted norm of the inverse matrix and a method of optimal synthesis is presented.

The author's monograph [12] presents the results of researches on developing of synthesis methods and new kinematic schemes of planar manipulators, which are isotropic (in terms of force transfer) in the entire working area.

Consider a robot/manipulator with $W_q = n$ d.o.f. and generalized coordinates $\vec{q} = [q_1, q_2, \ldots, q_n]^T$. Assume that the output object also has $W_x = n$ d.o.f. Then the singular positions of the manipulator or robot, which are important in the analysis and synthesis of manipulators, are determined by the Jacobian matrix ($\boldsymbol{J}$): in such positions, its determinant is equal to zero: $\det \boldsymbol{J} = 0$. In the vicinity of such configurations, there is no unique solution to the direct and inverse kinematics. These positions define the boundaries of the working area of the manipulator [13], [14] and are worst in terms of force transfer. The same Jacobian matrix is used in [12] to define the best configurations in terms of force and motion transfer:

$$\boldsymbol{J}^T\boldsymbol{J} = \alpha^2\boldsymbol{E}, \tag{1.1}$$

or

$$\boldsymbol{J}\boldsymbol{J}^T = \alpha^2\boldsymbol{E}. \tag{1.2}$$

The configurations, satisfying (1.1) or (1.2), where E is the identity matrix and α is some real number, are called "isotropic", which are the furthest configurations from singularity.

In Section II provided are the brief description of a walking robot (WR) structure and its kinematic-equivalent scheme that simplifies the study of turning modes. The isotropy criterion (1.2) is applied to the quasi-planar WR in Section III and on its basis, in Section IV, the optimal geometrical parameters of the robot were defined. An experimental prototypes of the WR with decoupled motion are presented. The experimental studies of the synthesized turning mechanism will be provided in the future. In this research we considered only one structural scheme of the WR. Future directions include also investigation of different structural schemes.

II. STRUCTURAL MODEL OF THE ROBOT

Number of works were dedicated to investigation of six-legged walking robot turning mechanisms [15], [16], [17] however, in general case the number of drives in traditional bio-inspired systems turns out to be redundant, thus, it becomes necessary a "hard" coordination of movements of all drive motors. Errors in the control system and inaccuracies in kinematic transmissions cause inconsistency in movements and, as a result, increase loads in mechanical transmissions and drives [12], [18]. In [18] we presented an alternative design, based on the motion decoupling principle and justified the efficiency of this approach in terms of power consumption and control. Fig. 1a demonstrates the structure of one d.o.f. WR using rectilinear-guiding leg mechanism. The main (forward and backward) motion and the adaptation mechanism were tested on a prototype, shown in Fig. 1b, Fig.1c and the turning mechanism is under development on the same layout.

The additional joints $\boldsymbol{O_i}, (\boldsymbol{i} = \boldsymbol{1}, \ldots, \boldsymbol{6})$ with vertical axes are introduced as shown in Fig.2a to carry out turning. To simplify the study of the turning modes, an **equivalent** kinematic scheme of the WR is also presented (Fig.2b, 2c), where entire rectilinear guiding propellers of the robot were modeled as prismatic pairs $\boldsymbol{P_i}(\boldsymbol{i} = \boldsymbol{1}, \ldots, \boldsymbol{6})$.

Actuating the joints $\boldsymbol{O_i}$ to turn the robot will lead to structural redundancy that mentioned above. Thus, turning is carried out due to the difference in velocities of $\boldsymbol{P_i}$. An experimental prototype shown in Fig.1b, Fig.1c correspond to the kinematic-equivalent scheme in Fig.3a, Fig.3b.

III. DERIVATION OF THE ISOTROPY CRITERION

This section is devoted to defining criteria for the WR, which ensure optimal movement of the robot in terms of force/motion transmission. Consider a tripod gait, i.e., a common method, when three legs are in the support, three are in the transfer (lifted up) phase at all times. In the equivalent scheme (Fig.4) the first, third and fifth legs are in the support phase (feet S_1, S_3, S_5), C is the center of mass of the robot body/hull, $O_0\xi\eta\zeta$ is a global coordinate system fixed with the bearing surface, CXY is a coordinate system fixed with the robot body/hull. $O_iP_i(i = 1, 3, 5)$ is a local coordinate system, fixed with a link O_iP_i. The local coordinates of the joint S_i in this coordinate system are $x_{S_i} = a_i, y_{S_i} = q_i$, where q_i are generalized coordinates, $i = 1, 3, 5$.

For each foot S_i the following vector equation holds:

$$\overrightarrow{O_0S_i} = \overrightarrow{O_0C} + \overrightarrow{CO_i} + \overrightarrow{O_iS_i}, \quad i = 1, 3, 5, \tag{3.1}$$

or in terms of radius vectors of the joint centers,

$$\overrightarrow{R_{S_i}} = \overrightarrow{R_C} + \Gamma(\theta)\,\overrightarrow{r_{O_i}} + \Gamma(\theta + \alpha_i)\,\overrightarrow{r_{S_i}}, \tag{3.2}$$

where $\Gamma(\theta)$ is a rotation matrix:

$$\Gamma(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}, \quad \text{and } \overrightarrow{R_{S_i}} = \left[\xi_{S_i}, \eta_{S_i}\right]^T,$$

$\overrightarrow{R_C} = [\xi_C, \eta_C]^T$, $\overrightarrow{r_{O_i}} = \left[X_{O_i}, Y_{O_i}\right]^T$, $\overrightarrow{r_{S_i}} = [a_i, q_i]^T$ are the radius-vectors of the mass center C of the body/hull,

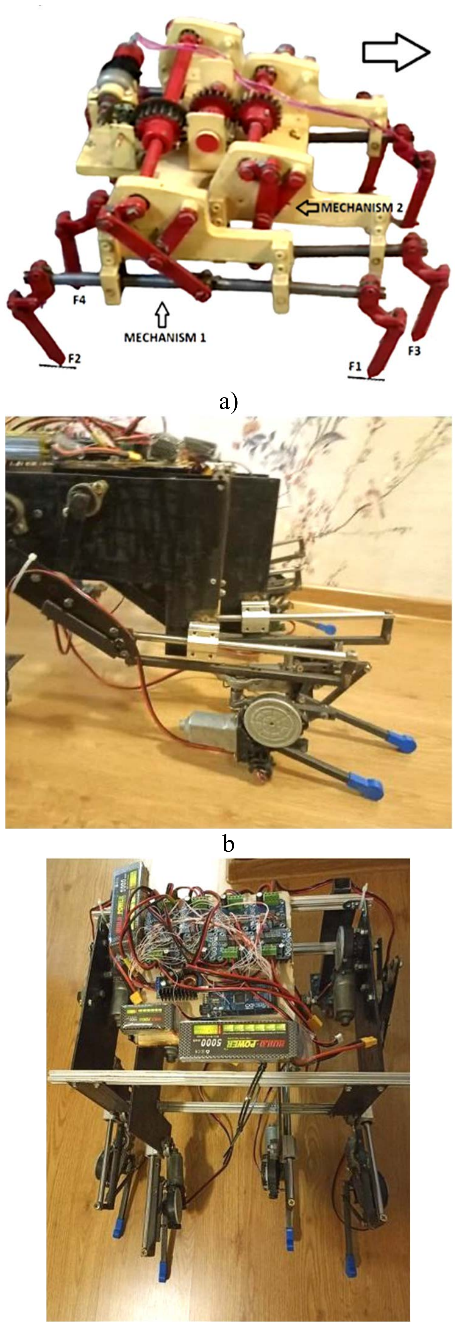


FIGURE 1. Prototypes of the WR with decoupled motion.

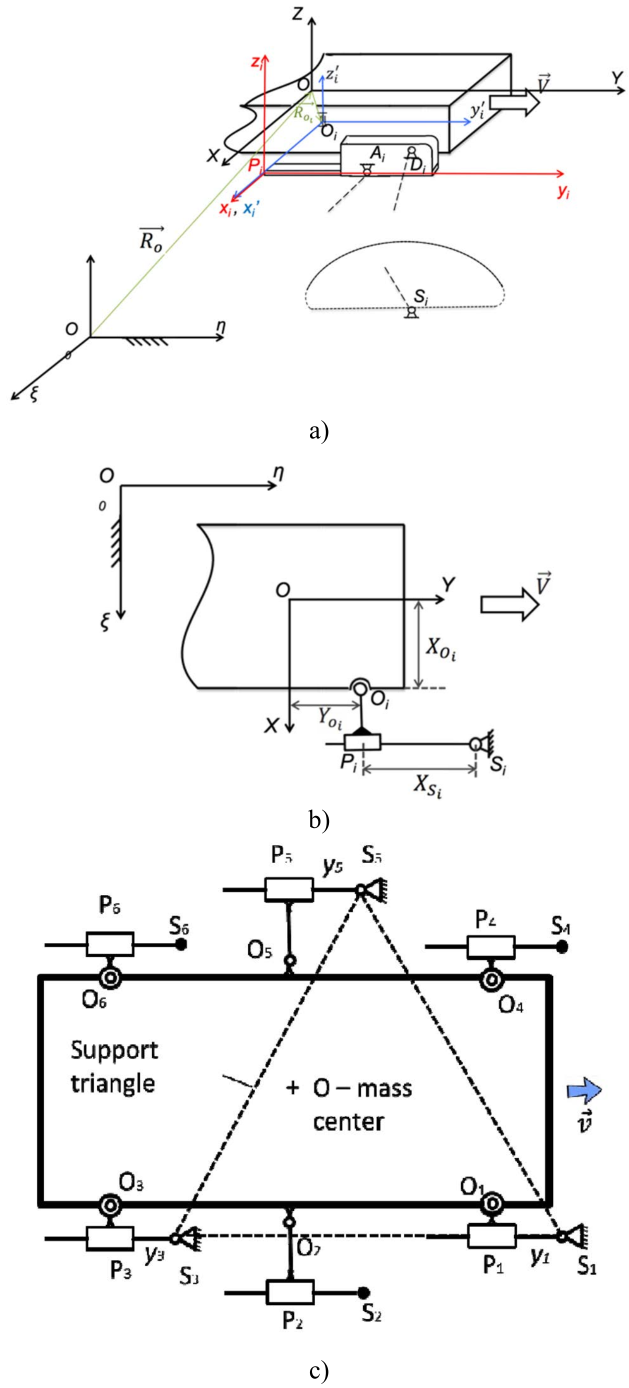


FIGURE 2. WR structural scheme on the plane $O\xi\eta$ with equivalent kinematics [18].

and the centers of the joints O_i, S_i in the coordinate systems $O_0\xi\eta\zeta$, CXY, $O_ix_iy_iz_i$ respectively. θ is a rotation angle of the robot body/hull with respect to the absolute coordinate system $O_0\xi\eta\zeta$.

Differentiation of (3.2) with respect to time gives the following correlation:

$$\vec{0} = \dot{\overrightarrow{R_C}} + \dot{\theta}\Gamma\left(\theta + \frac{\pi}{2}\right)\overrightarrow{r_{O_i}} + \left(\dot{\theta} + \dot{\alpha}_i\right)\Gamma\left(\theta + \alpha + \frac{\pi}{2}\right)\overrightarrow{r_{S_i}} + \Gamma\left(\theta + \alpha_i\right)\cdot\dot{\overrightarrow{r_{S_i}}}, \quad (3.3)$$

since $\frac{d\overrightarrow{R_{S_i}}}{dt} = \vec{0}$.

From (3.3) a Jacobian matrix J_q of the system can be found, which is defined as

$$\dot{\vec{x}} = J_q\dot{\vec{q}}$$

or

$$J_q = \frac{d\vec{x}}{d\vec{q}}, \quad (3.4)$$

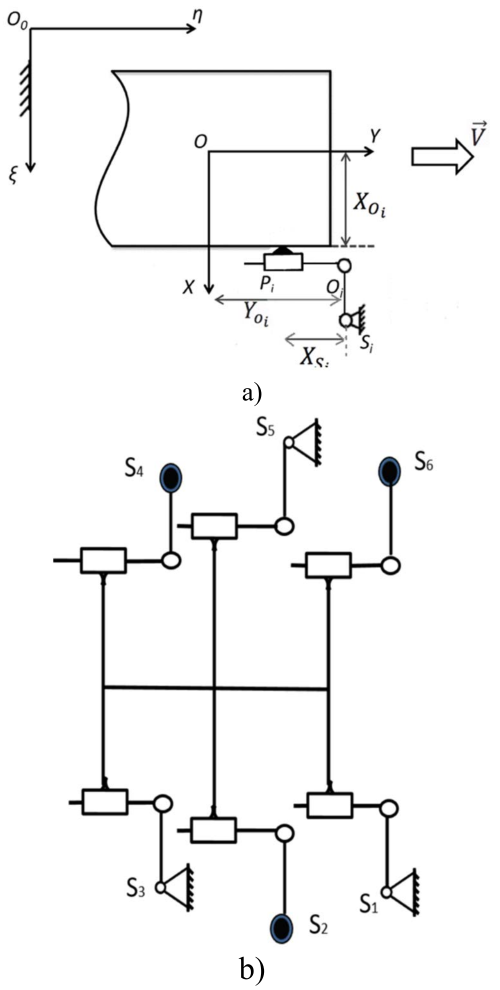


FIGURE 3. Kinematic-equivalent scheme of the WR.

$\overrightarrow{x} = \left[\overrightarrow{R_C}^T, L_\theta\theta\right]^T = [\xi_C, \eta_C, L_\theta\theta]^T$ – output coordinates, where L_θ – characteristic length, and input coordinates are $\overrightarrow{q} = [q_1, q_3, q_5]^T$.

To eliminate $\dot{\alpha}_i$, from the equation (3.3), we multiply the equation from the left by the vector $\overrightarrow{O_iS_i} = \Gamma(\theta + \alpha_i)\overrightarrow{r_{S_i}}$. Then since,

$$\left(\dot{\theta} + \dot{\alpha}_i\right)\overrightarrow{r_{S_i}}^T \cdot \Gamma^T(\theta + \alpha_i) \cdot \Gamma\left(\theta + \alpha_i + \frac{\pi}{2}\right) \cdot \overrightarrow{r_{S_i}} = 0,$$

the equation (3.3) can be rewritten as follows:

$$\overrightarrow{r_{S_i}}^T \cdot \Gamma^T(\theta + \alpha_i) \cdot \dot{\overrightarrow{R_C}} + \dot{\theta} \cdot \overrightarrow{r_{S_i}}^T \cdot \Gamma^T\left(\frac{\pi}{2} - \alpha_i\right) \cdot \overrightarrow{r_{O_i}} = -q_i\dot{q}_i, \quad i = 1, 3, 5. \tag{3.5}$$

Then the Jacobian matrix

$$J_q = A^{-1}B, \tag{3.6}$$

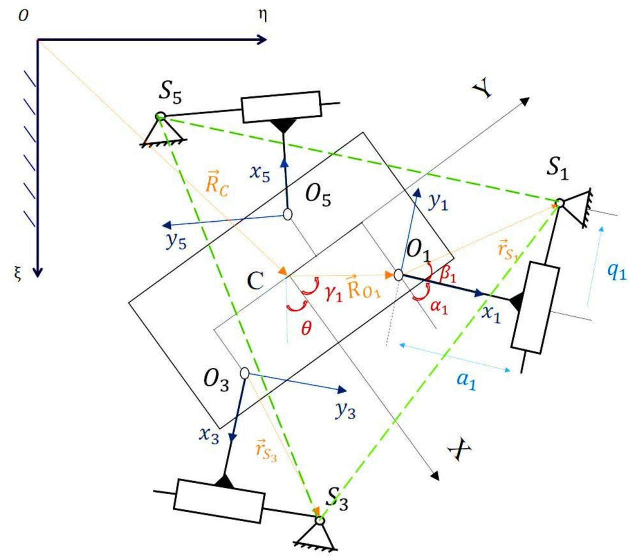


FIGURE 4. Kinematic-equivalent scheme of the WR with a "tripod"-gate.

where

$$A = \begin{bmatrix} \overrightarrow{r_{S_1}}^T \cdot \Gamma^T(\theta + \alpha_1) & \frac{1}{L_\theta}\overrightarrow{r_{S_1}}^T \cdot \Gamma^T\left(\frac{\pi}{2} - \alpha_1\right) \cdot \overrightarrow{r_{O_1}} \\ \overrightarrow{r_{S_3}}^T \cdot \Gamma^T(\theta + \alpha_3) & \frac{1}{L_\theta}\overrightarrow{r_{S_3}}^T \cdot \Gamma^T\left(\frac{\pi}{2} - \alpha_3\right) \cdot \overrightarrow{r_{O_3}} \\ \overrightarrow{r_{S_5}}^T \cdot \Gamma^T(\theta + \alpha_5) & \frac{1}{L_\theta}\overrightarrow{r_{S_5}}^T \cdot \Gamma^T\left(\frac{\pi}{2} - \alpha_5\right) \cdot \overrightarrow{r_{O_5}} \end{bmatrix},$$

$$B = -\begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_3 & 0 \\ 0 & 0 & q_5 \end{bmatrix}.$$

The isotropy condition (1.1) can be transformed to another form [12]:

$$\left(J_q^{-1}\right)^T \cdot J_q^{-1} = \frac{1}{\lambda^2}E, \tag{3.7}$$

where E is the identity matrix, $\dim E = 3 \times 3$, and

$$J_q^{-1} = \begin{bmatrix} \frac{1}{q_1}\overrightarrow{r_{S_1}}^T \cdot \Gamma^T(\theta + \alpha_1) & \frac{1}{q_1L_\theta}\overrightarrow{r_{S_1}}^T \cdot \Gamma^T\left(\frac{\pi}{2} - \alpha_1\right) \cdot \overrightarrow{r_{O_1}} \\ \frac{1}{q_3}\overrightarrow{r_{S_3}}^T \cdot \Gamma^T(\theta + \alpha_3) & \frac{1}{q_3L_\theta}\overrightarrow{r_{S_3}}^T \cdot \Gamma^T\left(\frac{\pi}{2} - \alpha_3\right) \cdot \overrightarrow{r_{O_3}} \\ \frac{1}{q_5}\overrightarrow{r_{S_5}}^T \cdot \Gamma^T(\theta + \alpha_5) & \frac{1}{q_5L_\theta}\overrightarrow{r_{S_5}}^T \cdot \Gamma^T\left(\frac{\pi}{2} - \alpha_5\right) \cdot \overrightarrow{r_{O_5}} \end{bmatrix} \tag{3.8}$$

The following equation can be derived from (3.8):

$$\frac{1}{q_i}\overrightarrow{r_{S_i}}^T \cdot \Gamma^T(\theta + \alpha_i) = \frac{1}{q_i}\left[\overrightarrow{e_\xi}^T\Gamma(\theta + \alpha_i)\overrightarrow{r_{S_i}} \quad \overrightarrow{e_\eta}^T\Gamma(\theta + \alpha_i)\overrightarrow{r_{S_i}}\right],$$

$$\frac{1}{q_iL_\theta}\overrightarrow{r_{S_i}}^T \cdot \Gamma^T\left(\frac{\pi}{2} - \alpha_i\right) \cdot \overrightarrow{r_{O_i}} = \frac{1}{q_iL_\theta}\overrightarrow{e_\zeta}^T\left[\overrightarrow{r_{O_i}} \times \Gamma(\alpha_i)\overrightarrow{r_{S_i}}\right],$$

where $\overrightarrow{e_\xi}, \overrightarrow{e_\eta}, \overrightarrow{e_\zeta}$ are the basis vectors of the coordinate system $O_0\xi\eta\zeta$.

Thus (3.9), as shown at the bottom of the page. The isotropy condition can be represented in a more compact form, using the variable β_i, the angle between the vectors $\overrightarrow{O_iP_i}$ and $\overrightarrow{r_{S_i}}$, i.e. $tg\beta_i = \frac{q_i}{a_i}$ (see figure 1). To this end we can obtain

$$\begin{aligned}&\frac{1}{q_i}\Gamma(\theta+\alpha_i)\overrightarrow{r_{S_i}}\\ &= \frac{r_{S_i}}{q_i}\Gamma(\theta+\alpha_i)\cdot\begin{bmatrix}\cos\beta_i\\ \sin\beta_i\end{bmatrix}\\ &= = \frac{1}{\sin\beta_i}\begin{bmatrix}\cos(\theta+\alpha_i+\beta_i)\\ \sin(\theta+\alpha_i+\beta_i)\end{bmatrix},\end{aligned}$$

where $r_{S_i} = \sqrt{a_i^2+q_i^2}$ – magnitude of the vector $\overrightarrow{r_{S_i}}$, and $\cos\beta_i = \frac{a_i}{r_{S_i}}$, $\sin\beta_i = \frac{q_i}{r_{S_i}}$. Hence,

$$\frac{1}{q_i}\overrightarrow{e_{\xi_i}}^T\cdot\Gamma(\theta+\alpha_i)\overrightarrow{r_{S_i}} = \frac{\cos(\theta+\alpha_i+\beta_i)}{\sin\beta_i};$$

$$\frac{1}{q_i}\overrightarrow{e_{\eta_i}}^T\cdot\Gamma(\theta+\alpha_i)\overrightarrow{r_{S_i}} = \frac{\sin(\theta+\alpha_i+\beta_i)}{\sin\beta_i};$$

Also, the right column in the expression (3.9) can be simplified as follows:

$$\begin{aligned}&\frac{1}{q_iL_\theta}\overrightarrow{e_\zeta}^T\left[\overrightarrow{r_{O_i}}\times\Gamma(\alpha_i)\overrightarrow{r_{S_i}}\right]\\ &= \frac{r_{S_i}}{L_\theta q_i}\overrightarrow{e_\zeta}^T\cdot\begin{vmatrix}X_{O_i} & Y_{O_i}\\ \cos(\alpha_i+\beta_i) & \sin(\alpha_i+\beta_i)\end{vmatrix}\overrightarrow{e_\zeta}\\ &= \frac{r_{O_i}}{L_\theta\sin\beta_i}\sin(\alpha_i+\beta_i-\gamma_i),\end{aligned}$$

r_{O_i}, γ_i – polar coordinates of the center of the joint O_i:

$$r_{O_i} = \sqrt{X_{O_i}^2+Y_{O_i}^2},\ tg\gamma_i = \frac{Y_{O_i}}{X_{O_i}}.$$

Then the expression (3.9), (3.10) as shown at the bottom of the page. Now, from (3.7)

$$\left(J_q^{-1}\right)^T\cdot J_q^{-1} = \begin{bmatrix}j_{11} & j_{12} & j_{13}\\ j_{21} & j_{22} & j_{23}\\ j_{31} & j_{32} & j_{33}\end{bmatrix} = \frac{1}{\lambda^2}E, \quad (3.11)$$

where

$$j_{11} = \sum_{i=1,3,5}\frac{\cos^2(\theta+\alpha_i+\beta_i)}{\sin^2\beta_i};$$

$$j_{12} = \sum_{i=1,3,5}\frac{\sin 2(\theta+\alpha_i+\beta_i)}{2\sin^2\beta_i};$$

$$j_{13} = \sum_{i=1,3,5}\frac{r_{O_i}\cos(\theta+\alpha_i+\beta_i)\sin(\alpha_i+\beta_i-\gamma_i)}{L_\theta\sin^2\beta_i};$$

$$j_{21} = \sum_{i=1,3,5}\frac{\sin 2(\theta+\alpha_i+\beta_i)}{2\sin^2\beta_i};$$

$$j_{22} = \sum_{i=1,3,5}\frac{\sin^2(\theta+\alpha_i+\beta_i)}{\sin^2 i};$$

$$j_{23} = \sum_{i=1,3,5}\frac{r_{O_i}\sin(\theta+\alpha_i+\beta_i)\sin(\alpha_i+\beta_i-\gamma_i)}{L_\theta\sin^2\beta_i};$$

$$j_{31} = \sum_{i=1,3,5}\frac{r_{O_i}\cos(\theta+\alpha_i+\beta_i)\sin(\alpha_i+\beta_i-\gamma_i)}{L_\theta\sin^2\beta_i};$$

$$j_{32} = \sum_{i=1,3,5}\frac{r_{O_i}\sin(\theta+\alpha_i+\beta_i)\sin(\alpha_i+\beta_i-\gamma_i)}{L_\theta\sin^2\beta_i};$$

$$j_{33} = \sum_{i=1,3,5}\frac{r_{O_i}\sin^2(\alpha_i+\beta_i-\gamma_i)}{L_\theta^2\sin^2\beta_i}.$$

Therefore, we get 6 **isotropy conditions**:

$$\sum_{i=1,3,5}\frac{\cos^2(\theta+\alpha_i+\beta_i)}{\sin^2\beta_i} = \frac{1}{\lambda^2}; \quad (3.12)$$

$$\sum_{i=1,3,5}\frac{\sin^2(\theta+\alpha_i+\beta_i)}{\sin^2\beta_i} = \frac{1}{\lambda^2}; \quad (3.13)$$

$$\sum_{i=1,3,5}\frac{r_{O_i}\sin^2(\alpha_i+\beta_i-\gamma_i)}{\sin^2\beta_i} = \frac{1}{\lambda^2}; \quad (3.14)$$

$$\sum_{i=1,3,5}\frac{\sin 2(\theta+\alpha_i+\beta_i)}{\sin^2\beta_i} = 0; \quad (3.15)$$

$$J_q^{-1} = -\begin{bmatrix}\frac{1}{q_1}\overrightarrow{e_\xi}^T\Gamma(\theta+\alpha_1)\overrightarrow{r_{S_1}} & \frac{1}{q_1}\overrightarrow{e_\eta}^T\Gamma(\theta+\alpha_1)\overrightarrow{r_{S_1}} & \frac{1}{q_1L_\theta}\overrightarrow{e_\zeta}^T\left[\overrightarrow{r_{O_1}}\times\Gamma(\alpha_1)\overrightarrow{r_{S_1}}\right]\\ \frac{1}{q_3}\overrightarrow{e_\xi}^T\Gamma(\theta+\alpha_3)\overrightarrow{r_{S_3}} & \frac{1}{q_3}\overrightarrow{e_\eta}^T\Gamma(\theta+\alpha_3)\overrightarrow{r_{S_3}} & \frac{1}{q_3L_\theta}\overrightarrow{e_\zeta}^T\left[\overrightarrow{r_{O_3}}\times\Gamma(\alpha_3)\overrightarrow{r_{S_3}}\right]\\ \frac{1}{q_5}\overrightarrow{e_\xi}^T\Gamma(\theta+\alpha_5)\overrightarrow{r_{S_5}} & \frac{1}{q_5}\overrightarrow{e_\eta}^T\Gamma(\theta+\alpha_5)\overrightarrow{r_{S_5}} & \frac{1}{q_5L_\theta}\overrightarrow{e_\zeta}^T\left[\overrightarrow{r_{O_5}}\times\Gamma(\alpha_5)\overrightarrow{r_{S_5}}\right]\end{bmatrix}. \quad (3.9)$$

$$J_q^{-1} = \begin{bmatrix}\frac{cos(\theta+\alpha_1+\beta_1)}{sin\beta_1} & \frac{sin(\theta+\alpha_1+\beta_1)}{sin\beta_1} & \frac{r_{O_1}}{L_\theta sin\beta_1}sin(\alpha_1+\beta_1-\gamma_1)\\ \frac{cos(\theta+\alpha_3+\beta_3)}{sin\beta_3} & \frac{sin(\theta+\alpha_3+\beta_3)}{sin\beta_2} & \frac{r_{O_3}}{L_\theta sin\beta_2}sin(\alpha_3+\beta_3-\gamma_3)\\ \frac{cos(\theta+\alpha_5+\beta_5)}{sin\beta_5} & \frac{sin(\theta+\alpha_5+\beta_5)}{sin\beta_5} & \frac{r_{O_5}}{L_\theta sin\beta_5}sin(\alpha_5+\beta_5-\gamma_5)\end{bmatrix}. \quad (3.10)$$

$$\sum_{i=1,3,5} \frac{r_{O_i} \cos(\theta + \alpha_i + \beta_i) \sin(\alpha_i + \beta_i - \gamma_i)}{\sin^2 \beta_i} = 0; \tag{3.16}$$

$$\sum_{i=1,3,5} \frac{r_{O_i} \sin(\theta + \alpha_i + \beta_i) \sin(\alpha_i + \beta_i - \gamma_i)}{\sin^2 \beta_i} = 0. \tag{3.17}$$

For convenience in the further studies the derived conditions were used in different forms. From (3.11), (3.12)

$$\frac{1}{\lambda^2} = \frac{1}{2}\left(\frac{1}{\sin^2 \beta_1} + \frac{1}{\sin^2 \beta_3} + \frac{1}{\sin^2 \beta_5}\right). \tag{3.18}$$

Since,

$$\sum_{i=1,3,5} \frac{\cos^2(\theta + \alpha_i + \beta_i)}{\sin^2 \beta_i} = \sum_{i=1,3,5} \frac{\sin^2(\theta + \alpha_i + \beta_i)}{\sin^2 \beta_i}, \tag{3.19}$$

$$\sum_{i=1,3,5} \frac{\cos 2(\theta + \alpha_i + \beta_i)}{\sin^2 \beta_i} = 0. \tag{3.20}$$

And from (3.15), (3.20)

$$\sum_{i=1,3,5} \frac{1}{\sin^2 \beta_i} \begin{bmatrix} \cos 2(\theta + \alpha_i + \beta_i) \\ \sin 2(\theta + \alpha_i + \beta_i) \end{bmatrix} = \sum_{i=1,3,5} \frac{1}{\sin^2 \beta_i} \Gamma(2\theta) \cdot \begin{bmatrix} \cos 2(\alpha_i + \beta_i) \\ \sin 2(\alpha_i + \beta_i) \end{bmatrix} = \vec{0}; \tag{3.21}$$

Since the last equality holds for any θ, the angle θ can be eliminated from the conditions (3.12) and (3.13):

$$\frac{\cos 2(\alpha_1 + \beta_1)}{\sin^2 \beta_1} + \frac{\cos 2(\alpha_3 + \beta_3)}{\sin^2 \beta_3} + \frac{\cos 2(\alpha_5 + \beta_5)}{\sin^2 \beta_5} = 0 \tag{3.22}$$

$$\frac{\sin 2(\alpha_1 + \beta_1)}{\sin^2 \beta_1} + \frac{\sin 2(\alpha_3 + \beta_3)}{\sin^2 \beta_3} + \frac{\sin 2(\alpha_5 + \beta_5)}{\sin^2 \beta_5} = 0 \tag{3.23}$$

Similarly, the rotation angle can be excluded from the equations (3.16), (3.17).

If we denote $u_i = r_{O_i} \sin(\alpha_i + \beta_i - \gamma_i)$, $i = 1, 3, 5$, then the equations (3.14), (3.16), (3.17), taking into account (3.18) will get the following forms:

$$\sum_{i=1,3,5} \frac{u_i^2}{sin^2 \beta_i} = \frac{L_\theta^2}{2} \sum_{i=1,3,5} \frac{1}{sin^2 \beta_i}; \tag{3.24}$$

$$\begin{bmatrix} \sum_{i=1,3,5} \frac{u_i cos(\theta + \alpha_i + \beta_i)}{sin^2 \beta_i} \\ \sum_{i=1,3,5} \frac{u_i sin(\theta + \alpha_i + \beta_i)}{sin^2 \beta_i} \end{bmatrix} = \sum_{i=1,3,5} \frac{u_i}{sin^2 \beta_i} \Gamma(\theta) \cdot \begin{bmatrix} cos(\alpha_i + \beta_i) \\ sin(\alpha_i + \beta_i) \end{bmatrix} = \vec{0}.$$

$$\sum_{i=1,3,5} \frac{u_i cos(\alpha_i + \beta_i)}{sin^2 \beta_i} = 0, \tag{3.25}$$

$$\sum_{i=1,3,5} \frac{u_i sin(\alpha_i + \beta_i)}{sin^2 \beta_i} = 0. \tag{3.26}$$

And from (3.22), (3.23)

$$\frac{\cos^2 2(\alpha_5 + \beta_5)}{\sin^4 \beta_5} = \frac{\cos^2 2(\alpha_3 + \beta_3)}{\sin^4 \beta_3} + \frac{\cos^2 2(\alpha_1 + \beta_1)}{\sin^4 \beta_1} + 2\frac{\cos 2(\alpha_3 + \beta_3) \cos 2(\alpha_1 + \beta_1)}{\sin^2 \beta_3 \sin^2 \beta_1};$$

$$\frac{\sin^2 2(\alpha_5 + \beta_5)}{\sin^4 \beta_5} = \frac{\sin^2 2(\alpha_3 + \beta_3)}{\sin^4 \beta_3} + \frac{\sin^2 2(\alpha_1 + \beta_1)}{\sin^4 \beta_1} + 2\frac{\sin 2(\alpha_3 + \beta_3) \sin 2(\alpha_1 + \beta_1)}{\sin^2 \beta_3 \sin^2 \beta_1}.$$

Hence,

$$\frac{1}{\sin^4 \beta_5} = \frac{1}{\sin^4 \beta_3} + \frac{1}{\sin^4 \beta_1} + 2\frac{\cos 2(\alpha_3 + \beta_3 - \alpha_1 - \beta_1)}{\sin^2 \beta_3 \sin^2 \beta_1}. \tag{3.27a}$$

$$\cos 2(\alpha_3 + \beta_3 - \alpha_1 - \beta_1) = \frac{\sin^2 \beta_3 \sin^2 \beta_1}{2}\left(\frac{1}{\sin^4 \beta_5} - \frac{1}{\sin^4 \beta_3} - \frac{1}{\sin^4 \beta_1}\right). \tag{3.27b}$$

Due to the symmetricity of equations (3.22), (3.23), using the cyclic permutation of indices (1 – 3 – 5 – 1),

$$\cos 2(\alpha_5 + \beta_5 - \alpha_1 - \beta_1) = \frac{\sin^2 \beta_5 \sin^2 \beta_1}{2}\left(\frac{1}{\sin^4 \beta_3} - \frac{1}{\sin^4 \beta_5} - \frac{1}{\sin^4 \beta_1}\right). \tag{3.28}$$

IV. STUDY OF THE ISOTROPY CONDITIONS

A. DESERVE MORE ATTENTION THE SYMMETRIC SOLUTIONS

Let's consider the case $\boldsymbol{\beta_i = \frac{\pi}{2}, i = 1, 3, 5}$. From (3.18),

$$\frac{1}{\lambda^2} = \frac{3}{2}, \tag{4.1}$$

The following can be derived from (3.12), (3.13):

$$\sin^2 \alpha_1 + \sin^2 \alpha_3 + \sin^2 \alpha_5 = \frac{3}{2}; \tag{4.2}$$

$$\cos^2 \alpha_1 + \cos^2 \alpha_3 + \cos^2 \alpha_5 = \frac{3}{2}. \tag{4.3}$$

And (3.14) gives

$$\sin 2\alpha_1 + \sin 2\alpha_3 + \sin 2\alpha_5 = 0. \tag{4.4}$$

From (4.2) and (4.3) it follows

$$\cos 2\alpha_1 + \cos 2\alpha_3 + \cos 2\alpha_5 = 0. \tag{4.5}$$

A simple solution can be found from last formulas:

$$\cos 2(\alpha_3 - \alpha_1) = -\frac{1}{2};$$

Analogically, we can get

$$\cos 2(\alpha_i - \alpha_j) = -\frac{1}{2}, \quad i, j = 1, 3, 5, \ i \neq j. \tag{4.6}$$

The parameters r_{O_i}, γ_i can be found using (3.14), (3.16) and (3.17):

$$\sum_{i=1,3,5} r_{O_i} \cos^2(\alpha_i - \gamma_i) = \frac{L_\theta^2}{\lambda^2}$$

Knowing that (see equation (4.1)

$$\sum_{i=1,3,5} r_{O_i} \cos^2(\alpha_i - \gamma_i) = \frac{3L_\theta^2}{2}, \tag{4.7}$$

we find

$$\sum_{i=1,3,5} r_{O_i} \sin\alpha_i \cos(\alpha_i - \gamma_i) = 0, \tag{4.8}$$

and

$$\sum_{i=1,3,5} r_{O_i} \cos\alpha_i \cos(\alpha_i - \gamma_i) = 0. \tag{4.9}$$

The last equations are also true for $\beta_i = -\frac{\pi}{2}$.

Denote $r_{O_i}\cos(\alpha_i - \gamma_i) = x_i$, then

$$\begin{cases} x_1^2 + x_3^2 + x_5^2 = \frac{3}{2}L_\theta^2 \\ x_1 \sin\alpha_1 + x_3 \sin\alpha_3 + x_5 \sin\alpha_5 = 0 \\ x_1 \cos\alpha_1 + x_3 \cos\alpha_3 + x_5 \cos\alpha_5 = 0 \end{cases} \tag{4.10}$$

According to the Cramer's rule, the solution of the last two equations:

$$\Delta = \sin(\alpha_3 - \alpha_5),$$

$$x_3 = -\frac{x_1 \begin{vmatrix} \sin\alpha_1 & \sin\alpha_5 \\ \cos\alpha_1 & \cos\alpha_5 \end{vmatrix}}{\Delta} = \frac{x_1 \sin(\alpha_5 - \alpha_1)}{\sin(\alpha_3 - \alpha_1)} \tag{4.11}$$

$$x_5 = -\frac{x_1 \begin{vmatrix} \sin\alpha_3 & \sin\alpha_1 \\ \cos\alpha_3 & \cos\alpha_1 \end{vmatrix}}{\Delta} = \frac{x_1 \sin(\alpha_1 - \alpha_3)}{\sin(\alpha_3 - \alpha_5)} \tag{4.12}$$

Thus,

$$x_1^2 = \frac{L_\theta^2}{2}. \tag{4.13}$$

For example, when $L_\theta = 1\frac{m}{rad}$,

$$r_{O_i} \cos(\alpha_i - \gamma_i) = \pm\frac{1}{\sqrt{2}}m;$$

Or if $L_\theta = \sqrt{2}\frac{m}{rad}$,

$$r_{O_i} \cos(\alpha_i - \gamma_i) = \pm 1.$$

B. CONSIDER A MORE GENERAL CASE: $\beta_1 = \beta_3 = \beta_5$

Rewrite the equations (3.22), (3.23) as follows:

$$\left(\frac{\cos x_1}{\sin^2\beta_1} + \frac{\cos x_3}{\sin^2\beta_3}\right)^2 = \left(-\frac{\cos x_5}{\sin^2\beta_5}\right)^2, \tag{4.14}$$

$$\left(\frac{\sin x_1}{\sin^2\beta_1} + \frac{\sin x_3}{\sin^2\beta_3}\right)^2 = \left(-\frac{\sin x_5}{\sin^2\beta_5}\right)^2. \tag{4.15}$$

Then we get

$$\cos(x_1 - x_5) = \frac{\sin^2\beta_5 \sin^2\beta_1}{2}\left(\frac{1}{\sin^4\beta_3} - \frac{1}{\sin^4\beta_5} - \frac{1}{\sin^4\beta_1}\right) \tag{4.16}$$

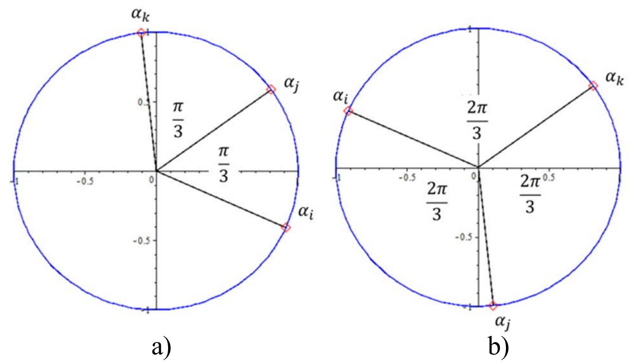


FIGURE 5. Illustration of combinations $\{\alpha_3 - \alpha_1, \alpha_5 - \alpha_1\}$.

And making cyclic permutation (1 – 3 – 5 – 1),

$$\cos(x_1 - x_5) = \frac{\sin^2\beta_5 \sin^2\beta_1}{2}\left(\frac{1}{\sin^4\beta_3} - \frac{1}{\sin^4\beta_5} - \frac{1}{\sin^4\beta_1}\right). \tag{4.17}$$

The case of equal angles $\boldsymbol{\beta_1 = \beta_3 = \beta_5}$ will lead to a simple solution, which coincides with (4.6):

$$\begin{cases} \cos 2(\alpha_3 - \alpha_1) = -\frac{1}{2}, \\ \cos 2(\alpha_5 - \alpha_1) = -\frac{1}{2}. \end{cases} \tag{4.18}$$

Hence,

$$\begin{cases} \alpha_3 = \alpha_1 \pm \frac{\pi}{3} + 2\pi n, & n = 0, 1, \\ \alpha_5 = \alpha_1 \mp \frac{\pi}{3} + 2\pi k, & k = 0, 1 \end{cases} \tag{4.19}$$

This solution gives 8 combinations of $\boldsymbol{\{\alpha_3 - \alpha_1, \alpha_5 - \alpha_1\}}$, each of which satisfies (3.22) and (3.23):

$$\left\{\frac{\pi}{3}, -\frac{\pi}{3}\right\}; \left\{\frac{\pi}{3}, -\frac{2\pi}{3}\right\}; \left\{-\frac{\pi}{3}, \frac{\pi}{3}\right\}; \left\{-\frac{\pi}{3}, -\frac{2\pi}{3}\right\}; \left\{-\frac{2\pi}{3}, -\frac{\pi}{3}\right\}; \left\{-\frac{2\pi}{3}, \frac{2\pi}{3}\right\}; \left\{\frac{2\pi}{3}, \frac{\pi}{3}\right\}; \left\{\frac{2\pi}{3}, -\frac{\pi}{3}\right\}, \tag{4.20}$$

Fig.5a illustrates 1-, 2-, 3-, 4-, 5-, 7-th solutions and the Fig.5b represents combinations $\boldsymbol{\{\alpha_3 - \alpha_1, \alpha_5 - \alpha_1\}} = \left\{-\frac{2\pi}{3}, \frac{2\pi}{3}\right\}$, and $\boldsymbol{\{\alpha_3 - \alpha_1, \alpha_5 - \alpha_1\}} = \left\{\frac{2\pi}{3}, -\frac{\pi}{3}\right\}$.

To search for the parameters $\boldsymbol{\beta_i, r_{O_i}}$, we set $\boldsymbol{\beta_1 = \beta_3 = \beta_5 = \beta}$ again. Equation (24) in this case will take the form:

$$\sum_{i=1,2,3} u_i^2 = \frac{3L_\theta^2}{2}, \tag{4.21}$$

where

$$\boldsymbol{u_i = r_{O_i} sin(\alpha_i + \beta - \gamma_i)}.$$

Equations (3.25) and (3.26) give a system of two linear equations in the unknowns $\{\boldsymbol{u_3}, \boldsymbol{u_5}\}$:

$$\sum_{i=1,3,5} u_i \Gamma(\beta) \cdot \begin{bmatrix} \cos(\alpha_i) \\ \sin(\alpha_i) \end{bmatrix} = 0,$$

from where

$$\sum_{i=1,3,5} u_i \cos(\alpha_i) = 0, \quad i = 1, 3, 5; \tag{4.22}$$

$$\sum_{i=1,3,5} u_i \sin(\alpha_i) = 0, \quad i = 1, 3, 5. \tag{4.23}$$

Solving the last equations by Cramer's method with respect to $\boldsymbol{u_3}$ and $\boldsymbol{u_5}$,

$$u_3 = u_1 \frac{\sin(\alpha_1 - \alpha_5)}{\sin(\alpha_5 - \alpha_3)},$$
$$u_5 = u_1 \frac{\sin(\alpha_3 - \alpha_1)}{\sin(\alpha_5 - \alpha_3)}. \tag{4.24}$$

It is known from (3.18) that

$$\sin^2(\alpha_i - \alpha_j) = \frac{1 - \cos 2(\alpha_i - \alpha_j)}{2} = \frac{3}{4}. \tag{4.25}$$

Then the solution of (4.21) is

$$u_i = \pm \frac{L_\theta}{\sqrt{2}}, \quad i = 1, 3, 5. \tag{4.26}$$

or

$$sin(\alpha_i + \beta - \gamma_i) = \pm \frac{L_\theta}{\sqrt{2}\, r_{O_i}}, \quad i = 1, 3, 5. \tag{4.27}$$

The equality $u_1^2 = u_3^2 = u_5^2$ follows from (4.20) and (4.21). The solutions correspond to 8 combinations of $\{u_1, u_3, u_5\}$:

$$\left\{\frac{L_\theta}{\sqrt{2}}, \frac{L_\theta}{\sqrt{2}}, \frac{L_\theta}{\sqrt{2}}\right\}, \quad \left\{\frac{L_\theta}{\sqrt{2}}, \frac{L_\theta}{\sqrt{2}}, -\frac{L_\theta}{\sqrt{2}}\right\},$$
$$\left\{\frac{L_\theta}{\sqrt{2}}, -\frac{L_\theta}{\sqrt{2}}, \frac{L_\theta}{\sqrt{2}}\right\}, \quad \left\{\frac{L_\theta}{\sqrt{2}}, -\frac{L_\theta}{\sqrt{2}}, -\frac{L_\theta}{\sqrt{2}}\right\},$$
$$\left\{-\frac{L_\theta}{\sqrt{2}}, \frac{L_\theta}{\sqrt{2}}, \frac{L_\theta}{\sqrt{2}}\right\}, \quad \left\{-\frac{L_\theta}{\sqrt{2}}, \frac{L_\theta}{\sqrt{2}}, -\frac{L_\theta}{\sqrt{2}}\right\},$$
$$\left\{-\frac{L_\theta}{\sqrt{2}}, -\frac{L_\theta}{\sqrt{2}}, \frac{L_\theta}{\sqrt{2}}\right\}, \quad \left\{-\frac{L_\theta}{\sqrt{2}}, -\frac{L_\theta}{\sqrt{2}}, -\frac{L_\theta}{\sqrt{2}}\right\} \tag{4.28}$$

As an example consider the case $\boldsymbol{u_1 = u_3 = u_5}$, i.e. when

$$sin(\alpha_i + \beta - \gamma_i) = \frac{L_\theta}{\sqrt{2}\, r_{O_i}}, \quad i = 1, 3, 5$$

or

$$sin(\alpha_i + \beta - \gamma_i) = -\frac{L_\theta}{\sqrt{2}\, r_{O_i}}, \quad i = 1, 3, 5.$$

Only two of eight combinations $\{\boldsymbol{\alpha_3 - \alpha_1}, \boldsymbol{\alpha_5 - \alpha_1}\}$ satisfy the conditions (3.25), (3.26) in this case:

1) $\alpha_3 = \alpha_1 - \frac{2\pi}{3}, \alpha_5 = \alpha_1 + \frac{2\pi}{3}$;
2) $\alpha_3 = \alpha_1 + \frac{2\pi}{3}, \alpha_5 = \alpha_1 - \frac{2\pi}{3}$.

Fig.6a demonstrate the configurations corresponding to the first, and Fig.6b correspond to the second solutions in symmetric case when

$$r_{O_1} = r_{O_3} = r_{O_5} = \pm \frac{L_\theta}{\sqrt{2}\sin(\alpha_1 + \beta - \gamma_1)};$$
$$\gamma_3 - \gamma_1 = \alpha_3 - \alpha_1;$$
$$\boldsymbol{\gamma_5 - \gamma_1 = \alpha_5 - \alpha_1}.$$

For $\boldsymbol{\gamma_1 = \frac{\pi}{3}}$,

1) for the first solution, $\boldsymbol{\gamma_3 = \gamma_1 - \frac{2\pi}{3} = -\frac{\pi}{3}}$; $\boldsymbol{\gamma_5 = \gamma_1 + \frac{2\pi}{3} = \pi}$.
2) and for the second solution $\boldsymbol{\gamma_3 = \gamma_1 + \frac{2\pi}{3} = \pi}$; $\boldsymbol{\gamma_5 = \gamma_1 - \frac{2\pi}{3} = -\frac{\pi}{3}}$.

Note that the second configuration can be obtained from the first by swapping leg mechanisms with numbers 3 and 5.

And for $\boldsymbol{\gamma_1 = -\frac{\pi}{3}}$,

1) the first solution is $\boldsymbol{\gamma_3 = -\pi}$; $\boldsymbol{\gamma_5 = \frac{\pi}{3}}$,
2) and the second is $\boldsymbol{\gamma_3 = \frac{\pi}{3}}$; $\boldsymbol{\gamma_5 = -\pi}$,

i.e. swapped are the legs with numbers 1 and 3.

When choosing $\boldsymbol{\alpha_1, \gamma_1}$, it is necessary that $\boldsymbol{sin(\alpha_i + \beta - \gamma_i), i = 1, 3, 5}$ have the same signs. In the example above, this follows from condition (4.1):

$$\alpha_1 + \beta - \gamma_1 = \alpha_3 + \beta - \gamma_3 = \alpha_5 + \beta - \gamma_5.$$

for the first solution, $\boldsymbol{\gamma_3 = \gamma_1 - \frac{2\pi}{3} = -\frac{\pi}{3}}$;

As can be seen in the Fig.6, in the isotropic configuration the lines $\boldsymbol{P_iS_i}$ form an equilateral triangle. Another advantage is that the center of mass of body C is located in the center of the supporting triangle $\boldsymbol{\Delta S_1S_3S_5}$, which ensures "equal" movement (the same ability of movement) in all directions and an equal margin of stability.

Remark: There is a disadvantage in these two configurations. Let's call "the main movement" the uniform translational motion of the WR. During the main movement, when the main engines rotate uniformly and the angular speeds $\omega_1, \omega_3, \omega_5$ reach the nominal value ω_{nom},, the robot operates in an energy-optimal mode. During such a movement, the guides P_iS_i of our model will be parallel (Figure *4*a). And if $\beta_1 = \beta_3 = \beta_5$ and $a_1 = a_3 = a_5$, then the lines O_iS_i will be parallel, which means that the mechanism is in a singular position. To avoid the singularity, a_5 *can be chosen* differently: $a_5 \neq a_1, a_5 \neq a_3$. But a more advantageous solution is the case $\beta_3 = \beta_1, \beta_5 = -\beta_1$ (Figure 4b) or $\beta_3 = -\beta_1, \beta_5 = \beta_1$.

Thus, the expressions are obtained that determine the parameters $\mathrm{P} = \gamma_3, \gamma_5, \alpha_3, \alpha_5, r_{O_1}, r_{O_3}, r_{O_5}$ for given values of $\mathrm{X} = \alpha_1, \gamma_1, \beta_1, \beta_3, \beta_5, a$. As noted earlier, during the movement of the robot, two conditions must be maintained: the absence of a singularity, as well as the stability of the robot (the center of mass of the WR body should be inside the support triangle). After numerical studies of different solutions of the isotropy equations, for each solution were found the boundary values of the generalized coordinates satisfying both mentioned conditions. The step length of the WR is

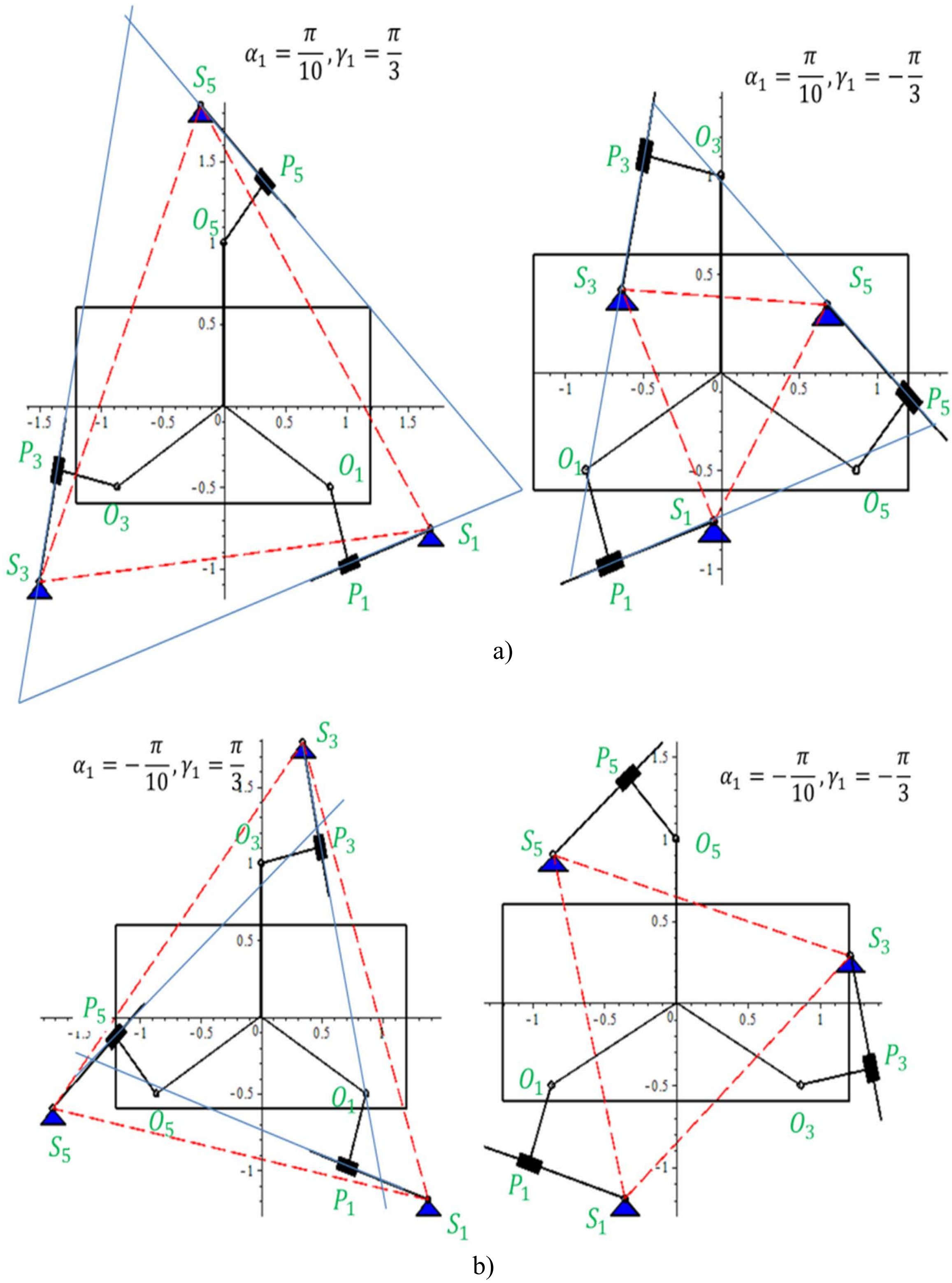


FIGURE 6. Isotropic configurations: $r_{O_i} = 1$, $a_i = 0.5$, $i = 1, 3, 5$.

defined as $L_0 = \min\left(\left|q_1^* - q_1^{**}\right|, \left|q_3^* - q_3^{**}\right|, \left|q_5^* - q_5^{**}\right|\right)$. The optimal solution corresponds to $L_0 \rightarrow max$. Such a solution is $u_1 = u_3 = u_5$, $\{\alpha_3 - \alpha_1, \alpha_5 - \alpha_1\} = \left\{-\frac{2\pi}{3}, -\frac{\pi}{3}\right\}$.

E.g., with the given parameters $\beta_0 = \beta_{1_0} = \beta_{3_0} = \beta_{5_0} = \frac{\pi}{4}$, $\gamma_1 = \frac{\pi}{3}$, $a_1 = 10cm$, $a_3 = 10cm$, $a_5 = 7cm$,

$$r_{O_i} = \frac{L_\theta}{\sqrt{2}sin\left(\alpha_1 + \beta - \gamma_1\right)}, \quad L_\theta = 0.1m;$$

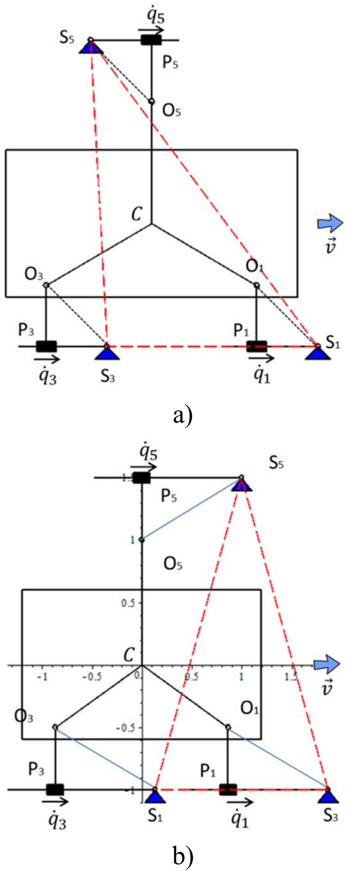


FIGURE 7. Configurations at the main movement: a) $\beta_1 = \beta_3 = \beta_5$; b) $\beta_1 = \beta_3 = -\beta_5$.

the step length can be up to $L_{0max} = 3.610688817$ m. This indicates that the solution ensures a sufficient ''remoteness'' from singularity and instability.

V. CONCLUSION

Turning modes of the WR were studied and a parametric synthesis of the turning mechanism has provided. The rotation of the WR is carried out due to the difference in the velocities of the main drives. The method of synthesis of parallel manipulators based on the isotropy criterion is applied for optimization of the WR turning mechanism. The isotropy conditions for robots with orthogonal propulsion are derived. Solutions of isotropy equations are defined. The analysis of the solutions of the isotropy equations was carried out and on their basis the metric parameters of the robot were obtained, which ensure the optimal transmission of forces and motion. One of the symmetric solutions ensures the stability and absence of singularity for the step length 3.6 m, while the characteristic length of the robot is 10 cm.

ACKNOWLEDGMENT

The authors thank Dr. M. Shahab for comments that helped to improve the article.

REFERENCES

[1] A. F. Kraynev and V. A. Glazunov, ''New mechanisms of relative manipulation,'' *Problems Mech. Eng. Rel. Mach.*, vol. 5, pp. 106–117, 1994.

[2] Y. Takeda and H. Funabashi, ''Motion transmissibility of in-parallel actuated manipulators,'' *JSME Int. J. C, Dyn., Control, Robot., Design Manuf.*, vol. 38, no. 4, pp. 749–755, 1995.

[3] Y. Takeda, H. Funabashi, and H. Ichimaru, ''Development of spatial in-parallel actuated manipulators with six degrees of freedom with high motion transmissibility,'' *JSME Int. J. C*, vol. 40, no. 2, pp. 299–308, 1997.

[4] T. Yoshikawa, ''Manipulability of robotic mechanisms,'' *Int. J. Robot. Res.*, vol. 4, no. 2, pp. 3–9, 1985.

[5] J. Lee, J. Duffy, and M. Keler, ''The optimum quality index for the stability of in-parallel planar platform devices,'' in *Proc. Int. Design Eng. Tech. Conf. Comput. Inf. Eng. Conf.*, vol. 97584. New York, NY, USA: American Society of Mechanical Engineers, Aug. 1996, Art. no. V02BT02A068.

[6] K. Y. Tsai and K. D. Huang, ''The manipulability and transmissivity of manipulators,'' *Int. J. Robot. Autom.*, vol. 19 no. 4, pp. 132–136, 1998.

[7] J. Angeles, Ed. *Fundamentals of Robotic Mechanical Systems: Theory, Methods, and Algorithms*. New York, NY, USA: Springer, 2003.

[8] J. Angeles and C. S. López-Cajún, ''Kinematic isotropy and the conditioning index of serial robotic manipulators,'' *Int. J. Robot. Res.*, vol. 11, no. 6, pp. 560–571, 1992.

[9] J. Schoenherr and Bemessen, ''Bewerten und optimieren von parallelstrukturen,'' in *Chemnitzer Parallelstruktur-Seminar*, 1998, pp. 85–96.

[10] J. Schoenherr, ''Design and evaluation of mechanisms using static performance indices,'' in *Proc. 10th World Congr.*, TMM, Oulu, 1999, pp. 747–753.

[11] J. Schoenherr and F. Weidermann, ''Bewertung und optimale auslegung von bewegungssystemen MIT parallelkinematik,'' in *VDI Berichte*, vol. 1427. Duesseldorf, Germany: VDI-Verlag, 1998, pp. 35–50.

[12] S. M. Ibraev, *Approximation Synthesis of Lever Mechanisms. Methods and Numerical Implementation: Monograph*. Almaty: Economics, 2014, pp. 205–216.

[13] J. Sefrioui and C. M. Gosselin, ''On the quadratic nature of the singularity curves of planar three-degree-of-freedom parallel manipulators,'' *Mechanism Mach. Theory*, vol. 30, no. 4, pp. 533–551, 1995.

[14] D. Chablat, P. Wenger, and J. Angeles, ''The kinematic design of a 3-DOF hybrid manipulator,'' in *Integrated Design and Manufacturing in Mechanical Engineering*. Dordrecht, Germany: Springer, 1999, pp. 225–232.

[15] D. Wettergreen and C. Thorpe, in *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst.*, vol. 2, 1992, pp. 1413–1420.

[16] A. Ryan and K. Hunt, ''Adjustable straight-line linkages—Possible legged-vehicle applications,'' *J. Mech. Transmiss. Automat.*, vol. 107, pp. 256–261, Jun. 1985.

[17] S. Song, V. Vohnout, K. Waldron, and G. Kinzel, ''Computer-aided design of a leg for an energy efficient walking machinela conception assitee par ordinateur d'une jambe pour une machine marchante energetiquement efficace,'' *Mechanisms Mach. Theory*, vol. 19, no. 1, pp. 17–24, 1984.

[18] S. Ibrayev, ''Optimal structural synthesis of agricultural legged robot with minimal damage on soil,'' in *Proc. E3S Web Conf.*, vol. 135. Les Ulis, France: EDP Sciences, 2019.

[19] R. J. Hijmans and J. van Etten, *Raster: Geographic analysis and Modeling With Raster Data*, R Package Version 2.0–12, Jan. 2012. [Online]. Available: http://CRAN.R-project.org/package=raster

[20] Teralyzer. *Lytera UG*. Kirchhain, Germany. Accessed: Jun. 5, 2014. [Online]. Available: http://www.lytera.de/Terahertz_THz_Spectroscopy.php?id=home

[21] *U.S. House. 102nd Congress, 1st Session. H. Con. Res. 1, Sense of the Congress on Approval of Military Action*, LEXIS Library, Genfed File, Bills, Jan. 1991.

[22] Mr. Brooks, ''Musical toothbrush with mirror,'' Patent D 326 189, May 19, 1992.

[23] D. B. Payne and J. R. Stern, ''Wavelength-switched passively coupled single-mode optical network,'' in *Proc. IOOC-ECOC*, Boston, MA, USA, 1985, pp. 585–590.

[24] D. Eberhard and E. Voges, ''Digital single sideband detection for interferometric sensors,'' in *Proc. SPIE*, Stuttgart, Germany, Nov. 1984.

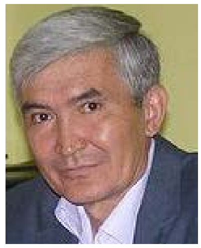
SAYAT IBRAYEV received the degree (Hons.) from the Republican School of Physics and Mathematics (Almaty, Kazakh SSR), in 1983, the Diploma degree from the Faculty of Mechanics and Mathematics, Lomonosov Moscow State University (Moscow, USSR), in 1988, and the master's degree from the Institute of Mathematics and Mechanics of the Academy of Sciences of the Kazakh SSR (Almaty, USSR), in 1992. In 1992, he defended his Ph.D. thesis (Candidate of Technical Sc.).

He started his career as an Engineer at Al-Farabi Kazakh State University, in 1988. Since 1991, he has been working at the Institute of Mechanics and Engineering of the National Academy of Sciences of the Republic of Kazakhstan (Researcher, Senior, Chief Researcher) and defended there his Ph.D. dissertation in 1996 (Doctor of Technical Sciences, Almaty, Kazakhstan). From 1998 to 2000, with the support of the International Alexander von Humboldt Foundation (Alexander von Humboldt Stiftung/Foundation, Bonn, Germany), he worked at the Technical University Chemnitz (Germany), and the Fraunhofer-Institut fuer Werkzeugmaschinen und Umformbautechnik IWU (Chemnitz, Germany), in 2000. From 2001 to 2010, he worked as a Professor and the Head of the Department of Theoretical and Applied Mechanics of KazNTU named after K. Satpayev. He is currently a Chief Researcher and the Head of the Laboratory Mechanics of Robots and Manipulators with the Institute of Mechanics and Engineering (Almaty). He is the author of 140 scientific publications, including five monographs, ten patents, also of the popular-science book *Aqylsyz Bolsa Ghylym Tul*. Under his guidance five candidates of sciences and one Ph.D. were defended. He was the author and presenter of educational programs in the Republican television and Radio Corporation Kazakhstan.

Dr. Ibrayev received the Gold Medal from the Republican School of Physics and Mathematics, Almaty, Kazakh SSR. He was elected as a Deputy of the Maslikhat of Almaty of the III Convocation.

ARMAN IBRAYEVA received the B.S. and M.S. degrees in mechanical engineering from Al-Farabi Kazakh National University, Almaty, Kazakhstan, in 2018 and 2020, respectively, where she is currently pursuing the Ph.D. degree.

From 2014 to 2015, she was a Laboratory Assistant at the Fluid and Gas Mechanics Laboratory, Kazakh National University. From 2019 to 2021, she was an Engineer at LLP—KOLSAJ STROJ. From 2020 to 2021, she was an Engineer at the Institute of Mechanics and Engineering, National Academy of Sciences, Kazakhstan. She is currently a Research Scientist with the Institute of Mechanics and Engineering, National Academy of Sciences, Almaty.



NUTPULLA JAMALOV received the degree in mechanics from the Faculty of Mechanics and Applied Mathematics, Kazakh State University (Almaty, Kazakhstan, USSR), in 1984. In 1990, he defended his thesis on the specialty February 2005–Robots, Manipulators and Robotic Systems.

After graduating from the university, he was sent to the Republican Scientific and Methodological Center for Robotics at KazSU. In 1987, he entered a Postgraduate School at KazSU. In subsequent years, he actively participated in the training of young personnel in robotics at universities, taught several courses on robotics. From 2009 to 2013, he worked at the Institute of Space Engineering and Technology (Kazakhstan) as a leading Researcher, participated in the creation of a spacecraft simulation model and other developments of the Institute, lectured on space technology at universities. Since 2018, he has been working as a leading Researcher with the Institute of Mechanics and Mechanical Engineering (Kazakhstan). He is the author of more than 130 scientific works, including 16 copyright certificates of the USSR, 14 patents of the Republic of Kazakhstan, five patents of the Eurasian Patent Office, one German, and one British patent for an invention. He was awarded the commendation of the Minister of Education and Science, in 2020, an Honorary Diploma of the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan, in 2016, and a Commemorative Medal named after Academician Zh. Yerzhanov, in 2017.

SAROSH H. PATEL received the B.E. degree (Hons.) in electrical engineering from Osmania University, India, and the M.S. degree in electrical engineering and technology management and the Ph.D. degree in computer science from the University of Bridgeport, Bridgeport, CT, USA. He is currently an Assistant Professor with the School of Engineering, University of Bridgeport. His research interests include manipulator prototyping, industrial control, modular morphological robots, and robotic swarms.