Received 26 September 2022, accepted 8 October 2022, date of publication 12 October 2022, date of current version 19 October 2022.
Digital Object Identifier 10.1109/ACCESS.2022.3213938

RESEARCH ARTICLE

Dynamic Event-Triggered Fault Detection for Discrete Networked Control System With Time-Delay

YANFENG WANG[1,2], YUQIN HOU[2], AND PEILIANG WANG[2]
[1]School of Mechanical and Electrical Engineering, Suqian University, Suqian 223800, China
[2]School of Engineering, Huzhou University, Huzhou 313000, China

Corresponding author: Peiliang Wang (wpl@zjhu.edu.cn)

This work was supported by the National Natural Science Foundation of China under Grant 61573136, Grant 61573137, and Grant 61503136.

ABSTRACT The dynamic event-triggered fault detection problem for discrete networked control systems with time-delay is investigated in this paper. Firstly, for reducing the triggering times, a new dynamic event-trigged is proposed. Secondly, an observer is constructed on the controller node to generate fault residual, and the mathematical model of the closed-loop systems is established by analyzing the sequence of the transmission signal. Thirdly, by constructing a suitable Lyapunov-Krasovskii functional, the sufficient stability conditions for the closed-loop networked control systems are derived. The computing method of the observer gain matrix, the controller gain matrix and the minimal disturbance repression index is provided. Finally, a casy study on the dynamic cart is employed to illustrate the merits of the proposed approach.

INDEX TERMS Time-delay, event-triggered mechanism, networked control system, fault detection.

I. INTRODUCTION

Coupled by the progression of communication technology and the increasing scale of control systems, the introduction of computer communication networks into control systems has become a trend in the field of industrial control. The closed-loop feedback systems formed by the shared networks are networked control systems (NCSs) [1], [2], [3], [4]. NCSs own the strength of easily expandable structure, low installation and maintenance costs, etc., and have been widely used in the field of industrial automation [5], [6], [7], [8]. However, some inevitable issues like network-induced time-delay, data packet dropout have appeared due to the introduction of the networks, making system analysis more complicated, and making it more demanding on safety and reliability in engineering applications [9], [10], [11].

With the increasing structure and scale of NCSs, due to the aging of components, external disturbances and changes in the working environment, the NCSs will inevitably have various failures [12], [13]. Therefore, the problem of NCSs fault detection (FD) is a subject of theoretical and practical significance, which has received extensive attention from the academic community and has appeared many results.

The existing literature on FD of NCSs can be divided into three kinds. The first kind literature only considers time-delay. For example, the sum of the data transmission time-delay in the sensor-to-controller (S-to-C) link and the data transmission time-delay in the controller-to-actuator (C-to-A) link was taken as a discrete Markovain chain, thereby NCSs subject to time-delay were modeled by Markovain jump systems (MJSs) [14]. The stability conditions for the closed-loop NCSs were derived. Moreover, the FD observer design method was provided. The S-to-C time-delay and C-to-A time-delay were described with two irreverent discrete Markovain chains respectively, and the NCSs FD method was proposed [15]. The second kind literature only considers data packet dropout. For instance, assuming that data packet dropout satisfied the Bernoulli distribution character, the FD issue for NCSs subject to S-to-C and C-to-A data packet dropout was researched [16]. The stable conditions for the closed-loop system with a certain robust $\mathcal{H}_\infty$ disturbance repression performance were obtained and

The associate editor coordinating the review of this manuscript and approving it for publication was Claudio Zunino.

This work is licensed under a Creative Commons Attribution 4.0 License. For more information, see https://creativecommons.org/licenses/by/4.0/

the FD filter design approach was given. The NCSs with S-to-C and C-to-A data packet dropout were described as MJSs owning four running modes, converting the NCSs FD problem into certain kind of $\mathcal{H}_\infty$ filter design issue [17]. The third kind literature takes both time-delay and packet dropout into consideration. For example, the conditions for NCSs subject to time-delay and packet dropout owning a certain level of $\mathcal{H}_\infty$ disturbance repression performance were obtained [18], [19].

Industrial control systems usually sample periodically to control and monitor the controlled plants. However, this traditional periodic sampling technique not only leads to a waste of energy but also increases the communication burden on the network. In this context, the event-triggered mechanisms (ETMs) are proposed, the idea of which is to sample or transmit data only when certain pre-specified conditions are satisfied. Therefore, ETMs can greatly save network bandwidth, thereby help to reduce network-induced time-delay and data packet dropout. Many ETMs have appeared for control or state estimation of various complex systems [20], [21], [22]. According to the different triggering conditions, the ETMs can be mainly divided into static and dynamic ETMs. The static ETMs are currently the most common ones, the triggering threshold of which is a constant [23], [24].

The common triggering condition is as follows:

$$\left[y_{p_i} - y_l\right]^T \left[y_{p_i} - y_l\right] - \mu y_l^T y_l > 0 \tag{1}$$

with y_{p_i} being the last transmitted system output, y_l being the current system output and $0 < \mu < 1$ being the triggering threshold. The above condition is based on relative error, and is called relative ETM. The following triggering condition is called absolute event-triggering, because the triggering condition is based on absolute error.

$$\left[y_{p_i} - y_l\right]^T \left[y_{p_i} - y_l\right] - \mu > 0. \tag{2}$$

With the purpose of further reducing the triggering times, some dynamic ETMs have been proposed, the characteristic of which is that the triggering threshold can be adjusted according to the evolution of the system, reducing the triggering times and thus saving system resources. Dynamic ETMs can be mainly divided into two types. One is to introduce auxiliary variables described by dynamic equations into the triggering conditions to adjust the triggering threshold [25], [26]. The scalar dynamic equation contains information about the system state, so the obtained threshold changes following the state of the system. The other is that the designed threshold does not evolve according to the dynamic equation, but decays at a certain rate until it reaches a given threshold [27], [28].

From the perspective of FD, the ETMs lose part of the NCSs information due to its non-uniform transmission mode, which increases the difficulty of FD for NCSs. Currently, there are few investigation on the FD of NCSs based on ETMs, and most literatures use the FD filter method. For example, considering the time-delay from the FD object to the FD filter, the problem of FD for NCSs under static ETM based on the FD filter was investigated [29], [30], [31], [32]. By expanding the states of the FD object and the FD filter, the FD issue was converted into a filtering problem, and the design method of the FD filter under ETM was proposed. However, the networks are not only in the S-C channel but also between the C-A channel for typical NCSs. The method proposed by [29], [30], [31], and [32] only took the time-delay or data packet dropout from the FD object to the FD filter into account, and only designed the FD filter, without realizing the co-design of the FD filter and the controller.

To the best of our knowledge, considering both S-to-C and C-to-A time-delay, the co-design of the FD filter and the controller for discrete NCSs under dynamic ETM has not been solved, which is the motivation of this paper. The contributions of this investigation can be exhibited as following points:

1) A new dynamic ETM is proposed, which can effectively reduce the triggering times. Compared with existing methods, the proposed method does not have any restriction on the sampling period.
2) An observer is constructed on the controller node to generate residuals and realize the output feedback control. Through the time sequence analysis of the signal, the closed-loop model of NCSs under the proposed dynamic ETM is established by using the time-delay system approach.
3) A proper Lyapunov-Krasovskii functional which can deal with the S-to-C and C-to-A time-delay independently is constructed. The co-design algorithm of observer gain matrix, the controller gain matrix is provided.

In this paper, through the signal timing analysis under the proposed dynamic ETM, the closed-loop systems subject to S-to-C and C-to-A time-delay are described with equivalent time-delay systems owning two time-delay variables. Furthermore, the co-design approach of the FD observer and the controller is provided by means of dealing with equation inequalities.

The reminder of this paper is arranged as follows. A new dynamic ETM is proposed and the mathematical model of NCSs is established in Section II. Section III provides the co-design approach of the FD observer and the controller. Section IV shows the merits and the effectiveness of the proposed approach by a numerical example. We conclude this paper in Section V.

II. PROBLEM FORMULATION

The considered NCSs' configuration is illustrated in Figure 1, in which an event generator is installed at the sensor node. The event generator's function is to decide whether to send system output to the controller at each sampling instant.

The following is the controlled plant's state space equation:

$$\begin{cases} x_{l+1} = Ax_l + B_u u_l + B_\omega \omega_l + B_f f_l \\ y_l = Cx_l \end{cases} \tag{3}$$

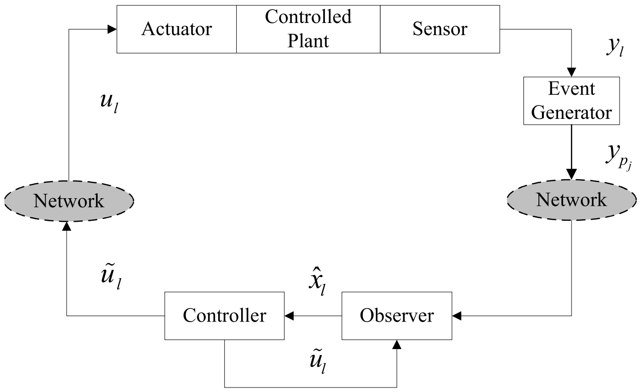

FIGURE 1. Configuration of NCSs with time-delay.

where $x_l \in R^n$ represents the plant's state, $y_l \in R^g$ stands for the plant's output, $u_l \in R^m$ means the plant's control input, $\omega_l \in R^p$ indicates the disturbance, $f_l \in R^q$ is the signal of fault. A, B_u, B_ω, B_f and C are all real matrices with suitable dimensions.

The current sampled output of the system (3) is set as y_l and the last transmitted sampled output is marked as $y_{p_i}(i = 0, 1, 2, \cdots, \infty)$. The event generator would send the current sampled output y_l, only when y_l and $y_{p_i}(i = 0, 1, 2, \cdots, \infty)$ satisfy the following proposed dynamic ETM.

$$\left[y_{p_i} - y_l\right]^T \left[y_{p_i} - y_l\right] - \sigma\phi_l - \mu y_l^T y_l > 0, \tag{4}$$

where $p_i \in Z_+$ is the triggering instant, $\mu \geq 0$ is the triggering threshold, $0 < \sigma < 0.5$ is a known scalar and ϕ_l represents an auxiliary variable with the following dynamic equation:

$$\phi_{l+1} = \sigma\phi_l + \mu y_l^T y_l \tag{5}$$

with $\phi_0 \geq 0$ being the initial condition of ϕ_l.

Remark 1: Apparently we can see from (4) that, when the σ approaches zero, the dynamic ETM (4) becomes (1), and $\phi_l \geq 0$ for $l \in [0, \infty)$, which means that the triggering times can be decreased effectively compared to that of the static ETM (1).

Remark 2: The following dynamic ETM has been proposed in existing literature [33], [34].

$$\left[y_{p_i} - y_l\right]^T \left[y_{p_i} - y_l\right] - \phi_l/\sigma - \mu > 0, \tag{6}$$

where the internal dynamical variable ϕ_l satisfies

$$\phi_{l+1} = \delta\phi_l + \mu - \left[y_{p_i} - y_l\right]^T \left[y_{p_i} - y_l\right]. \tag{7}$$

with $\delta \in (0, 1)$ being a given constant.

For $l \in [p_i + s_{p_i}, p_{i+1} + s_{p_{i+1}} - 1]$, one has

$$\left[y_l - y_{p_i}\right]^T \left[y_l - y_{p_i}\right] - \phi_l/\sigma - \mu \leq 0. \tag{8}$$

It can be obtained that $\phi_l \geq 0$ from (6) and (8) on condition that $\delta > 1/\sigma$ and $\phi_0 \geq 0$. It should be pointed out that $\phi_0 \geq 0$ only holds for $l \in [p_i + s_{p_i}, p_{i+1} + s_{p_{i+1}} - 1]$, rather that $l \in [0, \infty)$ since (8) does not hold at the triggering instants.

Let s_l and c_l represent S-to-C and C-to-A time-delay respectively, assuming that s_l and c_l are both bounded: $s_l \in [0, s_M]$, $c_l \in [0, c_M]$, where s_M and c_M are both non-negative integers.

Considering the influence of the S-to-C time-delay s_l, the instant when the data released by the event generator $y_{p_i}(i = 0, 1, 2, \cdots, \infty)$ reaches the observer is $p_i + s_{p_i}$. Due to the function of the zero-order holder, the system output received by the observer $\tilde{y}_l$ can be expressed as:

$$\tilde{y}_l = y_{p_i}, l \in \left[p_i + s_{p_i}, p_{i+1} + s_{p_{i+1}} - 1\right]. \tag{9}$$

Similar to [35] and [36], consider two cases bellow:

Case 1: If $p_{i+1} + s_{p_{i+1}} - 1 \leq p_i + 1 + s_M$, define a function η_l as

$$\eta_l = l - p_i, l \in \left[p_i + s_{p_i}, p_{i+1} + s_{p_{i+1}} - 1\right]. \tag{10}$$

It can be seen that

$$s_{p_i} \leq \eta_l \leq p_{i+1} - p_i + s_{p_{i+1}} - 1 \leq 1 + s_M. \tag{11}$$

Case 2: If $p_{i+1} + s_{p_{i+1}} - 1 > p_i + 1 + s_M$, apparently, there exists an integer $N \geq 0$ such that

$$p_i + N + s_M < p_{i+1} + s_{p_{i+1}} - 1 = p_i + N + 1 + s_M.$$

It is easy to see that

$$\begin{aligned}&\left[p_i + s_{p_i}, p_{i+1} + s_{p_{i+1}} - 1\right] \\ &\quad = \left[p_i + s_{p_i}, p_i + s_M + 1\right) \cup \cdots \\ &\qquad \cup \left[p_i + s_M + d, p_i + s_M + d + 1\right) \\ &\qquad \cup \left[p_i + s_M + N, p_i + s_M + N + 1\right],\end{aligned}$$

where $d \in 1, 2, \cdots, N$.

Define η_l as follows

$$\eta_l = \begin{cases} l - p_i, l \in \Omega_1, \\ l - p_i - d, l \in \Omega_2, \end{cases} \tag{12}$$

where,

$$\begin{aligned}\Omega_1 &= \left[p_i + s_{p_i}, p_i + s_M + 1\right), \\ \Omega_2 &= [p_i + s_M + d,\ p_i + s_M + d + 1],\ d \in 1, 2, \cdots, N.\end{aligned}$$

From (12), we can obtain

$$\begin{cases} s_{p_i} \leq \eta_l \leq 1 + s_M \triangleq \eta_M, l \in \Omega_1, \\ s_{p_i} \leq s_M \leq \eta_l \leq \eta_M, l \in \Omega_2. \end{cases} \tag{13}$$

For Case 1, define $h_{il} = 0, l \in [p_i + s_{p_i}, p_{i+1} + s_{p_{i+1}} - 1]$.

For Case 2, define

$$h_{il} = \begin{cases} 0, l \in \Omega_1, \\ y_{p_i} - y_{p_i+d}, l \in \Omega_2. \end{cases} \tag{14}$$

From the above definition of h_{il}, the following holds:

$$\begin{aligned} y_{p_i} &= y_{l-\eta_l} + h_{il}, \\ & l \in [p_i + s_{p_i}, p_{i+1} + s_{p_{i+1}} - 1]. \end{aligned} \tag{15}$$

Furthermore,

$$h_{il}^T h_{il} \leq \sigma\phi_l + \mu y_{l-\eta_l}^T y_{l-\eta_l}. \tag{16}$$

At the controller node, construct an observer with the following structure:

$$\begin{cases} \hat{x}_{l+1} = A\hat{x}_l + B_u\tilde{u}_l + L\left(y_{l-\eta_l} + h_{il} - \hat{y}_{l-\eta_l}\right) \\ \hat{y}_l = C\hat{x}_l \\ r_l = W\left(y_{l-\eta_l} - \hat{y}_{l-\eta_l}\right) \end{cases} \tag{17}$$

where $\hat{x}_l \in R^n$ stands for the observer' state, $r_l \in R^q$ means the residual signal and $\hat{y}_l \in R^g$ represents the observer' output. $L \in R^{n\times g}$ and $W \in R^{q\times g}$ are gain matrices of the observer and the residual, respectively.

Adopt the following observer-based feedback control law:

$$\tilde{u}_l = K\hat{x}_l. \tag{18}$$

Remark 3: Result from the time-delay in C-to-A link, the controlled plant's control input in (3) differs from the observer control input in (17), and the following holds:

$$u_l = \tilde{u}_{l-c_l} = K\hat{x}_{l-c_l}. \tag{19}$$

Define the following state estimation error, residual error signal and augmentation vector:

$$e_l = x_l - \hat{x}_l, e_{rl} = r_l - f_l,$$
$$\tau_l^T = \begin{bmatrix} x_l & e_l \end{bmatrix}, \chi_l^T = \begin{bmatrix} \omega_l & f_l \end{bmatrix}.$$

Based on the above analysis, when $l \in [p_i + s_{p_i}, p_{i+1} + s_{p_{i+1}} - 1]$, the closed-loop NCSs can be expressed by the following time-delay system:

$$\begin{cases} \tau_{l+1} = (A_1 + B_1KI_1)\tau_l + I_2Lh_{il} + B_2KI_1\tau_{l-c_l} \\ \qquad + B_d\chi_l + I_2LC_1\tau_{l-\eta_l} \\ e_{rl} = WC_1\tau_{l-\eta_l} - I_3\chi_l \\ \tau_l = \varphi_l, l \in \{-\max(\eta_M, c_M), \cdots, 0\} \end{cases} \tag{20}$$

where,

$$A_1 = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ -B_u \end{bmatrix}, B_2 = \begin{bmatrix} B_u \\ B_u \end{bmatrix},$$
$$B_d = \begin{bmatrix} B_\omega & B_f \\ B_\omega & B_f \end{bmatrix}, C_1 = \begin{bmatrix} 0 & C \end{bmatrix},$$
$$I_1 = \begin{bmatrix} I & -I \end{bmatrix} \in R^{n\times 2n}, I_2 = \begin{bmatrix} 0 & -I \end{bmatrix}^T \in R^{2n\times n},$$
$$I_3 = \begin{bmatrix} 0 & I \end{bmatrix} \in R^{q\times(p+q)}.$$

The objective of this paper is to design FD observer (17) and the control law (18) under the proposed dynamic ETM (4) with the consideration of S-to-C and C-to-A time-delay, such that

1) When $\chi_l = 0$, the closed-loop NCS (20) is asymptotically stable;
2) The following $\mathcal{H}_\infty$ disturbance suppression performance holds under zero initial conditions.

$$\sum_{l=0}^{\infty} e_{rl}^T e_{rl} < \gamma^2 \sum_{l=0}^{\infty} \chi_l^T \chi_l, \tag{21}$$

with $\gamma > 0$ being the disturbance repression performance index. Choose the following evaluation function of residual:

$$J_l = \sum_{z=\psi_0}^{\psi_0+l} \sqrt{r_z^T r_z}. \tag{22}$$

The corresponding FD threshold is as follows

$$J_{th} = \sup_{\chi_l=0} \sum_{z=\psi_0}^{\psi_0+L_0} \sqrt{r_z^T r_z}, \tag{23}$$

with ψ_0 being the initial instant of residual evaluation, and L_0 being the residual evaluation window length. The FD logic is designed as

$$\begin{cases} J_l \le J_{th} \Rightarrow normal, \\ J_l > J_{th} \Rightarrow fault. \end{cases}$$

III. MAIN RESULTS

We need the following lemma to proceed further.

Lemma 1 ([37]): For scalars α, α_0 with $\alpha \ge \alpha_0 \ge 1$ and positive definite matrix $G > 0$, the following always holds

$$\sum_{j=\alpha_0}^{\alpha} \upsilon_j^T G \sum_{j=\alpha_0}^{\alpha} \upsilon_j \le (\alpha - \alpha_0 + 1) \sum_{j=\alpha_0}^{\alpha} \upsilon_j^T G \upsilon_j.$$

The following theorem illustrates sufficient conditions for system (20) to be asymptotically stable.

Theorem 1: When $\chi_l = 0$, for given scalars $0 < \mu < 1$, $0 < \sigma < 0.5$, and residual gain matrix W, if there exist positive definite matrices $P > 0$, $S_1 > 0$, $S_2 > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Q_4 > 0$ and matrices L, K such that

$$\Gamma = \begin{bmatrix} \Gamma_{11} & * & * & * & * & * \\ \Gamma_{21} & \Gamma_{22} & * & * & * & * \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & * & * & * \\ \Gamma_{41} & \Gamma_{42} & \Gamma_{43} & \Gamma_{44} & * & * \\ 0 & S_2 & 0 & 0 & -Q_4 - S_2 & * \\ 0 & 0 & S_1 & 0 & 0 & -Q_3 - S_1 \end{bmatrix} < 0, \tag{24}$$

where,

$$\begin{aligned} \Gamma_{11} = {} & (1+\eta_M)Q_1 + (1+c_M)Q_2 + Q_3 + Q_4 - S_1 - S_2 - P \\ & + (A_1 + B_1KI_1)^T P(A_1 + B_1KI_1) + \mu(CI_4)^T CI_4 \\ & + \eta_M^2 (A_1 + B_1KI_1 - I)^T S_1 (A_1 + B_1KI_1 - I) \\ & + c_M^2 (A_1 + B_1KI_1 - I)^T S_2 (A_1 + B_1KI_1 - I), \\ \Gamma_{21} = {} & S_2 + (B_2KI_1)^T P(A_1 + B_1KI_1) \\ & + \eta_M^2 (B_2KI_1)^T S_1 (A_1 + B_1KI_1) \\ & + c_M^2 (B_2KI_1)^T S_2 (A_1 + B_1KI_1), \\ \Gamma_{22} = {} & -Q_2 - 2S_2 + (B_2KI_1)^T PB_2KI_1 \\ & + \eta_M^2 (B_2KI_1)^T S_1 B_2KI_1 + c_M^2 (B_2KI_1)^T S_2 B_2KI_1, \\ \Gamma_{31} = {} & S_1 + (I_2LC_1)^T P(A_1 + B_1KI_1) \\ & + \eta_M^2 (I_2LC_1)^T S_1 (A_1 + B_1KI_1) \\ & + c_M^2 (I_2LC_1)^T S_2 (A_1 + B_1KI_1), \\ \Gamma_{32} = {} & (I_2LC_1)^T PB_2KI_1 + \eta_M^2 (I_2LC_1)^T S_1 B_2KI_1 \\ & + c_M^2 (I_2LC_1)^T S_2 B_2KI_1, \end{aligned}$$

$$\begin{aligned}
\Gamma_{33} &= -Q_1 - 2S_1 + \mu(CI_4)^T CI_4 + (I_2LC_1)^T PI_2LC_1 \\
&\quad + \eta_M^2 (I_2LC_1)^T S_1 I_2LC_1 + c_M^2 (I_2LC_1)^T S_2 I_2LC_1, \\
\Gamma_{41} &= (I_2L)^T P(A_1 + B_1KI_1) + \eta_M^2 (I_2L)^T S_1(A_1 + B_1KI_1) \\
&\quad + c_M^2 (I_2L)^T S_2(A_1 + B_1KI_1), \\
\Gamma_{42} &= (I_2L)^T PB_2KI_1 + \eta_M^2 (I_2L)^T S_1B_2KI_1 \\
&\quad + c_M^2 (I_2L)^T S_2B_2KI_1, \\
\Gamma_{43} &= (I_2L)^T PI_2LC_1 + \eta_M^2 (I_2L)^T S_1I_2LC_1 \\
&\quad + c_M^2 (I_2L)^T S_2I_2LC_1, \\
\Gamma_{44} &= -I + (I_2L)^T PI_2L + \eta_M^2 (I_2L)^T S_1I_2L \\
&\quad + c_M^2 (I_2L)^T S_2I_2L, \\
I_4 &= \begin{bmatrix} I & 0 \end{bmatrix} \in R^{n\times 2n},
\end{aligned}$$

then the NCS (20) is asymptotically stable. ■

Proof: Construct the following Lyapunov-Krasovskii functional:

$$V_{\tau_l,\phi_l} = \sum_{j=1}^{5} V_{j\tau_l} + \phi_l, \tag{25}$$

where

$$\begin{aligned}
V_{1\tau_l} &= \tau_l^T P\tau_l, \\
V_{2\tau_l} &= \sum_{v=l-\eta_l}^{l-1} \tau_v^T Q_1\tau_v + \sum_{v=l-c_l}^{l-1} \tau_v^T Q_2\tau_v, \\
V_{3\tau_l} &= \sum_{v=l-\eta_M}^{l-1} \tau_v^T Q_3\tau_v + \sum_{v=l-c_M}^{l-1} \tau_v^T Q_4\tau_v, \\
V_{4\tau_l} &= \sum_{b=-\eta_M+1}^{0} \sum_{a=l+b}^{l-1} \tau_a^T Q_1\tau_a \\
&\quad + \sum_{b=-c_M+1}^{0} \sum_{a=l+b}^{l-1} \tau_a^T Q_2\tau_a, \\
V_{5\tau_l} &= \sum_{b=-\eta_M+1}^{0} \sum_{a=l+b-1}^{l-1} \eta_M \varepsilon_a^T S_1\varepsilon_a \\
&\quad + \sum_{b=-c_M+1}^{0} \sum_{a=l+b-1}^{l-1} c_M \varepsilon_a^T S_2\varepsilon_a, \\
\varepsilon_l &= \tau_{l+1} - \tau_l.
\end{aligned}$$

When $l \in [p_i + s_{p_i}, p_{i+1} + s_{p_{i+1}} - 1]$, along the trajectory of the NCSs (20), we can obtain

$$\begin{aligned}
&\Delta V_{1\tau_l} \\
&= \tau_{l+1}^T P\tau_{l+1} - \tau_l^T P\tau_l \\
&= \big((A_1 + B_1KI_1)\tau_l + B_2KI_1\tau_{l-c_l} \\
&\quad + I_2LC_1\tau_{l-\eta_l} + I_2Lh_{il}\big)^T P \\
&\quad \big((A_1 + B_1KI_1)\tau_l + B_2KI_1\tau_{l-c_l} \\
&\quad + I_2LC_1\tau_{l-\eta_l} + I_2Lh_{il}\big) - \tau_l^T P\tau_l \\
&= \tau_l^T \Big((A_1 + B_1KI_1)^T P(A_1 + B_1KI_1) - P\Big) \tau_l \\
&\quad + \tau_l^T (A_1 + B_1KI_1)^T PB_2KI_1\tau_{l-c_l} \\
&\quad + \tau_l^T (A_1 + B_1KI_1)^T PI_2LC_1\tau_{l-\eta_l} \\
&\quad + \tau_l^T (A_1 + B_1KI_1)^T PI_2Lh_{il} \\
&\quad + \tau_{l-c_l}^T (B_2KI_1)^T P(A_1 + B_1KI_1)\tau_l \\
&\quad + \tau_{l-c_l}^T (B_2KI_1)^T PB_2KI_1\tau_{l-c_l} \\
&\quad + \tau_{l-c_l}^T (B_2KI_1)^T PI_2LC_1\tau_{l-\eta_l} \\
&\quad + \tau_{l-c_l}^T (B_2KI_1)^T PI_2Lh_{il} \\
&\quad + \tau_{l-\eta_l}^T (I_2LC_1)^T P(A_1 + B_1KI_1)\tau_l \\
&\quad + \tau_{l-\eta_l}^T (I_2LC_1)^T PB_2KI_1\tau_{l-c_l} \\
&\quad + \tau_{l-\eta_l}^T (I_2LC_1)^T PI_2LC_1\tau_{l-\eta_l} \\
&\quad + \tau_{l-\eta_l}^T (I_2LC_1)^T PI_2Lh_{il} \\
&\quad + h_{il}{}^T (I_2L)^T P(A_1 + B_1KI_1)\tau_l \\
&\quad + h_{il}{}^T (I_2L)^T PB_2KI_1\tau_{l-c_l} \\
&\quad + h_{il}{}^T (I_2L)^T PI_2LC_1\tau_{l-\eta_l} \\
&\quad + h_{il}{}^T (I_2L)^T PI_2Lh_{il}.
\end{aligned} \tag{26}$$

$$\begin{aligned}
&\Delta V_{2\tau_l} \\
&= \sum_{v=l+1-\eta_{l+1}}^{l} \tau_v^T Q_1\tau_v + \sum_{v=l+1-c_{l+1}}^{l} \tau_v^T Q_2\tau_v \\
&\quad - \sum_{v=l-\eta_l}^{l-1} \tau_v^T Q_1\tau_v - \sum_{v=l-c_l}^{l-1} \tau_v^T Q_2\tau_v \\
&= \tau_l^T Q_1\tau_l + \sum_{v=l+1-\eta_{l+1}}^{l-1} \tau_v^T Q_1\tau_v \\
&\quad - \left(\sum_{v=l+1-\eta_l}^{l-1} \tau_v^T Q_1\tau_v + \tau_{l-\eta_l}^T Q_1\tau_{l-\eta_l} \right) \\
&\quad + \tau_l^T Q_2\tau_l + \sum_{v=l+1-c_{l+1}}^{l-1} \tau_v^T Q_2\tau_v \\
&\quad - \left(\sum_{v=l+1-c_l}^{l-1} \tau_v^T Q_2\tau_v + \tau_{l-c_l}^T Q_2\tau_{l-c_l} \right) \\
&= \tau_l^T Q_1\tau_l - \tau_{l-\eta_l}^T Q_1\tau_{l-\eta_l} \\
&\quad + \sum_{v=l+1-\eta_{l+1}}^{l-1} \tau_v^T Q_1\tau_v - \sum_{v=l+1-\eta_l}^{l-1} \tau_v^T Q_1\tau_v \\
&\quad + \tau_l^T Q_2\tau_l - \tau_{l-c_l}^T Q_2\tau_{l-c_l} \\
&\quad + \sum_{v=l+1-c_{l+1}}^{l-1} \tau_v^T Q_2\tau_v - \sum_{v=l+1-c_l}^{l-1} \tau_v^T Q_2\tau_v \\
&= \tau_l^T Q_1\tau_l - \tau_{l-\eta_l}^T Q_1\tau_{l-\eta_l} \\
&\quad + \sum_{v=l+1-\eta_l}^{l-1} \tau_v^T Q_1\tau_v + \sum_{v=l+1-\eta_{l+1}}^{l-\eta_l} \tau_v^T Q_1\tau_v \\
&\quad - \sum_{v=l+1-\eta_l}^{l-1} \tau_v^T Q_1\tau_v \\
&\quad + \tau_l^T Q_2\tau_l - \tau_{l-c_l}^T Q_2\tau_{l-c_l}
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{v=l+1-c_l}^{l-1}\tau_v^T Q_2\tau_v+\sum_{v=l+1-c_{l+1}}^{l-c_l}\tau_v^T Q_2\tau_v \\
&-\sum_{v=l+1-c_l}^{l-1}\tau_v^T Q_2\tau_v \\
\leq\ &\tau_l^T Q_1\tau_l-\tau_{l-\eta_l}^T Q_1\tau_{l-\eta_l} \\
&+\sum_{v=l+1-\eta_M}^{l}\tau_v^T Q_1\tau_v+\tau_l^T Q_2\tau_l \\
&-\tau_{l-c_l}^T Q_2\tau_{l-c_l}+\sum_{v=l+1-c_M}^{l}\tau_v^T Q_2\tau_v. \qquad (27)
\end{aligned}
$$

$$
\begin{aligned}
&\Delta V_{3\tau_l} \\
=\ &\sum_{v=l+1-\eta_M}^{l}\tau_v^T Q_3\tau_v+\sum_{v=l+1-c_M}^{l}\tau_v^T Q_4\tau_v \\
&-\sum_{v=l-\eta_M}^{l-1}\tau_v^T Q_3\tau_v-\sum_{v=l-c_M}^{l-1}\tau_v^T Q_4\tau_v \\
=\ &\tau_l^T Q_3\tau_l-\tau_{l-\eta_M}^T Q_3\tau_{l-\eta_M} \\
&+\tau_l^T Q_4\tau_l-\tau_{l-c_M}^T Q_4\tau_{l-c_M}. \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
&\Delta V_{4\tau_l} \\
=\ &\sum_{b=-\eta_M+1}^{0}\sum_{a=l+1+b}^{l}\tau_a^T Q_1\tau_a \\
&+\sum_{b=-c_M+1}^{0}\sum_{a=l+1+b}^{l}\tau_a^T Q_2\tau_a \\
&-\sum_{b=-\eta_M+1}^{0}\sum_{a=l+b}^{l-1}\tau_a^T Q_1\tau_a \\
&+\sum_{b=-c_M+1}^{0}\sum_{a=l+b}^{l-1}\tau_a^T Q_2\tau_a \\
=\ &\sum_{b=-\eta_M+1}^{0}\left(\sum_{a=l+1+b}^{l}\tau_a^T Q_1\tau_a-\sum_{a=l+b}^{l-1}\tau_a^T Q_1\tau_a\right) \\
&+\sum_{b=-c_M+1}^{0}\left(\sum_{a=l+1+b}^{l}\tau_a^T Q_2\tau_a-\sum_{a=l+b}^{l-1}\tau_a^T Q_2\tau_a\right) \\
=\ &\sum_{b=-\eta_M+1}^{0}\left(\tau_l^T Q_1\tau_l-\tau_{l+b}^T Q_1\tau_{l+b}\right) \\
&+\sum_{b=-c_M+1}^{0}\left(\tau_l^T Q_2\tau_l-\tau_{l+b}^T Q_2\tau_{l+b}\right) \\
=\ &\eta_M\tau_l^T Q_1\tau_l-\sum_{v=l-\eta_M+1}^{l}\tau_v^T Q_1\tau_v \\
&+c_M\tau_l^T Q_2\tau_l-\sum_{v=l-c_M+1}^{l}\tau_v^T Q_2\tau_v. \qquad (29)
\end{aligned}
$$

$$
\begin{aligned}
&\Delta V_{5\tau_l} \\
=\ &\sum_{b=-\eta_M+1}^{0}\sum_{a=l+b}^{l}\eta_M\varepsilon_a^T S_1\varepsilon_a \\
&+\sum_{b=-c_M+1}^{0}\sum_{a=l+b}^{l}c_M\varepsilon_a^T S_2\varepsilon_a \\
&-\sum_{b=-\eta_M+1}^{0}\sum_{a=l+b-1}^{l-1}\eta_M\varepsilon_a^T S_1\varepsilon_a \\
&-\sum_{b=-c_M+1}^{0}\sum_{a=l+b-1}^{l-1}c_M\varepsilon_a^T S_2\varepsilon_a \\
=\ &\sum_{b=-\eta_M+1}^{0}\left(\sum_{a=l+b}^{l}\eta_M\varepsilon_a^T S_1\varepsilon_a-\sum_{a=l+b-1}^{l-1}\eta_M\varepsilon_a^T S_1\varepsilon_a\right) \\
&+\sum_{b=-c_M+1}^{0}\left(\sum_{a=l+b}^{l}c_M\varepsilon_a^T S_2\varepsilon_a-\sum_{a=l+b-1}^{l-1}c_M\varepsilon_a^T S_2\varepsilon_a\right) \\
=\ &\sum_{b=-\eta_M+1}^{0}\left(\eta_M\varepsilon_l^T S_1\varepsilon_l-\eta_M\varepsilon_{l+b-1}^T S_1\varepsilon_{l+b-1}\right) \\
&+\sum_{b=-c_M+1}^{0}\left(c_M\varepsilon_l^T S_2\varepsilon_l-c_M\varepsilon_{l+b-1}^T S_2\varepsilon_{l+b-1}\right) \\
=\ &\eta_M^2\varepsilon_l^T S_1\varepsilon_l-\eta_M\sum_{v=l-\eta_M}^{l-1}\varepsilon_v^T S_1\varepsilon_v \\
&+c_M^2\varepsilon_l^T S_2\varepsilon_l-c_M\sum_{v=l-c_M}^{l-1}\varepsilon_v^T S_2\varepsilon_v. \qquad (30)
\end{aligned}
$$

Making use of Lemma 1, we have

$$
\begin{aligned}
-\eta_M&\sum_{v=l-\eta_M}^{l-1}\varepsilon_v^T S_1\varepsilon_v-c_M\sum_{v=l-c_M}^{l-1}\varepsilon_v^T S_2\varepsilon_v \\
&\leq-\left[\tau_l-\tau_{l-\eta_l}\right]^T S_1\left[\tau_l-\tau_{l-\eta_l}\right] \\
&\quad-\left[\tau_l-\tau_{l-c_l}\right]^T S_2\left[\tau_l-\tau_{l-c_l}\right] \\
&\quad-\left[\tau_{l-\eta_l}-\tau_{l-\eta_M}\right]^T S_1\left[\tau_{l-\eta_l}-\tau_{l-\eta_M}\right] \\
&\quad-\left[\tau_{l-c_l}-\tau_{l-c_M}\right]^T S_2\left[\tau_{l-c_l}-\tau_{l-c_M}\right]. \qquad (31)
\end{aligned}
$$

$$
\begin{aligned}
\Delta\phi_l&=\phi_{l+1}-\phi_l \\
&=(\sigma-1)\phi_l+\mu\tau_l^T(CI_4)^T CI_4\tau_l. \qquad (32)
\end{aligned}
$$

From (26) - (32), under the condition that $0<\sigma<0.5$, we can get

$$
\begin{aligned}
\Delta V_{\tau_l,\phi_l}&+\mu y_{l-\eta_l}^T y_{l-\eta_l}+\sigma\phi_l-h_{il}^T h_{il} \\
&\leq\xi_l^T\Gamma\xi_l+(2\sigma-1)\phi_l \\
&\leq\xi_l^T\Gamma\xi_l, \qquad (33)
\end{aligned}
$$

where,

$$
\begin{aligned}
\xi_l&=\left[\zeta_l^T\ \lambda_l^T\right]^T, \\
\zeta_l&=\left[\tau_l^T\ \tau_{l-c_l}^T\ \tau_{l-\eta_l}^T\right]^T,
\end{aligned}
$$

$$\lambda_l = \left[h_{il}^T \ \tau_{l-c_M}^T \ \tau_{l-\eta_M}^T \right]^T.$$

Therefore, if (24) holds, then the NCS (20) is asymptotically stable, and this completes the proof. □

To obtain the K and L, the inequality constraints in Theorem 1 should be further processed and we get the following theorem.

Theorem 2: When $\chi_l \neq 0$, for given scalars $0 < \mu < 1$, $0 < \sigma < 0.5$, and residual gain matrix W, if there exist matrices K, L and positive definite matrices $P > 0$, $S_1 > 0$, $S_2 > 0$, $Y > 0$, $T_1 > 0$, $T_2 > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Q_4 > 0$ such that

$$\Psi = \begin{bmatrix} \Psi_{11} & * & * \\ \Psi_{21} & \Psi_{22} & * \\ \Psi_{31} & \Psi_{32} & \Psi_{33} \end{bmatrix} < 0, \quad (34)$$

$$PY = I, S_jT_j = I, j \in \{1, 2\}, \quad (35)$$

where,

$$\Psi_{11} = \begin{bmatrix} \Theta_{11} & * & * \\ S_2 & -Q_2 - 2S_2 & * \\ S_1 & 0 & -Q_1 - 2S_1 \end{bmatrix},$$

$$\Theta_{11} = (1 + \eta_M)Q_1 + (1 + c_M)Q_2 + Q_3 + Q_4 - S_1 - S_2 - P,$$

$$\Psi_{21} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_1 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_{22} = Diag\{-I, -Q_4 - S_2, -Q_3 - S_1, -\gamma^2 I\},$$

$$\Psi_{31} = \begin{bmatrix} A_1 + B_1KI_1 & B_2KI_1 & I_2LC_1 \\ \eta_M(A_1 + B_1KI_1 - I) & \eta_M B_2KI_1 & \eta_M I_2LC_1 \\ c_M(A_1 + B_1KI_1 - I) & c_M B_2KI_1 & c_M I_2LC_1 \\ 0 & 0 & \sqrt{\mu}CI_4 \\ 0 & 0 & WC_1 \\ \sqrt{\mu}CI_4 & 0 & 0 \end{bmatrix},$$

$$\Psi_{32} = \begin{bmatrix} I_2L & 0 & 0 & B_d \\ \eta_M I_2L & 0 & 0 & \eta_M B_d \\ c_M I_2L & 0 & 0 & c_M B_d \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I_3 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_{33} = Diag\{-Y, -T_1, -T_2, -I, -I, -I\},$$

then the NCS (20) satisfies $\mathcal{H}_\infty$ performance (21). ■

Proof: when $\chi_l \neq 0$, we have

$$\begin{aligned}
&\Delta V_{\tau_l,\phi_l} + \mu y_{l-\eta_l}^T y_{l-\eta_l} + \sigma\phi_l - h_{il}^T h_{il} + e_{rl}^T e_{rl} - \gamma^2 \chi_l^T \chi_l \\
&\leq \tau_l^T \left((A_1 + B_1KI_1)^T P (A_1 + B_1KI_1) - P \right) \tau_l \\
&+ \tau_l^T (A_1 + B_1KI_1)^T PB_2KI_1\tau_{l-c_l} \\
&+ \tau_l^T (A_1 + B_1KI_1)^T PI_2LC_1\tau_{l-\eta_l} \\
&+ \tau_l^T (A_1 + B_1KI_1)^T PI_2Lh_{il} \\
&+ \tau_l^T (A_1 + B_1KI_1)^T PB_d\chi_l \\
&+ \tau_{l-c_l}^T (B_2KI_1)^T P(A_1 + B_1KI_1)\tau_l \\
&+ \tau_{l-c_l}^T (B_2KI_1)^T PB_2KI_1\tau_{l-c_l} \\
&+ \tau_{l-c_l}^T (B_2KI_1)^T PI_2LC_1\tau_{l-\eta_l} \\
&+ \tau_{l-c_l}^T (B_2KI_1)^T PI_2Lh_{il} \\
&+ \tau_{l-c_l}^T (B_2KI_1)^T PB_d\chi_l \\
&+ \tau_{l-\eta_l}^T (I_2LC_1)^T P(A_1 + B_1KI_1)\tau_l \\
&+ \tau_{l-\eta_l}^T (I_2LC_1)^T PB_2KI_1\tau_{l-c_l} \\
&+ \tau_{l-\eta_l}^T (I_2LC_1)^T PI_2LC_1\tau_{l-\eta_l} \\
&+ \tau_{l-\eta_l}^T (I_2LC_1)^T PI_2Lh_{il} \\
&+ \tau_{l-\eta_l}^T (I_2LC_1)^T PB_d\chi_l \\
&+ h_{il}^T (I_2L)^T P(A_1 + B_1KI_1)\tau_l \\
&+ h_{il}^T (I_2L)^T PB_2KI_1\tau_{l-c_l} \\
&+ h_{il}^T (I_2L)^T PI_2LC_1\tau_{l-\eta_l} \\
&+ h_{il}^T (I_2L)^T PI_2Lh_{il} \\
&+ h_{il}^T (I_2L)^T PB_d\chi_l \\
&+ \chi_l^T B_d{}^T P(A_1 + B_1KI_1)\tau_l \\
&+ \chi_l^T B_d{}^T PB_2KI_1\tau_{l-c_l} \\
&+ \chi_l^T B_d{}^T PI_2LC_1\tau_{l-\eta_l} \\
&+ \chi_l^T B_d{}^T PI_2Lh_{il} \\
&+ \chi_l^T B_d{}^T PB_d\chi_l \\
&+ \tau_l^T Q_1\tau_l - \tau_{l-\eta_l}^T Q_1\tau_{l-\eta_l} \\
&+ \tau_l^T Q_2\tau_l - \tau_{l-c_l}^T Q_2\tau_{l-c_l} \\
&+ \tau_l^T Q_3\tau_l - \tau_{l-\eta_M}^T Q_3\tau_{l-\eta_M} \\
&+ \tau_l^T Q_4\tau_l - \tau_{l-c_M}^T Q_4\tau_{l-c_M} \\
&+ \eta_M\tau_l^T Q_1\tau_l + c_M\tau_l^T Q_2\tau_l \\
&+ \eta_M^2 \varepsilon_l^T S_1\varepsilon_l + c_M^2 \varepsilon_l^T S_2\varepsilon_l \\
&- \left[\tau_l - \tau_{l-\eta_l}\right]^T S_1 \left[\tau_l - \tau_{l-\eta_l}\right] \\
&- \left[\tau_{l-\eta_l} - \tau_{l-\eta_M}\right]^T S_1 \left[\tau_{l-\eta_l} - \tau_{l-\eta_M}\right] \\
&- \left[\tau_l - \tau_{l-c_l}\right]^T S_2 \left[\tau_l - \tau_{l-c_l}\right] \\
&- \left[\tau_{l-c_l} - \tau_{l-c_M}\right]^T S_2 \left[\tau_{l-c_l} - \tau_{l-c_M}\right] \\
&+ \mu\tau_{l-\eta_l}^T I_4^T C^T CI_4\tau_{l-\eta_l} - h_{il}^T h_{il} \\
&+ \left[WC_1\tau_{l-\eta_l} - I_3\chi_l\right]^T \left[WC_1\tau_{l-\eta_l} - I_3\chi_l\right] \\
&- \gamma^2\chi_l^T\chi_l + \mu\tau_l^T (CI_4)^T CI_4\tau_l \\
&= \left[\xi_l \ \chi_l\right] \tilde{\Gamma} \left[\xi_l^T \ \chi_l^T\right]^T,
\end{aligned}$$

where,

$$\tilde{\Gamma} = \begin{bmatrix} \Gamma_{11} & * & * & * & * & * & * \\ \Gamma_{21} & \Gamma_{22} & * & * & * & * & * \\ \Gamma_{31} & \Gamma_{32} & \tilde{\Gamma}_{33} & * & * & * & * \\ \Gamma_{41} & \Gamma_{42} & \Gamma_{43} & \Gamma_{44} & * & * & * \\ 0 & S_2 & 0 & 0 & -Q_4 - S_2 & * & * \\ 0 & 0 & S_1 & 0 & 0 & -Q_3 - S_1 & * \\ \tilde{\Gamma}_{71} & \tilde{\Gamma}_{72} & \tilde{\Gamma}_{73} & \tilde{\Gamma}_{74} & 0 & 0 & \tilde{\Gamma}_{77} \end{bmatrix},$$

$$\tilde{\Gamma}_{33} = \Gamma_{33} + C_1^T W^T WC_1,$$

$$\tilde{\Gamma}_{71} = B_d^T P(A_1 + B_1KI_1) + \eta_M^2 B_d^T S_1(A_1 + B_1KI_1)$$

$$+c_M^2 B_d^T S_2(A_1+B_1KI_1),$$
$$\tilde{\Gamma}_{72}=B_d^T PB_2KI_1+\eta_M^2 B_d^T S_1B_2KI_1+c_M^2 B_d^T S_2B_2KI_1,$$
$$\tilde{\Gamma}_{73}=B_d^T PI_2LC_1-I_3^T WC_1+\eta_M^2 B_d^T S_1I_2LC_1$$
$$+c_M^2 B_d^T S_2I_2LC_1,$$
$$\tilde{\Gamma}_{74}=B_d^T PI_2L+\eta_M^2 B_d^T S_1I_2L+c_M^2 B_d^T S_2I_2L,$$
$$\tilde{\Gamma}_{77}=-\gamma^2 I+I_3^T I_3+B_d^T PB_d+\eta_M^2 B_d^T S_1B_d+c_M^2 B_d^T S_2B_d.$$

Using Schur's complement lemma, $\tilde{\Gamma}<0$ is equivalent to

$$\begin{bmatrix}\Psi_{11} & * & * \\ \Psi_{21} & \Psi_{22} & * \\ \Psi_{31} & \Psi_{32} & \tilde{\Psi}_{33}\end{bmatrix}<0, \tag{36}$$

where $\tilde{\Psi}_{33}=Diag\{-P^{-1},-S_1^{-1},-S_2^{-1},-I,-I,-I\}$. Letting $P^{-1}=Y$, $S_1^{-1}=T_1$, $S_2^{-1}=T_2$, (34) and (35) can be obtained. Besides, we have

$$\Delta V_{\tau_l,\phi_l}+\mu y_{l-\eta_l}^T y_{l-\eta_l}+\sigma\phi_l-h_{il}^T h_{il}$$
$$+e_{rl}^T e_{rl}-\gamma^2\chi_l^T\chi_l<0. \tag{37}$$

From (37) and (16), it is can be seen that

$$\Delta V_{\tau_l,\phi_l}+e_{rl}^T e_{rl}-\gamma^2\chi_l^T\chi_l<0. \tag{38}$$

For (38), summing up l from 0 to ∞, we have

$$\sum_{l=0}^{\infty}e_{rl}^T e_{rl}<\gamma^2\sum_{l=0}^{\infty}\chi_l^T\chi_l+V_{\tau_0,\phi_0}-V_{\tau_\infty,\phi_\infty}.$$

It can be obtained from the zero initial conditions of the system that

$$\sum_{l=0}^{\infty}e_{rl}^T e_{rl}<\gamma^2\sum_{l=0}^{\infty}\chi_l^T\chi_l,$$

which ends the proof. □

Remark 4: Result from the inverse constraints in (35), we cannot obatain L, K, and $\gamma_{\min}$ conveniently with the Matlab linear matrix inequality (LMI) toolbox. However, we can convert the inequality constrains in Theorem 2 into the following minimization problem with the aid of the cone complementarity linearization method [38].

$$Min\ \ Tr\left(PY+\sum_{j=1}^{2}S_jT_j\right)\ s.t.(34),(39)$$
$$\begin{bmatrix}P & I\\ I & Y\end{bmatrix}>0,\begin{bmatrix}S_j & I\\ I & T_j\end{bmatrix}>0, j\in\{1,2\}. \tag{39}$$

The algorithm for computing L, K, and $\gamma_{\min}$ is given in Algorithm 1.

IV. A CASE STUDY ON DYNAMIC CART SYSTEM

In this section, to show the advantage of the proposed dynamic event-triggered FD method, the obtained results will be used to the dynamic cart system [39] shown in Figure 2, where m means the cart mass, u stands for the force exerted on the cart, s represents the cart displacement, h refers to the buffer viscous friction coefficient, and k_s is the spring coefficient.

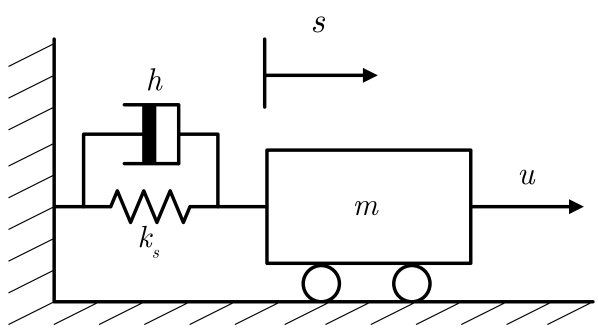

FIGURE 2. Cart dynamic system.

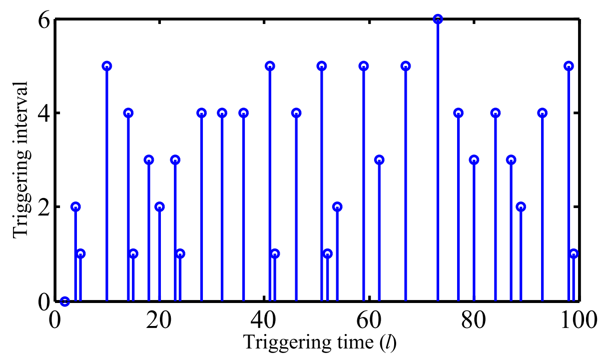

FIGURE 3. Triggering instant and triggering interval under normal conditions.

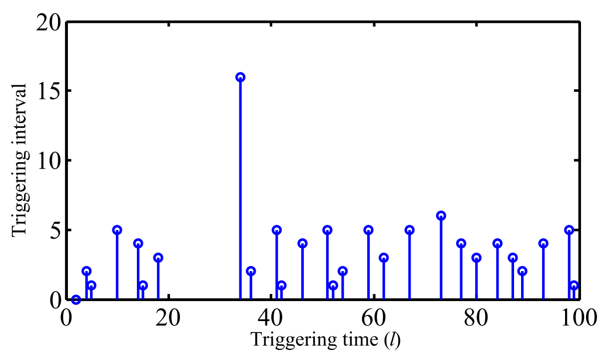

FIGURE 4. Triggering instant and triggering interval in case of failure.

The triggering times under different event-triggered mechanisms are illustrated in Table 1. From Table 1, we can see that the proposed dynamic ETM can efficiently cut the triggering times down, thereby saving the use of computing and communication resources. When $m=2kg$, $h=3N\cdot s/cm$, $k_s=100N/cm$, by selecting the state of the dynamic cart system as $x=\left[s\ \dot{s}\right]^T$ and choosing the sampling period as $0.1s$, the parameters of the discrete state space model can be obtained as follows

$$A=\begin{bmatrix}0.7717 & 0.0853\\ -4.2658 & 0.6437\end{bmatrix}, B_u=\begin{bmatrix}0.0023\\ 0.0427\end{bmatrix},$$

Algorithm 1 Computing Procedure of L, K and $\gamma_{\min}$

1: Let $\gamma = \gamma_0$, set the maximal iterations times $R_{\max}$
2: Obtain a feasible solution $\left(S_1^0, T_1^0, S_2^0, T_2^0, P^0, Y^0, K^0, L^0\right)$ satisfying (34) and (39), and set $l = 0$
3: Solve the optimization issue below:
$$Min \;\; Tr\left(\sum_{j=1}^{2} S_j T_j + PY\right) \text{ s.t. (34) and (39)}$$
4: Set $S_1^l = S_1, T_1^l = T_1, S_2^l = S_2, T_2^l = T_2, P^l = P, Y^l = Y, K^l = K, L^l = L$
5: **while** iterations times $< R_{\max}$ **do**
6: **if** (34), (35) hold **then**
7: $\gamma = \gamma - \delta$, $l = l + 1$, go to 3th step
8: **else**
9: $l = l + 1$, go to 3th step
10: **end if**
11: **end while**
12: **if** $\gamma < \gamma_0$ **then**
13: $\gamma_{\min} = \gamma + \delta$
14: **else**
15: No solution can be obtained the optimization problem within the number of iterations $R_{\max}$
16: **end if**

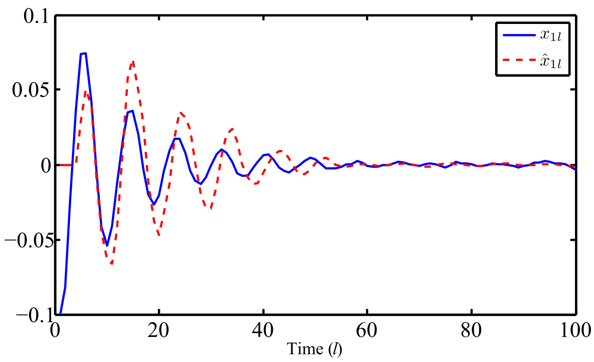

FIGURE 5. System state x_{1l} and its estimated value $\hat{x}_{1l}$.

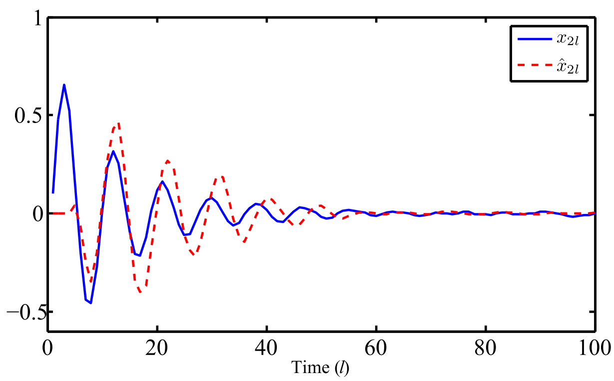

FIGURE 6. System state x_{2l} and its estimated value $\hat{x}_{2l}$.

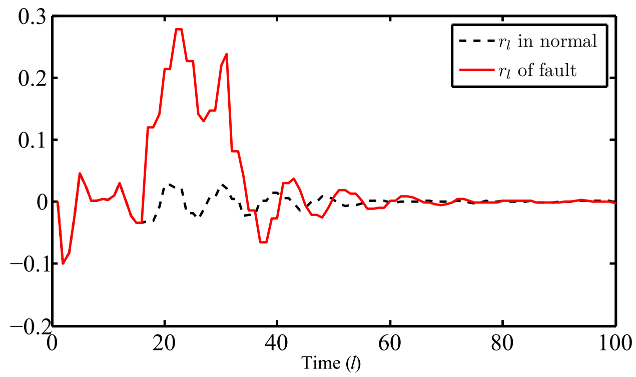

FIGURE 7. Residual signal r_l.

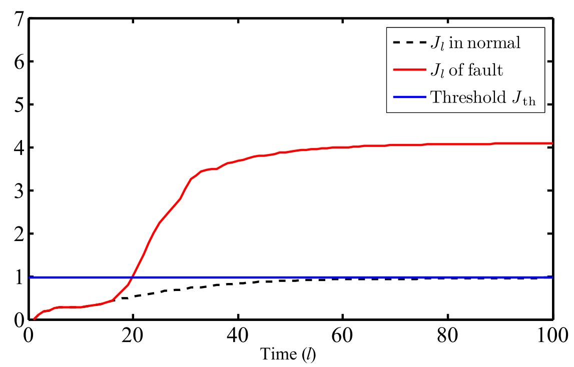

FIGURE 8. Residual evaluation function J_l and FD threshold J_{th}.

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Assuming $B_\omega = \begin{bmatrix} 0.005 \\ 0.065 \end{bmatrix}$, $B_f = \begin{bmatrix} 0.003 \\ 0.006 \end{bmatrix}$, the event-triggered threshold is set as $\mu = 0.5$, the residual gain matrix is given as $W = 1$ and $\sigma = 0.3$. Suppose $s_M = c_M = 1$, and the signal of fault is as follows

$$f_l = \begin{cases} 5, \; l = 15, \cdots, 30 \\ 0, \; otherwise \end{cases}$$

According to Theorem 2, the observer gain matrix L, controller gain matrix K, and minimum disturbance suppression

TABLE 1. The triggering times under different mechanisms.

Mechanism	Triggering times (normal)	Triggering times (fault)
Static mechanism	40	37
Proposed mechanism	29	25

index $\gamma_{\min}$ are computed as follows

$$L = \begin{bmatrix} -0.0531 \\ -0.3468 \end{bmatrix}, K = \begin{bmatrix} 1.2556 & -0.0406 \end{bmatrix},$$
$$\gamma_{\min} = 1.0005.$$

Assuming the initial state $x_0 = \begin{bmatrix} -0.1012 & 0.1001 \end{bmatrix}^T$, $\hat{x}_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$, $\phi_0 = 0$ and the external disturbance signal is random with the amplitude no more than 0.001. Select $J_l = \sum_{z=0}^{l} \sqrt{r_z^T r_z}$ and the FD window length $L_0 = 100$. When no fault occurs, the triggering instant and triggering interval of the event generator are shown in Figure 3. The states of closed-loop NCS and the estimated values are shown in Figure 5 and Figure 6.

When a fault occurs, the triggering instant and triggering interval of the event generator are shown in Figure 4. The fault residual r_l and function J_l are illustrated respectively in Figure 7 and Figure 8, from which it can be seen that both r_l and J_l change dramatically when a fault occurs. Besides, we have $J_{19} = 0.8741 < J_{th} = 0.8805 < J_{20} = 1.0439$, which indicates that the fault signal f_l is detected at the instant $l = 20$.

V. CONCLUSION

The FD issue for NCSs is investigated with consideration of both the impact of the ETM and time-delay based on observer. With the purpose of reducing the triggering times, a new dynamic ETM is proposed. The NCSs subject to both S-to-C and C-to-A time-delay are converted into equivalent time-delay systems. By the method of Lyapunov-Krasovskii functional, the stability conditions of the closed-loop NCSs are established, and the FD observer and the controller gain matrices computation approach is provided, and the co-design of the FD observer and the controller is realized. The problem of FD for NCSs under possible network attacks subject to time-delay and data packet dropout in both S-to-C and C-to-A links with dynamic ETM will be researched in the future.

REFERENCES

[1] L. Su and G. Chesi, "Robust stability analysis and synthesis for uncertain discrete-time networked control systems over fading channels," *IEEE Trans. Autom. Control*, vol. 62, no. 4, pp. 1966–1971, Apr. 2017.

[2] Y. Sadi and S. C. Ergen, "Joint optimization of wireless network energy consumption and control system performance in wireless networked control systems," *IEEE Trans. Wireless Commun.*, vol. 16, no. 4, pp. 2235–2248, Apr. 2017.

[3] M. S. Mahmoud and M. M. Hamdan, "Fundamental issues in networked control systems," *IEEE/CAA J. Autom. Sinica*, vol. 5, no. 5, pp. 902–922, Sep. 2018.

[4] C. Wu, J. Liu, X. Jing, H. Li, and L. Wu, "Adaptive fuzzy control for nonlinear networked control systems," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 47, no. 8, pp. 2420–2430, Aug. 2017.

[5] Y. Long, S. Liu, and L. Xie, "Stochastic channel allocation for networked control systems," *IEEE Control Syst. Lett.*, vol. 1, no. 1, pp. 176–181, Jul. 2017.

[6] B. Lian, Q. Zhang, and J. Li, "Sliding mode control for non-linear networked control systems subject to packet disordering via prediction method," *IET Control Theory Appl.*, vol. 11, no. 17, pp. 3079–3088, Nov. 2017.

[7] C.-Y. Chen, W. Gui, L. Wu, Z. Liu, and H. Yan, "Tracking performance limitations of MIMO networked control systems with multiple communication constraints," *IEEE Trans. Cybern.*, vol. 50, no. 7, pp. 2982–2995, Jul. 2020.

[8] N. Noroozi, R. Geiselhart, S. H. Mousavi, R. Postoyan, and F. R. Wirth, "Integral input-to-state stability of networked control systems," *IEEE Trans. Autom. Control*, vol. 65, no. 3, pp. 1203–1210, Mar. 2020.

[9] Y.-L. Wang, T.-B. Wang, and Q.-L. Han, "Fault detection filter design for data reconstruction-based continuous-time networked control systems," *Inf. Sci.*, vol. 328, pp. 577–594, Jan. 2016.

[10] F. Boem, R. M. G. Ferrari, C. Keliris, T. Parisini, and M. M. Polycarpou, "A distributed networked approach for fault detection of large-scale systems," *IEEE Trans. Autom. Control*, vol. 62, no. 1, pp. 18–33, Jan. 2017.

[11] X. He, Z. Wang, L. Qin, and D. Zhou, "Active fault-tolerant control for an internet-based networked three-tank system," *IEEE Trans. Control Syst. Technol.*, vol. 24, no. 6, pp. 2150–2157, Nov. 2016.

[12] F. Fang, H. Ding, Y. Liu, and J. H. Park, "Fault tolerant sampled-data H_∞ control for networked control systems with probabilistic time-varying delay," *Inf. Sci.*, vol. 544, pp. 395–414, 2021.

[13] L. Li, L. Yao, H. Jin, and J. Zhou, "Fault diagnosis and fault-tolerant control based on Laplace transform for nonlinear networked control systems with random delay," *Int. J. Robust Nonlinear Control*, vol. 30, no. 3, pp. 1223–1239, Feb. 2020.

[14] Q. Wang, Z. Wang, C. Dong, and E. Niu, "Fault detection and optimization for networked control systems with uncertain time-varying delay," *J. Syst. Eng. Electron.*, vol. 26, no. 3, pp. 544–556, Jun. 2015.

[15] K. Huang and F. Pan, "Fault detection for nonlinear networked control systems with sensor saturation and random faults," *IEEE Access*, vol. 8, pp. 92541–92551, 2020.

[16] H. Dong, Z. Wang, J. Lam, and H. Gao, "Fuzzy-model-based robust fault detection with stochastic mixed time delays and successive packet dropouts," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 42, no. 2, pp. 365–376, Apr. 2012.

[17] L. Cui, Y. Yang, and S. Xu, "Fuzzy model based fault detection for nonlinear NCS with packet dropout," in *Proc. IEEE ICCA*, Jun. 2010, pp. 2189–2194.

[18] W. Ding, Z. Mao, B. Jiang, and W. Chen, "Fault detection for a class of nonlinear networked control systems with Markov transfer delays and stochastic packet drops," *Circuits, Syst., Signal Process.*, vol. 34, no. 4, pp. 1211–1231, 2015.

[19] X. He, Z. Wang, and D. H. Zhou, "Robust fault detection for networked systems with communication delay and data missing," *Automatica*, vol. 62, no. 4, pp. 1966–1971, 2009.

[20] M. C. F. Donkers and W. P. M. H. Heemels, "Output-based event-triggered control with guaranteed $\mathcal{L}_\infty$-gain and improved and decentralized event-triggering," *IEEE Trans. Autom. Control*, vol. 57, no. 6, pp. 1362–1376, Jun. 2012.

[21] D. Ding, Z. Wang, D. W. C. Ho, and G. Wei, "Observer-based event-triggering consensus control for multiagent systems with lossy sensors and cyber-attacks," *IEEE Trans. Cybern.*, vol. 47, no. 8, pp. 1936–1947, Aug. 2017.

[22] D. Yang, W. Ren, X. Liu, and W. Chen, "Decentralized event-triggered consensus for linear multi-agent systems under general directed graphs," *Automatica*, vol. 69, pp. 242–249, Jul. 2016.

[23] J. Zhang, S. Li, C. K. Ahn, and Z. Xiang, "Decentralized event-triggered adaptive fuzzy control for nonlinear switched large-scale systems with input delay via command-filtered backstepping," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 6, pp. 2118–2123, Jun. 2022.

[24] S. Song, J. Hu, D. Chen, W. Chen, and Z. Wu, "An event-triggered approach to robust fault detection for nonlinear uncertain Markovian jump systems with time-varying delays," *Circuits, Syst., Signal Process.*, vol. 39, no. 7, pp. 3445–3469, Jul. 2020.

[25] X. Ge, Q.-L. Han, and Z. Wang, "A dynamic event-triggered transmission scheme for distributed set-membership estimation over wireless sensor networks," *IEEE Trans. Cybern.*, vol. 49, no. 1, pp. 171–183, Jan. 2017.

[26] Q. Li, B. Shen, Z. Wang, T. Huang, and J. Luo, "Synchronization control for a class of discrete time-delay complex dynamical networks: A dynamic event-triggered approach," *IEEE Trans. Cybern.*, vol. 49, no. 5, pp. 1979–1986, May 2019.

[27] K. Zhu, Y. Song, D. Ding, G. Wei, and H. Liu, "Robust MPC under event-triggered mechanism and round-robin protocol: An average dwell-time approach," *Inf. Sci.*, vols. 457–458, pp. 126–140, Aug. 2018.

[28] Y. Dong, Y. Song, J. Wang, and B. Zhang, "Dynamic output-feedback fuzzy MPC for Takagi–Sugeno fuzzy systems under event-triggering–based try-once-discard protocol," *Int. J. Robust Nonlinear Control*, vol. 30, no. 4, pp. 1394–1416, Mar. 2020.

[29] H. Li, Z. Chen, L. Wu, H.-K. Lam, and H. Du, "Event-triggered fault detection of nonlinear networked systems," *IEEE Trans. Cybern.*, vol. 47, no. 4, pp. 1041–1052, Apr. 2017.

[30] X. Wang and G. Yang, "Event-based fault detection for non-linear networked systems with multi-data transmission and output quantisation," *IET Control Theory Appl.*, vol. 11, no. 16, pp. 2698–2706, Nov. 2017.

[31] S. Xiao, Y. Zhang, and B. Zhang, "Event-triggered networked fault detection for positive Markovian systems," *Signal Process.*, vol. 157, pp. 161–169, Apr. 2019.

[32] M. Li, S. Li, C. K. Ahn, and Z. Xiang, "Adaptive fuzzy event-triggered command-filtered control for nonlinear time-delay systems," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 4, pp. 1025–1035, Apr. 2022.

[33] L. Huang, J. Guo, and H. Zhang, "Observer-based dynamic event triggering control for periodic denial of service attacks in networked systems," *Control Theory Appl.*, vol. 38, no. 6, pp. 851–861, 2021.

[34] Q. Li, B. Shen, Z. Wang, T. Huang, and J. Luo, "Synchronization control for a class of discrete time-delay complex dynamical networks: A dynamic event-triggered approach," *IEEE Trans. Cybern.*, vol. 49, no. 5, pp. 1979–1986, May 2019.

[35] H. Wang, P. Shi, C.-C. Lim, and Q. Xue, "Event-triggered control for networked Markovian jump systems," *Int. J. Robust Nonlinear Control*, vol. 25, no. 17, pp. 3422–3438, Nov. 2015.

[36] D. Yue, E. Tian, and Q.-L. Han, "A delay system method for designing event-triggered controllers of networked control systems," *IEEE Trans. Autom. Control*, vol. 58, no. 2, pp. 475–481, Feb. 2013.

[37] X. Jiang, Q.-L. Han, and X. Yu, "Stability criteria for linear discrete-time systems with interval-like time-varying delay," in *Proc. Amer. Control Conf.*, Jun. 2005, pp. 2817–2822.

[38] L. El Ghaoui, F. Oustry, and M. AitRami, "A cone complementarity linearization algorithm for static output-feedback and related problems," *IEEE Trans. Autom. Control*, vol. 42, no. 8, pp. 1171–1176, Aug. 1997.

[39] Y. Wang, P. He, P. Shi, and H. Zhang, "Fault detection for systems with model uncertainty and disturbance via coprime factorization and gap metric," *IEEE Trans. Cybern.*, vol. 52, no. 8, pp. 7765–7775, Aug. 2022.

YANFENG WANG was born in Liaocheng, Shandong, China, in 1980. He received the M.S. degree in operations research and cybernetics, in 2007, and the Ph.D. degree in control theory and control engineering from Northeastern University, in 2013.

From 2013 to 2022, he was an Associate Professor at the School of Engineering, Huzhou University, Huzhou, Zhejiang, China. He is currently a Full Professor with the School of Mechanical and Electrical Engineering, Suqian University, Suqian, China. He has authored of three books and more than 30 articles. He was supported by the National Natural Science Funds of China and Natural Science Funds of Zhejiang Province. His main research interests include networked control systems, fault detection, and fault-tolerant control.

YUQIN HOU was born in Binzhou, Shandong, China, in 1997. She received the B.S. degree in engineering from the Yantai Institute of Technology, Shandong, in 2020. She is currently pursuing the graduate degree with the School of Control Engineering, Huzhou University, Huzhou, Zhejiang, China.

Her research interests include network control system fault detection and fault tolerant control.

PEILIANG WANG received the B.Sc. degree in industrial electrical automation and the M.S. degree in control theory and control engineering from Zhejiang University, Hangzhou, China, in 1986 and 2005, respectively.

From 2008 to 2009, he was a Visiting Scholar at Zhejiang University and the University of Duisburg-Essen, Germany, in 2015. He is currently a Professor with the School of Engineering, Huzhou University, China, and a part-time Master Tutor with Hangzhou Dianzi University and the Zhejiang University of Technology, China. He is currently a member of the Committee on the Fault Diagnosis and Safety of Technical Process of the Chinese Association of Automation, the Executive Director and the Deputy Director of the Teaching Committee of the Zhejiang Association of Automation, and the Chairperson of the Huzhou Association of Automation. His research interests include pattern recognition and intelligent control, process monitoring and diagnosis, and industrial automation.