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### **RESEARCH ARTICLE**

# Electronically Tunable 3D Autonomous Chaotic Oscillator Employing Single CCCFA and Its Extension to 4D

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**ABSTRACT** This work presents the realization of a novel electronically tuneable third-order autonomous chaotic oscillator using a current-controlled current conveyor feedback amplifier (CCCFA). The proposed circuit consists of a single CCCFA, two grounded passive capacitors, one inductor, and two diodes. The chaotic oscillator possesses smooth symmetrical sin-hyperbolic nonlinearity through two antiparallelly connected diodes. The electronically tunable intrinsic resistance at the X terminal serves as the bifurcation control parameter in the circuit and is controlled through bias current. Tuning of bias current enables the formation of different periodic and chaotic attractors. The proposed chaotic oscillator exhibits rich nonlinear dynamical behavior such as periodicity, antimonotonicity, and coexistence of attractors. Multistability is investigated through circuit-level simulation. The chaotic oscillator can easily be implemented in an integrated circuit as the configuration is simple, resistorless, and uses the minimum number of components in the count. Further, a higher dimensional (4D) chaotic oscillator is proposed as an extended circuit of the 3D chaotic oscillator and illustrated as an application in chaos encryption. Simulations are done using TSMC 180nm CMOS technology in PSpice. Experimental results are presented to verify the theoretical and simulation analyses of the proposed circuits.

**INDEX TERMS** 3D, 4D, autonomous, CCCFA, chaotic, electronic tuneability, multistability, oscillator.

#### I. INTRODUCTION

At present, nonlinear chaotic systems contribute a captivating gateway into the world of chaos research. In the past few years, chaotic oscillator designs have received rising attention due to chaotic signal's possible commercial applications, mainly in secured communication systems, EEG signal analysis, cryptography, and many more [1], [2]. In the current scenario, analog IC designers have extended the traditional dynamical system designs to a high-performance design in terms of low power consumption, wider bandwidth, simple circuitry, and suitability for VLSI integration. Some basic chaos generators using non-autonomous and autonomous chaotic oscillator circuits are classical Chua's circuit [3], [4], [5], jerk circuit [6], and autonomous oscillator [7].

The continuous improvements of chaotic oscillator designs have attracted considerable attention. Previous chaotic system designs have been based on different realizations using an operational amplifier (Op-amp)/current feedback operational amplifier (CFOA) [3], [4], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18]. The most realized configuration in chaotic oscillator design is classical Chua's oscillator [3], [4], which has served as the guide for most implementations. Further, the different chaotic oscillators have been derived from conventional sinusoidal oscillators and other configurations [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18]. It can be inferred that the most challenging part of synthesizing a chaotic circuit is the design of active nonlinearity [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18]. Nonlinearity is categorized broadly in two forms; asymmetric and symmetric. Most chaotic oscillators' initial realizations comprise asymmetric and segmented piecewise nonlinearity. In [14], [15], [16], [17], and [18], Chaotic oscillators have

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been designed with asymmetric nonlinearity, whereas Opamp and CFOA-based oscillators in [3], [4], [19], [20], [21], and [22] comprise segmented piecewise nonlinearity.

The segmented piecewise nonlinear function is nonsmooth and non-differentiable; hence it has been substituted with various smooth nonlinear functions. The presence of a smooth symmetric function yields a circuit capable of rich and exciting nonlinear behaviors. The smooth nonlinear functions used for designs are arbitrary polynomial [23], Jerk function [24], and conically shaped functions [25]. Chua's circuit implements a cubic nonlinearity in [26], [27], [28], and [29]. In [26], cubic-like nonlinearity is generated through cubic polynomial realization. The transistor-level nonlinearity generated in [27], [28], and [29] resulted in an approximation of cubic-like nonlinearity. However, designs reported in [23], [24], [25], and [26] reveal that implementations of exactly smooth nonlinear functions need several analog multipliers and voltage mode building blocks powered with high supply voltages, and very high active and passive components count, making the resulting circuit very complex. Designs [27], [28], [29] present an approximation of smooth nonlinearity; hence do not exhibit advanced dynamical phenomena such as periodicity, anti-monotonicity, and coexistence of attractor [30], [31].

Further, transistor-based chaotic oscillators are reported in [32], [33], [34], and [35]. Two transistors-based chaotic oscillator is presented in [32]; however, it possesses many limitations, such as an excessive number of resistors in the circuit, the asymmetry between upper and lower bands of double scroll attractors due to different average collector current of the transistor, and limited oscillation frequency in kHz. The existence of chaos in a second-order transistorbased oscillator is presented in [33]; however, the oscillator is nonautonomous and does not employ symmetric nonlinearity. Hence cannot generate double scroll attractors and advanced dynamical behavior. The transistor-based Colpitts and Hartley oscillators have been modified for chaos in [34] and [35]; however, these oscillators do not possess symmetric nonlinearity, and behavior is limited up to the formation of single scroll attractors. Moreover, these circuits cannot exhibit advanced dynamical phenomena, e.g., multistability and antimonotonicity, and are not able to generate complex double scroll attractors.

Furthermore, the high-frequency chaos generators for VHF/UHF communication have been illustrated in [36], [37], and [38]. A vacuum tube-based delayed feedback klystron chaotic oscillator is presented in [36]. Reference [37] presents a spin wave transmission line based chaos generator, and [38] presents a micro-stripline chaos generator. These generators produce chaotic oscillations in the high-frequency range from MHz to GHz. However, implementation of these generators is complex and very bulky as; chaos is controlled through electron beams [36], the efficiency of generated chaos is affected by the attenuation losses of spin wave transmission line [37], and the dimensionality of chaos generator depends on the

dimension of micro strip transmission line [38]. Moreover, power consumption in these generators is very high.

The simplicity of a chaotic oscillator may be assessed in terms of three major criteria as follows: (a) simple mathematical equations involving nonlinear function for smooth and symmetric nonlinearity, (b) simple circuitry employing minimum possible active/passive components, and (c) easy practical implementation using off the shelf ICs. Therefore, because of the simplest chaotic oscillator circuit design with smooth symmetric nonlinearity, the literature [13], [39], [40], [41], [42], [43], [44] illustrated the use of two antiparallelly connected diodes generating sin-hyperbolic like smooth symmetric nonlinearity. The chaotic oscillator in [39] uses one CFOA, two capacitors, one inductor, one resistor, and two diodes, whereas the circuit in [40] uses one Op-Amp, two capacitors, one inductor, three resistors, and two diodes. The chaotic oscillator in [41] uses one Op-Amp, two capacitors, one inductor, one resistor, and two diodes.

During the past few decades, the current-mode approach in analog integrated circuit design has been receiving enormous attention over its voltage mode counterparts due to their potential advantages, such as inherently wide bandwidth, higher slew rate, greater linearity, wider dynamic range, simple circuitry, and low power consumption [14], [20], [21], [22], [25], [45], [46], [47]. Chaotic oscillator in [45] uses one current mode block, namely an operational trans-resistance amplifier (OTRA), two capacitors, one inductor, two resistors, and two diodes, whereas [46] uses one CFOA, one inverter, two capacitors, one inductor, one resistor, and two diodes. However, it is revealed from [39], [40], [41], [45], [46] that designs use an excessive number of components and/or do not possess advanced dynamical behaviors.

One of the desirable phenomena in current mode circuit design is the feature of electronic tuneability using intrinsic resistance of the current mode building block (CMBB). Chaotic oscillator designs having electronic tuneability features are reported in a few of the literature [16], [24]; however, none of them possesses the tunability through electronically controlled intrinsic resistance of the employed active block, which serves as the main bifurcation control parameter in the circuit. The chaotic oscillator in [16] employs electronic tuneability through external bias current to control the bifurcation parameter in the circuit, whereas tuneability in [24] is achieved through transconductance of the active block. Moreover, tunability for chaos control in chaotic systems is also achieved through techniques like adaptive symmetry and control [48], [49], hybrid synchronization in synchronized hybrid generators [50], and backstepping control [51]. In these techniques, the chaotic behavior of the system is controlled through adaptive control of symmetry coefficient or bifurcation parameters. However, tuneability in these techniques is achieved through synchronization between two chaotic systems; therefore, these techniques involve two chaotic systems either identical or one analog and another discrete to generate complex and random behaviour.

Moreover, it is known [39], [40], [41], [45], [46] that many analog 3D chaotic oscillators have limitations in providing some complex dynamic behaviors like complex phase portraits, a high degree of unpredictability, a complex chaotic regime for a wide range of control parameters, and a high degree of secure transmission. However, this statement is not precisely true, especially for digital or hybrid chaos generators based on minimal discrete chaotic maps, adaptive symmetry [48], and synchronized hybrid chaos generation with hybrid synchronization [50]. These characteristics can be achieved by a higher-dimensional analog chaotic oscillator like 4D. Considerable numbers of analog 4D chaotic oscillators have been designed in the literature [42], [43], [44], [45], [46], [47], [52], [53], [54], [55], [56], [57], [58], and references cited therein. The topologies [42], [43], [44], [47], [52], [53], [54], [55], [56], [57], [58] realize only a 4D chaotic oscillator; however, [45], [46], as discussed in the previous paragraph, realize 3D and then extend to 4D. The reported 4D oscillators possess one or more of the following shortcomings: non-availability of smooth symmetric nonlinearity/nonformation of complex attractors in all planes/use of excessive numbers of ABBs/use of multipliers/use of excessive passive components/non-availability of intrinsic electronic tuning.

An exhaustive literature survey motivated the authors to propose a novel, simple chaotic oscillator configuration with minimum possible active and passive components and advanced chaotic phenomena features. The proposed chaotic oscillator is resistorless and employs a single current mode CMOS-based building block, CCCFA, with two capacitors, one inductor, and two diodes. The circuit is electronically tuneable with the intrinsic resistance ( $R_X$ ) at the X terminal of CCCFA, which serves as the bifurcation parameter, on being controlled through bias current. Further, a higher dimensional (4D) chaotic circuit is realized by simply adding one capacitor and one resistor as an extension of the proposed 3D chaotic oscillator to illustrate the more complex dynamical behavior. An application of chaos encrypted ECG signal for secured communication in 3D, and 4D oscillators is provided.

This paper is organized into various sections. Section 1 is this section; herein, an introduction is given. Section 2 presents the proposed third-order chaotic oscillator (3D), followed by Section 3 wherein illustrates the dynamical analysis. Section 4 presents the simulation results and discussion. In section 5, phenomena of antimonotonicity and coexisting attractors are presented. Section 6 presents the extended 4D chaotic oscillator and its simulation results. Section 7 illustrates the application of proposed chaotic circuits in chaos encryption. Section 8 shows the experimental results, followed by Section 9, wherein a comparison is given. Section 10 is the conclusion.



FIGURE 1. CCCFA (a) symbolic representation (b) CMOS realization.

#### **II. CHAOTIC OSCILLATOR USING A SINGLE CCCFA**

This section proposes a new autonomous chaotic oscillator using a single CCCFA, two diodes, and a few passive components.

#### A. CURRENT CONTROLLED CURRENT CONVEYOR FEEDBACK AMPLIFIER (CCCFA)

The current feedback operational amplifier (CFA or CFOA) is one of the most versatile active blocks in current mode circuit designs. The structure of CFOA combines a second-generation current conveyor (CCII) and a buffered amplifier (BA) [59], [60]. There are various CMOS-based CFOAs available in the literature [59], [60]. However, CFOAs do not possess the electronic tuneability feature. Therefore, CCCFA [61], [62], [63] has been proposed to offer electronic tuneability in CFOAs. In CCCFA, electronically tunable intrinsic resistance  $(R_X)$  at the X terminal is available, which is controlled through tuning of bias current [61], [62], [63]. CCCFA combines a second-generation current controlled current conveyor (CCCII) and a BA [61], [62], [63]. This paper presents a fully CMOS-based structure of CCCFA. The symbolic representation of the CCCFA is shown in Fig. 1(a). The CMOS realization of CCCFA is shown in Fig. 1(b), combining the CCCII [63] formed of  $M_1$ - $M_{13}$  and a BA [59] formed of M<sub>14</sub>-M<sub>21</sub>. R<sub>X</sub> is controlled through bias current, Ib1. Port relationships are as follows:

$$\begin{pmatrix} I_{Y} \\ V_{X} \\ I_{Z} \\ V_{W} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ R_{X} & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} I_{X} \\ V_{Y} \\ V_{Z} \\ V_{W} \end{pmatrix}$$
(1)

where in (1),

$$R_{\rm X} = \frac{1}{\sqrt{8kI_{\rm b1}}} \tag{2}$$

where,  $\mathbf{k} = \mu_{n} C_{OX} \left(\frac{\mathbf{W}}{\mathbf{L}}\right)_{6,7} = \mu_{p} C_{OX} \left(\frac{\mathbf{W}}{\mathbf{L}}\right)_{8,9}$ .

#### **B. PROPOSED CHAOTIC OSCILLATOR**

The proposed third-order autonomous chaotic oscillator is shown in Fig. 2. A chaotic oscillator is derived from a conventional sinusoidal oscillator by connecting a nonlinear element/composite in the configuration [64]. In the proposed circuit, a passive LC resonator formed by L and C<sub>1</sub> is connected to antiparallelly connected diodes through an active RC network formed by CCCFA, R<sub>X</sub>, and C<sub>2</sub>. The C<sub>1</sub>-L-C<sub>2</sub> configuration with CCCFA forms a linear third-order sinusoidal oscillator, and diodes are used to induce the nonlinear behavior. CCCFA with R<sub>X</sub> acts as a NIC (negative impedance converter) to make the circuit satisfy the necessary oscillation condition. Electronically controllable intrinsic resistance, R<sub>X</sub>, serves as a bifurcation control parameter and is tuned through bias current, I<sub>b1</sub>, to result in different nonlinear chaotic phenomena.

The dynamics of the proposed 3D chaotic oscillator are described by a mathematical model, which is defined in a set



FIGURE 2. Proposed chaotic oscillator circuit.

of three first-order autonomous differential equations as;

$$C_{1} \frac{dV_{C2}}{dt} = -V_{C1}\sqrt{8kI_{b1}} + I_{NL} + I_{L};$$
  

$$C_{2} \frac{dV_{C1}}{dt} = -V_{C1}\sqrt{8kI_{b1}}, \quad L\frac{dI_{L}}{dt} = V_{C2} - V_{C1} \quad (3)$$

From (2) and (3), the model equations result as follows:

$$C_{1} \frac{dV_{C2}}{dt} = -\frac{V_{C1}}{R_{X}} + I_{NL} + I_{L}; \quad C_{2} \frac{dV_{C1}}{dt} = -\frac{V_{C1}}{R_{X}};$$
$$L \frac{dI_{L}}{dt} = V_{C2} - V_{C1}$$
(4)

where  $V_{C1}$  and  $V_{C2}$  are the voltages across capacitors  $C_1$  and  $C_2$ , respectively.  $I_L$  is the current in the inductor, L.  $I_{NL}$  corresponds to the nonlinear current of two antiparallel diodes, having sin-hyperbolic like characteristics, and is obtained as;

$$\begin{split} I_{NL} &= I_{D1} - I_{D2} = I_{S} e^{\left(\frac{V_{diff}}{V_{r}}\right) - 1} - I_{S} e^{\left(-\frac{V_{diff}}{V_{r}}\right) - 1} \\ &= 2I_{S} sinh\left(\frac{V_{diff}}{V_{r}}\right) \end{split} \tag{5}$$

where,  $V_{diff} = V_{C2} - V_{C1}$ ,  $V_r = \eta V_T$ , thermal voltage,  $V_T = 26mV$  at room temperature, and  $I_S$  is the reverse saturation current of the diode.

Further, (4) is represented in the form of a standard jerk equation [24] of a third-order chaotic oscillator as;

$$\frac{d^{3}V_{C2}}{dt^{3}}LC_{1}C_{2}R_{X} + \frac{d^{2}V_{C2}}{dt^{2}}LC_{1} + \frac{dV_{C2}}{dt}R_{X}C_{1} + V_{C2} + f_{NL}$$

$$= 0 \quad (6)$$

where  $f_{NL}$  is the nonlinear function of  $I_{NL}$ .

Dynamic properties and other measures of chaos are examined through the analytical method to verify that the system's flow (3) is chaotic. For this, (3) is transformed into dynamic state equations having a set of dimensionless variables (x, y, and z) and control parameters ( $\beta$ , a, b, and  $\varphi$ ). The transformed state equations result as;

$$\dot{x} = -ax + z + \varphi \sinh(y - x)$$
$$\dot{y} = -abx$$
$$\dot{z} = y - x \tag{7}$$

where  $\dot{x}$ ,  $\dot{y}$ , and  $\dot{z}$  are the derivatives of x, y, and z, respectively, with respect to  $\tau$ . Moreover,

$$\mathbf{x} = \frac{\mathbf{V}_{C1}}{\mathbf{V}_{r}}, \quad \mathbf{y} = \frac{\mathbf{V}_{C2}}{\mathbf{V}_{r}}, \ \mathbf{z} = \frac{\beta \mathbf{I}_{L}}{\mathbf{V}_{r}}, \ \mathbf{a} = \beta \sqrt{8k\mathbf{I}_{b1}} = \frac{\beta}{\mathbf{R}_{X}},$$

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$$\mathbf{b} = \frac{\mathbf{C}_1}{\mathbf{C}_2}, \quad \beta = \sqrt{\frac{\mathbf{L}}{\mathbf{C}_1}}, \ \varphi = \frac{2\mathbf{a}\mathbf{I}_S}{\mathbf{V}_r}, \ \text{and} \ \mathbf{t} = \tau\sqrt{\mathbf{L}\mathbf{C}_1} \quad (8)$$

Based on (7) and (8), numerical and simulation analyses of the proposed circuit are discussed in sections 3 and 5.

#### **III. DYNAMICAL ANALYSIS**

#### A. SYMMETRY AND INVARIANCE

System (7) is invariant, for co-ordinate substitution (x, y, z) $\Leftrightarrow$  (-x, -y, -z). If (x, y, z) is the solution of the system, then (-x, -y, -z) is also a solution for the same set of parameter values. The two trajectories have symmetry, and origin (0, 0, 0) is the equilibrium point and a trivial static solution of (7).

#### **B. EXISTENCE OF ATTRACTORS**

The existence of developed attractors in the designed system (7) is verified by calculating the volume (V) contraction/expansion rate of the system at any given point of the state space [30], [31]. For verification of this, divergence ( $\nabla$ ) of the system (7) is found and is obtained as;

$$\nabla V = \frac{\delta \dot{\mathbf{x}}}{\delta \mathbf{x}} + \frac{\delta \dot{\mathbf{y}}}{\delta \mathbf{y}} + \frac{\delta \dot{\mathbf{z}}}{\delta \mathbf{z}}$$
(9)

On solving (7) using (9), we get

$$\nabla V = -a - \varphi \cosh(y - x) < 0 \tag{10}$$

The value of  $(-\varphi \cosh(y - x))$  at any given point (x, y) in the state space will always be negative. Therefore, it confirms from (10) that  $-a - \varphi \cosh(y - x) < 0$ , i.e., the divergence of the system will always be less than zero anywhere in state space. It verifies that system (7) is dissipative.  $\nabla V < 0$ reflects that the initial volume of state space of (7) will be contracted endlessly, and the convergence of asymptotic motion of all orbits settles to an attractor.

#### C. FIXED POINT ANALYSIS

The chaotic system's complex dynamics are attained by investigating the equilibrium state's stability [30], [31]. System (7) has a single equilibrium point or fixed point at the origin (0, 0, 0); hence eigenvalues for the system at (0, 0, 0) are calculated. For this, the Jacobian matrix (M<sub>J</sub>) of (7) is obtained at any arbitrary point of coordinate (x, y, z) as;

$$M_{J} = \begin{pmatrix} -a - \varphi \cosh(y - x) & \varphi \cosh(y - x) & 1 \\ -ab & 0 & 0 \\ -1 & 1 & 0 \end{pmatrix}$$
(11)

Consequently, the eigenvalues of the matrix (11) at the origin (0, 0, 0) are found by solving the characteristic equation as;

$$\det(\mathbf{M}_{\mathbf{J}} - \lambda \mathbf{I}) = 0 \tag{12}$$

where 'I' is a  $3 \times 3$  identity matrix.

Since the eigenvalues are calculated at the equilibrium point i.e. origin (0,0,0), therefore the term ' $\cosh(y - x)$ ' for x=0, y=0 results in unity. Hence, on considering



**FIGURE 3.** Plot for (a) bifurcation of 'a' vs. 'x'. (b) bifurcation plots from circuit and numerical simulation (c) lyapunov exponent vs. 'a'.

cosh(y - x) = 1 in (11), the characteristic equation from (11) and (12), results as;

$$\lambda^3 + \lambda^2(a + \varphi) + \lambda(1 + ab\varphi) + ab = 0$$
(13)

As an example, considering the control parameters, (a, b) = (1.12, 50) and  $\varphi = 0.58 * 10^{-4}$ , the corresponding eigenvalues evaluated at the origin from (13) are obtained as  $\lambda_1 = -4.1422$ ,  $\lambda_{2,3} = 1.511 \pm j3.3519$ . Outcome  $\lambda_{2,3}$  indicates that the origin is unstable in the chaotic regime due to the presence of a positive real part in the eigenvalue. This is the characteristic of a self-excited chaotic oscillatory system.

#### **D. BIFURCATION**

Further, to observe the effect of bifurcation control parameter 'a' on the system (7) and to measure the transition to chaos, the bifurcation diagram is shown in Fig. 3(a), which has

 TABLE 1. Aspect ratio of transistors in CCCFA.

| Transistor  | W(um) | L(um) |  |
|---|-------|-------|--|
| $M_1 - M_3, M_6, M_7, M_{10}, M_{11}, M_{18}, M_{19}$ | 18    | 0.36  |  |
| $M_4, M_5, M_{8,}M_9, M_{12}, M_{13}$                 | 36    | 0.36  |  |
| $M_{16}, M1_7$  | 36    | 0.36  |  |
| $M_{14}, M_{15}, M_{20}, M_{21}$                      | 25    | 0.36  |  |

been plotted numerically in MATLAB. For fixed parameters (b=50,  $\beta = 0.8944^*10^3$ ), 'a' is varied, and the corresponding 'x' is obtained without resetting the initial condition. Fig. 3(a) shows the development of a rich bifurcation process indicating local maxima for 'x' over a range of 'a'. Moreover, the bifurcation diagram is plotted for the circuit's acquired data, obtained from multiple time series responses of the proposed circuit in PSpice, and is compared with numerically simulated responses, as shown in Fig. 3(b). It is observed that the response obtained from the circuit's acquired data is in close agreement with the response obtained through numerical simulation. However, slight shift in diagrams is due to the two different integration schemes employed in numerical and circuit simulations i.e., Runge-Kutta in MATLAB and implicit integration in PSpice.

#### E. ROUTE TO CHAOS

To attain additional proof of chaoticity, the Lyapunov exponents are obtained. The values of the Lyapunov exponent less than zero correspond to regular motions, while positive values correspond to chaotic states. The Lyapunov exponents of the designed system (7) are plotted in MATLAB with the standard Runge-Kutta integration scheme for fixed parameters (b=50,  $\beta = 0.8944^*10^3$ ) and compared with the values obtained from the circuit's acquired data. As shown in Fig 3(c), the largest positive Lyapunov exponent is 1.48 (red color); this confirms the proposed oscillator's chaotic behavior.

## IV. SIMULATION RESULTS FOR THE CHAOTIC OSCILLATOR

The chaotic oscillator is simulated using TSMC 0.18 $\mu$ m CMOS process technology in PSpice with technology model parameters as; C<sub>OX</sub> = 8.422fF/ $\mu$ m<sup>2</sup>,  $\mu$ <sub>O(N)</sub> = 259.53cm<sup>2</sup>/V-s,  $\mu$ <sub>O(P)</sub> = 109.97cm<sup>2</sup>/V-s, and t<sub>OX</sub> = 4.1nm. The results are obtained for parameter values as; V<sub>DD</sub> =  $-V_{SS} = 3V$ , I<sub>b2</sub> = 10 $\mu$ A, C<sub>1</sub> = 50pF, C<sub>2</sub> = 1pF, L = 40uH, and I<sub>b1</sub> = 148 $\mu$ A to 195 $\mu$ A. The aspect ratio of transistors in CCCFA is shown in Table 1.

The simulated time-domain responses of  $V_{C1}$  and  $V_{C2}$  of the chaotic oscillator of Fig. 2 for different values of  $I_{b1}$  (i.e.,  $R_X$ ) are depicted in Fig. 4(a)-(d). It is observed in Fig. 4(a)-(d) that on the tuning of  $I_{b1}$ , the circuit produces rich and interesting nonlinear behaviors of  $V_{C1}$  and  $V_{C2}$ . Further, the phase portraits of  $V_{C1}$  vs.  $V_{C2}$  signals are shown in Fig. 5(a)-(i) for varying bias current,  $I_{b1}$ . The phase trajectories of a single period, period-2, and period-4, are obtained, as shown in Fig. 5(a)-(c) for the I<sub>b1</sub> of 195 $\mu$ A, 189 $\mu$ A, and 184 $\mu$ A, respectively. The single-scroll attractor is obtained for I<sub>b1</sub> = 171 $\mu$ A, as shown in Fig. 5(d), followed by the double scroll attractor through bifurcation for I<sub>b1</sub> = 164 $\mu$ A, as shown in Fig. 5(e). On further decreasing I<sub>b1</sub>, single scroll, period-4, period-2, and single period are obtained at 154.7 $\mu$ A, 151 $\mu$ A, 148 $\mu$ A, and 144 $\mu$ A, respectively, as shown in Fig. 5(f)-(i). Fig. 5(f)-(i) depict similar types of attractors as that of Fig. 5(a)-(d), but the direction of trajectories is different.

Similarly, different phase portraits of V<sub>C2</sub> versus I<sub>L</sub> for different I<sub>b1</sub> are shown in Fig. 6(a)-(h). The phase trajectories of a single period, period-2, and period-4, are obtained, as shown in Fig. 6(a)-(c) for the  $I_{b1}$  of  $195\mu$ A,  $189\mu$ A, and 184 $\mu$ A, respectively. The single-scroll attractor is obtained for  $I_{b1} = 171 \mu A$ , as shown in Fig. 6(d), followed by the double scroll attractor formed for  $I_{b1} = 164 \mu A$ , as shown in Fig. 6(e). On further decreasing  $I_{b1}$ , single scroll, period-4, period-2, and single period are obtained at  $154.7\mu A$ ,  $151\mu A$ , 148 $\mu$ A, and 144 $\mu$ A, respectively, as shown in Fig. 6(f)-(i). Further, different phase portraits of  $V_{C1}$  versus  $I_L$  are obtained, as shown in Fig. 7(a)-(d). The phase trajectories of single period and period-2 attractors are shown in Fig. 7(a) and (b) for  $I_{b1}$  of  $195\mu A$  and  $189\mu A$ , respectively. The single-scroll attractor is obtained at  $I_{b1} = 171 \mu A$ , as shown in Fig. 7(c), and the double scroll attractor is obtained at  $I_{b1} = 164 \mu A$ , as shown in Fig. 7(d).

Further, the frequency spectrum of chaotic oscillations for  $V_{C1}$  and  $V_{C2}$  are shown in Fig. 8(a) and 8(b), respectively. The proposed circuit exhibits high-frequency chaotic oscillations as Fig. 8(a) and (b) depict; the dominant operating frequency of chaotic oscillation corresponds to 8.4MHz. However, it is also verified to operate over a wide frequency range of 18 kHz to 16.8 MHz (not shown).

#### **V. ADVANCED DYNAMICAL PHENOMENA**

#### A. ANTIMONOTONICITY

An interesting nonlinear phenomenon that has been investigated in symmetric dynamical systems is antimonotonicity. This measures the richness of the chaotic system and is the concurrent formation of forward periodic orbits, and then their destruction via a reverse period-doubling as a bifurcation control parameter is slowly monitored [30]. It should be noted that to exhibit this phenomenon, a nonlinear system must have the presence of periodic orbits in the parameter space as obtained in Fig. 5(a)-(c), (g)-(i). As expressed in (8), 'a' is a bifurcation control parameter tuned through I<sub>b1</sub>. For system (7), 'a' varies in the range,  $0.5 \le a \le 3.2$ , and correspondingly bifurcations for V<sub>C2</sub> are plotted. Bifurcations are shown in Fig. 9(a)-(c) for a single period, period-2, and double scroll attractor, respectively, for  $\beta = 0.8944^*10^3$  and different values of b.

#### B. COEXISTING ATTRACTORS AND MULTISTABILITY

One of the most striking phenomena in symmetric chaotic system designs is finding various regions in the control



**FIGURE 4.** Time domain responses; (a) single period  $(I_{b1}=195\mu A)$  (b) period-2  $((I_{b1}=189\mu A))$  (c) single scroll  $(I_{b1}=171\mu A)$  (d) double scroll attractor  $((I_{b1}=164\mu A))$ .

parameter's space, where the oscillator develops different coexisting attractors. The coexistence of different attractors in a space domain is identified as multistability, and to investigate this, regions of coexisting attractors are traced over a range of control parameter's space with different initial conditions, (x(0), y(0), z(0)). Some advanced dynamic systems have investigated such a phenomenon [30], [31]. Multistability is a parameter-independent phenomenon in



**FIGURE 5.** Phase portraits of V<sub>C1</sub> vs. V<sub>C2</sub> for; (a) single period ( $l_{b1}$ =195 $\mu$ A), (b) period-2 ( $l_{b1}$ =189 $\mu$ A), (c) period-4 ( $l_{b1}$ =184 $\mu$ A), (d) single scroll ( $l_{b1}$ =171 $\mu$ A), (e) double scroll ( $l_{b1}$ =164 $\mu$ A), (f) single scroll ( $l_{b1}$ =154.7 $\mu$ A), (g) period-4 ( $l_{b1}$ =151 $\mu$ A), (h) period-2 (( $l_{b1}$ =148 $\mu$ A), (i) single period ( $l_{b1}$ =144 $\mu$ A).

advanced chaotic systems for forming different attractors by selecting different initial conditions. In the proposed oscillator, this exciting phenomenon is investigated through circuit-level simulations with different initial conditions and control parameter 'a' (i.e., I<sub>b1</sub>). In the proposed circuit-level simulation, different initial conditions are selected with different values of I<sub>b1</sub>. Multiple coexisting attractors of V<sub>C2</sub> vs. I<sub>L</sub> and V<sub>C1</sub> vs. I<sub>L</sub> are obtained with fixed parameter values as b = 50 and  $\beta$  = 0.8944\*10<sup>3</sup>. As depicted in Fig. 10-13, a set



**FIGURE 6.** Phase portraits of V<sub>C2</sub> vs. I<sub>L</sub> for; (a) single period (I<sub>b1</sub>=195 $\mu$ A), (b) period-2 (I<sub>b1</sub>=189 $\mu$ A), (c) period-4 (I<sub>b1</sub>=184 $\mu$ A), (d) single scroll (I<sub>b1</sub>=171 $\mu$ A), (e) double scroll (I<sub>b1</sub>=164 $\mu$ A), (f) single scroll (I<sub>b1</sub>=154.7 $\mu$ A), (g) period-4 (I<sub>b1</sub>=151 $\mu$ A), (h) period-2 ((I<sub>b1</sub>=148 $\mu$ A), (i) single period (I<sub>b1</sub>=144 $\mu$ A).

of four disconnected co-existing attractors for  $V_{C2}$  vs.  $I_L$  are obtained.

Similarly, as depicted in Fig. 14 and 15, four disconnected co-existing attractors for  $V_{C1}$  vs.  $I_L$  are obtained. It can be observed that, for two initial conditions, the attractors exhibited by the system are precisely similar, except the directions of the trajectories are different. It may be noted that investigation of multistability at the circuit level with intrinsic electronic tuneability has not been found in reported chaotic oscillator designs.



**FIGURE 7.** Phase portraits of V<sub>C1</sub> vs. I<sub>L</sub> for; (a) single period (( $I_{b1}$ =195 $\mu$ A) (b) period 2 (( $I_{b1}$ =189 $\mu$ A) (c) single scroll (( $I_{b1}$ =171 $\mu$ A) (d) double scroll ( $I_{b1}$ =164 $\mu$ A).



FIGURE 8. Frequency spectrum for chaotic oscillations; (a) V<sub>C1</sub> (b) V<sub>C2</sub>.

#### **VI. 4D CHAOTIC OSCILLATOR**

High dimensional chaotic oscillators are in enormous demand due to their potential applications in secure communication, such as cryptography and chaos synchronization [42], [43], [44], [47], [52], [53], [54], [55], [56], [57], [58]. Some 4D or hyperjerk circuits have been reported in [42], [43], [44], [47], [52], [53], [54], [55], [56], [57], and [58], exhibiting complex dynamical behaviours through phase portraits. In this section, a new four-dimensional (4D) chaotic oscillator circuit is proposed as an extension of the 3D circuit in Fig. 2. The proposed 4D chaotic oscillator is shown in Fig. 16, which results by connecting one grounded capacitor  $C_3$  and a resistance  $R_0$ in the circuit of Fig. 2. It is a simple 4D chaotic oscillator employing minimum possible components and possessing electronic tuneability feature.



**FIGURE 9.** Bifurcation diagram for  $V_{C2}$  versus 'a' for (a) single period for b=0.8 (b) period-2 for b=4 (c) double scroll for b=48.



**FIGURE 10.** Coexistence of period-2 attractors ( $V_{C2}$  vs.  $I_L$ ) for  $I_{b1}$ =186 $\mu$ A, and (x(0), y(0), z(0)) as, (a) (0, 0.8, 0) (b) (0, -0.8, 0).



**FIGURE 11.** Coexistence of period-4 attractors  $(V_{C2} \text{ vs. } I_L)$  for  $I_{b1} = 186 \mu \text{ A}$ , and (x(0), y(0), z(0)) as, (a) (0, 1, 0), (b) (0, -1, 0).

Governing equations of the proposed 4D chaotic circuit, as shown in Fig. 16, are a set of four first-order differential equations as follows:

$$C_{1} \frac{dV_{C2}}{dt} = -\frac{V_{C1}}{R_{X}} + I_{NL} + \frac{V_{C3} - V_{C1}}{R_{o}};$$

$$C_{2} \frac{dV_{C1}}{dt} = -\frac{V_{C1}}{R_{X}};$$

$$L \frac{dI_{L1}}{dt} = V_{C2} - V_{C3};$$

$$C_{3} \frac{dI_{L2}}{dt} = I_{L} - \frac{V_{C3} - V_{C1}}{R_{o}};$$
(14)



**FIGURE 12.** Coexistence of attractors ( $V_{C2}$  vs.  $I_L$ ) for  $I_{b1}$ =174 $\mu$ A, and (x(0), y(0), z(0)) as, (a) (0, 0.8, 0) (b) (0, -0.8, 0).



**FIGURE 13.** Coexistence of single scroll attractors ( $V_{C2}$  vs.  $I_L$ ) for  $I_{b_1}=174 \mu A$ , and (x(0), y(0), z(0)) as, (a) (0, 1, 0), (b) (0, -1, 0).

On expressing model equations in dimensionless state variable form, (14) results as;

$$\begin{split} \dot{\mathbf{x}} &= -a\mathbf{x} + \mathbf{c} \left( \mathbf{w} - \mathbf{x} \right) + \varphi \sinh(\mathbf{y} - \mathbf{x}); \\ \dot{\mathbf{y}} &= -ab\mathbf{x}; \\ \dot{\mathbf{z}} &= \mathbf{y} - \mathbf{w}; \\ \dot{\mathbf{w}} &= \rho \mathbf{z} - \rho \mathbf{c} (\mathbf{w} - \mathbf{x}); \end{split} \tag{15}$$

where,

$$x = \frac{V_{C1}}{V_r}, \quad y = \frac{V_{C2}}{V_r}, \quad z = \frac{\beta I_{L1}}{V_r}, \quad w = \frac{V_{C3}}{V_r},$$
$$a = \frac{\beta}{R_X}, \quad c = \frac{\beta}{R_O} b = \frac{C_1}{C_2}, \quad \beta = \sqrt{\frac{L}{C_1}}, \quad \rho = \frac{C_1}{C_3},$$
$$\varphi = \frac{2aI_S}{V_r}, \text{ and } t = \tau \sqrt{LC_1}$$
(16)

#### A. DYNAMICAL ANALYSIS OF 4D CHAOTIC OSCILLATOR

#### 1) DISSIPATIVITY

In order to verify the existence of attractors in the proposed 4D system as given in (15), divergence ( $\nabla$ ) of volume (V) of the system (15) is taken as per (9) in line with section III, B, and it results as;

$$\nabla V = -a - c - \rho c - \varphi \cosh(y - x) < 0$$
 (17)

At any given point (x, y), the term ' $-\varphi \cosh(y - x)$ ' will always be negative. Therefore, (17) depicts that  $\nabla V$  will always be less than zero; it proves that system (15) is dissipative.

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**FIGURE 14.** Coexistence of single periodic attractors  $(V_{C1} \text{ vs. } I_L)$  for  $I_{b1}=186 \mu A$ , with (x(0), y(0), z(0)) as, (a) (.01, 0,0), (b) (-.01, 0,0); and period-2 attractors with, (c) (.04, 0,0), (d) (-.04, 0,0).

#### 2) EQUILIBRIUM POINT AND STABILITY

The equilibrium point of system (15) is the origin (0, 0, 0, 0), and eigenvalues of (15) are calculated by det  $(M_J - \lambda I) = 0$ , where Jacobian matrix  $(M_J)$  for (15) is obtained with a 4 × 4 identity matrix 'I' as per (11). The resulting characteristic equation is obtained as follows:

$$\lambda^{4} + \lambda^{3} n_{3} + \lambda^{2} n_{2} + \lambda n_{1} + n_{0} = 0$$
 (18)

where,  $n_3 = \rho c + a + c + \varphi$ ;  $n_2 = \rho a c + \varphi a b + \rho + \rho c \varphi$ ;  $n_1 = \rho \varphi + a b c \rho \varphi + a \rho + c \rho$ ;

$$n_{\rm O} = abc\rho + \varphi\rho ab \tag{19}$$

For a case of parameter values as, (a, b, c,  $\rho$ ) = (1.12, 50, 0.2236, 1), and  $\varphi$  = 0.58\*10<sup>-4</sup>, using (19), the eigenvalues for (18) are obtained as,  $\lambda_{1,2} = -1.6802 \pm j1.2886$ ,  $\lambda_{3,4} = 0.8966 \pm j1.4101$ . The positive real part in the eigenvalue ( $\lambda_{3,4}$ ) strongly indicates that the origin is unstable in the chaotic regime.

#### 3) BIFURCATION AND ROUTE TO CHAOS

The bifurcation diagram showing the transition to chaos in the proposed 4D system (15) on varying bifurcation control parameter 'a' is shown in Fig. 17(a). The plot from numerical simulation in MATLAB and the plot obtained from circuit simulation's time series data in PSpice are compared in Fig. 17(a). For fixed parameters (b=50, c = 0.2236,  $\rho = 1$ ), 'a' is being varied, and corresponding 'x' is obtained without resetting the initial condition. There is the development of a rich bifurcation process indicating local maxima for 'x' over a wide range of 'a'. Further, the plots for Lyapunov exponents are depicted in Fig. 17(b). The highest positive exponent corresponds to 1.58 (red color), which confirms the chaoticity in the proposed 4D circuit. It is observed that numerical simulated responses depict close agreement to responses obtained from the circuit's data.



**FIGURE 15.** Coexistence of single scroll attractors  $(V_{C1} vs. l_L)$  for  $l_{b_1}=174 \mu A$ , with (x(0), y(0), z(0)) as, (a) (.01, 0,0), (b) (-.01, 0,0), and (c) (.04, 0,0), (d) (-.04, 0,0).



FIGURE 16. Proposed 4D chaotic oscillator circuit.

#### 4) LYAPUNOV STABILITY DIAGRAM

Lyapunov stability diagram depicting dynamical regions with variation in two bifurcation parameters, 'a' and 'b' for the proposed 4D system, is shown in Fig. 17(c). Blue corresponds to a stable state, green to a periodic region, yellow to quasiperiodic, and red to a chaotic region. It indicates proposed 4D system possesses periodicity and chaotic behaviour over a considerable range of parameter values, and the chaotic characteristics of the proposed system are more complex and difficult to predict. Fig. 17(c) also depicts the Lyapunov exponents corresponding to chaotic and periodic regions.

#### **B. SIMULATION RESULTS FOR 4D CHAOTIC OSCILLATOR**

The complex attractors of different varieties are obtained for 4D topology of Fig. 16, for  $C_1 = 50pF$ ,  $C_2 = 1pF$ ,  $L=40\mu$ H,  $C_3 = 50pF$ , and  $R_0 = 4k\Omega$  as depicted in Fig. 18-25. Complex structures of phase portraits are characteristics of a higher dimensional chaotic system, as the simulated attractors exhibit. 4D double scroll attractors corresponding to the phase portrait of  $V_{C1}$  vs.  $V_{C3}$  are shown in Fig. 18(a) and (b) for  $I_{b1} = 195\mu$ A and  $I_{b1} =$  $166.7\mu$ A, respectively. Double scroll attractors corresponding to  $V_{C3}$  vs.  $V_{C2}$  are shown in Fig. 19(a) and (b), whereas



FIGURE 17. Plots for (a) bifurcation diagram, (b) Lyapunov exponents, (c) Lyapunov stability diagram for 'a' vs. 'b'.

corresponding to  $V_{C1}$  vs.  $V_{C2}$  are shown in Fig. 20(a) and (b). Further, to observe periodicity and route to chaos, various periodic and single scroll attractors corresponding to  $V_{C1}$  vs.  $V_{C3}$ ,  $V_{C2}$  vs.  $V_{C3}$ , and  $V_{C1}$  vs.  $V_{C2}$  are obtained as shown in Fig. 21(a)-(d), 22(a)-(d) and 23(a)-(d) respectively. Moreover, phase portraits of I<sub>L</sub> vs.  $V_{C1}$  and I<sub>L</sub> vs.  $V_{C2}$  depicting periodic, single scroll, and double scroll attractors are shown in Fig. 24(a)-(e) and 25(a)-(e), respectively. It is observed from Fig. 18-25 that the proposed 4D circuit develops a wide



**FIGURE 18.** 4D double scroll attractors of  $V_{C1}$  vs.  $V_{C3}$  for (a)  $I_{b1}$ =195 $\mu$ A, (b)  $I_{b1}$ =166.7 $\mu$ A.



FIGURE 19. 4D double scroll attractors of V<sub>C3</sub> vs. V<sub>C2</sub> for (a)  $I_{b1}$ =195 $\mu$ A, (b)  $I_{b1}$ =166.7 $\mu$ A.



**FIGURE 20.** 4D double scroll attractors of  $V_{C1}$  vs.  $V_{C2}$  for (a)  $I_{b1}$ =195 $\mu$ A, (b)  $I_{b1}$ =166.7 $\mu$ A.

variety of complex attractors in all the planes. It may also be noted that the phenomenon of forming various complex attractors in all the planes through electronic tuneability in a 4D chaotic oscillator is not reported in the available literature.

#### C. MULTISTABILITY IN 4D CHAOTIC OSCILLATOR

Multistability is investigated in the proposed 4D chaotic oscillator circuit in Fig. 16 to illustrate the phenomenon of the coexistence of attractors. Circuit level investigation is done on selecting different set of initial conditions, (x(0), y(0), z(0), w(0)) and tuning of I<sub>b1</sub> (i.e. control parameter 'a'). Various sets of disconnected coexisting attractors are obtained for different initial conditions and I<sub>b1</sub>, as depicted in Fig. 26-29. It may be noted that circuit-level investigation of multistability in 4D chaotic oscillators through electronic tuneability is not reported in any available literature.

#### **VII. APPLICATION IN CHAOS ENCRYPTION**

In the communication system, there is a necessity for the secure transmission of the information signal, which



**FIGURE 21.** Phase portraits of V<sub>C1</sub> vs. V<sub>C3</sub> for (a) single period  $(I_{b1}=144\mu A)$ , (b) period-2  $(I_{b1}=148\mu A)$ , (c) period-4  $(I_{b1}=151\mu A)$ , (d) single scroll  $(I_{b1}=154.7\mu A)$  attractors.



**FIGURE 22.** Phase portraits of V<sub>C3</sub> vs. V<sub>C2</sub> for; (a) single period  $(I_{b1}=144\mu A)$ , (b) period-2  $(I_{b1}=148\mu A)$ , (c) period-4  $(I_{b1}=151\mu A)$ , (d) single scroll  $(I_{b1}=154.7\mu A)$  attractors.

raises the need for many cryptographic techniques. One of the most employed techniques is chaos encryption, which uses a chaotic signal as a carrier signal. High dimensional (4D) chaotic systems find wide application in these techniques over the 3D system due to their complex dynamical behaviour. Here the proposed 3D and 4D chaotic oscillators are demonstrated as an application in the encryption of electrocardiogram signal (ECG), as illustrated in Fig. 30. In this technique, chaotic (V<sub>C2</sub>) and ECG (V<sub>G</sub>) signals are added and then multiplexed with V<sub>C2</sub> [65]. This encrypted signal is transmitted over the channel and demultiplexed at the receiver. After this, the result gets subtracted, and the decrypted ECG signal is obtained through a low pass filter (LPF). Adder and subtractor circuits are shown



**FIGURE 23.** Phase portraits of V<sub>C1</sub> vs. V<sub>C2</sub> for (a) single period ( $I_{b1}$ =144 $\mu$ A), (b) period-2 ( $I_{b1}$ =148 $\mu$ A), (c) period-4 ( $I_{b1}$ =151 $\mu$ A), (d) single scroll ( $I_{b1}$ =154.7 $\mu$ A) attractors.



**FIGURE 24.** Phase portraits of V<sub>C1</sub> vs. I<sub>L</sub> for; (a) single period (I<sub>b1</sub>=144 $\mu$ A), (b) period-2 (I<sub>b1</sub>=148 $\mu$ A), (c) period-4 (I<sub>b1</sub>=151 $\mu$ A), (d) single scroll (I<sub>b1</sub>=154.7 $\mu$ A) (e) double scroll (I<sub>b1</sub>=164 $\mu$ A) attractors.

in Fig. 31(a) and (b). IC ADG659 implements multiplexer (MUX) and demultiplexer (DE-MUX), while LPF is a simple RC circuit. In this technique, the information signal is actually deducted from the transmitted signal instead of the estimation [65], however, for secure transmission of signal, the attacker should not have the DE-MUX with same select



**FIGURE 25.** Phase portraits of V<sub>C2</sub> vs. I<sub>L</sub> for; (a) single period (I<sub>b1</sub>=144 $\mu$ A), (b) period-2 (I<sub>b1</sub>=148 $\mu$ A), (c) period-4 (I<sub>b1</sub>=151 $\mu$ A), (d) single scroll (I<sub>b1</sub>=154.7 $\mu$ A) (e) double scroll (I<sub>b1</sub>=164 $\mu$ A) attractors.







**FIGURE 27.** Coexistence of quasi-periodic attractors  $(V_{C1} \text{ vs. } I_L)$  for  $I_{b_1} = 148 \mu A$  and (x(0), y(0), z(0), w(0)) as, (a) (0.4, 0, 0, 0) (b) (-0.4, 0, 0, 0).

line pulse signal as of the receiver. The simulated waveforms for 3D and 4D chaotic oscillators are shown in Fig. 32 and 33, respectively. It is observed that the encrypted signal, as shown



**FIGURE 28.** Coexistence of period-4 attractors ( $V_{C1}$  vs.  $I_L$ ) for  $I_{b1}$ =154 $\mu$ A and (x(0), y(0), z(0), w(0)) as, (a) (0.2, 0, 0, 0) (b) (-0.2, 0, 0, 0).



**FIGURE 29.** Coexistence of single scroll attractors  $(V_{C1} \text{ vs. } I_L)$  for  $I_{b1} = 154 \mu A$  and (x(0), y(0), z(0), w(0)) as, (a) (0.4, 0, 0, 0) (b) (-0.4, 0, 0, 0).



FIGURE 30. Chaos encryption scheme of ECG signal using a chaotic oscillator.



FIGURE 31. Op-amp based circuits for; (a) adder (b) subtractor.

in Fig. 33(b) for a 4D chaotic oscillator, depicts a higher degree of chaos and complexity as compared to 3D, shown in Fig. 32(c). This verifies the complex and unpredictable chaotic behaviour of the proposed 4D chaotic circuit and assures the application of the proposed oscillator in the secure transmission of the information signal in communication systems.

#### **VIII. EXPERIMENTAL RESULTS**

The proposed chaotic oscillator using commercially available IC, AD844, is shown in Fig. 34 for experimental verification. The CCCFA is implemented with one AD844 and resistance,  $R_1$  in place of intrinsic resistance,  $R_X$  at the X terminal of CCCFA. The supply voltage is set as  $\pm 5V$ , with selected values of components,  $C_1 = 50$  pF,  $C_2 = 1$ pF, and  $L = 40\mu$ H.







FIGURE 33. Encryption with 4D chaotic oscillator; (a) chaotic signal (b) encrypted ECG signal (c) decrypted ECG signal.



FIGURE 34. Implementation of the proposed chaotic circuit using AD844.

Fig. 35(a) shows the experimental setup. The response of the single scroll attractor of the 3D circuit is shown in Fig. 35(b) for  $R_1 = 300\Omega$ , whereas the double scroll attractor is shown in Fig. 35(c), obtained at  $R_1 = 440\Omega$ . Moreover, various double scroll attractors of the proposed 4D circuit are shown in Fig. 36(a)-(c). Double scroll attractors corresponding to  $V_{C1}$  vs.  $V_{C3}$  and  $V_{C2}$  vs.  $V_{C3}$  are shown in Fig. 36(a) and (b), respectively, for  $R_1 = 480\Omega$ . Double scroll attractor of  $V_{C1}$  vs.  $V_L$  is shown in Fig. 36(c), where  $V_L$  corresponds to the voltage across L. Parameter values for the 4D circuit are set as;  $C_3 = 50pF$ ,  $R_O = 8k\Omega$  as per circuit of Fig. 16.

#### IX. COMPARISON WITH THE AVAILABLE LITERATURE

The comparison of the reported literature is given in Table 2. It is observed that designs [4, 8, 9 (Fig. 1(b, c)), 11, 13 (Fig. 6a), 20-26, 46] use 2-12 ABBs, and designs [9 (Fig.2b), 10, 12, 13 (Fig. 2a), 14-18, 22, 39-41, 45] employ only one ABB as that of the proposed work. The number of passive







FIGURE 35. Experimental; (a) set up, (b) single scroll attractor ( $V_{C2}$  vs.  $V_{C1}$ ) for 3D, (c) double scroll attractor ( $V_{C2}$  vs.  $V_{C1}$ ) for 3D.

components in the count is excessive in [4], [8], [9], [10], [11], [12], [13], [14], [15], [19], [20], [21], [22], [23], [24], [25], [26], [28], [29], and [40]. The possession of smooth symmetrical nonlinearity is desirable for rich chaotic responses, and the topologies [13 (Fig. 6a), 23, 25, 26, 39-41, 45, 46] possess this property. The electronic tuning feature is available in [16] and [24], whereas the formation of attractors through electronic tuning is not available in [24]. Transistor-based chaotic oscillators [32], [34], [35] possess many shortcomings. The circuit in [32] has shortcomings as an excessive number of resistors in the circuit, low operating frequency, and asymmetry in upper and lower bands of double scroll attractors (i.e., not exactly symmetric nonlinearity). Moreover, [32] does not exhibit advanced dynamical phenomena of multistability. Designs in [34] and [35] have an absence of symmetric nonlinearity, hence cannot generate the double scroll attractors and are not able to exhibit advanced dynamical phenomena, e.g., multistability. Moreover, [32], [34], [35] have not extended the designs to 4D circuits to obtain complex attractors. Advanced dynamical phenomena

#### TABLE 2. Comparison of proposed 3D and 4D chaotic oscillators with available literature.

| Ref. | No. and type<br>of Analog<br>Building<br>Blocks (ABB)<br>/FETs/ Diodes | No. of<br>Passive<br>elements | Presence of<br>smooth<br>symmetric<br>nonlinearity | Availability of<br>Bifurcations<br>through<br>control<br>parameter (R) | Electronic<br>Tuning<br>feature | Advanced<br>Dynamical<br>phenomena<br>with<br>investigation of<br>Multistability | No. of ICs<br>used for<br>practical<br>realization | Dominant<br>Operating<br>Frequency<br>(Hz) | Extended to<br>new 4D<br>chaotic<br>oscillator<br>with<br>electronic<br>tuneability | Multistabili<br>ty and<br>complex<br>attractors in<br>all the<br>planes in<br>4D chaotic<br>oscillator |
|------|--|-------------------------------|--|--|---------------------------------|--|--|--|---|--|
| [4]  | 2 Op-Amp   | 2C, 1L,7R                     | No   | Yes  | No                              | No   | 2  | NA   | No  | No   |
| [8]  | 3 Op-Amp   | 3C, 10R                       | No   | Yes  | No                              | No   | 3  | NA   | No  | No   |
| [9]  | Fig. 1(b),(c): 2<br>VOAs/<br>CFOAs                                     | 2C, 1L, 4-<br>6R              | No   | Yes  | No                              | No   | 2  | NA   | No  | No   |
|      | Fig. 2(b): 1<br>CFOA<br>1 JFET   | 3C, 5R                        | No   | Yes  | No                              | No   | 1  | 20.5k                                      | No  | No   |
| [10] | 1 Op- Amp<br>1 JFET  | 3C, 3R                        | No   | Yes  | No                              | No   | 1  | NA   | No  | No   |
| [11] | Fig. 3, 4:<br>2 VOAs,<br>1 CFOA  | 3C, 11R                       | No   | Yes  | No                              | No   | 3  | NA   | No  | No   |
|      | Fig. 5, 6, 15:<br>2 VOAs /<br>1-2 CFOAs,<br>1 Diode                    | 2-3C, 1L,<br>4-8R             | No   | Yes  | No                              | No   | 2, 1   | NA   | No  | No   |
|      | Fig. 11, 13:<br>1 VOA /<br>2 CFOAs<br>1 JFET                           | 3C, 4R                        | No   | Yes  | No                              | No   | 1  | NA   | No  | No   |
| [12] | 1 CFOA,  | 2C, 1L,                       | No   | Yes  | No                              | No   | 1  | 1.35M                                      | No  | No   |
| [13] | Fig. 2a :<br>1 Op-Amp,   | 3C, 3-4 R                     | No   | Yes  | No                              | No   | 1  | 0.58M                                      | No  | No   |
|      | Fig. 6a :<br>2 Op-Amp,<br>2 Diodes                                     | 3C, 8R                        | Yes  | Yes  | No                              | No   | 2  | NA   | No  | No   |
| [14] | 1 CCII<br>1 JFET   | 3C, 3R                        | No   | Yes  | No                              | No   | 1  | NA   | No  | No   |
| [15] | 1 Op-Amp<br>1 Diode  | 2C, 1L,<br>4R                 | No   | Yes  | No                              | No   | 1  | 58k  | No  | No   |
| [16] | 1 Op-Amp<br>1 Diode  | 2C, 1L                        | No   | Yes  | Yes                             | No   | 1  | NA   | No  | No   |
| [17] | 1 Op-Amp<br>1 Diode  | 2C, 1L,<br>1R                 | No   | Yes  | No                              | No   | 1  | NA   | No  | No   |
| [18] | 1 Op-Amp<br>1 JFET   | 2C, 1L,<br>1R                 | No   | Yes  | No                              | No   | 1  | NA   | No  | No   |
| [19] | 2 CFOAs  | 2C, 1L,                       | No   | Yes  | No                              | No   | 2  | 522k                                       | No  | No   |
| [20] | 4 MOCCII   | 3C, 7R                        | No   | Yes  | No                              | No   | NA   | NA   | No  | No   |
| [21] | 3 DVCCTA   | 3C, 6R                        | No   | Yes  | No                              | No   | 12   | NA   | No  | No   |
| [22] | 1 VDTA   | 2C, 1L,2R                     | No   | Yes  | No                              | No   | 2  | 10M  | No  | No   |
| [23] | 1 Multiplier,<br>11 Op-Amps  | 3C, 24R                       | Yes  | Yes  | No                              | NA   | 12   | 5k   | No  | No   |
| [24] | Fig. 3,4,5,6 :<br>Multiplier, 3-7<br>OTAs,<br>1-5 VFA,<br>1 buffer     | 3C,<br>1-26 R                 | No   | No   | Yes                             | No   | 6 -10  | Few kHz                                    | No  | No   |
| [25] | Fig. 6, 7:<br>3 Multiplier,<br>4 Op-Amp,<br>9 CCII                     | 3C, 6-8R                      | Yes  | Yes  | No                              | No   | 7,12   | 1M   | No  | No   |

| [26]         | 2-Multipliers<br>1 Op-Amp          | 2C, 1L,<br>7R | Yes | Yes | No  | No  | 3  | Few kHz | No   | No  |
|--------------|------------------------------------|---------------|-----|-----|-----|-----|----|---------|------|-----|
| [27]         | Differential pair                  | 2C, 1L,<br>1R | No  | Yes | No  | No  | NA | 1.5M    | No   | No  |
| [28]         | 2 Inverters                        | 2C, 1L,2R     | No  | Yes | No  | No  | NA | 1 M     | No   | No  |
| [29]         | 2 Inverters                        | 2C,1L,3R      | No  | Yes | No  | No  | NA | 1 M     | No   | No  |
| [32]         | 2 Transistors                      | 4C, 7R        | *No | Yes | No  | No  | NA | kHz     | No   | No  |
| [34]         | 1 Transistor                       | 2 L,          | No  | No  | Yes | No  | NA | NA      | No   | No  |
| [35]         | 1 Transistor                       | 2C,1L, 2R     | No  | Yes | Yes | No  | NA | NA      | No   | No  |
| [39]         | 1 CFOA,<br>2 Diodes                | 2C, 1L,<br>1R | Yes | No  | No  | No  | 1  | NA      | No   | No  |
| [40]         | 1 OpAmp,<br>2 Diodes               | 2C, 1L,<br>3R | Yes | Yes | No  | No  | 1  | NA      | No   | No  |
| [41]         | 1 OpAmp,<br>2 Diodes               | 2C, 1L,<br>1R | Yes | Yes | No  | Yes | 1  | NA      | No   | No  |
| [45]         | 1 OTRA,<br>2 Diodes                | 2C, 1L,<br>2R | Yes | Yes | No  | No  | 2  | 7.8M    | *Yes | No  |
| [46]         | 1 CFOA, 1<br>Inverter,<br>2 Diodes | 2C, 1L,<br>1R | Yes | No  | No  | No  | 2  | NA      | *Yes | No  |
| This<br>Work | 1 CCCFA<br>2 Diodes                | 2C, 1L        | Yes | Yes | Yes | Yes | 1  | 8.4M    | Yes  | Yes |

TABLE 2. (Continued.) Comparison of proposed 3D and 4D chaotic oscillators with available literature.

Note: \*Yes: Tuning is not through intrinsic resistance of ABB; \*No: Not exactly symmetric.



FIGURE 36. Double scroll attractors of 4D circuit; (a)  $V_{C1}\,$  vs.  $V_{C3}\,$  (b)  $V_{C3}\,$  vs.  $V_{C2}\,$  (c)  $V_L\,$  vs.  $V_{C1}.$ 

of multistability/coexisting attractors are reported only in [41]. Topologies [39], [40], [45] employ one ABB and possess smooth symmetrical nonlinearity, however, [39] is

controllable through L and does not have bifurcation parameter R, [40] uses excessive resistors and [45] use 2R and two ICs for practical implementation. In [46], the formation of different attractors is through variation in C and L, which is not desirable; moreover, electronic tuneability is not possible, and the advanced multistability phenomenon has also not been investigated. Further, it may also be noted that two active blocks are used to implement a chaotic circuit in [46]. The proposed chaotic oscillator uses minimum active and passive components, employing only one ABB, two Cs, and one L. It is resistorless and possesses smooth symmetric nonlinearity. The circuit is electronically tuneable through intrinsic resistance of ABB and presents the circuitlevel investigation of the advanced chaotic phenomenon of multistability. Moreover, the dominant operating frequency of chaotic oscillations, as depicted in Fig. 8(a) and (b), is the highest among all the earlier available chaotic oscillators designed with smooth symmetric nonlinearity. Furthermore, a new electronically tuneable 4D chaotic oscillator is proposed by simple extension of 3D topology employing minimum active/passive components. The proposed 4D circuit develops various periodic and complex chaotic attractors in all the planes with an exhibition of coexisting attractors (i.e., the phenomenon of multistability), whereas 4D circuits in [45] and [46] neither exhibited various complex attractors in all the planes nor the multistability has been investigated and use more number of components. The proposed circuit is useful in the frequency range of 18 kHz to 16.8 MHz range.

#### **X. CONCLUSION**

This work presents a new electronically tuneable third-order autonomous chaotic oscillator using a single CCCFA. The presence of an electronically tuneable bifurcation parameter  $(R_X)$  in the circuit results in an exciting variety of periodic and

chaotic attractors and bifurcations. It generates chaos with rich dynamical behaviours such as antimonotonicity and the coexistence of attractors. Further, a new 4D chaotic oscillator is obtained by Extension of 3D topology. It exhibits rich chaotic behaviour. The chaos encryption technique for secure communication presents an application of both 3D and the 4D chaotic oscillator. The proposed electronically tuneable 3D and 4D chaotic oscillators are simple circuits employing minimum possible active and passive components with smooth symmetric nonlinearity and exhibit circuit-level investigation of the advanced dynamical phenomenon of multistability. The oscillator exhibits a high dominant operating frequency. Experimental results are presented to verify the proposed chaotic oscillators' simulation results and workability.

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