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RESEARCH ARTICLE

Construction and Assignment of Orthogonal Sequences and Zero Correlation Zone Sequences Over GF(p)

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ABSTRACT Orthogonal sequences can be assigned to a regular tessellation of hexagonal cells, typical for synchronised code-division multiple-access (S-CDMA) systems. The sequences within any cell should both be orthogonal in the adjacent cells and require a sufficient number of users in each cell. However, for binary sequences, the capacity of communication system is limited. So, a new family of non-binary orthogonal sequences is constructed, which has more sequences per cell than binary sequences in the network. Next, an efficient assignment of these orthogonal sequence sets to a regular tessellation of hexagonal cells is given, and also the sequence sets have large re-use distance. Finally, based on above the construction, we construct a family of orthogonal sequences with zero correlation zone (ZCZ) property which can reduce the interference among users in a multi-path environment. The constructed ZCZ sequences can be applied to the quasi-synchronous CDMA (QS-CMDA) spread spectrum systems. And the assignment of sequences has the same re-use distance.

INDEX TERMS Orthogonal sequences, zero correlation zone, Boolean functions, semi-bent functions.

I. INTRODUCTION

Orthogonal sequences are used in many applications, such as synchronous code division multiple access (S-CDMA) spread spectrum systems, detection signals in satellite communications and mobile devices in cloud computing [1], [2]. To prevent interference from the neighboring cells, the regular tessellation of hexagonal cells be used as a model [3], [4], [5]. The sequences within any cell should both be orthogonal in the adjacent cells and require a sufficient number of users in each cell [6]. Employing correlation-constrained sets of Hadamard matrices to construct spreading codes in these systems are a usual way such that the maximal cross-correlation of sequences lies in the range $[2^{m/2}, 2^{\lfloor (m+2)/2 \rfloor}]$ [7], [8]. Akansu et al. presented a class of Walsh-like nonlinear binary orthogonal sequence sets which can be used in asynchronous direct sequence CDMA communications system [9]. However, the number of binary sequences in each cell is 2^{m-3} or 2^{m-2} and it is difficult to increase in S-CDMA systems. This raises a challenge to how to increase the capacity of each cell in S-CDMA systems. In order to increase the number of users in each cell, we propose a class of sequences over $GF(p)$ such that each cell has more sequences. To more sufficient using the sequence sets, the re-use distance was proposed. The so-called re-use distance D reflects the ability to use the same codewords in non-adjacent cells that are far away from the cell where these codewords have originally been placed. At the same time, to prevent interference from the users in

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neighbouring cells, a standard requirement for the assignment of orthogonal sequences in the network is that the sequences within any cell should be orthogonal to the sequences in the neighbouring cells.

In S-CDMA systems, orthogonal sequences, such as well-known Walsh sequences, can guarantee that the correlation of any pairwise sequence is zero without shifting. However, whenever there is a relative shift between the sequences which will loss their orthogonality. To reduce the interference among users in a multipath environment or in a qusai-synchronised CDMA (QS-CDMA) system, Fan et al. proposed the concept of so-called zero correlation zone (ZCZ) property [10]. Tang and Fan obtained the lower bound of the sequence set of ZCZ [11]. Tang and Mow presented a systematic construction of new families of sequences with zero correlation zone [12]. In these systems it is of importance that two sequences have zero cross-correlation within a certain range time-shifts. Such the the time-shifts are called ZCZ width of the sequences. Tang and Wai proposed a new systematic construction of zero correlation zone sequences based on interleaved perfect sequences [13]. Tang et al. obtained a class of ZCZ sequences based on complete complementary codes [14]. Hu and Gong provided a construction ZCZ sequence sets based on interleaving technique [15]. Li and Xu presented a construction of ZCZ sequence sets over Gaussian integers [16], [17]. Zhou et al. proposed a class of optimal ZCZ sequence sets based on interleaving technique and perfect nonlinear functions [18], [19]. Gu et al. constructed a new family of polyphase sequences with low correlation [20]. Wang et al. obtained a class of M-sequences, which have good auto-correlation properties [21], [22].

The main contribution this paper is an efficient method to construct a large set of p-phase orthogonal sequences, using (vectorial) semi-bent functions such that there are p^{m-1} sequences (users) in each cell. The problem of allocating these cells of orthogonal sequences to a regular tessellation of hexagonal cells is also addressed, thus allowing an efficient practical implementation of our method. Based on the results above, a family of p-phase ZCZ sequence sets is constructed based on p-phase complementary triads and p-phase complete complementary codes. We also compare the results with those of previous results. Our constructed ZCZ sequence sets have new parameters. And the assignment of sequences has the same re-use distance.

This article is organized as follows. In Section II, the useful notations are given. In Section III, we construct a new class of p-ary orthogonal sequences, and a family of orthogonal sequences with ZCZ is presented in Section IV. Finally, the conclusion is presented in Section V.

II. PRELIMINARIES

For a prime p, let $\mathbf{F}_{p^m}$ and $\mathbf{F}_p^m$ denote the finite field $GF(p^m)$ and the corresponding m-dimensional vector space, respectively. Let $\mathbf{B}_m$ denote the set of all functions in m-variables. The set of all linear functions in m variables is defined by $L = \{\lambda \cdot \boldsymbol{x} \mid \lambda \in \mathbf{F}_p^m\}$, where $\lambda = (\lambda_1, \cdots, \lambda_m) \in \mathbf{F}_p^m$, $\boldsymbol{x} = (x_1, \cdots, x_m) \in \mathbf{F}_p^m$.

The Fourier transform of function f at λ is denoted by $W_f(\lambda)$ and it is computed as

$$W_f(\lambda) = \sum_{\boldsymbol{x}\in\mathbf{F}_p^m} \omega^{f(\boldsymbol{x})-\lambda\cdot\boldsymbol{x}}, \tag{1}$$

where $\omega = e^{2\pi i/p}$.

The function f is called a bent function if $|W_f(\lambda)| = p^{m/2}$ for any $\lambda \in \mathbf{F}_p^m$. If $|W_f(\lambda)| \in \{0, p^{\lfloor (m+2)/2 \rfloor}\}$ for all $\lambda \in \mathbf{F}_p^m$, then f is called semi-bent function.

Let $f_1, f_2, \cdots, f_p$ be p functions with length p^m. The concatenation of $f_1, f_2, \cdots, f_p$, denoted by $f = f_1 \| f_2 \| \cdots \| f_p$, is a function with length p^{m+1}. The true table of the function f is divided into p equipartitions such that the first part of function f corresponds to f_1, the second part to f_2 and the last part to f_p, the rest can be done in the same manner.

The sequence of a function $f \in \mathbf{B}_m$ is defined as

$$f = \left(\omega^{f(0,\cdots,0,0)}, \omega^{f(0,\cdots,0,1)}, \cdots, \omega^{f(p-1,\cdots,p-1,p-1)}\right), \tag{2}$$

where the period of sequence is p^m.

Definition 1: Let f_1, f_2 be the sequences of functions f_1 and f_2 over the vector space $\mathbf{F}_p^m$, sequences f_1 and f_2 are called orthogonal, denoted by $f_1 \perp f_2$, if

$$f_1 \cdot f_2^* = \sum_{\boldsymbol{x}\in\mathbf{F}_p^m} \omega^{f_1(\boldsymbol{x})-f_2(\boldsymbol{x})} = 0, \tag{3}$$

where f^* denotes the conjugate of f.
Noticing that

$$W_{f_1-f_2}(0_m) = 0 \Leftrightarrow \sum_{\boldsymbol{x}\in\mathbf{F}_p^m} \omega^{f_1(\boldsymbol{x})-f_2(\boldsymbol{x})} = 0,$$

so we have

$$W_{f_1-f_2}(0_m) = 0 \Leftrightarrow f_1 \cdot f_2^* = 0, \tag{4}$$

then the following important characterization of orthogonal sequences is obtained.

Lemma 1: Let f_1, f_2 be the functions over the vector space $\mathbf{F}_p^m$, then $f_1 \perp f_2$ if and only if $W_{f_1-f_2}(\mathbf{0}_m) = 0$.
According to the Lemma 1, it is easily deducing that the set of linear functions $\mathcal{L}_m$ over the vector space $\mathbf{F}_p^m$ is a set of orthogonal sequences.

Definition 2: Let $a = (a_0, a_1, \cdots, a_{N-1})$ and $b = (b_0, b_1, \cdots, b_{N-1})$ be two complex sequences, the aperiodic correlation function of a and b at shift τ is given as follows:

$$R_{a,b}(\tau) = \begin{cases} \sum_{i=0}^{N-1-\tau} a_i b_{i+\tau}^*, & 0 \le \tau \le N-1, \\ 0, & \tau \ge N. \end{cases} \tag{5}$$

$R_{a,b}(\tau)$ is called aperiodic cross-correlation function (ACCF) if $a \neq b$; otherwise, it is called the aperiodic auto-correlation function (AACF). For simplicity, the AACF of a will be written as $R_a(\tau)$.

Definition 3: Let $\mathcal{A} = \{A_1, A_2, \cdots, A_K\}$ be a sequences set, let the length of each sequence of $\mathcal{A}$ is L,

$$A_i = (a_0^i, a_1^i, \cdots, a_{L-1}^i), \quad 1 \le i \le K.$$

$\mathcal{A}$ is said to be a zero correlation zone (ZCZ) sequence set with ZCZ width Z if and only if the set $\mathcal{A}$ satisfies the following two conditions: i)

1) $R_{A_i}(\tau) = 0$ holds for any $1 \le i \le K$ and $1 \le |\tau| \le Z$;
2) $R_{A_i,A_j}(\tau) = 0$ holds for any $i \neq j$ and $0 \le |\tau| \le Z$.

III. CONSTRUCTION OF A LARGE FAMILY OF p-ARY ORTHOGONAL SEQUENCES

In this section we obtain the orthogonal sequences such that the number of users equals p^{m-1} per cell based on linear functions.

Construction 1: Let m, s and t be three positive integers with $m = s+t$, where $s = \lfloor (m-1)/2 \rfloor$ and $t = \lfloor (m+2)/2 \rfloor$. Let p be a prime with $p \ge 3$. Let γ be a primitive element of $\mathbf{F}_{p^{t-1}}$, and $\{1, \gamma, \cdots, \gamma^{t-2}\}$ be a polynomial basis of $\mathbf{F}_{p^{t-1}}$ over $\mathbf{F}_p$. Define the isomorphism $\pi\colon \mathbf{F}_{p^{t-1}} \mapsto \mathbf{F}_p^{t-1}$ by

$$\pi(b_1 + b_2\gamma + \cdots + b_{t-1}\gamma^{t-2}) = (b_1, b_2, \cdots, b_{t-1}).$$

Let $\phi_c : \mathbf{F}_p^s \to \mathbf{F}_p^{t-1}$ be a mapping defined by

$$\phi_c(y) = \begin{cases} 0_{t-1}, & y = 0_s \\ \pi(\gamma^{[y]+c}), & y \in \mathbf{F}_p^s \backslash 0_s \end{cases} \tag{6}$$

where $[y]$ denotes the integer representation of y, and $c \in \{1, 2, \cdots, p^{t-1}\}$ is an integer. For $y \in \mathbf{F}_p^s$, $x \in \mathbf{F}_p^t$, the semi-bent functions can be obtained as follows,

$$\begin{aligned} f_c(y,x) = \phi_c(y)\cdot x &= 0_t \cdot x \| (\pi(\gamma^c), 0)\cdot x \| \\ &\times (\pi(\gamma^{c+1}), 0)\cdot x \| \cdots \| (\pi(\gamma^{c+p^s-2}), 0)\cdot x. \end{aligned} \tag{7}$$

Let $(\beta, \alpha_1, \alpha) \in \mathbf{F}_p^s \times \mathbf{F}_p^{t-1} \times \mathbf{F}_p$, for any fixed $\alpha \in \mathbf{F}_p$, define

$$L_\alpha = \{l = (\beta, \alpha_1, \alpha)\cdot (y,x) \mid \beta \in \mathbf{F}_p^s, \alpha_1 \in \mathbf{F}_p^{t-1}\}. \tag{8}$$

We denote $H_0 = L_0$, $H_1 = L_1, \cdots, H_{p-1} = L_{p-1}$. For any $c \in \{1, 2, \cdots, p^{t-1}\}$, p^t disjoint sequence sets are constructed as follows:

$$S_{c,i} = \{f_c + l \mid l \in H_i\}, \quad \text{for } i \in \{0, 1, \cdots, p-1\}. \tag{9}$$

According to (7), note that the $f_c(y,x)$ is the concatenation of p^s t-variable functions, the length of the function $f_c(y,x)$ is $p^s \cdot p^t = p^m$. These sequences can be easily assigned to hexagonal cells in such a way that the sequences assigned to adjacent cells is orthogonal, while the correlation between sequences assigned to non-adjacent cells is small.

Theorem 1: Let the sequence set $S_{c,i}$ be defined by (9) as in Construction 1. Then, we always have

1) For $c \in \{1, 2, \cdots, p^{t-1}\}$, $\#S_{c,i} = p^{m-1}$, where $\#S$ denotes the number of a sequence set S;
2) All the sequences in $S_{c,i}$ are semi-bent sequences;
3) Let $c, c' \in \mathbf{F}_{p^{t-1}}$, $i, i' \in \{0, 1, \cdots, p-1\}$. If $i \neq i'$, $S_{c,i} \perp S_{c',i'}$ holds.

Proof:

1) Note that $\#L_\alpha = p^{m-1}$, which implies that 1) hold.

2) For the sequence set $S_{c,i}$, without loss of generality, we only consider the case of $i = 0$. Then the sequence set $S_{c,0} = \{f_c + l \mid l \in H_0\}$, where f_c is the sequence of a function f_c, and l is the sequence of a linear function l. For a nonzero $c \in \{1, 2, \cdots, p^s\}$ and for $y \in \mathbf{F}_p^s$, $x \in \mathbf{F}_p^t$,

$$\begin{aligned} f_c(y,x) = 0_t \cdot x \| (\pi(\gamma^c), 0)\cdot x \| (\pi(\gamma^{c+1}), 0)\cdot x \| \\ \cdots \| (\pi(\gamma^{c+p^s-2}), 0)\cdot x. \end{aligned}$$

According to (7), note that the $f_c(y,x)$ is the concatenation of p^s t-variable linear functions, the length of the function $f_c(y,x)$ is $p^s \cdot p^t = p^m$.

For any $(\beta, \alpha_1, \alpha) \in \mathbf{F}_p^s \times \mathbf{F}_p^{t-1} \times \mathbf{F}_p$, we have

$$|W_{f_c}(\beta, \alpha_1, \alpha)| = |\textstyle\sum_{(y,x)\in \mathbf{F}_p^m} \omega^{f_c(y,x) - \beta\cdot y - (\alpha_1,\alpha)\cdot x}| \tag{10}$$

From (10), $\beta \cdot y + (\alpha_1, \alpha)\cdot x$ is an m-variable linear function, where $(\beta, \alpha_1, \alpha) \in \mathbf{F}_p^m$. An m-variable linear function $\beta \cdot y + (\alpha_1, \alpha)\cdot x$ can be regarded as the concatenation of a t-variable linear function $(\alpha_1, \alpha)\cdot x$ and the corresponding affine functions $(\alpha_1, \alpha)\cdot x + c$, where $c \in \mathbf{F}_p$.

In order to get the magnitude of the fourier transform of the function $f_c(y,x)$, we just need to know if the linear function $(\alpha_1, \alpha)\cdot x$ appears in the function $f_c(y,x)$. When $\alpha_1 = \pi(\gamma^{c+k})$, and $\alpha = 0$, where $k \in \{0, 1, \cdots, p^s - 2\}$, $|W_{f_c}(\beta, \alpha_1, \alpha)| = p^t$ holds; otherwise, $|W_{f_c}(\beta, \alpha_1, \alpha)| = 0$. Then all the sequences in $S_{c,i}$ are semi-bent sequences.

3) Let $f_c + l \in S_{c,i}$ and $f_{c'} + l' \in S_{c',i'}$, where $l = \beta\cdot y + (\alpha_1, \alpha)\cdot x \in T_i$ and $l' = \beta'\cdot y + (\alpha_1', \alpha')\cdot x \in T_{i'}$. To analyze the orthogonality between $f_c + l$ and $f_{c'} + l'$, we consider

$$h = (f_c + l) - (f_{c'} + l') = f_{c-c'} + (l - l'). \tag{11}$$

From the analysis of the 2), when $i \neq i'$, we can obtain $\alpha \neq \alpha'$, then $W_h(0_m) = 0$. From Lemma 1, we have $S_{c,i} \perp S_{c',i'}$. □

Remark 1: The assignment of the sequences is consistent with the above assignment such that the correlation between sequences assigned to adjacent cells is zero.

Example 1: Let $m = 4$, $p = 3$ and $\gamma \in \mathbf{F}_{3^2}$ be a root of the primitive polynomial $z^2 + z + 2$. Define the isomorphism $\pi\colon \mathbf{F}_{3^2} \mapsto \mathbf{F}_3^2$ by

$$\pi(b_1 + b_2\gamma) = (b_1, b_2).$$

For $y \in \mathbf{F}_3^1$, $x \in \mathbf{F}_3^3$, then the function $f_c(y,x)$ can be denoted as follows,

$$f_c(y,x) = 0_3 \cdot x \| (\pi(\gamma^c), 0)\cdot x \| (\pi(\gamma^{c+1}), 0)\cdot x,$$

where $c \in \{1, 2, \cdots, 9\}$. We can get 9 semi-bent functions. For simplicity, we only list two functions f_1 and f_2 as follows,

$$\begin{aligned} f_1(y,x) &= 0_t \cdot x || x_2 || x_3, \\ f_2(y,x) &= 0_t \cdot x || x_3 || 2x_3 + x_2, \end{aligned}$$

where $y \in \mathbf{F}_3$, $x = (x_3, x_2, x_1) \in \mathbf{F}_3^3$.

For $c \in \{1, 2, \cdots, 9\}$ and $i \in \mathbf{F}_3$, a set of orthogonal sequences can be defined as follows,

$$S_{c,i} = \{f_c + l \mid l \in H_i\}, \tag{12}$$

where $H_0 = \{\alpha_1 x_3 + \alpha_2 x_2 \mid \alpha_1, \alpha_2 \in \mathbf{F}_3^2\}$, $H_1 = \{\alpha_1 x_3 + \alpha_2 x_2 + x_1 \mid \alpha_1, \alpha_2 \in \mathbf{F}_3^2\}$, and $H_2 = \{\alpha_1 x_3 + \alpha_2 x_2 + 2x_2 \mid \alpha_1, \alpha_2 \in \mathbf{F}_3^2\}$.

From (11), then we have

$$S_{c,i} \perp S_{c',i'} \Leftrightarrow f_{c-c'} \perp H_{i-i'}. \tag{13}$$

The orthogonality between f_c and H_i is shown in the following Table 1. Then we construct 27 sets of orthogonal sequences $\{S_{c,i} \mid c \in \{1, 2, \cdots, 9\}, i \in \mathbf{F}_3\}$. All the 27 sets of orthogonal sequences can be used to get an assignment with the re-use distance $D = \sqrt{27}$ as depicted in Fig. 1.

TABLE 1. Orthogonality between f_c and H_i.

	H_0	H_1	H_2
$\mathbf{f}_{00}$		$\perp$	$\perp$
$\mathbf{f}_{01}$		$\perp$	$\perp$
$\mathbf{f}_{02}$		$\perp$	$\perp$
$\mathbf{f}_{10}$		$\perp$	$\perp$
$\mathbf{f}_{11}$		$\perp$	$\perp$
$\mathbf{f}_{12}$		$\perp$	$\perp$
$\mathbf{f}_{20}$		$\perp$	$\perp$
$\mathbf{f}_{21}$		$\perp$	$\perp$
$\mathbf{f}_{22}$		$\perp$	$\perp$

Construction 2: Let $m, k \geq 2$ be two positive integers with $m = 2k + 2$. Let γ be a primitive element of $\mathbf{F}_{p^k}$, and $\{1, \gamma, \cdots, \gamma^{k-1}\}$ be a polynomial basis of $\mathbf{F}_{p^k}$ over $\mathbf{F}_p$. Define the isomorphism $\pi : \mathbf{F}_{p^k} \mapsto \mathbf{F}_p^k$ by

$$\pi(b_1 + b_2\gamma + \cdots + b_k\gamma^{k-1}) = (b_1, b_2, \cdots, b_k).$$

For $i = \{1, \ldots, p^k\}$, let $\phi_i : \mathbf{F}_p^k \to \mathbf{F}_p^k$ be a bijective mapping defined by

$$\phi_i(y) = \begin{cases} 0_k, & y = 0_k \\ \pi(\gamma^{[y]+i}), & y \in \mathbf{F}_p^k \backslash 0_k \end{cases} \tag{14}$$

where $[y]$ denotes the integer representation of y. Let $y \in \mathbf{F}_p^k$, $x \in \mathbf{F}_p^{k+2}$. For $i = 1, \ldots, k$, let

$$f_i(y, x) = (\phi_i(y), 00) \cdot x. \tag{15}$$

For any fixed $\alpha \in \mathbf{F}_p^2$, linear function can be defined as follows,

$$L_\alpha = \{l_\beta = (\beta, \alpha) \cdot (y, x) \mid \beta \in \mathbf{F}_p^{m-2}\}. \tag{16}$$

Let $H_0 = L_{00} \cup L_{01} \cup \cdots \cup L_{0(p-1)}$, $H_1 = L_{10} \cup L_{11} \cup \cdots \cup L_{1(p-1)}, \cdots, H_{p-1} = L_{(p-1)0} \cup L_{(p-1)1} \cup \cdots \cup L_{(p-1)(p-1)}$.

We construct disjoint sequence sets as follows:

$$S_{i,j} = \{f_i + l \mid l \in H_j\}, \quad \text{for } i \in \mathbf{F}_{p^k},\ j \in \{0, 1, \cdots, p-1\}. \tag{17}$$

Theorem 2: Let the sequence set $S_{c,i}$ be defined by (17) as in Construction 2. Then, we always have

1) For $i \in \mathbf{F}_p^k$, $\#S_{i,j} = p^{m-1}$, where $j \in \{0, 1, \cdots, p-1\}$;
2) All the sequences in $S_{i,j}$ are semi-bent sequences, where c is nonzero;
3) Let $i, i' \in \mathbf{F}_{p^{t}-1}$, $j, j' \in \{0, 1, \cdots, p-1\}$. $S_{i,j} \perp S_{i',j'}$ if and only if $j \neq j'$.

Proof: 1) Note that $\#L_\alpha = p^{m-2}$, and $\#S_{i,j} = p^{m-2} \cdot p = p^{m-1}$, which implies that 1) holds.

2) For the sequence set $S_{i,j}$, without loss of generality, considering the case of $i = j = 0$, we have $S_{0,0} = \{f_0 + l \mid l \in H_0\}$.

$$f_0(y, x) = (\phi_0(y), 00) \cdot x.$$

For any $(\beta, \alpha_1, \alpha_2) \in \mathbf{F}_p^k \times \mathbf{F}_p^k \times \mathbf{F}_p^2$ and $(y, x_1, x_2) \in \mathbf{F}_p^k \times \mathbf{F}_p^k \times \mathbf{F}_p^2$, we have

$$\begin{aligned} W_{f_0}(\beta, \alpha_1, \alpha_2) &= \sum_{(y,x_1,x_2) \in \mathbf{F}_p^m} \omega^{f_c(y,x) - \beta \cdot y - (\alpha_1,\alpha_2) \cdot x} \\ &= \sum_{y \in \mathbf{F}_p^k} \omega^{\beta \cdot y} \cdot \sum_{x_1 \in \mathbf{F}_p^k} \omega^{(\phi_0(y) - \alpha_1)x_1} \cdot \\ &\quad \times \sum_{x_2 \in \mathbf{F}_p^2} \omega^{\alpha_2 x_2}. \end{aligned} \tag{18}$$

Noticing that

$$|\sum_{x_2 \in \mathbf{F}_p^2} \omega^{\alpha_2 x_2}| = \begin{cases} p^2, & \alpha_2 = 0, \\ 0, & otherwise. \end{cases} \tag{19}$$

Note that π is bijective and there exists a unique $y \in \mathbf{F}_p^k$ such that $\phi_0(y) = \alpha_1$, we have $\sum_{x_1 \in \mathbf{F}_p^k} \omega^{(\phi_0(y) - \alpha_1)x_1} = p^k$. For $\alpha_2 = 0$, then $|W_{f_0}(\beta, \alpha_1, \alpha_2)| = p^{k+2}$ holds, which implies that 2) holds.

3) Let $f_i + l \in S_{i,j}$ and $f_{i'} + l' \in S_{i',j'}$, where the Boolean function corresponding to the sequence l is $l = \beta \cdot y + (\alpha_1, \alpha_2) \cdot x \in H_j$ and the Boolean function corresponding to the sequence l' is $l' = \beta' \cdot y + (\alpha_1', \alpha_2') \cdot x \in H_{j'}$. To analyze the orthogonality between $f_i + l$ and $f_{i'} + l'$, we consider

$$h = (f_i + l) - (f_{i'} + l') = f_{i-i'} + (l - l'). \tag{20}$$

When $j \neq j'$, then $W_h(0_m) = 0$ holds. From Lemma 1, we have $S_{i,j} \perp S_{i',j'}$. □

The orthogonal sequences can be applied to the S-CDMA systems successfully. But orthogonal sequences can not be applied to the QS-CDMA systems directly. This problem is solved in the next section by constructing large sets of orthogonal sequences with ZCZ property.

IV. A FAMILY OF ORTHOGONAL SEQUENCES WITH ZCZ

Based on the orthogonal sequences obtained in Section III, we next construct a family of sequences satisfying ZCZ properties, which implies that the sequences can be applied to QS-CDMA systems. Furthermore, the re-use distance D is the same with the orthogonal sequences obtained in Section III.

Definition 4: Let $b = (b_0, b_1, \cdots, b_{N-1})$ be a sequence with length N, and $\mathcal{A} = (A_0, A_1, \cdots, A_{N-1})$ be a row vector consisting of N sequences A_i, $0 \leq i < N$. We define

$$b \otimes \mathcal{A} = (b_0 \cdot A_0, b_1 \cdot A_1, \cdots, b_{N-1} \cdot A_{N-1})$$

is the Kronecker product operation.

Definition 5 ([24], [25]): A length N sequence $A = (a_0, a_1, \cdots, a_{N-1})$ is a 3-phase sequence if for $0 \leq i < N$, $a_i \in \{1, \omega, \omega^2\}$, where $\omega = e^{2\pi i/3}$. The set of three 3-phase sequence triad $\{A, B, C\}$ is a Golay complementary triad (GCT) if $R_A(\tau) + R_B(\tau) + R_C(\tau) = 0$, for $1 \leq \tau \leq N-1$.

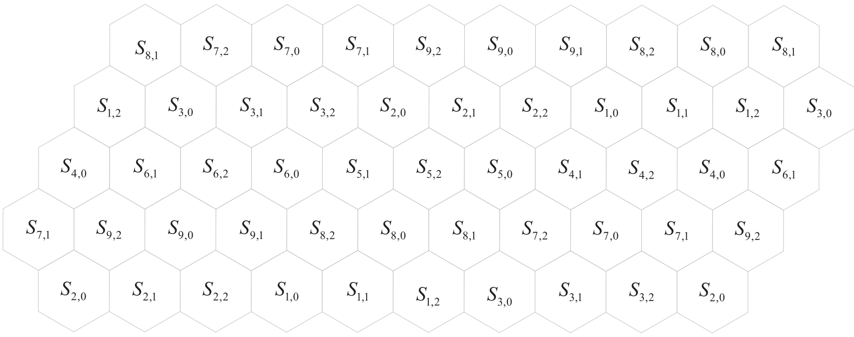

FIGURE 1. **Assignment of orthogonal sets to a lattice of regular hexagonal cells.**

Table 2 gives the total number of 3-phase GCTs and total number of sequences available till date for various lengths. For example, for $N = 5, A = (1\ 1\ 1\ \omega\ 1), B = (1\ \omega\ \omega^2\ \omega^2\ \omega)$ and $C = (1\ 1\ \omega^2\ \omega\ \omega^2)$, we have $R_A(\tau)+R_B(\tau)+R_C(\tau) = 0$, for $1 \le \tau \le 4$; For $N = 6$, $A = (1\ \omega^2\ 1\ 1\ \omega^2\ 1)$, $B = (1\ \omega\ \omega^2\ \omega^2\ \omega^2\ \omega)$ and $C = (1\ \omega\ \omega\ \omega\ 1\ \omega^2)$, we have $R_A(\tau) + R_B(\tau) + R_C(\tau) = 0$, for $1 \le \tau \le 5$.

TABLE 2. **Existing 3-phase GCTs.**

Length N	Total number of GCTs	Total number of sequences
2	27	9
3	243	27
4	0	0
5	1944	108
6	4536	288
7	16848	792
8	4536	306
9	75168	3708
10	0	0
11	73872	4832
12	9072	756
13	72576	5850
14	103032	8334
15	336312	27576
16	0	0
17	52920	2160
18	1097712	89532
19	18144	1458
20	12960	1080
21	2575152	202932
22	0	0

Suppose that both $\{A_1, B_1, C_1\}$ and $\{A_2, B_2, C_2\}$ are 3-phase GCTs with length N. 3-phase sequence triad $\{A_2, B_2, C_2\}$ is called the mate of GCT $\{A_1, B_1, C_1\}$, if for all $0 \le \tau \le N-1$, $R_{A_1,A_2}(\tau) + R_{B_1,B_2}(\tau) + R_{C_1,C_2}(\tau) = 0$.

For example, for $\{A_1, B_1, C_1\}$ and $\{A_2, B_2, C_2\}$ are 3-phase length $N = 7$ GCTs, we have

$$\begin{aligned}
A_1 &= (1\ 1\ \omega\ 1\ \omega^2\ \omega^2\ 1)\\
B_1 &= (1\ 1\ 1\ \omega^2\ 1\ \omega\ \omega)\\
C_1 &= (1\ 1\ \omega^2\ 1\ \omega\ 1\ \omega^2)\\
A_2 &= (1\ \omega\ \omega\ \omega\ 1\ \omega\ \omega^2)\\
B_2 &= (1\ \omega\ 1\ \omega^2\ \omega\ 1\ 1)\\
C_2 &= (1\ \omega\ \omega^2\ \omega^2\ \omega^2\ \omega^2\ \omega)
\end{aligned}$$

then we can obtain $R_{A_1,A_2}(\tau) + R_{B_1,B_2}(\tau) + R_{C_1,C_2}(\tau) = 0$, for $0 \le \tau \le 6$.

Construction 3: Let $S_{c,\alpha}$ be the sequence set generated by the section III, where $p = 3$. Let $\{A_1, B_1, C_1\}$ be a GCT with length N defined as in Definition 5. The sequence set $A = (A_1, A_1, \cdots, A_1)$ is p^m-tuples, $B = (B_1, B_1, \cdots, B_1)$, $C = (C_1, C_1, \cdots, C_1)$. We construct 3^{k+1} disjoint sequence sets of $(3^{m+1}N + 2N - 2)$-length with ZCZ width N as follows:

$$\begin{aligned}
T_{c,\alpha} &= \{T^i_{c,\alpha} \mid 0 \le i \le 3^{m-1} - 1\}\\
&= \{(S^i_{c,\alpha} \otimes A,\ 0_{N-1},\ S^i_{c,\alpha} \otimes B,\ 0_{N-1},\ S^i_{c,\alpha} \otimes C)|\\
&\quad \times 0 \le i \le 3^{m-1} - 1\}, \qquad (21)
\end{aligned}$$

Theorem 3: Let $p = 3$, for $c \in \{1, 2 \cdots, 3^k\}$, $\alpha \in \mathbf{F}_3$, let the sequence set $T_{c,\alpha}$ be defined by (21) as in Construction 3. Then $T_{c,\alpha}$ forms a $(3^{m+1}N+2N-2)$-length ZCZ sequence set with $Z_{cz} = N$. Let $T^i_{c,\alpha}, T^j_{c',\alpha'}$ be two sequences taken from the sequence sets $T_{c,\alpha}$ and $T_{c',\alpha'}$, respectively. Then we can obtain

$$\begin{aligned}
&R_{T^i_{c,\alpha},T^j_{c',\alpha'}}(\tau) = 0, \quad \text{if } 0 < \tau < N,\\
&R_{T^i_{c,\alpha},T^j_{c',\alpha'}}(0) = 0, \quad \text{if } c = c',\ \alpha = \alpha' \text{ and } i \ne j.
\end{aligned}$$

When $T_{c,\alpha}$ and $T_{c',\alpha'}$ are in two adjacent cells, we have

$$R_{T^i_{c,\alpha},T^j_{c',\alpha'}}(0) = 0.$$

When $T_{c,\alpha}$ and $T_{c',\alpha'}$ are in two non-adjacent cells, we have

$$|R_{T^i_{c,\alpha},T^j_{c',\alpha'}}(0)| \le 3^{\lfloor (m+2)/2 \rfloor + 1}N.$$

Proof: Let $S^i_{c,\alpha} = (s_0, s_1, \cdots, s_{3^m-1})$ and $S^j_{c',\alpha'} = (s'_0, s'_1, \cdots, s'_{3m-1})$ be the sequences defined as Construction 3. For $\tau = 0$, the correlation of $T^i_{c,\alpha}$, and $T^j_{c',\alpha'}$ is denoted as follows,

$$R_{T^i_{c,\alpha},T^j_{c',\alpha'}}(0) = 3N \sum_{i=0}^{3^m-1} s_i s_i'^{*}. \qquad (22)$$

Note that s_i and s'_i are orthogonal, $\sum_{i=0}^{3^m-1} s_i s_i'^{*} = 0$ holds, where $l \in L_\alpha$, $l' \in L_{\alpha'}$.

For $\tau = 0$, according to (22), we have

$$R_{T^i_{c,\alpha},T^j_{c',\alpha'}}(0) = 0$$

if $T_{c,\alpha}$ and $T_{c',\alpha'}$ are in two adjacent cells.

If $T_{c,\alpha}$ and $T_{c',\alpha'}$ are in two non-adjacent cells, then

$$|R_{T^i_{c,\alpha},T^j_{c',\alpha'}}(0)| \leq 3^{\lfloor (m+2)/2 \rfloor +1}N.$$

When $\tau \neq 0$, we consider the following case, $T^i_{c,\alpha} = (S^i_{c,\alpha} \otimes A,\ 0_{N-1},\ S^i_{c,\alpha} \otimes B,\ 0_{N-1},\ S^i_{c,\alpha} \otimes C)$ and $T^j_{c',\alpha'} = (S^j_{c',\alpha'} \otimes A,\ 0_{N-1},\ S^j_{c',\alpha'} \otimes B,\ 0_{N-1},\ S^j_{c',\alpha'} \otimes C)$.

Note that

$$\begin{aligned} R_{T^i_{c,\alpha},T^j_{c',\alpha'}}(\tau) &= \sum_{i=0}^{3^m-1} s_i s_i'^{*} R_{A_1}(\tau) + \sum_{i=0}^{3^m-2} s_{i+1}{s'_{i+1}}^{*} R_{A_1}(N-\tau) \\ &\quad + \sum_{i=0}^{3^m-1} s_i s_i'^{*} R_{B_1}(\tau) + \sum_{i=0}^{3^m-2} s_{i+1}{s'_{i+1}}^{*} R_{B_1}(N-\tau) \\ &\quad + \sum_{i=0}^{3^m-1} s_i s_i'^{*} R_{C_1}(\tau) + \sum_{i=0}^{3^m-2} s_{i+1}{s'_{i+1}}^{*} R_{C_1}(N-\tau) \end{aligned} \tag{23}$$

Since the set $\{A_1, B_1, C_1\}$ is a three 3-phase Golay triad, for $0 < \tau < N$, and $0 < N-\tau < N$, $R_{A_1}(\tau)+R_{B_1}(\tau)+R_{C_1}(\tau) = 0$ holds. Then for $0 < \tau < N$, we can deduce

$$R_{T^i_{c,\alpha},T^j_{c',\alpha'}}(\tau) = 0.$$

We can determine that $T_{c,\alpha}$ is a ZCZ sequence set with $Z_{cz} = N$. □

Remark 2: The p^{k+1} sets $T_{c,\alpha}$, $c \in \{1, 2, \cdots, p^k\}$, $\alpha \in \mathbf{F}_p$, can be arranged as Fig. 1 in a similar way, where we just replace $S_{c,\alpha}$ with $T_{c,\alpha}$. At the same time, the orthogonality is updated to ZCZ property accordingly. Note that the number of sequences is p^{m-1} in each $T_{c,\alpha}$.

Example 2: Let $N = 6$, the set of three 3-phase sequences $\{A_1, B_1, C_1\}$ is a Golay sequence triad defined in Definition 5, where $A_1 = (1\ \omega^2\ 1\ 1\ \omega^2\ 1)$, $B_1 = (1\ \omega\ \omega^2\ \omega^2\ \omega^2\ \omega)$ and $C_1 = (1\ \omega\ \omega\ \omega\ 1\ \omega^2)$. We have $R_{A_1}(\tau)+R_{B_1}(\tau)+R_{C_1}(\tau) = 0$, for $1 \leq \tau \leq 5$.

Let $m = 4$, $p = 3$ and $\gamma \in \mathbf{F}_{3^2}$ be a root of the primitive polynomial $z^2 + z + 2$. We can obtain 27 sets of orthogonal sequences $\{S_{c,\alpha} \mid c \in \{1, 2, \cdots, 9\}, \alpha \in \mathbf{F}_3\}$ as in Example 1. Then a ZCZ sequence set $T_{c,\alpha}$ is obtained with length $N = 3^5 \cdot 6 + 10 = 1468$ as follows,

$$\begin{aligned} T_{c,\alpha} &= \{T^i_{c,\alpha} \mid 0 \leq i \leq 3^3 - 1\} \\ &= \{(S^i_{c,\alpha} \otimes A,\ 0_{N-1},\ S^i_{c,\alpha} \otimes B,\ 0_{N-1},\ S^i_{c,\alpha} \otimes C) | \\ &\quad \times 0 \leq i \leq 3^3 - 1\}. \end{aligned} \tag{24}$$

The constructed sequence set $T_{c\alpha}$ has ZCZ width N, and also can be arranged as Fig. 1 in a similar way with the re-use distance $D = \sqrt{27}$, where we just replace $S_{c,\alpha}$ with $T_{c,\alpha}$.

Corollary 1: Let $S_{c,\alpha}$ be the sequence set generated by the section III, where $p = 3$. Let $\{A_1, B_1, C_1\}$ be a GCT with length N, and $\{A_2, B_2, C_2\}$ be a mate of $\{A_1, B_1, C_1\}$. We define a new sequence triads as follows:

$$\begin{aligned} A &= (A_1, A_2, A_1, A_2, \cdots, A_1, A_2) \\ B &= (B_1, B_2, B_1, B_2, \cdots, B_1, B_2) \\ C &= (C_1, C_2, C_1, C_2, \cdots, C_1, C_2). \end{aligned}$$

TABLE 3. Parameters of ZCZ sequences.

literature	length	ZCZ	p-phase sequences
[12]	$2^{m+1} \cdot N + N - 1$	N	2-phase
[23]	$2^{m+1} \cdot N + N - 1$	N	2-phase
Th. 3	$3^{m+1} \cdot N + 2N - 2$	N	3-phase
Col. 1	$3^{m+1} \cdot N + 4N - 2$	$2N$	3-phase
Col. 2	$p^{m+2} + (p-1)^2$	p	p-phase,$p \geq 3$

We construct 3^{k+1} disjoint sequence sets of $(3^{m+1}N + 4N - 2)$-length with ZCZ width $2N$ as follows:

$$\begin{aligned} T_{c,\alpha} &= \{T^i_{c,\alpha} \mid 0 \leq i \leq 3^{m-1} - 1\} \\ &= \{(S^i_{c,\alpha} \otimes A,\ 0_{2N-1},\ S^i_{c,\alpha} \otimes B,\ 0_{2N-1},\ S^i_{c,\alpha} \otimes C) | \\ &\quad \times 0 \leq i \leq 3^{m-1} - 1\}. \end{aligned} \tag{25}$$

Corollary 1 has a similar proof to Theorem 3 and is therefore omitted here.

Remark 3: Corollary 1 obtain a class of ZCZ sequence sets with $Z_{cz} = 2N$, which has larger ZCZ width compared with Theorem 3.

Definition 6: ([26], [27]) Let the sequence set $\mathcal{A} = \{A_0, A_1, \cdots, A_{M-1}\}$, and $A_i = \{a_{i,0}, a_{i,1}, \cdots, a_{i,M-1}\}$, where $a_{i,l}$ is a sequence with length N. We call $\mathcal{A}$ a (M, M, N)-complete complementary code (CCC), if

$$R_{A_i,A_j}(\tau) = \sum_{l=0}^{M-1} R_{a_{i,l},a_{j,l}}(\tau) = \begin{cases} 0, & 0 < \tau \leq N-1, i = j, \\ 0, & 0 \leq \tau \leq N-1, i \neq j. \end{cases}$$

Corollary 2: Let $S_{c,\alpha}$ be the sequence set generated by the section III. Let $\mathcal{A} = \{A_0, A_1, \cdots, A_{p-1}\}$ be a (p, p, p)-CCC, and $A_j = \{a_{j,0}, a_{j,1}, \cdots, a_{j,p-1}\}$, where $a_{j,l}$ is a sequence with length p, and $0 \leq j, l \leq p-1$. Defining $A_{j,l} = (a_{j,l}, a_{j,l}, \cdots, a_{j,l})$ with p^m-tuples, we construct p^{k+2} disjoint sequence sets of $(p^{m+2}+(p-1)^2)$-length with ZCZ as follows:

$$\begin{aligned} T_{c,\alpha} &= \{T^i_{c,\alpha,j} \mid 0 \leq i \leq p^{m-1} - 1, 0 \leq j \leq p-1\} \\ &= \{(S^i_{c,\alpha} \otimes A_{j,0},\ 0_{p-1},\ S^i_{c,\alpha} \otimes A_{j,1},\ 0_{p-1}, \cdots, \\ &\quad \times S^i_{c,\alpha} \otimes A_{j,p-1}) \mid 0 \leq i \leq p^{m-1} - 1, 0 \leq j \leq p-1\}. \end{aligned}$$

In each ZCZ sequence set, we have p^m constituent sequence. Corollary 2 has a similar proof to Theorem 3 and is therefore omitted here.

Remark 4: Corollary 2 obtains a class of ZCZ sequence sets with $Z_{cz} = N$, and the number of constituent sequences is p^m in each sequence set, which has a larger number of sequences compared with Theorem 3.

Remark 5: In Table 3, we compare the main parameters of ZCZ sequence. Comparison with the literature [12] and [23]. A class of p-phase ZCZ sequence sets is constructed, which the length of sequence is $3^{m+1} \cdot N + 2N - 2$ in Theorem 3. This is the first time we obtain a ZCZ sequence with new length. In Corollary 1, a class of ZCZ sequence sets is presented with ZCZ width $2N$ of $3^{m+1} \cdot N + 4N - 2$ length. The ZCZ sequence sets in Corollary 1 have larger ZCZ width than sequence sets in Theorem 3. In Corollary 2, a family of ZCZ sequence sets is proposed based on p-phase CCC, which has $p^{m+2} + (p-1)^2$ length.

In regular tessellation of hexagonal cells, the constructed ZCZ sequence sets have the same reuse distance as orthogonal sequence sets constructed in Theorem 1.

V. CONCLUSION

In this article, we construct a class of orthogonal sequences over GF(p), and assign them to a tessellation of hexagonal cells. The method increases the number of sequences to be p^{m-1} per cell in the network. Compared with binary sequences, non-binary sequences have more sequences in each sequence set. Next we extended the orthogonal sequences to ZCZ sequences which can be applied to QS-CDMA system, and analyzed the properties of the new sequence sets. For the sequence set in each cell, the ZCZ sequence sets not only have the same re-use distance as the orthogonal sequences, but also have large zero correlation zone which can be applied to QS-CDMA system.

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