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RESEARCH ARTICLE

Improved on Extended Dissipative Analysis for Sampled-Data Synchronization of Complex Dynamical Networks With Coupling Delays

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ABSTRACT This paper concerns the issue of extended dissipative analysis for complex dynamical networks with coupling delays under a sampled-data control scheme. Firstly, we derive the input delay method and combine it with an appropriate Lyapunov functional, which can make full use of the information of the sampling period. Secondly, novel sufficient synchronization criteria are established by applying Jensen's inequality, Wirtinger's integral inequality, a new integral inequality, free-weighting matrix technique, and convex combination method. Moreover, we focus on the extended dissipative analysis issue, which includes $\mathcal{L}_2 - \mathcal{L}_\infty$, $\mathcal{H}_\infty$, passivity, and dissipativity performance in a unified formulation. These conditions can express in Linear matrix inequalities (LMIs) restrictions, which can solve with readily accessible software. Finally, two numerical examples illustrate the effectiveness and reduced conservatism of our developed method.

INDEX TERMS Extended dissipative, sampled-data control, synchronization, complex dynamical networks, coupling delays.

I. INTRODUCTION

During the last few decades, complex dynamical networks (CDNs) have been applied in real-world systems such as social networks, the internet, brain networks, food chains, and disease-spreading networks [1], [2], [3]. There are many connected nodes in this system, each of which represents a dynamic system. Depending on the network topology, some of these nodes are typically connected. A graph is a mathematical concept that describes these networks. In such diagrams, vertices represent system membership, whereas edges represent their interaction. In addition, the time delay is unavoidable in many scenarios, including neural networks, electrical engineering systems, chemical or process control systems, the internet, transportation systems, etc. It can cause instability, oscillations, and poor system performance. Because coupling delay is a prevalent time delay in CDNs, it is critical to think about how time delay affects CDNs.

Furthermore, synchronization is vital in CDNs because it can explain various phenomena, such as synchronous data exchange on the World Wide Web and the internet. As a result, it has become a hot topic that has been thoroughly researched. Li and Chen [4] presented CDNs with coupling delay for continuous- and discrete-time systems in the last decade. Gao et al. [5] studied synchronization for a general class of CDNs with coupling delays to discover a novel criterion for ensuring that the system is asymptotically stable. In the case that all nodes of networks cannot achieve synchronization by themselves. So, it is challenging to design a practical controller that can synchronize the networks. Various control schemes can be applied in CDNs, such as the pinning control [6], [7], [8], the impulsive control [9], [10], [11], the intermittent control [12], [13], [14], and the adaptive control [15], [16], [17]. Recently, communication networks and computers have advanced to a high-speed version, and practically digital signal techniques have replaced analog signal methods, which provide a more consistent result. Thus, the sampled-data system has received much attention [18], [19], [20], [21].

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As is known, a vital point in sampled-data systems is to place importance on the desired controller under a more considerable sampling period. Li et al. [22] investigated the synchronization of regular CDNs for the first time, including coupling time-varying delay, where sampled-data controllers were designed. An adequate condition was obtained using the Jensen inequality to achieve the exponential stability of the synchronization error system. Wu et al. [23] presented the exponential synchronization of CDNs with coupling time-varying delay. The above authors reduced a synchronization criterion using the same Lyapunov functional and a new integral inequality that takes full merit of the detail on delays. In [24], such a problem was further addressed by applying a reciprocally convex technique with an improved inequality that can yield a tighter upper bound than Jensen inequality. The previously reported results [22], [23], [24] were enhanced further in [25] by using a new time-dependent Lyapunov functional method and Wirtinger-based integral inequality [26]. Chen et al. [27] introduced Wirtinger-based double integral inequalities [28] to reduce the conservatism of the synchronization criterion by incorporating the new Lyapunov functional in triple integral inequalities. Obtaining a more sampling period has proven to be a significant issue in earlier studies [22], [23], [24], [25], [27]. As a result, implementing less conservative synchronization requirements for CDNs with sampled-data control has become a hot topic, and designing CDNs with sampled-data control is worthwhile. However, it should be noted that while these studies focus on using the inequality methodology to improve results, the information on the sawtooth structure on the input delay is not fully utilized, leaving room for improvement. In addition, the works mentioned above take no account of external disturbances, which is unworkable in practice. Thus, to implement sampled-data synchronization in CDNs, it is required to account for external disturbances and fully benefit the sawtooth function class of sampling period, which is the inspiration for our paper.

Furthermore, the topic of network performance analysis, which is based on the relationship between input and output, is essential in practical systems. Over the last few decades, analyzing performance has been essential in engineering and science applications [30], [31], [32], [33], [34], [35]. Zhang et al. [35] introduced a generic performance called extended dissipativity, which effectively encompasses the passivity, $\mathcal{L}_2 - \mathcal{L}_\infty$, $\mathcal{H}_\infty$, and dissipativity index and it has received additional research in the last few years. In [30], the extended dissipative performance was utilized in neural network systems with continuous time-varying delay. For the first time, Yang et al. [36] studied the extended dissipative for synchronization of CDNs with coupling delay. However, The vital information of $t - t_k$ and $t_{k+1} - t$, $\forall t \in [t_k, t_{k+1})$ is not fully maximum potential. As a result, it is only reasonable to seek a different perspective to construct a less conservative condition for synchronizing sampled-data CDNs with coupling delays. From those mentioned above, we dedicate ourselves to overcoming the issue of extended dissipative analysis for sampled-data synchronization for CDNs with coupling delays. The main contribution of this article highlights as follows

- The proposed CDNs with coupling time-varying delays are comprehensive models for the other existing CDNs [22], [23], [25], [29], [37], [38]. We take the external disturbances into each node which is not considered in [22], [23], [25], [27], [29], [37], and [38].
- We construct a time-dependent Lyapunov functional different from the references in [22], [23], [24], [25], and [27], and the advantage information on discrete sample point t_k is fully used.
- It is worth mentioning that the positive definitiveness of the proposed Lyapunov functional requires only sampling times that are not necessarily throughout the sampling periods. So, we derive a new inequality that applies to prove the exponential synchronization of CDNs.
- A less conservative criterion is obtained under the novel Lyapunov functional using Jensen's inequality, Wirtinger's integral inequality, a new integral inequality, the free-weighting matrix technique, and the convex combination approach. Moreover, our results can verify Chua's circuit system.

This paper is separated in the following way. The problem formulation, definitions, assumptions, and lemmas provide in Section 2. The main results are present in Section 3, which considers the exponential synchronization via sampled-data control and the extended index analysis. Two numerical examples manifest the effectiveness and reduced conservatism of the existing results in section 4. Finally, we sum up this letter in Section 5.

Notation: This paper contains the following notations, $\mathcal{R}^n$ stands for n-dimensional Euclidean space and $\mathcal{R}^{m\times n}$ is the set of $m \times n$ real matrices. The symmetric matrix $\mathcal{P} > 0$ is that the matrix $\mathcal{P}$ is positive definite. $\lambda_{\min}(\mathcal{P})$ and $\lambda_{\max}(\mathcal{P})$ mean the minimum and maximum eigenvalues of $\mathcal{P}$, $sym\{\mathcal{P}\}$ defines $\mathcal{P} + \mathcal{P}^T$ The superscript T is the transpose. The symbol $*$ represents the symmetric entries of the symmetric matrix. The symbol $\otimes$ denotes the Kronecker product, and diag$\{\dots\}$ is the block diagonal matrix.

II. PRELIMINARIES

Consider the complex dynamical networks, which consist of N identical linked nodes, each of which is an n- dimensional dynamical system:

$$\begin{aligned}\dot{x}_i(t) &= f(x_i(t)) + c\sum_{j=1}^{N}\mathcal{G}_{ij}\Gamma^{(1)}x_j(t) \\ &\quad + c\sum_{j=1}^{N}\mathcal{G}_{ij}\Gamma^{(2)}x_j(t-\tau(t)) + u_i(t) + \omega_i(t), \\ z_i(t) &= \mathcal{J}x_i(t), \quad i = 1, 2, \dots, N, \end{aligned} \tag{1}$$

where $x_i(t)$ and $u_i(t)$ are, respectively the state variable and the control input of each node. $f(x_i(t)) \in \mathcal{R}^n$ is a nonlinear vector valued function defining the dynamics of ith nodes.

$c > 0$ is the coupling strength. The internal-coupling matrices are $\Gamma^{(1)}, \Gamma^{(2)} \in \mathcal{R}^{n\times n}$ and the matrix $\mathcal{G} = (\mathcal{G}_{ij})_{N\times N} \in \mathcal{R}^{N\times N}$ is the outer-coupling configuration matrix, in which $\mathcal{G}_{ij}$ is satisfied as follows: if there exists a string from node i to node j $(i \neq j), \mathcal{G}_{ij} > 0$; otherwise, $\mathcal{G}_{ij} = 0$. While, the diagonal elements of $\mathcal{G}$ is defined as $\mathcal{G}_{ii} = -\sum_{j=1,j\neq i}^{N} \mathcal{G}_{ij}$. $\omega_i(t) \in \mathcal{R}^n$ is the external perturbation, which belongs to $\mathcal{L}_2[0,\infty)$, $z_i(t) \in \mathcal{R}^n$, is the output, and $\mathcal{J}$ is a given matrix with proper dimensions. The function $\tau(t)$ represents the coupling time-varying delay that satisfies:

$$0 \leq \tau(t) \leq \tau_u, \quad 0 \leq \dot{\tau}(t) \leq \tau_d. \tag{2}$$

Let $\dot{\nu}(t) = f(\nu(t))$, where $\nu(t) \in \mathcal{R}^n$ is the state response of the unforced isolate node. Then the error vector is $r_i(t) = x_i(t) - \nu(t)$. Let $\eta_i(t)$ be the output of isolate node, which defines as $\eta_i(t) = \mathcal{J}\nu_i(t)$. Then, the output error $\hat{z}_i(t)$ is described as follows

$$\hat{z}_i(t) = z_i(t) - \eta_i(t), \qquad i = 1, 2, \ldots, N. \tag{3}$$

At the same time, a zero-order-hold (ZOH) function with a sequence of hold times of $0 = t_0 < t_1 < \cdots t_k < \cdots$, $lim_{k\to+\infty} t_k = +\infty$ is assumed to be used to create the control signal. Then, the sampled-data feedback controller constructs in the form as:

$$u_i(t) = K_i r_i(t_k), \qquad t_k \leq t < t_{k+1}, \qquad i = 1, 2, \ldots, N, \tag{4}$$

where K_i denotes a set of feedback controller gain matrices that can be designed. $r_i(t_k)$ is a discrete mearsurement of $r_i(t)$ at the sampling time t_k. Here, we assume that $t_{k+1} - t_k = d_k \leq h_u$ for all integer $k \geq 0$, where $h_u > 0$ stands for the largest sampling period. Then, we have the following error closed-loop system:

$$\begin{aligned}
\dot{r}_i(t) &= g(r_i(t)) + c\sum_{j=1}^{N} \mathcal{G}_{ij}\Gamma^{(1)} r_j(t) \\
&\quad + c\sum_{j=1}^{N} \mathcal{G}_{ij}\Gamma^{(2)} r_j(t-\tau(t)) + K_i r_i(t_k) + \omega_i(t), \\
\hat{z}_i(t) &= \mathcal{J}x_i(t) - \mathcal{J}\nu_i(t) = \mathcal{J}r_i(t), \quad i = 1, 2, \ldots, N,
\end{aligned} \tag{5}$$

where $g(r_i(t)) = f(x_i(t)) - f(\nu(t))$. Let $r(t) = [r_1^T(t), r_2^T(t), \ldots, r_N^T(t)]^T$, $g(r(t)) = [g^T(r_1(t)), g^T(r_2(t)), \ldots, g^T(r_N(t))]^T$, $K = \text{diag}\{K_1, K_2, \ldots, K_N\}$. Then, error dynamical (5) can be transformed as ways:

$$\begin{aligned}
\dot{r}(t) &= g(r(t)) + c(\mathcal{G}\otimes\Gamma^{(1)})r(t) + c(\mathcal{G}\otimes\Gamma^{(2)})r(t-\tau(t)) \\
&\quad + Kr(t_k) + \omega(t), \\
\hat{z}(t) &= \mathcal{J}r(t), \quad t \in [t_k, t_{k+1}).
\end{aligned} \tag{6}$$

Here, we give some assumptions, definitions, and lemmas that are required to obtain the new synchronization criterion.

Assumption 1 [39] The continuous function $f : \mathcal{R}^n \to \mathcal{R}^n$ satisfies the following condition:

$$\begin{aligned}
&[f(x) - f(y) - U(x-y)]^T \\
&\quad \times [f(x) - f(y) - V(x-y)] \leq 0, \ \forall x, y \in \mathcal{R}^n,
\end{aligned} \tag{7}$$

where U and V are known constant matrices of compatible dimensions.

Remark 1: It is to be noted that such a nonlinear function of characterization is the sector-bounded condition, which is derived in [39] and includes the commonly Lipschitz conditions. It is clearly to show that for any nonlinear function f satisfying (7), there exists a scalar $\rho > 0$ such that

$$\|f(x) - f(y)\|^2 \leq \rho\|x-y\|^2. \tag{8}$$

Definition 1: [36] The CDNs (1) *is said to be exponentially synchronized if the error dynamic* (6) *is exponentially stable, i.e., there exist two constants $\beta > 0$ and $\alpha > 0$ such that*

$$\|r(t)\|^2 \leq \alpha e^{-\beta t} \sup_{-\max\{h_u,\tau_u\}\leq\theta\leq 0} \{\|r(\theta)\|^2, \|\dot{r}(\theta)\|^2\}. \tag{9}$$

Definition 2: [36] Given matrices Φ_1, Φ_2, Φ_3, and Φ_4 with symmetric matrices Φ_1, Φ_3, and Φ_4, system (6) *is achieved to be extended dissipative if for all $t_f \geq 0$ and any $\omega(t) \in \mathcal{L}_2[0,\infty)$, under zero initial condition, the following inequality holds*:

$$\int_0^{t_f} J(t)dt \geq \sup_{0\leq t\leq t_f} \hat{z}^T(t)\Phi_4\hat{z}(t), \tag{10}$$

where $J(t) = \hat{z}^T(t)\Phi_1\hat{z}(t) + 2\hat{z}^T(t)\Phi_2\omega(t) + \omega^T(t)\Phi_3\omega(t)$. Throughout this paper, the general assumptions on Φ_1, Φ_2, Φ_3, and Φ_4 are utilized.

Assumption 2. [35] For given real symmetric matrices Φ_1, Φ_2, Φ_3 and Φ_4 the following conditions hold:

(1) $\Phi_1 \leq 0, \Phi_3 > 0,$ and $\Phi_4 \geq 0$;

(2) $(\|\Phi_1\| + \|\Phi_2\|) \cdot \|\Phi_4\| = 0$.

Remark 2: Notably, in Eq. (10) is called an extended index that provides a more extensive performance by adjusting the matrix parameters $\Phi_i, i = 1, 2, 3, 4$. In particular, (10) turns into the $\mathcal{L}_2 - \mathcal{L}_\infty$ performance when $\Phi_1 = \Phi_2 = 0$, $\Phi_3 = \delta^2 I$, and $\Phi_4 = I$; Eq. (10) is the $\mathcal{H}_\infty$ performance when $\Phi_1 = -I$, $\Phi_2 = \Phi_4 = 0$, and $\Phi_3 = \delta^2 I$, (10) diminished to the $(\mathcal{Q}, \mathcal{S}, \mathcal{R})-$ property index when $\Phi_1 = \mathcal{Q}, \Phi_2 = \mathcal{S}, \Phi_3 = \mathcal{R} - \delta I$, and $\Phi_4 = 0$. In each of the four cases, the scalar δ represents the corresponding performance index.

Lemma 1: (Reciprocally Convex Lemma [40]). For any vectors x_1, x_2, matrices $M > 0, S$, and scalars $\epsilon_1 > 0, \epsilon_2 > 0$, satisfying $\epsilon_1 + \epsilon_2 = 1$, the following inequality holds:

$$-\frac{1}{\epsilon_1}x_1^T M x_1 - \frac{1}{\epsilon_2}x_2^T M x_2 \leq \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} M & S \\ * & M \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{11}$$

subject to

$$0 < \begin{bmatrix} M & S \\ * & M \end{bmatrix}.$$

Lemma 2: [26] For any matrix $R > 0$. Then for any continuous function x in $[a,b] \to \mathcal{R}^n$ the inequality *holds as below*:

$$\int_a^b x^T(u)Rx(u)du \geq \frac{1}{b-a}\left(\int_a^b x(u)du\right)^T R\left(\int_a^b x(u)du\right) + \frac{3}{b-a}\Omega^T R\Omega,$$

where $\Omega = \int_a^b x(s)ds - \frac{2}{b-a}\int_a^b\int_s^b x(r)drds$.

Lemma 3: (Schur Complement [41]). Let X, Y, Z *be given matrices such that* $Z > 0$, *then*

$$\begin{bmatrix} Y & X^T \\ X & -Z \end{bmatrix} < 0 \Leftrightarrow Y + X^T Z^{-1} X < 0.$$

Lemma 4: Consider the dynamical system (6). Then there exist two scalars φ_1 *and* φ_2 *satisfying*

$$\|r(t)\|^2 \leq \varphi_1\|r(t_k)\|^2 + \varphi_2\int_{t_k-\tau_u}^{t_k}\|r(\alpha)\|^2 d\alpha, \quad t_k \leq t < t_{k+1}, \tag{12}$$

where

$$\varphi_1 = 5(1+\|K\|^2 h_u^2)e^{5(\|c(\mathcal{G}\otimes\Gamma^{(1)})\|^2+\rho+\|c(\mathcal{G}\otimes\Gamma^{(2)})\|^2)h_u^2}$$
$$\varphi_2 = 5\|c(\mathcal{G}\otimes\Gamma^{(2)})\|^2 h_u e^{5(\|c(\mathcal{G}\otimes\Gamma^{(1)})\|^2+\rho+\|c(\mathcal{G}\otimes\Gamma^{(2)})\|^2)h_u^2}.$$

Proof: From (6), we have the following inequality for any $t \in [t_k, t_{k+1})$,

$$\begin{aligned}\|r(t)\| \leq{}& \|r(t_k)\| + \left\|\int_{t_k}^t c(\mathcal{G}\otimes\Gamma^{(1)})r(\alpha)d\alpha\right\| \\ &+ \left\|\int_{t_k}^t c(\mathcal{G}\otimes\Gamma^{(2)})r(\alpha-\tau(\alpha))d\alpha\right\| \\ &+ \left\|\int_{t_k}^t Kr(t_k)d\alpha\right\| + \left\|\int_{t_k}^t g(r(\alpha))d\alpha\right\|. \end{aligned}\tag{13}$$

By utilizing the Cauchy–Schwarz inequality, we obtain from that (13)

$$\begin{aligned}\|r(t)\|^2 \leq{}& 5\|r(t_k)\|^2 + 5\left\|\int_{t_k}^t c(\mathcal{G}\otimes\Gamma^{(1)})r(\alpha)d\alpha\right\|^2 \\ &+ 5\left\|\int_{t_k}^t c(\mathcal{G}\otimes\Gamma^{(2)})r(\alpha-\tau(\alpha))d\alpha\right\|^2 \\ &+ 5\left\|\int_{t_k}^t Kr(t_k)d\alpha\right\|^2 + 5\left\|\int_{t_k}^t g(r(\alpha))d\alpha\right\|^2. \end{aligned}\tag{14}$$

Using the Cauchy–Schwarz inequality once more, we can deduce from (14) that

$$\begin{aligned}\|r(t)\|^2 \leq{}& 5\|r(t_k)\|^2 + 5h_u\int_{t_k}^t \|c(\mathcal{G}\otimes\Gamma^{(1)})r(\alpha)\|^2 d\alpha \\ &+ 5h_u\int_{t_k}^t \|c(\mathcal{G}\otimes\Gamma^{(2)})r(\alpha-\tau(\alpha))\|^2 d\alpha \\ &+ 5h_u\int_{t_k}^t \|Kr(t_k)\|^2 d\alpha \\ &+ 5h_u\int_{t_k}^t \|g(r(\alpha))\|^2 d\alpha. \end{aligned}\tag{15}$$

Furthermore, it is clear from (8) that

$$\|g(r(t))\|^2 \leq \rho\|r(t)\|^2.$$

Therefore

$$\begin{aligned}\|r(t)\|^2 \leq{}& 5(\|c(\mathcal{G}\otimes\Gamma^{(1)})\|^2+\rho)h_u\int_{t_k}^t \|r(\alpha)\|^2 d\alpha \\ &+ 5\|r(t_k)\|^2 + 5\|K\|^2 h_u\int_{t_k}^t \|r(t_k)\|^2 d\alpha \\ &+ 5\|c(\mathcal{G}\otimes\Gamma^{(2)})\|^2 h_u\int_{t_k}^t \|r(\alpha-\tau(\alpha))\|^2 d\alpha \\ \leq{}& 5(1+\|K\|^2 h_u^2)\|r(t_k)\|^2 \\ &+ 5\|c(\mathcal{G}\otimes\Gamma^{(2)})\|^2 h_u\int_{t_k-\tau_u}^{t_k} \|r(\alpha)\|^2 d\alpha \\ &+ 5(\|c(\mathcal{G}\otimes\Gamma^{(1)})\|^2 + \rho \\ &+ \|c(\mathcal{G}\otimes\Gamma^{(2)})\|^2)h_u\int_{t_k}^t \|r(\alpha)\|^2 d\alpha. \end{aligned}\tag{16}$$

We can immediately obtain (12) by utilizing the Gronwall–Bellman Lemma to (16). The proof is now complete. □

Remark 3: The proof of Lemma 4 is similar to that of Lemma 2 of [42]. However, it should be noted that the mentioned paper assumed that time delays are expected to be constant. In general, the case of time-varying delays is more practical than constant delays. This paper considers time-varying coupling delays, which are more general than constant time delays.

III. MAIN RESULTS

This section will show you how to solve the problem using the design method. Firstly, a less conservative synchronization assures the system (6) is exponentially stable with the sampled-data control (4). Secondly, the disturbance is input to satisfy extended dissipative analysis. For the sake of fluent presentation, we provide some notations as follows

$$\mathcal{X} = \begin{bmatrix} sym\{\frac{1}{2}X_1\} & -X_1+X_2 & X_3 \\ * & sym\{\frac{1}{2}X_1 - X_2\} & X_4 \\ * & * & sym\{\frac{1}{2}X_5\} \end{bmatrix},$$

$$\begin{aligned}\zeta(t) = \Big[& r^T(t),\ r^T(t_k),\ r^T(t-h_u),\ r^T(t-\tau(t)), \\ & r^T(t-\tau_u),\ \dot{r}^T(t),\ \left(\int_{t_k}^t r(s)ds\right)^T, \\ & \left(\frac{1}{(t-t_k)}\int_{t_k}^t\int_u^t r(s)dsdu\right)^T, \\ & \left(\frac{1}{\tau(t)}\int_{t-\tau(t)}^t r(s)ds\right)^T, \\ & \left(\frac{1}{\tau_u-\tau(t)}\int_{t-\tau_u}^{t-\tau(t)} r(s)ds\right)^T,\ g^T(r(t))\Big]^T,\end{aligned}$$

$e_i = \left[0_{nN\times(i-1)nN},\ I_{nN},\ 0_{nN\times(11-i)nN}\right]$ for $i = 1, 2, \ldots, 11$,

$$\Omega_1 = sym\{e_1^T P e_6\} + 2\beta e_1^T P e_1,$$

$$\begin{aligned}
\Omega_2 &= e_1^T T_1 e_1 - e^{-2\beta\tau_u} e_5^T T_1 e_5,\\
\Omega_3 &= e_1^T T_2 e_1 - (1-\tau_d) e^{-2\beta\tau_u} e_4^T T_2 e_4 + e_1^T T_3 e_1\\
&\quad - e^{-2\beta\tau_u} e_3^T T_3 e_3,\\
\Omega_4 &= -\begin{bmatrix} e_1\\ e_2\\ e_7\end{bmatrix}^T \mathcal{X} \begin{bmatrix} e_1\\ e_2\\ e_7\end{bmatrix},\\
\Omega_5 &= -\frac{e^{-2\beta h_u}}{h_u}\left(e_7^T S e_7 + 3(e_7-2e_8)^T S(e_7-2e_8)\right),\\
\Omega_6 &= -sym\left\{(e_1-e_2)Y_1 + 3\left(\frac{2}{h_u}e_7\right)Y_2\right\}\\
&\quad + \frac{3}{h_u^2} sym\left\{(e_1+e_2)^T R 2 e_7\right\},\\
\Omega_7 &= sym\left\{N_1\begin{bmatrix} e_1-e_2\\ e_7\end{bmatrix}\right\},\\
\Omega_8 &= sym\left\{N_2 e_7\right\},\\
\Omega_9 &= \tau_u^2 e_6^T Q_1 e_6 - e^{-2\beta h_u}\begin{bmatrix} e_1-e_4\\ e_1+e_4-2e_9\\ e_4-e_5\\ e_4+e_5-2e_{10}\end{bmatrix}^T\\
&\quad \times \begin{bmatrix} \mathrm{diag}\{Q_1, 3Q_2\} & S_1\\ * & \mathrm{diag}\{Q_1, 3Q_2\}\end{bmatrix}\\
&\quad \times \begin{bmatrix} e_1-e_4\\ e_1+e_4-2e_9\\ e_4-e_5\\ e_4+e_5-2e_{10}\end{bmatrix},\\
\Omega_{10} &= h_u e_6^T Q_2 e_6 - \frac{e^{-2\beta h_u}}{h_u}\begin{bmatrix} e_1-e_2\\ e_2-e_3\end{bmatrix}^T \begin{bmatrix} Q_2 & S_2\\ * & Q_2\end{bmatrix}\\
&\quad \times \begin{bmatrix} e_1-e_2\\ e_2-e_3\end{bmatrix},\\
\Omega_{11} &= -\varepsilon\begin{bmatrix} e_1\\ e_{11}\end{bmatrix}^T\\
&\quad \times \begin{bmatrix} sym\left\{I\otimes U^T V\right\} & -I_N\otimes(U^T+V^T)\\ * & 2I\end{bmatrix}\begin{bmatrix} e_1\\ e_{11}\end{bmatrix},\\
\Omega_{12} &= sym\left\{(e_1^T+e_6^T)M^T(-e_6+e_{11}+c(\mathcal{G}\otimes\Gamma^{(1)})e_1\right.\\
&\quad \left. + c(\mathcal{G}\otimes\Gamma^{(2)})e_4 + Ke_2)\right\},\\
\theta_1 &= \frac{h_u}{2}\beta e_2^T T_4 e_2 - e_2^T T_4 e_2,\\
\theta_2 &= \frac{h_u}{2}\beta e_2^T T_4 e_2 + e_2^T T_4 e_2,\\
\theta_3 &= 2\beta\begin{bmatrix} e_1\\ e_2\\ e_7\end{bmatrix}^T \mathcal{X}\begin{bmatrix} e_1\\ e_2\\ e_7\end{bmatrix} + 2\begin{bmatrix} e_1\\ e_2\\ e_7\end{bmatrix}^T \mathcal{X}\begin{bmatrix} e_6\\ 0\\ e_1\end{bmatrix},\\
\theta_4 &= e_1^T S e_1,\\
\theta_5 &= e_6^T R e_6,\\
\theta_6 &= -\frac{3}{h_u^2}(e_1+e_2)^T R(e_1+e_2),\\
\theta_7 &= \begin{bmatrix} e_6\\ e_1\end{bmatrix}^T \begin{bmatrix} W_1 & W_2\\ * & W_3\end{bmatrix}\begin{bmatrix} e_6\\ e_1\end{bmatrix},\\
\theta_8 &= \frac{h_u}{4} e_6^T Z e_6,\\
\theta_9 &= \frac{h_u}{4} e_6^T Z e_6,\\
\theta_{10} &= sym\left\{N_2 e_1\right\},\ d_1(t) = t - t_k,\ d_2(t) = t_{k+1} - t.
\end{aligned} \tag{17}$$

A. STABILITY ANALYSIS

The first theorem gives an exponential synchronization condition for CDNs (6) with a non-disturbance.

Theorem 1: For given scalars h_u, τ_u and τ_d, if there are matrices $P > 0, T_i > 0$ $(i = 1, 2, 3, 4), S > 0, R > 0, Z > 0, Q_1 > 0, Q_2 > 0$ and any matrices $W_1, W_2, W_3, X_i, (i = 1, 2, 3, 4, 5), N_1, N_2, Y_1, Y_2, S_1 = \begin{bmatrix} S_{11} & S_{12}\\ * & S_{22}\end{bmatrix}, S_2$ such that $\mathcal{W} = \begin{bmatrix} W_1 & W_2\\ * & W_3\end{bmatrix} > 0$, $\mathcal{X} > 0$, $M = diag\{M_1, M_2, \ldots, M_N\}, H = diag\{H_1, H_2, \ldots, H_N\}$ satisfying the following LMIs:

$$\sum_{i=1}^{12}\Omega_i < 0, \tag{18}$$

$$\begin{bmatrix} \sum_{i=1}^{12}\Omega_i + h_u(\theta_1+\theta_6+\theta_9+\theta_{10}) & \sqrt{h_u}N_1 & \Sigma^{(1)}\\ * & -e^{-2\beta h_u}\mathcal{W} & 0\\ * & * & \Sigma^{(2)}\end{bmatrix} < 0, \tag{19}$$

$$\sum_{i=1}^{12}\Omega_i + \theta_2+\theta_3+\theta_4+\theta_5+\theta_7+\theta_8 < 0, \tag{20}$$

$$\begin{bmatrix} diag\{Q_1, 3Q_1\} & S_1\\ * & diag\{Q_1, 3Q_1\}\end{bmatrix} > 0, \tag{21}$$

$$\begin{bmatrix} Q_2 & S_2\\ * & Q_2\end{bmatrix} > 0, \tag{22}$$

where

$$\begin{aligned}
\Sigma^{(1)} &= \left[h_u N_2\ h_u Y_1\ h_u Y_2\right],\\
\Sigma^{(2)} &= diag\left\{-2e^{-2\beta h_u}Z, -h_u e^{2\beta h_u}R, -3h_u e^{2\beta h_u}R\right\},
\end{aligned}$$

then, the error system (6) is exponentially synchronized and the feedback controller gain matrix can be obtained as

$$K = M^{-T}H^T. \tag{23}$$

Proof: Consider the following Lyapunov functional represent for system (6):

$$V(t) = \sum_{i=1}^{11} V_i(t), \qquad t\in[t_k, t_{k+1}),$$

where

$$\begin{aligned}
V_1(t) &= e^{2\beta t} r^T(t) P r(t),\\
V_2(t) &= \int_{t-\tau_u}^{t} e^{2\beta s} r^T(s) T_1 r(s) ds,\\
V_3(t) &= \int_{t-\tau(t)}^{t} e^{2\beta s} r^T(s) T_2 r(s) ds
\end{aligned}$$

$$
\begin{aligned}
&+\int_{t-h_u}^{t} e^{2\beta s} r^T(s)T_3 r(s)ds,\\
V_4(t) &= (t_{k+1}-t)(t-t_k)e^{2\beta t} r^T(t_k)T_4 r(t_k),\\
V_5(t) &= (t_{k+1}-t)\int_{t_k}^{t} e^{2\beta s}\begin{bmatrix}\dot r(s)\\ r(s)\end{bmatrix}^T\begin{bmatrix}W_1 & W_2\\ * & W_3\end{bmatrix}\begin{bmatrix}\dot r(s)\\ r(s)\end{bmatrix}ds,\\
V_6(t) &= (t_{k+1}-t)e^{2\beta t}\begin{bmatrix} r(t)\\ r(t_k)\\ \int_{t_k}^{t} r(s)ds\end{bmatrix}^T \mathcal{X}\begin{bmatrix} r(t)\\ r(t_k)\\ \int_{t_k}^{t} r(s)ds\end{bmatrix},\\
V_7(t) &= (t_{k+1}-t)\int_{t_k}^{t} e^{2\beta s} r^T(s)Sr(s)ds,\\
V_8(t) &= (t_{k+1}-t)\int_{t_k}^{t} e^{2\beta s}\dot r^T(s)R\dot r(s)ds,\\
V_9(t) &= (t_{k+1}-t)\int_{t_k}^{t}\int_{u}^{t} e^{2\beta s}\dot r^T(s)Z\dot r(s)dsdu,\\
V_{10}(t) &= \tau_u\int_{t-\tau_u}^{t}\int_{u}^{t} e^{2\beta s}\dot r^T(s)Q_1\dot r(s)dsdu,\\
V_{11}(t) &= \int_{-h_u}^{0}\int_{t+u}^{t} e^{2\beta s}\dot r^T(s)Q_2\dot r(s)dsdu.
\end{aligned}
$$

Initially, $\dot V_1(t)$, $\dot V_2(t)$ and $\dot V_3(t)$ are computed as

$$
\begin{aligned}
\dot V_1(t) &= 2e^{2\beta t} r^T(t)P\dot r(t) + 2\beta e^{2\beta t} r^T(t)Pr(t)\\
&= e^{2\beta t}\zeta^T(t)\left(sym\left\{e_1^T Pe_6\right\} + 2\beta e_1^T Pe_1\right)\zeta(t)\\
&= e^{2\beta t}\zeta^T(t)\Omega_1\zeta(t) \qquad (24)\\
\dot V_2(t) &= e^{2\beta t} r^T(t)T_1 r(t) - e^{2\beta(t-\tau_u)} r^T(t-\tau_u)T_1 r(t-\tau_u)\\
&= e^{2\beta t}\zeta^T(t)\left(e_1^T T_1 e_1 - e^{-2\beta\tau_u} e_5^T T_1 e_5\right)\zeta(t)\\
&= e^{2\beta t}\zeta^T(t)\Omega_2\zeta(t) \qquad (25)\\
\dot V_3(t) &= e^{2\beta t} r^T(t)T_2 r(t)\\
&\quad -(1-\dot\tau(t))e^{2\beta(t-\tau(t))} r^T(t-\tau(t))T_2 r(t-\tau(t))\\
&\quad + e^{2\beta t} r^T(t)T_3 r(t)\\
&\quad - e^{2\beta(t-h_u)} r^T(t-h_u)T_3 r(t-h_u)\\
&\le e^{2\beta t} r^T(t)T_2 r(t)\\
&\quad -(1-\tau_d)e^{2\beta(t-\tau_u)} r^T(t-\tau(t))T_2 r(t-\tau(t))\\
&\quad + e^{2\beta t} r^T(t)T_3 r(t)\\
&\quad - e^{2\beta(t-h_u)} r^T(t-h_u)T_3 r(t-h_u)\\
&= e^{2\beta t}\zeta^T(t)\Big(e_1^T T_2 e_1 - (1-\tau_d)e^{-2\beta\tau_u} e_4^T T_2 e_4\\
&\quad + e_1^T T_3 e_1 - e^{-2\beta\tau_u} e_3^T T_3 e_3\Big)\zeta(t)\\
&= e^{2\beta t}\zeta^T(t)\Omega_3\zeta(t) \qquad (26)\\
\dot V_4(t) &= (t_{k+1}-t)(t-t_k)2\beta e^{2\beta t} r^T(t_k)T_4 r(t_k)\\
&\quad + (t_{k+1}-t)e^{2\beta t} r^T(t_k)T_4 r(t_k)\\
&\quad - (t-t_k)e^{2\beta t} r^T(t_k)T_4 r(t_k)\\
&\le \frac{((t_{k+1}-t)+(t-t_k))^2}{4}2\beta e^{2\beta t} r^T(t_k)T_4 r(t_k)\\
&\quad + (t_{k+1}-t)e^{2\beta t} r^T(t_k)T_4 r(t_k)\\
&\quad - (t-t_k)e^{2\beta t} r^T(t_k)T_4 r(t_k)\\
&\le \frac{h_u((t_{k+1}-t)+(t-t_k))}{4}2\beta e^{2\beta t} r^T(t_k)T_4 r(t_k)\\
&\quad + (t_{k+1}-t)e^{2\beta t} r^T(t_k)T_4 r(t_k)\\
&\quad - (t-t_k)e^{2\beta t} r^T(t_k)T_4 r(t_k)\\
&= \frac{h_u}{2}(t_{k+1}-t)\beta e^{2\beta t}\zeta^T(t)e_2^T T_4 e_2\zeta(t)\\
&\quad + \frac{h_u}{2}(t-t_k)\beta e^{2\beta t}\zeta^T(t)e_2^T T_4 e_2\zeta(t)\\
&\quad + (t_{k+1}-t)e^{2\beta t}\zeta^T(t)e_2^T T_4 e_2\zeta(t)\\
&\quad - (t-t_k)e^{2\beta t}\zeta^T(t)e_2^T T_4 e_2\zeta(t)\\
&= e^{2\beta t}\zeta^T(t)(d_1(t)\theta_1 + d_2(t)\theta_2)\zeta(t). \qquad (27)
\end{aligned}
$$

$\dot V_5(t)$ is calculated as

$$
\begin{aligned}
\dot V_5(t) &= (t_{k+1}-t)e^{2\beta t}\begin{bmatrix}\dot r(t)\\ r(t)\end{bmatrix}^T\begin{bmatrix}W_1 & W_2\\ * & W_3\end{bmatrix}\begin{bmatrix}\dot r(t)\\ r(t)\end{bmatrix}\\
&\quad - \int_{t_k}^{t} e^{2\beta s}\begin{bmatrix}\dot r(s)\\ r(s)\end{bmatrix}^T\begin{bmatrix}W_1 & W_2\\ * & W_3\end{bmatrix}\begin{bmatrix}\dot r(s)\\ r(s)\end{bmatrix}ds\\
&\le (t_{k+1}-t)e^{2\beta t}\zeta^T(t)\begin{bmatrix}e_6\\ e_1\end{bmatrix}^T\begin{bmatrix}W_1 & W_2\\ * & W_3\end{bmatrix}\begin{bmatrix}e_6\\ e_1\end{bmatrix}\zeta(t)\\
&\quad - e^{2\beta t}\int_{t_k}^{t} e^{-2\beta h_u}\begin{bmatrix}\dot r(s)\\ r(s)\end{bmatrix}^T\begin{bmatrix}W_1 & W_2\\ * & W_3\end{bmatrix}\begin{bmatrix}\dot r(s)\\ r(s)\end{bmatrix}ds. \qquad (28)
\end{aligned}
$$

$\dot V_6(t)$ is obtained as

$$
\begin{aligned}
\dot V_6(t) &= -e^{2\beta t}\begin{bmatrix} r(t)\\ r(t_k)\\ \int_{t_k}^{t} r(s)ds\end{bmatrix}^T \mathcal{X}\begin{bmatrix} r(t)\\ r(t_k)\\ \int_{t_k}^{t} r(s)ds\end{bmatrix}\\
&\quad + 2\beta(t_{k+1}-t)e^{2\beta t}\begin{bmatrix} r(t)\\ r(t_k)\\ \int_{t_k}^{t} r(s)ds\end{bmatrix}^T \mathcal{X}\begin{bmatrix} r(t)\\ r(t_k)\\ \int_{t_k}^{t} r(s)ds\end{bmatrix}\\
&\quad + 2(t_{k+1}-t)e^{2\beta t}\begin{bmatrix} r(t)\\ r(t_k)\\ \int_{t_k}^{t} r(s)ds\end{bmatrix}^T \mathcal{X}\begin{bmatrix}\dot r(t)\\ 0\\ r(t)\end{bmatrix}\\
&= -e^{2\beta t}\zeta^T(t)\begin{bmatrix}e_1\\ e_2\\ e_7\end{bmatrix}^T \mathcal{X}\begin{bmatrix}e_1\\ e_2\\ e_7\end{bmatrix}\\
&\quad + 2\beta(t_{k+1}-t)e^{2\beta t}\begin{bmatrix}e_1\\ e_2\\ e_7\end{bmatrix}^T \mathcal{X}\begin{bmatrix}e_1\\ e_2\\ e_7\end{bmatrix}\\
&\quad + 2(t_{k+1}-t)e^{2\beta t}\zeta^T(t)\begin{bmatrix}e_1\\ e_2\\ e_7\end{bmatrix}^T \mathcal{X}\begin{bmatrix}e_6\\ 0\\ e_1\end{bmatrix}\zeta(t)\\
&= e^{2\beta t}\zeta^T(t)(\Omega_4 + d_2(t)\theta_3)\zeta(t). \qquad (29)
\end{aligned}
$$

$\dot V_7(t)$ can be presented as

$$
\begin{aligned}
\dot V_7(t) &= (t_{k+1}-t)e^{2\beta t} r^T(t)Sr(t) - \int_{t_k}^{t} e^{2\beta s} r^T(s)Sr(s)ds\\
&\le (t_{k+1}-t)e^{2\beta t} r^T(t)Sr(t)\\
&\quad - e^{2\beta(t-h_u)}\int_{t_k}^{t} r^T(s)Sr(s)ds.
\end{aligned}
$$

Utilizing Lemma 2, we obtain

$$\begin{aligned} &-\int_{t_k}^{t} r^T(s)Sr(s)ds \\ &\le -\frac{1}{(t-t_k)}\left(e_7^T S e_7 + 3(e_7-2e_8)^T S(e_7-2e_8)\right) \\ &\le -\frac{1}{h_u}\left(e_7^T S e_7 + 3(e_7-2e_8)^T S(e_7-2e_8)\right). \end{aligned}$$

So, $\dot{V}_7(t)$ is bounded by

$$\dot{V}_7(t) \le e^{2\beta t}\zeta^T(t)(d_2(t)\theta_4 + \Omega_5)\zeta(t). \tag{30}$$

An estimation of $\dot{V}_8(t)$ is gained by

$$\begin{aligned} \dot{V}_8(t) &= (t_{k+1}-t)e^{2\beta t}\dot{r}^T(t)R\dot{r}(t) \\ &\quad - \int_{t_k}^{t} e^{2\beta s}\dot{r}^T(s)R\dot{r}(s)ds \\ &\le (t_{k+1}-t)e^{2\beta t}\dot{r}^T(t)R\dot{r}(t) \\ &\quad - e^{2\beta(t-h_u)}\int_{t_k}^{t}\dot{r}^T(s)R\dot{r}(s)ds. \end{aligned} \tag{31}$$

Use Lemma 2 to obtain

$$\begin{aligned} &-\int_{t_k}^{t}\dot{r}^T(s)R\dot{r}(s)ds \\ &\le -\frac{1}{d_1(t)}\zeta^T(t)(e_1-e_2)^T R(e_1-e_2)\zeta(t) \\ &\quad -\frac{3}{d_1(t)}\zeta^T(t)(e_1+e_2-\frac{2}{d_1(t)}e_7)^T R \\ &\quad \times(e_1+e_2-\frac{2}{d_1(t)}e_7)\zeta(t) \\ &= -\frac{1}{d_1(t)}\zeta^T(t)(e_1-e_2)^T R(e_1-e_2)\zeta(t) \\ &\quad -\frac{3}{d_1(t)}\frac{d_1^2(t)}{d_1^2(t)}\zeta^T(t)(e_1+e_2-\frac{2}{d_1(t)}e_7)^T R \\ &\quad \times(e_1+e_2-\frac{2}{d_1(t)}e_7)\zeta(t) \\ &\le -\frac{1}{d_1(t)}\zeta^T(t)(e_1-e_2)^T R(e_1-e_2)\zeta(t) \\ &\quad -\frac{3}{d_1(t)h_u^2}\zeta^T(t)(d_1(t)(e_1+e_2)-2e_7)^T R \\ &\quad \times(d_1(t))(e_1+e_2)-2e_7)\zeta(t) \\ &= -\frac{1}{d_1(t)}\zeta^T(t)\Big((e_1-e_2)^T R(e_1-e_2) \\ &\quad +3\left(\frac{2}{h_u}e_7\right)^T R\left(\frac{2}{h_u}e_7\right)\Big)\zeta(t) \\ &\quad +\zeta^T(t)\left((d_1(t))\left(-\frac{3}{h_u^2}(e_1+e_2)^T R(e_1+e_2)\right)\right)\zeta(t) \\ &\quad -\frac{3}{h_u^2}\zeta^T(t)sym\left\{-(e_1+e_2)^T R(2e_7)\right\}\zeta(t). \end{aligned} \tag{32}$$

Note that the inequalities,

$$-\frac{1}{t-t_k}\varpi_i^T R\varpi_i \le (t-t_k)Y_i R^{-1}Y_i^T - Y_i\varpi_i - \varpi_i^T Y_i^T$$

for given matrices, ϖ_i, any matrices $Y_i \in \mathcal{R}^{n\times 11n}(i=1,2)$ hold. It follows that

$$\begin{aligned} &-\frac{1}{d_1(t)}\zeta^T(t)\left((e_1-e_2)^T R(e_1-e_2) + 3\left(\frac{2}{h_u}e_7\right)^T R\left(\frac{2}{h_u}e_7\right)\right)\zeta(t) \\ &\le \zeta^T(t)\Big(-sym\left\{(e_1-e_2)Y_1 + 3\left(\frac{2}{h_u}e_7\right)Y_2\right\} \\ &\quad +(t-t_k)Y_1^T R^{-1}Y_1 + 3(t-t_k)Y_2^T R^{-1}Y_2\Big)\zeta(t). \end{aligned} \tag{33}$$

From Eq.(31)-(33), $\dot{V}_8(t)$ is bounded by

$$\begin{aligned} \dot{V}_8(t) &\le e^{2\beta t}\zeta^T(t)\Big(d_2(t)\theta_5 + e^{-2\beta h_u}(\Omega_6 + d_1(t)Y_1^T R^{-1}Y_1 \\ &\quad + d_1(t)3Y_2^T R^{-1}Y_2 + d_1(t)\theta_6)\Big)\zeta(t). \end{aligned} \tag{34}$$

An expression of $\dot{V}_9(t)$ is followed by

$$\begin{aligned} \dot{V}_9(t) &= (t_{k+1}-t)(t-t_k)e^{2\beta t}\dot{r}^T(t)Z\dot{r}(t) \\ &\quad - \int_{t_k}^{t}\int_{u}^{t} e^{2\beta s}\dot{r}^T(s)Z\dot{r}(s)dsdu \\ &\le \frac{((t_{k+1}-t)+(t-t_k))^2}{4}e^{2\beta t}\dot{r}^T(t)Z\dot{r}(t) \\ &\quad - \int_{t_k}^{t}\int_{u}^{t} e^{2\beta s}\dot{r}^T(s)Z\dot{r}(s)dsdu \\ &\le \frac{h_u((t_{k+1}-t)+(t-t_k))}{4}e^{2\beta t}\dot{r}^T(t)Z\dot{r}(t) \\ &\quad - e^{2\beta(t-h_u)}\int_{t_k}^{t}\int_{u}^{t}\dot{r}^T(s)Z\dot{r}(s)dsdu \\ &= \frac{h_u}{4}(t_{k+1}-t)e^{2\beta t}\dot{r}^T(t)Z\dot{r}(t) \\ &\quad + \frac{h_u}{4}(t-t_k)e^{2\beta t}\dot{r}^T(t)Z\dot{r}(t) \\ &\quad - e^{2\beta(t-h_u)}\int_{t_k}^{t}\int_{u}^{t}\dot{r}^T(s)Z\dot{r}(s)dsdu. \end{aligned} \tag{35}$$

By using the free-weighting matrix method [43], [44], the below equations hold for any matrices N_1 and N_2 with compatible dimensions.

$$\begin{aligned} 0 &= 2\zeta^T(t)N_1\left[\begin{bmatrix} r(t)-r(t_k) \\ \int_{t_k}^{t} r(s)ds \end{bmatrix} - \int_{t_k}^{t}\begin{bmatrix}\dot{r}(s) \\ r(s)\end{bmatrix}ds\right] \\ &= 2\zeta^T(t)N_1\left[\begin{bmatrix} e_1-e_2 \\ e_7 \end{bmatrix}\zeta(t) - \int_{t_k}^{t}\begin{bmatrix}\dot{r}(s) \\ r(s)\end{bmatrix}ds\right], \end{aligned} \tag{36}$$

and

$$\begin{aligned} 0 &= 2\zeta^T(t)N_2\Big[(t-t_k)r(t) - \int_{t_k}^{t} r(s)ds \\ &\quad - \int_{t_k}^{t}\int_{u}^{t}\dot{r}(s)dsdu\Big]. \end{aligned} \tag{37}$$

Furthermore, rely on the well-known inequality for any matrix $\mathcal{N} > 0$, $-2x^T y \le x^T\mathcal{N}^{-1}x + y^T\mathcal{N}y$, it is obvious to given that

$$-2\zeta^T(t)N_1\int_{t_k}^{t}\begin{bmatrix}\dot{r}(s) \\ r(s)\end{bmatrix}ds$$

$$\leq e^{-2\beta h_u} \int_{t_k}^{t} \begin{bmatrix} \dot{r}(s) \\ r(s) \end{bmatrix}^T \begin{bmatrix} W_1 & W_2 \\ * & W_3 \end{bmatrix} \begin{bmatrix} \dot{r}(s) \\ r(s) \end{bmatrix} ds + (t-t_k) e^{2\beta h_u} \zeta^T(t) N_1 \begin{bmatrix} W_1 & W_2 \\ * & W_3 \end{bmatrix}^{-1} N_1^T \zeta(t) \quad (38)$$

$$-2\zeta^T(t) N_2 \int_{t_k}^{t} \int_{u}^{t} \dot{r}(s) ds du \leq \frac{(t-t_k)^2}{2} e^{2\beta h_u} \zeta^T(t) N_2 Z^{-1} N_2^T \zeta(t) + \int_{t_k}^{t} \int_{u}^{t} e^{-2\beta h_u} \dot{r}^T(s) Z \dot{r}(s) ds du. \quad (39)$$

Combining (28), (36) and (38), we obtain the new estimation of $\dot{V}_5(t)$ as follows

$$\dot{V}_5(t) \leq e^{2\beta t} \zeta^T(t) \left(d_2(t)\theta_7 + \Omega_7 + d_1(t) e^{2\beta h_u} N_1 \times \begin{bmatrix} W_1 & W_2 \\ * & W_3 \end{bmatrix}^{-1} N_1^T \right) \zeta(t). \quad (40)$$

Combining (35), (37) and (39), we obtain all estimations of $\dot{V}_9(t)$ as follows

$$\dot{V}_9(t) \leq e^{2\beta t} \zeta^T(t) \left(d_2(t)\theta_8 + d_1(t)\theta_9 + d_1(t)\theta_{10} - \Omega_8 + \frac{h_u d_1(t)}{2} e^{2\beta h_u} N_2 Z^{-1} N_2^T \right) \zeta(t). \quad (41)$$

An expression of $\dot{V}_{10}(t)$ is calculated by

$$\begin{aligned} \dot{V}_{10}(t) &= \tau_u^2 e^{2\beta t} \dot{r}^T(t) Q_1 \dot{r}(t) - \tau_u \int_{t-\tau_u}^{t} e^{2\beta s} \dot{r}^T(s) Q_1 \dot{r}(s) ds \\ &\leq \tau_u^2 e^{2\beta t} \dot{r}^T(t) Q_1 \dot{r}(t) - \tau_u e^{2\beta(t-\tau_u)} \int_{t-\tau_u}^{t} \dot{r}^T(s) Q_1 \dot{r}(s) ds. \end{aligned} \quad (42)$$

Applying Lemmas 1 and 2, based on the inequality (21), a new calculating of $\dot{V}_{10}(t)$ transforms to

$$\begin{aligned} -\tau_u \int_{t-\tau_u}^{t} \dot{r}^T(s) Q_1 \dot{r}(s) ds &= -\tau_u \int_{t-\tau(t)}^{t} \dot{r}^T(s) Q_1 \dot{r}(s) ds - \tau_u \int_{t-\tau_u}^{t-\tau(t)} \dot{r}^T(s) Q_1 \dot{r}(s) ds \\ &\leq -\zeta^T(t) \begin{bmatrix} e_1 - e_4 \\ e_1 + e_4 - 2e_9 \\ e_4 - e_5 \\ e_4 + e_5 - 2e_{10} \end{bmatrix}^T \begin{bmatrix} Q_1 & S_1 \\ * & 3Q_1 \end{bmatrix} \times \begin{bmatrix} e_1 - e_4 \\ e_1 + e_4 - 2e_9 \\ e_4 - e_5 \\ e_4 + e_5 - 2e_{10} \end{bmatrix} \zeta(t). \end{aligned}$$

Thus, we get all estimations of $\dot{V}_{10}(t)$ as follows

$$\dot{V}_{10}(t) \leq e^{2\beta t} \zeta^T(t) \Omega_9 \zeta(t). \quad (43)$$

An expression of $\dot{V}_{11}(t)$ is calculated by

$$\begin{aligned} \dot{V}_{11}(t) &= e^{2\beta t} h_u \dot{r}^T(t) Q_2 \dot{r}(t) - \int_{t-h_u}^{t} e^{2\beta s} \dot{r}^T(s) Q_2 \dot{r}(s) ds \\ &\leq e^{2\beta t} h_u \dot{r}^T(t) Q_2 \dot{r}(t) - e^{2\beta t} \int_{t-h_u}^{t} e^{-2\beta h_u} \dot{r}^T(s) Q_2 \dot{r}(s) ds. \end{aligned}$$

Applying Lemmas 1 and Jensen's inequality, relied on the inequality (22), a new calculating of $\dot{V}_{11}(t)$ transforms to

$$-\int_{t-h_u}^{t} \dot{r}^T(s) Q_2 \dot{r}(s) ds \leq -\frac{1}{h_u} \begin{bmatrix} e_1 - e_2 \\ e_2 - e_3 \end{bmatrix}^T \begin{bmatrix} Q_2 & S_2 \\ * & Q_2 \end{bmatrix} \times \begin{bmatrix} e_1 - e_2 \\ e_2 - e_3 \end{bmatrix}. \quad (44)$$

So, $\dot{V}_{11}(t)$ can be gained as

$$\dot{V}_{11}(t) \leq \zeta^T(t) \Omega_{10} \zeta(t). \quad (45)$$

Moreover, we can obtain from condition (7) for any $\varepsilon > 0$

$$\begin{aligned} 0 &\geq -\varepsilon \begin{bmatrix} r(t) \\ g(r(t)) \end{bmatrix}^T \times \begin{bmatrix} sym\{I \otimes U^T V\} & -I_N \otimes (U^T + V^T) \\ * & 2I \end{bmatrix} \begin{bmatrix} r(t) \\ g(r(t)) \end{bmatrix} \\ &\geq \zeta^T(t) e^{2\beta t} \Omega_{11} \zeta(t). \end{aligned} \quad (46)$$

For any proper dimension M, the following equation is clearly to be obtained

$$\begin{aligned} 0 &= 2e^{2\beta t} \left[r^T(t) + \dot{r}^T(t) \right] M^T \left[-\dot{r}(t) + g(r(t)) + c(\mathcal{G} \otimes \Gamma^{(1)}) r(t) + c(\mathcal{G} \otimes \Gamma^{(2)}) r(t-\tau(t)) + K r(t_k) \right] \\ &= e^{2\beta t} \zeta^T(t) sym\left\{ (e_1^T + e_6^T) M^T (-e_6 + e_{11} + c(\mathcal{G} \otimes \Gamma^{(1)}) e_1 + c(\mathcal{G} \otimes \Gamma^{(2)}) e_4 + K e_2) \right\} \zeta(t) \\ &= \zeta^T(t) e^{2\beta t} \Omega_{12} \zeta(t). \end{aligned} \quad (47)$$

Adding Eq.(24)-(47) to the right-hand sides of $\dot{V}(t)$ yields

$$\begin{aligned} \dot{V}(t) &\leq e^{2\beta t} \Big(\zeta^T(t) \Omega \zeta(t) + d_1(t) \zeta^T(t) \Big(\theta_1 + \theta_6 + \theta_9 + \theta_{10} + e^{-2\beta h_u} Y_1 R^{-1} Y_1^T + 3e^{-2\beta h_u} Y_2 R^{-1} Y_2^T \\ &\quad + e^{2\beta h_u} N_1 \begin{bmatrix} W_1 & W_2 \\ * & W_3 \end{bmatrix}^{-1} N_1^T + \frac{h_u}{2} e^{2\beta h_u} N_2 Z^{-1} N_2^T \Big) \zeta(t) \\ &\quad + d_2(t) \zeta^T(t) \Big(\theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_7 + \theta_8 \Big) \zeta(t) \Big) \\ &= e^{2\beta t} \zeta^T(t) \Big(\frac{d_1(t)}{d_k} \Big(\Omega + d_k(\theta_1 + \theta_6 + \theta_9 + \theta_{10} + e^{-2\beta h_u} Y_1 R^{-1} Y_1^T + 3e^{-2\beta h_u} Y_2 R^{-1} Y_2^T \\ &\quad + e^{2\beta h_u} N_1 \begin{bmatrix} W_1 & W_2 \\ * & W_3 \end{bmatrix}^{-1} N_1^T \end{aligned}$$

$$+\frac{h_u}{2}e^{2\beta h_u}N_2Z^{-1}N_2^T\Big) + \frac{d_2(t)}{d_k}\Big(\Omega + d_k(\theta_2+\theta_3+\theta_4+\theta_5+\theta_7+\theta_8)\Big)\Big)\zeta(t), \tag{48}$$

where $\Omega = \sum_{i=1}^{12}\Omega_i$ and the other notations are provided in (17). From (48), it can be rearranged condition for system (6) as follows:

$$= e^{2\beta t}\zeta^T(t)\Big(\frac{d_1(t)}{d_k}\Big(\Psi^{(1)}_{[d_k]}\Big) + \frac{d_2(t)}{d_k}\Big(\Psi^{(2)}_{[d_k]}\Big)\Big)\zeta(t) < 0, \tag{49}$$

where

$$\begin{aligned}\Psi^{(1)}_{[d_k]} &= \Omega + d_k(\theta_1+\theta_6+\theta_9+\theta_{10} + e^{-2\beta h_u}Y_1R^{-1}Y_1^T \\ &\quad + 3e^{-2\beta h_u}Y_2R^{-1}Y_2^T \\ &\quad + e^{2\beta h_u}N_1\begin{bmatrix} W_1 & W_2 \\ * & W_3\end{bmatrix}^{-1}N_1^T \\ &\quad + \frac{h_u}{2}e^{2\beta h_u}N_2Z^{-1}N_2^T), \\ \Psi^{(2)}_{[d_k]} &= \Omega + d_k(\theta_2+\theta_3+\theta_4+\theta_5+\theta_7+\theta_8).\end{aligned}$$

The inequality (49) presents a convex combination of $d_1(t)$ and $d_2(t)$. It is notably that

$$\begin{aligned}\Psi^{(1)}_{[d_k]} &= \frac{h_u-d_k}{h_u}\Psi^{(1)}_{[d_k=0]} + \frac{d_k}{h_u}\Psi^{(1)}_{[d_k=h_u]}, \\ \Psi^{(2)}_{[d_k]} &= \frac{h_u-d_k}{h_u}\Psi^{(2)}_{[d_k=0]} + \frac{d_k}{h_u}\Psi^{(2)}_{[d_k=h_u]}.\end{aligned} \tag{50}$$

Applying Schur complement to (19) and from conditions (18)-(20) result in

$$\Psi^{(1)}_{[d_k]} < 0, \qquad \Psi^{(2)}_{[d_k]} < 0. \tag{51}$$

Now, from (51), we have

$$\dot V(t) < 0, \quad t \in [t_k, t_{k+1}). \tag{52}$$

Therefore, it can be readily presented that, for $t \in [t_k, t_{k+1})$

$$V(t) \le V(t_k) \le V(t_{k-1}) \le \cdots \le V(0). \tag{53}$$

We can conclude from Lemma 4 and (53) that for $t_k \le t < t_{k+1}$

$$\begin{aligned}\|r(t)\|^2 &\le \varphi_1\|r(t_k)\|^2 + \varphi_2\int_{t_k-\tau_u}^{t_k}\|r(\alpha)\|^2 d\alpha \\ &= \frac{\varphi_1}{\lambda_{\min}(P)e^{2\beta t_k}}e^{2\beta t_k}\lambda_{\min}(P)\|r(t_k)\|^2 \\ &\quad + \frac{\varphi_2}{\lambda_{\min}(T_1)e^{2\beta t_k}}e^{2\beta t_k}\lambda_{\min}(T_1) \\ &\quad \times \int_{t_k-\tau_u}^{t_k}\|r(\alpha)\|^2 d\alpha \\ &\le \frac{\varphi_1}{\lambda_{\min}(P)e^{2\beta t_k}}e^{2\beta t_k}r^T(t_k)Pr(t_k) \\ &\quad + \frac{\varphi_2 e^{2\beta\tau_u}}{\lambda_{\min}(T_1)e^{2\beta t_k}}\int_{t_k-\tau_u}^{t_k}e^{2\beta s}r^T(s)T_1r(s)ds \\ &\le \frac{\max\left\{\frac{\varphi_1}{\lambda_{\min}(P)}, \frac{\varphi_2 e^{2\beta\tau_u}}{\lambda_{\min}(T_1)}\right\}}{e^{2\beta t_k}}(V_1(t_k)+V_2(t_k)) \\ &\le \frac{\max\left\{\frac{\varphi_1}{\lambda_{\min}(P)}, \frac{\varphi_2 e^{2\beta\tau_u}}{\lambda_{\min}(T_1)}\right\}}{e^{2\beta t_k}}(V(t_k)) \\ &\le \frac{\max\left\{\frac{\varphi_1}{\lambda_{\min}(P)}, \frac{\varphi_2 e^{2\beta\tau_u}}{\lambda_{\min}(T_1)}\right\}}{e^{2\beta t_k}}(V(0)) \\ &= \max\left\{\frac{\varphi_1}{\lambda_{\min}(P)}, \frac{\varphi_2 e^{2\beta\tau_u}}{\lambda_{\min}(T_1)}\right\}e^{-2\beta t}e^{2\beta(t-t_k)} \\ &\quad \times (V(0)) \\ &\le e^{2\beta h_u}\max\left\{\frac{\varphi_1}{\lambda_{\min}(P)}, \frac{\varphi_2 e^{2\beta\tau_u}}{\lambda_{\min}(T_1)}\right\}e^{-2\beta t}(V(0)).\end{aligned} \tag{54}$$

On the other hand, it should be pointed that $V_4(0) - V_9(0) = 0$ and hence

$$\begin{aligned}V(0) &= \sum_{i=1}^{11}V_i(0) \\ &\le \lambda_{\max}(P)\|r(0)\|^2 \\ &\quad + \tau_u\lambda_{\max}(T_1)\sup_{-\max\{\tau_u,h_u\}\le\theta\le 0}\{\|r(\theta)\|^2\} \\ &\quad + \tau_u\lambda_{\max}(T_2)\sup_{-\max\{\tau_u,h_u\}\le\theta\le 0}\{\|r(\theta)\|^2\} \\ &\quad + h_u\lambda_{\max}(T_3)\sup_{-\max\{\tau_u,h_u\}\le\theta\le 0}\{\|r(\theta)\|^2\} \\ &\quad + \tau_u^3\lambda_{\max}(Q_1)\sup_{-\max\{\tau_u,h_u\}\le\theta\le 0}\{\|\dot r(\theta)\|^2\} \\ &\quad + h_u^2\lambda_{\max}(Q_2)\sup_{-\max\{\tau_u,h_u\}\le\theta\le 0}\{\|\dot r(\theta)\|^2\} \\ &\le b_1\sup_{-\max\{\tau_u,h_u\}\le\theta\le 0}\{\|r(\theta)\|^2\} \\ &\quad + b_2\sup_{-\max\{\tau_u,h_u\}\le\theta\le 0}\{\|\dot r(\theta)\|^2\} \\ &\le (b_1+b_2)\sup_{-\max\{\tau_u,h_u\}\le\theta\le 0}\{\|r(\theta)\|^2, \|\dot r(\theta)\|^2\},\end{aligned} \tag{55}$$

where

$$\begin{aligned}b_1 &= \lambda_{\max}(P) + \tau_u\lambda_{\max}(T_1) + \tau_u\lambda_{\max}(T_2) + h_u\lambda_{\max}(T_3) \\ b_2 &= \tau_u^3\lambda_{\max}(Q_1) + h_u^2\lambda_{\max}(Q_2).\end{aligned}$$

Using (54) and (55), we can get

$$\begin{aligned}\|r(t)\|^2 &\le e^{2\beta h_u}\max\left\{\frac{\varphi_1}{\lambda_{\min}(P)}, \frac{\varphi_2 e^{2\beta\tau_u}}{\lambda_{\min}(T_1)}\right\}e^{-2\beta t} \\ &\quad \times \sup_{-\max\{\tau_u,h_u\}\le\theta\le 0}\{\|r(\theta)\|^2, \|\dot r(\theta)\|^2\}.\end{aligned} \tag{56}$$

From Definition 1, the error system (6) is exponentially stable. The proof is now complete. □

Remark 4: It is vital to seek an appropriate Lyapunov functional, which used to derive a less conservative condition. Thus, we introduce novel six (t_k, t_{k+1})-dependent terms

$V_4(t) - V_9(t)$, which is different from the construction of the Lyapunov functional in [22], [23], [24], [25], and [27]. We not only consider the information on the upper bound of the time-varying delay but also take the full merit of the available information about the actual sampling instant t_k.

Remark 5: To compare the designed controller in [24] and [27], we employed the one free-matrix parameter M in Eq. (47) instead of two parameters in the zero equation in [24] and [27]. By means that two weighting parameters β_1 and β_2 are required to obtain the sampled-data feedback controller gain matrix K in [24] and [27]. However, we can get it without introducing more variables.

B. PERFORMANCE ANALYSIS

The below theorem introduces the extended dissipative performance criterion for CDNs (6) with nonzero $\omega(t) \in \mathcal{L}_2[0, \infty)$, under the zero initial condition.

Theorem 2: For given scalars h_u, τ_u and τ_d, if there are matrices $P > 0, T_i > 0$ $(i = 1, 2, 3, 4), S > 0, R > 0, Z > 0, Q_1 > 0, Q_2 > 0$ and any matrices $W_1, W_2, X_i, (i = 1, 2, 3, 4, 5), N_1, N_2, Y_1, Y_2, S_1 = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix}, S_2$ such that $\mathcal{W} = \begin{bmatrix} W_1 & W_2 \\ * & W_3 \end{bmatrix} > 0, \mathcal{X} > 0, M = diag\{M_1, M_2, \ldots, M_N\}, H = diag\{H_1, H_2, \ldots, H_N\}$ satisfying the following LMIs (21), (22) and

$$\sum_{i=1}^{12} \Omega_i - \Pi < 0 \tag{57}$$

$$\begin{bmatrix} \sum_{i=1}^{12} \Omega_i + h_u(\theta_1 + \theta_6 + \theta_9 + \theta_{10}) - \Pi & \Sigma^{(1)} \\ * & \Sigma^{(2)} \end{bmatrix} < 0, \tag{58}$$

$$\sum_{i=1}^{12} \Omega_i + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_7 + \theta_8 - \Pi < 0, \tag{59}$$

$$P - \mathcal{J}^T \Phi_4 \mathcal{J} > 0, \tag{60}$$

where

$$\begin{aligned} \Sigma^{(1)} &= \left[\sqrt{h_u} N_1 \; h_u N_2 \; h_u Y_1 \; h_u Y_2\right], \\ \Sigma^{(2)} &= diag\left\{-e^{-2\beta h_u}\mathcal{W}, -2e^{-2\beta h_u} Z, \right. \\ &\left. -h_u e^{2\beta h_u} R, -3h_u e^{2\beta h_u} R\right\}, \\ \Pi &= e_1^T \mathcal{J}^T \Phi_1 \mathcal{J} e_1 + 2e_1^T \mathcal{J}^T \Phi_2 e_{12} + e_{12}^T \Phi_3 e_{12}, \end{aligned} \tag{61}$$

and $\tilde{e}_i \in \mathcal{R}^{nN \times 12nN}$ is denoted that $\tilde{e}_i = \left[0_{nN \times (i-1)nN}, \;\; I_{nN}, \;\; 0_{nN \times (12-i)nN}\right]$ for $i = 1, 2, \ldots, 12$ and the other parameters are similar to those in Theorem 1.

Proof: Firstly, we show that the system (6) is stable.

By Lemma 3, it is clearly that for all nonzero vector $\tilde{\zeta}(t) = [\zeta^T(t) \;\; \omega^T(t)]^T$, (57)-(59) is equivalent to

$$\tilde{\zeta}^T(t)\left(\frac{d_1(t)}{d_k}\left(\bar{\Psi}^{(1)}_{[d_k]}\right) + \frac{d_2(t)}{d_k}\left(\bar{\Psi}^{(2)}_{[d_k]}\right)\right)\tilde{\zeta}(t) < 0, \tag{62}$$

where $\bar{\Psi}^{(1)}_{[d_k]} = \Psi^{(1)}_{[d_k]} - \tilde{e}_1^T \mathcal{J}^T \Phi_1 \mathcal{J} \tilde{e}_1 - 2\tilde{e}_1^T \mathcal{J}^T \Phi_2 \tilde{e}_{12} - \tilde{e}_{12}^T \Phi_3 \tilde{e}_{12}$, $\bar{\Psi}^{(2)}_{[d_k]} = \Psi^{(2)}_{[d_k]} - \tilde{e}_1^T \mathcal{J}^T \Phi_1 \mathcal{J} \tilde{e}_1 - 2\tilde{e}_1^T \mathcal{J}^T \Phi_2 \tilde{e}_{12} - \tilde{e}_{12}^T \Phi_3 \tilde{e}_{12}$.
Note that if we take $\zeta(t) \neq 0$ and $\omega(t) = 0$, then (62) now also holds and we have

$$\zeta^T(t)\left(\frac{d_1(t)}{d_k}\left(\hat{\Psi}^{(1)}_{[d_k]}\right) + \frac{d_2(t)}{d_k}\left(\hat{\Psi}^{(2)}_{[d_k]}\right)\right)\zeta(t) < 0, \tag{63}$$

where $\hat{\Psi}^{(1)}_{[d_k]} = \Psi^{(1)}_{[d_k]} - e_1^T \mathcal{J}^T \Phi_1 \mathcal{J} e_1$, $\hat{\Psi}^{(2)}_{[d_k]} = \Psi^{(2)}_{[d_k]} - e_1^T \mathcal{J}^T \Phi_1 \mathcal{J} e_1$.
Since $\Phi_1 \leq 0$ in Assumption 2, we get the below inequality

$$\zeta^T(t)\left(\frac{d_1(t)}{d_k}\left(\Psi^{(1)}_{[d_k]}\right) + \frac{d_2(t)}{d_k}\left(\Psi^{(2)}_{[d_k]}\right)\right)\zeta(t) < 0. \tag{64}$$

Applying Lemma 3 and noting $\zeta(t) \neq 0$, we gain the LMIs (18), (19) and (20) in Theorem 1. Then the system (6) is exponentially stable.
From (49), we obtain

$$\dot{V}(t) \leq \zeta^T(t)\left(\frac{d_1(t)}{d_k}\left(\Psi^{(1)}_{[d_k]}\right) + \frac{d_2(t)}{d_k}\left(\Psi^{(2)}_{[d_k]}\right)\right)\zeta(t), \tag{65}$$

and it is easy that

$$\begin{aligned} &\tilde{\zeta}^T(t)\left(\frac{d_1(t)}{d_k}\left(\bar{\Psi}^{(1)}_{[d_k]}\right) + \frac{d_2(t)}{d_k}\left(\bar{\Psi}^{(2)}_{[d_k]}\right)\right)\tilde{\zeta}(t) \\ &= \zeta^T(t)\left(\frac{d_1(t)}{d_k}\left(\Psi^{(1)}_{[d_k]}\right) + \frac{d_2(t)}{d_k}\left(\Psi^{(2)}_{[d_k]}\right)\right)\zeta(t) - J(t), \end{aligned} \tag{66}$$

where $J(t)$ is definded in Definition 2.
Then, we have

$$\begin{aligned} \dot{V}(t) &\leq \tilde{\zeta}^T(t)\left(\frac{d_1(t)}{d_k}\left(\bar{\Psi}^{(1)}_{[d_k]}\right) + \frac{d_2(t)}{d_k}\left(\bar{\Psi}^{(2)}_{[d_k]}\right)\right)\tilde{\zeta}(t) \\ &+ J(t) \leq J(t). \end{aligned} \tag{67}$$

Taking the integration both sides of inequality (67) from 0 to t $(t \geq 0)$, we get result in

$$\int_0^t J(s)ds \geq V(t) - V(0) \geq r^T(t)Pr(t). \tag{68}$$

Now, we only focus on two cases of $\Phi_4 = 0$ and $\Phi_4 \geq 0$, since the extended dissipative performance can unified the strictly $(\mathcal{Q}, \mathcal{R}, \mathcal{S})$-dissipativity, $\mathcal{H}_\infty$ and the passive performance when $\Phi_4 = 0$ or the $\mathcal{L}_2 - \mathcal{L}_\infty$ performance when $\Phi_4 > 0$.
Considering $\Phi_4 = 0$, from (68) we have

$$\int_0^{t_f} J(s)ds \geq 0. \tag{69}$$

At the same time, when $\Phi_4 > 0$, as noted in Assumption 2, this implies that the matrices $\Phi_1 = 0, \Phi_2 = 0$ and $\Phi_3 > 0$.

Then, for $t \in [0, t_f]$, (68) result in $\int_0^{t_f} J(s)ds \geq \int_0^t J(s)ds \geq r^T(t)Pr(t)$. Thus, from (60), we obtain

$$\hat{z}^T(t)\Phi_4\hat{z}(t) = r^T(t)\mathcal{J}^T\Phi_4\mathcal{J}r(t) \leq r^T(t)Pr(t) \leq \int_0^{t_f} J(s)ds \tag{70}$$

By combining (69) and (70), the system (6) is described as extended dissipative. The proof is now complete. □

Remark 6: The novelty of our method is that we take the external disturbances into each node which is not considered in [22], [23], [25], [27], [29], [37], and [38]. Moreover, we obtain newly exponential synchronization with extended dissipative containing, passive, $\mathcal{H}_\infty$, $\mathcal{L}_2-\mathcal{L}_\infty$ and dissipative performance. The conditions are more general than those in [22], [23], [25], [27], [29], [37], and [38]. Therefore, we can notice that their conditions cannot be simulated to our examples. Moreover, we construct a new time-dependent Lyapunov functional, which is different from the proposed in [22], [23], [24], [25], and [27], and the advantage information on discrete sample point t_k is fully used. Moreover, the positive definitiveness of the proposed Lyapunov functional is required only at sampling times that are not necessarily throughout the sampling periods. So, we derive a new inequality that applies to prove the exponential synchronization of CDNs.

Remark 7: Compared to the findings in [22], [23], [24], [25], [27], [29], [36], and [38], the result in this study is less conservative by constructing the appropriate Lyapunov functional and the technique for estimating the upper bound of its derivative. In contrast to the Lyapunov function in [40], [41], [42], and [43], we fully consider the critical information $t - t_k$ and $t_{k+1} - t$, $\forall t \in [t_k, t_{k+1})$. Additionally, compared to the convex combination technique and Jensen inequality, the inverse of the first-order approach, mixed convex combination and some effective integral inequalities can offer a more precise upper bound.

Remark 8: In this study, we address the sampled-data synchronization problem of CDNs with fixed coupling and time-varying delay in each dynamical system and obtained less conservative results. Because the sampled-data controller gain matrices must be obtained using system parameters, the proposed method is unsuitable for CDNs with time-varying coupling and multiple time delays. As a result, there is still much room for further investigation into obtaining the sampled-data synchronization criteria of CDNs with time-varying coupling. Some more effective methods, such as the proposed in [45] and [46], inspire us to conduct additional research.

Remark 9: Some free matrices are introduced in this paper using a time-dependent Lyapunov functional with complete information on the actual sampling instant t_k, a new inequality, and a convex combination approach. As a result, the construction and computation technique of the Lyapunov functional are the primary keys to improving the outcomes of this work. All of this leads to a reduction in our results's conservatism compared to recent works and, in particular, numerical examples.

IV. NUMERICAL EXAMPLES

This part focuses on using two numerical examples to manifest the effectiveness of the suggested method in the above theorems.

Example 1: Chua's well-known circuit (Fig. 1 [49]) consists of a linear inductor L_1, a linear resistor R, two linear capacitors C_1, C_2, and a nonlinear resistor N_R named Chua's diode. υ_1 and υ_2 are the voltages across the capacitors C_1 and C_2, respectively, i_1 is the current through the inductor L_1. Moreover, Chua's circuit is chosen as the isolated node of the system (6), which is given by the following dynamical system

$$\dot{\nu}_1 = \rho_1(-\nu_1 + \nu_2 - \sigma(\nu_1)),$$
$$\dot{\nu}_2 = \nu_1 - \nu_2 + \nu_3,$$
$$\dot{\nu}_3 = -\rho_2\nu_2,$$

where $\rho_1 = 10, \rho_2 = 14.87$, *and* $\sigma(\nu_1) = -0.68\nu_1 + 0.5(-1.27 + 0.68)(|\nu_1 + 1| - |\nu_1 - 1|)$.

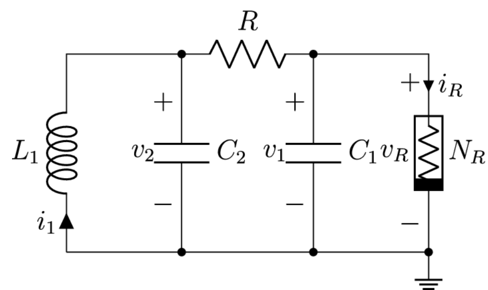

FIGURE 1. Chua's circuit as an isolated node in example 1.

From condition (7), it can be satisfied as follows

$$U = \begin{bmatrix} 2.7 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -14.87 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} -3.2 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -14.87 & 0 \end{bmatrix}.$$

The inner and outer-coupling matrices are specified as

$$\Gamma^{(1)} = 0, \ \Gamma^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathcal{G} = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}.$$

Then, we choose $c = 0.9$ *and the time-varying coupling delay is considered as* $\tau(t) = 0.03 + 0.01\sin(t)$, *which implies that* $\tau_u = 0.04$ *and* $\tau_d = 0.01$. *As shown in Table 1, we can see that the largest sampling period by theorem 1 is* 0.1982, *which is larger than the ones proposed in [23], [24], [27], and [38]. It means that our result is less conservative than the existing ones in [23], [24], [27], and [38].*

To depict the effectiveness of our approach, we choose $h_u = 0.1982$, *using Matlab software to calculate the LMIs in Theorem 1 the desired controller gains matrices can be presented as follows*

$$K_1 = \begin{bmatrix} -1.9436 & -1.5180 & 0.6179 \\ -0.7088 & -4.0341 & 0.2254 \\ -0.5361 & 1.7484 & -4.3617 \end{bmatrix},$$

TABLE 1. The largest upper bound of sampling period h_u.

Methods	h_u	improved by
[23]	0.0711	178.76%
[38]	0.1120	76.96%
[24]	0.1327	49.36%
[27]	0.1536	29.04%
Theorem 1	0.1982	

$$K_2 = \begin{bmatrix} -6.6941 & 0.5959 & 2.1041 \\ 0.3328 & -4.4092 & 0.2676 \\ 5.4029 & 0.0934 & -5.2084 \end{bmatrix}$$
$$K_3 = \begin{bmatrix} -4.8812 & -1.4433 & 0.3209 \\ 0.1603 & -2.5156 & 2.0958 \\ 1.7657 & 4.1299 & -1.1285 \end{bmatrix}.$$

Setting the initial condition as $\nu(0) = [1 \ \ -1 \ \ -2]^T$, $x_1(0) = [1 \ -3 \ 1]^T, x_2(0) = [2 \ -2 \ 1]^T, x_3(0) = [-5 \ 1 \ -1]^T$ *and using the above-designed controller gain matrices, Fig. 2 shows the chaotic behavior of Chua's circuit. Moreover, Fig. 3 depicts the error state of the system (6) without control. The controlled error CDNs and the state trajectories of the controller are demonstrated in Figs. 4 and 5. As can be seen, the developed sampled-data controller matrices can successfully synchronize the error system (6).*

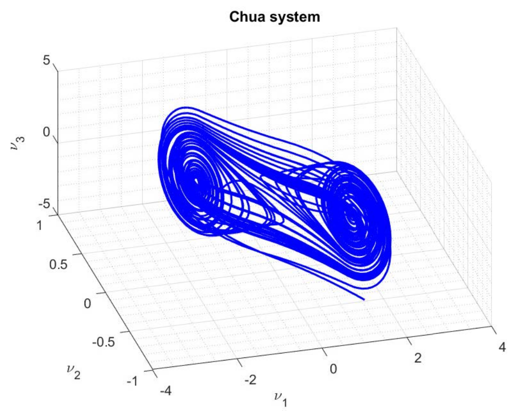

FIGURE 2. The state trajectories of the isolated node in example 1.

Remark 10: In recent years, there has been a lot of interest in chaos control and chaos synchronization of dynamic systems. A chaotic system has complex dynamical behaviors with unique characteristics, such as being extremely sensitive to small changes in initial conditions and having bounded trajectories in phase space. Nonlinear systems such as Chua's, Lure's, and Chen's system have been studied to control chaos. Furthermore, applications of Chua's circuit are remarkable as a standard for various strange attractors in analyses of chaos control, image encryption, signals, and neural networks [47], [48], [49], [50]. So, Chua's circuit is employed as the unforced isolated nodes of (6) to show the effectiveness and practical example.

Example 2: Consider three-node CDNs with $\omega(t) = 0$. *The internal-coupling matrices and outer-coupling matrices*

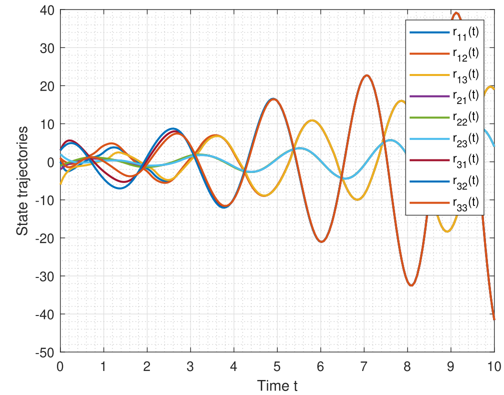

FIGURE 3. The state trajectories of the uncontrol CDNs.

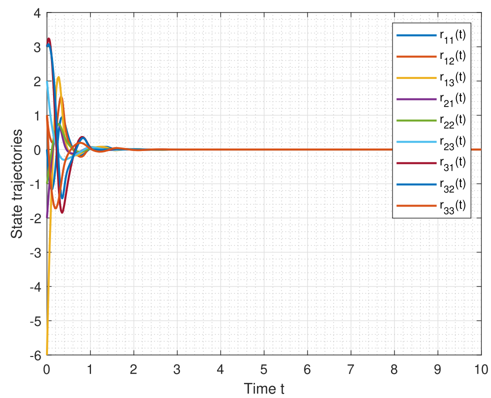

FIGURE 4. The state trajectories $r_i(t)$ of CDNs with the control (4).

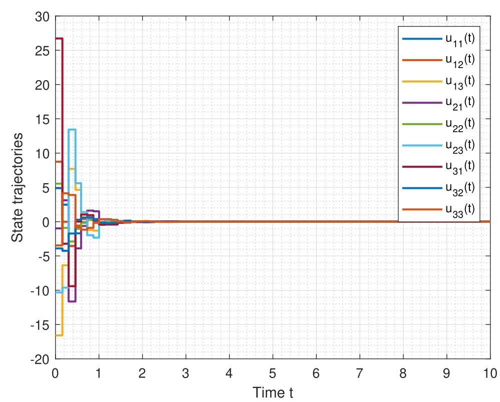

FIGURE 5. The state trajectories of the control $u_i(t)$.

are defined as follows:

$$\Gamma^{(1)} = 0, \quad \Gamma^{(2)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathcal{G} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}.$$

The nonlinear function f *is described as*

$$f(x_i(t)) = \begin{bmatrix} -0.5x_{i1} + \tanh(0.2x_{i1}) + 0.2x_{i2} \\ 0.95x_{i2} - \tanh(0.75x_{i2}) \end{bmatrix}.$$

It can be calculated that the following matrices are satisfied in condition (7)

$$U = \begin{bmatrix} -0.5 & 0.2 \\ 0 & 0.95 \end{bmatrix}, \quad V = \begin{bmatrix} -0.3 & 0.2 \\ 0 & 0.2 \end{bmatrix}.$$

We select $c = 0.5, \tau_u = 0.25$ *and* $\tau_d = 0.5$ *and the list of the maximum allowable value of sampling period* h_u *contains in Table* 2. *From Table 2, we show that the largest sampling periods using the approach described in [22], [23], [24], [25], [27], [29], [36], and [38] are* 0.5409, 0.5573, 0.8767, 0.9016, 0.9225, 1.1564, 1.3756, 1.3978 *and* 1.4222, *respectively. However, the largest sampling period* h_u *by Theorem 1 is* 1.4222, *which is greater than the references therein [22], [23], [24], [25], [27], [29], [36], and [38]. This concludes that our result is less conservative than those found in [22], [23], [24], [25], [27], [29], [36], and [38].*

TABLE 2. The largest upper bound of sampling period h_u.

Methods	h_u	improved by
[22]	0.5409	162.93%
[23]	0.5573	155.19%
[24]	0.8767	62.22%
[29]	0.9016	57.72%
[38]	0.9225	54.17%
[36]	1.1564	22.99%
[25]	1.3756	3.39%
[27]	1.3978	1.75%
Theorem 1	1.4222	

The effectiveness of our method is demonstrated via the following simulation. By choosing $\tau(t) = 0.125 + 0.125\sin(4t)$, *and* $h_u = 1.4222$, *and using Matlab software to solve the LMIs in Theorem 1, the desired controller gains matrices can be presented as follows*

$$K_1 = \begin{bmatrix} -0.8781 & -0.0896 \\ -0.0816 & -1.5110 \end{bmatrix}, K_2 = \begin{bmatrix} -1.0636 & -0.0625 \\ -0.0514 & -1.5556 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -0.6280 & 0.0014 \\ 0.0104 & -0.8830 \end{bmatrix}.$$

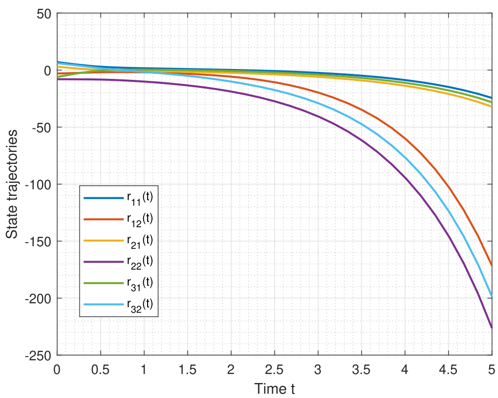

FIGURE 6. The state trajectories of the uncontrol CDNs.

Here, the initial condition is $v(0) = [2 \;\; -1]^T$, $x_1(0) = [9 \;\; -4]^T$, $x_2(0) = [5 \;\; -9]^T$, $x_3(0) = [-4 \;\; 5]^T$. *Then, using the above-designed controller gain matrices, Fig. 6 shows the state trajectories of the uncontrol error system. The controlled error CDNs and the state vectors of the controller are illustrated in Figs. 7 and 8.*

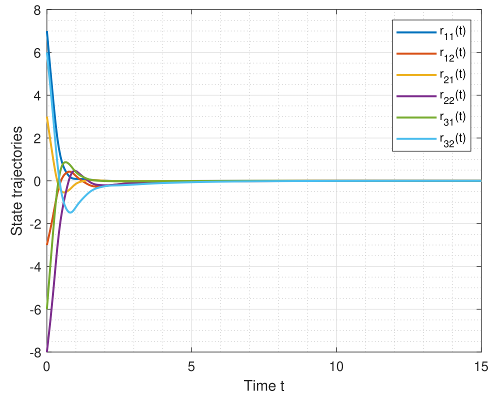

FIGURE 7. The state trajectories $r_i(t)$ of CDNs with the control (4).

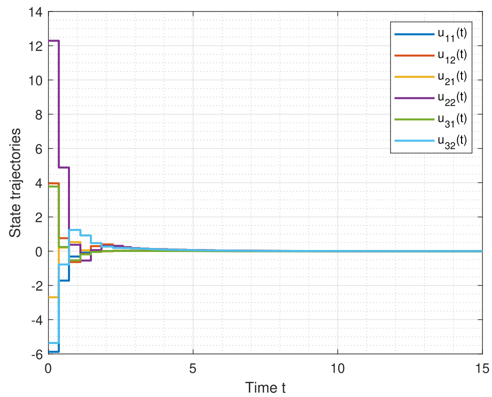

FIGURE 8. The state trajectories of the control $u_i(t)$.

To consider extended dissipative performance, we define the variables $c = 1, \tau_u = 0.25, \tau_d = 0.5, \mathcal{J} = I_3 \otimes I_2$ *and* $\omega(t) = \frac{1}{(1+t^2)}$. *Then, the internal-coupling matrices and the outer-coupling matrices are used by the following matrices*:

$$\Gamma^{(1)} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \; \Gamma^{(2)} = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix},$$
$$\mathcal{G} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}.$$

By performance scalar $\delta = 0.5$, *and in the* $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-*dissipativity property, we choose* $\Phi_1 = -I, \Phi_2 = I, \Phi_3 = 2I$, *and* $\Phi_4 = 0$. *From Table 3, we show that the largest sampling periods using the approach described in [36] are* 0.7548, 0.4375, 0.2219, *and* 0.2962, *which rely on* $\mathcal{L}_2 - \mathcal{L}_\infty$, $\mathcal{H}_\infty$, *Passivity,*$(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-*dissipativity respectively. Whereas the maximum sampling period* h_u *by Theorem 2 is* 0.91372, 1.0272, 1.001, 0.9561 *which is greater than the reference therein [36]. It concludes that our result is less conservative than the reference in [36].*

TABLE 3. The largest upper bound of sampling period h_u.

Performance	$\mathcal{L}_2-\mathcal{L}_\infty$	$\mathcal{H}_\infty$	Passivity	dissipativity
[36]	0.7548	0.4375	0.2219	0.2962
Theorem 2	0.9137	1.0272	1.001	0.9561
improved by	21.05%	134.79%	351.10%	222.79%

On the other hand, the controller gains matrices with $h_u = 1.0272$ and $\delta = 0.5$ in $\mathcal{H}_\infty$ performance can be calculated as follows:

$$K_1 = \begin{bmatrix} -0.7358 & -0.0902 \\ -0.0692 & -1.4936 \end{bmatrix}, K_2 = \begin{bmatrix} -1.0332 & -0.0599 \\ -0.0448 & -1.5110 \end{bmatrix}, K_3 = \begin{bmatrix} -0.6215 & -0.0068 \\ 0.0031 & -0.8862 \end{bmatrix}.$$

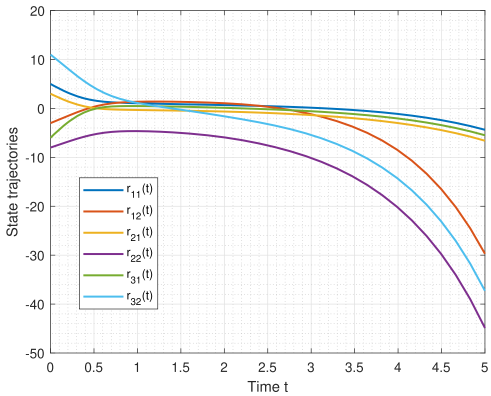

FIGURE 9. The state trajectories of the output $\hat{z}_i(t)$ without control.

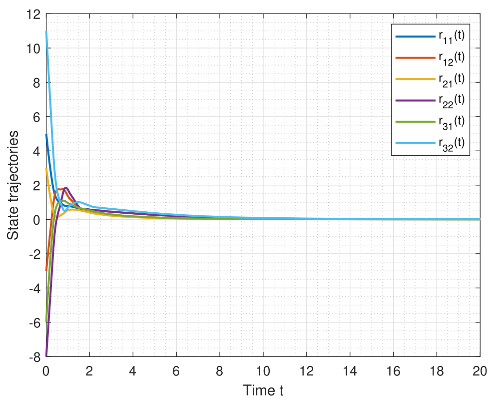

FIGURE 10. The state trajectories of the output $\hat{z}_i(t)$ with the control (4).

By using the above parameters, Fig. 10 *shows the state trajectories of the error system (6), which converges to zero under an exterior disturbance and the state trajectories control $u_i(t)$ are depicted in Fig.* 11*, where $x_1(0) = [7 \ \ -4]^T, x_2(0) = [5 \ \ -9]^T, x_3 = [-4 \ \ 10]^T, \nu(0) = [2 \ \ -1]^T$. Notably, the CDNs cannot successfully synchronize when the controller is not applied, which can be seen in Fig* 9.

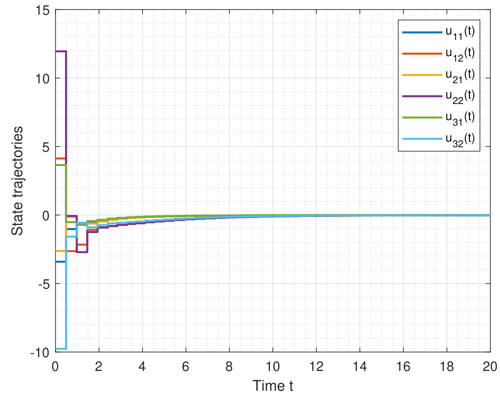

FIGURE 11. The state trajectories of the control $u_i(t)$.

V. CONCLUSION

This study uses a sampled-data controller to handle the problem of extended dissipative exponential synchronization of CDNs with time-varying coupling delays. An aperiodic sampled-data control scheme has been devised to solve this challenge, with the sampling period specified as time-varying but bounded. Then, using an enhanced Lyapunov function, integral inequalities, the free-weighting matrix technique, and the convex combination approach, a new adequate condition for strict LMIs was discovered. These conditions can also be used to investigate the extended dissipativity analysis issue, which includes the passivity, $\mathcal{L}_2-\mathcal{L}_\infty$, $\mathcal{H}_\infty$, and dissipativity performance in a unified formulation. Finally, two numerical examples indicate that our finding is better than the previous references, highlighting how this paper has developed. It is worthwhile to mention that the method in this article can be investigated further and applied to more complicated systems such as neutral-type CDNs [51], stochastic CDNs [52], T–S fuzzy CDNs [53].

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