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RESEARCH ARTICLE

Adaptation of Population Size in Differential Evolution and Its Effects on Localization of Target Nodes

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ABSTRACT The differential evolution (DE) is a well known population-based evolutionary algorithm that has shown capabilities for solving real-world problems such as resource allocation, multicast routing, and localization of target nodes. However, the accuracy of the DE, like other evolutionary algorithms, depends on the settings of its control parameters. The localization of target nodes is highly nonlinear and multi-modal, which may trap the DE in a local optimum. A local optimum may be avoided by a proper selection of the control parameters. One of the key control parameters is the population size (PS), which affects directly the localization accuracy and computational complexity. Finding an adequate PS throughout the evolution process is a challenging task. Even if an adequate PS is found it may not be the adequate PS anymore when the scenario of a problem changes. Although several approaches have been proposed for adapting the PS, they have not been evaluated when solving the localization problem. In this paper, a comprehensive comparison in terms of accuracy and computational demand is conducted among the state-of-the-art PS adaptation techniques when employed with the DE for solving the localization problem of target nodes in various scenarios. We also propose three new PS adaptation techniques, namely, exponential, parabolic, and logistic reduction. The results from extensive numerical simulations show that, after setting the initial PS properly, there is no technique that outperforms the others in practically all the scenario of the localization problem. Additionally, the DE with the proposed techniques provides competitive localization accuracy with considerably less computational complexity. Specifically, The proposed approaches reduce the computational demand by approximately 50% over the standard DE in all the scenarios considered here.

INDEX TERMS Differential evolution, evolutionary algorithms, localization, population size control, wireless sensor networks.

I. INTRODUCTION

Target nodes (TNs) localization in wireless sensor networks (WSNs) is of pivotal importance in several real-world applications such as healthcare monitoring, surveillance, and robotics. To estimate the position of TNs, several approaches based on linear least squares, semidefinite programming, and unscented transformation are commonly employed [1]. Although such conventional approaches improve the localization accuracy over their predecessors, they usually approximate the maximum likelihood (ML) cost function, resulting in suboptimal solutions. Interestingly, it was shown recently that approaches based on evolutionary algorithm provided considerably improvement over conventional approaches [2], [3]. This is because evolution based approaches do not require any approximation of the localization problem, and thus

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results in a better localization accuracy. Although evolutionary algorithms have been applied for solving several problems, its application to the localization problem has been barely explored. There are important questions that need to be answered: for instance, do the existing population size (PS) control techniques enhance (or reduce) the quality of the solution as in other problems [4], [5]? This paper tries to answer this question.

Over the past years, several evolutionary algorithms have been developed such as the differential evolution (DE) [6], genetic algorithm [7], particle swarm optimization [8], and ant colony optimization [9]. Among them, the DE is known to provide excellent solutions to several complex problems with reasonable robustness and computational demand [10], [11]. Despite the robustness and relative simplicity of the DE, however, it has also been shown that the accuracy of the DE and its variants are highly dependent on their control parameters [12].

A standard DE has three main control parameters: PS, mutation scaling factor, and crossover probability. To obtain reasonable solutions when employing the DE, it is usually necessary to tune the control parameters. Since this is a significant task in practice, several adaptive techniques have been developed [13].

Among the three control parameters, the PS on the DE plays a key role in balancing the exploration and exploitation of the algorithm, which has an impact on the convergence rate and solution quality [14]. A large PS in the DE can encourage wider exploration of the solution space which may help to avoid stagnation, but at the cost of delaying the convergence to the global optimal solution. On the other hand, a small PS usually results in a faster convergence with a higher risk of convergence to a local optimal solution. In essence, finding an optimal PS is of great importance in the DE and any evolutionary algorithm.

A. RELATED WORK

When dealing with the PS, three categories of techniques can be devised: fixed population, reduction-based, and self-adaptive techniques. Fixed population (FiP) techniques maintain the same PS throughout the evolution process, and reduction-based and self-adaptive techniques adapt the PS based on certain rules. For instance, the iterative halving (ItH) technique reduces the PS in half at predetermined intervals. Each of the individuals in the first half is compared with the second half by one-to-one correspondence and, then, the best individuals are chosen for the next interval evolution process. In [15], it was shown that the ItH technique helps to improve the solution quality of the standard DE in most of the cases considered therein although only for high dimensional functions. The linear reduction (LiR) technique reduces the PS linearly as a function of the number of fitness evaluations or equivalently as a function of the number of iterations. The LiR was introduced in [4] to improve the convergence rate of an earlier algorithm called SHADE [16]. Therein, it was shown that SHADE with the LiR technique outperforms, and in some cases was outperformed, by the SHADE and other evolutionary techniques [4]. Unlike the LiR technique, the inverse parabolic (InP) technique reduces the PS following an InP function defined based on the number of fitness evaluations. A variant of the DE called PaDE, which uses the InP technique, was compared with the SHADE with the LiR technique for problems of dimension 30 [5]. Therein, it was shown that the PaDE with the InP technique performs better and worse than the SHADE with LiR depending of the problem at hand. The linear staircase (LiS) technique reduces the PS at a predefined rate and interval [17]. In the LiS technique, as in the other techniques, only the best individuals are selected to move to the next evolution process.

There also exist techniques that increase and decrease the PS based on information related to the population and/or predefined rules. For instance, the population diversity (PD) technique measures the diversity of individuals in the population for increasing or decreasing the PS [18]. Comparison between the DE and variants of the DE with and without the PD technique has shown that it is beneficial to use the PD technique, in most cases, especially for high-dimensional problems [18]. In contrast, the pulse wave (PuW) technique increases or decreases the PS at pre-defined intervals by taking into account variations in the fitness values [17]. Interestingly, it is shown that the PuW technique is more effective than the PD technique [17]. More recently, the entropy control (EnC) technique was introduced for dynamically adapting the PS [19]. The EnC technique measures the entropy in the population of the current and previous generations, which are used for creating a ratio that is later employed in a control parameter for increasing or decreasing the PS. The EnC technique was shown to outperform the PD technique when employed together with the SHADE algorithm [19].

In some works, the PS adaptation techniques were compared. For instance, the FiP, ItH, LiR, and PD together with the DE and its variants were compared in [20] for solving optimization problems from the CEC2014 [21]. The comparison was only in terms of solution quality and it was shown that in most of the cases the PD technique was more effective than the ItH and LiR. A more comprehensive comparison between PS techniques was performed in [17], but with another evolutionary algorithm, i.e., the sine cosine algorithm [22]. Therein, the FiP, LiS, LiR, ItH, PuW, and PD were employed with the sine cosine algorithm for solving unimodal, multimodal, and composite functions similar to those in the CEC2014. It was shown that the PuW performed better than the FiP, LiS, LiR, ItH, and PD techniques at the cost of higher computational demand in most cases.

B. MOTIVATIONS AND KEY CONTRIBUTIONS

The works mentioned above demonstrate, in general, that PS control techniques improve the performance of evolutionary algorithms. However, it is not clear which of the PS control techniques is the best since it was reported that an increase in the performance of the evolutionary algorithm due to the PS techniques can vary significantly depending on the problem at hand. Furthermore, use of a certain PS technique may be counterproductive in some cases. Therefore, a comprehensive comparison of the PS control techniques for a particular problem is of great importance.

In this paper, we aim at exploring the performance of the PS adaptation techniques when employed together with the DE for solving the localization problem of the TN based on received signal strength (RSS) measurements in WSNs. The DE with several PS adaptation techniques will be compared in terms of localization accuracy and computational complexity. Although there exist some works where PS adaptation techniques are compared, the comparisons are limited to a few PS adaptation techniques [20] or with a different evolutionary algorithm [17]: these works in addition do not deal with the TN localization problem, but deal with other problems such as those presented in the CEC2014 [21]. Furthermore, they made the comparison only for high dimensional problems, *i.e.*, dimensions not smaller than 10.

In addition to conducting a comparison among popular PS adaptation techniques, we also propose three new PS adaptation techniques: exponential reduction (ExR), logistic reduction (LoR), and parabolic reduction (PaR). The reason for suggesting new PS adaptation techniques is twofold. First, according to the results in [4] and [5] on LiR and InP techniques, the accuracy of DE can be improved by reducing the PS in some scenarios, implying the possibility of more efficient PS reduction techniques. Second, we would like to test if a drastic or smooth PS reduction affects the accuracy of the DE for solving the localization problem.

The main contributions of this article can be summarized as follows:

- To the best of our knowledge, this article is the first study in which the state-of-the-art PS adaptation techniques are tested and compared when solving the localization problem.
- The comparison is conducted exhaustively in several scenarios of the localization problem in terms of localization accuracy and computational demand.
- We propose, and include in the comparison, three new PS adaptation techniques that reduce the computational burden without compromising the localization accuracy.
- Unlike most of the existing PS adaptation techniques, the proposed techniques in this work require only one or no control parameter for reducing the PS. This is important since an evolutionary algorithm with less control parameters is commonly preferred.

The remainder of this paper is organized as follows. In Section II, the TN localization problem based on RSS in WSNs is presented. In Section III, the DE with PS adaptation is introduced. Section IV presents PS adaptation techniques for the DE, where three new PS adaptation techniques are introduced in addition to conventional techniques. Section V is devoted to the comparison of DE with the proposed and other popular PS adaptation techniques when solving the localization problem presented in Section II. The comparisons are based on the localization accuracy and computational complexity in a variety of scenarios. Discussion on the comparison results is also presented. Finally, in Section VI, concluding remarks are included.

II. LOCALIZATION PROBLEM OF TARGET NODE

For simplicity and without loss of generality, two-dimensional (2-D) WSNs are assumed. The extension to three-dimensional (3-D) WSN should be straightforward. Consider N anchor nodes (ANs) with known positions $\boldsymbol{x}_n = [x_n, y_n]^T$ for $n = 1, 2, \cdots, N$, and one TN with unknown position $\boldsymbol{x}_0 = [x_0, y_0]^T$.

Under the path-loss model in [23], the received power P_n at the n-th AN from the TN can be expressed as

$$P_n = P_0 - 10\gamma \log_{10} \|\boldsymbol{x}_n - \boldsymbol{x}_0\| + v_n \quad \text{(dB)} \tag{1}$$

for $n = 1, 2, \cdots, N$, where $\|\boldsymbol{\phi}\| = \sqrt{\phi_1^2 + \phi_2^2}$ denotes the Euclidean norm of a vector $\boldsymbol{\phi} = [\phi_1, \phi_2]$ in the two-dimensional space $\mathbb{R}^2$. Here, γ denotes the path loss exponent and P_0 denotes the power of the signal transmitted from the TN. The additive noise term v_n represents the log-shadowing effect and follows a Gaussian distribution with mean zero and standard deviation σ_n.

The localization problem consists in finding the unknown location $\boldsymbol{x}_0$ of the TN by

$$\hat{\boldsymbol{x}}_0 = \arg\min_{\boldsymbol{x}} f(\boldsymbol{x}) \tag{2}$$

using the RSS measurements $\{P_n\}_{n=1}^N$, where

$$f(\boldsymbol{x}) = \sum_{n=1}^{N} \frac{1}{\sigma_n^2} \left(P_n - P_0 + 10\gamma \log_{10} \|\boldsymbol{x}_n - \boldsymbol{x}\|\right)^2 \tag{3}$$

is the ML cost function with $\boldsymbol{x} \in \mathbb{R}^2$.

To solve (2), several approaches have been proposed and refined including those based on the linear least squares [24], convex optimization [25] and unscented transformation [26]. Recently, approaches based on evolutionary algorithms are gaining attention because they do not approximate (3) and tend to provide competitive localization accuracy [2], [3], [11].

III. DIFFERENTIAL EVOLUTION WITH POPULATION SIZE ADAPTATION

The DE with PS adaptation is a population based algorithm, a stochastic search optimization tool. The population in the DE with PS adaptation evolves through G generations. At each generation, the population of the DE with PS adaptation passes through four main processes: mutation, crossover, selection, and PS adaptation.

For $g = 0, 1, \cdots, G-1$, the population $\left\{\boldsymbol{I}_l^{(g)}\right\}_{l=1}^{L^{(g)}}$ is a set of $L^{(g)}$ individuals at the g-th generation, where each of the two entries of $\boldsymbol{I}_l^{(g)} = \left[I_{l,1}^{(g)}, I_{l,2}^{(g)}\right]$ is called a gene. In the case of dealing with a three-dimensional localization problem, the individual $\boldsymbol{I}_l^{(g)}$ is a three-dimensional vector. The number $L^{(g)}$ of individuals can be bounded as

$$\underrightarrow{L} \leq L^{(g)} \leq \overleftarrow{L}. \tag{4}$$

In (4), $\underrightarrow{L}$ and $\overleftarrow{L}$ denote the lower and upper bounds of $L^{(g)}$, respectively: $\underrightarrow{L}$ can be obtained by analyzing the minimum number of individuals required in the evolution process. On the other hand, $\overleftarrow{L}$ can be set up based on the suggestion, for example, that a PS higher than $10D$, where D represents the dimension of the problem, does not have a considerable effect on enhancing the quality of the solution of a problem [6].

The DE with PS adaptation creates the initial population $\left\{\boldsymbol{I}_l^{(0)}\right\}_{l=1}^{L^{(0)}}$ randomly in the solution space. Here, the solution space, also called the area of interest, is defined as

$$\boldsymbol{Z} = \{(z_1, z_2) : a_1 \leq z_1 \leq b_1, a_2 \leq z_2 \leq b_2\}, \tag{5}$$

where a_d and b_d denote the lower and upper bounds, respectively, on z_d for $d = 1, 2$. After the initial population is generated, the population evolves through the four processes of the DE with PS adaptation:

A. MUTATION

A mutant population $\left\{\tilde{\boldsymbol{I}}_l^{(g)} = \left[\tilde{I}_{l,1}^{(g)}, \tilde{I}_{l,2}^{(g)}\right]\right\}_{l=1}^{L^{(g)}}$ is generated as

$$\tilde{\boldsymbol{I}}_l^{(g)} = \boldsymbol{I}_k^{(g)} + \alpha\left(\boldsymbol{I}_p^{(g)} - \boldsymbol{I}_q^{(g)}\right) \tag{6}$$

from the original population $\left\{\boldsymbol{I}_l^{(g)}\right\}_{l=1}^{L^{(g)}}$, where $k, p, q \in \left\{1, 2, \cdots, L^{(g)}\right\}$ such that $k \neq p$, $k \neq q$, and $p \neq q$. Here, α denotes the scaling factor which regulates the diversity in the population.

B. CROSSOVER

A trial population $\left\{\check{\boldsymbol{I}}_l^{(g)} = \left[\check{I}_{l,1}^{(g)}, \check{I}_{l,2}^{(g)}\right]\right\}_{l=1}^{L^{(g)}}$ is generated by combining the elements of the individuals in the mutated and original populations with a crossover probability $p_C \in [0, 1]$ as

$$\check{I}_{l,d}^{(g)} = \begin{cases} \tilde{I}_{l,d}^{(g)}, & \text{if } c_l \leq p_C \text{ or } d = d_R, \\ I_{l,d}^{(g)}, & \text{otherwise}, \end{cases} \tag{7}$$

where $c_l \sim U(0, 1)$ and $d_R \in \{1, 2\}$ is a randomly selected index.

C. SELECTION

The selection process compares individuals in the trial and original populations. An individual with the lowest fitness value is selected as

$$\boldsymbol{I}_l^{(g+1)} = \underset{\boldsymbol{I}_l^{(g)}, \check{\boldsymbol{I}}_l^{(g)}}{\arg\min} \left\{f\left(\boldsymbol{I}_l^{(g)}\right), f\left(\check{\boldsymbol{I}}_l^{(g)}\right)\right\}. \tag{8}$$

Then, the selected individual is employed as the initial individual in the next generation.

Algorithm 1: Pseudo Code of the DE With PS Adaptation

Input: ML cost function (3), RSS measurements $\{P_n\}_{n=1}^N$, number N of ANs, initial population size $L^{(0)}$, scaling factor α, crossover probability p_C, maximum number G of generations, lower bounds a_d, and upper bounds b_d for $d = 1, 2$
Output: Estimated value of the position of TN
Initialize the population randomly;
while $g \leq G - 1$ **do**
  for $l = 1; l \leq L; l = l + 1$ **do**
    Create mutant individuals with (6);
    Generate trial individuals via (7);
    Select the best individual via (8);
  end
  Adapt the PS as in (9) with (11), (13), (14), (17), (18), (22), (23), (28), (32), (30), or (31);
  When required:
  → Increase the PS by generating random individuals;
  → Reduce the PS by eliminating the least favorable individuals;
  Increase $g = g + 1$;
end
Return the estimated value of the position of TN;

D. ADAPTATION OF PS

The PS adaptation is performed after the three main processes of the DE have been concluded. The adaptation of the PS can be achieved at any stage and based on available information, *e.g.*, Euclidean distance among the individuals or entropy of the population.

This process adapts the PS and assures that the PS does not violate the lower and upper bounds of the PS as

$$L^{(g+1)} = \begin{cases} \overleftarrow{L}, & \text{if } L^{(g)} > \overleftarrow{L} \\ \underrightarrow{L}, & \text{if } L^{(g)} < \underrightarrow{L} \\ \text{'Adaptation technique'}, & \text{Otherwise.} \end{cases} \tag{9}$$

A formal description of the 'Adaptation technique' will be provided in detail in Section IV.

Finally, after G generations, the best individual

$$\boldsymbol{I} = \underset{\boldsymbol{I}_l^{(G)}}{\arg\min} \left\{f\left(\boldsymbol{I}_l^{(G)}\right)\right\}_{l=1}^{L^{(G)}} \tag{10}$$

among $\left\{\boldsymbol{I}_l^{(G)}\right\}_{l=1}^{L^{(G)}}$ is selected as the estimate $\hat{\boldsymbol{x}}_0$ of the unknown location $\boldsymbol{x}_0$ of the TN. Algorithm 1 summarizes the pseudo-code of the DE with several PS adaptation techniques.

IV. ADAPTATION TECHNIQUES FOR POPULATION SIZE

Several techniques for adapting the PS in various evolutionary algorithms have been reported in [4], [5] [13], [15], [17], [18], and [19]. The techniques can be divided into three categories: fixed population, reduction-based, and self-adaptive techniques.

A. FIXED POPULATION TECHNIQUES

The FiP techniques keep the same PS over all the generations: that is, for $g = 0, 1, \cdots, G-1$,

$$L^{(g+1)} = L^{(g)}. \tag{11}$$

There exists a general guideline that in most cases provides good results [6], *i.e.*, $5D \leq L^{(g)} \leq 10D$. High values of fixed PS usually provide a better accuracy of the DE, especially when the dimension of the problem is high, *e.g.*, $D > 10$. Interestingly, however, for the localization problem where $D \leq 3$ it will be shown that high values of PS do not provide considerable improvement in the localization accuracy. Instead, setting PS to high values, *i.e.*, near $10D$, only incurs increased computational complexity.

B. REDUCTION-BASED TECHNIQUES

The reduction-based techniques reduce the PS monotonically to the minimum PS $\underrightarrow{L}$ in successive generations. When reducing the PS, the least favorable individuals are eliminated.

1) ITERATIVE HALVING

The ItH technique reduces the PS in half at intervals defined by [15]

$$\delta_{\mathrm{ItH}} = \left\lfloor \frac{G}{N_p} \right\rfloor, \tag{12}$$

where $\lfloor * \rfloor$ denotes the floor function rounding $*$ to the greatest integer less than or equal to $*$ and N_p denotes the number of interval that needs to be set based on the user expertise [15]. Once δ_{ItH} is obtained, $L^{(g)}$ is updated as

$$L^{(g+1)} = \begin{cases} \left\lfloor \frac{L^{(g)}}{2} \right\rfloor, & \text{if } \mathrm{mod}(g, \delta_{\mathrm{ItH}}) = 0, \\ L^{(g)}, & \text{otherwise,} \end{cases} \tag{13}$$

where $\mathrm{mod}(c_1, c_2)$ is the modular operator. The ItH technique adjusts the PS at an interval δ_{ItH} of generations by taking only the best half individuals in the current generation [15].

2) LINEAR REDUCTION

The LiR technique decreases the PS at each generation. It was proposed to enhance the accuracy of a variant of the DE algorithm [4]. The PS in the LiR technique is reduced as

$$L^{(g+1)} = \left[\frac{\underrightarrow{L} - L^{(0)}}{G} \cdot g + L^{(0)} \right], \tag{14}$$

where $[*]$ rounds $*$ to the closest integer.

3) LINEAR STAIRCASE

In the LiS technique a reduction factor Δ is employed for establishing the number N_r of reductions as [17]

$$N_r = \left\lfloor \frac{L^{(0)} - \underrightarrow{L}}{\Delta} \right\rfloor. \tag{15}$$

The reduction of the PS in the LiS technique is then executed at an interval of δ_{LiS} defined by

$$\delta_{\mathrm{LiS}} = \left\lfloor \frac{G}{N_r + 1} \right\rfloor. \tag{16}$$

The PS is updated as

$$L^{(g+1)} = \begin{cases} L^{(g)} - \Delta, & \text{if } \mathrm{mod}(g, \delta_{\mathrm{LiS}}) = 0, \\ L^{(g)}, & \text{otherwise.} \end{cases} \tag{17}$$

4) INVERSE PARABOLIC

The InP technique reduces the PS as [5]

$$L^{(g+1)} = \left[\frac{\underrightarrow{L} - L^{(0)}}{\left(G - L^{(0)}\right)^2} \left(g - L^{(0)}\right)^2 + L^{(0)} \right]. \tag{18}$$

C. SELF-ADAPTIVE TECHNIQUES

Self-adaptive techniques increase or decrease the PS according to some information extracted from the population. When an increase in the PS is required, additional individuals are generated randomly within the solution space $\mathbf{Z}$. When a reduction is needed, the least favorable individuals are eliminated.

1) POPULATION DIVERSITY

The PD technique employs the parameter

$$R^{(g)} = \frac{\rho^{(g)}}{\rho^{(0)}}, \tag{19}$$

called the relative diversity. In (19), $\rho^{(g)}$ denotes the population diversity at the g-th generation calculated as

$$\rho^{(g)} = \sqrt{\frac{1}{L^{(g)}} \sum_{l=1}^{L^{(g)}} \sum_{j=1}^{2} \left(I_{l,j}^{(g)} - \bar{I}_j^{(g)} \right)^2} \tag{20}$$

with

$$\bar{I}_j^{(g)} = \frac{1}{L^{(g)}} \sum_{l=1}^{L^{(g)}} I_{l,j} \tag{21}$$

being the arithmetic mean of the j-th gene over all the individuals at the g-th generation. Taking into account the relative diversity $R^{(g)}$, the PD technique updates the PS as

$$L^{(g+1)} = \begin{cases} L^{(g)} + 1, & \text{if } R^{(g)} < 0.9\, R^{(g-1)}, \\ L^{(g)} - 1, & \text{if } R^{(g)} > 1.1\, R^{(g-1)}, \\ L^{(g)}, & \text{otherwise.} \end{cases} \tag{22}$$

2) PULSE WAVE

The PuW technique makes the changes in the PS by taking into account variations in the best fitness values at the g-th and $(g-1)$-st generations. The PuW technique employs a control parameter δ_{PuW} which is used as a factor for increasing or decreasing the PS and as an interval of adaptation. The PS adaptation with the PuW technique is conducted as

$$L^{(g+1)} = \begin{cases} L^{(g)} + \left[\dfrac{\delta_{\text{PuW}} \left(\overset{\leftarrow}{L} - L^{(g)} \right)}{G} \right], & \text{if } \operatorname{mod}(g, \delta_{\text{PuW}}) = 0 \\ & \text{and } \Delta_f \leq \kappa, \\ L^{(g)} - \left[\dfrac{\delta_{\text{PuW}} \left(L^{(g)} - \underset{\rightarrow}{L} \right)}{G} \right], & \text{if } \operatorname{mod}(g, \delta_{\text{PuW}}) = 0 \\ & \text{and } \Delta_f > \kappa, \\ L^{(g)}, & \text{otherwise,} \end{cases} \tag{23}$$

where κ is a small number that serves a threshold for measuring the difference Δ_f between the smallest values of the ML cost function shown in (3) at the current and previous generations.

3) ENTROPY CONTROL

The EnC technique is based on the concept of entropy introduced by Shannon [27]. The EnC technique analyzes the existing entropy in the population and, according to it, increases or decreases the PS [19]. The entropy of the population at the g-th generation is calculated as

$$\tilde{E}^{(g)} = \prod_{j=1}^{2} E_j^{(g)}, \tag{24}$$

where

$$E_j^{(g)} = -\sum_{l=1}^{L^{(g)}} p_{l,j}^{(g)} \log \left(p_{l,j}^{(g)} \right) \tag{25}$$

denotes the entropy of the j-th gene at the g-th generation for $j = 1, 2$. In (25),

$$p_{l,j}^{(g)} = \frac{N_{l,j}^{(g)}}{L^{(g)}} \tag{26}$$

is the probability that an individual falls into certain decision space evenly divided into $L^{(g)}$ intervals, where $N_{l,j}^{(g)}$ is the number of individuals with their j-th gene falling into the l-th interval at the g-th generation.

The EnC technique creates the ratio

$$R_{\text{EnC}}^{(g)} = \frac{\tilde{E}^{(g)}}{\tilde{E}^{(g-1)}} \tag{27}$$

at each generation using the entropy of the population at the current and previous generations. Then, the PS is adjusted at certain intervals δ_{EnC} as

$$L^{(g+1)} = \begin{cases} L^{(g)} + \left[r_1 \cdot L^{(g)} \right], & \text{if } \operatorname{mod}(g, \delta_{\text{EnC}}) = 0, \\ & g < 0.2G, \text{ and} \\ & \mu > \bar{R}_{\text{EnC}}, \\ L^{(g)} - \left[r_2 \cdot L^{(g)} \right], & \text{if } \operatorname{mod}(g, \delta_{\text{EnC}}) = 0, \\ & g \geq 0.2\,G, \text{ and} \\ & \mu < \bar{R}_{\text{EnC}}, \\ L^{(g)}, & \text{otherwise,} \end{cases} \tag{28}$$

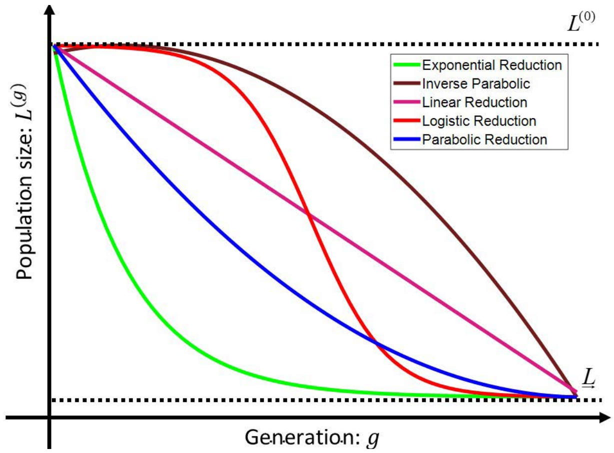

FIGURE 1. **Reduction of the population size of the proposed and some popular reduction-based techniques.**

where $\mu \sim U(0, 1)$, and

$$\bar{R}_{\text{EnC}} = \frac{1}{\delta_{\text{EnC}}} \sum_{i=g-\delta_{\text{EnC}}}^{g} R_{\text{EnC}}^{(i)} \tag{29}$$

is the average value of the ratio of entropy. In the EnC technique, the parameters δ_{EnC}, r_1, and r_2 need to be provided by the users.

D. PROPOSED TECHNIQUES

Based on the LiR technique and the observation that reducing the PS linearly may not be the best option [5], we now propose three new techniques for reducing the PS. It will be shown later in Section V that nonlinear reduction such as those proposed in this section can provide a better trade-off between the localization accuracy and computational complexity. When reducing the PS, the proposed techniques also eliminate the least favorable individuals.

1) EXPONENTIAL REDUCTION

In the ExR technique, the reduction of the PS follows an exponential decay as

$$L^{(g+1)} = \left[\left(L^{(0)} - \underset{\rightarrow}{L} \right) \beta^{g} + \underset{\rightarrow}{L} \right], \tag{30}$$

where $0 \leq \beta \leq 1$ is the control parameter of the ExR technique controlling how fast the PS reduction happens. For small values of β, the PS is rapidly reduced from the initial PS $L^{(0)}$ to the minimum PS $\underset{\rightarrow}{L}$. This diminishes the exploration capability of the DE, which results in a poor quality solution. On the contrary, at higher values of β, the reduction of the PS is slowed down, which results in an improvement of the exploration capability and, consequently, the performance of the algorithm. Note that at the highest value of $\beta = 1$ the ExR becomes FiP.

2) LOGISTIC REDUCTION

The main idea of the LoR technique is to have a smooth transition from exploration, where having a larger PS is necessary,

TABLE 1. Comparison of Theoretical Computational Complexity.

Technique	Complexity	Note
Entropy Control [19]	$O\left(L_{\text{EnC}}^2 + K_{1\text{EnC}}N + K_{2\text{EnC}}(L_{\text{EnC}}\log(L_{\text{EnC}}))\right)$	L_{EnC} denotes the PS at the g-th generation, and it varies according to (28). $K_{1\text{EnC}}$ and $K_{2\text{EnC}}$ are binary parameters: $K_{1\text{EnC}} = 1$ and $K_{2\text{EnC}} = 0$ if $\text{mod}(g, \delta_{\text{EnC}}) = 0, g < 0.2G$, and $\mu > \bar{R}_{\text{EnC}}$. $K_{1\text{EnC}} = 0$ and $K_{2\text{EnC}} = 1$ if $\text{mod}(g, \delta_{\text{EnC}}) = 0, g \geq 0.2G$, and $\mu < \bar{R}_{\text{EnC}}$. $K_{1\text{EnC}} = 0$ and $K_{2\text{EnC}} = 0$ in other cases.
Iterative Halving [15]	$O\left(K_{\text{ItH}}\frac{L_{\text{ItH}}}{2}\right)$	L_{ItH} denotes the PS at the g-th generation, and it varies according to (13). K_{ItH} is a binary parameter: $K_{\text{ItH}} = 1$ if $\text{mod}(g, \delta_{\text{ItH}}) = 0$, $K_{\text{ItH}} = 0$ otherwise.
Inverse Parabolic [5]	$O\left(L_{\text{InP}}\log(L_{\text{InP}})\right)$	L_{InP} denotes the PS at the g-th generation, and it varies according to (18).
Linear Reduction [4]	$O\left(L_{\text{LiR}}\log(L_{\text{LiR}})\right)$	L_{LiR} denotes the PS at the g-th generation, and it varies according to (14).
Linear Staircase [17]	$O\left((K_{\text{LiS}}L_{\text{LiS}}\log(L_{\text{LiS}}))\right)$	L_{LiS} denotes the PS at the g-th generation, and it varies according to (17). K_{LiS} is a binary parameter: $K_{\text{LiS}} = 1$ if $\text{mod}(g, \delta_{\text{LiS}}) = 0$, $K_{\text{LiS}} = 0$ otherwise.
Population Diversity [18]	$O\left(2L_{\text{PD}} + K_{1\text{PD}}N + K_{2\text{PD}}(L_{\text{PD}}\log(L_{\text{PD}}))\right)$	L_{PD} denotes the PS at the g-th generation, and it varies according to (22). $K_{1\text{PD}}$ and $K_{2\text{PD}}$ are binary parameters: $K_{1\text{PD}} = 1$ and $K_{2\text{PD}} = 0$ if $R^{(g)} < 0.9R^{(g-1)}$. $K_{1\text{PD}} = 0$ and $K_{2\text{PD}} = 1$ if $R^{(g)} > 1.1R^{(g-1)}$. $K_{1\text{PD}} = 0$ and $K_{2\text{PD}} = 0$ in other cases.
Proposed - Exponential	$O\left(L_{\text{ExR}}\log(L_{\text{ExR}})\right)$	L_{ExR} denotes the PS at the g-th generation, and it varies according to (30).
Proposed - Logistic	$O\left(L_{\text{LoR}}\log(L_{\text{LoR}})\right)$	L_{LoR}denotes the PS at the g-th generation, and it varies according to (31).
Proposed - Parabolic	$O\left(L_{\text{PaR}}\log(L_{\text{PaR}})\right)$	L_{PaR} denotes the PS at the g-th generation, and it varies according to (32).
Pulse Wave [17]	$O\left(K_{1\text{PuW}}N + K_{2\text{PuW}}(L_{\text{PuW}}\log(L_{\text{PuW}}))\right)$	L_{PuW} denotes the PS at the g-th generation, and it varies according to (23). $K_{1\text{PuW}}$ and $K_{2\text{PuW}}$ are binary parameters: $K_{1\text{PuW}} = 1$ and $K_{2\text{PuW}} = 0$ if $\text{mod}(g, \delta_{\text{PuW}}) = 0$ and $\Delta_f \leq \kappa$. $K_{1\text{PuW}} = 0$ and $K_{2\text{PuW}} = 1$ if $\text{mod}(g, \delta_{\text{PuW}}) = 0$ and $\Delta_f > \kappa$. $K_{1\text{PuW}} = 0$ and $K_{2\text{PuW}} = 0$ in other cases.

to exploitation, where a small PS may be more suitable [4]. To do so, the reduction of the PS is conducted according to an inverse logistic function as

$$L^{(g+1)} = \left[\left(L^{(0)} - \underrightarrow{L}\right)\left(1 - \frac{1}{1 + e^{\left(\frac{G}{2} - g\right)\lambda}}\right) + \underrightarrow{L}\right], \quad (31)$$

where $0.1 \leq \lambda$ denotes the control parameter of the LoR technique controlling how smoothly the PS changes from the initial value $L^{(0)}$ to the lower bound $\underrightarrow{L}$. Note that among the proposed techniques the LoR is the one that allows a smoother transition from the initial to the minimum PS. For small values of λ, the LoR tends to behave like the LiR technique. On the other hand, for high values of λ, the LoR tends to behave as the LiS, but with only one stair. Interestingly, it will be shown in Section V-E that the performance of the DE with the LoR is rather insensitive to the values of λ.

3) PARABOLIC REDUCTION

In the PaR technique, the reduction of the PS follows a parabolic decay as the evolution process evolves. This means that as g increases the PS is reduced parabolically as

$$L^{(g+1)} = \left[\frac{L^{(0)} - \underrightarrow{L}}{G^2}(g - G)^2 + \underrightarrow{L}\right]. \quad (32)$$

It is worthwhile mentioning that the PaR technique does not require control parameters unlike most of the techniques introduced above.

It is noteworthy that the proposed approaches, like the reduction-based techniques such as the ItH, LiR, LiS, and InP, do not require to measure the diversity in the current generation. Consequently, the proposed approaches significantly simplify the process of PS control without affecting the quality of the solution, especially over adaptive methods.

Fig. 1 depicts the PS reduction patterns of the ExR, LoR, and PaR techniques with respect to the LiR and InP techniques. Note that the ExR and PaR are more drastic reduction techniques than the LiR and InP. The LoR technique can be considered a smoother way of reducing the PS. The InP technique is the most cautious when reducing the PS, and the ExR technique is the boldest and reduces the PS at a higher rate compared to the other techniques. Although the PaR technique also reduces the PS much faster than the LiR

TABLE 2. Control parameters of PS adaptation techniques.

Techniques	Parameters	Values
Entropy Control [19]	$\delta_{\text{EnC}}, r_1, r_2$	$3, 0.3, 0.17$
Iterative Halving [15]	N_p	3
Linear Staircase [17]	Δ	3
Proposed - Exponential	β	0.9
Proposed - Logistic	λ	0.2
Pulse Wave [17]	$\delta_{\text{PuW}}, \kappa$	$3, 10^{-4}$

and InP techniques, it is more conservative than the ExR technique. In contrast, the LoR technique follows a pattern that can be considered as a median between the InP and PaR techniques.

E. COMPUTATIONAL COMPLEXITY

Table 1 shows the theoretical computational complexity of the PS control techniques introduced in this section. In general, it is observed that the computational complexity of all the techniques depends mainly on the PS size at the current generation, which is intrinsically linked to the characteristics of each of the techniques. Therefore, it is difficult to state which of the techniques is the one with the highest or lowest computational complexity. Nevertheless, it can be observed that the self-adaptive techniques are more complex than those based on reduction techniques. This statement will be confirmed in Section V.

V. COMPARISON AMONG THE ADAPTATION TECHNIQUES

In this section, results from numerical simulations are presented for comparing the accuracy of the DE with several PS adaptation techniques when solving the localization problem of a TN in WSNs. The effects of the PS adaptation techniques described in Section IV are carefully analyzed in terms of the localization accuracy and computational complexity. The DE algorithms with PS adaptation techniques are all implemented in MATLAB - R2017b. We also include the Cramer-Rao lower bound (CRLB) as a benchmark, and employ the root-mean-square error (RMSE) as the parameter for comparing the localization accuracy. The RMSE is defined as

$$\text{RMSE} = \sqrt{\frac{1}{M}\sum_{m=1}^{M}\left\|\hat{\boldsymbol{x}}_{0,m} - \boldsymbol{x}_{0,m}\right\|^2}, \tag{33}$$

where M is the number of Monte Carlo trials, and $\boldsymbol{x}_{0,m} = \left[x_{0,m}, y_{0,m}\right]^T$ and $\hat{\boldsymbol{x}}_{0,m} = \left[\hat{x}_{0,m}, \hat{y}_{0,m}\right]^T$ denote the actual and estimated positions, respectively, of the TN at the m-th Monte Carlo trial.

We have assumed nine ANs deployed deterministically and randomly, and a TN deployed randomly on a 2-dimensional area of 50 m × 50 m with the lower and upper bounds $a_1 = a_2 = 0$ and $b_1 = b_2 = 50$, respectively. For the path-loss model (1), we have considered −10 dBm as the power P_0 of the transmitted signal from the TN, $\gamma = 3$ as the path-loss exponent, and $\sigma = 5$ dB as the standard deviation of the log-shadowing noise. The required control parameters of six PS adaptation techniques are presented in Table 2. The remaining five PS adaptation techniques considered here do not require control parameters.

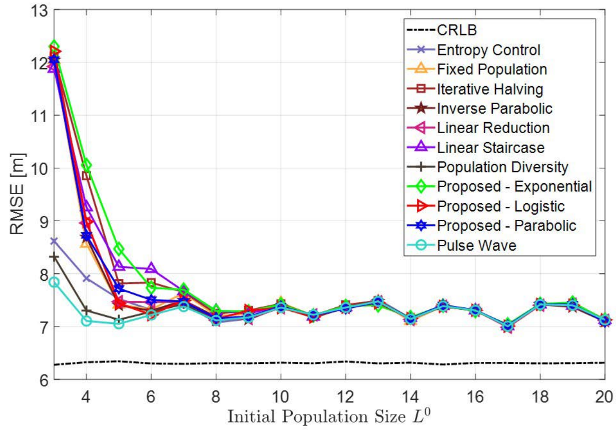

FIGURE 2. Localization accuracy versus initial population size $L^{(0)}$ (minimum PS $\underrightarrow{L} = 3$, maximum PS $\overleftarrow{L} = 20$).

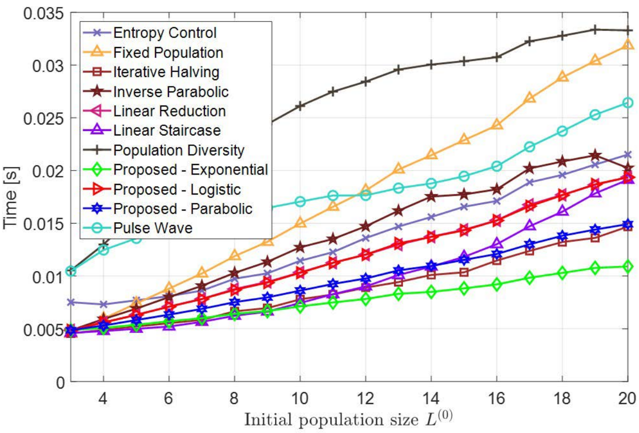

FIGURE 3. Average running time versus initial population size $L^{(0)}$ (minimum PS $\underrightarrow{L} = 3$, maximum PS $\overleftarrow{L} = 20$).

A. EFFECT OF THE INITIAL POPULATION SIZE

The initial PS is a key parameter since it is directly related to the exploration and exploitation paradigm in any evolutionary algorithm [14]. Here, we analyze the effect of the initial PS in terms of the accuracy and computational complexity of the DE with PS adaptation techniques. Nine ANs are deployed deterministically at (0, 0), (0, 25), (0, 50), (25, 0), (25, 25), (25, 50), (50, 0), (50, 25), and (50, 50). The maximum number G of iterations is set to 50: the reason for this setting will be evident in the next subsection.

Fig. 2 shows the RMSE of the DE with PS adaptation techniques as a function of the initial PS $L^{(0)}$. In general, it is observed that as $L^{(0)}$ increases the localization accuracy of the DE is enhanced regardless of the PS adaptation

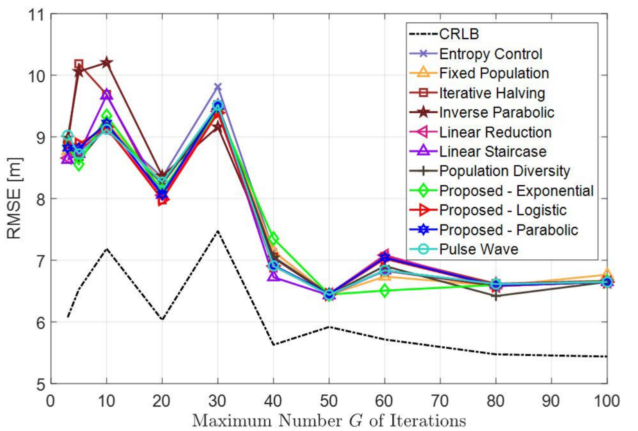

FIGURE 4. **Localization accuracy versus maximum number *G* of iterations (minimum PS $\underrightarrow{L} = 3$, maximum PS $\overleftarrow{L} = 20$).**

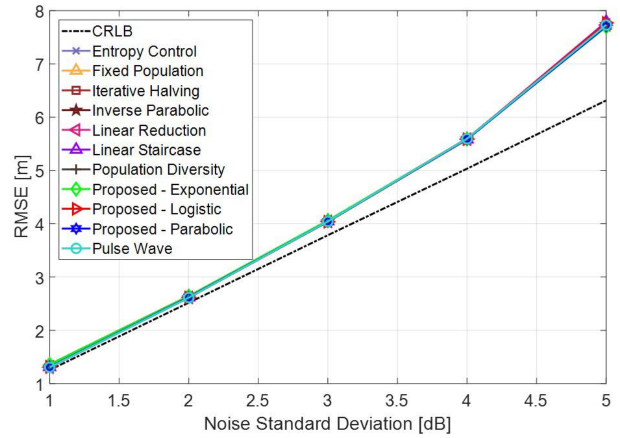

FIGURE 6. **Localization accuracy versus standard deviation of the log-shadowing noise (minimum PS $\underrightarrow{L} = 3$, maximum PS $\overleftarrow{L} = 20$).**

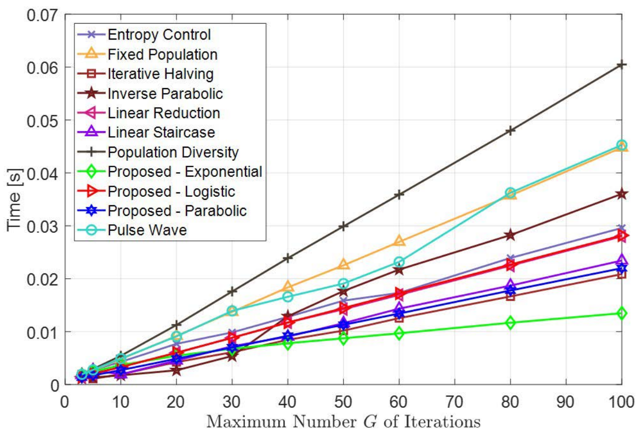

FIGURE 5. **Average running time versus maximum number *G* of iterations (minimum PS $\underrightarrow{L} = 3$, maximum PS $\overleftarrow{L} = 20$).**

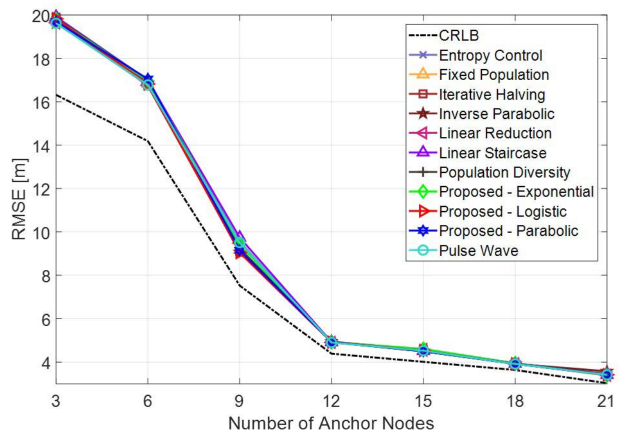

FIGURE 7. **Localization accuracy versus number of anchor nodes (standard deviation of the log-shadowing noise $\sigma = 5$ dB, minimum PS $\underrightarrow{L} = 3$, maximum PS $\overleftarrow{L} = 20$).**

technique. Interestingly, Fig. 2 also shows that, when $L^{(0)}$ is larger than 10 approximately, the performance of DE with any PS adaptation technique provides similar localization accuracy, which in some extent validates the suggestion of setting the value of the initial PS between $5D$ and $10D$ [6]. One may think that by reducing the PS through the evolution processes may deteriorate the accuracy: on the contrary, Fig. 2 shows that the DE algorithms with PS reduction techniques provide similar localization accuracy to the others.

Fig. 3 depicts the average computational time of the DE with eleven PS adaptation techniques as a function of the initial PS $L^{(0)}$. It is observed that as the initial PS $L^{(0)}$ increases the computational time increases. Additionally, the DE algorithms with the PD, PuW, and FiP are among the techniques with the highest computational complexity. In contrast, the DE algorithms with the proposed PS adaptation techniques are among the least computationally demanding. In fact, the DE algorithm with the proposed ExR technique is the least computationally demanding among all the algorithms compared when $L^{(0)} > 8$.

B. EFFECT OF THE NUMBER OF ITERATION

In this subsection, we analyze the effect of the maximum number G of iterations on the localization accuracy and computational complexity. Here, the ANs and TN are randomly deployed inside the area of interest: the reason is to test if less favorable distributions of ANs affect the accuracy of the DE with PS adaptation techniques. We set the initial PS $L^{(0)}$ to 15 for all the algorithms considered.

Fig. 4 depicts the RMSE of the DE with PS techniques as a function of the maximum number of iterations. Although it is not completely clear, there is a tendency for better accuracy as G increases. However, the improvement obtained beyond $G > 40$ seems to be negligible. This figure also shows that none of the algorithms perform uniformly best in all the range of values of G. For instance, at $G = 60$ it is clearly

TABLE 3. Mean and Standard Deviation of Absolute Localization Error of PS Techniques.

Technique:	Mean in x/y axis (m)	Standard Deviation in x/y axis (m)
Entropy Control [19]	3.9529 / 4.0191	3.5836 / 3.9177
Fixed Population [6]	3.9502 / 4.0200	3.5817 / 3.9152
Iterative Halving [15]	3.9482 / 4.0291	**3.5615** / 3.9070
Inverse Parabolic [5]	3.9506 / 4.0195	3.5816 / 3.9156
Linear Reduction [4]	3.9477 / 4.0156	3.5723 / 3.9120
Linear Staircase [17]	3.9791 / 4.0203	3.6525 / 3.9166
Population Diversity [18]	3.9509 / 4.0201	3.5815 / 3.9155
Proposed - Exponential	3.9717 / 3.9941	3.5943 / **3.8222**
Proposed - Logistic	3.9783 / 4.0153	3.6193 / 3.9144
Proposed - Parabolic	**3.6884 / 3.7733**	3.9765 / 3.9655
Pulse Wave [17]	3.9409 / 4.0192	3.5690 / 3.9157

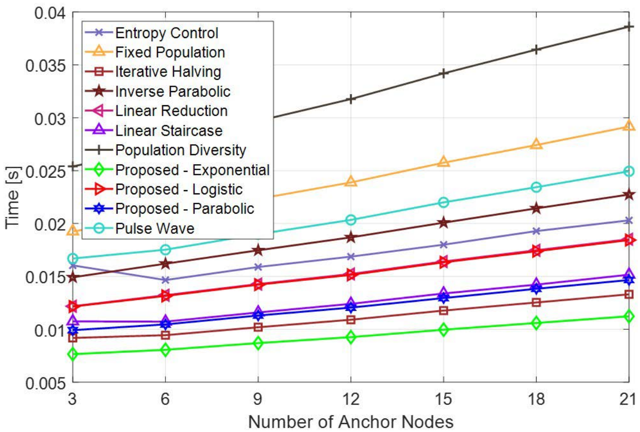

FIGURE 8. Average running time versus number of ANs (CPU: Intel (R) Core (TM) i5-6600 3.30 GHz. RAM: 16.0 GB, minimum PS $\underrightarrow{L} = 3$, maximum PS $\overleftarrow{L} = 20$).

observed that the DE with the ExR technique provides the best accuracy while the DE with the PD technique provides the best accuracy at $G = 80$.

Fig. 5 shows the average computational time of several algorithms as a function of the maximum number G of iterations. In general, it is observed that the computational time of all the algorithms increases with a larger value of G. The DE with the PD, FiP, and PuW techniques are the most computationally demanding. In contrast, the DE with the ExR, LoR, and PaR techniques together with the ItH and LiS techniques are in the group of the least computationally demanding algorithms. Within the second group, the DE with the ExR technique is the least computationally demanding when $G > 40$.

C. EFFECT OF NOISE VARIANCE

Let us consider the accuracy of the DE with several PS adaptation techniques for solving the localization problem when the standard deviation of the log-shadowing noise varies. The ANs and TN are distributed as in Section V-A. The initial PS $L^{(0)}$ and the maximum number G of iterations are set to 15 and 50, respectively, for all the algorithms: these values are chosen based on the results in Sections V-A and V-B, respectively.

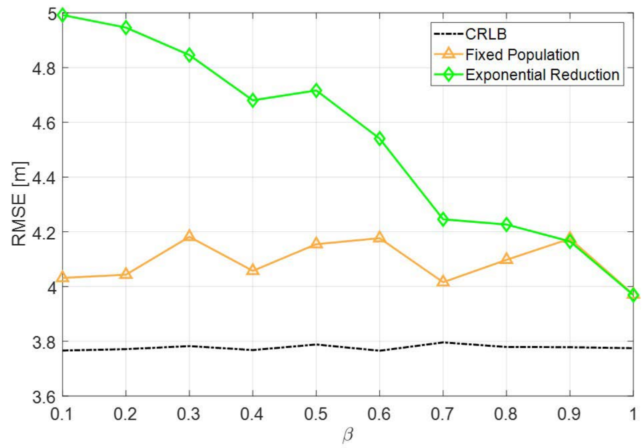

FIGURE 9. Localization accuracy of the DE with the ExR technique versus β (standard deviation of the log-shadowing noise $\sigma = 3$ dB, minimum PS $\underrightarrow{L} = 3$, maximum PS $\overleftarrow{L} = 20$).

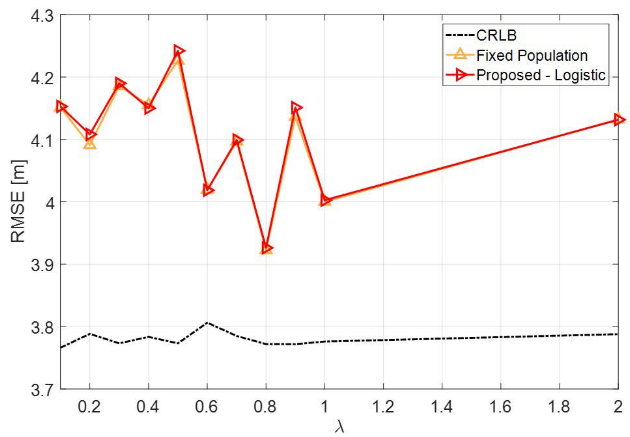

FIGURE 10. Localization accuracy of the DE with the LoR technique versus λ (standard deviation of the log-shadowing noise $\sigma = 3$ dB, minimum PS $\underrightarrow{L} = 3$, maximum PS $\overleftarrow{L} = 20$).

Fig. 6 shows that, as the standard deviation of noise increases, the accuracy of all algorithms deteriorates. Interestingly, this figure also shows that, at each value of the standard deviation of the log-shadowing noise, all the algorithms provide similar localization accuracy. This means that the DE algorithms with all the PS adaptation techniques are equally sensitive to the variations of the noise strength. To validate this observation, we calculate the mean and standard deviation of the absolute estimation error $|\hat{x}_m - x_m|$. Table 3 shows the mean and standard deviation of the absolute estimation error for the algorithms when the standard deviation of the log-shadowing noise is 5 dB. It is observed that all the PS adaptation techniques provide similar values of the mean and standard deviation of the localization error in the x- and y-axes of the solution space.

TABLE 4. Performance of Several Algorithms for Different Dimensions of the TN Localization Problem $\left(\text{2-D: } L^{(0)} = 15,\ G = 50;\ \text{3-D: } L^{(0)} = 20,\ G = 80;\ \text{5-D: } L^{(0)} = 30,\ G = 100\right)$.

Technique	2-D		3-D		5-D	
	RMSE (m)	Time (ms)	RMSE (m)	Time (ms)	RMSE (m)	Time (ms)
Entropy Control [19]	4.0472	16.0	**11.3277**	29.4	11.0119	63.3
Fixed Population [6]	4.0443	22.3	11.3779	46.4	**10.4256**	114.1
Iterative Halving [15]	4.0501	10.1	11.3761	22.0	10.9118	52.0
Inverse Parabolic [5]	4.0404	17.4	11.3784	36.0	10.4288	83.3
Linear Reduction [4]	4.0446	14.2	11.3788	28.0	10.4768	64.9
Linear Staircase [17]	4.0486	11.6	11.3844	20.3	11.2729	42.4
Population Diversity [18]	4.0437	29.8	11.3784	65.3	10.4995	123.4
Proposed - Exponential	4.0615	**8.6**	11.4094	**13.0**	14.2229	**23.4**
Proposed - Logistic	**4.0375**	14.1	11.3821	28.1	10.6425	65.3
Proposed - Parabolic	4.0422	11.2	11.3799	21.5	10.5813	47.9
Pulse Wave [17]	4.0438	19.0	11.3637	40.3	10.4493	96.2

D. EFFECT OF NUMBER OF ANCHOR NODES

In this subsection, we investigate the effect of the number of ANs on the localization accuracy of algorithms. We consider $N = 3, 6, \cdots, 21$ ANs. At each number of ANs, the ANs and TN are deployed randomly inside the area of interest, and $M = 1000$ Monte Carlo trials are considered.

Fig. 7 shows the RMSE as a function of the number of ANs. This figure shows that, as the number of ANs increases, the localization accuracy is enhanced and all the algorithms perform closer to the CRLB. Additionally, the figure shows that at any number of ANs the localization accuracy of the algorithms is practically the same in general.

Assuming the same values of parameters as those chosen for Fig. 7, Fig. 8 depicts the average running time of the algorithms as a function of the number of ANs. It is observed that the DE with the ExR technique is computationally the least demanding among the eleven algorithms. When $N = 15$ the execution times of the DE with the ExR, LoR, PaR, EnC, FiP, ItH, InP, LiR, LiS, PD, and PuW are approximately 10, 16.3, 13, 18, 25.7, 11.8, 20.1, 16.4, 13.4, 34.2, and 22 milliseconds, respectively. These results demonstrate that the DE with the ExR technique reduces more than half the computational demand of the DE with the FiP technique, which can be regarded as the standard DE.

E. EFFECT OF CONTROL PARAMETERS

Assuming the same distribution of ANs and TN as in Section V-A, here we analyze the effect of the control parameters of the ExR and LoR techniques when the standard deviation of the log-shadowing noise is $\sigma = 3$ dB. The CRLB and DE with the FiP technique are included as benchmarks.

Fig. 9 shows the localization accuracy as a function of the control parameter β. It is clearly observed that as β increases the accuracy of the DE with the ExR technique improves and approaches the CRLB. Note that the ExR technique with $\beta = 1$ is the same as the FiP.

Fig. 10 depicts the RMSE of the DE with LoR technique as a function of the control parameter λ. It is observed that the RMSE of the DE with LoR oscillates around RMSE $= 4.1$ m. However, this oscillation is not due to the variations of its control parameter λ, but due to the geometrical configuration among the ANs and TN. This assertion is based on the observation that the CRLB also shows oscillation. Additionally, the DE with the FiP technique, which does not depend on the control parameter λ, has almost the same pattern of oscillation as the DE with the LoR technique. Base on these observations, we can infer that the DE with LoR technique is not sensitive to its control parameter λ, and regardless of the value of λ, the DE with LoR technique provides competitive localization accuracy.

F. EFFECT OF THE DIMENSION OF THE PROBLEM

In this section, we analyze the performance of the algorithms when the dimension of the localization problem is increased. We consider the localization problem in 2-D, 3-D, and five-dimensional (5-D) spaces. For the 2-D localization problem, the setup of parameters is the same as that in Section V-C. For the 3-D localization problem, the positions of the eight ANs are fixed in each of the vertices of a cube of 50 m $\times$ 50 m $\times$ 50 m. For the 5-D localization problem, in addition to the unknown 3-D position of the TN, we include the transmit power and path loss exponent as unknown parameters.

The performance of several algorithms in terms of localization accuracy and average computational time is presented in Table 4 when the standard deviation of the log-shadowing noise is $\sigma = 3$ dB. It is observed that, for the 2-D and 3-D localization problems, the localization accuracy of all the algorithms is similar. However, some differences can be observed when the algorithms are tested in the 5-D localization problem. For instance, The DE with the PaR and LoR are among the algorithms with the highest localization accuracy. In contrast, the DE with the ExR provides the lowest localization accuracy. It is also observed that the DE with the ExR, LiS, and PaR are the ones with the lowest average computational time. Among them, the DE with the ExR is the least computationally demanding.

G. CONVERGENCE CURVES

Considering the same scenario of the localization problem with the same distribution of ANs and TN as in Section V-C,

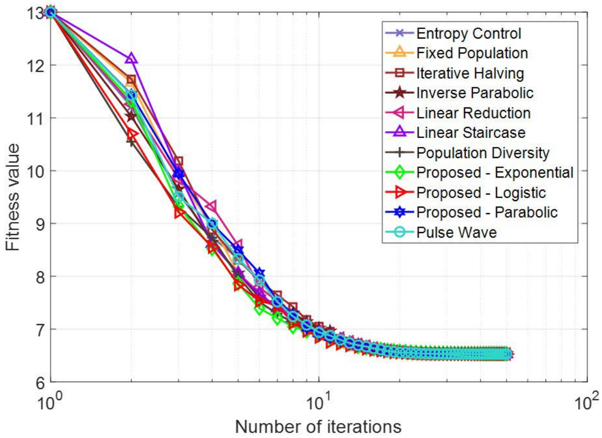

FIGURE 11. **Convergence of the best-so-far solution (standard deviation of the log-shadowing noise $\sigma = 5$ dB, minimum PS $\underrightarrow{L} = 3$, maximum PS $\overleftarrow{L} = 20$).**

Fig. 11 shows the averaged convergence curves as a function of the number of iterations in log-scale. For each algorithm, $M = 100$ Monte Carlo runs is considered when the standard deviation of the noise is $\sigma = 5$ dB. For each run, we record the convergence curve of each of the algorithms based on the dynamics of the best-so-far fitness value. It is observed that all the algorithms converge after approximately 20 iterations and that there is no significant difference in the fitness values of the algorithms over 20 iterations.

H. DISCUSSION

It has been demonstrated in [4] and [5] that a careful reduction of the PS in the DE could provide us with an improvement of the quality of solutions. However, in the specific problem of TN localization, the results from our simulations show that there is no such improvement in the solution quality, *i.e.*, localization accuracy, of the reduction-based PS adaptation techniques. In fact, the simulations results show that, after selecting properly the initial PS, *i.e.*, between $5D$ and $10D$, the quality of the solutions provided by the DE algorithms does not show considerable differences depending on the reduction-based PS adaptation techniques. Even more surprisingly, the DE algorithms with self-adaptive techniques also do not show any considerable advantage with respect to the DE algorithms with fixed PS and reduction-based techniques when the initial PS is in the range of $5D \sim 10D$. The DE algorithms with self-adaptive techniques show better localization accuracy only when the initial PS is small. This result can be expected since, contrary to the fixed population and reduction-based techniques, the self-adaptive techniques can increase and decrease the PS at any stage of the evolution process of the DE. Based on these results, we can conclude that to ensure a reasonable localization accuracy of the DE, regardless of the PS adaptation technique employed, the initial PS should be set in the range of $5D \sim 10D$, which is in line with the suggestions made in [16] and [14].

The ExR, PaR, and LoR techniques belong to the group of reduction-based PS adaptation techniques. Compared to the LiR and InP techniques, the ExR and PaR reduce the PS more drastically generation by generation. One might think that a faster reduction in the PS could result in a degradation of the localization accuracy. However, interestingly, our simulation results for the 2-D and 3-D localization problems show that the PaR and ExR do not degrade the accuracy of the DE algorithms, and consequently, the DE algorithms with PaR and ExR provide competitive localization accuracy with respect to the other algorithms. Likewise, the DE algorithm with the LoR technique, a technique that provides a smoother transition from the maximum PS to the minimum PS, also provides a competitive, though not significantly better, localization accuracy over the other techniques. In contrast, when the algorithms are tested in the 5-D localization problem, a degradation of the localization accuracy was observed, especially for the DE with the ExR and LiS techniques. This result suggests that a drastic reduction in the PS such as the ones provided by the ExR and LiS, for example, may not be suitable for localization problems of a high dimension. Consequently, techniques that provide moderate PS reduction rates as the PaR and LiR are recommended.

The comparison of localization accuracy of the DE with several adaptation techniques in this paper was conducted with several variations in the localization scenario such as deterministic and random placement of ANs, values of standard deviation of the log-shadowing noise, and numbers of ANs. In such variations of the localization problem, none of the PS adaptation techniques in the DE algorithm provides significant advantage in terms of localization accuracy. This is quite an interesting observation considering that other works suggested that some PS adaptation techniques may perform better than the others [17], [20]. However, it is worth mentioning that, when the comparison was conducted in the 5-D localization problem, some algorithms performed better than the others: for instance, the DE with the PaR, LoR, LiR, and FiP outperformed the DE with the EnC, LiS, and ExR.

Although the PS adaptation techniques in the DE do not show any considerable difference in terms of localization accuracy, especially in the 2-D and 3-D localization problems, major differences have been observed in terms of computational complexity. In general, the DE with fixed PS and self-adaptive techniques are the ones with higher computational complexity. The DE with reduction-based adaptation techniques, which includes the three techniques proposed in this paper, incur lower computational complexity. Among all the algorithms considered, the DE with the PD and the DE with the ExR are the highest and lowest computationally demanding, respectively.

VI. CONCLUSION

In this article, we have compared the accuracy and computational complexity of the differential evolution with several adaptation techniques of population size when solving the localization problem based on received signal strength in wireless sensor networks. In addition to the existing techniques, we have proposed and included in the comparison three new adaptation techniques of population size.

Simulations results show that, once the initial population size is set properly, all the algorithms compared provide similar localization accuracy regardless of the variations on the parameters of the localization problem, especially for lower dimensional localization problems. When the comparison is conducted in terms of computational complexity, the proposed algorithms incur less computational requirement. Among them, the DE with the exponential reduction technique is the least computationally demanding.

In future work, we expect it interesting to analyze the performance of the proposed techniques together with methods that adapt other control parameters such as the mutation rate and crossover probability. Additionally, employing and testing other evolutionary algorithms for solving the localization problem will receive our attention.

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