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RESEARCH ARTICLE

Performance Assessment of MIMO System With Non-Gaussian Disturbances

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ABSTRACT Performance assessment of control loops is of great importance for industrial production. This paper proposes a novel performance assessment and controller tuning method for non-Gaussian MIMO feedback control systems. First, an algorithm based on minimum entropy and mutual information projection to latent structure (ME-PLS) was proposed to replace the canonical correlation analysis algorithm (CCA). The ME-PLS algorithm decomposes the system data into independent components related to inputs and outputs, and this algorithm applies to both Gaussian and non-Gaussian systems. Each pair of principal components represents a virtual non-Gaussian control loop. Next, the performance of each virtual loop is calculated separately with the non-Gaussian minimum entropy method. Finally, to identify the parameters of each virtual loop, the author gives a least absolute deviation iterative algorithm based on the CARMA model (CARMA-LADI). When the control does not work well, based on the ME-PLS algorithm's relationship and the CARMA-LADI algorithm's identification results can give the controller tuning direction.

INDEX TERMS Control loop performance assessment, independent components, MIMO system, maximum mutual information, minimum entropy.

I. INTRODUCTION

Recently, there has been much research on control performance assessment (CPA). The earliest idea of performance assessment based on minimum variance was proposed by Harris [1]. This method uses minimum variance (MV) as a benchmark to assess the performance of single-input single-output (SISO) feedback control systems. The control method that achieves the best output for the system is called minimum variance control (MVC). This approach was later applied to the assessment of multivariate control systems [2]. The minimum variance benchmark of a multivariate feedback control system can be calculated based on the interactor matrix [3]. In order to extend the MV index of the MIMO system, the researchers conducted further studies [4], [5], [6], [7]. Similarly many other evaluation benchmarks exist, such as: generalized minimum variance (GMV) benchmark [8], [9], generalized Hurst exponent based benchmark [10], [11], [12], [13], linear quadratic Gaussian (LQG) benchmark [14], [15], etc.

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In these performance evaluation benchmarks, MVC has been widely used. However, the direct use of variance as an indicator when the disturbances are non-Gaussian distributed is no longer applicable. Performance assessment methods based on minimum entropy (ME) have received much attention in recent years to address this problem. Meng [16] proposed using the minimum information entropy as a benchmark for performance assessment of non-Gaussian systems. After that, many research efforts have expanded this concept. Zhang [17] derived a feedback control algorithm based on minimum entropy, called MEC (minimum entropy control). The idea of MEC is also applied to a non-Gaussian cascade control system [18]. Zhou [19] proposed a new uncertainty measure, rational entropy, by analyzing the Shannon entropy in continuous random variables (CRV) and gave several benchmarks for controller performance evaluation of control systems with a random distribution of outputs. Researchers integrated dynamic data reconciliation (DDR) into minimum rational entropy control (MREC) to improve management performance [20]. Zhang [21] proposed a new benchmark combining entropy and output mean to address the inconsistency of the minimum variance benchmark in evaluating non-Gaussian perturbed systems. However, MEC methods applicable to multivariate systems still require continued research.

The commonly used methods for MIMO system performance assessment require much prior knowledge, which is more difficult to achieve in practice. Then, MIMO systems affected by non-Gaussian noises need a new benchmark for assessment. These problems motivate the authors to analyze MIMO systems from a data-driven perspective to obtain a suitable performance assessment method.

Therefore, the idea of decoupling control is introduced in this paper. Decoupling control requires decoupling the relationship between input and output. If decoupling of the MIMO system can be achieved, the decoupling relationship can be obtained, but it is also very convenient for performance assessment. Further, there is a specific target for controller tuning in case of poor performance. Thus, getting the input-output decoupling relationship is an essential part of MIMO system performance evaluation in the MIMO system. There are few results on stochastic decoupling control, and to the best of the authors' knowledge, only Prof. Wang Hong's team has done some work in this area. For example, the articles [22], [23] use the concept of coupling decay to approximate stochastic decoupling. It essentially allows the absolute values of the non-diagonal elements of the covariance matrix to decay to zero as much as possible. We extend this concept further to establish decoupling between inputs and outputs from a data-driven perspective. The essence of decoupling is to obtain a correspondence between the control input and the output quality. And many modeling methods in data-driven methods, such as least squares (LS) and canonical correlation analysis (CCA), can establish a similar correspondence. Hence, the problem translates into establishing a one-to-one correspondence between input and output for non-Gaussian disturbances. The authors first draw on the ideas of the CCA algorithm to find a suitable method to describe the latent structure of multiple-input multiple-output systems and then propose a performance evaluation method applicable to non-Gaussian systems based on the latent structure. Finally, the MIMO system can be adjusted based on the assessment of the latent structure. The whole process is shown in Fig. 1.

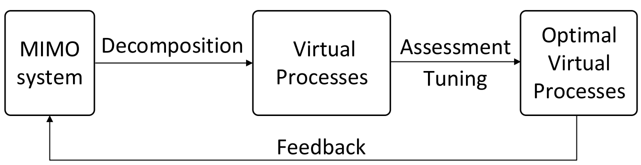

FIGURE 1. Non-Gaussian MIMO system assessment and tuning flow chart.

The rest of this article is outlined as follows: The basic theories are given in Section II. In Section III, the based minimum entropy and mutual information projection to latent structure (ME-PLS) algorithm are proposed. In Section IV, A performance assessment method based on the potential structure of non-Gaussian MIMO systems and a least absolute deviation iterative algorithm based on the CARMA model (CARMA-LADI) are given. In Section V, perform simulation. In Section VI, the conclusion.

II. BASIC THEORY

The canonical correlation analysis algorithm (CCA) is a common method for principal component extraction among multidimensional variables. This algorithm [24] extracts the principal components t_1 and m_1 from the system data using correlations, where t_1 and m_1 are linear combinations of the input and output data, respectively.

$$t_1 = X_0 w_1, m_1 = Y_0 c_1 \tag{1}$$

where X_0 and Y_0 are the results of normalizing the original datasets X, Y, and w_1, c_1 are the weight vectors. The criterion function of CCA is:

$$\begin{aligned} \arg\max r(t_1, m_1) &= \frac{E\left[t_1^T m_1\right]}{\sqrt{E\left[t_1^T t_1\right] E\left[m_1^T m_1\right]}} \\ &= \frac{E\left[(X_0 w_1)^T Y_0 c_1\right]}{\sqrt{E\left[(X_0 w_1)^T X_0 w_1\right] E\left[(Y_0 c_1)^T Y_0 c_1\right]}} \end{aligned} \tag{2}$$

where r denotes the Pearson correlation coefficient.

From (2), it is clear that the CCA extracts the principal components by maximizing the Pearson correlation coefficient. This objective relies on the calculation of second-order moments to achieve. However, second-order moments are just one of the many properties of random variables. When the variable obeys a non-Gaussian distribution, the second-order moments do not fully reflect its distribution. In this case, non-Gaussian data need to be described by other methods such as higher order moments, distribution functions, etc. Compared with other indicators, entropy and mutual information can better explain the information of random variables from the distribution perspective. Therefore, these two statistics are used to extract the principal components of the non-Gaussian data.

The Shannon entropy of a continuous random variable is defined as [25]:

$$H = -\int_a^b \gamma(x) \ln(\gamma(x)) dx \tag{3}$$

The Shannon entropy of a discrete random variable is defined as:

$$H = -\sum_{i=1}^{n} p_i \ln p_i \tag{4}$$

The information measure of the jointly distributed random variables XY (two-dimensional sources) is called the joint entropy and is defined as:

$$H(XY) = -\sum_{k=1}^{K} \sum_{j=1}^{J} p(x_k, y_j) \ln(x_k, y_j) \tag{5}$$

Extending to the joint entropy of N random variables $X_1X_2\ldots X_N$ there are:

$$H(X_1\ldots X_N)=-\sum_{i_1,\ldots,i_N}^{K} p\left(x_{i_1},\ldots,x_{i_N}\right)\ln p\left(x_{i_1},\ldots,x_{i_N}\right) \tag{6}$$

The mutual information $I(X;Y)$ of the random variables X and Y are calculated by:

$$\begin{aligned} I(X;Y) &= H(Y)+H(X)-H(XY) \\ &= \sum_{k=1}^{K}\sum_{j=1}^{J} p(x_k,y_j)\ln\left[\frac{p(x_k,y_j)}{p(x_k)p(y_j)}\right] \end{aligned} \tag{7}$$

where $p(x_k, y_j)$ is the joint distribution between x_k and y_j. The smaller $I(X;Y)$ is, the weaker the correlation between the random variables X and Y, and vice versa, the stronger the correlation. X and Y are independent of each other when $I(X;Y) = 0$. Mutual information extended to multidimensional inputs and outputs with [26]:

$$\begin{aligned} &I(X_1,\ldots,X_n;Y_1,\ldots,Y_m) \\ &= \sum_{x_1\in A_1}\cdots\sum_{y_m\in B_m} p(x_1,\ldots,x_n,y_1,\ldots,y_m) \\ &\quad\times \ln\frac{p(x_1,\ldots,x_n,y_1,\ldots,y_m)}{p(x_1,\ldots,x_n)p(y_1,\ldots,y_m)} \end{aligned} \tag{8}$$

III. ME-PLS ALGORITHM

The mutual information can describe the correlation between the variables, the same as the Pearson correlation coefficient. It is not affected by the non-Gaussian characteristics of the variables [27]. Therefore, different from the CCA algorithm the ME-PLS algorithm uses mutual information to extract the principal components with the following optimization objectives:

$$\begin{aligned} J = \max\quad & I\left(t_1; m_1\right) = \max I\left(X_0w_1; Y_0c_1\right) \\ s.t.\quad & w_1^Tw_1 = 1, c_1^Tc_1 = 1 \end{aligned} \tag{9}$$

Equation (9) yields the first pair of principal components t_1, m_1 by maximizing the mutual information, and the next pair of principal components is to be extracted from the residual matrix. The residual matrix is calculated as shown below:

$$X_1 = X_0 - t_1p_1^T \tag{10}$$

$$Y_1 = Y_0 - m_1q_1^T \tag{11}$$

where X_1, Y_1 are residual matrices and p_1, q_1 are load vectors. The ME-PLS algorithm aims to obtain the latent structure of the non-Gaussian MIMO system, which requires:

a) maximum correlation between t_i and m_i;

b) the principal component pairs should be independent of each other.

Equation (9) achieves objective a, which is more general than the Pearson correlation coefficient. Objective b can be achieved indirectly by making the residual matrix and the data represented by the principal components independent of each other, as follows:

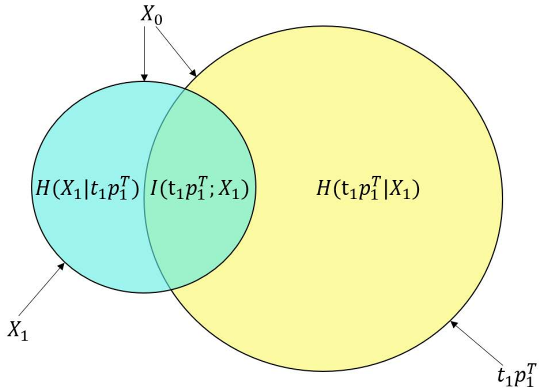

FIGURE 2. Relationship between data sets.

$$P(t_1p_1^T, X_1) = P(t_1p_1^T)P(X_1) \tag{12}$$

For Gaussian data, independent and uncorrelated are equivalent so that orthogonalization can ensure independence between datasets. However, this is not applicable for non-Gaussian data, so independence between data is still achieved using mutual information, as follows:

$$\arg\min I\left(t_1p_1^T; X_1\right) \tag{13}$$

But the calculation of multidimensional mutual information is expensive and time-consuming, so the authors propose another method.

The relationship between $X_0, t_1p_1^T, X_1$ is shown in the Fig. 2, where X_0 represents the original data set kept constant, X_1 and $t_1p_1^T$ represent the residual data and principal component data that change with the loading vector p_1, respectively.

Lemma 1: When Y = X + Z, where X and Z are both independent random variables [25]:

$$I\left(X;Y\right) = H(Y) - H(Z) \tag{14}$$

Lemma 2: Conditional entropy is less than unconditional entropy:

$$H(X) \geq H(X|Y) \tag{15}$$

Considering Fig. 2 and Lemma 2 together, it is obtained that:

$$H\left(X_1\right) \geq H\left(X_1|t_1p_1^T\right) \tag{16}$$

The equal sign holds when $H(X_1)$ is extremely small. When the equal sign holds $I(t_1p_1^T; X_1) = 0$, X_1 and $t_1p_1^T$ are independent of each other. At this point, it can be obtained that:

$$I\left(X_0; t_1p_1^T\right) = H(X_0) - H(X_1) \tag{17}$$

The entropy value $H(X_0)$ of the original data is fixed, so that $I(X_0; t_1p_1^T)$ takes a great value. This indicates that minimizing the residual entropy is equivalent to seeking a suitable p such that $t_1 p_1^T$ contains as much information about X_0 as possible.

Remark: the least squares method calculates the load vector p_1, $p_1 = X_0^T t_1 / t_1^T t_1$, which essentially minimizes the variance of X_1. Under the Gaussian distribution, entropy and variance have a one-to-one mapping relationship. There is no difference between the load vector found by the minimum variance solution and the minimum entropy solution. In this sense, the minimum entropy is more general than the minimum variance and can be used to find the independent components.

In summary, the objective function for calculating the residual matrix and the load vector is:

$$\arg\min\ H(X_1) = H\left(X_0 - t_1 p_1^T\right) \tag{18}$$

$$\begin{aligned} &\arg\min\ H(Y_1) = H\left(Y_0 - m_1 q_1^T\right) \\ &s.t.\quad p_1^T p_1 = 1, q_1^T q_1 = 1 \end{aligned} \tag{19}$$

t_1 has the strongest interpretation of m_1. Thus, the residual output matrix can be computed with the following constraint:

$$\begin{aligned} &\arg\min\ H\left(Y_1^*\right) = H\left(Y_0 - t_1 q_1^{*T}\right) \\ &s.t.\qquad q_1^{*T} q_1^* = 1 \end{aligned} \tag{20}$$

As the information of the original data is fixed, the entropy value of the residual matrix will keep getting smaller with the continuous extraction of the principal components. The ME-PLS algorithm should stop when the entropy of the residual matrix is less than the threshold ε_1. There are no explicit formulas for calculating mutual information and entropy, so all of the above objectives can be calculated using optimization algorithms, such as particle swarm optimization algorithm, genetic algorithm, etc.

To reduce the computational effort, the authors propose a method for estimating the distribution of multidimensional random variables using histograms, as follows:

Take the three-dimensional random variable $X = [X_1, X_2, X_3]_{S\times 3}$ as an example, where X_1, X_2, X_3 are three random variables, each row of X is the value obtained by sampling X_1, X_2, X_3 at the same moment, and S represents the total number of samples.

a) Choose a suitable set of intervals $N = [n_1, n_2, n_3]$ that contains all the data of X. Then make a cube O in space with n_1, n_2, n_3 as side lengths and X_1, X_2, X_3 as axis.

b) Determine a fixed interval w that cuts the cube O into v sub-cubes.

c) Each row of X is put into the cube O as a point, and counting the number of points in different sub-cubes as the frequencies of other groups, denoted as $(l_1, l_2, \ldots, l_v)$.

d) The probability is obtained by dividing l_i by S and is denoted as P.

$$P = (\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_v) = \left(\frac{l_1}{S}, \frac{l_2}{S}, \ldots, \frac{l_v}{S},\right) \tag{21}$$

P is an estimate of the multidimensional data distribution. The estimation steps are the same when the number of data dimensions is not three.

The algorithm steps are shown in the table.

Algorithm ME-PLS

Input: control input X, output Y.

Output: Principal component pairs t_i, m_i.

1) Normalize the data X and Y, giving $X_0 = [x_1, x_2, \ldots, x_f]_{S\times f}$ and $Y_0 = [y_1, y_2, \ldots, y_f]_{S\times f}$.

2) Optimize (9) to obtain the first pair of principal components t_1, m_1, and weight vectors w_1, c_1.

3) Establish the regression of X_0, Y_0 on t_1 according to (10)) and (11).

$$\begin{aligned} X_1 &= X_0 - t_1 p_1^T \\ Y_1^* &= Y_0 - t_1 q_1^{*T} \end{aligned}$$

4) Optimize (18)) and (20)) to obtain the residual matrix X_1, Y_1, and the load vectors p_1, q_1^*.

5) Compare whether the joint entropy of the residual matrix is less than ε_1. Yes, terminate the algorithm to get the principal components; otherwise, skip to step 2 and replace X_0, Y_0 with X_1, Y_1 to extract the next pair of principal components.

The ME-PLS algorithm is used to get the latent structure of non-Gaussian MIMO systems from a data-driven perspective. This approach can be applied in more directions, such as performance assessment, data reduction, fault diagnosis, etc.

IV. NON-GAUSSIAN MIMO SYSTEM PERFORMANCE ASSESSMENT METHOD

The ME-PLS algorithm is first used to extract the principal components of the non-Gaussian MIMO system data before performing the performance assessment. It can be seen from the above that each pair of principal components represents an independent virtual single-loop system. The performances of the non-Gaussian virtual loop are evaluated separately to obtain the operation of the whole MIMO system.

The system's data reflects the system's operation, so the joint entropy of the residual matrix should be zero at the termination of the ME-PLS algorithm to completely extract the information of the data set.

A. MIMO CONTROL SYSTEM MODLE

A MIMO closed-loop feedback discrete control system with N inputs and N outputs is shown in Fig. 3. G_p is an $N \times N$ process function, the input is u, and the output is y. G_c represents the feedback controller, and $e(k)$ is the input to the controller. $at(k)$ is a matrix of mutually independent noise sequences, which is generally assumed in the study to be white noise with mean zero and covariance matrix of Σ_w. G_w is an $N \times N$ matrix of noise process functions.

The system depicted in Fig. 3 is such that the set value is 0, $y^{sp} = 0$, and the input to the system is given entirely by the noise in the disturbance path. The function of the whole

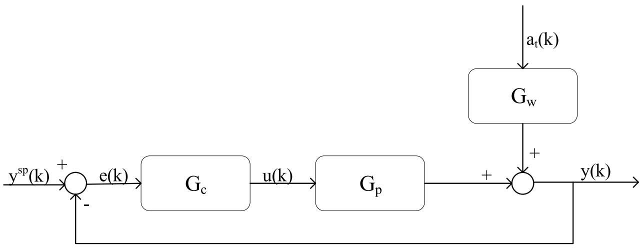

FIGURE 3. MIMO feedback control system.

closed-loop system is:

$$y = (I + G_pG_c)^{-1}G_w a_t \tag{22}$$

The process model is:

$$y = G_p u + G_w a_t \tag{23}$$

The ME-PLS algorithm decomposes the system into

$$u = t_1p_1^T + t_2p_2^T + \ldots + t_np_n^T \tag{24}$$
$$y = m_1q_1^T + m_2q_2^T + \ldots + m_nq_n^T \tag{25}$$

where t_i and m_i are one-to-one principal components, and each pair of principal components represents a virtual loop. According to the linear relationship in (24) and (25), the virtual controller for different loops can be obtained as shown in the following equation:

$$\begin{aligned} \widehat{G}_{C1} &= G_C w_1 \\ \widehat{G}_{C2} &= G_C(I - w_1p_1^T)w_2 \\ &\ldots \end{aligned} \tag{26}$$

B. PERFORMANCE ASSESSMENT FOR NON-GAUSSIAN VIRTUAL LOOPS

For the first virtual loop represented by t_1, m_1 there are:

$$u = t_1p_1^T + u_1 \tag{27}$$
$$y = m_1q_1^T + y_1 \tag{28}$$

t_1 is the control input to the virtual single loop, and m_1 is the output. u_1, y_1 are the residual matrices. The process function of this virtual loop is:

$$m_1 = \widehat{G}_{p1}t_1 + \widehat{N}_1\hat{a}_{t1} \tag{29}$$

where $\widehat{G}_{p1}$ is the virtual process function, $\widehat{G}_{p1} = z^{-d}\widetilde{G}_{p1}$, d is the virtual process time delay, $\widetilde{G}_{p1}$ is the process delay-free part. $\widehat{N}_1$ is the virtual noise function, and $\hat{a}_{t1}$ is the virtual noise. The minimum variance performance assessment method uses the variance of the system feedback invariants as the evaluation benchmark [1]. The interference model $\widehat{N}_1$ can be decomposed as follows:

$$\begin{aligned} \widehat{N}_1 &= F_1 + R_1z^{-d} \\ &= \underbrace{f_0 + f_1z^{-1} + \ldots + f_{d-1}z^{-(d-1)}}_{F_1} + R_1z^{-d} \end{aligned} \tag{30}$$

Substituting (30)) into (29) leads to:

$$\begin{aligned} m_1 &= \widehat{G}_{p1}t_1 + \left(F_1 + R_1z^{-d}\right)\hat{a}_{t1} \\ &= F_1\hat{a}_{t1} + z^{-d}\left(\widetilde{G}_{p1}t_1 + R_1\hat{a}_{t1}\right) \end{aligned} \tag{31}$$

The output of the system is minimized when $\widetilde{G}_{p1}t_1 + R_1\hat{a}_{t1} = 0$ and $F_1\hat{a}_{t1}$ is the feedback invariant of the system. The optimal control input in the case of minimum variance is:

$$t_{1_{opt}} = -\frac{R_1\hat{a}_{t1}}{\widetilde{G}_{p1}} \tag{32}$$

The minimum variance controller structure of the system can be obtained as:

$$\begin{aligned} G_{C_{opt1}} &= \frac{t_{1_{opt}}}{r - y} = -\frac{R_1\hat{a}_{t1}}{-y\widetilde{G}_{p1}} \\ &= \frac{R_1\hat{a}_{t1}}{F_1\hat{a}_{t1}\widetilde{G}_{p1}} = \frac{R_1}{F_1\widetilde{G}_{p1}} \end{aligned} \tag{33}$$

The minimum variance performance index is:

$$\begin{aligned} \eta_{MV} &= \frac{\sigma_{MV}^2}{{\sigma_{m_1}}^2} \\ &= \frac{\left(f_0^2 + f_1^2 + \ldots + f_{d-1}^2\right)\sigma_{\hat{a}_{t1}}^2}{\sigma_{m_1}^2} \end{aligned} \tag{34}$$

The variance in the non-Gaussian case is no longer suitable to reflect the data distribution. Therefore, the entropy of the feedback invariant $F_1\hat{a}_{t1}$ is used as a benchmark for performance evaluation. From the description above, the entropy of output can be written:

$$H(m_1) = H\left(F_1\hat{a}_{t1} + z^{-d}\left(\widetilde{G}_{p1}t_1 + R_1\hat{a}_{t1}\right)\right) \tag{35}$$

The minimum entropy output of the system is obtained when $\widetilde{G}_{p1}t_1 + R_1\hat{a}_{t1} = 0$. At this time, $H(m_1)_{ME} = H(F_1\hat{a}_{t1})$. The performance index of the minimum entropy benchmark is formulated as:

$$\eta_{ME} = \frac{H\left(F_1\hat{a}_{t1}\right)}{H(m_1)} \tag{36}$$

The same definition as the minimum variance performance benchmark, $\eta_{ME} \in [0, 1]$. When η_{ME} is closer to 1, the better the performance of the loop, the closer the controller is to the minimum entropy controller; if η_{ME} is closer to 0, it means that the performance of the loop is poor and the system has more room for improvement.

The time delay d plays an important role in the performance assessment. It can be estimated by detecting the peak of the cross-correlation function, i.e.:

$$\hat{d} = \arg\max_{\tau} E\{m(k)t_1(k - \tau)\} \tag{37}$$

The following equation approximates the actual calculation:

$$\hat{d} = \arg\max_{\tau} \sum_k m(k)t_1(k - \tau) \tag{38}$$

When $d = 0$, the noise function $\widehat{N}_1$ does not need to be decomposed, then:

$$\widehat{N}_1 = F_1 \tag{39}$$

In this case, the system's output is its feedback invariants, and performance assessment is unnecessary.

To calculate the estimates of the feedback invariants, we used the ARMA model to model the system, structured as:

$$m_1(k+d) = (\frac{R_1 - F_1\widehat{G}_{p1}\widehat{G}_{C1}}{1+\hat{G}_{v1}\widehat{G}_{C1}})m_1(k) + F_1\hat{a}_{t1}(k+d) \tag{40}$$

where $F_1\hat{a}_{t1}(k+d)$ are feedback invariants. The first term to the right of the equals sign of (40) can be approximated by an AR model, rewritten as:

$$\begin{aligned} m_1(k) &= b_1 m_1(k-d) + \ldots + b_n m_1(k-d-n+1) \\ &\quad + F_1\hat{a}_{t1}(k) \\ &= \sum_{i=1}^{n} b_i m_1(k-d-i+1) + F_1\hat{a}_{t1}(k) \end{aligned} \tag{41}$$

Write (41) in matrix form.

$$M = X\beta + F_1 A \tag{42}$$

In the paper [28], it is shown that the least absolute deviation (LAD) outperforms the least-squares (LS) method for the identification of non-Gaussian noise perturbations, so the least absolute deviation is also used in this paper to identify the parameter sequence of (42). The estimation of the residual series can be expressed as:

$$\widehat{\phi}_1 = F_1 A = M - X\hat{\beta} \tag{43}$$

where $\widehat{\phi}_1$ is an estimate of $F_1\hat{a}_{t1}(k)$ and $\hat{\beta}$ is an estimate of the coefficient b_i. The entropy of $\widehat{\phi}_1$ is calculated to obtain the performance assessment value of the minimum entropy benchmark as follows:

$$\hat{\eta}_{ME} = \frac{H(\widehat{\phi}_1)}{H(m_1)} \tag{44}$$

The performance of the other virtual loops is evaluated in the same way as above.

C. PARAMETER ESTIMATION

Controllers need to be tuned when the control performance is poor, in which case the parameters of the virtual system need to be identified. Equation (29) can be written as a CARMA model.

$$\begin{aligned} &A(z^{-1})m_1(k) = B(z^{-1})t_1(k-d) + C(z^{-1})\hat{a}_{t1}(k) \\ &\begin{cases} A\left(z^{-1}\right) = 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_{n1} z^{-n1} \\ B\left(z^{-1}\right) = b_0 + b_1 z^{-1} + b_2 z^{-2} \ldots + b_{n2} z^{-n2} \\ C\left(z^{-1}\right) = 1 + c_1 z^{-1} + c_2 z^{-2} + \ldots + c_{n3} z^{-n3} \end{cases} \end{aligned} \tag{45}$$

The AIC information criterion estimates the order of the system. The LAD method for identifying non-Gaussian systems is superior to the LS method. The authors propose a least absolute deviation iterative algorithm (CARMA-LADI) based on the CARMA model. Let:

$$\begin{aligned} M(L) &= [m_1(L), m_1(L-1), \ldots, m_1(1)]^T \\ \phi(L) &= \left[\varphi^T(L), \varphi^T(L-1), \ldots, \varphi^T(1)\right]^T \\ V(L) &= \left[\hat{a}_{t1}(L), \hat{a}_{t1}(L-1), \ldots, \hat{a}_{t1}(1)\right]^T \end{aligned} \tag{46}$$

$$\theta = [a_1, \ldots, a_{n1}, b_0, \ldots, b_{n2}, c_1, \ldots, c_{n3}]^T \tag{47}$$

where L is the length of the data, $M(L)$ is the stacked output vector, $\phi(L)$ is the stacked information matrix, $V(L)$ is the stacked noise vector, and:

$$\begin{aligned} \varphi(t) = [&-m_1(t-1), -m_1(t-2), \ldots, -m_1(t-n1) \\ &t_1(t-1-d), t_1(t-2-d), \ldots, t_1(t-n2-d), \\ &\hat{a}_{t1}(t-1), \hat{a}_{t1}(t-2), \ldots, \hat{a}_{t1}(t-n3)\Big]^T \end{aligned} \tag{48}$$

In summary, we can obtain:

$$M(L) = \phi(L)\theta + V(L) \tag{49}$$

Define the least absolute deviation criterion function:

$$J(\theta) = \arg\min \|M(L) - \phi(L)\theta\|_1 \tag{50}$$

Minimizing the criterion function $J(\theta)$ yields an estimate of the parameters. It can be calculated using the relaxation algorithm. It is important to note that (50) does not directly yield an estimate of the parameter because $\varphi(L)$ contains the unmeasurable noise $\hat{a}_{t1}$. Introduction of the iterative identification principle: Let $k = 1, 2, 3, \ldots$ be an iterative variable, $\dot{\theta}_k(t-i)$ be used as an iterative estimate of θ, and the unknown term $\hat{a}_{t1}(t-i)$ in the information vector $\varphi(t)$ is replaced by its $k-1$ iterative estimate $\dot{a}_{t1|k-1}(t-i)$. The substituted $\dot{\varphi}(t)$ is denoted as:

$$\begin{aligned} \dot{\varphi}_k(t) = [&-m_1(t-1), -m_1(t-2), \ldots, -m_1(t-n1) \\ &t_1(t-1-d), t_1(t-2-d), \ldots, t_1(t-n2-d), \\ &\dot{a}_{t1|k-1}(t-1), \dot{a}_{t1|k-1}(t-2), \ldots, \dot{a}_{t1|k-1}(t-n3)\Big]^T \end{aligned} \tag{51}$$

And:

$$\dot{a}_{t1|k}(t) = m_1(t) - \dot{\varphi}_k^T(t)\dot{\theta}_k \tag{52}$$

At this point, the stacked information matrix is:

$$\dot{\phi}_k(L) = \left[\dot{\varphi}_k^T(L), \dot{\varphi}_k^T(L-1), \ldots, \dot{\varphi}_k^T(1)\right]^T \tag{53}$$

Equation (49) can be rewritten as:

$$M(L) = \dot{\phi}_k(L)\dot{\theta}_k + \dot{V}_k(L) \tag{54}$$

where:

$$\dot{V}_k(L) = \left[\hat{a}_{t1|k}(L), \hat{a}_{t1|k}(L-1), \ldots, \hat{a}_{t1|k}(1)\right]^T \tag{55}$$

Equation (50) can be rewritten as:

$$J(\dot{\theta}_k) = \min \|M(L) - \dot{\phi}_k(L)\dot{\theta}_k\|_1 = \min \|\dot{V}_k(L)\|_1 \tag{56}$$

Keep iterating when $J(\theta)$ converges to obtain the estimates of the system parameters $\dot{\theta}_k$. The algorithm steps are shown in the table.

Algorithm CARMA-LADI

Input: virtual control input t_1, virtual output m_1.

Output: parameter estimation $\dot{\theta}_k$.

1) Collect the input and output data $t_1(t), m_1(t) \; : \; t = 1, 2, \ldots, L$, construct $M(L)$ using (47), and give the parameter estimation accuracy ε_2.

2) Let $k = 1$ and assign the initial value $\dot{a}_{t1|0}(t)$ as a random.

3) Construct $\dot{\varphi}_k(t), \dot{\phi}_k(t)$ using (51) and (53).

4) Using (54) to calculate the parameter estimate $\dot{\phi}_k$.

5) Using (55) to calculate $\dot{V}_k(L)$.

6) Compare $J(\dot{\theta}_k)$ with $J(\dot{\theta}_{k-1})$, if $|J(\dot{\theta}_k) - J(\dot{\theta}_{k-1})| < \varepsilon_2$, terminate the algorithm to get the parameter estimate; otherwise $k+1$ go to step 3.

The CARMA-LADI algorithm is used to calculate the noise process function $\widehat{N}_i$ and process function $\widetilde{G}_{pi}$ for different virtual loops, then the optimal controller $G_{C_{opti}}$ for each virtual loop can be obtained by (33). Finally, the linear relationship obtained by (26) combined with $G_{C_{opti}}$ can give the actual controller tuning direction to make the system work better.

V. SIMULATION EXPERIMENTS

A. EXPERIMENTAL MODEL AND PARAMETERS

Take the following MIMO system as an example [12], its process transfer function and perturbation transfer function are:

$$G_{\mathrm{p}} = \begin{bmatrix} \frac{q^{-1}}{1-0.4\,q^{-1}} & \frac{q^{-2}}{1-0.1\,q^{-1}} \\ \frac{0.3\,q^{-1}}{1-0.1\,q^{-1}} & \frac{q^{-2}}{1-0.8\,q^{-1}} \end{bmatrix} \tag{57}$$

$$G_{\mathrm{w}} = \begin{bmatrix} \frac{1}{1-0.5\,q^{-1}} & \frac{-0.6}{1-0.1\,q^{-1}} \\ \frac{0.5}{1-0.5\,q^{-1}} & \frac{1}{1-0.5\,q^{-1}} \end{bmatrix} \tag{58}$$

The controller is set to:

$$G_{\mathrm{c}} = \begin{bmatrix} \frac{1-0.2\mathrm{q}^{-1}}{1-0.5\,q^{-1}} & 0 \\ 0 & \frac{1-0.2\,q^{-1}}{(1-0.5\,q^{-1})(1+0.5\,q^{-1})} \end{bmatrix} \tag{59}$$

Noises are introduced after forming a closed-loop loop, as follows:

a. $w \sim f(\mathrm{x}) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$, where $\mu = 0, \sigma = 1$.

b. $w \sim f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$, where $\theta_1 = 3, \theta_2 = 0.3$.

c. $w \sim f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$, where $\alpha_1 = 2, \beta_1 = 5, \alpha_2 = 2, \beta_2 = 0.5$.

d. $w \sim Be(\alpha, \beta)$, where the main interference parameters are $\alpha = 9, \beta = 4$.

B. ME-PLS ALGORITHM RESULTS

The ME-PLS algorithm applies not only to non-Gaussian MIMO systems but also to Gaussian systems. The variation

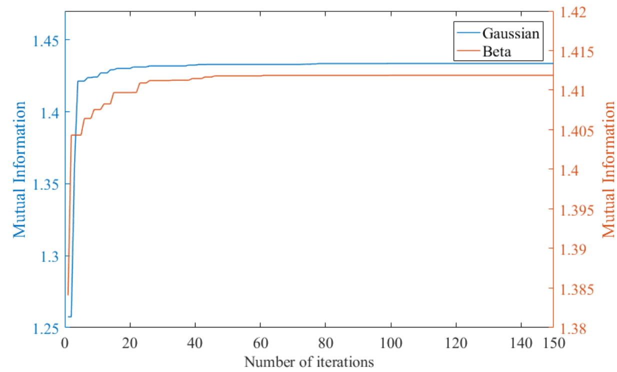

FIGURE 4. Mutual information between t_1 and m_1.

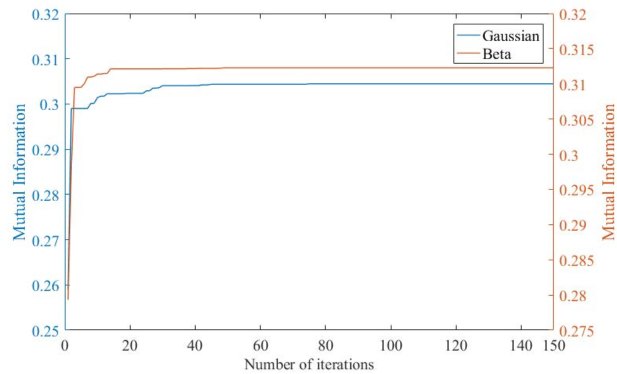

FIGURE 5. Mutual information between t_2 and m_2.

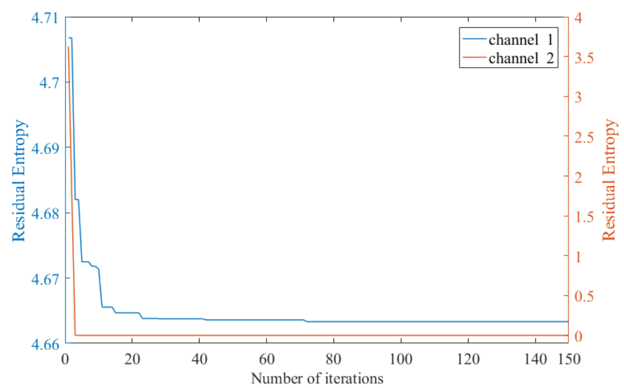

FIGURE 6. Entropy of residual data under the influence of Gaussian noises.

of the mutual information between principal components and the entropy of the residual matrix under the influence of two Gaussian noises or two beta noises is shown in Fig. 4 – Fig. 7.

Fig.4 and Fig.5 show the changes in mutual information among principal components during the optimization of (9). In both figures, the blue line represents the system affected by Gaussian noise, and the orange line represents the system affected by beta noise. It can be seen from the figures that the

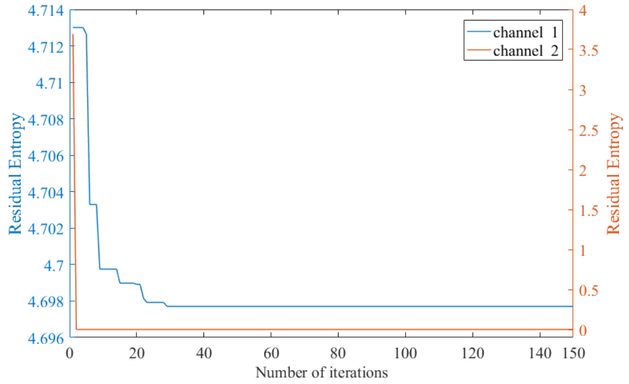

FIGURE 7. Entropy of residual data under the influence of Beta noises.

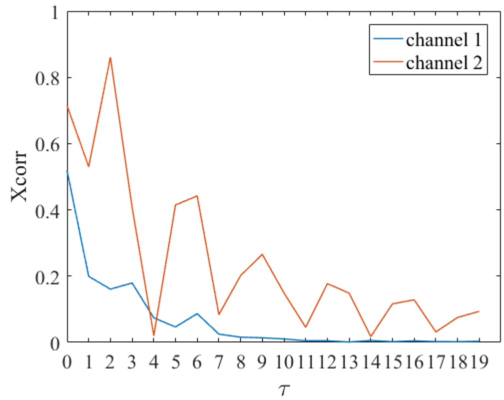

FIGURE 8. Virtual loop time delay estimation.

maximum value of mutual information is reached, at which time t_i has the strongest interpretation of m_i. It means that we find the input data with the most potent control effect on the output data and achieve the separation of the latent structure of the MIMO system.

The blue lines in Fig.6 and Fig.7 indicate the change in the joint entropy of the input signal residual matrix after extracting the principal components t_1, m_1 according to the ME-PLS algorithm. From the two figures, it can be seen that the joint entropy at this time is more significant, which means that there is still some information in the input signal, and the algorithm needs to continue. The orange line represents the joint entropy of the residual matrix of the input signal after the extraction of the second pair of principal components. It can be found that the convergence values all converge to zero, indicating that the information of the input data has been extracted completely to obtain the complete latent structure of the system at this time.

C. PERFORMANCE ASSESSMENT AND CONTROLLER TUNING RESULTS

From Fig. 4 – Fig. 7, it can be seen that the ME-PLS algorithm extracts two pairs of principal components from the system as follows:

$$u = t_1 p_1^T + t_2 p_2^T \tag{60}$$

$$y = m_1 q_1^T + m_2 q_2^T \tag{61}$$

The time delay d of the two virtual loops is first estimated before the performance evaluation. Taking the system affected by two beta noises as an example, the time delay of the virtual loops is estimated as shown in Fig. 8.

The correlation analysis in Fig. 8 shows that the process delay is zero for the t_1, m_1 loop and 2 for the t_2, m_2 loop. The system performance is essentially the ability of the controller to attenuate noises at a steady state, and the virtual output m_1 of virtual loop 1 are feedback invariants that do not require performance assessment. The controller performance assessment of virtual loop two is performed using the method proposed in Section 4.2. The performance assessment values of the controller under the influence of different non-Gaussian noise are shown in Table 1.

TABLE 1. Performance assessment values for virtual loops.

	virtual loop 1	virtual loop 2
two Gaussian noises	/	0.8294
two exponential noises	/	0.8078
two gamma noises	/	0.8231
two beta noises	/	0.8329
Gaussian-exponential noises	/	0.8109
beta-exponential noises	/	0.8009

As seen from Table 1, the performance assessment results of the virtual single loop are all within the interval [0,1], and the controller needs to be adjusted.

The linear relationship between the virtual loop controller and the actual controller can be obtained according to (26), as follows:

$$\hat{G}_{C1} = G_c * \begin{bmatrix} 0.8876 \\ 0.4606 \end{bmatrix} \tag{62}$$

$$\hat{G}_{C2} = G_c * \begin{bmatrix} -0.4394 \\ 0.8832 \end{bmatrix} \tag{63}$$

It is not possible to write the optimal controller structure for virtual loop one, so only virtual loop two is considered to tune the actual controller. The following is obtained by identification:

$$m_2 = \frac{0.5925\ q^{-2}}{1-0.1036\ q^{-1}} t_2 + \frac{1+0.077\ q^{-1}}{1-0.1036\ q^{-1}} \hat{a}_{t2} \tag{64}$$

Decomposition of the virtual noise process function:

$$\begin{aligned} \hat{N}_2 &= \frac{1+0.077\ q^{-1}}{1-0.1036\ q^{-1}} \\ &= 1 + 0.1806\ q^{-1} + \frac{0.0187\ q^{-2}}{1-0.1036\ q^{-1}} \end{aligned} \tag{65}$$

The optimal controller for virtual loop two is obtained as follows:

$$\widehat{G}_{C_{\text{opt}\,2}} = \frac{0.0187}{0.5925 + 0.107\ q^{-1}}$$

TABLE 2. Performance assessment values for virtual loops.

	virtual loop 1	virtual loop 2
two Gaussian noises	/	0.9674
two exponential noises	/	0.9455
two gamma noises	/	0.9600
two beta noises	/	0.9655
Gaussian-exponential noises	/	0.9505
beta-exponential noises	/	0.9452

$$= \frac{0.0316}{1 + 0.1806\, q^{-1}} \tag{66}$$

The requirements for controller adjustment are as follows:

$$G_c * \begin{bmatrix} -0.4394 \\ 0.8832 \end{bmatrix} = G_{C_{\text{opt}\,2}} = \frac{0.0316}{1+0.1806\, q^{-1}} \tag{67}$$

Comparing (59) and (66) reveals that the optimal controller structure of the virtual loop is different from the actual controller structure. The structure of the controller needs to be kept unchanged when tuned. Therefore, (67) can be rewritten as

$$G_c * \begin{bmatrix} -0.4394 \\ 0.8832 \end{bmatrix} = \frac{0.0316\times(1-0.1806\, q^{-1})}{(1+0.1806\, q^{-1})\times(1-0.1806\, q^{-1})} \tag{68}$$

Let the denominator of the actual controller be the same as that of the optimal controller, and the coefficients of the numerator are obtained by computing the least squares solution of the (68). The structure of the adjusted controller is given in the following equation:

$$G_{c_{opt}} = \begin{bmatrix} \frac{-0.0119}{1-0.1806\, q^{-1}} & 0 \\ 0 & \frac{0.0298-0.0326\, q^{-1}}{(1-0.1806\, q^{-1})(1+0.1806\, q^{-1})} \end{bmatrix} \tag{69}$$

The new performance evaluation results obtained by introducing the same noises into the system are shown in Table 2.

Comparing the evaluation results in Table 1 and Table 2, the control effectiveness of the system is improved. It shows that the controller tuning scheme based on a linear relationship is effective.

VI. CONCLUSION

A novel latent structure extraction algorithm, ME-PLS, for non-Gaussian MIMO systems is proposed. The ME-PLS achieves the extraction of principal components by maximizing mutual information. It also ensures that each pair of main features is independent and the complete extraction of information from the original data by minimizing the joint entropy of the residual matrix. Then, the performance assessment method for non-Gaussian systems based on minimum entropy gives the performance evaluation results for mutually independent virtual loops. Finally, the tuning direction of the actual controller is given based on the linear relationship derived from the ME-PLS algorithm and the parameters obtained from the proposed CARMA-LADI identification method. The results of the simulation experiments verify the effectiveness of the technique. The following work will apply this paper's research content to actual production data to validate further and refine the proposed method.

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