IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 18 August 2022, accepted 2 October 2022, date of publication 5 October 2022, date of current version 12 October 2022.
Digital Object Identifier 10.1109/ACCESS.2022.3212081

RESEARCH ARTICLE

An Archive-Based Multi-Objective Arithmetic Optimization Algorithm for Solving Industrial Engineering Problems

NIMA KHODADADI[1], (Member, IEEE), LAITH ABUALIGAH[2,3], (Senior Member, IEEE), EL-SAYED M. EL-KENAWY[4,5], (Senior Member, IEEE), VACLAV SNASEL[6], (Senior Member, IEEE), AND SEYEDALI MIRJALILI[7,8], (Senior Member, IEEE)

[1]Department of Civil and Environmental Engineering, Florida International University, Miami, FL 33199, USA
[2]Hourani Center for Applied Scientific Research, Al-Ahliyya Amman University, Amman 19328, Jordan
[3]Faculty of Information Technology, Middle East University, Amman 11831, Jordan
[4]Faculty of Artificial Intelligence, Delta University for Science and Technology, Mansoura 35712, Egypt
[5]Department of Communications and Electronics, Delta Higher Institute for Engineering and Technology (DHIET), Mansoura 35111, Egypt
[6]Faculty of Electrical Engineering and Computer Science, VŠB-Technical University of Ostrava, 70032 Ostrava, Czech Republic
[7]Centre for Artificial Intelligence Research and Optimization, Torrens University Australia, Adelaide, SA 5000, Australia
[8]Yonsei Frontier Laboartory (YFL), Yonsei University, Seoul 03722, South Korea

Corresponding author: Nima Khodadadi (nkhod002@fiu.edu)

ABSTRACT This research proposes an Archive-based Multi-Objective Arithmetic Optimization Algorithm (MAOA) as an alternative to the recently established Arithmetic Optimization Algorithm (AOA) for multi-objective problems (MAOA). The original AOA approach was based on the distribution behavior of vital mathematical arithmetic operators, such as multiplication, division, subtraction, and addition. The idea of the archive is introduced in MAOA, and it may be used to find non-dominated Pareto optimum solutions. The proposed method is tested on seven benchmark functions, ten CEC-2020 mathematic functions, and eight restricted engineering design challenges to determine its suitability for solving real-world engineering difficulties. The experimental findings are compared to five multi-objective optimization methods (Multi-Objective Particle Swarm Optimization (MOPSO), Multi-Objective Slap Swarm Algorithm (MSSA), Multi-Objective Ant Lion Optimizer (MOALO), Multi-Objective Genetic Algorithm (NSGA2) and Multi-Objective Grey Wolf Optimizer (MOGWO) reported in the literature using multiple performance measures. The empirical results show that the proposed MAOA outperforms existing state-of-the-art multi-objective approaches and has a high convergence rate.

INDEX TERMS Arithmetic optimization algorithm (AOA), archive-based multi-objective arithmetic optimization algorithm (MAOA), multi-objective problems, engineering optimization.

I. INTRODUCTION

Metaheuristics are efficient in solving complex real-world problems mainly due to their black-box optimization nature and gradient-free mechanism. Recently, various metaheuristic methods have been proposed: based on natural processes, collective behavior, or scientific rules, such as Differential Evolution (DE) [1], Ant Colony Optimization (ACO) [2], Particle Swarm Optimization (PSO) [3], Reptile Search Algorithm (RSA) [4], Dynamic Arithmetic Optimization Algorithm (DAOA) [5], Transient Search Optimization (TSO) [6], Dynamic Water Strider Algorithm (DWSA) [7], Aquila Optimizer (AO) [8], Stochastic Paint Optimizer (SPO) [9] and Group Teaching Optimization (GTO) [10] algorithm. Furthermore, several researchers used these algorithms to solve various problems [11], [12], [13], [14], [15], [16], [17]. These studies show that metaheuristic algorithms can solve problems with high precision and a reasonable amount of

The associate editor coordinating the review of this manuscript and approving it for publication was Yudong Zhang.

time when used to solve complex optimization problems. Moreover, metaheuristic algorithms have many advantages over analytical methods, including ease of implementation, a straightforward structure, good accuracy, and an appropriate execution time.

Uncertainties in weights, geometry, structural properties, manufacturing processes, and operating conditions, among other things, are still present in engineering problems [18]. A deterministic optimization can yield a feasible solution for problems with poor uncertainties. On the other hand, deterministic optimization may result in inefficient or inconsistent solutions for issues with high uncertainties, raising the probability of design failure. Several nondeterministic optimization algorithms have been designed to address this matter and successfully solve various design problems in real-world engineering [19], [20], [21].

Since the advent of computing, the field of computer-aided design has arisen, allowing engineers to use computers to solve engineering problems. Designers will use a computer to model the dilemma and its variables in the early stages. As a result, there was no need to create a prototype and test it on real-world problems. Computer software, for example, can make the 3D outline of a car's body when simulating wind drag [22]. In this scenario, the programmers were more concerned with architecture than simulation. This resulted in a substantial reduction in the overall expense of the design method and human interaction.

The vast majority of the real-world optimization problems are multi-objective by nature [23]. Multi-objective optimization is a branch of decision-making that balances various frequently disputed goals [24]. It has been used in various research fields, including physics, economics, and logistics, where optimum choices must be considered in balancing between two or more competing goals. A single-objective approach will only represent one part of a multi-objective problem if used to solve it. A single-objective algorithm, for example, cannot wholly describe the hydrological mechanism and function in optimizing hydrological design variables [25].

The task of solving one objective function problem is easy. The optimal solution is the product of such problems, and the optimization methods are assessed on the nature of the current solutions using the specified objective functions [26]. However, many objective functions compete with one another in multi-objective problems. There is no one-size-fits-all approach that is optimal for all of them simultaneously. Therefore, the decision-makers are searching for the established alternative instead of the best solution in these situations. The Pareto solutions are a subset of these types of solutions. As a result, in multi-objective problems, optimality is renewed by the method of Pareto optimality. The Pareto optimal (besides known as effective, non-dominated, or non-inferior) solutions cannot increase their efficiency in one objective function without affecting the achievement of at least one of the others; this is the strategy of dealing with more

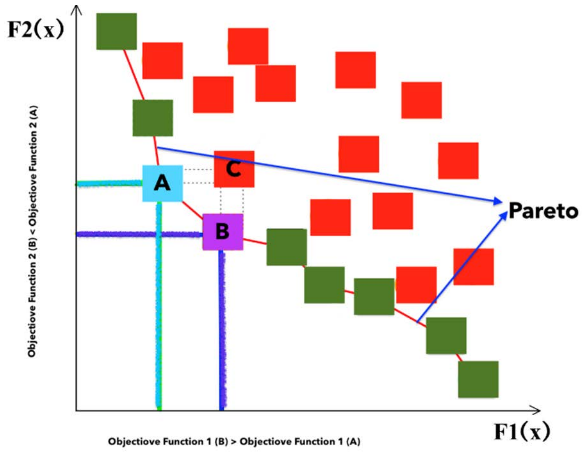

FIGURE 1. Pareto dominance.

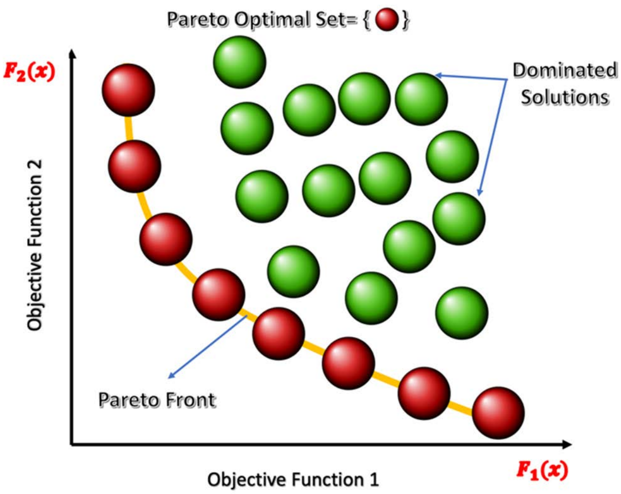

FIGURE 2. Pareto optimal solutions.

than objean ctive function. The Pareto set is the collection of Pareto optimal solutions [27].

The following is an example of a general multi-objective optimization problem:

$$minimize\ F(x) = ((f_1(x), f_2(x), f_i(x), \ldots, f_t(x))^T, \quad x \in \Omega \tag{1}$$

where Ω is the possible decision values, $x \in R^k$ is the decision variable, k is the number of decisions, $F\colon \Omega \to R^m$ is a scalar objective function with multidimensional objective components $fi(x)$, $i = 1, 2, \ldots, t \geq 2$, and R^m is the objective range. Thus, F(x) contains t objectives, and the given problem includes k decisions. The decision range $\Omega \subset R^k$ is a linked and closed area. Moreover, all objective functions are continuous throughout the decision space; thus, a multi-objective function in any real optimization problem is a continuous multivariable problem [28].

Most recent algorithms have rules in place to solve problems with multiple goals. However, the famous

TABLE 1. Parameters setting of all algorithms.

Parameters	MOPSO	MSSA	MOALO	NSGA2	MOGWO	MAOA
Mutation Probability (P_w, *or* pro)	0.5	-	-	1/D	-	-
Population Size (N_{pop})	100	100	100	100	100	100
Archive Size (N_{rep}, *or* TM)	100	100	100	100	100	100
Number of Adaptive Grid (N_{grid})	30	30	30	-	30	30
Personal Learning Coefficient (C_1)	1	-	-	-	-	-
Global Learning Coefficient (C_2)	2	-	-	-	-	-
Inertia weight (w)	0.4	-	-	-	-	-
Beta	4	4	4	-	4	4
Gamma	2	2	2	-	2	2
Crossover probability (P_c)	-	-	-	0.9	-	

TABLE 2. Multimodal benchmark functions with fixed-dimension.

Function	Mathematical formulation	D	Range
ZDT1	$F_1 = x_1, \quad F_2 = g\left(1-\sqrt{F_1/g}\right), \quad g = 1+\frac{9}{d-1}\sum_{i=2}^{d} x_i$	30	$x_i \in [0,1]$
ZDT2	$F_1 = x_1, \quad F_2 = g\left(1-(F_1/g)^2\right), \quad g = 1+\frac{9}{d-1}\sum_{i=2}^{d} x_i$	30	$x_i \in [0,1]$
ZDT3	$F_1 = x_1, \quad F_2 = g\left(1-\sqrt{F_1/g} - F_1/g\,\sin(10\pi F_1)\right),$ $g = 1+\frac{9}{d-1}\sum_{i=2}^{d} x_i$	30	$x_i \in [0,1]$
ZDT4	$F_1 = x_1, \quad F_2 = g\left(1-\sqrt{F_1/g}\right),$ $g = 1+10(d-1)+\sum_{i=2}^{d}(x_i^2 - 10\cos(4\pi x_i))$	10	$x_1 \in [0,1]$ $x_i \in [-5,5]$ $i = 1, \ldots, D$
ZDT6	$F_1 = 1-\exp(-4x_1)\sin^6(6\pi x_1), \quad F_2 = g\left(1-\left(F_1/g\right)^2\right),$ $g = 1+9\left(\frac{\sum_{i=2}^{d} x_i}{d-1}\right)^{0.25}$	10	$x_i \in [0,1]$
DTLZ2	$F_1 = (1+g)\cos\left(x_1\left(\frac{\pi}{2}\right)\right)\cos\left(x_2\left(\frac{\pi}{2}\right)\right), F_2 = (1+g)\cos\left(x_1\left(\frac{\pi}{2}\right)\right)\sin\left(x_2\left(\frac{\pi}{2}\right)\right),$ $F_3 = (1+g)\sin\left(x_1\left(\frac{\pi}{2}\right)\right), g = \sum_{i=3}^{d}(x_i-0.5)^2$	12	$x_i \in [0,1]$
DTLZ4	$F_1 = (1+g)\cos\left(x_1^{\pi}\left(\frac{\pi}{2}\right)\right)\cos\left(x_2^{\pi}\left(\frac{\pi}{2}\right)\right), F_2 = (1+g)\cos\left(x_1^{\pi}\left(\frac{\pi}{2}\right)\right)\sin\left(x_2^{\pi}\left(\frac{\pi}{2}\right)\right),$ $F_3 = (1+g)\sin\left(x_1^{\pi}\left(\frac{\pi}{2}\right)\right), g = \sum_{i=3}^{d}(x_i-0.5)^2$	12	$x_i \in [0,1]$

No-Free-Lunch (NFL) [29] theorem theoretically proves that no one best method or algorithm can solve all optimization problems. This means existing algorithms can be improved or new ones proposed to help solve particular problems. However, new algorithms are best suited to unconstrained problems and cannot handle various constraints without

Pseudo-Code of the MAOA Algorithm

```
Initialize the MAOA parameters
Initialize the soLutions' positions
Evaluate the fitness function for given soLutions
Obtain the non-dominated answers and create the archive
While (Iteration < Max-Iteration)
    Update MOA and MOP with Eqs (10) and (11)
      For each soLution
          Find Leader
              for each position
                Create a random number for rl, r2, r3 between [0,1]
              if r1>MOA then the exploration step will be active
                  If r2>0.5
                      Update the position with the first ruie of Eq (12)
                Else
                      Update the position with the second ruLe of Eq (12)
              End if
          Else
              if r3>M0A then the exploitation Step will be active
                  If r2>0.5
                      Update the position with the first ruLe of Eq (13)
                Else
                      Update the position with second ruLe of Eq (13)
                  End if
              End if
          End for
      End for
Evaluate the objective values for aLL popuLation
Obtain the non-dominated answers
Update the archive according to the found non-dominated answers
      If the archive is compLeted
          Use the grid mechanism to deLete the current archives
          Add the new answer to the archive
      Endif
      If any of the newLy added answers to the archive is pLaced outside of the hypercubes
          Update the grids
      End if
End while;
Return Archive
```

using specific components. The multi-objective variant of the recently introduced Arithmetic Optimization Algorithm (AOA) is proposed in this paper to solve both unconstrained and constrained problems. The mathematical operators in math sciences became the main inspiration for the conventional AOA. The Archive-based Multi-objective Arithmetic Optimization Algorithm (MAOA) uses an archive and leader selection process used in Multi-objective Particle Swarm Optimization. The experiments are conducted on several various real-world engineering problems. The proposed MAOA proved its ability to solve multiple complex problems compared to other state-of-the-art multi-objective methods. The remainder of the article is laid out as follows:

Most recent algorithms have rules in place to solve problems with multiple goals. However, the well-known No-Free-Lunch (NFL) theorem proves that none of such algorithms can solve all optimization problems. This means existing algorithms can be improved or new ones proposed to help solve particular problems. New algorithms are best suited to unconstrained problems and cannot handle various kinds of constraints without the use of specific components. The multi-objective variant of the recently introduced Arithmetic Optimization Algorithm (AOA) is proposed in this paper to solve both unconstrained and constrained problems. The mathematical operators in math sciences became the main inspiration for the conventional AOA. The Archive-based Multi-objective Arithmetic Optimization Algorithm

TABLE 3. The statistical results of mathematical functions for GD performance metric.

Functions		MOPSO	MSSA	MOALO	MAOA
ZDT1	Ave	1.9932E-03	1.2035E-02	1.9644E-03	**1.8467E-03**
	SD	5.0496E-03	1.9168E-03	**1.7358E-04**	2.5549E-03
	WRT	+	+	-	
ZDT2	Ave	1.4749E-01	1.1652E-02	4.0621E-04	**2.1622E-04**
	SD	6.5211E-02	2.1916E-03	1.9757E-05	**1.6201E-05**
	WRT	+	+	+	
ZDT3	Ave	**2.0489E-04**	1.6015E-02	1.7253E-03	2.1406E-03
	SD	**2.7322E-05**	2.0859E-03	1.1636E-03	3.4858E-03
	WRT	-	+	+	
ZDT4	Ave	4.1299E+00	6.0099E-01	2.0045E+00	**2.6285E-04**
	SD	3.9519E+00	3.1030E-01	9.6098E-01	**3.2461E-05**
	WRT	+	+	+	
ZDT6	Ave	2.5805E-02	2.1047E-02	3.3034E-02	**1.6268E-03**
	SD	5.8008E-02	9.6106E-03	2.0337E-02	**5.0708E-03**
	WRT	+	+	+	
DTLZ2	Ave	**6.8600E-03**	9.2866E-03	2.1134E-02	1.8112E-01
	SD	**9.0878E-04**	2.4582E-03	1.0861E-02	1.0018E-02
	WRT	-	+	+	
DTLZ4	Ave	**1.0097E-02**	1.3120E-02	3.6328E-02	2.1398E-01
	SD	**2.4570E-03**	3.8723E-03	1.9865E-02	2.3231E-02
	WRT	-	+	+	
W^+/W^-		8/6	14/0	14/1	
$+/-/=$		4/3/0	7/0/0	7/0/0	

(MAOA) uses an archive and leader selection process used in Multi-objective Particle Swarm Optimization. The experiments are conducted on several various real-world engineering problems. The proposed MAOA proved its ability to solve multiple complex problems compared to other well-known multi-objective methods. The remainder of the article is laid out as follows:

Section 2 presents the multi-objective optimization (MOO) theory. Section 3 introduces Arithmetic Optimization Algorithm and its multi-objective version. Performance metrics and results of test functions and engineering problems are discussed in Section 4. Finally, this study is concluded in the final Section.

II. LITERATURE REVIEW

The Multi-objective Optimization (MOO) theory and current meta-heuristic techniques are discussed in this Section.

Due to the unary goal and the existence of just one most satisfactory solution in single-objective problems, the global optimum in single-objective problems is only one solution. When only one target is in consideration, comparing solutions with relational operators is easy. Because of the characteristics of such problems, evaluating potential solutions and selecting the best one is easy for optimization problems. Multi-objective evaluation is a branch of multiple approaches to solving those deal with mathematical programming issues that require the evaluation of numerous objective functions at the same time.

As a result, research into effective and reliable multi-objective optimization algorithms is needed. Many interesting experiments have been conducted in multi-objective optimization to deal with complicated multi-objective nonlinear problems. A novel congestion control method is proposed for Wireless Sensor Networks in [30]. Congestion management is conducted by determining the optimum rate by a simple Poisson method. The current techniques used to solve this problem have high complications and power consumption due to retransmission. To address these problems, the multi-objective optimization technique (PSOGSA) is used to suggest a congestion control strategy [30]. For speed optimization and controlling the data arrival rate from each child node to the parent node. The proposed method used two main search mechanisms; Particle Swarm Optimization (PSO) and Gravitational Search Algorithm (GSA).

TABLE 4. The statistical results of mathematical functions for IGD performance metric.

Functions		Algorithm			
		MOPSO	*MSSA*	*MOALO*	*MAOA*
ZDT1	Ave	8.0879E-04	4.5908E-03	1.5144E-02	**5.0981E-04**
	SD	1.4764E-03	1.1201E-03	2.1178E-03	**6.5912E-05**
	WRT	+	+	+	
ZDT2	Ave	5.2023E-02	5.2059E-03	2.1623E-02	**5.0588E-04**
	SD	9.5007E-03	1.6220E-03	1.7638E-03	**4.2200E-05**
	WRT	+	+	+	
ZDT3	Ave	2.6241E-04	5.1290E-03	3.7118E-03	**2.5010E-04**
	SD	3.9884E-05	8.9560E-04	1.6638E-03	**3.4233E-05**
	WRT	+	+	+	
ZDT4	Ave	7.0747E-01	1.8230E-01	6.1546E-01	**9.0817E-04**
	SD	3.1784E-01	9.5457E-02	2.4280E-01	**1.7711E-04**
	WRT	+	+	+	
ZDT6	Ave	8.2281E-03	2.0670E-03	3.4649E-03	**6.2074E-04**
	SD	2.4862E-02	8.0527E-04	1.8983E-03	**9.2925E-05**
	WRT	+	+	+	
DTLZ2	Ave	**4.6940E-04**	3.0425E-03	3.2518E-03	6.8738E-03
	SD	**2.6098E-05**	3.7517E-04	4.6848E-04	3.1549E-04
	WRT	-	+	+	
DTLZ4	Ave	**1.6840E-03**	8.3377E-03	1.1027E-02	2.1954E-02
	SD	**9.4057E-05**	1.1001E-03	2.9491E-03	1.2775E-03
	WRT	-	+	+	
W^+/W^-		10/4	14/0	14/0	
$+/-/=$		5/2/0	7/0/0	7/0/0	

A multi-objective optimization function considers the energy of the connection in its fitness value. Priority-dependent transmission is allowed because the optimization strategy controls the arrival rate based on priority: available performance bandwidth and child node capacity. Adjusting the speed to the desired value is used to alleviate congestion. The results of simulations show that the proposed method outperforms current methods.

Due to its considerable impact on grid security and power dispatching, researchers have recently paid growing awareness to reliable and consistent wind speed estimation. On the other hand, most current research has concentrated solely on improving precision or stability, with only a few studies covering both problems simultaneously. This job is difficult due to the intermittency and dynamic variations of wind speed. This paper proposes an innovative hybrid method based on the Multi-Objective Whale Optimization Algorithm [31], called MOWOA, consisting of 4 modules: optimization, data preprocessing, forecasting, and assessment. The suggested MOWOA's optimization module is used to configure the weights and thresholds of the neural network utilized in the predicted module to achieve high precision and confidence for wind speed prediction while overcoming the shortcomings of single-objective algorithms. The findings show that the proposed MOWOA outperforms the other sixteen methods.

Cloud computing service arrangements are a complicated optimization problem for a big company, given many users and owned services. Sheikholeslami and Navimipour [32] considered three competing goals: optimizing resources for consumers and suppliers and identifying the best solution at the right time. Since the highly efficient converging to the Pareto front produces a well-distributed collection of non-dominated solutions, a Crowding Distance is used in the Multi-Objective Particle Swarm Optimization (MOPSO-CD) to address the problem. Furthermore, the best-obtained solution is often determined using the fuzzy set approach. Again, the suggested method's efficiency is related to two other multi-objective algorithms' performance to demonstrate that it is powerfully efficient against them. Finally, the experiment findings revealed that the system increases the speed at which the services distribution algorithm is executed while also creating significant income for consumers and suppliers and increasing resource efficiency.

TABLE 5. The statistical results of mathematical functions for MS performance metric.

Functions		MOPSO	MSSA	MOALO	MAOA
ZDT1	Ave	1.0070E+00	9.0233E-01	3.0049E-01	**1.0828E+00**
	SD	2.3779E-02	1.0585E-01	5.1645E-02	**1.7251E-02**
	WRT	+	+	+	
ZDT2	Ave	7.4311E-02	7.7491E-01	3.6133E-02	**1.0000E+00**
	SD	2.3499E-01	1.0567E-01	4.3001E-02	**0.0000E+00**
	WRT	+	+	+	
ZDT3	Ave	9.9865E-01	8.7507E-01	7.1037E-01	**1.0303E+00**
	SD	**1.7888E-03**	6.5264E-02	1.1021E-01	6.0899E-02
	WRT	-	+	+	
ZDT4	Ave	4.6863E-01	1.4795E-01	2.3408E-01	**1.0000E+00**
	SD	1.4819E+00	1.7705E-02	4.8105E-02	**0.0000E+00**
	WRT	+	+	+	
ZDT6	Ave	1.1170E+00	1.1361E+00	1.1199E+00	**1.2932E+00**
	SD	5.3983E-01	1.8535E-01	5.8815E-01	**1.4093E-01**
	WRT	+	+	+	
DTLZ2	Ave	1.0976E+00	1.8014E-01	1.5348E-01	**2.7164E+00**
	SD	**3.0869E-02**	1.6986E-01	6.5189E-02	7.8217E-02
	WRT	**-**	+	+	
DTLZ4	Ave	1.2437E+00	2.5352E-01	3.3046E-01	**2.7172E+00**
	SD	1.4187E-01	2.5166E-01	1.8953E-01	**1.0237E-01**
	WRT	+	+	+	
W^+/W^-		14/0	14/0	14/0	
+/−/=		7/0/0	7/0/0	7/0/0	

A multi-objective method-based energy-aware, called EA-MOA, is proposed in [33] to solve a complex problem (flow shop scheduling) when considering service energy consumption. Simultaneously, two goals are considered: the reduction of the makespan and the reduction of energy demand. First, each solution is expressed by two vectors in the proposed EA-MOA: the priority vector for machine assignment and the scheduling vector. Second, four different decoding techniques are tested to account for these goals. Third, to use the origin solutions from the archive of the Pareto package, two dynamic crossover mechanisms, Single-point Pareto-based crossover (SPBC) and Two-point Pareto-based crossover (TPBC), is performed. After that, eight neighborhood search processes and an adaptive neighborhood selection approach are constructed based on the problem structure. Besides, a right-shifting protocol is used to reduce the running time for all devices, thus increasing the specified solution's energy consumption target. Finally, the proposed EA-MOA is analyzed against several well-known benchmark cases. Finally, the highly successful proposed EA-MOA method is tested and analyzed with other qualified algorithms by studying experimental effects.

Mirjalili et al. [34] proposed the Multi-Objective version of Ant Lion Optimizer (MOALO), which is modeled by the hunting behavior of antlions and the way ants interact with them. The non-dominated Pareto optimum answers thus far are stored in a repository first. In this case, a roulette wheel mechanism is used to choose the best solutions from this repository. The roulette wheel chooses the best multi-objective search areas based on how many antlions each solution has. Several standard unconstrained and restricted test functions demonstrate the method's effectiveness under consideration. The approach is also used to solve a number of multi-objective engineering design problems.

A multi-objective version of the Crystal Structure Algorithm (MOCryStAl) was recently presented by Khodadadi et al. [35], and it was motivated by the ideas of crystal structure creation. This algorithm's three primary mechanisms were the grid, leader selection, and archive. The efficacy of the presented method is evaluated using mathematic problems and real-world engineering design challenges. The findings suggest that the proposed solutions can produce remarkable results when applied to the multi-objective challenges under examination.

TABLE 6. The statistical results of mathematical functions for S performance metric.

Functions		Algorithm			
		MOPSO	*MSSA*	*MOALO*	*MAOA*
ZDT1	Ave	1.1319E-02	1.2165E-02	**5.1688E-03**	1.4712E-02
	SD	**1.0762E-03**	3.3123E-03	1.9499E-03	5.9223E-03
	WRT	-	+	+	
ZDT2	Ave	7.3295E-02	1.2250E-02	8.6826E-02	**6.2168E-02**
	SD	2.3178E-03	6.3466E-03	1.9868E-03	**1.9380E-03**
	WRT	+	+	+	
ZDT3	Ave	1.3270E-02	1.4228E-02	1.4655E-02	**1.2602E-02**
	SD	2.3561E-03	6.5709E-03	9.2716E-03	**2.1021E-03**
	WRT	+	+	+	
ZDT4	Ave	4.3804E-03	**2.2121E-03**	1.4977E-02	2.1446E-02
	SD	1.3852E-02	1.9168E-03	2.5415E-02	**1.8980E-03**
	WRT	+	+	+	
ZDT6	Ave	1.9109E-02	1.8684E-02	2.4910E-02	**1.9105E-02**
	SD	3.0071E-02	1.2219E-02	1.2571E-02	**1.0440E-02**
	WRT	+	+	+	
DTLZ2	Ave	5.7741E-02	**9.1604E-03**	1.1726E-02	2.0007E-01
	SD	4.9507E-03	5.5147E-03	**5.3284E-03**	2.7188E-02
	WRT	+	+	-	
DTLZ4	Ave	6.1316E-02	**1.5061E-02**	1.8314E-02	2.4654E-01
	SD	**5.3260E-03**	7.5177E-03	1.0509E-02	4.4854E-02
	WRT	-	+	+	
W^+/W^-		14/0	8/6	12/2	
+/−/=		7/0/0	4/3/0	6/1/0	

Francis Ysidro et al. [36] compared two solutions to multi-objective problems in 1881, and Vilfredo Pareto [37] expanded on this in 1964. The following is multi-objective optimization (MOO) for minimization problems [38]:

$$\textit{Maximization}: \ F(\vec{x}) = \{f_1(\vec{x}), f_2(\vec{x}), \ldots, f_z(\vec{x})\} \quad (2)$$

$$\textit{Subject to}: \ g_i(\vec{x}) \geq 0 \quad i = 1, 2, \ldots, m \quad (3)$$

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \ldots, p \quad (4)$$

$$L_i \leq x_i \leq U_i \quad i = 1, 2, \ldots, n \quad (5)$$

where, the number of variables, inequality and equality constraints, objective functions are described as m, p, n and z, respectively. The boundaries of ith variable are indicated by L_i and U_i.

A. PARETO DOMINANCE

In this part, there are a couple of solutions (i.e., $\vec{x}$ and $\vec{y}$) with fitness functions values $\vec{x} = (x_1, x_2, \ldots, x_k)$ and $\vec{y} = (y_1, y_2, \ldots, y_k)$ that accomplished the given constraints of a multi-objective problem. The main aim here is to determine the smallest objective value, the solution $\vec{x}$ is found to dominate $\vec{y}$(*denote as* $\vec{x} \prec \vec{y}$), if the current values of $\vec{y}$ are not smaller than the current cost value of $\vec{x}$, and at least one component of $\vec{x}$ must be smaller than that of $\vec{y}$ (see Fig. 1).

This definition may be expressed numerically as follows [39]:

$$\forall i \in \{1, 2, \ldots, k\}: \quad f_i(\vec{x}) \leq f_i(\vec{y}) \wedge \exists i \in \{1, 2, \ldots, k\}: \quad f_i(\vec{x}) \leq f_i(\vec{y}) \quad (6)$$

The concept of Pareto optimality, as per this concept of Pareto domination, is as follows:

B. PARETO OPTIMALITY (PO)

A solution $\vec{x} \in X$ is called Pareto-optimal if and only if:

$$\nexists \vec{y} \in X | \vec{y} \prec \vec{x} \quad (7)$$

The Pareto optimum is a set of non-dominated solutions to complex problems that are presented as follows:

C. PARETO OPTIMAL SET (POS)

The Pareto optimum set (Ps) for a given MOO is described as Eq. (8). This set comprises all viable solutions that cannot be controlled by anyone else plausible option. The collection of Pareto optimum solutions is depicted in Fig. 2.

$$P_s := \{\vec{x}, \vec{y} \in X | \nexists \vec{y} \prec \vec{x}\} \quad (8)$$

The following is the definition and description of the Pareto optimal front:

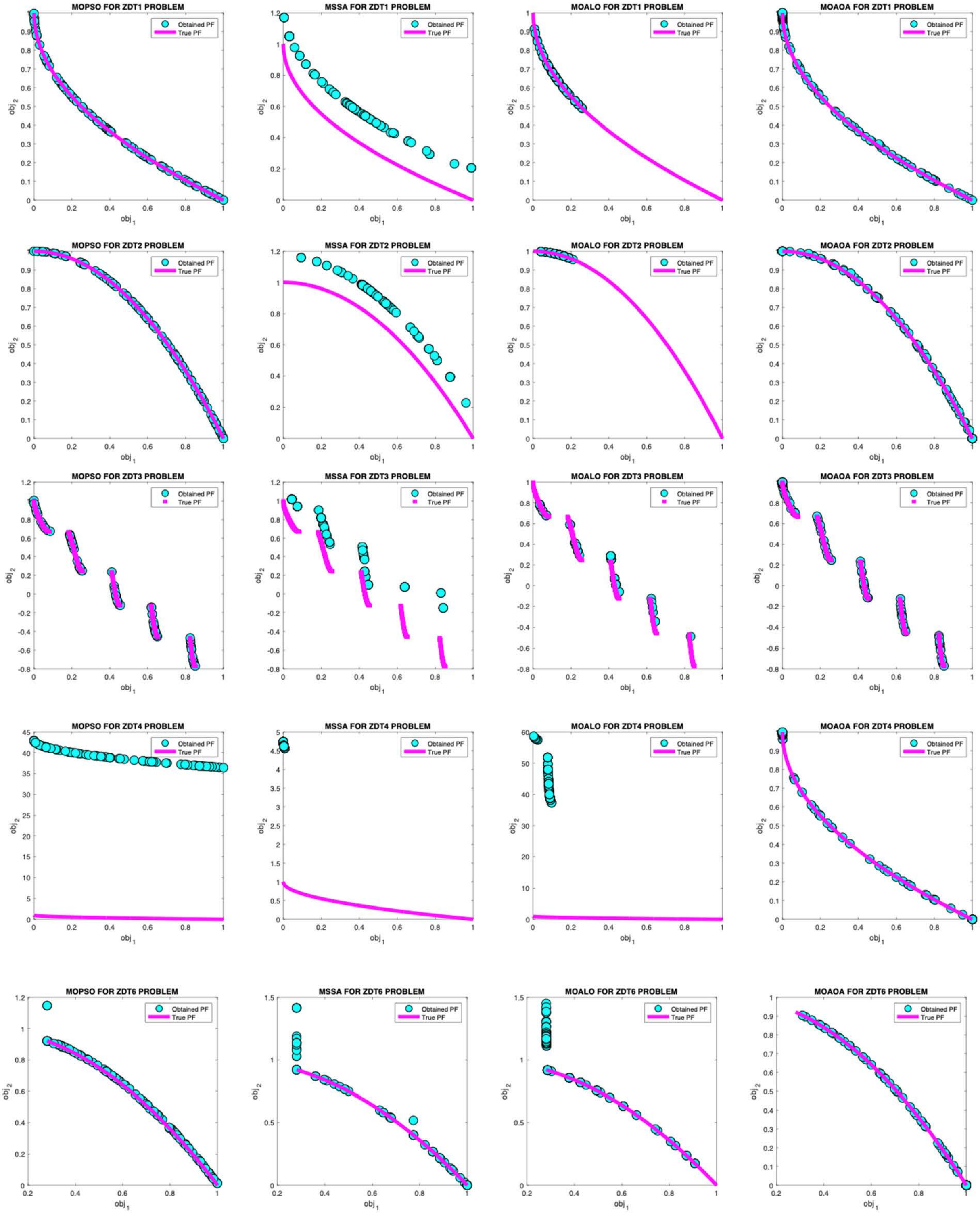

FIGURE 3. True and obtained Pareto front for ZDT benchmarks.

D. PARETO OPTIMAL FRONT (POF)

A Pareto front (P_f) illustrates the Pareto optimal set in the objective space as shown in Fig. 2. According to the above definitions, it can be expressed as Eq. (9).

As illustrated in Fig. 2, a Pareto front (P_f) depicts the Pareto optimum subset in the goal space. It may be stated as Eq. as per preceding Eq. (9).

$$P_f = \{F(\vec{x}), \vec{x} \in P_s\} \tag{9}$$

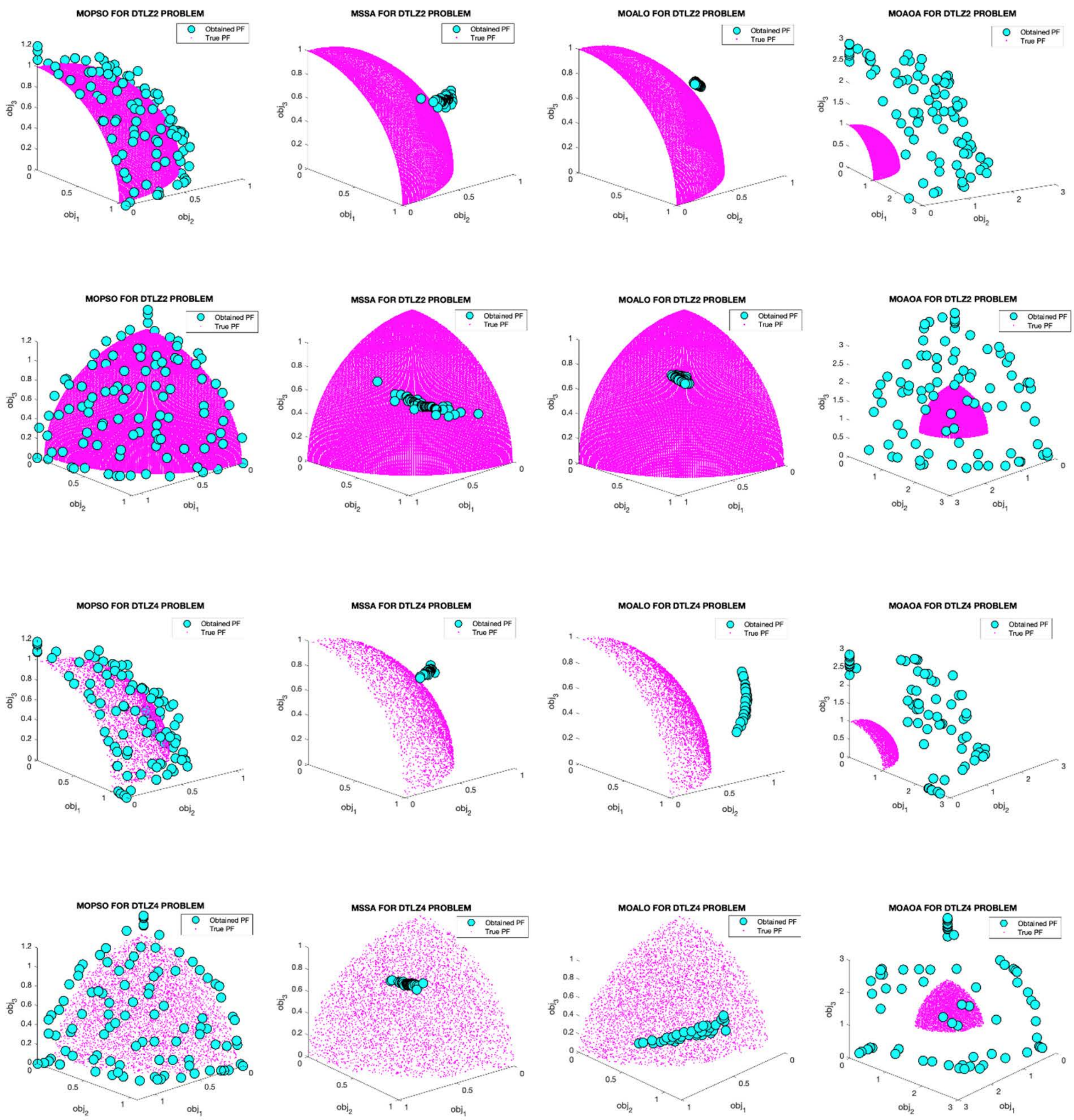

FIGURE 4. True and obtained Pareto front for DTLZ2 and DTLZ4.

To solve a multi-objective problem, one must determine the Pareto optimal set, a collection of solutions representing the best equilibrium between goals.

III. MULTI-OBJECTIVE ARITHMETIC OPTIMIZATION ALGORITHM

In the following Section, the conventional AOA method is presented first. After that, a novel multi-objective MAOA is developed and proposed for addressing multi-objective optimization s.

A. ARITHMETIC OPTIMIZATION ALGORITHM

Based on mathematical formulas and procedures, Abualigah et al. [40] suggested this approach in 2020. Like other metaheuristic algorithms, the AOA method starts with a set of random values. Each iteration determines the objective value of each solution. Two governing variables in this method, MOA and MOP, need to be adjusted before updating the position of solutions:

$$MOA\,(t) = min + t \times \left(\frac{Max - Min}{T}\right) \tag{10}$$

where t is the latest iteration, T represents the maximum iterative process, and *Max/Min* represents the maximum and lowest values used to bind MOA.

$$MOP\,(t) = 1 - (\frac{t}{T})^{\frac{1}{\alpha}} \tag{11}$$

where α is a controlling parameter.

TABLE 7. The statistical results of engineering problems for GD performance metric.

Functions		Algorithm			
		MOPSO	*MSSA*	*MOALO*	*MAOA*
BNH	Ave	**3.3894E-02**	6.4575E-02	5.3758E-02	1.5266E-01
	SD	**2.1392E-03**	1.5035E-02	1.7132E-02	6.0530E-02
	WRT	-	+	+	
CONSTR	Ave	8.3098E-04	1.6878E-03	1.5711E-03	**6.7091E-04**
	SD	3.3369E-04	5.8950E-04	7.7806E-04	**1.9336E-04**
	WRT	+	+	+	
DISK BREAKE	Ave	2.3045E-03	6.2545E-03	4.1887E-02	**2.1020E-03**
	SD	5.6273E-04	3.3423E-03	5.4951E-03	**5.3082E-04**
	WRT	+	+	+	
4-BAR TRUSS	Ave	**1.4095E+01**	7.7966E+00	2.5901E+00	2.1487E+01
	SD	**5.1580E-01**	3.5486E+00	2.2161E+00	1.7221E+00
	WRT	-	+	+	
WELDED BEAM	Ave	1.0846E-02	6.3467E-03	4.4399E-03	**4.2899E-03**
	SD	1.9956E-03	3.0983E-03	**6.9956E-04**	1.3605E-03
	WRT	+	+	-	
OSY	Ave	3.5765E+00	9.4637E-01	**7.0760E-01**	1.9774E+00
	SD	2.5250E+00	**1.9612E-01**	2.4932E-01	1.5031E+00
	WRT	+	-	+	
SPEED REDUCER	Ave	1.8394E+01	7.8657E+00	7.6817E+00	**6.8854E+00**
	SD	5.3101E+00	3.6189E+00	3.4697E+00	**2.9191E+00**
	WRT	+	+	+	
SRN	Ave	3.1617E-02	4.1566E-02	**1.5798E-02**	4.8994E-01
	SD	1.0695E-02	2.4971E-02	**3.2892E-03**	1.3375E-01
	WRT	+	+	-	
W^+/W^-		12/4	16/0	12/4	
+/−/=		6/2/0	8/0/0	6/2/0	

A random number called *r1* is created after MOA and MOP have been updated to shift between exploration and exploitation. The exploration search processes are used to find the available solutions and improve them according to the mathematical of AOA. Additionally, during this improvement stage, the exploration operators (D and M) were used to help the exploitation step of the search phase by enhancing communication between both. The following equation is used for exploration:

$$x_{i,j}(t+1) = \begin{cases} \dfrac{best(x_j)}{MOP+\epsilon} \times (UB_j - LB_j) \times \mu + LB_j & if\ r2 < 0.5 \\ best(x_j) \times MOP \times (UB_j - LB_j) \times \mu + LB_j & if\ r2 \geq 0.5 \end{cases} \tag{12}$$

where μ is a parameter, ϵ is a small number, and $r2$ is a random value. $Best(x_j)$ is the best *jth* value in the best-obtained solution so far. UB_j and LB_j are the lower and upper boundaries of the underlying problem.

The exploitation search processes of the AOA are used to search for a new solution according to the available candidate solutions. AOA's exploitation techniques (S and A) investigate the search area carefully in many dense places and use a two-pronged approach to find a better solution. The following equation is utilized for the exploitation process:

$$x_{i,j}(t+1) = \begin{cases} best(x_j) - MOP \times (UB_j - LB_j) \times \mu + LB_j & if\ r3 < 0.5 \\ best(x_j) + MOP \times (UB_j - LB_j) \times \mu + LB_j & if\ r3 \geq 0.5 \end{cases} \tag{13}$$

B. ARCHIVE-BASED MULTI-OBJECTIVE ARITHMETIC OPTIMIZATION ALGORITHM (MAOA)

In order to execute multi-objective optimization, AOA has received two further upgrades. The functions utilized are identical to those in MOPSO [41]. The first is an archive containing all of the non-dominated Pareto optimum solutions thus far. The following function is the leader selection feature,

TABLE 8. The statistical results of engineering problems for IGD performance metric.

Functions		Algorithm			
		MOPSO	MSSA	MOALO	MAOA
BNH	Ave	9.6868E-03	3.3817E-03	7.5631E-03	**1.0889E-03**
	SD	1.7147E-04	1.1513E-03	4.5344E-03	**1.4497E-04**
	WRT	+	+	+	
CONSTR	Ave	**5.1838E-04**	7.4460E-04	2.1706E-03	2.9843E-03
	SD	**5.5618E-05**	1.7697E-04	7.3799E-04	1.7713E-03
	WRT	-	+	+	
DISK BRAKE	Ave	5.8831E-04	7.2091E-04	1.7399E-03	**6.5336E-04**
	SD	**5.5995E-05**	1.1206E-04	7.7421E-04	9.1198E-05
	WRT	-	+	+	
4-BAR TRUSS	Ave	2.0410E-02	2.1409E-02	2.1434E-02	**2.0225E-02**
	SD	3.9632E-05	3.5445E-04	3.8292E-04	**2.6571E-04**
	WRT	+	+	+	
WELDED BEAM	Ave	**5.9705E-04**	1.3441E-03	4.8242E-03	1.0993E-03
	SD	**4.6341E-05**	3.9377E-04	3.5852E-03	2.5765E-04
	WRT	-	+	+	
OSY	Ave	1.4663E-02	7.6220E-03	7.7532E-03	**7.1949E-03**
	SD	8.6917E-03	**6.4006E-04**	1.2021E-03	2.8459E-03
	WRT	+	-	+	
SPEED REDUCER	Ave	6.6530E-02	1.4243E-02	1.4933E-02	**1.3731E-02**
	SD	3.1310E-02	**3.2032E-03**	5.7981E-03	9.1710E-03
	WRT	+	-	+	
SRN	Ave	**4.5146E-04**	2.4823E-03	2.2308E-03	6.4963E-04
	SD	**1.1656E-04**	1.0614E-03	1.5266E-03	2.3927E-04
	WRT	-	+	+	
W^+/W^-		10/6	16/0	16/0	
$+/-/=$		5/3/0	8/0/0	8/0/0	

which aids in selecting the best available place solutions from the repository as the search process's leaders.

The archive is a simple storing mechanism for non-dominated Pareto optimum solutions produced up to this point. Because it controls the archive when a solution is meant to be archived or when the archive is complete, an archive controller is a vital component. Consider that only a tiny number of people can contribute to the archive. Non-dominated solutions produced up to this point are compared against the existing archive during the training process of an iteration. Three different situations might happen:

- The search agents trade details about the search space, rapidly moving toward the genuine Pareto optimal front.
- Multi-objective strategies aid in obtaining a single run that approximates the whole genuine Pareto optimal front.
- Keeping a problem's multi-objective solution allows for problem investigation across various design requirements and operational environments.

The likelihood of removing an answer increases directly to the number of responses in the hypercube (Section). If the archive is complete, one of the most densely inhabited areas is chosen for elimination solutions first. In order to create room for the new solution, a solution is randomly eliminated from one of them. When a solution is found outside the hypercubes, a unique scenario arises. As a result, different alternative solutions' components can be modified.

According to the second feature, the leader selection function in MAOA, the best answers obtained thus far are used as the best position. This function leads the other search agents to exciting parts of the search space to find a solution close to the global optimum. However, because of the Pareto optimality ideas discussed in the preceding subsection, the answers in a multi-objective search space cannot be quickly compared. The leader selection function was created to address this problem. As previously mentioned, there is a repository of the best non-dominated answers gathered thus far. The leader selection part selects the least-crowded sections of the search space and supplies among its non-dominated answers. For each hypercube, a roulette-wheel method determines the hypercube with the following probability:

$$P_i = \frac{C}{N_i} \tag{14}$$

In this part, a constant integer greater than one is the number of acquired Pareto optimum responses. Eq. (14) shows that hypercubes with a lower population are more likely to

TABLE 9. The statistical results of engineering problems for MS performance metric.

Functions		Algorithm			
		MOPSO	MSSA	MOALO	MAOA
BNH	Ave	**1.0000E+00**	7.6222E-01	5.4090E-01	**1.0000E+00**
	SD	**0.0000E+00**	1.3378E-01	1.0692E-01	**0.0000E+00**
	WRT	=	+	+	
CONSTR	Ave	**9.9384E-01**	9.0536E-01	7.7822E-01	8.7677E-01
	SD	**6.8603E-03**	4.8380E-02	7.6343E-02	9.9189E-02
	WRT	-	+	+	
DISK BRAKE	Ave	9.9928E-01	7.9510E-01	9.1779E-01	**9.9981E-01**
	SD	**1.1417E-03**	1.3013E-01	2.0904E-01	5.7629E-03
	WRT	-	+	+	
4-BAR TRUSS	Ave	1.4876E+00	1.2019E+00	8.4793E-01	**1.4879E+00**
	SD	5.4502E-04	1.8532E-01	1.2454E-01	**2.3406E-06**
	WRT	+	+	+	
WELDED BEAM	Ave	1.0072E+00	7.9085E-01	6.2463E-01	**1.0216E+00**
	SD	6.1053E-02	1.4645E-01	6.9354E-02	**5.8530E-02**
	WRT	+	+	+	
OSY	Ave	3.2390E-01	6.2048E-01	5.7530E-01	**8.2944E-01**
	SD	3.4284E-01	8.0930E-02	**2.6351E-02**	3.5814E-01
	WRT	+	+	-	
SPEED REDUCER	Ave	1.0000E-05	7.2004E-01	6.6868E-01	**7.5636E-01**
	SD	2.0000E-04	**6.9034E-02**	6.0939E-02	4.0006E-02
	WRT	+	-	+	
SRN	Ave	9.0900E-01	7.0583E-01	3.9177E-01	**9.3221E-01**
	SD	**4.5463E-02**	1.5109E-01	8.2165E-02	6.4793E-02
	WRT	-	+	+	
w^+/w^-		12/2	16/0	16/0	
$+/-/=$		6/1/1	8/0/0	8/0/0	

recommend new leaders. The probability of selecting a hypercube from which to select leaders grows as the hypercube's range of discovered solutions decreases.

The MAOA algorithm gets its convergence from the AOA algorithm. Suppose we pick one of the archiving solutions. In that case, the AOA algorithm will almost surely be able to improve on its already great consistency. On the other hand, finding the Pareto optimum responses created a huge variety. This issue was solved by combining the leader function collection with archive maintenance. Regarding computational complexity, when n is the population's total number of individuals and m is the total number of objectives, the computing complexity of MAOA is $O(mn^2)$. The computational complexity is superior to techniques that have $O(mn^3)$ complexity. We employed the leader function collection and archive maintenance to solve this problem. After all, the pseudo-code for MAOA is as follows:

All of the AOA capability is acquired by the MAOA algorithm, suggesting that search agents can explore and utilize the search space similarly. The primary distinction is that AOA multi-objective preserves non-dominated services in an external database and executes searches based on a group of archive members.

IV. RESULTS AND DISCUSSION

Performance measurements and case studies, which comprise unconstrained and constrained bi- and tri-objective mathematics and real-world engineering design problems, are used to assess the efficacy of the suggested technique in this Section. Multi-objective optimizers must tackle non-convex and non-linear problems, among other characteristics, to be tested by these challenges and mathematical calculations. The algorithm was written in

MATLAB 2021a. The computer's features are configured as follows: The CPU on a Macintosh computer is 2.3 GHz (an Intel Core i9 computing platform), with 16 GB 2400 MHz DDR4 RAM (MacOS Monterey).

A. PERFORMANCE METRICS

To evaluate the algorithms' results, the following four metrics are used [42], [43]:

1) GENERATIONAL DISTANCE (GD)

The total of the distances between solution candidates produced using several approaches is an intelligent indicator for analyzing the convergence characteristics of meta-heuristic

TABLE 10. The statistical results of engineering problems for S performance metric.

Functions		Algorithm			
		MOPSO	MSSA	MOALO	MAOA
BNH	Ave	1.0901E+00	1.2546E+00	8.8402E-01	**1.0196E+00**
	SD	**1.3631E-01**	3.9410E-01	4.4648E-01	6.7887E-01
	WRT	-	+	+	
CONSTR	Ave	5.8740E-02	5.5908E-02	6.9071E-02	**5.1010E-02**
	SD	**5.8936E-03**	1.4886E-02	1.8623E-02	1.2462E-02
	WRT	-	+	+	
DISK BRAKE	Ave	1.2452E-01	1.2768E-01	1.4457E-01	**1.1983E-01**
	SD	1.3022E-02	7.0776E-02	1.1754E-02	**7.5928E-03**
	WRT	+	+	+	
4-BAR TRUSS	Ave	5.3605E+00	6.1160E+00	**4.6988E+00**	9.3337E+00
	SD	**2.6169E-01**	1.6605E+00	1.1721E+00	1.9859E+00
	WRT	-	+	+	
WELDED BEAM	Ave	2.3432E-01	**1.8912E-01**	2.2431E-01	2.0362E-01
	SD	2.5702E-02	8.7588E-02	1.3595E-01	**2.4473E-02**
	WRT	+	+	+	
OSY	Ave	**1.1278E+00**	1.3502E+00	1.4382E+00	4.6936E+00
	SD	1.4675E+00	6.2967E-01	**4.8498E-01**	3.6191E+00
	WRT	+	+	-	
SPEED REDUCER	Ave	3.1045E+02	**1.3952E+01**	3.6722E+01	4.7177E+01
	SD	3.0044E+00	9.8199E+00	9.3623E+00	**1.0700E+00**
	WRT	+	+	+	
SRN	Ave	2.2396E+00	2.2721E+00	**1.7386E+00**	2.2164E+00
	SD	**4.7324E-01**	9.3573E-01	8.6872E-01	1.2625E+00
	WRT	-	+	+	
W^+/W^-		14/2	12/4	12/4	
+/−/=		7/1/0	6/2/0	6/2/0	

algorithms with multiple objectives.

$$GD = \left(\frac{1}{n_{pf}}\sum_{i=1}^{n_{pf}} dis_i^2\right)^{\frac{1}{2}} \quad (15)$$

2) SPACING (S)

A measure to show how far candidates are from each other when compared to different sets of results achieved by multiple algorithms.

$$S = \left(\frac{1}{n_{pf}}\sum_{i=1}^{n_{pf}} (d_i - \bar{d})^2\right)^{\frac{1}{2}} \quad Where\ \bar{d} = \frac{1}{n_{pf}}\sum_{i=1}^{n_{pf}} d_i \quad (16)$$

3) MAXIMUM SPREAD (MS)

It depicts the distribution of candidates among other obtained sets by considering the various optimal choices.

$$MS = \left[\frac{1}{m}\sum_{i=1}^{m}\left[\frac{\min\left(f_i^{max}, F_i^{max}\right) - \max(f_i^{min}, F_i^{min})}{F_i^{max} - F_i^{min}}\right]^2\right]^{\frac{1}{2}} \quad (17)$$

4) INVERTED GENERATIONAL DISTANCE (IGD)

Multiple objective optimization techniques can be used to accurately estimate the performance of Pareto front approximations by this measure [44].

$$IGD = \frac{\sqrt{\sum_{i=1}^{n} d_i^2}}{n} \quad (18)$$

Wilcoxon's Rank-Sum Test (WRT): In order to evaluate whether two or more datasets are from the same dispersed population, Wilcoxon's rank-sum test is a non-parametric statistical test with a 5% significance level. The performance of the algorithm is thoroughly assessed using the Wilcoxon rank-sum test. According to the null hypothesis, the two algorithms have no difference in performance because their mean metrics are identical. According to the alternative hypothesis, the mean metrics obtained by the two examined algorithms are different. The symbols "−," "+," and "=" are used to compare the algorithms suggested in this article to other algorithms. The algorithm performs poorly, substantially better than that, and there is no visible difference between the two.

The IGD and GD performance measures quantify the convergence, and the S and MS measure the coverage of Pareto optimum solutions predicted by the algorithms. The

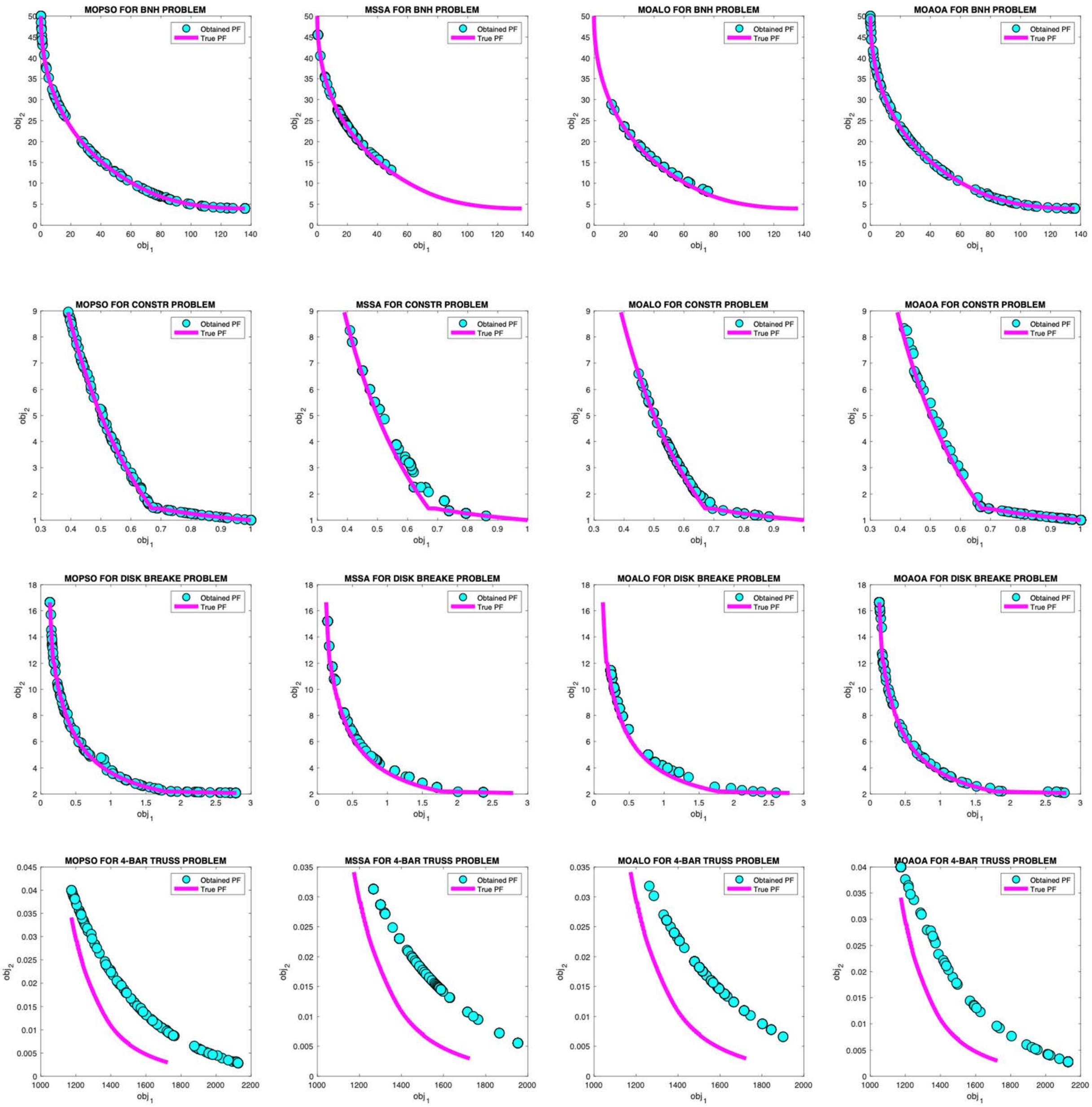

FIGURE 5. True and obtained Pareto front for engineering design problems (BNH, CONSTR, DISK BRAKE and 4-BAR TRUSS).

Wilcoxon rank-sum test was used to assess algorithm performance based on the mean value, and it successfully illustrated the algorithm's high degree of competitiveness and effectiveness.

B. EXPERIMENTAL SETUP

The MOPSO [41], MSSA [45], MOALO [34], NSGA2 [46] and MOGWO [47] are compared to MAOA in this Section, and the best figure of a set of Pareto optimal is shown. Table 1 summarizes all of the listed algorithms' initial parameters. It is worth noting that 100 populations and a maximum of 1000 iterations were used for each experiment. This Section tests the efficiency of the proposed algorithm on twenty-five experiments, including seven unconstrained, constrained functions, eight engineering design problems, and ten CEC-2020 mathematic functions.

The proposed MAOA's efficiency is evaluated using ZDT, DTLZ, and eight multi-objective engineering design problems. Each of the five evaluation problems in the ZDT benchmark has two goals as shown in Table 2. DTLZ comprises two test problems with three goals and twelve variables for each function, as shown in Table 2. Engineering design problems contain eight multi-objective optimization problems; each problem has several variables given in Appendix A.

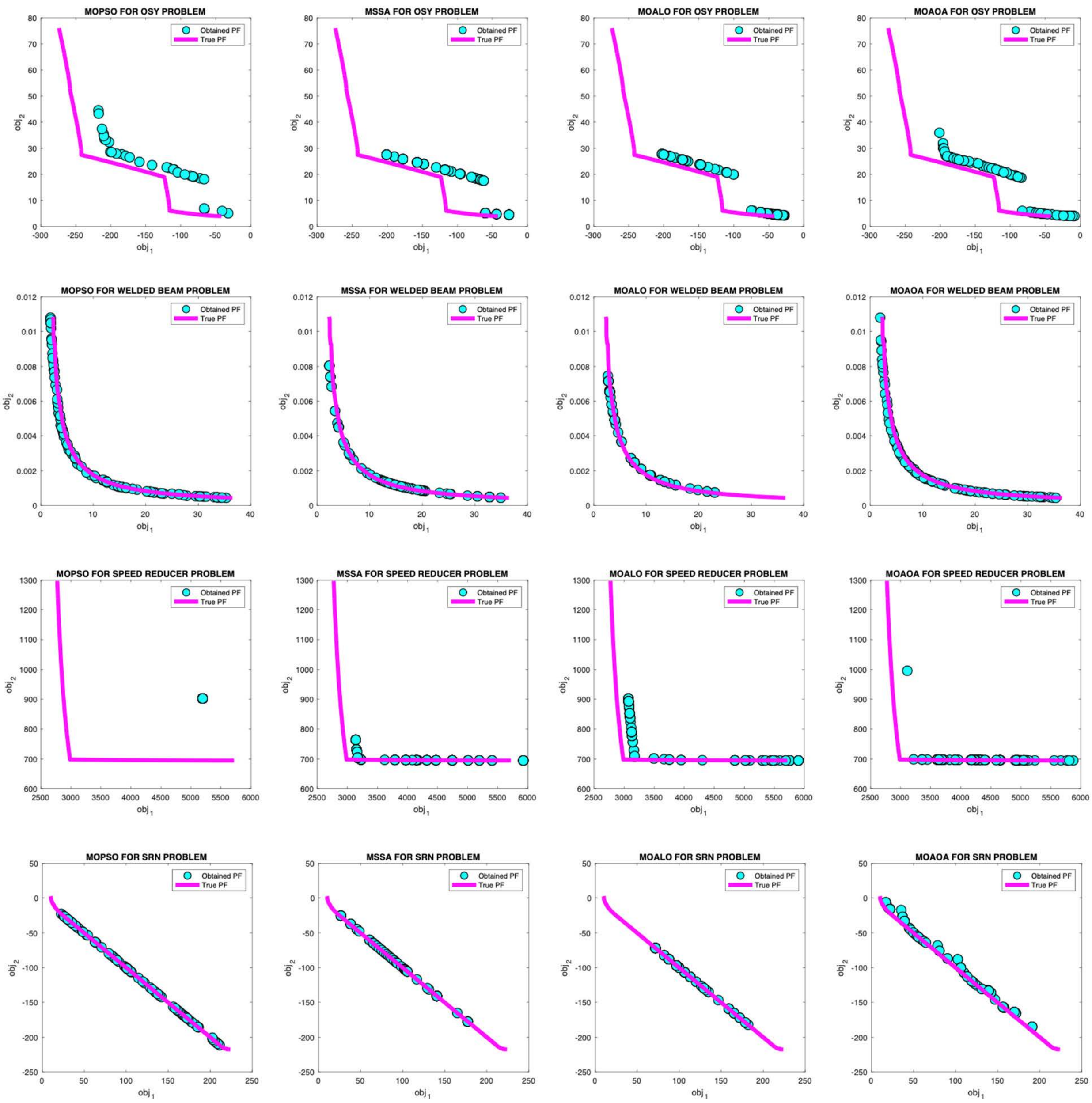

FIGURE 6. True and obtained Pareto front for engineering design problems (WELDED BEAM, OSY, SPEED REDUCER and SRN).

These benchmarks include the most recent and complex test functions: convex, non-convex, and disconnected Pareto ideal forms.

C. DISCUSSION OF THE MATHEMATICAL TEST FUNCTIONS

Table 3 shows the statistical results of mathematical functions (i.e., ZDT and DTLZ) for the generational distance performance metric. It is clear that the proposed MAOA has better results compared to other comparative methods (MOPSO, MSSA, and MOALO). The proposed MAOA acquired the most excellent results in four out of seven test cases (i.e., ZDT1, ZDT2, ZDT4, and ZDT6), followed by MOPSO, which brought the best results in three out of seven test cases (i.e., ZDT3, DTLZ2, and DTLZ4). Thus, the results illustrated the ability of the proposed MAOA to solve complex mathematical multi-objective optimization problems. It acquired excellent results in most problems tested with low SD values.

Table 4 shows the statistical results of mathematical functions (i.e., ZDT and DTLZ) for the inverted generational distance performance metric. The proposed MAOA has better results than other comparative methods (MOPSO, MSSA, and MOALO). The proposed MAOA acquired the most excellent results in five out of seven test cases (i.e.,

TABLE 11. The statistical results of the CEC 2020 problems for performance metric *GD*.

Function		Algorithm					
		MOPSO	MSSA	MOALO	NSGA2	MOGWO	MAOA
M1: MMF1	Ave	1.5852E-03	5.5321E-03	3.8109E-03	3.7211E-02	3.7425E-03	**1.4005E-03**
	SD	1.1756E-03	4.0271E-03	2.3997E-03	2.8343E-02	4.3844E-03	**4.4849E-04**
	WRT	+	+	+	+	+	
M2: MMF2	Ave	8.3130E-04	4.0660E-02	7.8547E-02	9.0907E-02	**4.1874E-04**	2. 5421E-03
	SD	2.0555E-04	7.4608E-02	1.4097E-01	1.7739E-03	**7.4551E-05**	3.4512E-04
	WRT	+	+	+	+	-	
M3: MMF4	Ave	5.5744E-03	2.4294E-03	6.1889E-03	2.8266E-03	2.6278E-03	**2.3245E-03**
	SD	7.4168E-03	2.0292E-03	9.3279E-03	3.6950E-03	3.2182E-03	**1.3201E-03**
	WRT	+	+	+	+	+	
M4: MMF5	Ave	5.4699E-03	2.7860E-02	3.9652E-03	2.0062E-03	8.1579E-03	**1.9954E-03**
	SD	2.4279E-03	2.2339E-02	3.8404E-03	2.3322E-03	9.2404E-03	**1.0019E-03**
	WRT	+	+	+	+	+	
M5: MMF7	Ave	**1.3656E-03**	1.8240E-03	1.4219E-03	3.6400E-03	1.4260E-03	2.8795E-03
	SD	2.6484E-03	6.7194E-04	7.8765E-04	5.2526E-03	5.4680E-04	**2.8741E-04**
	WRT	-	+	+	+	+	
M6: MMF8	Ave	**6.5325E-04**	2.1142E-01	8.0254E-04	9.5964E-03	3.4481E-02	3.3295E-03
	SD	**4.1323E-05**	3.0208E-01	6.2608E-04	3.5307E-03	7.5762E-02	1.4044E-03
	WRT	-	+	+	+	+	
M7: MMF10	Ave	3.6524E-03	2.3929E-02	8.0650E-03	**2.4629E-03**	3.3425E-03	7.1412E-03
	SD	**3.8649E-04**	4.9091E-02	1.0934E-02	1.4128E-02	1.7242E-03	2.0019E-03
	WRT	+	+	+	-	+	
M8: MMF11	Ave	5.2576E-03	3.0179E-03	2.2348E-03	1.1193E-02	2.9190E-03	**2.0094E-03**
	SD	2.6945E-04	1.8586E-03	1.1111E-03	8.2984E-01	6.2066E-04	**2.2841E-04**
	WRT	+	+	+	+	+	
M9: MMF12	Ave	5.6761E-04	5.0576E-04	5.1757E-04	4.4113E-03	4.2095E-04	**3.9825E-04**
	SD	6.1558E-05	4.7320E-05	1.3891E-04	2.8398E-03	5.9355E-05	**4.3251E-05**
	WRT	+	+	+	+	+	
M10: MMF13	Ave	2.1242E-03	1.8325E-03	1.7252E-03	7.8191E-03	1.8095E-03	**1.2341E-03**
	SD	1.9934E-04	8.0983E-04	1.3299E-03	1.5743E-04	4.4849E-04	**1.4254E-04**
	WRT	+	+	+	+	+	
W^+/W^-		16/4	20/0	20/0	18/2	18/2	
+/−/=		8/2/0	10/0/0	10/0/0	9/1/0	9/1/0	

ZDT1-ZDT6), followed by MOPSO, which provided the best results in two out of seven test cases (i.e., DTLZ2 and DTLZ4). Thus, the results illustrated the ability of the proposed MAOA to solve complex mathematical multi-objective optimization problems. The proposed MAOA produced the best results in most problems tested with low SD values. The other comparative methods (i.e., MSSA and MOALO) did not obtain any best results regarding the inverted generational distance performance metric.

Table 5 shows the statistical results of mathematical functions (i.e., ZDT and DTLZ) for the maximum spread performance metric. The proposed MAOA has better results than other comparative methods (MOPSO, MSSA, and MOALO) in almost all the tested cases. The proposed MAOA acquired the most excellent results in seven out of seven test cases (i.e., ZDT1-ZDT6, DTLZ2, and DTLZ4). The proposed MAOA produced the best results in most problems tested with low SD values, except ZDT3 and DTLZ2. The other comparative methods (i.e., MOPSO, MSSA, and MOALO) did not obtain any best results regarding the maximum spread performance metric.

Table 6 shows the statistical results of mathematical functions (i.e., ZDT and DTLZ) for the spacing performance metric. The proposed MAOA has better results than other comparative methods (MOPSO, MSSA, and MOALO). The proposed MAOA acquired the most excellent results in three out of seven test cases (i.e., ZDT2, ZDT3, and ZDT6), followed by MSSA, which acquired the most excellent results in three out of seven test cases (i.e., ZDT4, DTLZ2, and DTLZ4), and MOALO acquired the most excellent results in one out of seven test cases (i.e., ZDT1). Thus, the results illustrated the proposed MAOA's ability to solve complex mathematical multi-objective optimization problems. The proposed MAOA produced the best results in most problems tested with low SD values, except ZDT1, DTLZ2, and DTLZ4. The MOPSO does not obtain any best results regarding the spacing performance metric.

Figs. 3 and 4 show the true and obtained Pareto front for ZDT and DTLZ benchmark problems for all the comparative methods. The Pareto front results of the proposed MAOA and other comparative methods are seen in this figure, with MAOA achieving greater convergence and coverage than

TABLE 12. The statistical results of the CEC 2020 problems for performance metric *IGD*.

Function		Algorithm					
		MOPSO	MSSA	MOALO	NSGA2	MOGWO	MAOA
M1: MMF1	Ave	9.2459E-04	4.0338E-03	3.4017E-03	**6.3276E-04**	2.7547E-03	9.0124E-04
	SD	1.2258E-04	7.8337E-04	4.0413E-04	**1.1765E-04**	2.3995E-04	3.6524E-04
	WRT	+	+	+	-	+	
M2: MMF2	Ave	2.2238E-03	4.4213E-03	2.2902E-03	9.2533E-03	**9.4570E-04**	2.5741E-03
	SD	8.4385E-04	1.5387E-03	6.1735E-04	1.7930E-03	**1.1935E-04**	5.6866E-04
	WRT	+	+	+	+	-	
M3: MMF4	Ave	8.8933E-04	4.9496E-03	4.8819E-03	4.3677E-04	1.7148E-03	**3.6225E-04**
	SD	1.2397E-04	2.5771E-03	1.7092E-03	**2.4241E-05**	3.3043E-04	8.7402E-04
	WRT	+	+	+	+	+	
M4: MMF5	Ave	8.7820E-04	3.3427E-03	2.1757E-03	8.0141E-04	2.2009E-03	**7.6001E-04**
	SD	9.3769E-04	6.0112E-04	1.7789E-04	4.1688E-04	1.0885E-03	**9.8527E-05**
	WRT	+	+	+	+	+	
M5: MMF7	Ave	5.5406E-04	4.2735E-03	7.6473E-03	**4.1358E-04**	1.9071E-03	1.1013E-03
	SD	1.2011E-04	1.8876E-03	5.8726E-03	9.6031E-05	**7.7299E-05**	5.3301E-03
	WRT	+	+	+	-	+	
M6: MMF8	Ave	**4.6497E-04**	2.2485E-03	2.0296E-03	1.6972E-02	9.5161E-04	2.3001E-03
	SD	**1.7000E-05**	4.2443E-04	5.4512E-04	1.2889E-02	4.2348E-04	2.3341E-04
	WRT	-	+	+	+	+	
M7: MMF10	Ave	**8.8248E-04**	1.0774E-02	1.0450E-02	1.0269E-01	1.3264E-03	1.0122E-03
	SD	**1.2160E-04**	6.9750E-03	5.0625E-03	7.2147E-03	4.3239E-04	4.4717E-03
	WRT	-	+	+	+	+	
M8: MMF11	Ave	8.3217E-04	8.1063E-03	1.3618E-02	9.3848E-02	4.3671E-03	**6.7458E-04**
	SD	1.2475E-04	4.8219E-03	3.8071E-03	1.0381E-03	3.1204E-03	**1.1017E-04**
	WRT	+	+	+	+	+	
M9: MMF12	Ave	5.2314E-04	6.6534E-03	6.6576E-03	7.1622E-03	1.5235E-03	**4.9854E-04**
	SD	**3.9743E-05**	7.1555E-03	2.7721E-03	7.6951E-03	5.6604E-04	3.4307E-04
	WRT	+	+	+	+	+	
M10: MMF13	Ave	4.4906E-04	4.5576E-03	9.0777E-03	5.8677E-02	1.9444E-03	**3.9321E-04**
	SD	7.0224E-04	2.1694E-03	5.8111E-03	1.8127E-02	6.5347E-04	**5.5865E-04**
	WRT	+	+	+	+	+	
W^{+}/W^{-}		16/4	20/0	20/0	16/4	18/2	
+/−/=		8/2/0	10/0/0	10/0/0	8/2/0	9/1/0	

the different comparative algorithms. Furthermore, MAOA's solutions are widely spread around the entire Pareto front. Where MSSA gives a non-uniform distribution of solutions at the curve's ends, the proposed MAOA algorithm is superior. MAOA generated more convergent and widely spread solutions on the real Pareto optimal. Other comparative methods could not cover any of the true Pareto optimal solutions for these functions, but they could cover one end of the true Pareto. Simultaneously, it crashes on the other end. Based on the Wilcoxon rank sum test findings shown in Tables 3 to 6, it is clear that the MAOA is significantly more efficient than the other methods.

D. DISCUSSION OF THE ENGINEERING PROBLEMS

In this Section, the proposed MAOA method is employed to solve several different multi-objective engineering design problems and compared with other well-known multi-objective methods.

Table 7 shows the statistical results of the tested engineering problems for the generational distance performance metric. The proposed MAOA got promising results for all the tested problems. MAOA acquired the most excellent results for four out of eight tested problems: CONSTR, DISK BRAKE, WELDED BEAM, and SPEED REDUCER problems. Followed by MOPSO, which acquired the most excellent results on two out of eight test problems (i.e., BNH and 4-BAR TRUSS), and MOALO acquired the most excellent results on two out of eight test problems (i.e., OSY and SRN). Moreover, the proposed MAOA got stable results according to the SD measure, which reflects the ability of the proposed MAOA over several runs. We conclude from the obtained results that the proposed MAOA method has achieved stimulating results and has a high and balanced capacity to solve complex engineering problems with multiple objectives.

Table 8 presents the statistical results of the tested engineering problems for the inverted generational distance performance metric. The proposed MAOA produced encouraging results for all the tested problems. MAOA acquired the best results for five out of eight tested problems: BNH, DISK BRAKE, 4-BAR TRUSS, OSY, and SPEED REDUCER. Followed by MOPSO, it obtained the best results on three out of eight test problems (i.e., CONSTR, WELDED BEAM, and SRN). Besides, the proposed MAOA got well-built results according to the SD measure, which reflects the ability of the proposed MAOA over various runs. From these results, we concluded that the proposed MAOA method has reached

TABLE 13. The statistical results of the CEC 2020 problems for performance metric *MS*.

Function		Algorithm					
		MOPSO	MSSA	MOALO	NSGA2	MOGWO	MAOA
M1: MMF1	Ave	1.0405E+00	1.0292E+00	9.6095E-01	1.0008E+00	1.1469E+00	**1.1889E+00**
	SD	3.8703E-02	7.4761E-02	7.9028E-02	4.6539E-01	2.8675E-01	**3.6523E-02**
	WRT	+	+	+	+	+	
M2: MMF2	Ave	1.0000E+00	1.5837E+00	**2.4489E+00**	4.8560E-01	9.9776E-01	8.9984E-01
	SD	**0.0000E+00**	1.1692E-01	2.1803E-01	9.1533E-01	5.0063E-03	1.5142E-02
	WRT	+	+	-	+	+	
M3: MMF4	Ave	1.1564E+00	8.6683E-01	8.9243E-01	7.1184E-01	1.1074E+00	**1.2006E+00**
	SD	2.3744E-01	8.7815E-02	9.4367E-02	1.4056E-01	1.8442E-01	**6.3253E-02**
	WRT	+	+	+	+	+	
M4: MMF5	Ave	1.1281E+00	1.0736E+00	1.0848E+00	1.0893E+00	**1.2667E+00**	9.9851E-01
	SD	5.3298E-02	4.3644E-01	1.1268E-01	1.7275E-01	2.1784E-01	**4.6521E-02**
	WRT	+	+	+	+	-	
M5: MMF7	Ave	1.0621E+00	7.5755E-01	6.7909E-01	**1.1233E+00**	9.5926E-01	9.3541E-01
	SD	6.0923E-02	9.0877E-02	1.8167E-01	2.0187E-01	1.3433E-02	**1.0141E-02**
	WRT	+	+	+	-	+	
M6: MMF8	Ave	9.9669E-01	1.5597E+00	9.3247E-01	1.3965E+00	**2.9770E+00**	9.7548E-01
	SD	**3.6031E-03**	1.0455E+01	6.3542E-02	1.5714E-01	9.5271E-01	2.2541E-02
	WRT	+	+	+	+	-	
M7: MMF10	Ave	**1.0000E+00**	8.5590E-01	7.6156E-01	6.5685E-01	9.8906E-01	7.2518E-01
	SD	**7.8300E-03**	2.2228E-01	1.2705E-01	1.0628E-01	3.4240E-02	9.3253E-03
	WRT	-	+	+	+	+	
M8: MMF11	Ave	9.6992E-01	7.4196E-01	5.1036E-01	2.7847E-02	8.7184E-01	**9.8516E-01**
	SD	2.3850E-02	8.4684E-02	1.1371E-01	2.6141E-02	7.4918E-02	**1.0393E-02**
	WRT	+	+	+	+	+	
M9: MMF12	Ave	9.0156E-01	8.4380E-01	8.4393E-01	9.1950E-01	9.7502E-01	**9.8369E-01**
	SD	**1.2421E-03**	2.1330E-01	6.3666E-02	1.4823E-01	2.3631E-02	1.0258E-02
	WRT	+	+	+	+	+	
M10: MMF13	Ave	9.9921E-01	7.5131E-01	5.2827E-01	6.1534E-01	8.7876E-01	**9.9976E-01**
	SD	3.0561E-02	1.2026E-01	2.1729E-01	9.3616E-02	4.0393E-02	**3.0393E-02**
	WRT	+	+	+	+	+	
W^+/W^-		18/2	20/0	18/2	18/2	16/4	
+/−/=		9/1/0	10/0/0	9/1/0	9/1/0	8/2/0	

exciting developments and has a high and stable capability to tackle complex engineering problems with multi-objective functions.

Table 9 shows the results of the tested industrial engineering problems for the maximum spread performance metric. The proposed MAOA got better outcomes for all the tested problems compared to other comparative methods. MAOA acquired the most excellent results on seven out of eight tested problems. Followed by MOPSO, it acquired the most excellent results on two out of eight test problems (i.e., BNH and CONSTR).

Table 10 shows the statistical results of the tested engineering problems for the spacing performance metric. The proposed MAOA got better results for all the tested problems compared to other multi-objective methods. MAOA acquired the most excellent results on three out of eight tested problems: BNH, CONSTR, and DISK BRAKE problems. Followed by MOALO, it acquired the most excellent results on two out of eight test problems (i.e., 4-BAR TRUSS and SRN), MSSA acquired the most excellent results on two out of eight test problems (i.e., WELDED BEAM and SPEED REDUCER), and finally, MOPSO acquired the most excellent results on one out of eight test problems (i.e., OSY). Moreover, the proposed MAOA got stable results according to the SD measure, which reflects the ability of the proposed MAOA over several runs. Finally, the proposed MAOA method has proven its effectiveness in solving various complex engineering problems.

Figs. 5 and 6 show the true and obtained Pareto front for eight engineering design problems (i.e., BNH, DISK BRAKE, 4-BAR TRUSS, WELDED BEAM, OSY, SPEED REDUCER, SRN, and CONSTR) using all the comparative methods. The achieved Pareto front results of the proposed MAOA method and other comparative methods are viewed in this figure, with MAOA achieving greater convergence and coverage than the different comparative methods. The proposed MAOA Pareto solutions are very similar to Pareto solutions to engineering design problems, as shown in this figure. The MAOA gives better results for almost all the tested engineering problems.

Figs. 5 and 6 show the true and obtained Pareto front for eight engineering design problems (i.e., BNH, DISK BRAKE, 4-BAR TRUSS, WELDED BEAM, OSY, SPEED REDUCER, SRN, and CONSTR) using all the comparative methods. The achieved Pareto front results of the proposed MAOA method, and other comparative methods are viewed

TABLE 14. The statistical results of the CEC 2020 problems for performance metric *S*.

Function		Algorithm					
		MOPSO	MSSA	MOALO	NSGA2	MOGWO	MAOA
M1: MMF1	Ave	2.7041E-03	1.6387E-02	4.9232E-03	2.0626E-03	3.5578E-02	**2.0087E-03**
	SD	1.7504E-02	**1.1864E-02**	4.5731E-02	1.3061E-01	1.2108E-02	1.3725E-01
	WRT	+	+	+	+	+	
M2: MMF2	Ave	**3.0686E-03**	8.7544E-03	4.0518E-02	2.9205E-02	1.6747E-03	3.2514E-02
	SD	**1.1091E-02**	1.6772E-02	1.3560E-02	6.6782E-02	2.6699E-02	5.6524E-01
	WRT	-	+	+	+	+	
M3: MMF4	Ave	2.0380E-03	1.1620E-02	6.2102E-03	1.1259E-03	2.0243E-02	**1.0047E-03**
	SD	1.1973E-02	1.1296E-02	1.3611E-02	1.3390E-02	3.5138E-02	**1.1125E-02**
	WRT	+	+	+	+	+	
M4: MMF5	Ave	6.2267E-03	6.5895E-03	1.1871E-02	7.9434E-03	1.8720E-02	**5.1587E-03**
	SD	1.1600E-02	1.2613E-02	**9.5569E-03**	1.1462E-02	1.5283E-02	2.5417E-01
	WRT	+	+	+	+	+	
M5: MMF7	Ave	1.6462E-03	5.2882E-03	6.1320E-03	**1.4521E-03**	5.1392E-03	9.8881E-03
	SD	**9.2245E-03**	9.9820E-03	1.5631E-02	1.0510E-01	1.7856E-01	5.8541E-02
	WRT	+	+	+	-	+	
M6: MMF8	Ave	**3.8884E-04**	3.4389E-03	4.6702E-03	6.3614E-02	3.7117E-01	1.0148E-03
	SD	4.1672E-02	**3.6111E-02**	5.3845E-02	7.6412E-02	8.3050E-02	4.2511E-02
	WRT	-	+	+	+	+	
M7: MMF10	Ave	**3.7347E-03**	2.1464E-02	1.9358E-02	9.0087E-02	1.9687E-02	4.9865E-02
	SD	6.6300E-02	1.0746E-01	**4.7241E-02**	3.2719E-01	1.1160E-01	2.0015E-01
	WRT	-	+	+	+	+	
M8: MMF11	Ave	8.8577E-03	9.8470E-02	5.8224E-03	2.7027E-02	2.5791E-02	**4.3257E-03**
	SD	1.1781E-02	1.1163E-02	3.8159E-03	4.1492E-02	1.3140E-02	**1.0705E-03**
	WRT	+	+	+	+	+	
M9: MMF12	Ave	1.3056E-03	6.0189E-03	4.0508E-03	1.8690E-02	6.5432E-03	**1.0532E-03**
	SD	8.7333E-02	1.0319E-01	7.6104E-02	7.2539E-01	1.3725E-01	**6.9864E-02**
	WRT	+	+	+	+	+	
M10: MMF13	Ave	5.5400E-03	6.2981E-02	1.7521E-02	1.5878E-02	5.1587E-02	**5.1142E-03**
	SD	**1.7041E-03**	1.6387E-02	4.9232E-03	2.0626E-03	3.5578E-02	4.1182E-03
	WRT	+	+	+	+	+	
W^+/W^-		14/6	20/0	20/0	18/2	20/0	
$+/-/=$		7/3/0	10/0/0	10/0/0	9/1/0	10/0/0	

in this figure, with MAOA achieving greater convergence and coverage than the different comparative methods. The proposed MAOA Pareto solutions are very similar to Pareto solutions to engineering design problems, as shown in this figure. The MAOA gives better results for almost all the tested engineering problems. The findings of the Wilcoxon rank sum test, which are provided in Tables 7 to 10, make it abundantly clear that the MAOA is a strategy that is significantly more successful than the other methods.

E. DISCUSSION OF THE CEC-2020 MMO TEST PROBLEMS

In this Section, the capability of the proposed multi-objective strategy, known as MAOA, to address a wide variety of issues related to CEC-2020 will be tested. The functions of the MMO test, including all of their specifics, are described in [48].

Tables 11-14 show the obtained results for all the comparative methods in various evaluation measures. The average, standard deviation and Wilcoxon signed-rank test sign are calculated for each measure to explain the results clearly.

Table 11 presents the results of GD for the comparative methods. The proposed MAOA got the best results in most of the tested cases (i.e., MMF1, MMF4, MMF5, MMF11, MMF12, and MMF13). The performance of the proposed method is promising compared to the comparative methods like MOPSO, MSSA, MOALO, NSGA2, and MOGWO. According to the Wilcoxon signed-rank test, the proposed MAOA almost overcame all the comparative methods.

Table 12 presents the results of IGD for the comparative methods. The proposed MAOA obtained the best outcomes in most of the tested cases (i.e., MMF4, MMF5, MMF11, MMF12, and MMF13). The performance of the proposed MAOA method is good compared to the comparative methods like MOPSO, MSSA, MOALO, NSGA2, and MOGWO. According to the Wilcoxon signed-rank test, the proposed MAOA almost overcame all the comparative methods. For example, the proposed MAOA overwhelms MOPSO in 8 cases, MSSA in 10 cases, MOALO in 10 cases, NSGA2 in 8 cases, and MOGWO in 9 cases.

Table 13 presents the results of MS for the comparative methods. The proposed MAOA got the best results in most of the tested cases (i.e., MMF1, MMF4, MMF11, MMF12, and MMF13). The performance of the proposed MAOA method is sound likened to comparative methods like MOPSO, MSSA, MOALO, NSGA2, and MOGWO. The proposed MAOA almost overwhelmed all the comparative

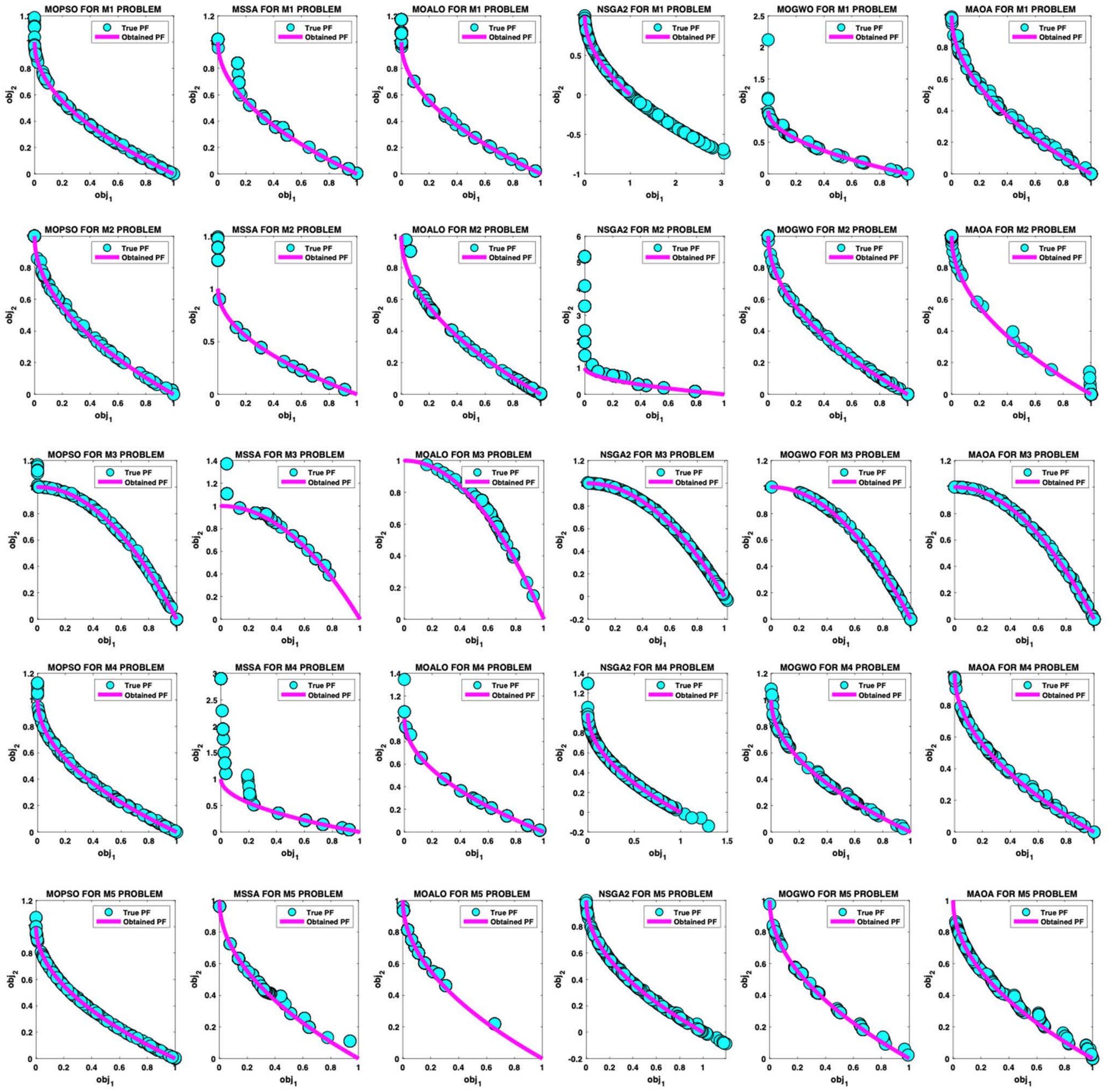

FIGURE 7. True and obtained Pareto fronts for the CEC 2020 problems (M1-M5).

methods in the Wilcoxon signed-rank test. For example, the proposed MAOA overwhelms MOPSO in 9 cases, MSSA in 10 cases, MOALO in 9 cases, NSGA2 in 9 cases, and MOGWO in 8 cases.

Table 14 shows the results in terms of the S measure for the comparative methods. The proposed MAOA got the best results in most of the tested cases (i.e., MMF1, MMF4, MMF5, MMF11, MMF12, and MMF13). The performance of the proposed MAOA method is sound likened to comparative methods like MOPSO, MSSA, MOALO, NSGA2, and MOGWO. The proposed MAOA almost overwhelmed all the comparative methods in the Wilcoxon signed-rank test. For example, the proposed MAOA overwhelms MOPSO in 7 cases, MSSA in 10 cases, MOALO in 10 cases, NSGA2 in 9 cases, and MOGWO in 10 cases. From the given results, we concluded that the performance of the proposed MOAOA is excellent, and it can get better results compared to other similar methods in solving various multi-objective problems.

Figures 7 and 8 show the true and obtained Pareto fronts for the CEC 2020 problems (M1-M5) and the true and

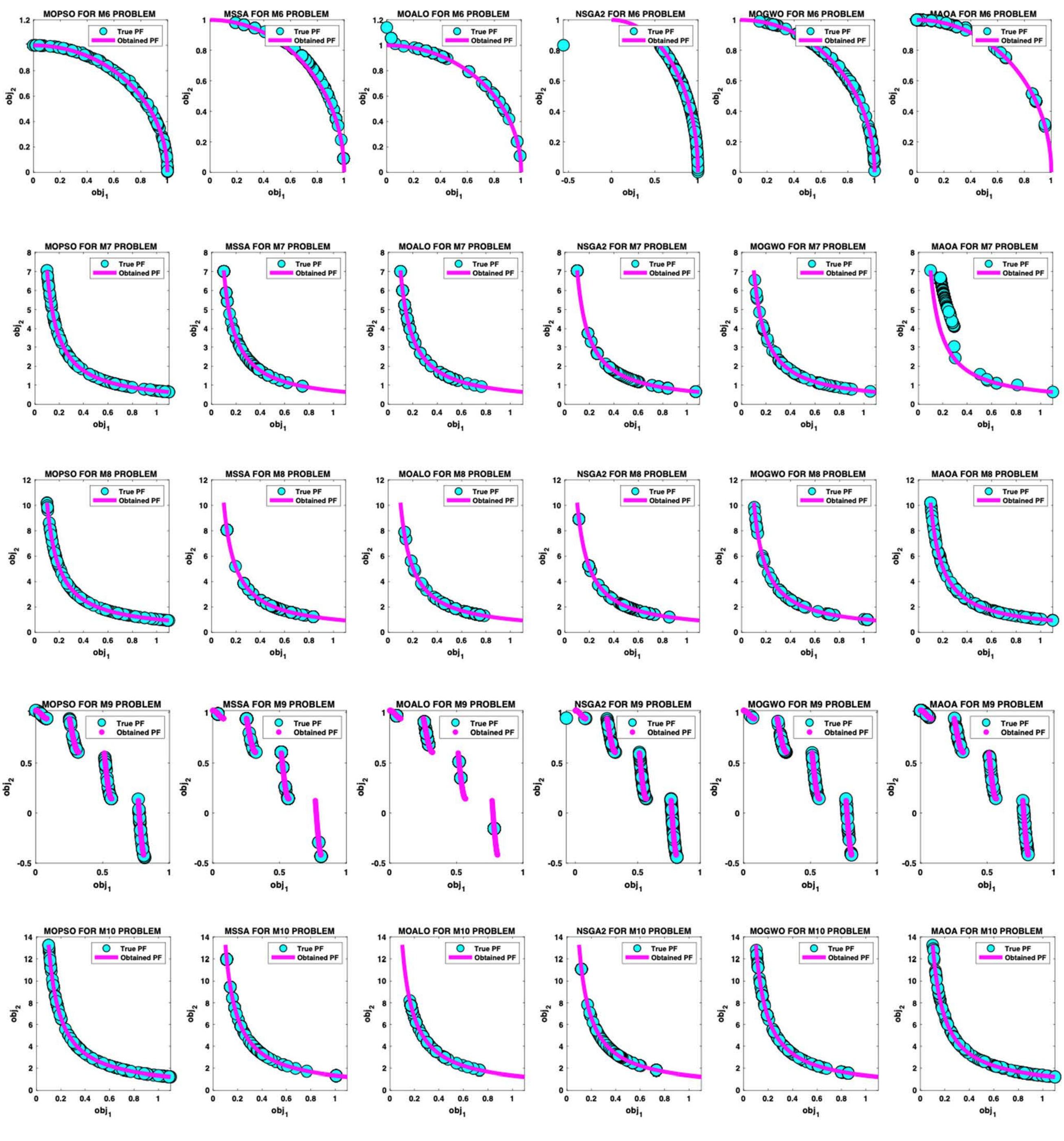

FIGURE 8. True and obtained Pareto fronts for the CEC 2020 problems (M6-M10).

obtained Pareto fronts for the CEC 2020 problems (M6-M13), respectively. These figures show that the obtained results by the proposed MAOA are closer to the optimal solution and got almost the best results compared to similar methods. Moreover, the performance of the proposed MOAO is promising according to the presented curves in Figures 7 and 8.

V. CONCLUSION

In order to assist decision-makers, the field of multi-objective optimization provides solutions for a variety of complex problems with many conflicting objectives. The previously developed Arithmetic Optimization Algorithm (AOA) was modified to create the Archive-based Multi-objective Arithmetic Optimization Algorithm (MAOA) in order to address different multi-objective issues. The suggested MAOA is based on the distribution behavior of the most common mathematical arithmetic operators, such as multiplication, division, subtraction, and addition. In MAOA, non-dominated Pareto optimal solutions are discovered using an archived concept. To determine whether the suggested MAOA approach can be used to address actual engineering challenges, it is tested on eight constrained engineering design problems and seven benchmark test functions. Its performance is compared to those of the five well-known multi-objective techniques MOPSO, MSSA, MOALO, NSGA2 and MOGWO. Several metrics, including Generational Distance (GD), Inverted Generational Distance (IGD), Maximum Spread (MS), and Spacing (S), are used to assess the results. The experimental study demonstrated the superior performance and high degree of convergence of the proposed MAOA technique compared to other existing algorithms. The MAOA gives the best outcomes in terms of computing costs when compared to other existing competing algorithms, according to a number of experimental studies in this respect. The binary version of the MAOA method will be developed for use in upcoming studies to address a variety of challenging, intricate real-world problems in various fields. Additionally, the future contribution of the suggested method can be seen in its many-objective form.

APPENDIX A CONSTRAINED MULTI-OBJECTIVE TEST PROBLEMS USED IN THIS PAPER

This issue [49] contains two constraints and two design variables with a convex Pareto front.

$$Minimize:\ f_1(x) = x_1 \quad (A.1)$$
$$Minimize:\ f_2(x) = (1 + x_2)/x_1 \quad (A.2)$$
$$where:\ g_1(x) = 6 - (x_2 + 9x_1) \quad (A.3)$$
$$g_2(x) = 1 + (x_2 - 9x_1) \quad (A.4)$$
$$0.1 \le x_1 \le 1, \quad 0 \le x_2 \le 5$$

A. SRN

Srinivas and Deb [50] proposed the following continuous Pareto optimum front for the following issue:

$$Minimize:\ f_1(x) = 2 + (x_1 - 2)^2 + (x_2 - 1)^2 \quad (A.5)$$
$$Minimize\ f_2(x) = 9x_1(x_2 - 1)^2 \quad (A.6)$$
$$where:\ g_1(x) = x_1^2 + x_2^2 - 255 \quad (A.7)$$
$$g_2(x) = x_1 - 3x_2 + 10 \quad (A.8)$$
$$-20 \le x_1 \le 20, \quad -20 \le x_2 \le 20$$

B. BNH

Binh and Korn [51] provided this example for the first time as follows:

$$Minimize:\ f_1(x) = 4x_1^2 + 4x_2^2 \quad (A.9)$$
$$Minimize:\ f_2(x) = (x_1 - 5)^2 + (x_2 - 5)^2 \quad (A.10)$$
$$where:\ g_1(x) = (x_1 - 5)^2 + x_2^2 - 25 \quad (A.11)$$
$$g_2(x) = 7.7 - (x_1 - 8)^2 - (x_2 + 3)^2 \quad (A.12)$$
$$0 \le x_1 \le 5, \quad 0 \le x_2 \le 3$$

C. OSY

It was suggested by Osyczka and Kundu [52] that there should be five separate regions. In addition, the following six restrictions and six design variables should be taken into account:

$$\text{Minimize: } f_1(x) = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 \quad (A.13)$$
$$\text{Minimize : } f_2(x) = [25(x_1 - 2)^2 + (x_2 - 1)^2 + (x_3 - 1)^2(x_4 - 4)^2 + (x_5 - 1)^2] \quad (A.14)$$
$$\text{Where: } g_1(x) = 2 - x_1 - x_2 \quad (A.15)$$
$$g_2(x) = -6 + x_1 + x_2 \quad (A.16)$$
$$g_3(x) = -2 - x_1 + x_2 \quad (A.17)$$
$$g_4(x) = -2 + x_1 - 3x_2 \quad (A.18)$$
$$g_5(x) = -4 + x_4 + (x_3 - 3)^2 \quad (A.19)$$
$$g_6(x) = 4 - x_6 - (x_5 - 3)^3 \quad (A.20)$$
$$0 \le x_1 \le 10, \quad 0 \le x_2 \le 10,\ 1 \le x_3 \le 5 \quad (A.21)$$
$$0 \le x_4 \le 6, \quad 1 \le x_5 \le 5,\ 0 \le x_6 \le 10$$

APPENDIX B CONSTRAINED MULTI-OBJECTIVE ENGINEERING PROBLEMS USED IN THIS PAPER

A. THE FOUR-BAR TRUSS DESIGN PROBLEM

In this 4-bar truss issue [53], there are two objectives as structural volume (f_1) and displacement (f_2), which are considered to be minimized. This issue has four design variables ($x_1 - x_4$) according to the cross-sectional area of members 1, 2, 3, and 4. This problem may be mathematically formulized as below:

$$Minimize:\ f_1(x) = 200 \times (2 \times x(1)) + sqrt(2 \times x(2)) + sqrt(x(3)) + x(4) \quad (B.1)$$
$$Minimize:\ f_2(x) = 0.01 \times \left(\frac{2}{x(1)}\right) + \left(\frac{2 \times sqrt(2)}{x(2)}\right) - ((2 \times sqrt(2))/x(3)) + (2/x(1))) \quad (B.2)$$
$$1 \le x_1 \le 3, \quad 1.4142 \le x_2 \le 3$$
$$1.4142 \le x_3 \le 3, \quad 1 \le x_4 \le 3$$

B. THE WELDED BEAM DESIGN PROBLEM

Ray and Liew [54] first offered this welded beam issue with two objectives, namely the fabrication cost (f_1) and beam deflection (f_2), which is to be reduced and four constraints. This issue has four numbers of variables, named weld thickness (x_1), clamped bar length (x_2), bar height (x_3), and bar thickness (x_4) as follows:

$$Minimize:\ f_1(x) = 1.10471 \times x(1)^2 \times x(2) + 0.04811$$

$$\times x(3) \times x(4) \times (14 + x\,(2)) \quad \text{(B.3)}$$

$$Minimize:\ f_2\,(x) = 65856000/(30 \times 10^6 \times x(4) \times x\,(3)^3) \quad \text{(B.4)}$$

$$where:\ g_1\,(x) = \tau - 13600 \quad \text{(B.5)}$$

$$g_2(x) = \sigma - 30000 \quad \text{(B.6)}$$

$$g_3\,(x) = x\,(1) - x(4) \quad \text{(B.7)}$$

$$g_4\,(x) = 6000 - P \quad \text{(B.8)}$$

$$0.125 \le x_1 \le 5, \quad 0.1 \le x_2 \le 10$$

$$0.1 \le x_3 \le 10, \quad 0.125 \le x_4 \le 5$$

$$where:\ q = 6000 * \left(14 + \frac{x(2)}{2}\right);$$

$$D = sqrt\left(\frac{x(2)^2}{4} + \frac{x(1) + x(3))^2}{4}\right) \quad \text{(B.9)}$$

$$J = 2 * \left(x(1) * x(2) * sqrt(2) * \left(\frac{x(2)^2}{12} + \frac{(x(1) + x(3))^2}{4}\right)\right) \quad \text{(B.10)}$$

$$\alpha = \frac{6000}{sqrt(2) * x(1) * x(2)} \quad \text{(B.11)}$$

$$\beta = Q * \frac{D}{J} \quad \text{(B.12)}$$

C. DISK BRAKE DESIGN PROBLEM

This disc brake design issue has been figured out by Ray and Liew [54] with two objectives, namely stopping time (f_1) and brake mass (f_2) to minimize and five constraints for a disc brake. This problem contains five numbers of variables: disc inner radius (x_1), outer disc radius (x_2), engaging force (x_3), and friction surfaces number (x_4). The following equations may represent this problem:

$$Minimize:\ f_1\,(x) = 4.9 \times (10)^{(-5)} \times (x(2)^{(2)} - x\,(1)^{(2)}) \times (x\,(4) - 1) \quad \text{(B.13)}$$

$$Minimize:\ f_2\,(x) = (9.82 \times (10)^{(6)})) \times (x(2))^{(2)} - x\,(1)^{(2)}))/((x(2))^{(3)} - x\,(1)^{(3)}) \times x\,(4) \times x\,(3)) \quad \text{(B.14)}$$

$$g_1\,(x) = 20 + x\,(1) - x(2) \quad \text{(B.15)}$$

$$g_2\,(x) = 2.5 + (x\,(4) + 1) - 30 \quad \text{(B.16)}$$

$$g_3\,(x) = (x\,(3))/(3.14 \times \left(x\,(2)^2 - x\,(1)^2\right)^2) - 0.4 \quad \text{(B.17)}$$

$$g_4\,(x) = (2.22 \times (10)^{(-3)} \times x\,(3) \times \left(x\,(2)^3 - x\,(1)^3\right))/(\left(x\,(2)^2 - x\,(1)^2\right)^2) - 1 \quad \text{(B.18)}$$

$$g_5\,(x) = 900 - (2.66 \times (10)^{(-2)} \times x\,(3) \times x\,(4) \times \left(x\,(2)^3 - x\,(1)^3\right))/ \times (\left(x\,(2)^2 - x\,(1)^2\right)^2) \quad \text{(B.19)}$$

$$55 \le x_1 \le 80, \quad 75 \le x_2 \le 110$$

$$1000 \le x_3 \le 3000, \quad 2 \le x_4 \le 20$$

D. SPEED REDUCER DESIGN PROBLEM

This issue [53], [55] contains two objectives such as weight (f_1) and stress (f_2), which are to be minimized. The problem may represent with a diagram as given in Fig. 9. Also, the problem has eleven constraints with seven numbers of design variables such as the width of the gear face (x_1), teeth module (x_2), pinion teeth number (x_3 numeral variable), a distance between of bearings 1 (x_4), a distance of bearings 2 (x_5), shaft 1 diameter (x_6), and shaft 2 diameter (x_7). The equations may clearly represent this problem as below:

$$Minimize:\ f_1\,(x) = 0.7854 \times x\,(1) \times x(2)^2 \times (3.3333 \times x\left(3)^2 + 14.9334 \times x\,(3)\right) \ldots - 43.0934) - 1.508 \times x(1) \times (x(6)^2 + x(7)^2 \quad \text{(B.20)}$$

$$\text{Minimize}:f_2(x) = ((sqrt(((745 * x(4))/x(2) * x(3)))^2 + 19.9e6)/(0.1 * x(6)^3)) \quad \text{(B.21)}$$

$$where:\ g_1\,(x) = 27/(x\,(1) \times x\left(2)^2 \times x\,(3)\right) - 1 \quad \text{(B.22)}$$

$$g_2\,(x) = 397.5/(x\,(1) \times x(2)^2 \times x\left(3)^2\right) - 1 \quad \text{(B.23)}$$

$$g_3\,(x) = (1.93 \times (x\left(4)^3\right)/(x(2) \times x(3) \times x\left(6)^4\right) - 1 \quad \text{(B.24)}$$

$$g_4\,(x) = (1.93 \times (x\left(5)^3\right)/(x(2) \times x(3) \times x\left(7)^4\right) - 1 \quad \text{(B.25)}$$

$$g_5\,(x) = ((sqrt((745 \times x\,(4))/x(2) \times x\left(3)))^2 + 16.9e6\right))/(110 \times x\left(6)^3\right)) - 1 \quad \text{(B.26)}$$

$$g_6\,(x) = ((sqrt((745 \times x\,(4))/x(2) \times x\left(3)))^2 + 157.5e6\right))/(85 \times x\left(7)^3\right)) - 1 \quad \text{(B.27)}$$

$$g_7\,(x) = ((x(2) \times x(3))/40)1 \quad \text{(B.28)}$$

$$\tau = sqrt\left(\alpha^2 + 2 \times \alpha \times \beta \times \frac{x(2)}{2 \times D} + \beta^2\right) \quad \text{(B.29)}$$

$$\sigma = \frac{504000}{x(4) \times x(3)^2} \quad \text{(B.30)}$$

$$tmpf = 4.013 \times \frac{30 \times 10^6}{196} \quad \text{(B.31)}$$

$$P = tmpf \times sqrt\left(x(3)^2 \times \frac{x(4)^6}{36}\right) \times \left(1 - x(3) \times \frac{sqrt(\frac{30}{48})}{28}\right) \quad \text{(B.32)}$$

REFERENCES

[1] K. V. Price, "Differential evolution," in *Handbook of Optimization* (Intelligent Systems Reference Library), vol. 38, I. Zelinka, V. Snášel, and A. Abraham, Eds. Berlin, Germany: Springer, 2013.

[2] M. Dorigo and C. Blum, "Ant colony optimization theory: A survey," *Theor. Comput. Sci.*, vol. 344, nos. 2–3, pp. 243–278, Nov. 2005.

[3] J. Kennedy and R. Eberhart, "Particle swarm optimization," in *Proc. Int. Conf. Neural Netw. (ICNN)*, vol. 4, 1995, pp. 1942–1948.

[4] L. Abualigah, M. A. Elaziz, P. Sumari, Z. W. Geem, and A. H. Gandomi, "Reptile search algorithm (RSA): A nature-inspired meta-heuristic optimizer," *Expert Syst. Appl.*, vol. 191, Apr. 2022, Art. no. 116158.

[5] N. Khodadadi, V. Snasel, and S. Mirjalili, "Dynamic arithmetic optimization algorithm for truss optimization under natural frequency constraints," *IEEE Access*, vol. 10, pp. 16188–16208, 2022.

[6] M. H. Qais, H. M. Hasanien, and S. Alghuwainem, "Transient search optimization: A new meta-heuristic optimization algorithm," *Int. J. Speech Technol.*, vol. 50, no. 11, pp. 3926–3941, Nov. 2020.

[7] A. Kaveh, A. D. Eslamlou, and N. Khodadadi, "Dynamic water strider algorithm for optimal design of skeletal structures," *Periodica Polytechnica Civil Eng.*, vol. 64, no. 3, pp. 904–916, Jun. 2020.

[8] L. Abualigah, D. Yousri, M. A. Elaziz, A. A. Ewees, M. A. A. Al-Qaness, and A. H. Gandomi, "Aquila optimizer: A novel meta-heuristic optimization algorithm," *Comput. Ind. Eng.*, vol. 157, Jul. 2021, Art. no. 107250.

[9] A. Kaveh, S. Talatahari, and N. Khodadadi, "Stochastic paint optimizer: Theory and application in civil engineering," *Eng. Comput.*, vol. 38, pp. 1921–1952, Oct. 2020.

[10] Y. Zhang and Z. Jin, "Group teaching optimization algorithm: A novel metaheuristic method for solving global optimization problems," *Expert Syst. Appl.*, vol. 148, Jun. 2020, Art. no. 113246.

[11] A. Kaveh, S. Talatahari, and N. Khodadadi, "Hybrid invasive weed optimization-shuffled frog-leaping algorithm for optimal design of truss structures," *Iranian J. Sci. Technol., Trans. Civil Eng.*, vol. 44, no. 2, pp. 405–420, Jun. 2020.

[12] A. Hu and N. Razmjooy, "Brain tumor diagnosis based on metaheuristics and deep learning," *Int. J. Imag. Syst. Technol.*, vol. 31, no. 2, pp. 657–669, Jun. 2021.

[13] A. Kaveh, S. Talatahari, and N. Khodadadi, "The hybrid invasive weed optimization-shuffled frog-leaping algorithm applied to optimal design of frame structures," *Periodica Polytechnica Civil Eng.*, vol. 63, no. 3, pp. 882–897, Aug. 2019.

[14] M. Kaur, V. Kumar, V. Yadav, D. Singh, N. Kumar, and N. N. Das, "Metaheuristic-based deep COVID-19 screening model from chest X-ray images," *J. Healthcare Eng.*, vol. 2021, pp. 1–9, Mar. 2021.

[15] N. Khodadadi and S. Mirjalili, "Truss optimization with natural frequency constraints using generalized normal distribution optimization," *Appl. Intell.*, vol. 52, pp. 10384–10397, Jan. 2022.

[16] C. Mageshkumar, S. Karthik, and V. P. Arunachalam, "Hybrid metaheuristic algorithm for improving the efficiency of data clustering," *Cluster Comput.*, vol. 22, no. S1, pp. 435–442, Jan. 2019.

[17] A. Kaveh, N. Khodadadi, and S. Talatahari, "A comparative study for the optimal design of steel structures using CSS and ACSS algorithms," *Iran Univ. Sci. Technol.*, vol. 11, no. 1, pp. 31–54, 2021.

[18] G. Sun, H. Zhang, J. Fang, G. Li, and Q. Li, "A new multi-objective discrete robust optimization algorithm for engineering design," *Appl. Math. Model.*, vol. 53, pp. 602–621, Jan. 2018.

[19] N. Khodadadi, S. Talatahari, and A. D. Eslamlou, "MOTEO: A novel multi-objective thermal exchange optimization algorithm for engineering problems," *Soft Comput.*, vol. 26, pp. 6659–6684, Apr. 2022.

[20] L. M. Abualigah, A. T. Khader, and E. S. Hanandeh, "A combination of objective functions and hybrid Krill herd algorithm for text document clustering analysis," *Eng. Appl. Artif. Intell.*, vol. 73, pp. 111–125, Aug. 2018.

[21] A. Kaveh, N. Khodadadi, B. F. Azar, and S. Talatahari, "Optimal design of large-scale frames with an advanced charged system search algorithm using box-shaped sections," *Eng. Comput.*, vol. 37, no. 4, pp. 2521–2541, 2021.

[22] S. Mirjalili, P. Jangir, S. Z. Mirjalili, S. Saremi, and I. N. Trivedi, "Optimization of problems with multiple objectives using the multi-verse optimization algorithm," *Knowl.-Based Syst.*, vol. 134, pp. 50–71, Oct. 2017.

[23] J. Huo and L. Liu, "An improved multi-objective artificial bee colony optimization algorithm with regulation operators," *Information*, vol. 8, no. 1, p. 18, Feb. 2017.

[24] S. Z. Mirjalili, S. Mirjalili, S. Saremi, H. Faris, and I. Aljarah, "Grasshopper optimization algorithm for multi-objective optimization problems," *Appl. Intell.*, vol. 48, no. 4, pp. 805–820, 2018.

[25] H. Zille, H. Ishibuchi, S. Mostaghim, and Y. Nojima, "A framework for large-scale multiobjective optimization based on problem transformation," *IEEE Trans. Evol. Comput.*, vol. 22, no. 2, pp. 260–275, Apr. 2018.

[26] V. Savsani and M. A. Tawhid, "Non-dominated sorting moth flame optimization (NS-MFO) for multi-objective problems," *Eng. Appl. Artif. Intell.*, vol. 63, pp. 20–32, Aug. 2017.

[27] A. Babalik, A. Ozkis, S. A. Uymaz, and M. S. Kiran, "A multi-objective artificial algae algorithm," *Appl. Soft Comput.*, vol. 68, pp. 377–395, Jul. 2018.

[28] J. Liang, W. Xu, C. Yue, K. Yu, H. Song, O. D. Crisalle, and B. Qu, "Multimodal multiobjective optimization with differential evolution," *Swarm Evol. Comput.*, vol. 44, pp. 1028–1059, Feb. 2019.

[29] D. H. Wolper and W. G. Macready, "No free lunch theorems for optimization," *IEEE Trans. Evol. Comput.*, vol. 1, no. 1, pp. 67–82, Apr. 1997.

[30] K. Singh, K. Singh, L. H. Son, and A. Aziz, "Congestion control in wireless sensor networks by hybrid multi-objective optimization algorithm," *Comput. Netw.*, vol. 138, pp. 90–107, Jun. 2018.

[31] A. Özkış and A. Babalık, "A novel Metaheuristic for multi-objective optimization problems: The multi-objective vortex search algorithm," *Inf. Sci.*, vol. 402, pp. 124–148, Sep. 2017.

[32] F. Sheikholeslami and N. J. Navimipour, "Service allocation in the cloud environments using multi-objective particle swarm optimization algorithm based on crowding distance," *Swarm Evol. Comput.*, vol. 35, pp. 53–64, Aug. 2017.

[33] J.-Q. Li, H.-Y. Sang, Y.-Y. Han, C.-Q. Wang, and K.-Z. Gao, "Efficient multi-objective optimization algorithm for hybrid flow shop scheduling problems with setup energy consumptions," *J. Clean. Prod.*, vol. 181, pp. 584–598, Apr. 2018.

[34] S. Mirjalili, P. Jangir, and S. Saremi, "Multi-objective ant lion optimizer: A multi-objective optimization algorithm for solving engineering problems," *Appl. Intell.*, vol. 46, no. 1, pp. 79–95, Jan. 2017.

[35] N. Khodadadi, M. Azizi, S. Talatahari, and P. Sareh, "Multi-objective crystal structure algorithm (MOCryStAl): Introduction and performance evaluation," *IEEE Access*, vol. 9, pp. 117795–117812, 2021.

[36] F. Y. Edgeworth, *F.Y. Edgeworth Mathematical Psychics and Further Papers on Political Economy*. London, U.K.: Oxford Univ. Press, 2003.

[37] V. Pareto, *Cours d'Économie Politique*, vol. 1. Geneva, Switzerland: Librairie Droz, 1964.

[38] C. A. C. Coello, "Evolutionary multi-objective optimization: Some current research trends and topics that remain to be explored," *Frontiers Comput. Sci. China*, vol. 3, no. 1, pp. 18–30, Mar. 2009.

[39] P. Ngatchou, A. Zarei, and A. El-Sharkawi, "Pareto multi objective optimization," in *Proc. 13th Int. Conf. Intell. Syst. Appl. Power Syst.*, 2005, pp. 84–91.

[40] L. Abualigah, A. Diabat, S. Mirjalili, M. A. Elaziz, and A. H. Gandomi, "The arithmetic optimization algorithm," *Comput. Methods Appl. Mech. Eng.*, vol. 376, Apr. 2021, Art. no. 113609.

[41] C. A. C. Coello and M. S. Lechuga, "MOPSO: A proposal for multiple objective particle swarm optimization," in *Proc. Congr. Evol. Comput. (CEC)*, vol. 2, 2002, pp. 1051–1056.

[42] A. J. Nebro, J. J. Durillo, and C. A. C. Coello, "Analysis of leader selection strategies in a multi-objective particle swarm optimizer," in *Proc. IEEE Congr. Evol. Comput.*, Jun. 2013, pp. 3153–3160.

[43] J. Knowles and D. Corne, "Approximating the nondominated front using the Pareto archived evolution strategy," *Evol. Comput.*, vol. 8, no. 2, pp. 149–172, Jan. 2000.

[44] M. R. Sierra and C. A. C. Coello, "Improving PSO-based multi-objective optimization using crowding, mutation and ϵ-dominance," in *Proc. Int. Conf. Evol. Multi-Criterion Optim.*, 2005, pp. 505–519.

[45] S. Mirjalili, A. H. Gandomi, S. Z. Mirjalili, S. Saremi, H. Faris, and S. M. Mirjalili, "Salp swarm algorithm: A bio-inspired optimizer for engineering design problems," *Adv. Eng. Softw.*, vol. 114, pp. 163–191, Dec. 2017.

[46] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Trans. Evol. Comput.*, vol. 6, no. 2, pp. 182–197, Apr. 2000.

[47] S. Mirjalili, S. Saremi, S. M. Mirjalili, and L. D. S. Coelho, "Multi-objective grey wolf optimizer: A novel algorithm for multi-criterion optimization," *Expert Syst. Appl.*, vol. 47, pp. 106–119, Apr. 2016.

[48] J. J. Liang, B. Y. Qu, D. W. Gong, and C. T. Yue, "Problem definitions and evaluation criteria for the CEC 2020 special session on multimodal multiobjective optimization," Comput. Intell. Lab., Zhengzhou Univ., Zhengzhou, China, Tech. Rep. 201912, 2019.

[49] H. Eskandar, A. Sadollah, A. Bahreininejad, and M. Hamdi, ''Water cycle algorithm—A novel metaheuristic optimization method for solving constrained engineering optimization problems,'' *Comput. Struct.*, vols. 110–111, pp. 151–166, Nov. 2012.

[50] N. Srinivasan and K. Deb, ''Multi-objective function optimisation using non-dominated sorting genetic algorithm,'' *Evol. Comput.*, vol. 2, no. 3, pp. 221–248, 1994.

[51] T. T. Binh and U. Korn, ''MOBES: A multiobjective evolution strategy for constrained optimization problems,'' in *Proc. 3rd Int. Conf. Genet. Algorithms (Mendel)*, vol. 25, 1997, p. 27.

[52] A. Osyczka and S. Kundu, ''A new method to solve generalized multicriteria optimization problems using the simple genetic algorithm,'' *Struct. Optim.*, vol. 10, no. 2, pp. 94–99, 1995.

[53] C. C. A. Coello and G. Toscano, ''Multiobjective structural optimization using a microgenetic algorithm,'' *Struct. Multidisciplinary Optim.*, vol. 30, no. 5, pp. 388–403, 2005.

[54] T. Ray and K. M. Liew, ''A swarm metaphor for multiobjective design optimization,'' *Eng. Optim.*, vol. 34, no. 2, pp. 141–153, Jan. 2002.

[55] A. Kurpati, S. Azarm, and J. Wu, ''Constraint handling improvements for multiobjective genetic algorithms,'' *Struct. Multidisciplinary Optim.*, vol. 23, no. 3, pp. 204–213, Apr. 2002.

NIMA KHODADADI (Member, IEEE) received the bachelor's degree in civil engineering and the master's degree in structural civil engineering from the University of Tabriz (one of the ten top universities in Iran). He is currently pursuing the Ph.D. degree in wind engineering with Florida International University (FIU), Miami, FL, USA. In addition, he is also a Research Assistant at the NSF-NHERI Wall of Wind Experimental Facility. Previously, he was a Research Scientist at the Iran University of Science and Technology (IUST). His main research interests include the field of wind engineering, steel structures, and particularly in the experimental and numerical investigation of steel braced frames. In addition, he has been actively involved in the area of engineering optimization, especially in evolutionary algorithms. Using comprehensive finite element analyses, he has also investigated the use of different shaped sections in real steel frames. Recently, he has been engaged in research in the area of single and multi-objective engineering optimization, especially in solving large-scale and practical structural design problems.

LAITH ABUALIGAH (Senior Member, IEEE) received the first degree in computer information system and the master's degree in computer science from Al-Albayt University, Jordan, in 2011 and 2014, respectively, and the Ph.D. degree from the School of Computer Science, University Sains Malaysia (USM), Malaysia, in 2018. He is currently an Associate Professor at Al-Ahliyya Amman University, Jordan. He is also a Distinguished Researcher at the School of Computer Science, USM. According to the report published by Stanford University in 2020, he is one of the 2% influential scholars, which depicts the 100,000 top scientists in the world. He has published more than 220 journal articles and books, which collectively have been cited more than 7000 times (H-index = 39). His main research interests include arithmetic optimization algorithm (AOA), bio-inspired computing, nature-inspired computing, swarm intelligence, artificial intelligence, meta-heuristic modeling, and optimization algorithms, evolutionary computations, information retrieval, text clustering, feature selection, combinatorial problems, optimization, advanced machine learning, big data, and natural language processing. He also serves as an Associate Editor for *Cluster Computing* (Springer), *Soft Computing* (Springer), and *Journal of King Saud University–Computer and Information Sciences* (Elsevier).

EL-SAYED M. EL-KENAWY (Senior Member, IEEE) is currently an Assistant Professor at the Delta Higher Institute for Engineering & Technology (DHIET), Mansoura, Egypt. He published more than 35 articles with more than 1200 citations and an H-index of 22. He has launched and pioneered independent research programs. He motivates and inspires his students in different ways by providing a thorough understanding of various computer concepts. He explains complex concepts in an easy-to-understand manner. His research interests include artificial intelligence, machine learning, optimization, deep learning, digital marketing, and data science. He is a Reviewer of *Computers, Materials & Continua* journal, IEEE Access, and other journals.

VACLAV SNASEL (Senior Member, IEEE) is currently a Professor with the Department of Computer Science, VSB—Technical University of Ostrava, Czech Republic. He works as a Researcher and a University Teacher. He is also the Dean of the Faculty of Electrical Engineering and the Computer Science Department. He is the Head of the Research Program IT4 Knowledge Management, European Center of Excellence IT4 Innovations. His research and development experience includes over 30 years in the industry and academia. He works in a multi-disciplinary environment involving artificial intelligence, social networks, conceptual lattice, information retrieval, semantic web, knowledge management, data compression, machine intelligence, neural networks, web intelligence, nature and bio-inspired computing, data mining, and applied to various real-world problems.

SEYEDALI MIRJALILI (Senior Member, IEEE) is currently a Professor and the Founding Director of the Centre for Artificial Intelligence Research and Optimization, Torrens University Australia. He is internationally recognized for his advances in optimization and swarm intelligence, including the first set of algorithms from a synthetic intelligence standpoint—a radical departure from how natural systems are typically understood—and a systematic design framework to reliably benchmark, evaluate, and propose computationally cheap robust optimization algorithms. He has published over 300 publications with over 40,000 citations and an H-index of 70. As the most cited researcher in robust optimization, he has been on the list of 1% highly-cited researchers and named as one of the most influential researchers in the world by Web of Science for three consecutive years, since 2019. In 2020, he was ranked 21st across all disciplines and 4th in artificial intelligence and image processing in the Stanford University's list of World's Top Scientists. In 2021, The Australian newspaper named him as the top researcher in Australia in three fields of artificial intelligence, evolutionary computation, and fuzzy systems. His research interests include optimization, swarm intelligence, evolutionary algorithms, and machine learning. He is an Associate Editor of several AI journals, including *Neurocomputing*, *Applied Soft Computing*, *Advances in Engineering Software*, *Computers in Biology and Medicine*, *Healthcare Analytics*, *Applied Intelligence*, and IEEE Access.