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APPLIED RESEARCH

Vessel Scheduling and Bunker Management With Speed Deviations for Liner Shipping in the Presence of Collaborative Agreements

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ABSTRACT Vessels face uncertain factors during a voyage, such as bad weather and harsh sea conditions along a route in container liner shipping. As such, the real vessel speed during each leg of a voyage often deviates from the planned one, which may lead to fluctuations in vessel schedule and bunker consumption. This paper investigates the problem of vessel scheduling and bunker management with speed deviations (VSBMSD) for liner shipping in the presence of collaborative agreements. By establishing the worst-case scenario of the maximum bunker consumption function with vessel sailing speed as an independent variable, we develop a mixed-integer nonlinear programming model to minimize the total liner shipping route service cost. A piecewise linear secant approximation method is designed, and then a CPLEX solver is used to solve the problem. The results of the computational experiments conducted for the AEMX route indicate that VSBMSD in the presence of collaborative agreements can enable shipping companies to design vessel schedules reasonably and reduce the total cost of liner shipping route service by at least 2.95% compared to the results from similar studies.

INDEX TERMS Liner shipping, vessel scheduling, bunker management, speed deviation, collaborative agreement, robust optimization.

I. INTRODUCTION

Containerized cargo (containers) mainly move consumer goods with a short life cycle, high unit value, and high time sensitivity, including manufactured products, food, and fashion goods [1]. As a result, liners travel at a higher speed (for example, 20-25 knots) when transporting containerized cargo, so the containers can be transported to their destinations in less time [2]. Attracting potential customers and improving transport services are very important in the current downturn of the shipping market [3]. Therefore, liner shipping companies may experience huge economic losses if they do not set effective vessel schedules [4]. Vessel scheduling (VS) for liner shipping services is a tactical-level planning decision made every three to six months [5]. For liner shipping companies, VS includes determining the optimal vessel speed of the voyage, the arrival time at each port, the handling rates of each port, the departure time from each port, the optimal refueling amount of each port, and the number of vessels required for the route [6].

A collaborative agreement between the liner shipping company and marine container terminal operators is critical for both supply chain players [7]. Marine container terminal operators offer liner shipping companies multiple vessel arrival time windows (TWs) and handling rates, providing liner companies with flexible port arrival scenarios, sailing times, and port handling time options. This can help reduce vessel bunker consumption and total port handling cost [8]. For marine container terminal operators, a collaborative agreement improves the efficiency of terminal operation planning and relieves port congestion [9]. This highlights the value of studying VS in the presence of collaborative agreements.

The bunker consumption of a vessel is approximately proportional to the cubed value of the vessel speed, and container vessels consume more bunker fuels than other types

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of vessels [10]. Previous studies have found that bunker costs make up 30% to 70% of the total liner shipping operational cost. This cost is closely related to bunker fuel prices [11]. To maintain liner shipping services, container vessels must refuel their bunker fuel at certain ports of call during round-trip voyages. The problem of bunker management (BM) is urgent because profit margins in the shipping industry are extremely low, with some liner shipping companies suffering losses [12]. The approaches that liner shipping companies use to determine refueling strategies and effectively manage bunker fuel to reduce the total bunker cost is a vital and urgent problem to address for these businesses. BM is critical for optimizing vessel speeds because it provides an ideal balance between bunker consumption on a particular leg of a voyage and the difference in bunker inventory levels at two consecutive ports. In addition, fluctuating vessel speeds significantly impact bunker consumption [13]. An increase in the vessel speed increases bunker consumption and the amount of fuel needed at ports. Adopting an appropriate refueling policy with fluctuating bunker consumption is an effective way to accurately reduce bunker costs [14]. Therefore, it is of great practical significance for shipping companies to study the joint optimization problem of VS and BM in the presence of collaborative agreements.

In the context of this background, this study solves an optimization problem of VS and BM by jointly determining the optimal vessel speed between consecutive ports and developing a robust refueling strategy in the presence of collaborative agreements. The goal is to minimize the total liner shipping route service cost. Solving these problems supports the operation of liner shipping companies in several ways. First, at a tactical level, this method can help design a vessel schedule that can benefit both supply chain players. Second, the approach can be used to formulate medium-term and long-term bunker fuel procurement plans for each port [15] and facilitate the development of detailed bunker fuel procurement contracts with bunker fuel suppliers [16], [17]. Finally, at an operational level, this method provides a useful reference for liner shipping companies by considering the fluctuation of bunker consumption and port bunker fuel prices, determining different vessel speeds at different legs of a voyage, and determining the refueling amount at each port.

A. LITERATURE REVIEW

VS is closely related to the amount of cargo that vessels handle at the port and the vessel speed at sea. Perakis and Jaramillo [18] first established an integer linear programming model to study this problem, assuming that the freight demand is predetermined, and developed a schedule for vessel arrivals and departures from the port. Ng [19] presented a nonlinear mixed-integer programming model for VS in the case of the stochastic fluctuation of demand, where the mean and variance are known. The bunker cost accounted for a large proportion of the total liner shipping route service cost. Brouer et al. [20] found that vessel bunker consumption was directly proportional to the cubed value of the vessel's speed. The problem of VS based on speed optimization has important practical significance for liner shipping companies. For that reason, many scholars have researched ways to reduce vessel bunker consumption by optimizing vessel speed [6], [21], [22], [23], [24].

However, these studies have not considered important BM and refueling strategies. The bunker fuel price may vary significantly at different ports, so correctly selecting refueling ports and reasonably estimating refueling amounts are important for reducing the bunker cost and optimizing the vessel speed, which then affects VS and handling cost. Kim et al. [25] determined the optimal vessel speed and required refueling amount for a given liner shipping route, evaluated the performance of refueling ports, and conducted a sensitivity analysis for refueling strategies. The study identified a positive effect of a slow steaming policy on the environment and analyzed changes in refueling strategy. Yao et al. [26] stressed the importance of optimizing three components: the selection of the refueling port, the estimated refueling amounts, and adjustments in the vessel speeds during different legs of the voyage. These were jointly evaluated to calculate an optimal refueling strategy.

In another study, Kim [27] proposed a Lagrangian heuristic to determine the variable speed and refueling ports for a container vessel, allowing the vessel to arrive at ports at any time. Sheng et al. [28] developed a multistage dynamic model, considering the stochastic nature of bunker prices to dynamically determine the refueling strategy for liner shipping. Sheng et al. [29] investigated an effective dynamic (s,S) policy in a refueling and speed determination problem, given uncertainties in both bunker fuel prices and bunker consumption. Meng et al. [30] developed a joint route and refueling problem for tramp vessels by considering differences in cargo types and bunker fuel prices. That study proposed a branch and price solution approach based on predictions of future cargo demand to obtain an optimal refueling strategy and effective route options. When addressing the speed optimization problem in liner shipping, Aydin et al. [31] considered the randomness of time and proposed a dynamic programming model to determine the impact of BM policies and bunker prices on vessel schedules.

In addition, different collaborative agreements have been established between marine container terminal operators and liner shipping companies [32], gradually attracting widespread attention in the shipping market. This highlights the need to discuss the problem of VS in the presence of a collaborative agreement. Wang et al. [3] proposed an overall solution that merged the available vessel service TWs at each port in a week and expressed it as a mixed-integer nonlinear nonconvex optimization model to solve a tactical problem of VS. The results showed that the designed schedule based on multiple vessel arrival TWs offered by marine container terminal operators can be applied in practice without or with only minor modifications. However, that study's model focused on arrival time windows and did not model and analyze other influencing factors. Alharbi et al. [33] studied the problem of VS, focusing on the availability of multiple vessel arrival TWs for container supply chain networks. They proposed a mixed-integer nonlinear nonconvex programming model with the objective function of minimizing the total route service cost and designed an iterative optimization approach. Simulations showed that the vessel arrival TWs selected significantly influenced the number of vessels, vessel speed, and route service.

Focusing even more clearly on collaboration, Liu et al. [9] applied an operational collaborative mechanism where marine container terminal operators offer multiple handling rates and liner shipping companies pay for the additional costs. Higher handling rates allow vessels more time to sail at sea, reducing bunker consumption. The reduced bunker costs for liner shipping companies may be more essential than the additional costs incurred by container terminals. To minimize the sum of bunker consumption and port handling cost, this study proposes a global optimization algorithm and conducts a set of calculation experiments on a CCX liner shipping route. The results showed that the proposed collaborative agreement helped liner shipping companies reduce bunker costs and improved the overall efficiency of container transportation. In another study, Dulebenets proposed a new collaborative agreement [7] where for each port along a given liner shipping route, the marine container terminal operators provided liner shipping companies with multiple arrival TWs, multiple TW start and end times, and multiple TW-specific handling rates. Several numerical simulations performed for the Pacific Atlantic 1 liner shipping route showed that the proposed collaborative agreement provided an effective alternative to liner shipping business improvements.

In the above studies on the problems of VS and BM, the factors to be considered ranged from a clear determination of transport demand to random demand, from no refueling strategy to a refueling strategy selection, from bunker fuel prices that followed a specific random distribution to prices that followed a defined stochastic process, and from no TW constraints to multiple TWs and multiple handling rate strategies. However, these reviewed studies were conducted with the assumption of a constant speed on each leg of a voyage. In practical applications, the vessel speed often deviates due to environmental and human factors, resulting in a series of chain reactions. Only a few studies considered speed deviation for the problem of VS. Wang and Meng [14] examined the vessel speed and refueling strategy in a liner shipping network, allowing the real speed of the vessel to deviate from the planned speed. They derived a closed-form expression for the worst-case bunker consumption by assuming real speed changes in a certain range. Arijit De et al. [34] expanded the research by considering stochastic bunker consumption, stochastic bunker fuel prices, and different bunker refueling policies to determine an optimal refueling strategy. They then proposed a novel approximate algorithm based on a mathematical formulation. Different from the above two studies, our work puts forward the VS and BM optimization problem in accordance with speed deviation and collaborative agreements jointly and adopts a robust optimization framework by defining interval uncertain sets for the nonlinear model. The key issues involved in our paper are shown in Table 1, which provides a setting comparison for the most relevant VS and BM studies of our paper.

B. CONTRIBUTION

The contribution of this study is threefold. First, the VSBMSD optimization problem for liner shipping in the presence of collaborative agreements is proposed jointly. It is a new research topic with practical significance because it has never been addressed comprehensively by the literature. In practice, real vessel speeds continuously change within a range above and below the planned speed. As such, we consider the change in real speed. Second, the maximum bunker consumption function under the worst-case speed deviation is derived, and an approximate expression of it is obtained. Then, a "big M" technique was exploited to transform the model into a MILP model based on the analysis of its structural properties to further accelerate the convergence of our algorithm. Third, we conduct extensive computational experiments based on realistic instances to validate the effectiveness of our model and the efficiency of the algorithm. Based on the results from the experiments, managerial insights are acquired.

The rest of this paper is organized as follows. Section 2 describes the problem, and Section 3 presents the mathematical model. Section 4 presents a closed-form expression for the worst-case bunker consumption and presents piecewise linear secant approximation methods for the model. Section 5 describes a numerical experiment of a real-case route to illustrate the model's effectiveness. Section 6 provides conclusions and outlines the prospects for future research.

II. PROBLEM DESCRIPTION

Consider a liner shipping company that deploys a certain number of vessels to provide container transportation service on a given route, which has several container ports of call. The shipping company has signed collaborative agreements with container terminal operators in the ports of call. When considering vessel speed deviation, the goal of the shipping company's VS and BM is to minimize the total liner shipping route service cost. Thus, this section presents the following concepts or expressions in the study problem: (a) collaborative agreement; (b) port handling cost; (c) late arrival penalty; (d) container inventory cost; (e) bunker consumption; (f) the worst-case bunker consumption model; and (g) refueling ports and bunker cost. The optimization model for the VSBMSD problem is formulated accordingly in the next section.

A. COLLABORATIVE AGREEMENT

We denote the set of ports where vessels visit as $I = \{1, 2, \cdots, n\}$. A vessel sails between two consecutive ports

TABLE 1. Setting comparison for VS and BM literature.

Related literature	Modeling method	Solution	VS	BM	Collaborative agreement	Speed optimization	Speed deviation
Perakis et al. (1991)	LP	Deterministic optimization	✓			✓	
Fagerholt et al. (2010)	NP	Heuristic algorithm		✓		✓	
Norstad et al. (2011)	NP	Heuristic algorithm	✓			✓	
Yao et al. (2012)	NP	Piecewise linear approximation		✓		✓	
Qi and Song (2012)	NCP	Stochastic approximation	✓	✓			
Kim et al. (2012)	LP	Epsilon-optimal algorithm		✓			
Wang et al. (2013)	MINLP	Autoconduction second-order cone programming		✓		✓	
Ng (2014)	MILP	Stochastic programming	✓	✓			
Brouer et al. (2014)	IP	Heuristic algorithm	✓	✓		✓	
Psaraftis et al. (2014)	NP	Dynamic programming	✓			✓	
Kim (2014)	NP	Lagrangian heuristic algorithm		✓		✓	
Sheng et al. (2014)	MIP	Modified rolling horizon method		✓		✓	
Wang et al. (2014)	MINLP	Holistic solution		✓	✓	✓	
Alhrabi et al. (2015)	ILP	Iterative optimization	✓		✓		
Wang et al. (2015)	MINLP	Robust optimization		✓		✓	✓
Sheng et al. (2015)	MINCP	Progressive hedging algorithm		✓		✓	
Meng et al. (2015)	ILP	Branch-and-price solution	✓	✓			
Liu et al. (2016)	MINLP	Global optimization algorithm		✓	✓	✓	
Aydin et al. (2017)	DP	Dynamic programming		✓		✓	
Dulebenets (2019)	MINLP	Discretization			✓		
De et al. (2021)	MINLP	Heuristic algorithm		✓		✓	✓
Our paper	MINLP	Piecewise linear secant approximation	✓	✓	✓	✓	✓

i and $i+1$ along voyage leg i. The liner shipping company negotiates a specific collaborative agreement with marine container terminal operators, with three main components: (1) a set of vessel arrival TWs $T_i = \{1, 2, \cdots, m_i\}$, where $i \in I$ is provided to the liner shipping company at each port of call; (2) a set of start times $S_{it} = \{1, 2, \cdots, g_{it}\}$, where $i \in I, t \in T_i$, and end times $E_{it} = \{1, 2, \cdots, o_{it}\}$, where $i \in I$, $t \in T_i$, are included for each TW; and (3) a set of handling rates $H_{it} = \{1, 2, \cdots, w_{it}\}$, where $i \in I, t \in T_i$ is provided to the liner shipping company during each TW.

The arrival TWs at each port are divided into two parts: TW^S_{its} is the start time s for TW t at port i, and TW^E_{ite} is the end time e for TW t at port i. A vessel should arrive at the port of call within these two TWs. This assumes that the marine container terminal operator of each port provides the liner shipping company with m vessel arrival TWs, g available start times for each TW, o available end times for each TW, and w available vessel handling rates during the available TWs. These variables are transformed into the collaborative agreement, where the marine container terminal operator of each port provides $(m \cdot g \cdot o)$ available vessel arrival TWs and w available vessel handling rates during the available TWs. The TW durations vary from port to port but generally range from 1 to 3 days. Figure 1 shows a scenario where the marine container terminal operator of port i provides 3 start times and 3 end times for each TW t. This is transformed as follows: the marine container terminal operator of port i provides 3·3·m=9m vessel arrival TWs (where m is the total vessel arrival TWs). The vessel must arrive within the TWs. Section 2.3 describes the impacts of arriving outside the TWs on the liner shipping service.

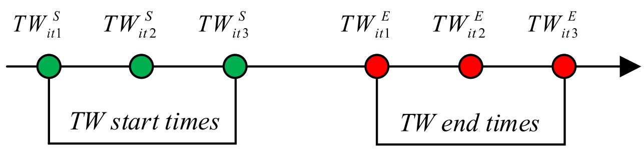

FIGURE 1. Port scenario with 3 start times and 3 end times.

B. PORT HANDLING COST

Along with TW selection, each marine container terminal operator offers a set of handling rates, with a corresponding handling productivity $HP_{ith}, i \in I, t \in T_i, h \in H_{it}$ (measured in TEU/hour). Handling productivities differ at different ports and depend on the amount of handling equipment available to the marine container terminal operator in the selected TW. The vessel handling time during the selected TW t at port i is estimated based on the number of containers to be handled at a given port D_i (TEU) and the handling rate h. This yields the following equation:

$$t_i^b = \sum_{t \in T_i} \sum_{h \in H_{it}} y_{ith} \frac{D_i}{HP_{ith}} \tag{1}$$

The vessel handling cost $C^{hc}_{ith}, i \in I, t \in T_i, h \in H_{it}$ (USD/TEU) gap depends mainly on the ports of call, selected TWs, and requested handling rates. Even if the handling productivity is the same, the vessel handling cost at one port may be higher compared to other ports of call. Vessel handling costs may vary due to equipment availability during the selected TW at each port. Furthermore, a higher handling productivity results in a higher vessel handling cost.

The expected total port handling cost is computed as:

$$TPC = \sum_{i \in I} \sum_{t \in T_i} \sum_{h \in H_{it}} C_{ith}^{hc} y_{ith} D_i \tag{2}$$

where TPC is the total port handling cost (USD) and y_{ith} is a binary variable used to determine the vessel handling rate.

C. LATE ARRIVAL PENALTY

Generally, vessels need to arrive at port i on time, within the selected arrival TW t. Vessels that arrive at port i before the TW range must wait in a dedicated area, and vessels that complete port operations outside the TW pay penalty charges. Vessels that arrive late cause congestion at ports, disrupting marine terminal operations and causing service delays for other vessels. The total penalty paid by the vessel arriving after the selected arrival TW ends is estimated as follows:

$$TLP = \sum_{i \in I} C_i^{lc} t_i^l \tag{3}$$

where TLP is the total late arrival penalty (USD), C_i^{lc} is the per unit late arrival penalty at port i (USD/h), and t_i^l is the number of hours that the vessel is late in arriving at port i (h).

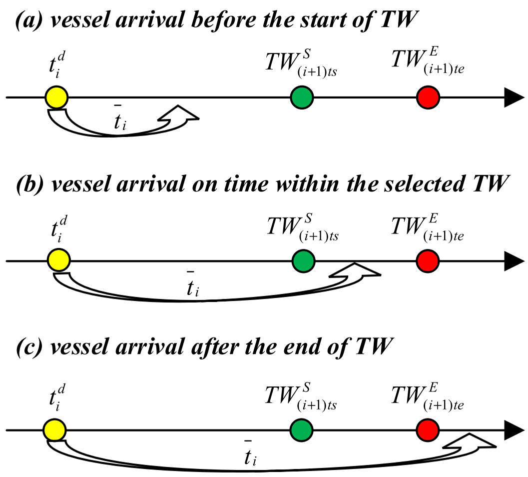

FIGURE 2. Three scenarios of a vessel arriving at the next port.

Figure 2 shows three scenarios where a vessel leaves from port i and arrives at the next port. The variable t_i^d denotes the vessel departure time from port i (h), and $\bar{t}_i$ denotes the planned sailing time of a vessel at voyage leg i (h).

D. CONTAINER INVENTORY COST

Generally, a higher vessel speed is associated with lower container inventory costs. The container inventory cost relates to the total sailing time and the number of containers. This cost is calculated as:

$$TIC = C^{ic} \sum_{i \in I} R_i \bar{t}_i \tag{4}$$

where TIC is the total container inventory cost (USD), C^{ic} is the unit container inventory cost (USD per TEU per hour), and R_i is the number of containers transported during voyage leg i (TEU).

E. BUNKER CONSUMPTION

During actual operations, the vessel speed is impacted by uncertain factors, including weather, sea conditions, and the route's environment. As such, the vessel cannot sail at a fixed speed during the voyage. We assume that a vessel's real sailing speed at time t during voyage leg i is $v_i(t)$, and $v_i(t)$ deviates from the planned speed $\bar{v}_i$ within a set range. The deviation range is expressed as:

$$\bar{v}_i - V^{err} \le v_i(t) \le \bar{v}_i + V^{err}, \quad \forall i \in I \tag{5}$$

Past studies have found that the bunker consumption $g_i(v)$ (ton/hour) is directly proportional to the power function of the real speed $v_i(t)$[9]. Therefore, the vessel bunker consumption function at time t on voyage leg i is expressed as:

$$g_i(v_i(t)) = \alpha v_i(t)^{\beta}, \alpha > 0, \beta > 2, \forall i \in I \tag{6}$$

where α and β are bunker consumption function coefficients.

Here, L_i is the length of voyage leg i (i.e., port i to port $i+1$, n miles). To ensure that the vessel arrives at the next port of call i at the scheduled time of arrival $t_i^d + \frac{L_i}{\bar{v}_i}$, the captain periodically adjusts the vessel speed to respond to bad weather or other uncertainties. Therefore, the real bunker consumption of a vessel during a particular leg varies over time throughout the voyage. To facilitate this expression, we introduce variable $\bar{t}_i$, representing the planned sailing time of voyage leg i. This variable replaces equation $\frac{L_i}{\bar{v}_i}$. Thus, the relationship between the total bunker consumption $Q_i(v_i(t))$ and the real sailing speed $v_i(t)$ during voyage leg i is calculated using the following formula.

$$Q_i(v_i(t)) = \int_0^{\bar{t}_i} g_i(v_i(t))\,dt, \quad \forall i \in I \tag{7}$$

F. THE WORST-CASE BUNKER CONSUMPTION MODEL

We propose a model to calculate the bunker consumption during a certain leg of the voyage; the model estimates the worst-case bunker consumption during a particular leg of the voyage along a certain route with the given planned speed $\bar{v}_i$. The maximum bunker consumption under the worst case of speed deviation is expressed as $Q_i^{\max}(\bar{v}_i)$ and depends on the change in the vessel speed $v_i(t)$. The vessel speed is evaluated at all possible vessel speed profiles $v_i(t)$ under given planned speed conditions $\bar{v}_i$, maximizing bunker consumption $Q_i(v_i(t))$. Therefore, the worst-case scenario of maximum bunker consumption $Q_i^{\max}(\bar{v}_i)$ is determined using the following formula:

$$Q_i^{max}(\bar{v}_i) = \max_{v_i(t)} \int_0^{\bar{t}_i} g_i(v_i(t))\,dt, \quad \forall i \in I \tag{8}$$

To meet the requirements of shipping schedules and customer transportation services, the vessel must arrive at the next port of call at a scheduled time. The following expression shows the constraint of the planned arrival time to ensure that the vessel arrives on time:

$$\int_0^{\bar{t}_i} v_i(t)dt = L_i, \quad \forall i \in I \tag{9}$$

As stated in the above constraint, the speed variability of $v_i(t)$ must fall within the range $\bar{v}_i - V^{err} \le v_i(t) \le \bar{v}_i + V^{err}$, which is consistent with real-world conditions. The higher the value of V^{err} is, the more bunker fuel the vessel must consume. The lower the value of V^{err} is, the more vessels may be developed on the route.

G. REFUELING PORTS AND BUNKER COST

We assume that the bunker inventory level when the vessel arrives at port i is z_i^1. The bunker inventory level when the vessel leaves port i is z_i^2. The relationship between z_i^1 and z_i^2 is as follows:

$$z_{i+1}^1 = z_i^2 - Q_i^{max}(\bar{v}_i), \quad \forall i \in I, i < n \tag{10}$$
$$z_1^1 = z_n^2 - Q_n^{max}(\bar{v}_n) \tag{11}$$

We denote z_i as the refueling amount at port i; it is calculated as $z_i = z_i^2 - z_i^1$. To facilitate the research, we introduce the refueling cost function $f(z_i)$, which is a piecewise linear function with three linear segments about the refueling amount z_i, as shown in Figure 3.

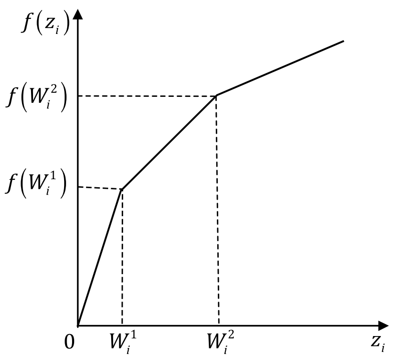

FIGURE 3. A piecewise linear function of the bunker cost.

Hence, the refueling cost function at port i is expressed as in (12), shown at the bottom of the page, where C_i^{f0} is the per unit fuel price at port i for regular bunkers when the refueling amount is within W_i^1 (USD/ton); and λ_i^1, λ_i^2 are the bunker fuel price discount factors for different refueling amounts at each port (%), where $0 < \lambda_i^2 \le \lambda_i^1 \le 1$. The variables W_i^1 and W_i^2 are the breakpoints in the refueling amounts, where the liner shipping company can receive a price discount, $W_i^1 \le W_i^2$ (tons).

Liner shipping companies pay a fixed fee for each refueling. Hence, the total bunker cost of all ports is $FC = \sum_{i\in I} [F + f(z_i)] b_i$. Here, F is the fixed cost to refuel the vessel (USD), and b_i is a binary variable used to decide whether to refuel at port i.

III. MODEL FORMULATION

In this section, we start by denoting the parameters and variables and then formulate a mixed-integer nonlinear mathematical model for the VSBMSD problem in the presence of collaborative agreement. Then, we present some important theoretical results of the model through reformulations and a "big M" technique, which are used for our algorithm design in the following section.

A. SYMBOLS AND NOTATIONS

- **Sets**

$I = \{1, 2, \cdots, n\}$: set of ports in the liner shipping route, indexed by i;

$T_i = \{1, 2, \cdots, m_i\}$, $i \in I$: set of arrival TWs available at port i, indexed by t;

$S_{it} = \{1, 2, \cdots, g_{it}\}$, $i \in I$, $t \in T_i$: set of start times for TW t at port i, indexed by s;

$E_{it} = \{1, 2, \cdots, o_{it}\}$, $i \in I$, $t \in T_i$: set of end times for TW t at port i, indexed by e;

$H_{it} = \{1, 2, \cdots, w_{it}\}$, $i \in I$, $t \in T_i$: set of handling rates available at port i during TW t, indexed by h.

- **Parameters**

C^{oc} : weekly vessel operational cost (USD/week);

C_i^{lc}, $i \in I$: unit vessel late arrival penalty at port i (USD/h);

C^{ic} : unit container inventory cost (USD/TEU/h);

C_i^{f0}, $i \in I$: unit regular bunker fuel price at port i when the refueling amount is within W_i^1 (USD/ton);

C_{ith}^{hc}, $i \in I$, $t \in T_i$, $h \in H_{it}$: handling cost at port i under handling rate h during TW t(USD/TEU);

W_i^1, $i \in I$: amount of refueling with the first discount at port i (tons);

W_i^2, $i \in I$: amount of refueling with the second discount at port i (tons);

λ_i^1, $i \in I$: bunker fuel price discount factor at port i when the refueling amount is in the range of $\left(W_i^1, W_i^2\right]$ (%);

λ_i^2, $i \in I$: bunker fuel price discount factor at port i when the refueling amount exceeds W_i^2 (%);

L_i, $i \in I$: length of voyage leg i (i.e., port i to port $i+1$, n miles), where L_n is the length between consecutive ports n and 1 (n miles);

R_i, $i \in I$: total amount of containers transported at voyage leg i (TEU);

D_i, $i \in I$: total amount of containers handled at port i (TEU);

$V^{\min}$: minimum vessel sailing speed (knots);

$V^{\max}$: maximum vessel sailing speed (knots);

$$f(z_i) = \begin{cases} C_i^{f0} z_i & 0 \le z_i \le W_i^1 \\ C_i^{f0}\left[W_i^1 + \lambda_i^1\left(z_i - W_i^1\right)\right] & W_i^1 < z_i \le W_i^2 \\ C_i^{f0}\left[W_i^1 + \lambda_i^1\left(W_i^2 - W_i^1\right) + \lambda_i^2\left(z_i - W_i^2\right)\right] & W_i^2 < z_i \le W \end{cases} \tag{12}$$

V^{err} : speed deviation from extreme value (knots);
W : vessel bunker tanker maximum capacity (tons);
F : fixed cost to refuel vessel (USD);
I_0 : vessel initial bunker inventory level (tons);
TW_{its}^{S}, $i \in I, t \in T_i, s \in S_{it}$: the start time s for TW t at port i (h);
TW_{ite}^{E}, $i \in I, t \in T_i, e \in E_{it}$: the end time e for TW t at port i (h);
HP_{ith}, $i \in I, t \in T_i, h \in H_{it}$: handling productivity for handling rate h at port i during TW t (TEU/h).

- **Decision Variables**

$\bar{v}_i$, $i \in I$: planned vessel sailing speed during voyage leg i (i.e., port i to port $i+1$, knots);
$b_i, i \in I$: $= 1$ if the vessel refuels at port i ($= 0$ otherwise);
x_{it}^{TW}, $i \in I, t \in T_i$: $= 1$ if the vessel arrives at port i within TW t ($= 0$ otherwise);
x_{its}^{S}, $i \in I, t \in T_i, s \in S_{it}$: $= 1$ if start time s is selected for TW t at port i ($= 0$ otherwise);
x_{ite}^{E}, $i \in I, t \in T_i, e \in E_{it}$: $= 1$ if end time e is selected for TW t at port i ($= 0$ otherwise);
y_{ith}, $i \in I, t \in T_i, h \in H_{it}$: $= 1$ if handling rate h is selected at port i during TW t ($= 0$ otherwise).

- **Auxiliary Variables**

p : number of vessels to be deployed for the liner shipping route (vessels);
t_i^a, $i \in I$: vessel arrival time at port i (h);
t_i^b, $i \in I$: vessel handling time at port i (h);
t_i^d, $i \in I$: vessel departure time from port i (h);
t_i^w, $i \in I$: vessel waiting time at port i (h);
t_i^l, $i \in I$: vessel late arrival time at port i (h);
$\bar{t}_i$, $i \in I$: sailing time of the vessel with planned vessel speed $\bar{v}_i$ during voyage leg i (h);
z_i^1, $i \in I$: bunker inventory level when the vessel arrives at port i(tons);
z_i^2, $i \in I$: bunker inventory level when the vessel leaves port i (tons);
z_i, $i \in I$: refueling amount at port i (tons);
$f(z_i)$, $i \in I$: refueling cost function of the vessel at port i;
$Q_i^{\max}(\bar{v}_i)$, $i \in I$: maximum bunker consumption under the worst-case speed deviation when sailing at planned speed $\bar{v}_i$ during voyage leg i.

B. MODELING THE VSBNSD PROBLEM

As we have discussed in the previous sections, liner shipping companies should make their decision to minimize the total cost of liner shipping route service in the presence of collaborative agreements. The VSBMSD model can be formulated as:

$$\min \left\{ C^{oc}p + \sum_{i \in I}[F + f(z_i)]b_i + \sum_{i \in I}\sum_{t \in T_i}\sum_{h \in H_{it}} C_{ith}^{hc} D_i y_{ith} + C^{ic}\sum_{i \in I}\bar{t}_i R_i + \sum_{i \in I} C_i^{lc} t_i^l \right\} \quad (13)$$

The model is subject to the following constraints:

$$0.1 b_i W \le z_i \le b_i W, \quad \forall i \in I \quad (14)$$
$$z_i = z_i^2 - z_i^1, \quad \forall i \in I \quad (15)$$
$$z_1^1 = I_0 \quad (16)$$
$$z_i^1 \ge 0.1W, \quad \forall i \in I \quad (17)$$
$$z_i^2 \le W, \quad \forall i \in I \quad (18)$$
$$z_{i+1}^1 = z_i^2 - Q_i^{\max}(\bar{v}_i), \quad \forall i \in I, i < n \quad (19)$$
$$z_1^1 = z_n^2 - Q_n^{\max}(\bar{v}_n) \quad (20)$$
$$\sum_{t \in T_i} x_{it}^{TW} = 1, \quad \forall i \in I \quad (21)$$
$$\sum_{t \in T_i}\sum_{s \in S_{it}} x_{its}^{S} = 1, \quad \forall i \in I \quad (22)$$
$$x_{its}^{S} \le x_{it}^{TW}, \quad \forall i \in I,\ t \in T_i,\ \forall s \in S_{it} \quad (23)$$
$$\sum_{t \in T_i}\sum_{e \in E_{it}} x_{ite}^{E} = 1, \quad \forall i \in I \quad (24)$$
$$x_{ite}^{E} \le x_{it}^{TW}, \quad \forall i \in I,\ t \in T_i,\ e \in E_{it} \quad (25)$$
$$\sum_{t \in T_i}\sum_{h \in H_{it}} y_{ith} = 1, \quad \forall i \in I \quad (26)$$
$$y_{ith} \le x_{it}^{TW}, \quad \forall i \in I,\ \forall t \in T_i,\ \forall h \in H_{it} \quad (27)$$
$$\bar{t}_i = \frac{L_i}{\bar{v}_i}, \quad \forall i \in I \quad (28)$$
$$t_i^b = \sum_{t \in T_i}\sum_{h \in H_{it}} y_{ith}\frac{D_i}{HP_{ith}}, \quad \forall i \in I \quad (29)$$
$$t_i^d = t_i^a + t_i^b + t_i^w, \quad \forall i \in I \quad (30)$$
$$\sum_{i \in I}\left(\bar{t}_i + t_i^b + t_i^w\right) = 168p \quad (31)$$
$$t_{i+1}^a = t_i^d + \bar{t}_i, \quad \forall i \in I,\ i < n \quad (32)$$
$$t_1^a = t_n^d + \bar{t}_n - 168p \quad (33)$$
$$t_i^l \ge t_i^a - \sum_{t \in T_i}\sum_{e \in E_i} TW_{ite}^{E} x_{ite}^{E}, \quad \forall i \in I \quad (34)$$
$$t_{i+1}^w \ge \sum_{t \in T_i}\sum_{s \in S_{it}} TW_{(i+1)ts}^{S} x_{(i+1)ts}^{S} - t_i^a - t_i^b - t_i^w - \bar{t}_i, \quad \forall i \in I,\ i < n \quad (35)$$
$$t_1^w \ge \sum_{t \in T_1}\sum_{s \in S_{1t}} TW_{1ts}^{S} x_{1ts}^{S} - t_n^a - t_n^b - t_n^w - \bar{t}_n + 168p \quad (36)$$
$$V^{\min} + V^{err} \le \bar{v}_i \le V^{\max} - V^{err}, \quad \forall i \in I \quad (37)$$
$$x_{it}^{TW}, x_{its}^{S}, x_{ite}^{E}, b_i, y_{ith} \in \{0, 1\}, \forall i \in I, \forall t \in T_i, \forall e \in E_{it}, \forall s \in S_{it}, \quad \forall h \in H_{it} \quad (38)$$

Objective Function (13) of the VSBMSD model minimizes the total liner shipping route service cost incurred by one vessel per cycle. This total cost includes the following components: vessel operational cost, bunker cost, port handling cost, container inventory cost, and vessel late arrival penalty.

The following constraints listed above apply in the model. Constraints (14) and (15) calculate the bunkers purchased at a port and provide the respective range. Constraint (16) represents the vessel's initial bunker fuel inventory level. Constraint (17) ensures that the minimum level of bunker fuel inventory is reached before every refueling of a vessel. Constraint (18) ensures that the maximum bunker fuel inventory is reached after every refueling of a vessel. Constraints (19) and (20) define the relationship between the worst-case scenario of maximum bunker consumption and the bunker inventory level before and after every refueling at the port. Constraint (21) ensures that only one service TW is selected from the vessel's available arrival TWs at each port. Constraints (22) to (25) ensure that the vessel has only one start time and one end time in the selected TW at each port. Constraints (26) and (27) ensure that the vessel selects only one handling rate for the operations within the selected TW at each port. Constraint (28) computes the planned sailing time of the vessel for each leg of the voyage. Constraint (29) computes the vessel handling time, operated using the selected handling rate at each port. Constraint (30) computes the vessel departure time from each port. Constraint (31) ensures that the weekly service is provided by each port, so a round-trip voyage is timed to be an integer that is a multiple of a week; this equals 168 hours (one week), multiplied by the number of vessels developed on the route. Constraints (32) and (33) compute the vessel's arrival time at each port. Constraint (34) estimates the vessel's late arrival time. Constraints (35) and (36) estimate the vessel's waiting time at a later port. Constraint (37) imposes the range limits of the vessel sailing speed. Constraint (38) is a binary constraint.

IV. METHODOLOGY

In this section, we first derive the maximum bunker consumption function under the worst-case speed deviation and obtain an approximate expression of it. After that, a linearization method is applied to transform the model into a MILP model based on the nonlinear characteristics of the objective function and model constraints. The MILP solver is then used to solve the problem.

A. DERIVATION OF THE WORST-CASE BUNKER CONSUMPTION FUNCTION

First, we analyze and verify the model defined in the problem description above and obtain an approximate expression of the function $Q_i^{\max}(\bar{v}_i)$.

Theorem 1: For speed $v_i(t)$, the vessel is assumed to sail at speed $\bar{v}_i - V^{err}$ in time period $\frac{\bar{t}_i}{2}$. It sails at speed $\bar{v}_i + V^{err}$ for the other time period $\frac{\bar{t}_i}{2}$. In this case, the maximum bunker consumption function under the worst-case speed deviation $Q_i^{max}(\bar{v}_i)$ is obtained as follows:

$$Q_i^{max}(\bar{v}_i) = \frac{\bar{t}_i}{2}\alpha\left[\left(\bar{v}_i - V^{err}\right)^{\beta} + \left(\bar{v}_i + V^{err}\right)^{\beta}\right] \quad (39)$$

Proof: We prove the theorem through contradiction. Assume the speed is $v_i^*(t)$ under the worst-case speed deviation (in this case, $v_i^*(t)$ is considered the variable speed); we then select a small time interval $[t_1, t_1 + \Delta t]$, with a constant mean speed, specified as $\bar{\bar{v}}_i$. Its range is $\bar{v}_i - V^{err} < \bar{\bar{v}}_i < \bar{v}_i + V^{err}$. The variable $\bar{\bar{v}}_i$ increases or decreases within a smaller value Δv_i, where $\Delta v_i = \min\left\{\bar{\bar{v}}_i - (\bar{v}_i - V^{err}), (\bar{v}_i + V^{err}) - \bar{\bar{v}}_i\right\}$. We then build a new speed piecewise function $\overrightarrow{v}_i(t)$:

$$\overrightarrow{v}_i(t) = \begin{cases} v_i^*(t), & t \in [0, t_1] \cup [t_1 + \Delta t, \bar{t}] \\ \bar{\bar{v}}_i + \Delta v_i, & t \in (t_1, t_1 + \frac{\Delta t}{2}] \\ \bar{\bar{v}}_i - \Delta v_i, & t \in (t_1 + \frac{\Delta t}{2}, t_1 + \Delta t] \end{cases} \quad (40)$$

Then, we compare the bunker consumption at speed $\overrightarrow{v}_i(t)$ and speed $v_i^*(t)$:

$$\begin{aligned} &\int_0^{\bar{t}} g_i\left(\overrightarrow{v}_i(t)\right)dt - \int_0^{\bar{t}} g_i\left(v_i^*(t)\right)dt \\ &= \int_{t_1}^{t_1+\Delta t} \alpha \overrightarrow{v}_i(t)^{\beta}\, dt \\ &\quad - \int_{t_1}^{t_1+\Delta t} \alpha v_i^*(t)^{\beta}\, dt = \frac{\Delta t}{2}\alpha[(\bar{\bar{v}}_i + \Delta v_i)^{\beta} \\ &\quad + (\bar{\bar{v}}_i - \Delta v_i)^{\beta}] - \Delta t \alpha \bar{\bar{v}}_i^{\beta} > 0 \end{aligned} \quad (41)$$

The bunker consumption is larger at speed $\overrightarrow{v}_i(t)$ compared to speed $v_i^*(t)$. Hence, $v_i^*(t)$ is not the speed that causes the maximum bunker consumption under the worst-case speed deviation. To determine the model solution, we verify the properties of the function $Q_i^{\max}(\bar{v}_i)$.

Theorem 2: $Q_i^{\max}(\bar{v}_i)$ is an increasing convex function with respect to the planned speed $\bar{v}_i$.

Proof: We calculate the first and second derivatives of the function $Q_i^{max}(\bar{v}_i) = \frac{\alpha L_i}{2\bar{v}_i}\left[(\bar{v}_i - V^{err})^{\beta} + (\bar{v}_i + V^{err})^{\beta}\right]$ with respect to $\bar{v}_i$ and analyze their properties.

The first derivative is as in (42), shown at the bottom of the next page.

This yields $\frac{Q_i^{max}(\bar{v}_i)}{d\bar{v}_i} > 0$.

Then, we calculate the second-order derivative as in (43), shown at the bottom of the next page.

We also determine that $\frac{d^2 Q_i^{max}(\bar{v}_i)}{d\bar{v}_i^2} > 0$. Hence, $Q_i^{\max}(\bar{v}_i)$ is an increasing convex function of the planned speed $\bar{v}_i$, where the range of $\bar{v}_i$ is $V^{\min} + V^{err} \leq \bar{v}_i \leq V^{\max} - V^{err}$.

B. MODEL TRANSFORMATION

After obtaining the closed-form expression of the VSBMSD model, we transform the model to derive the solution. In the objective function of the VSBMSD model, the bunker cost $f(z_i)$ and binary variable b_i have multiplicative relationships. Constraint (28) includes the reciprocal of the planned speed $\bar{v}_i$, while Constraints (19) and (20) contain the closed-form expression for $Q_i^{\max}(\bar{v}_i)$, which is an increasing convex function of $\bar{v}_i$. The VSBMSD model contains both continuous decision variables and 0-1 integer variables, representing the reciprocal terms of the decision variables and the product terms of the decision variables, respectively. Therefore, the VSBMSD model is a mixed-integer nonlinear programming model, and the exact optimal solution cannot be directly calculated.

Therefore, the VSBMSD model is linearized according to its characteristics.

First, the reciprocal method is adopted to make $\overline{u}_i = 1/\overline{v}_i, \forall i \in I$. Then, the decision variable $\overline{v}_i$ is replaced by its reciprocal $\overline{u}_i$. Thus, we reformulate the nonlinear term $\frac{L_i}{\overline{v}_i}$ by using the new decision variables $\overline{u}_i$, and Constraint (28) is linearized.

Then, the linear secant approximation method is adopted. $\overline{v}_i$ is replaced with its reciprocal $\overline{u}_i$. Then, the worst-case scenario of the maximum bunker consumption function $Q_i^{\max}(\overline{u}_i)$ is linearized. The specific steps are as follows [35].

Step 1: We determine the range of the independent variable $\overline{u}_i$ values and then divide the range into several small sections. The range of $\overline{u}_i$ in the worst-case scenario of the maximum bunker consumption function $Q_i^{\max}(\overline{u}_i)$ is the interval $\left[\overline{U}^{\min}, \overline{U}^{\max}\right]$, where $\overline{U}^{\min} = {}^{1}\!/_{V^{\max}} - V^{err}$, $\overline{U}^{\max} = {}^{1}\!/_{V^{\min}} + V^{err}$, and $\forall i \in I$. We divide the interval $\left[\overline{U}^{\min}, \overline{U}^{\max}\right]$ into N uniform segments and obtain $N+1$ equal points. The value of $\overline{u}_i$ at the equal point τ is $\overline{U}_i^{\tau} = \overline{U}^{\min} + (\tau - 1)\frac{\overline{U}^{\max} - \overline{U}^{\min}}{N}$.

Step 2: We adopt the piecewise linear secant approximation method. First, we calculate the value $Q_i^{\max\tau}\left(\overline{U}_i\right)$, $\tau \in \{1, 2, \cdots, N+1\}$ given the equal point τ on the interval $\left[\overline{U}^{\min}, \overline{U}^{\max}\right]$ of the worst-case scenario of the maximum bunker consumption function $Q_i^{\max}(\overline{u}_i)$. Then, we construct the piecewise linear secant function $\overline{Q}_i^{\max\tau}(\overline{u}_i)$. The range of $\overline{u}_i$ in the worst-case scenario of the maximum bunker consumption function $Q_i^{\max}(\overline{u}_i)$ is the small segment τ. The formula is as follows:

$$\overline{Q}_i^{\max\tau}(\overline{u}_i) = \frac{Q_i^{\max}\left(\overline{U}_i^{\tau+1}\right) - Q_i^{\max}\left(\overline{U}_i^{\tau}\right)}{\overline{U}_i^{\tau+1} - \overline{U}_i^{\tau}}\overline{u}_i + \frac{\overline{U}_i^{\tau} Q_i^{\max}\left(\overline{U}_i^{\tau+1}\right) - \overline{U}_i^{\tau+1} Q_i^{\max}\left(\overline{U}_i^{\tau}\right)}{\overline{U}_i^{\tau} - \overline{U}_i^{\tau+1}}, \quad \forall i \in I, \ \tau \in \{1, 2, \cdots, N\} \tag{44}$$

The linear piecewise functions of the worst-case scenario of the maximum bunker consumption function $Q_i^{\max}(\overline{u}_i)$ are composed of the piecewise linear secant function $\overline{Q}_i^{\max\tau}(\overline{u}_i)$ in each uniform segment τ. The linearization method in Figure 4 approximates the nonlinear function. Figure 4 shows that when the divided uniform segment N is sufficiently large, the piecewise linear secant function $\overline{Q}_i^{\max\tau}(\overline{u}_i)$, where $\tau \in \{1, 2, \cdots, N\}$ is approximately the same as the worst-case scenario of maximum bunker consumption function $Q_i^{\max}(\overline{u}_i)$. Using the "big M" piecewise linear secant approximations method ("M" is a large number) [36], the VSBMSD model can be transformed into the linearized VSBMSD model as follows:

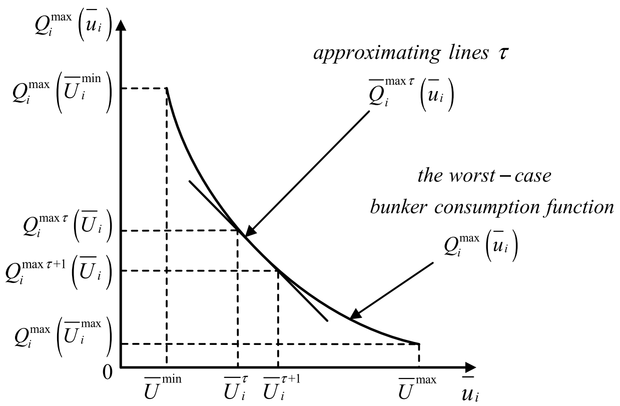

FIGURE 4. The linear secant approximation of the worst-case bunker consumption function.

The objective function is expressed as:

$$\min\left\{C^{oc}p + \sum_{i\in I}[F + f(z_i)]\,b_i + \sum_{i\in I}\sum_{t\in T_i}\sum_{h\in H_{it}} C_{ith}^{hc} D_i y_{ith} + C^{ic}\sum_{i\in I}\overline{t}_i R_i + \sum_{i\in I} C_i^{lc} t_i^l\right\} \tag{45}$$

with Constraints (14)-(18), (21)-(27), (29)-(37) and (44). The following new constraints also apply:

$$z_{i+1}^1 = z_i^2 - \overline{Q}_i^{\max\tau}(\overline{u}_i), \quad \forall i \in I, \quad i < n, \quad \forall \tau \in \{1, 2, \cdots, N\} \tag{46}$$

$$z_1^1 = z_n^2 - \overline{Q}_i^{\max\tau}(\overline{u}_n), \quad \forall \tau \in \{1, 2, \cdots, N\} \tag{47}$$

$$\overline{Q}_i^{\max\tau}(\overline{u}_i) \geq \frac{Q_i^{\max}\left(\overline{U}_i^{\tau+1}\right) - Q_i^{\max}\left(\overline{U}_i^{\tau}\right)}{\overline{U}_i^{\tau+1} - \overline{U}_i^{\tau}}\overline{u}_i + \frac{\overline{U}_i^{\tau} Q_i^{\max}\left(\overline{U}_i^{\tau+1}\right) - \overline{U}_i^{\tau+1} Q_i^{\max}\left(\overline{U}_i^{\tau}\right)}{\overline{U}_i^{\tau} - \overline{U}_i^{\tau+1}} - M\left(1 - \gamma_i^{\tau}\right), \forall i \in I, \tau \in \{1, 2, \cdots, N\} \tag{48}$$

$$\frac{dQ_i^{max}(\overline{v}_i)}{d\overline{v}_i} = \frac{\alpha L_i(\overline{v}_i - V^{err})^{\beta-1}[(\beta-1)\overline{v}_i + V^{err}] + \alpha L_i(\overline{v}_i + V^{err})^{\beta-1}[(\beta-1)\overline{v}_i - V^{err}]}{2\overline{v}_i^2} \tag{42}$$

$$\frac{d^2Q_i^{max}(\overline{v}_i)}{d\overline{v}_i^2} = \frac{\alpha L_i(\overline{v}_i - V^{err})^{\beta-2}\left[(\beta-1)(\beta-2)\overline{v}_i^2 + 2(\beta-2)\overline{v}_i V^{err} + 2\left(V^{err}\right)^2\right]}{\overline{v}_i^3} \tag{43}$$

$$\bar{t}_i = L_i \bar{u}_i, \quad \forall i \in I \tag{49}$$

$$1/V^{\max} - V^{err} \le \bar{u}_i \le 1/V^{\min} + V^{err}, \quad \forall i \in I \tag{50}$$

$$\sum_{\tau=1}^{N} \gamma_i^{\tau} = 1, \quad \forall i \in I \tag{51}$$

$$\gamma_i^{\tau} \in \{0, 1\}, \quad \forall i \in I, \quad \forall \tau \in \{1, 2, \cdots, N\} \tag{52}$$

In the linearized VSBMSD model, Objective Function (45) minimizes the total liner shipping route service cost. Constraints (46) and (47) use linear functions to redefine the relationship between the worst-case scenario associated with maximum bunker consumption and the bunker inventory level before and after every refueling, respectively. Constraint (48) expresses the closed-form expression for the worst-case scenario of maximum bunker consumption $Q_i^{\max}(\bar{v}_i)$, using a number of linear secant functions. Constraint (49) calculates the sailing time using the reciprocal of the planned speed. Constraint (50) shows the range of the planned speed reciprocal. Constraint (51) indicates that only one linear secant function $\overline{Q}_i^{\max\,\tau}(\bar{u}_i)$ is selected to approximate the worst-case scenario of maximum bunker consumption. Constraint (52) introduces a new binary variable. The linearized VSBMSD model can be efficiently solved using an off-the-shelf MILP solver (such as CPLEX) [36].

V. NUMERICAL EXPERIMENTS

This section presents numerical experiments for a specific simulated liner shipping route and identifies management insights with respect to vessel scheduling, refueling strategy, and collaborative agreement formulation.

A. INPUT DATA DESCRIPTION

The numerical simulations focus on the Asia–Northern Europe (AEMX) liner shipping route served by the China Ocean Shipping Co., Ltd. (COSCO) [37]. Figure 5 shows that the sequential ports visited weekly along the AEMX liner shipping route are as follows: Busan → Shanghai → Ningbo → Kaohsiung → Xiamen → Shekou → Singapore → Suez Canal → Beirut → Said → Piraeus → Evyap → Istanbul → Constanta → Odessa → Istanbul → Mersin → Said → Jeddah → Kelang → Busan. The ports of Said and Istanbul are visited twice; all other ports are visited only once. Two virtual nodes represent the ports of Said and Istanbul. As such, the port rotation for the AEMX liner shipping route includes 18+2=20 ports.

Table 2 presents the distances between two consecutive ports (n miles) [38] and bunker fuel prices at each port (USD/ton) [39].

To verify the validity of the linearized VSBMSD model, data from previous liner shipping studies are used as the parameter values required for numerical experiments (see Table 3).

Based on previous studies [7], marine container terminal operators offer 4 vessel arrival TWs to the liner shipping company at each port. Each TW has 3 start times and 3 end times. Table 4 shows the 4 arrival TWs provided by Busan Port, which is the first port on the route, and the start time s (hours) of each TW.

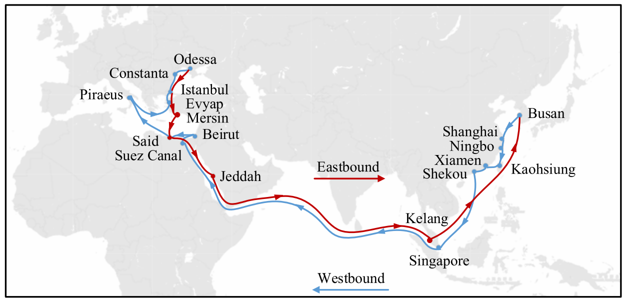

FIGURE 5. AEMX liner shipping route.

TABLE 2. Lengths of each leg of the voyage and bunker fuel prices at each port.

Port number	Port of call	Country	Length (n miles)	Bunker fuel price (USD/ton)
1	Busan	South Korea	464	488.0
2	Shanghai	China	131	480.0
3	Ningbo	China	497	482.0
4	Kaohsiung	China	180	487.0
5	Xiamen	China	335	467.0
6	Shekou	China	1 472	481.0
7	Singapore	Singapore	4 967	456.5
8	Suez Canal	Egypt	421	396.0
9	Beirut	Lebanon	242	437.0
10	Said	Egypt	610	425.0
11	Piraeus	Greece	398	451.0
12	Evyap	Turkey	55	460.0
13	Istanbul	Turkey	229	457.0
14	Constanta	Romania	201	472.0
15	Odessa	Ukraine	372	465.0
16	Istanbul	Turkey	800	457.0
17	Mersin	Turkey	368	455.0
18	Said	Egypt	810	425.0
19	Jeddah	Saudi Arabia	4 202	467.0
20	Kelang	Malaysia	2 706	462.0
1	Busan	South Korea	-	488.0

The starting time s of the four TWs at other ports is evaluated using the formula $TW_{(i+1)ts}^{S} = TW_{its}^{S} + \frac{L_i}{\bar{v}_i}$, $i \in I, t \in T_i, s \in S_{it}$(hours). The vessel's planned speed $\bar{v}_i$ (knots) on the given leg i is subjected to the uniform distribution $U\left[V^{\min} + V^{err}, V^{\max} - V^{err}\right]$. The duration of the TW (hours) at each port is randomly generated from the uniform distribution $TW_{ite}^{E} - TW_{its}^{S} = U\,[24, 74]$, $i \in I, t \in T_i, e \in E_{it}, s \in S_{it}$ (hours) [7].

Busan, Shanghai, Ningbo, Kaohsiung, Xiamen, Singapore, and Kelang are among the 20 largest ports in the world. The weekly amount of the containers handled (TEU) at these large ports is randomly generated from a uniform distribution $U\,[500, 2\,000]$ [36]. In contrast, the weekly amount of the containers handled (TEU) at the small ports is randomly generated from a different uniform distribution $U\,[200, 1000]$ [36]. We assume that the marine container terminal operators provide 4 container handling rates at each

TABLE 3. Related parameters.

Parameter	Value	Source
Number of port visits: n	20	-
Coefficients of vessel bunker consumption function: α, β	α=0.013, β=3	[36]
Vessel bunker tanker maximum capacity: W (tons)	5 000	[34]
Refueling amount when receiving a discount at port i: W_i^1, W_i^2 (tons)	1 000, 2 000	[26]
Fixed cost of vessel refueling: F (USD)	1 000	[34]
Vessel initial bunker inventory level: I_0 (tons)	1 000	[34]
Vessel weekly total operational cost: C^{oc} (USD/week)	300 000	[36]
Late arrival penalty at port i: C_i^{lc} (USD/h)	U[5 000,10 000]	[7]
Number of containers transported during voyage leg i: R_i (TEU)	U[8 000,11 000]	[37]
Unit container inventory cost: C^{ic} (USD/TEU/h)	0.5	[7]
Minimum vessel sailing speed: $V^{\min}$ (knots)	11	[10]
Maximum vessel sailing speed: $V^{\max}$ (knots)	26	[14]
Speed deviation from extreme value: V^{err} (knots)	3	[34]
Bunker fuel price discount factor: λ_i^1, λ_i^2 (%)	10%, 20%	[26]

TABLE 4. TWs and their start times offered by Busan Port.

TW	1	2	3	4
Start time	0:00 6:00 12:00	18:00 24:00 30:00	36:00 42:00 48:00	54:00 60:00 66:00

port. The container handling productivity HP_{ith}(TEUs/hour) for handling rate h at port i during TW t is generated using the formula $HP_{ith} = HP_{pth}^m \pm \Delta_h$, $i \in I$, $t \in T_i$, $h \in H_{it}$. In this expression, HP_{ith}^m is the average handling productivity, which varies based on the available equipment at each port. Therefore, the 4 values of HP_{ith}^m offered at large ports are set at 50, 75, 100, and 125, and the 4 values offered at small ports are set at 50, 60, 75, and 100 [36]. The difference in the handling productivity value Δ_h is randomly generated from the uniform distribution U [0, 20] [7]. The handling cost C_{ith}^{hc} (USD/TEU) under handling rate h at port i during TW t is calculated using the formula $C_{ith}^{hc} = C_{ith}^m \pm \Delta_{hc}$, $i \in I$, $t \in T_i$, $h \in H_{it}$. In this expression, C_{ith}^m is the unit average handling cost (TEU), with values at each port of 475, 550, 625, and 700 [7]. The difference in handling cost Δ_{hc} is randomly generated from the uniform distribution U [0, 50] [7].

B. EVALUATION OF SOLUTION METHODOLOGY

To evaluate the proposed solution methodology, 5 groups of calculation examples with the number of segments uniformly increasing from 10 to 50 are constructed by changing the number of uniform segments N of the piecewise linear secant function [36]. The median values of the uniform distribution parameters are used for the calculation and comparison. Based on the piecewise linear secant approximation of the bunker consumption function simulated by MATLAB 2016a, the linearized VSBMSD model is numerically analyzed by the ILOG CPLEX 12.6 solver on a Pentium (R) i5 3.10 GHz computer with 4 GB of RAM. This yields the total cost Y for a different number of uniform segments N. When Lingo is used to directly solve the VSBMSD model, it continues running after 24 hours, leading us to cut the Lingo calculations after that time point. Therefore, more time is required to solve the VSBMSD model, and the objective value Y^* is still not ideal. Table 5 shows the total costs and calculation times for different numbers of uniform segments.

TABLE 5. Total costs and calculation times for different numbers of uniform segments.

Instance	Number of uniform segments N	Total cost Y (10^3USD)	Total cost Y^* (10^3USD)	CPU time (sec)
1	10	16 606.289		1.12
2	20	16 606.085		1.21
3	30	16 606.081	16 614.151	1.33
4	40	16 606.074		1.35
5	50	16 606.074		1.54

Table 5 shows that an increase in the number of uniform segments N is associated with the increased optimization of total cost Y; however, this requires more calculation time. To identify the balance between calculation error and calculation time, we set the number of uniform segments N at 40 [36].

C. MANAGERIAL INSIGHTS

This section analyzes the experimental results from three perspectives, each with important management implications: (i) vessel schedule and robust refueling strategy; (ii) comparative analysis with other models; and (iii) sensitivity analysis of the linearized VSBMSD model. This analysis is followed by a discussion.

1) RESULTS FOR THE SIMULATED AEMX SHIPPING ROUTE

Based on the analysis above, 1 000 scenes are randomly generated using the numerical values above. The bunker consumption function is divided into 40 uniform segments to construct piecewise linear secant approximations. Then, the CPLEX solver is programmed on GAMS for the numerical analysis, generating the round-trip voyage vessel scheduling and refueling strategy shown in Table 6.

Table 6 shows that the total round-trip voyage time is 1 176 hours. As such, the vessel schedule along the liner shipping route is designed to be 1 176 hours (7 weeks), with 7 developed vessel scenarios. Vessels refuel at three different ports: Xiamen, Singapore, and the Suez Canal. The refueling decisions are based on bunker fuel prices at different ports and the inventory level of the bunker tanker. Therefore, most

TABLE 6. Results for the simulated AEMX shipping route.

Port of call	Leg of voyage	Arrival time (hours)	Departure time (hours)	Vessel sailing speed (knots)	Bunker consumption on sailing legs (tons)	Bunker inventory level arriving at the port (tons)	Refueling amount (tons)
Busan	Busan →Shanghai	0	22	22.46	133	1 000	-
Shanghai	Shanghai →Ningbo	42	62	22.62	38	867	-
Ningbo	Ningbo →Kaohsiung	68	87	22.75	147	829	-
Kaohsiung	Kaohsiung →Xiamen	110	128	22.83	54	682	-
Xiamen	Xiamen →Shekou	136	155	22.85	100	628	1 005
Shekou	Shekou →Singapore	169	180	22.83	439	1 533	-
Singapore	Singapore →Suez Canal	245	265	22.69	1 478	1 094	901
Suez Canal	Suez Canal →Beirut	482	494	22.99	127	517	3 783
Beirut	Beirut →Said	512	523	22.98	73	4 173	-
Said	Said →Piraeus	534	546	22.99	184	4 100	-
Piraeus	Piraeus →Evyap	572	584	22.97	120	3 916	-
Evyap	Evyap →Istanbul	601	613	22.98	17	3 796	-
Istanbul	Istanbul →Constanta	615	627	22.99	69	3 779	-
Constanta	Constanta →Odessa	637	649	22.96	61	3 710	-
Odessa	Odessa →Istanbul	658	669	22.99	112	3 649	-
Istanbul	Istanbul →Mersin	685	697	22.99	240	3 537	-
Mersin	Mersin →Said	732	745	22.98	111	3 297	-
Said	Said →Jeddah	760	773	22.99	244	3 186	-
Jeddah	Jeddah →Kelang	808	833	22.84	1 261	2 942	-
Kelang	Kelang →Busan	1 017	1 046	20.15	681	1 681	-
Busan	-	1 176	1 198	-	-	-	-

vessels refuel at ports where bunker fuel prices are low; the ports where the most bunker fuel is purchased also have the lowest prices. This numerical experiment determines the vessel schedules and refueling policies, considering collaborative agreements and speed deviations.

2) COMPARATIVE ANALYSIS

To verify the effectiveness and applicability of the proposed model, we compare the numerical experiment results with those from similar studies. The models in similar studies are listed as follows:

1) linearized VSBMSD: The model presented in this paper.
2) linearized VSBMSD-NCA: This model, presented in [14], involves VS, BM and speed deviation but does not involve collaborative agreements. Instead, it follows a different approach, where the liner shipping company is offered multiple handling rates but a single vessel arrival TW at each port of call.
3) linearized VSSD-NBM: This model, presented in [7], involves VS, speed deviation and collaborative agreements but does not involve BM.
4) linearized VSBM-NSD: This model involves VS, BM and collaborative agreements but does not involve speed deviation. This is an ideal hypothetical state, but in most cases, it does not match reality.

This paper compares the above four mathematical models using the following performance indicators: ① total liner shipping route service cost TRC; ② mean vessel sailing speed MS; ③ total port handling cost THC; ④ total number of vessels P; ⑤ total late arrival penalty TLC; ⑥ total bunker cost TBC; ⑦ total refueling amount TRT and ⑧ refueling amount at different ports RAP. Table 7 shows the average values of the performance indicators over 1 000 scenes for each model.

Table 7 shows that, compared with the linearized VSBMSD-NCA model, the linearized VSBMSD model yields a lower total liner shipping route service cost, port handling cost, late arrival penalty and a higher mean vessel sailing speed. Among these, it can reduce the total liner shipping route service cost by 11.33%. This is because the linearized VSBMSD model considers the collaborative agreement signed between the marine container terminal operator and the liner shipping company. The liner shipping company has multiple options, such as multiple TWs, multiple start and end times, and multiple port handling rates. This provides a buffer for uncertainties in the transportation process. Using this approach, liner shipping companies can reduce the total liner shipping route service cost, the total port handling cost, and the total late arrival penalty through global optimization. This helps determine the optimal refueling ports and the refueling amounts, enabling more flexible choices for the vessel arrival and departure times and the port handling rate.

Compared with the linearized VSSD-NBM model, most of the costs obtained in the linearized VSBMSD model are lower, with the same total number of vessels. Among these, it can reduce the total liner shipping route service cost by 2.95%. The reason is that in terms of the refueling strategy, the linearized VSBMSD model considers the bunker fuel price difference and discount factors of each port, allowing the liner shipping company to choose refueling ports with lower bunker fuel prices. The company may even choose to refuel more to receive bunker fuel price discounts, significantly reducing the total bunker cost.

Compared with the linearized VSBM-NSD model, the linearized VSBMSD model accounts for the speed deviation, reducing the mean vessel sailing speed. As such, an increase in the sailing time at sea leads to a decrease in the time at port. The total port handling cost and total late arrival penalty are higher, but the total bunker cost and the total refueling amount are lower. Although it will increase the total liner shipping

TABLE 7. Average values of performance indicators for different models.

Model	TRC (10^3USD)	MS (knots)	THC (10^3USD)	P (vessels)	TLC (10^3USD)	TBC (10^3USD)	TRT (tons)	RAP (tons)		
								Port 5	Port 7	Port 8
linearized VSBMSD	16 894	22.48	8 066	7	417	2 201	5 689	1 005	901	3 783
linearized VSBMSD-NCA	19 053	21.63	9 611	7	969	2 094	5 441	1 956	-	3 485
linearized VSSD-NBM	17 408	22.46	8 058	7	545	2 592	5 653	-	-	-
linearized VSBM-NSD	16 286	24.16	7 707	7	213	2 443	6 383	1 688	553	4 142

TABLE 8. Average values of performance indicators with different durations of TWs.

Vessel arrival TWs duration	TRC (10^3USD)	MS (knots)	THC (10^3USD)	P (vessels)	TLC (10^3USD)	TBC (10^3USD)	TRT (tons)	Refueling amount at port (tons)		
								Port 5	Port 7	Port 8
[24-29]	17 441	22.16	8 451	7.00	558	2 162	5 586	1 146	753	3 687
[29-34]	17 327	22.24	8 370	7.00	532	2 169	5 609	1 138	760	3 711
[34-39]	17 212	22.31	8 299	7.00	493	2 177	5 632	1 138	766	3 728
[39-44]	17 111	22.39	8 239	7.00	459	2 185	5 657	1 144	757	3 756
[44-49]	17 018	22.41	8 172	7.00	433	2 188	5 665	1 142	764	3 759
[49-54]	16 926	22.47	8 107	7.00	411	2 195	5 686	1 141	766	3 779
[54-59]	16 843	22.50	8 058	7.00	378	2 200	5 697	1 131	781	3 785
[59-64]	16 749	22.53	7 999	7.00	344	2 203	5 707	1 136	777	3 794
[64-69]	16 665	22.54	7 951	7.00	310	2 204	5 713	1 166	751	3 796
[69-74]	16 607	22.56	7 920	7.00	285	2 206	5 719	1 160	757	3 802

TABLE 9. Average values of performance indicators with different bunker fuel price rates.

Bunker fuel price changing rate	TRC (10^3USD)	MS (knots)	THC (10^3USD)	P (vessels)	TLC (10^3USD)	TBC (10^3USD)	TRT (tons)	Refueling amount at port (tons)		
								Port 5	Port 7	Port 8
30% decrease	16 222	22.55	8 065	7.00	412	1 546	5 722	1 188	743	3 791
20% decrease	16 441	22.53	8 064	7.00	413	1 764	5 712	1 165	757	3 790
10% decrease	16 667	22.52	8 069	7.00	413	1 981	5 702	1 146	766	3 790
Original	16 894	22.48	8 066	7.00	417	2 201	5 689	1 005	901	3 783
10% increase	17 113	22.43	8 065	7.00	420	2 408	5 670	1 151	753	3 766
20% increase	17 332	22.39	8 066	7.00	422	2 617	5 646	1 133	767	3 746
30% increase	17 551	22.27	8 065	7.00	422	2 817	5 604	1 122	771	3 711
40% increase	17 765	22.19	8 063	7.00	418	3 020	5 570	1 105	792	3 673
50% increase	17 975	22.10	8 061	7.00	415	3 219	5 539	1 124	766	3 649
60% increase	18 195	22.06	8 063	7.00	419	3 422	5 517	1 130	761	3 626

route service cost by 3.60%, it is more practical because it is more realistic.

3) SENSITIVITY ANALYSIS

In actual operations, there are differences in the duration of the arrival TWs, port bunker fuel prices, vessel bunker consumption rates, vessel bunker tanker capacity, and vessel speed deviations from extreme values. These also have different impacts on the VS and BM strategies of the liner shipping company. A sensitivity analysis explores the impact of these factors on the optimization results of the study model. Each set of calculations and experiments is based on 1 000 scenarios, randomly generated.

a: THE IMPACT OF CHANGES IN THE DURATION OF THE VESSEL'S ARRIVAL TWs

The duration of the vessel's arrival TW is an important term in the collaborative agreement signed between liner shipping companies and marine container terminal operators. This makes it an essential factor in simulations. This study constructs 10 sets of calculation examples according to the duration of the vessel's arrival TWs, randomly generated from the uniform distribution U [24, 29] to U [69, 74]. Table 8 shows the VS and BM strategies along the AEMX liner shipping route, with different durations of the vessel's arrival TWs.

Table 8 shows that as the duration of the vessel's arrival TWs is extended, the total liner shipping route service cost, the total port handling cost, and the total late arrival penalty all decrease. In contrast, the mean vessel sailing speed, the total bunker cost and the total refueling amount all slightly increase. This is because the liner shipping company can choose a lower port handling rate more flexibly to reduce the total port handling cost with the extension of the TW duration. In addition, to mitigate the extension of the vessel handling time at ports that are limited to a vessel fleet of a certain size, the vessel speed increases, and the total bunker cost and the total refueling amount increase accordingly.

b: THE IMPACT OF CHANGES IN PORT BUNKER FUEL PRICE

Bunker cost is the main component of the total liner shipping route service cost. Based on the bunker fuel price of each port in Table 2, this paper constructs 10 sets of calculation examples, reflecting changes in the bunker fuel price from -30% to +60%. Table 9 shows the VS and BM strategies of the AEMX liner shipping route with different bunker fuel price rates.

Table 9 shows that the total port handling cost and the total late arrival penalty do not significantly change as bunker fuel prices increase. However, the total bunker cost increases

TABLE 10. Average values of performance indicators with different bunker consumption coefficients.

Bunker consumption coefficient α	TRC (10^3USD)	MS (knots)	THC (10^3USD)	P (vessels)	TLC (10^3USD)	TBC (10^3USD)	TRT (tons)	Refueling amount at port (tons)		
								Port 5	Port 7	Port 8
0.010	16 540	22.58	8 108	7.00	511	1 727	4 394	751	723	2 920
0.011	16 586	22.65	8 072	7.00	436	1 898	4 857	814	830	3 213
0.012	16 758	22.66	8 059	7.00	465	2 055	5 302	814	940	3 548
0.013	16 894	22.48	8 066	7.00	417	2 201	5 689	1 005	901	3 783
0.014	17 161	22.52	8 075	7.00	532	2 349	6 115	959	1 064	4 092
0.015	17 565	22.42	8 081	7.00	783	2 480	6 489	977	1 163	4 349

TABLE 11. Average values of performance indicators with different bunker tanker capacities.

Bunker tanker capacity (tons)	TRC (10^3USD)	MS (knots)	THC (10^3USD)	P (vessels)	TLC (10^3USD)	TBC (10^3USD)	TRT (tons)	Refueling amount at port (tons)					
								Port 3	Port 5	Port 7	Port 8	Port 18	Port 20
3 000	16 980	22.41	8 102	7.00	281	2 374	5 749	-	1 446	435	2 430	993	445
4 000	16 856	22.50	8 083	7.00	309	2 256	5 743	-	1 313	949	3 481	-	-
5 000	16 894	22.48	8 066	7.00	417	2 201	5 689	-	1 005	901	3 783	-	-
6 000	16 969	22.63	8 061	7.00	502	2 223	5 727	-	1 015	964	3 748	-	-
7 000	17 027	22.51	8 081	7.00	508	2 233	5 678	1 102	-	967	3 609	-	-

TABLE 12. Average values of performance indicators with different speed deviations.

Speed deviation from extreme value (knots)	TRC (10^3USD)	MS (knots)	THC (10^3USD)	P (vessels)	TLC (10^3USD)	TBC (10^3USD)	TRT (tons)	Refueling amount at port (tons)		
								Port 5	Port 7	Port 8
0	16 286	24.16	7 707	7	213	2 443	6 383	1 688	553	4 142
1	16 554	23.75	7 719	7	496	2 351	6 131	1 186	843	4 102
2	16 618	23.36	7 834	7	445	2 283	5 934	1 122	845	3 967
3	16 894	22.48	8 066	7	417	2 201	5 689	1 005	901	3 783
4	17 416	21.81	8 415	7	527	2 137	5 540	800	1 003	3 737
5	18 057	20.93	8 972	7	481	2 088	5 412	742	1 001	3 669

rapidly, leading to a direct increase in the total liner shipping route service cost. In addition, the mean vessel speed and the total amount of refueling gradually decrease. This is because as the bunker fuel price rises, liner shipping companies need to slow down the vessel speed to reduce bunker consumption and the total fuel amount over a round-trip voyage.

c: THE IMPACT OF CHANGES IN VESSEL BUNKER CONSUMPTION COEFFICIENT α

Next, we construct 6 sets of calculation examples using an increasing range in the vessel bunker consumption coefficient, from 0.010 to 0.015. Table 10 shows the VS and BM strategies along the AEMX liner shipping route, with different vessel bunker consumption coefficients α.

Table 10 shows that this factor is similar to the port bunker fuel price and total port handling cost factors, with very few changes in values. However, as the bunker consumption coefficient α increases, bunker consumption also increases. This is associated with a significant increase in the total bunker cost and the total refueling amount, thereby increasing the total liner shipping route service cost.

d: THE IMPACT OF CHANGES IN BUNKER TANKER CAPACITY

Next, we construct 5 sets of calculation examples based on the bunker tanker capacity, ranging from 3 000 tons to 7 000 tons. Table 11 shows the VS and BM strategies along the AEMX liner shipping route based on different bunker tanker capacities.

Table 11 indicates that a lower bunker tanker capacity increases the refueling time and impacts the total bunker cost, resulting in a decreased vessel speed. A higher bunker tanker capacity may be associated with a decline in cargo capacity. The total liner shipping route service cost may increase accordingly, impacting freight revenues.

The refueling strategy differs at different bunker tanker capacities. When the bunker tanker capacity is 3 000, Said (Port 18) and Kelang (Port 20) become new refueling ports, sharing some of the refueling amounts otherwise conducted at Singapore (Port 7) or the Suez Canal (Port 8). This differs from the ports used with bunker tanker capacities of 4 000, 5 000, and 6 000. When the bunker tanker capacity increases to 7 000, new refueling ports include Ningbo (Port 3), Singapore (Port 7), and the Suez Canal (Port 8). Different bunker tanker capacities significantly impact the computational results. This highlights the need to design an appropriate bunker tanker capacity for the optimization problem of VS and BM.

e: THE IMPACT OF CHANGES IN VESSEL SPEED DEVIATION FROM EXTREME VALUES

Finally, 6 groups of numerical examples are constructed according to the vessel speed deviation from an extreme value, increasing from 0 to 5 knots. Table 12 shows the VS and BM strategies along the AEMX liner shipping route with different vessel speed deviations from extreme values.

Table 12 shows that the total liner shipping route service cost, the total port handling cost, and the total late arrival penalty increase rapidly as the speed deviations increase from extreme values. This leads directly to reductions in the mean vessel speed, the total bunker cost, and the total refueling amount. In addition, if the speed deviations from extreme values change, so do the refueling amounts at optimal refueling ports. As the speed deviations from extreme values increase, the total refueling amounts at Singapore (Port 7) also increase, while the total refueling amounts at Xiamen (Port 5) and the Suez Canal (Port 8) decrease. To balance the reduced bunker consumption and the total bunker cost due to speed deviations, liner shipping companies are more likely to reduce the vessel speed, adjust refueling strategies, and increase port handling rates.

VI. CONCLUSION

Designing a reasonable and effective shipping schedule and determining a realistic refueling strategy can help container liners better meet customer demands for container cargo transportation. This can also optimize the joint needs of shipping companies and marine container terminal operators. By evaluating the worst-case speed deviations during each leg of a voyage in the presence of a collaborative agreement between marine container terminal operators and liner shipping companies, this paper investigates the VSBMSD problem for container liner transportation. We present a nonlinear mixed-integer programming model to minimize the total liner shipping route service cost. The maximum bunker consumption function is derived under the worst-case speed deviation for each leg of the voyage. We also adopt a piecewise linear secant approximation method. Using the AEMX route as a case study, numerical experiments are presented using 1 000 simulation scenes.

The analysis yields the following conclusions and management insights:

- When facing the maximum bunker consumption under the worst-case speed deviation on a certain leg of a voyage, a vessel sails at the lowest speed deviation value during the first half of the planned sailing time and sails at the highest speed deviation value during the second half. Therefore, based on the maximum bunker consumption, liner shipping companies can determine the minimum safe bunker inventory level of the bunker tanker during every leg of the voyage.
- A longer duration of the vessel's arrival TWs in the collaborative agreement is associated with increased flexibility for the liner shipping company to choose the appropriate port handling rate and to reduce the total liner shipping route service cost. The duration of the vessel's arrival TWs does not equal the time duration that a vessel occupies a berth; however, an increase in the TW duration may negatively impact the use of terminal berth resource services by other shipping companies. This creates an opportunity cost for marine container terminal operators. Thus, determining reasonable vessel arrival TWs helps establish mutually beneficial and win-win collaborative agreements between shipping companies and marine container terminal operators.
- With increases in the port bunker fuel price and vessel speed deviations from extreme values on each leg of the voyage, the total liner shipping route service costs increase, while the mean vessel speed and the bunker consumption decrease. Therefore, liner shipping companies need to consider the differences in bunker fuel prices (including discounts) among different ports and the impact of speed deviations. This may help the liners formulate an optimal vessel schedule, minimize the total liner shipping service cost, and ensure the appropriate level of bunker fuel supply.
- In providing container liner transportation services, liner shipping companies should address changes in bunker fuel prices and speed deviations from extreme values and consider changes in bunker consumption coefficients and bunker tanker capacity. Liner shipping companies need to formulate scientific vessel scheduling and refueling strategies to reduce the total liner shipping route service cost and improve the voyage cargo revenue when engaged in liner shipping services on different routes (speed deviation from extreme values and the different port bunker fuel prices). It is also important to deploy different types of vessels (with different bunker consumption coefficients and bunker tanker capacities).

This paper mainly addresses the problems of vessel scheduling and bunker management to support container liner shipping by considering speed deviations in the presence of a collaborative agreement. Future research should consider the vessel scheduling and bunker management problems under the conditions of multiple routes and multiple vessel types.

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