

RESEARCH ARTICLE

Chattering-Free Terminal Sliding Mode Control Based on Adaptive Barrier Function for Chaotic Systems With Unknown Uncertainties

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ABSTRACT This paper designs and implements a chattering-free terminal sliding mode control approach for a class of chaotic systems with unknown uncertainties. It considers sliding mode control (SMC) to deal with the dynamic model uncertainties of the chaos system, and uses a combination of SMC with an adaptive control approach to solve the upper boundaries problem of unknown model uncertainties and their estimation. Chattering is completely eliminated without over estimating the control gains by adopting an adaptive continuous barrier function in the dynamic switching function. Using the Lyapunov's stability theory, it was shown that the proposed scheme can guarantee the convergence of system states to the vicinity of the sliding surface in finite time. Additionally, the adoption of a sliding surface with a nonlinear and integral switching function resulted in removing the reaching phase of the sliding surface and yielding a controller that is robust to uncertainties from the start. The effectiveness of the proposed control method was assessed using three scenarios implemented to a Liu's uncertain chaotic system in MATLABSimulink environment. The obtained results confirmed the ability of the proposed approach to achieve continuous and smooth control rules for such chaotic systems. Among the main attributes of the proposed control method are its ability to completely eliminate chattering and yield a robust performance against model uncertainties and unknown external disturbances; common issues in chaotic systems.

INDEX TERMS Terminal sliding mode control, chaotic systems, adaptive barrier function, chattering-free, unknown uncertainty.

I. INTRODUCTION

Chaos theory is a branch of mathematics that studies chaotic dynamic systems. These latter are a class of nonlinear dynamic systems that are very sensitive to their initial conditions so that small changes in the initial conditions of such systems cause large changes in their output [1]. This phenomenon is known as the "butterfly effect" in chaos theory, which is a metaphor for the behavior of a butterfly flapping

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its wings in Brazil (under certain circumstances) that could cause a storm in Texas [2]. Therefore, it is impossible to predict the long-term behavior of such systems. The behavior of chaotic systems appears to be random; however, there is no need for the accident element to cause chaotic behavior, and deterministic dynamic systems can also exhibit chaotic behavior [3], [4]. It can be shown that the necessary condition for chaotic behavior in continuous and time-independent dynamic systems is the existence of at least three state variables; in other words, it must be at least a third-order system.

Chaotic behavior is an attractive and pervasive nonlinear phenomenon that has received more attention in recent decades due to its many applications. Some of its industry applications include chemical reactions, electrical converters, data processing, secure telecommunication systems, etc. [5], [6], [7], [8]. The chaos control issue in chaotic systems has been extensively researched since the early 1990s [9]. So far, different control methods have been suggested to control chaos in such systems; including: the feedback linearization method [10], [11], optimal control [12], [13], adaptive control [1], [14], [15], backstepping control [16], [17], sliding mode control (SMC) [18], [19], [20], linear matrix inequalities [8], intelligent control methods based on neural networks [21], [22], stochastic delay methods [23], and barrier function-based SMC approaches [24], to list a few.

Among the above control methods, adaptive control is an effective and relatively suitable method to solve the problem of parameter uncertainties in nonlinear systems and has been well used in various aspects to improve controller performance and solve problems such as input saturation, immeasurable modes and so on [25], [26], [27]. However, for a nonlinear system with unknown nonlinear terms and unknown model uncertainties, simple adaptive control cannot guarantee ideal control performance. Moreover, the backstepping control method is an efficient and robust technique for high-order nonlinear systems in terms of theoretical and applied analysis in engineering. In [28], [29], and [30], adaptive backstepping controllers are designed for nonlinear systems by combining the backstepping technique and the adaptive control; However, due to the inherent problem of “terms explosion,” in the backstepping method, the application of this method in high-order nonlinear systems will be associated with drawbacks and complications [31].

Amongst the available control methods, the SMC is an efficient and robust method for chaos control in chaotic systems [32], [33]. Most common SMC approaches, however, suffer from the chattering phenomenon, which occurs due to the discontinuous character of the sign function in the control input signal. Recently in [34], a novel chattering-free SMC approach was proposed to remove the undesirable chattering phenomenon. The design was derived based on the assumption that the upper boundaries of the uncertainty term and their first derivatives are known. The situation where the upper boundaries of the uncertainty term and their derivatives are unknown has not been examined.

The SMC methods have been applied in industrial applications extensively. In [35], a fractional order sliding mode controller for the sensor-less tele-robotic system with uncertain time delay, model uncertainty, fractional calculus numerical approximation bias, and external disturbances was proposed and optimized based on the greedy algorithm. In [36] the authors have focused on the fast position tracking problem for the permanent magnet linear motor (PMLM) system under a logarithmic sliding mode control signal with reduced chattering. A novel fractional-order sliding mode control scheme based on a two-layer hidden recurrent neural network

(THLRNN) for single-phase shunt active power filter was proposed in [37]. Research [38] presented an adaptive nonsingular terminal sliding mode (ANTSM) method for motion tracking control of a nonlinear uncertain bilateral teleoperation system with time-varying delays and disturbances. In [39], some recent advances in SMC for the networked control systems (NCSs) are reviewed. In particular, first, some new SMC schemes for NCSs subject to time delay, packet losses, quantization, and uncertainty/disturbance are briefly outlined. Subsequently, the problem of SMC for NCSs under scheduling protocols was discussed, where different communication protocols are introduced for energy-saving purposes during the synthesis of NCSs. In addition, in [40] a SMC scheme is proposed for nonlinear systems with time-delays and packet losses and under uncertain missing probability.

Terminal sliding mode control (TSMC) has widely been used recently as a suitable approach for finite-time control of mechanical and chaotic systems with model uncertainties and external disturbances. In standard SMC, the system state trajectories converge to the origin in finite time, and the controller is not robust to the uncertainties in the reaching phase. In the TSMC method, on the other hand, due to the nonlinear term of the sliding surface, the system state trajectories converge to the origin in finite time, and the controller is quite robust to the uncertainties in the reaching phase [38]. In [41], an adaptive fractional high-order TSMC for the nonlinear robotic manipulator under alternating loads is provided. The work in [42] proposed a combined terminal dynamic sliding mode controller based on an adaptive observer for the stability of nonlinear uncertain SISO systems. Also in [43], an adaptive TSMC scheme for controlling chaotic systems is proposed, in which the proposed controller has fast finite-time stability and strong robustness against uncertainties. Controlling chaotic systems despite unknown disturbances using a nonsingular TSMC is presented in [44], in which the switching surface is technically designed to achieve fast convergence. Moreover, in [18], an adaptive TSMC scheme with a fractional-order sliding surface has been used to synchronization fractional-order uncertain chaotic systems with parameter uncertainties and external disturbances. An adaptive TSMC was proposed in [45] to stabilize the system states of port Hamiltonian chaotic systems while counteracting system chaotic behavior. Recent research works [46], [47], [48] have considered finite-time SMC (terminal SMC) in industrial, and, medical systems; thus further motivating the use of terminal SMC in this work.

Recently, a new control technique for removing the chattering problem in sliding mode controllers has been used as an excellent alternative to the saturation function method, the high-order derivative of the sliding surface, as well as other conventional approaches of removing chattering, which is simple, robust and practically efficient. This technique is known as “sliding mode control based on the adaptive continuous barrier function,” in which an adaptive continuous barrier function is used to provide a smooth and continuous

control rule for the switching surface and completely solve the chattering problem in SMC. The SMC based on the barrier function was first proposed in [49] by Plestan *et al.* for controlling an electro-pneumatic actuator system. An adaptive continuous barrier function TSMC scheme was proposed in [50] to control a Three-degrees of freedom manipulator subject to external disturbances. In [51], A barrier function adaptive nonsingular TSMC approach was designed to control quad-rotor unmanned aerial vehicles with external disturbances, in which a novel nonsingular terminal sliding surface is suggested to guarantee convergence of the sliding surface to the origin in a limited time. The work in [52] provides a quasi-adaptive sliding mode method based on the barrier function to control the motion of the hydraulic servo-mechanism with modeling uncertainty. Also, barrier function-based adaptive high-order SMC has been used for the fast stabilization of a perturbed chain of integrators with bounded uncertainties in [53]. Recently in [54], a TSMC based on the continuous barrier function along with a fuzzy estimator has been designed to control an inverted pendulum. The approach resulted in the creation of continuous and smooth control rules. In addition, a novel barrier function-based on the adaptive technique for the first-order SMC was proposed in [55]. This technique was applied to a class of first-order disturbed systems whose upper boundaries of the uncertainty are unknown. The proposed barrier technique can guarantee the convergence of the output variables independently of the uncertainty boundaries and keep it in a neighborhood of the origin without overestimating the control gain.

In [1], it was shown that using a dynamic sliding mode surface with both integral operator and differential operator significantly reduces chattering but does not eliminate it completely. Thus, in this paper, in addition to using a dynamic sliding mode surface with both integral and differential operators to reduce the chattering phenomenon, we propose using an adaptive continuous barrier function in the switching function to completely eliminate chattering, which is the main innovation of this article. Hence, we propose a chattering-free TSMC strategy for a complete class of uncertain chaotic systems with unknown upper boundaries of the uncertainty terms and their first derivatives; In a way that the chaotic system presented in [34] is considered as a special case. For this purpose, in order to overcome the problem caused by the upper boundaries of unknown uncertainty, the SMC method is combined with the adaptive control technique, and a novel strategy entitled “chatter-free TSMC based on adaptive continuous barrier function” is proposed. Its main contributions are as follows:

- A novel adaptive control to tune the controller’s adaptation gain parameters to estimate the boundaries of the uncertainty terms.
- An adaptation gain that is not overestimated, so only the convergence of the system state variables in a predefined neighborhood of the origin is guaranteed.

- A design that considers a barrier function as a simple switching function to completely eliminate the chattering phenomena.
- A design that guarantees the convergence and maintenance of the system state variables to a predefined neighborhood of the origin in finite time.
- A control design that covers a complete class of uncertain chaotic systems which upper boundaries of the uncertainty are assumed to be unknown.

The remainder of this article is organized as follows. In section 2, the chaotic system under study is described and formulated. Section 3 details the proposed TSMC approach based on the adaptive barrier function for controlling uncertain chaotic systems and proves its stability. The simulation results in the MATLAB/Simulink environment are shown in section 4 to display the efficiency of the proposed control method under tripl control scenarios. Finally, section 5 is devoted to the conclusion.

II. SYSTEM DESCRIPTION AND ITS FORMULATION

Chaotic systems are nonlinear systems whose dynamic equation can generally be represented as follows:

$$\dot{x} = f(t, x) + d(t, x) + u(t), \quad (1)$$

where $x = [x_1, x_2, \dots, x_n]^T \in R^n$ represents the system state vector, $f(t, x) : R^+ \times R^n \rightarrow R^n$ indicates a nonlinear vector field, $d(t, x) : R^+ \times R^n \rightarrow R^n$ is the unknown uncertainty term that denotes model uncertainties and unknown external disturbances and system unmodeled dynamics, and $u(t) \in R^n$ is the control input. The dynamic equation of the chaotic system (1) is based on the assumption that the number of state variables x is equal to the number of control inputs u .

The goal of this article is to design an adaptive TSMC rule for chaotic systems with model uncertainties and unknown external disturbances; so that the closed-loop system states are asymptotically stable for any specified initial condition, that means, the system states vector converges to zero:

$$\lim_{t \rightarrow \infty} \|X(t)\| = 0. \quad (2)$$

Note 1: It can be easily seen that if $f(t, x) = Bx + F(t, x)$ and $d(t, x) = d(t)$ consider; In fact, dynamic Eq. (1) is the reduced form of the dynamic following equation given as Eq. (1) in [34].

$$\dot{x} = Bx + F(t, x) + d(t) + u(t), \quad (3)$$

In this article, the problem is described and formulated based on the assumption that in the uncertainty term $d(x, t)$, the state variable vector x depends on the unknown time t , which is different from the uncertainty term $d(t)$ defined in [34] where the state variable x is independent of the time t .

Lemma 1 [38]: Consider if $x \in \mathcal{M} \subset R^n$ and $\dot{x} = \Omega(x)$, $\Omega : R^n \rightarrow R^n$ a continuous nonlinear system on the region \mathcal{M} as the open neighborhood of the origin and local Lipschitz at $\mathcal{M} \setminus \{0\}$ and $\Omega(0) = 0$; And also assuming that

there exists a continuous Lyapunov's function $V : \mathcal{M} \rightarrow R$ wherein the following conditions hold:

- (I) V is positive-definite; (II) \dot{V} is negative-definite on $\mathcal{M} \setminus \{0\}$; (III) There are positive real values m and $0 < a < 1$, and a neighborhood $M \subset \mathcal{M}$ of the origin such that:

$$\dot{V} + mV^a \leq 0; \text{ on the } M \setminus \{0\}. \tag{4}$$

Then, the origin is a stable finite-time equilibrium point for the nonlinear system $\dot{x} = \Omega(x)$ and in this case, the system state trajectories converge to the origin in a finite time.

Next, in the above lemma, for the initial time t_0 , let the Lyapunov's function $V : \mathcal{M} \rightarrow R$ converge to zero in a finite time. As a result, we have:

$$t_s = t_0 + \frac{V^{1-a}(t_0)}{r(1-a)}, \tag{5}$$

where r is a positive constant, and t_s is the system settling-time.

III. CONTROLLER DESIGN AND STABILITY ANALYSIS

In order to obtain continuous, smooth, and differentiable control inputs and further weaken the chattering phenomenon, the idea of diverting the switching term of the discontinuous sliding surface into the first derivative of the control input is adopted. As a result, the dynamic sliding mode surfaces resulting in a smooth adaptive SMC law are defined as follows:

$$\sigma_i(t) = \dot{s}_i(t) + \lambda_i s_i(t), \tag{6}$$

where $\lambda_i, i = 1, 2, \dots, n$ are positive constants, and sliding surfaces s_i with the integral operator are defined as follows:

$$s_i(t) = k_i \int_0^t x_i(\tau) d\tau + x_i(t), \tag{7}$$

where $k_i, i = 1, 2, \dots, n$ are positive constants, and $x_i(t), i = 1, 2, \dots, n$ are system state variables. The main advantage and feature of (7) is that the use of a sliding surface with this nonlinear and integral switching function makes the SMC become a global sliding mode control (GSMC) because the nonlinear sliding surface (7) has both GSMC conditions, i.e. $s(0) = 0$ and $s(\infty) = 0$. Accordingly, one of the main advantages of selecting the mentioned nonlinear and integral sliding surfaces is that the proposed controller becomes GSMC and the phase of reaching the sliding surface is removed. This results in placing the sliding surface at the initial moment, and yielding a controller that is robust to uncertainties from the beginning.

A. ANALYZING THE STABILITY AND CONVERGENCE OF SYSTEM STATES

In the proposed control method, SMC is used to deal with the dynamic model uncertainties of the chaos system, and a combination of SMC with the adaptive control approach is used to solve the upper boundaries problem of unknown model uncertainties and their estimation.

In this section, to analyze the stability of system states, we first make the following assumption.

Assumption 1: Here, we assume that the unknown positive constants c_i and $i = 1, 2, \dots, n$ exist as the upper boundaries of the system uncertainty term, so that the following inequality holds:

$$\left| \sum_{j=1}^n \frac{\partial d_i(t, x)}{\partial x_j} \dot{x}_j + \frac{\partial d_i(t, x)}{\partial t} \right| + (\lambda_i + k_i) |d_i(t, x)| \leq c_i, \tag{8}$$

where $d_i(t, x)$ and $i = 1, 2, \dots, n$ are the unknown uncertainty terms in Eq. (1).

Note 2: Contrary to the assumption in [34], here we allow the upper boundaries of the uncertainty term, i.e., c_i , to be unknown.

Theorem 1 [1]: Considering assumption 1, if the adaptive TSMC rule is written as follows:

$$\begin{aligned} \dot{u}_i = & -(\lambda_i + k_i)(u_i + f_i(t, x)) - \sum_{j=1}^n \frac{\partial f_i(t, x)}{\partial x_j} \dot{x}_j \\ & - \frac{\partial f_i(t, x)}{\partial t} - \lambda_i k_i x_i - \epsilon_i \hat{c}_i \text{sign}(\sigma_i), \end{aligned} \tag{9}$$

where $\epsilon_i > 1$, and $i = 1, 2, \dots, n$ are some constant parameters related to the controller design, \hat{c}_i is the upper boundary estimate of c_i , which displays the adaption gain, and is obtained from the following equation:

$$\dot{\hat{c}}_i = \epsilon_i |\sigma_i|; \quad \epsilon_i > 1, \tag{10}$$

Then, for each specified initial condition, the closed-loop system state vector $x(t)$ becomes asymptotically stable, leading to the convergence of the system states to the origin.

Proof: To prove Theorem 1, consider a Lyapunov's candidate function as follows:

$$V = \frac{1}{2} \sum_{i=1}^n [\sigma_i^2 + (\hat{c}_i - c_i)^2], \tag{11}$$

Therefore, the time derivative of Lyapunov's function will be as follows:

$$\dot{V} = \sum_{i=1}^n [\sigma_i \dot{\sigma}_i + (\hat{c}_i - c_i) \dot{\hat{c}}_i], \tag{12}$$

By combining Eq. (12) with Eqs. (6) and (7), the following equation is achieved:

$$\begin{aligned} \dot{V} = & \sum_{i=1}^n \left[\sigma_i \left(\sum_{j=1}^n \frac{\partial f_i(t, x)}{\partial x_j} \dot{x}_j + \frac{\partial f_i(t, x)}{\partial t} \right) \right. \\ & + \sum_{j=1}^n \frac{\partial d_i(t, x)}{\partial x_j} \dot{x}_j + \frac{\partial d_i(t, x)}{\partial t} + \dot{u}_i \\ & + (\lambda_i + k_i)(f_i(t, x) + d_i(t, x) + u_i) + \lambda_i k_i x_i \left. \right) \\ & + (\hat{c}_i - c_i) \dot{\hat{c}}_i \left. \right], \end{aligned} \tag{13}$$

By applying the inequality (8) of assumption 1, the following relation will be obtained:

$$\dot{V} \leq \sum_{i=1}^n \left[\sigma_i \left(\sum_{j=1}^n \frac{\partial f_i(t, x)}{\partial x_j} \dot{x}_j + \frac{\partial f_i(t, x)}{\partial t} \right) + \dot{u}_i \right]$$

$$\begin{aligned}
& + (\lambda_i + k_i) (f_i(t, x) + u_i) + \lambda_i k_i x_i) + c_i |\sigma_i| \\
& + (\hat{c}_i - c_i) \dot{\hat{c}}_i \Big], \quad (14)
\end{aligned}$$

Substituting the control rule of the adaptive sliding mode (9) in inequality (14), yields the following relation:

$$\begin{aligned}
\dot{V} & \leq \sum_{i=1}^n \left[-\epsilon_i \hat{c}_i \sigma_i \text{sign}(\sigma_i) + c_i |\sigma_i| + (\hat{c}_i - c_i) \dot{\hat{c}}_i \right] \\
& = \sum_{i=1}^n \left[-\epsilon_i \hat{c}_i |\sigma_i| + c_i |\sigma_i| + (\hat{c}_i - c_i) \dot{\hat{c}}_i \right] \\
& = \sum_{i=1}^n \left[-\epsilon_i \hat{c}_i |\sigma_i| + c_i |\sigma_i| + (\hat{c}_i - c_i) \epsilon_i |\sigma_i| \right] \\
& = - \sum_{i=1}^n (\epsilon_i - 1) c_i |\sigma_i|. \quad (15)
\end{aligned}$$

Thus, if $\epsilon_i > 1$ is selected according to the assumptions of Theorem 1, the derivative of the candidate Lyapunov function is less than or equal to zero. That is, the following inequality holds:

$$\dot{V} \leq - \sum_{i=1}^n (\epsilon_i - 1) c_i |\sigma_i| \leq 0. \quad (16)$$

Now, based on Eq. (16), we define the variable $\gamma(t)$ as the following relation:

$$\gamma(t) = \sum_{i=1}^n (\epsilon_i - 1) c_i |\sigma_i|. \quad (17)$$

Then, by integrating both parts of (17), we will have the following inequality:

$$\int_0^t \gamma(\tau) d\tau \leq V(0) - V(t) \leq V(0). \quad (18)$$

Bytaking the limits of both parts of relation (18) and considering the existence and finiteness of the limit $\int_0^t \gamma(\tau) d\tau$, using Barbalat's lemma, the following result will be obtained:

$$\lim_{t \rightarrow \infty} \gamma(t) = 0. \quad (19)$$

From Eq. (19) and Eqs. (6) and (7), the following result is obtained, which means that the system states eventually converge to zero, and the closed-loop system state vector $x(t)$ is asymptotically stable.

$$\lim_{t \rightarrow \infty} \sigma_i = 0 \Rightarrow \lim_{t \rightarrow \infty} \|x(t)\| = 0 \quad (20)$$

□

B. IMPROVING THE PROPOSED CONTROLLER WITH ADAPTIVE BARRIER FUNCTION

In this study, to improve and develop the proposed control method, the chattering-free TSMC based on the adaptive barrier function is used for the robust stability of chaotic systems with unknown external disturbances. For this purpose, a novel adaptive control rule based on the adaptive continuous barrier function is suggested in this section.

The block diagram of the proposed adaptive continuous barrier function-based chattering-free TSMC approach is illustrated in Fig.1.

Lemma 2 [50]: To improve the proposed control method, the unknown uncertainties of the system can be estimated

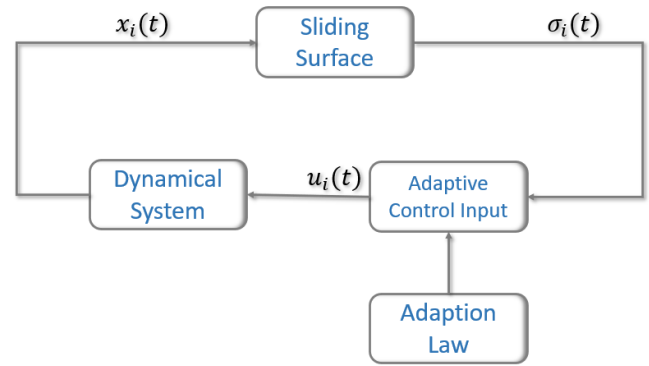


FIGURE 1. Block diagram of the proposed adaptive continuous barrier function-based TSMC approach.

more effectively by employing the on adaptive barrier function-based TSMC, in which case the closed-loop system will be more stable. For this purpose, the adaptation gain parameter \hat{c}_i in the adaptive TSMC rule (9) can be written as follows

$$\hat{c}_i(t) = \begin{cases} \hat{c}_{ia}(t), & \text{if } 0 < t \leq \bar{t} \\ \hat{c}_{ipsd}(t), & \text{if } t > \bar{t} \end{cases} \quad (21)$$

where \bar{t} represents the time when the system state trajectories converge to the neighborhood τ of the sliding surface dynamics $\sigma_i(t)$, and when $t = \bar{t}$, the smallest root of the equation $|\sigma_i(t)| \leq \tau$ is obtained. In the proposed control method, the adaptation rule and positive-semi-definite (PSD) continuous barrier function are created by the following equations, respectively:

$$\dot{\hat{c}}_{ia} = \epsilon_i |\sigma_i(t)|, \quad (22)$$

$$\hat{c}_{ipsd}(t) = \frac{|\sigma_i(t)|}{\tau_i - |\sigma_i(t)|}, \quad (23)$$

where in (23), τ is a positive scalar parameter.

Note 3: The adaption rule (22) is extracted from Eq. (10) in Theorem 1.

Employing the adaptation rule (22), the adaption gain is adjusted to increase until the state trajectories reach the neighborhood τ of the sliding surface at the time \bar{t} . For times greater than \bar{t} , the adaption gain shifts to the PSD barrier function to reduce the convergence region and hold the state trajectories in it.

For time $0 < t \leq \bar{t}$, the controller design is suggested by the control rule defined in Eq. (9) of Theorem 1, and for conditions when the time is longer than \bar{t} ($t > \bar{t}$), the controller is designed by the adaptive control rule based on the barrier function as follows:

$$\begin{aligned}
\dot{u}_i & = -(\lambda_i + k_i) (u_i + f_i(t, x)) - \sum_{j=1}^n \frac{\partial f_i(t, x)}{\partial x_j} \dot{x}_j \\
& \quad - \frac{\partial f_i(t, x)}{\partial t} - \lambda_i k_i x_i - \epsilon_i \hat{c}_{ipsd} \text{sign}(\sigma_i), \quad (24)
\end{aligned}$$

Then, the system state trajectories reach the convergence region $|\sigma_i(t)| \leq \tau$ in a finite time. As a result, the system state trajectories converge to the origin in a finite time, and the closed-loop system will reach stability in finite time.

Note 4: The process of designing and generating adaptive control rules (9) and (24) according to the method [34] is briefly illustrated in Appendix.

Note 5: According to lemma 2, using the adaptive barrier function in the SMC, the proposed controller will be automatically finite-time; in other words, the closed-loop control system will be stable in a finite time.

Proof: The controller stability with the control rule (9) was proven in the previous subsection. Here again, the proof is based on Lyapunov’s approach and is shown according to [49] that the system state trajectories reach the convergence region $|\sigma_i(t)| \leq \tau$ in a finite time. To prove the controller stability with the control rule (24), we can consider a Lyapunov’s candidate function which includes both sliding surface dynamics and adaption gain dynamics (barrier function), as follows:

$$V_i(t) = 0.5 \left(\sigma_i(t)^2 + (\hat{c}_{ipsd}(t) - \hat{c}_{ipsd}(0))^2 \right), \quad (25)$$

Taking the time derivative of the Lyapunov’s function (25), yields:

$$\dot{V}_i(t) = \sigma_i(t) \dot{\sigma}_i(t) + (\hat{c}_{ipsd}(t) - \hat{c}_{ipsd}(0)) \dot{\hat{c}}_{ipsd}(t), \quad (26)$$

Substituting the sliding surface derivative $\dot{\sigma}_i(t)$ and $\dot{\hat{c}}_{ipsd}(0) = 0$ in Eq. (26), yields:

$$\begin{aligned} \dot{V}_i(t) = & \sigma_i(t) \left(\sum_{j=1}^n \frac{\partial f_i(t,x)}{\partial x_j} \dot{x}_j + \frac{\partial f_i(t,x)}{\partial t} \right. \\ & + \sum_{j=1}^n \frac{\partial d_i(t,x)}{\partial x_j} \dot{x}_j + \frac{\partial d_i(t,x)}{\partial t} + \dot{u}_i \\ & \left. + (\lambda_i + k_i) (f_i(t,x) + d_i(t,x) + u_i) + \lambda_i k_i x_i \right) \\ & + \hat{c}_{ipsd}(t) \dot{\hat{c}}_{ipsd}(t), \end{aligned} \quad (27)$$

Applying the inequality (8) of assumption 1 and placing the control input \dot{u}_i of Eq. (24) in Eq. (27) yields:

$$\begin{aligned} \dot{V}_i(t) &= \sigma_i(t) \left(-\epsilon_i \hat{c}_{ipsd}(t) \operatorname{sgn}(\sigma_i(t)) + d(t,x) \right) + \hat{c}_{ipsd}(t) \dot{\hat{c}}_{ipsd}(t) \\ &\leq |\sigma_i(t)| \times \left\{ |d(t,x)| - \epsilon_i \hat{c}_{ipsd}(t) \right\} + \hat{c}_{ipsd}(t) \dot{\hat{c}}_{ipsd}(t) \\ &\leq |\sigma_i(t)| \times \left\{ |d(t,x)| - \epsilon_i \hat{c}_{ipsd}(t) \right\} + \hat{c}_{ipsd}(t) \\ &\quad \times \frac{\tau}{(\tau - |\sigma_i(t)|)^2} \left[d(t,x) - \epsilon_i \hat{c}_{ipsd}(t) \operatorname{sgn}(\sigma_i(t)) \right] \\ &\quad \times \operatorname{sgn}(\sigma_i(t)), \end{aligned} \quad (28)$$

From Eq. (28), the following inequality can be obtain:

$$\begin{aligned} \dot{V}_i(t) \leq & - \left\{ \epsilon_i \hat{c}_{ipsd}(t) - |d(t,x)| \right\} |\sigma_i(t)| - \hat{c}_{ipsd}(t) \\ & \times \frac{\tau}{(\tau - |\sigma_i(t)|)^2} \left[\epsilon_i \hat{c}_{ipsd}(t) - |d(t,x)| \right]. \end{aligned} \quad (29)$$

Since from (29), we have: $\epsilon_i \hat{c}_{ipsd}(t) > |d(t,x)|$ and $\frac{\tau}{(\tau - |\sigma_i(t)|)^2} > 0$; an upper boundary can be found as follows:

$$\dot{V}_i(t) \leq -\sqrt{2} \left\{ \epsilon_i \hat{c}_{ipsd}(t) - |d(t,x)| \right\} \frac{|\sigma_i(t)|}{\sqrt{2}}$$

$$\begin{aligned} & - \frac{\sqrt{2}\tau}{(\tau - |\sigma_i(t)|)^2} \left[\epsilon_i \hat{c}_{ipsd}(t) - |d(t,x)| \right] \frac{\hat{c}_{ipsd}(t)}{\sqrt{2}} \\ & \leq -Z \left(\frac{|\sigma_i(t)|}{\sqrt{2}} + \frac{\epsilon_i \hat{c}_{ipsd}(t)}{\sqrt{2}} \right) \leq -Z V_i(t)^{0.5}, \end{aligned} \quad (30)$$

where $Z = \sqrt{2} \left\{ \epsilon_i \hat{c}_{ipsd}(t) - |d(t,x)| \right\} \min \left\{ 1, \frac{\sqrt{2}\tau}{(\tau - |\sigma_i(t)|)^2} \right\}$ is defined.

Therefore, due to Eq. (30) and according to Lyapunov’s stability theorem and lemma 2, we can conclude that the proposed control system will be stable in a finite time.

IV. NUMERICAL SIMULATION OF THE PROPOSED CONTROL ON CHAOTIC SYSTEMS

The performance of the proposed approach is assessed in this section using three scenarios implemented in the MATLAB-Simulink environment. For this purpose, first, consider Liu’s uncertain chaotic system [56] with the following dynamic equation:

$$\dot{x} = f(x) + d(t,x) + u(t), \quad (31)$$

$$f(x) = \begin{bmatrix} -10x_1 + 10x_2 \\ 40x_1 - x_1x_3 \\ 4x_1^2 - 2.5x_3 \end{bmatrix}, \quad (32)$$

$$d(t,x) = \begin{bmatrix} \sin(x_1) \\ \sin(t) \\ \sin(x_1) + \sin(t) \end{bmatrix}, \quad (33)$$

where in Eq. (31), $x = [x_1, x_2, x_3]^T$ represents the system state vector, $f(x)$ is a known vector field and displays the dynamic system whose values are shown in Eq. (32) and $d(t,x)$ is the unknown uncertainty term related to the model uncertainties and the unknown external disturbances of the system whose values are shown in Eq. (33).

Note 6: Here, Liu’s chaotic system is a time-independent system, and as we know, time does not occur in dynamics; Accordingly, in Eqs. (31) and (32) instead of the function $f(x)$, the function $f(t,x)$ has been used.

The effect of chaos in Liu’s chaotic system is shown in Figs. 2 to 4, considering the initial states based on reference [1] as: $x(0) = [-0.2, 0.3, 0.2]^T$ has been brought. Fig. 2 shows the 3D phase diagram of Liu’s chaotic system states before applying the proposed controller. In addition, Fig. 3 depicts the phase diagrams of Liu’s chaotic system states in terms of time for each of the state variables each X_1, X_2 and X_3 separately, and the closed-loop system state trajectories before applying the proposed controller in Fig. 4 is displayed. From Figs. 2 to 4, it can be easy to observe and analyze the effect of chaos on each of the state variables X_1, X_2 , and X_3 in Liu’s chaotic system. Figs. 2 and 3 show how the state trajectories of the X_1, X_2 , and X_3 reaching the origin and the path these trajectories take to reach the origin.

Note 7: In this study, the variable values of the initial states of the system according to [1] are considered as: $x(0) = [-0.2, 0.3, 0.2]^T$ so that we can compare two controllers in a similar and fair condition. It is also necessary to explain that the values of the initial conditions of any chaotic system

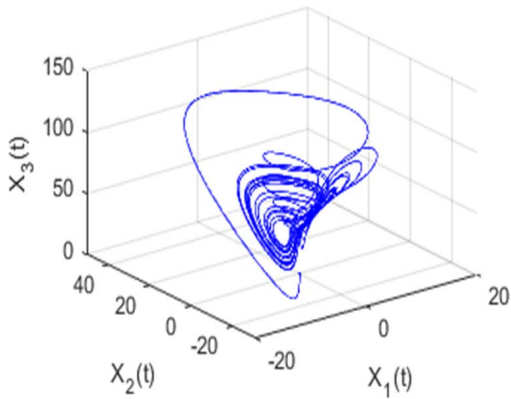


FIGURE 2. 3D phase diagram of Liu's chaotic system states before applying the proposed controller.

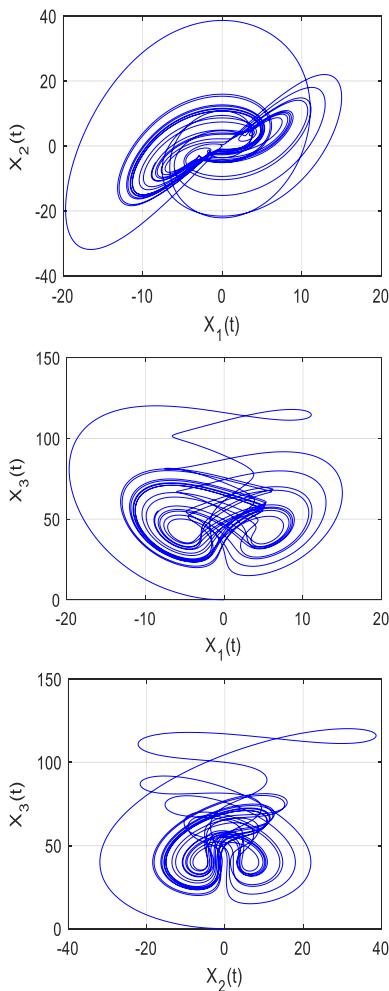


FIGURE 3. Phase diagrams of states X_1 , X_2 and X_3 of Liu's chaotic system in terms of time.

are inherently related to that system, and that the system becomes chaotic in that particular initial condition, and by changing the values of the initial conditions, the system may no longer become chaotic; Therefore, we are not allowed to change the values of the initial conditions of the chaotic system optionally.

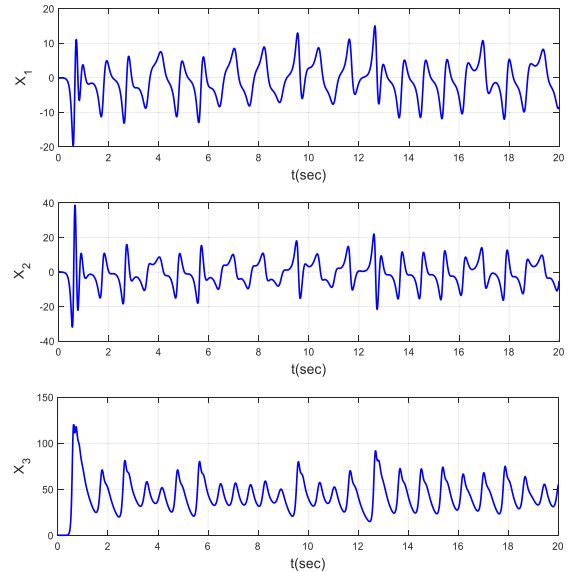


FIGURE 4. Closed-loop system state trajectories before applying the proposed controller.

By performing the necessary calculations according to Theorem 1, the rule of chattering-free TSMC based on adaptive barrier function is obtained as follows for the above-mentioned system:

$$\begin{aligned}
 \dot{u}_1 &= -(\lambda_1 + k_1)(u_1 - 10x_1 + 10x_2) + 10(\dot{x}_1 - \dot{x}_2) \\
 &\quad - x_1 - \epsilon_1 \hat{c}_{1psd} \text{sign}(\sigma_1), \\
 \dot{u}_2 &= -(\lambda_1 + k_1)(u_1 - 10x_1 + 10x_2) + 10(\dot{x}_1 - \dot{x}_2) \\
 &\quad - x_1 - \epsilon_1 \hat{c}_{1psd} \text{sign}(\sigma_1), \\
 \dot{u}_3 &= -(\lambda_3 + k_3)(u_3 + 4x_1^2 - 2.5x_3) - (8x_1\dot{x}_1 - 2.5\dot{x}_3) \\
 &\quad - x_3 - \epsilon_3 \hat{c}_{3psd} \text{sign}(\sigma_3), \\
 \hat{c}_{1psd}(t) &= \frac{|\sigma_1(t)|}{\tau_1 - |\sigma_1(t)|}, \\
 \hat{c}_{2psd}(t) &= \frac{|\sigma_2(t)|}{\tau_2 - |\sigma_2(t)|}, \\
 \hat{c}_{3psd}(t) &= \frac{|\sigma_3(t)|}{\tau_3 - |\sigma_3(t)|}.
 \end{aligned} \tag{34}$$

In the following, simulations of the proposed controller are designed and implemented based on three scenarios so that system responses can be better and more desirable in a step-by-step process. In scenario 1, in order to make it possible to compare the simulation results of the proposed control method with the adaptive sliding mode method presented in [1], the parameters of the control law have been chosen exactly in accordance with [1]. In scenario2, the control law parameters are chosen in such a way that the convergence rate to the origin of the system state trajectories and the sliding surface curves is increased. In scenario 3, the control law parameters are chosen in such a way that, in addition to increasing the convergence rate, the overshoot and undershoot of the system state trajectories and the sliding surface curves are eliminated as much as possible.

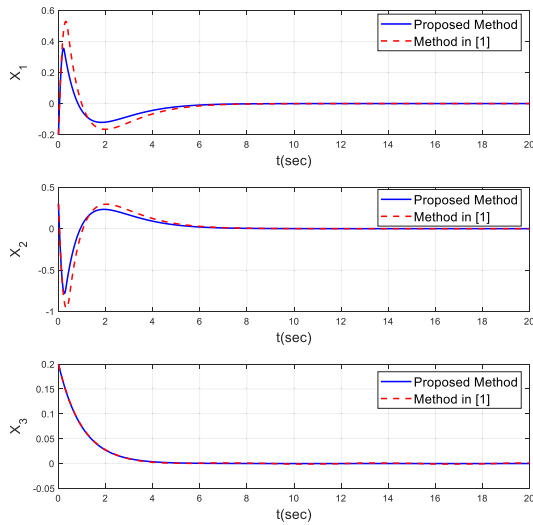


FIGURE 5. Comparison of closed-loop system state trajectories of the proposed method with the method in [1].

A. SCENARIO 1

In this scenario, the proposed control method is compared with the adaptive sliding mode method presented in [1] to observe the efficiency of the proposed control method; For this purpose, the control law parameters in accordance with the parameters in [1] are chosen as: $\lambda_i = 1, k_i = 1, i = 1, 2, 3, \epsilon_1 = 5, \epsilon_2 = 4, \epsilon_3 = 4$; and the initial states are considered as: $x(0) = [-0.2, 0.3, 0.2]^T$. Now, for simulation, we place the values of the above control parameters in Eq. (34) and apply unknown uncertainties and external disturbances to the proposed control system. Accordingly, based on scenario 1, the simulation results in the Simulink-MATLAB environment are obtained as Figs. 5 to 11 as follows; In which the transient and steady-state behavior of the system in the proposed control method is compared with the adaptive chattering-free TSMC method presented in [1].

The behavior of the closed-loop system state variables is given and compared in Fig. 5. Also, the behavior of state variables in the system steady-state is shown in Fig. 6, and Fig. 7 depicts the 3D phase diagram of Liu’s chaotic system states after applying the proposed controller. In Fig. 8, the control input signals are given. Fig. 9 shows the sliding surface curves, and the sliding surface dynamics are displayed in Fig. 10. Finally, the time responses of the adaptation gains are displayed in Fig.11.

In addition, to show the high efficiency of the proposed controller, Table 1 shows the transient system specifications for the quantitative comparison of the proposed control method with the control method presented in [1], and Table 2 shows the qualitative comparison of the proposed control method with the control method presented in [1] in all respects; These two tables clearly show the efficiency and practical superiority of our proposed control method.

According to the results obtained from the simulations, we can clearly see from Fig. 5 and the data in Table 1 that

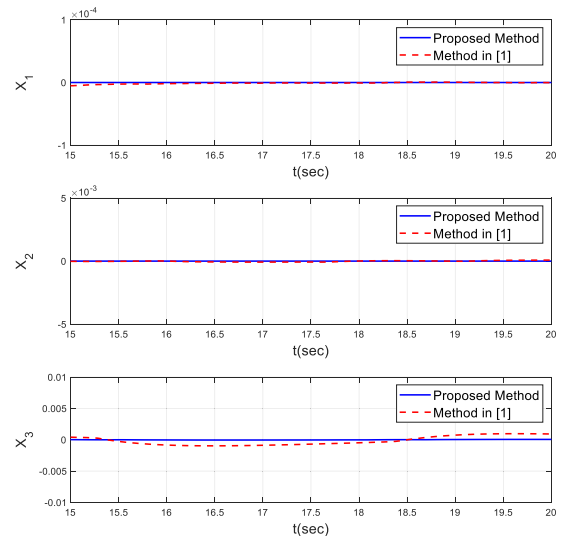


FIGURE 6. Comparison of closed-loop system state trajectories for the steady-state of the proposed method with the method in [1].

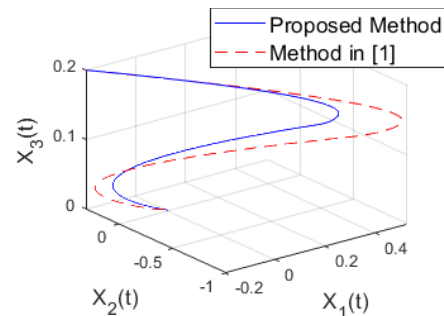


FIGURE 7. Comparison of the states phase diagram of Liu’s chaotic system of the proposed method with the method in [1].

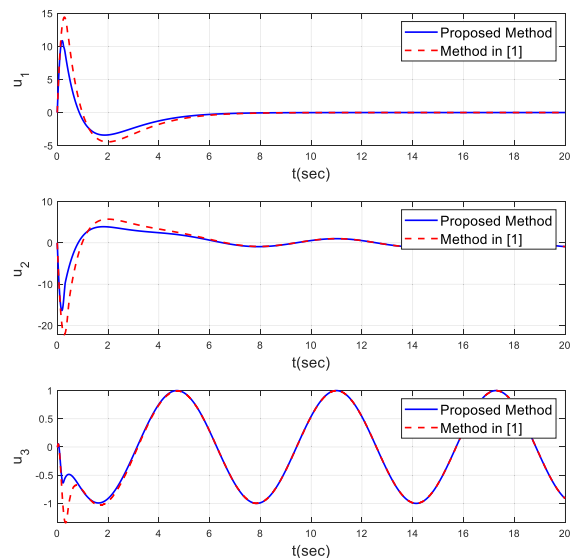


FIGURE 8. Comparison of sliding surface curves of the proposed method with the method in [1].

the state trajectories X_1, X_2 in the proposed control method converge to the origin after a suitable time and with less overshoot and undershoot compared to [1]; Also, the state trajectories X_3 converge to the origin almost the same as [1]

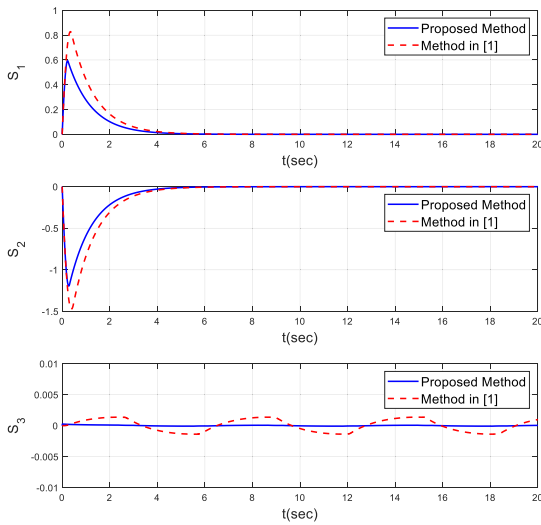


FIGURE 9. Comparison of the sliding surface dynamics of the proposed method with the method in [1].

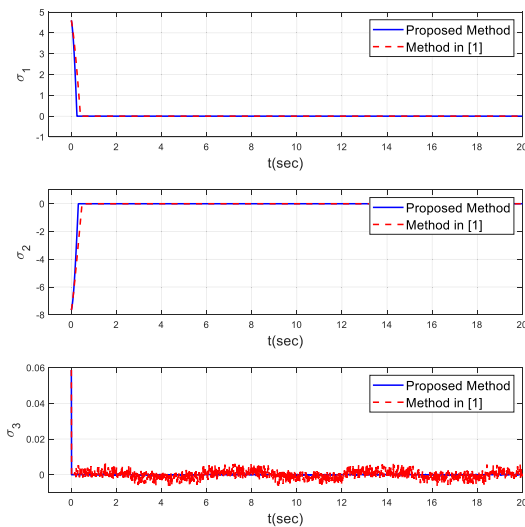


FIGURE 10. Comparison of the control input signals of the proposed method with the method in [1].

without any jumps. In addition, in Fig. 6, it can be observed that in the system steady-state, the state trajectories X_1, X_2, X_3 in the proposed control method, compared to the method presented in [1], converge to the origin almost without any oscillation.

Fig. 7 shows the effect of the proposed controller on the state trajectories of X_1, X_2, X_3 in Liu’s chaotic system after applying the proposed controller and the effect of the proposed controller on the stability of Liu’s chaotic system. By comparing Fig. 7 with Fig. 2, it is clearly observed that applying the proposed controller has stabilized Liu’s chaotic system so that the chaotic behavior of the system has been controlled. Due to Figs. 5 to 7, the transient and steady-state behavior of the system in the proposed control method in this paper, compared to the control method presented in [1], has been significantly improved, and the control system has become more stable.

TABLE 1. Transient system specifications for the quantitative comparison of control methods.

Chaotic System	Proposed Method			The Method In [1]		
	X_1	X_2	X_3	X_1	X_2	X_3
State Variables						
Overshoot	0.3526	0.2557	0	0.529	0.2947	0
Undershoot	-0.1182	-0.7716	0	-0.1646	-0.9588	0
Control Inputs	u_1	u_2	u_3	u_1	u_2	u_3
Overshoot	11.33	3.9	0.005	14.43	5.727	0.005
Undershoot	-3.633	-16.89	-0.6383	-4.436	-22.22	-1.347

In addition, due to the control input signals shown in Fig. 8 and the data in Table 1, the control inputs in the proposed control method have less overshoot and undershoot compared to [1], and their signals are smoother and flatter. Also, as can be clearly seen in Figs. 9 and 10, the sliding surface curves and sliding surface dynamics in the proposed control method are significantly improved compared to [1] and have less oscillation and jumps, and their chattering has been removed. Finally, it can be seen from Fig. 11 that since the time-derivative of the adaptation gains $\hat{c}_i(t)$ is an absolute value term; As a result, the estimation of system uncertainties is almost without slope. Also, the terminal values of the adaptation gains are obtained as $\hat{c}_i(t) = [2.775 \ 3.793 \ 0.8133]^T$.

As seen in Table 2, the results of the simulations display well the efficiency and practical superiority of “the chatter-free TSMC based on adaptive barrier function” in avoiding the chattering phenomenon and overcoming model uncertainties and unknown external disturbances in chaotic systems.

B. SCENARIO 2

This scenario has been designed and implemented to improve the convergence rate to the origin of the system state trajectories and sliding surface curves. With the help of scenario 2, we can observe the effect of changing the control law parameters of the λ_i and k_i on the convergence of the system state trajectories, sliding surface curves, and other system responses. For this purpose, the control law parameters are chosen as: $\lambda_i = 15, k_i = 15, i = 1, 2, 3, \epsilon_1 = 5, \epsilon_2 = 4, \epsilon_3 = 4$; and the initial states are considered as: $x(0) = [-0.2, 0.3, 0.2]^T$.

In scenario 2, like the previous scenario, after placing the value of the above control parameters in Eq. (34), the results of the simulations in the MATLAB-Simulink environment are obtained as follows, according to Figs. 12 to 17. The closed-loop system state trajectories and state variables behavior under scenario 2 is displayed in Fig. 12. Fig. 13 depicts the state phase diagram of Liu’s chaotic system of the proposed method under scenario 2. In Fig. 14, the control input signals under scenario 2 conditions are given. Fig. 15 indicates the sliding surfaces curves in scenario 2, and the sliding surface dynamics are shown in Fig. 16. Finally, the time responses of the adaptation gains are displayed in Fig. 17.

TABLE 2. Qualitative comparison of the proposed method and the method in [1].

Control Approach	Jump in the transient state of the state trajectories	Oscillation in steady-state of the state trajectories	Jump in the control input signals	Oscillation & Chattering in the control input signals	Oscillation & chattering in sliding surface curves	Oscillation & chattering in sliding surface dynamics
The method in [1]	Yes	Yes	High	No	Yes	Yes
Proposed method	Yes	No	Low	No	No	No

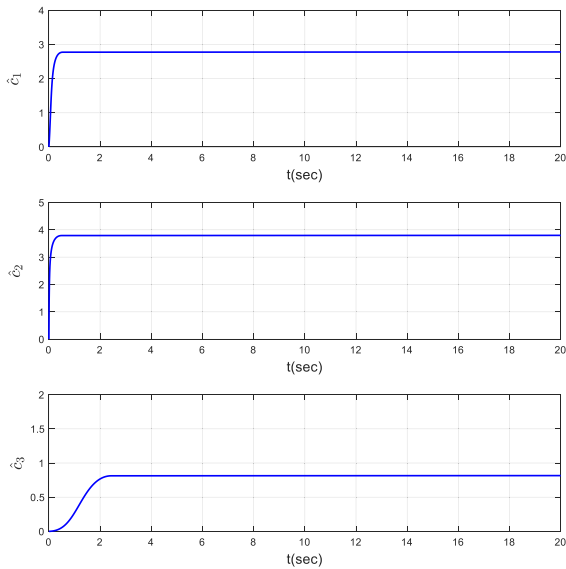


FIGURE 11. Dynamics of the adaptation gains.

In addition, as in scenario 1, in Table 3, the transient system specifications of the control system in the scenario 2 conditions are given quantitatively to show the high efficiency of the proposed controller in the new conditions.

According to the results obtained from the conditions of scenario 2, Fig. 12 shows that the state trajectories X_1, X_2, X_3 converge to the origin in less than one second; Whereas in Fig. 5 the system state trajectories X_1, X_2, X_3 converge to the origin in almost 3 seconds. Therefore, it can be concluded that the convergence rate to the origin of the system state trajectories in scenario 2 has improved significantly from scenario 1, and the control system has reached stability faster. Also, according to Table 3 and its comparison with Table 1, it can be seen that the overshoots and undershoots of the system state trajectories X_1, X_2, X_3 in scenario 2 have not changed much compared to scenario 1 and sometimes they have become worse.

In addition, similar to scenario 1, in scenario 2, by comparing Fig. 13 with Fig. 2, it can be seen that applying the proposed controller has led to the stabilization and control of chaotic behavior of Liu’s chaotic system.

Furthermore, due to the control input signals under scenario 2 in Fig. 14 and the data in Table 3, it can be concluded that the overshoots and undershoots of the control inputs and their level of smoothness have not changed much compared

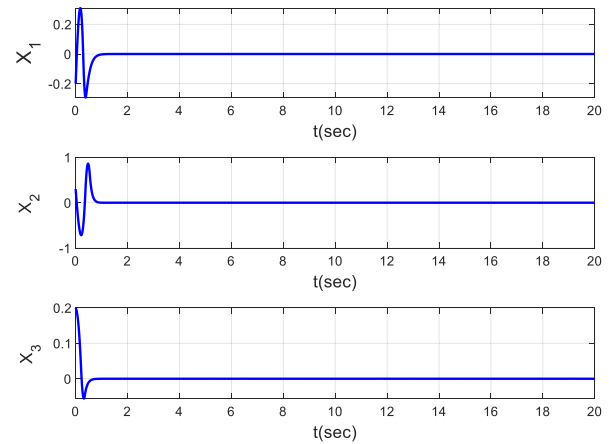


FIGURE 12. Closed-loop system state trajectories of the proposed method under the conditions of scenario 2.

to scenario 1 and sometimes they have become worse. Also, as can be seen in Figs. 15 and 16, the sliding surface curves and sliding surface dynamics in the proposed control method under scenario 2 conditions converge to the origin in less than one second, and compared to scenario 1, their convergence rate to origin has improved significantly and the control system has reached stability faster. Finally, like scenario 1, the terminal values of the adaptation gains are obtained as $\hat{c}_i(t) = [2.78 \ 3.812 \ 0.8945]$.

In general, in scenario 2, we were able to significantly improve the convergence rate of system state trajectories, sliding surface curves, and other system responses compared to scenario 1, by appropriately changing the control law parameters of λ_i and k_i . Therefore, it can be concluded that system state variables, sliding surfaces, and other system responses are sensitive to changing the parameters of λ_i and k_i . On the other hand, comparing the simulation results of scenario 2 with scenario 1, it can be said that although the convergence rate to origin system state trajectories, sliding surface curves, and other system responses has increased in scenario 2, but overshoots, undershoots and jumps have not changed much and sometimes they have become worse; As a result, the need to solve this problem in another scenario is strongly felt. Accordingly, we designed and presented scenario 3 as follows.

C. SCENARIO 3

This scenario has been designed and implemented so that, in addition to increasing the convergence rate to the origin

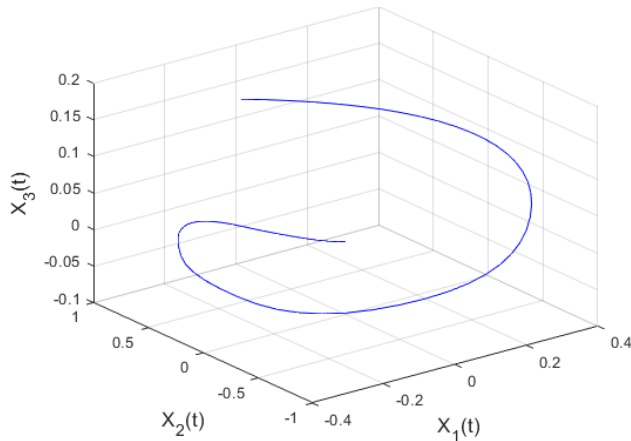


FIGURE 13. Phase diagram of Liu's chaotic system states of the proposed method under the conditions of scenario 2.

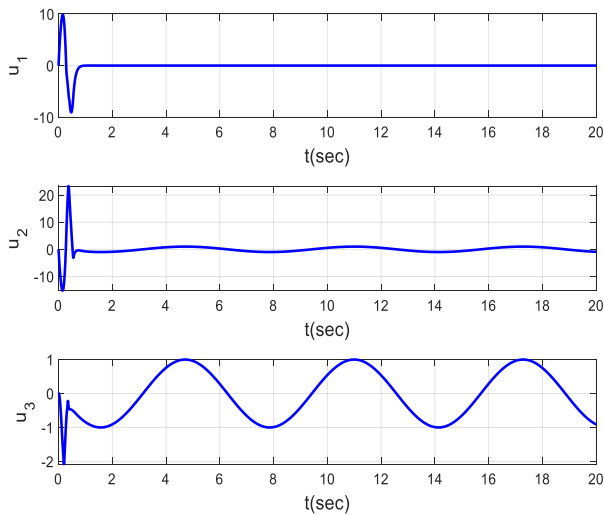


FIGURE 14. Control input signals of the proposed method under the conditions of scenario 2.

TABLE 3. Transient specifications of the proposed control system under the conditions of scenario 2.

Chaotic System	Proposed Method		
	X_1	X_2	X_3
State Variables			
Overshoot	0.3021	0.8781	0
Undershoot	-0.2767	-0.7986	-0.02889
Control Inputs	u_1	u_2	u_3
Overshoot	10	21.413	0
Undershoot	-8.815	-13.019	-2.08

of the system state trajectories and sliding surface curves and other system responses in simulations, can also be eliminated their overshoots, undershoots and jumps. For this purpose, the control law parameters are chosen as: $\lambda_i = 15$, $k_i = 15$, $i = 1, 2, 3$, $\epsilon_1 = 50$, $\epsilon_2 = 40$, $\epsilon_3 = 40$; and the initial states are considered as: $x(0) = [-0.2, 0.3, 0.2]^T$. Here, in addition to changing the parameters of λ_i and k_i , other parameters in the control law (34) including ϵ_i , have also been changed and more optimal values have been replaced them.

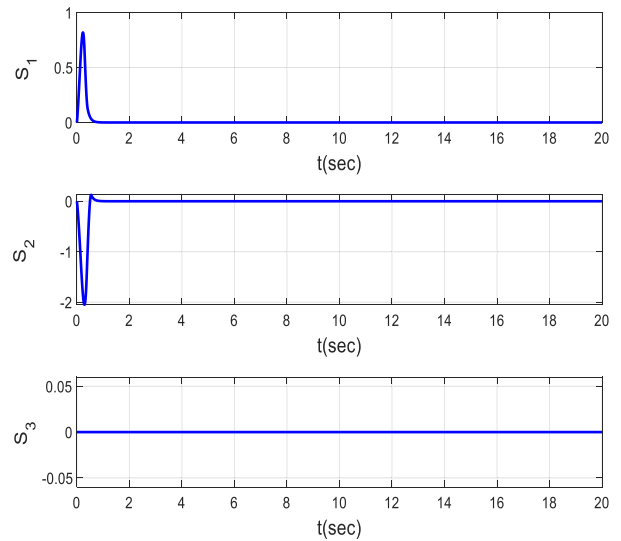


FIGURE 15. Sliding surface curves of the proposed method under the conditions of scenario 2.

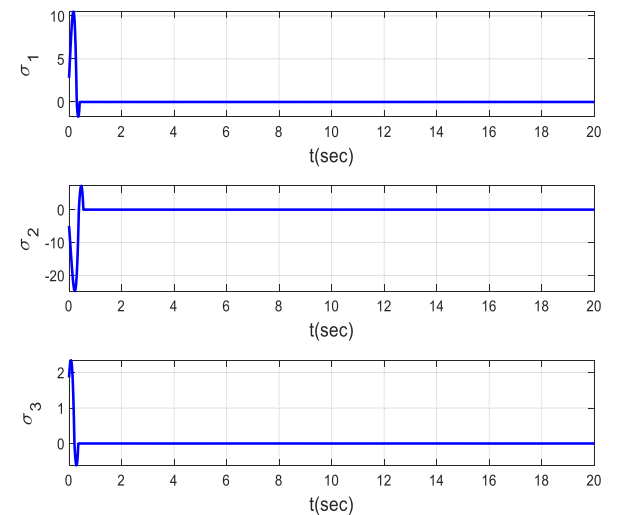


FIGURE 16. Sliding surface dynamics of the proposed method under the conditions of scenario 2.

Note 7: In this study, the optimal values of the control law parameters have been obtained by trial-and-error method based on the best and most optimal responses of the system in the simulation results.

In scenario 3, similar to the previous two scenarios, after placing the value of the above control parameters in Eq. (34), the simulation results are obtained as follows, in accordance with Figs. 18 through 23. The closed-loop system state trajectories and state variables behavior under scenario 3 is displayed in Fig. 18. Fig. 19 depicts the state phase diagram of Liu's chaotic system of the proposed method under scenario 3. In Fig. 20, the control input signals under scenario 3 conditions are given. Fig. 21 indicates the sliding surfaces curves in scenario 3, and the sliding surface dynamics are shown in Fig. 22. Finally, the time responses of the adaptation gains are displayed in Fig. 23. In addition, to show the superiority of scenario 3 over scenarios 1 and 2, the qualitative comparison

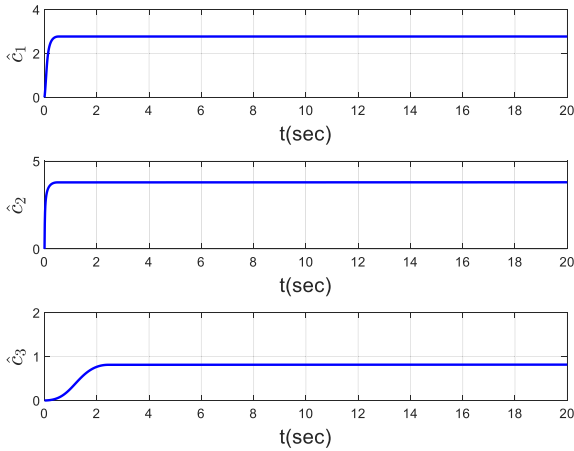


FIGURE 17. Time responses of system adaptation gains under the conditions of scenario 2.

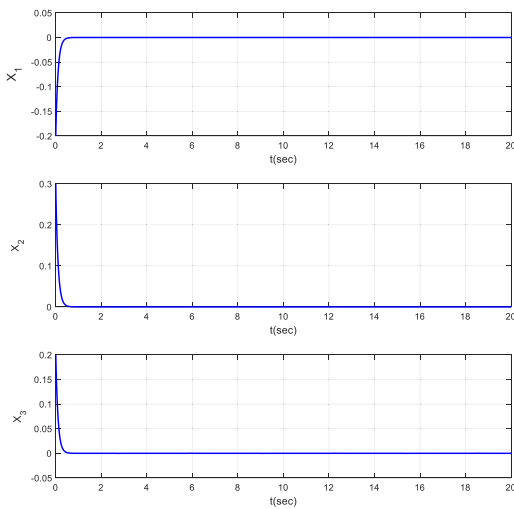


FIGURE 18. Closed-loop system state trajectories of the proposed method under the conditions of scenario 3.

of the triple scenarios for the proposed controller is given in Table 4.

According to the results obtained from the simulations under the conditions of scenario 3, we can see from Fig. 18, that not only the state trajectories X_1, X_2, X_3 have converged to the origin in less than one second, but also their overshoots and undershoots have been completely eliminated. Therefore, it can be concluded that scenario 3 has significantly improved in terms of the convergence rate to the origin of system state trajectories and in terms of removing their overshoots and undershoots compared to scenarios 1 and 2. From Fig. 19 and comparing it with Figs. 2, 7 and 13, it can be seen that in scenario 3, by applying the proposed controller, not only Liu’s chaotic system has been stabilized and its chaotic behavior has been controlled, but also the phase diagram has reached the origin in the shortest path and without deviation, which is very ideal.

In addition, according to the control input signals in Fig. 20, and comparing it with Figs. 8 and 14, it can be concluded that in scenario 3, the overshoots and undershoots of the control inputs are almost eliminated, and their

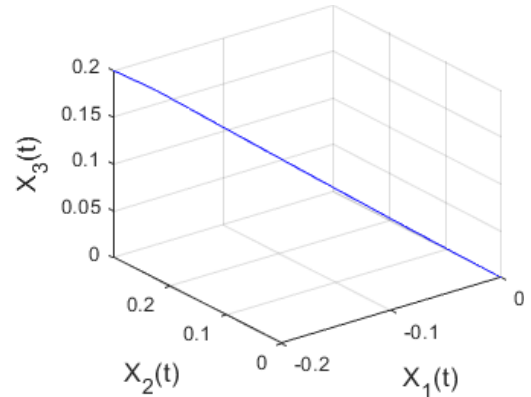


FIGURE 19. Phase diagram of Liu’s chaotic system states of the proposed method under the conditions of scenario 3.

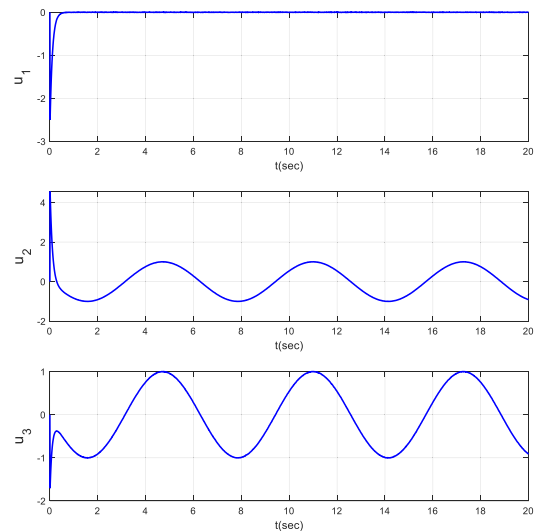


FIGURE 20. Control input signals of the proposed method under the conditions of scenario 3.

level of smoothness is relatively improved compared to scenarios 1 and 2. Furthermore, as it can be clearly seen in Figs. 21 and 22, not only the sliding surface curves and the sliding surface dynamics under scenario 3 conditions have converged to the origin much faster, but also their overshoots and undershoots have been completely eliminated. This shows a significant improvement in this part of the simulations compared to scenarios 1 and 2. Finally, from Fig. 23, the final values of the adaptation gains under scenario 3 conditions are obtained as $\hat{c}_i(t) = [2.78 \ 3.812 \ 0.8945]$.

In general, by designing and implementing scenario 3 and using the optimal values of the control law parameters of λ_i, k_i and ϵ_i ; Not only we were able to increase the convergence rate of system state trajectories and sliding surface curves, and other system responses, but also we were able to eliminate their overshoots, undershoots and jumps. In fact, at the same time, we have improved the convergence rate and the stability of system responses compared to scenario 1 and 2. Furthermore, from the information in Table 4, which briefly provides a qualitative comparison of the triple scenarios for the proposed controller, it can be concluded that scenario 3 is

TABLE 4. Qualitative comparison of three scenarios for the proposed controller.

Control Conditions	Jump in the state trajectories	Rate of convergence in the state trajectories	Jump in the control input signals	Jump in the sliding surface curves	Rate of convergence in the sliding surface curve	Jump in the sliding surface dynamics	Rate of convergence in the sliding surface dynamics	Deviation of the phase diagram in reaching the origin
Scenario 1	Yes	Low	Yes	Yes	Low	Yes	High	Yes
Scenario 2	Yes	High	Yes	Yes	High	Yes	High	Yes
Scenario 3	No	High	No	No	High	No	Very High	No

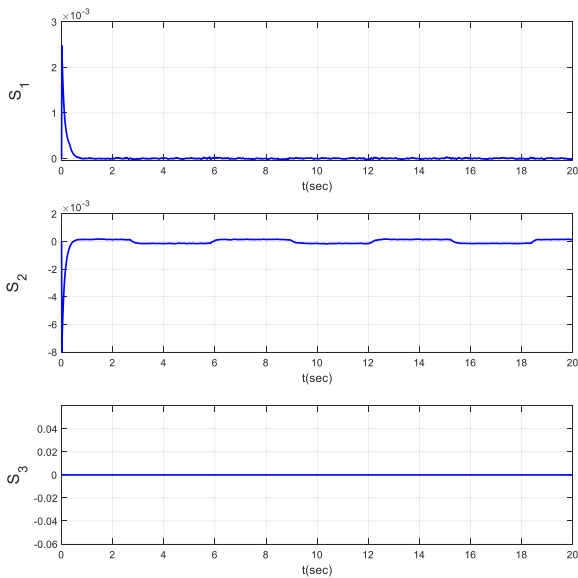


FIGURE 21. Sliding surface curves of the proposed method under the conditions of scenario 3.

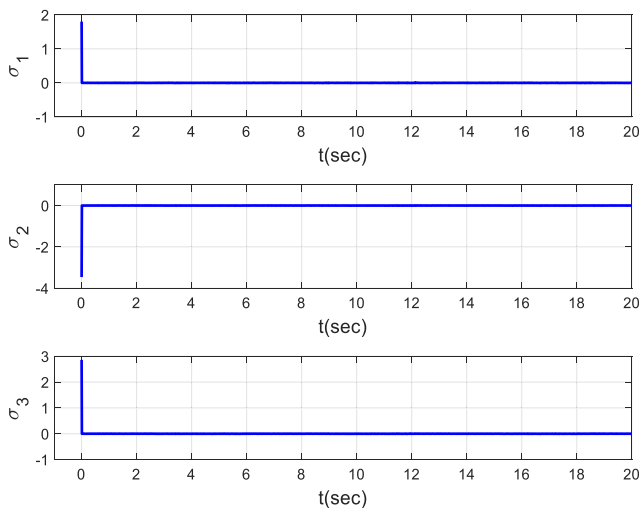


FIGURE 22. Sliding surface dynamics of the proposed method under the conditions of scenario 3.

superior to scenarios 1 and 2 both in terms of convergence rate and system stability. The simulation results in all three scenarios show the feasibility and effectiveness of the proposed control scheme under different conditions.

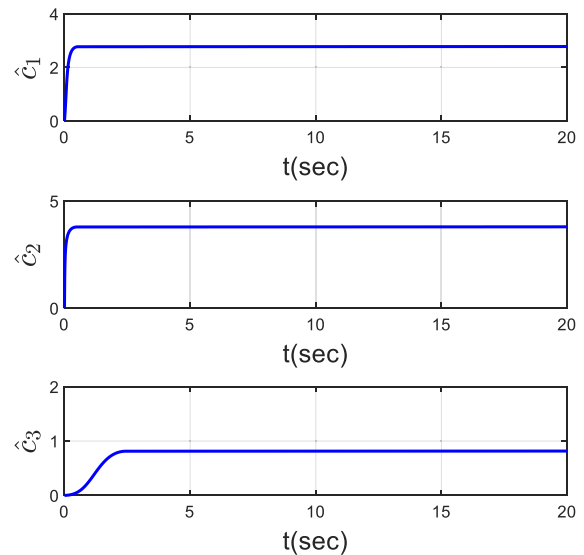


FIGURE 23. Time responses of system adaptation gains under the conditions of scenario 3.

Some of the major problems encountered in this research were: the problem of unbounded disturbances and the upper boundaries of unknown uncertainty, the problem of the control input not being smooth and continuous in SMC and the complete elimination of chattering, the problem of the convergence rate to the origin of system state trajectories and sliding surface curves and the problem of undershoots and overshoots. To overcome the mentioned problems, the following efforts have been taken. In order to overcome the problem caused by the upper boundaries of unknown uncertainty, the SMC method is combined with the adaptive control technique. The control inputs in most standard SMC suffer from the chattering phenomenon. In this research, a dynamic sliding mode controller (DSMC) is suggested to solve this problem. Using the dynamic sliding mode surface with the integral operator, along with the use of the adaptive continuous barrier function, has played an essential role in eliminating chattering in the controlled chaotic system. In addition, to solve the problem of the convergence rate to the origin of the system state trajectories and sliding surface curves, by changing the parameters λ_i and k_i , scenario 2 has been designed and implemented. To eliminate the problem of the existence of undershoots and overshoots in the system

state trajectories and the sliding surface curves, using the optimal values of parameters λ_i , k_i and ϵ_i , scenario 3 has been designed and implemented. The simulation results confirmed the effectiveness of the proposed control scheme to control or synchronize chaos in chaotic systems with uncertainties.

V. CONCLUSION AND FUTURE WORKS

This study proposed a chattering-free and finite-time SMC approach based on Lyapunov's stability theory and adaptive control for a class of uncertain chaotic systems subject to model uncertainties and unknown external disturbances, in conditions where the upper boundaries of the uncertainty term and their first-derivatives are unknown. According to the simulation results, it is clear that the proposed controller can control chaotic systems well, even with model uncertainties and unknown external disturbances. The proposed control scheme using a smooth and continuous control rule can eliminate the chattering phenomenon, which occurs due to the discontinuity of the sign function in the control input signal in ordinary sliding mode controllers. The obtained results confirmed the efficiency of the proposed control technique in eliminating the chattering phenomenon and maintaining effective and robust controller performance of chaotic systems albeit the unknown uncertainties and disturbances. In addition, the simulation results of the triple scenarios showed that the system responses can improve during a step-by-step process, and by using the optimal values of control parameters, the simulation results can further be improved both in terms of system stability convergence rate. Our future work will focus on extending the application of the proposed approach to time delay systems [38], [39], [40]. The aim would be to design a barrier function-based SMC approach to yield smooth and continuous control rules for uncertain chaotic systems with multiple time-delays.

APPENDIX

PROCESS OF GENERATING EQUATIONS (9) AND (24)

To design and generate the adaptive control rules (9) and (24) according to the method [34], first, consider the derivative of the sides of Eq. (6) as follows:

$$\dot{\sigma} = \ddot{s}_i + \lambda_i \dot{s}_i, \quad (35)$$

Then, substitute the sliding surfaces of s_i from Eq. (7) into Eq. (35); As a result, we will have:

$$\dot{\sigma} = \ddot{x}_i + (\lambda_i + k_i) \dot{x}_i(t) + \lambda_i k_i x_i(t), \quad (36)$$

Now, by placing the system states \dot{x}_i from Eq. (1) into Eq. (36), the following equation is obtained:

$$\begin{aligned} \dot{\sigma} = & A_i \dot{x} + \sum_{j=1}^n \frac{\partial F_i(t, x)}{\partial x_j} \dot{x}_j + \frac{\partial F_i(t, x)}{\partial t} \\ & + \sum_{j=1}^n \frac{\partial d_i(t, x)}{\partial x_j} \dot{x}_j + \frac{\partial d_i(t, x)}{\partial t} + \dot{u}_i \\ & + (\lambda_i + k_i) (A_i x + F_i(t, x) + d_i(t) + u_i) + \lambda_i k_i x_i, \end{aligned} \quad (37)$$

where $f_i(t, x)$ denotes the i^{th} row of the matrix or vector $f(t, x)$.

By generating the appropriate dynamical sliding mode surface (6), the next step is to design a SMC scheme to drive the system trajectories on the dynamical sliding mode surface $\sigma_i = 0 (i = 1, 2, \dots, n)$.

In the following, according to Note 1 and the relation $f(t, x) = Ax + F(t, x)$ and by placing $\dot{\sigma}_i = 0$ and transferring \dot{u}_i to the other side of the Eq. (37), the following equation will be generated:

$$\begin{aligned} \dot{u}_i = & - \sum_{j=1}^n \frac{\partial f_i(t, x)}{\partial x_j} \dot{x}_j - \frac{\partial f_i(t, x)}{\partial t} - \sum_{j=1}^n \frac{\partial d_i(t, x)}{\partial x_j} \dot{x}_j \\ & - \frac{\partial d_i(t, x)}{\partial t} - (\lambda_i + k_i) (f_i(t, x) + d_i(t, x) + u_i) \\ & - \lambda_i k_i x_i, \end{aligned} \quad (38)$$

Finally, according to Assumption 1 and Eq. (7), we can estimate the disturbance values d_i in Eq. (38) as follows:

$$\begin{aligned} \dot{u}_i = & - (\lambda_i + k_i) (u_i + f_i(t, x)) - \sum_{j=1}^n \frac{\partial f_i(t, x)}{\partial x_j} \dot{x}_j \\ & - \frac{\partial f_i(t, x)}{\partial t} - \lambda_i k_i x_i - c_i, \end{aligned} \quad (39)$$

In Eq. (39) above, by estimating c_i as $c_i = \epsilon_i \hat{c}_i \text{sign}(\sigma_i)$ and $c_i = \epsilon_i \hat{c}_{i,psd} \text{sign}(\sigma_i)$, Eqs. (9) and (24) appear respectively.

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