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METHODS

Reset-and-Hold Control of Systems With Time Delay

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ABSTRACT Reset control is a kind of hybrid control which is capable of overcoming fundamental limitations on the performance intrinsic to linear time invariant (LTI) systems. In this work, we develop a novel control strategy for first order LTI plants with time delay, based on the proportional-integral plus Clegg integrator (PI+CI) controller, a hybrid extension of the proportional-integral (PI) controller, augmented with a new mechanism for keeping its state constant during a given interval of time (reset-and-hold strategy). This strategy is capable of producing an approximation of a flat response which greatly improves the performance in comparison with non-hybrid linear strategies, extending previous results developed for first order systems without delay. Well-posedness and closed-loop stability of the resulting control system are analyzed under the Hybrid Inclusions (HI) framework, and a set of sufficient stability conditions are provided. Furthermore, a case study is developed that showcases the possibilities of this new approach.

INDEX TERMS Reset control, hybrid control systems, control design, PI control, nonlinear control systems, time delays.

I. INTRODUCTION

Reset control is a control approach in which controllers come equipped with mechanisms for resetting some of its states according to a given triggering event. Reset control systems constitute an important class of hybrid control systems, in which time evolution of the states can be both continuous and discrete. The main interest in reset systems lies in the fact that they are capable of overcoming fundamental performance limitations inherent to linear time invariant (LTI) control [1] by means of a simple mechanism.

In its original meaning, reset control involves linear time invariant controllers endowed with a means to reset their states to zero. A fundamental example of reset controller, now called the Clegg integrator (CI), was introduced by Clegg in his seminal work [2]. In the works of Horowitz and others [3], [4] another basic example of reset controller was introduced, the first order reset element (FORE), and design rules for the CI and FORE were developed. Since then, the meaning of reset system in practice has been expanded to encompass different triggering conditions such as error bands [5], [6] or periodic reset instants [7], as well as systems that are nonlinear or in which the resetted states do not necessarily go to zero.

The field of reset control has proven to be very fruitful, with a multitude of reset strategies having been successfully devised and applied in practice. More concretely, reset control strategies specific to systems with time delay have been developed and studied e.g. in [8], [9], [10], and [11].

In this work, we consider the problem of reset control for LTI first order plants with time delay (also called FOPDT systems), extending previous results pertaining to first order plants without delay [12]. With the introduction of delay, first order systems are sufficiently generic to capture the essential dynamics of many practical processes in industry [1]. Our approach is based on the proportional-integral plus Clegg integrator (PI+CI) controller, together with a new reset mechanism (reset-and-hold) in which the controller is augmented

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with the ability to hold its output constant for a certain interval of time. Under this strategy, we are able to develop tuning rules which result in a much greater performance compared to that of traditional reset control approaches.

The remainder of this article is structured as follows. In Section II, we introduce relevant definitions and basic results of the Hybrid Inclusions (HI) framework for systems with memory, as well as robust models for the CI and PI+CI, and general reset controller. Section III introduces the reset-and-hold strategy. Section IV discusses the proposed control structure, including a full description of the system, and the properties of well-posedness and closed-loop stability are analyzed. In Section V, design rules to achieve a flat response (to a good approximation) are derived, both for reference tracking and for disturbance rejection. Finally, Section VI presents a case study showcasing the capabilities of the proposed tuning rules by means of a simulated example.

Notation: $\mathbb{Z}_{\geq 0}$ ($\mathbb{Z}_{\leq 0}$) is the set of nonnegative (nonpositive) integers, and $\mathbb{R}_{\geq 0}$ ($\mathbb{R}_{\leq 0}$) nonnegative (nonpositive) real numbers set. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and $|\cdot|$ is the Euclidean norm. The transpose of a matrix A is $A^\top$, and $||A||$ is its norm. I and 0 denote identity and zero matrices. For a subset $X \subset \mathbb{R}^n$, $\bar{X}$ denotes its closure. The symbol $\times$ denotes Cartesian product and $\setminus$ denotes set difference. The convex hull of X is written $\operatorname{conv} X$. Finally, $\operatorname{floor}(x)$ is the greatest integer less than or equal to x.

A continuous function $f : [0, \infty) \to [0, \infty)$ belongs to class $\mathcal{K}_\infty$ (denoted $f \in \mathcal{K}_\infty$) if it is strictly increasing and unbounded, and $f(0) = 0$. If the unboundedness condition is dropped, it is said that f belongs to class $\mathcal{K}$. Similarly, a continuous function $f : [0, \infty) \times [0, \infty) \to [0, \infty)$ belongs to class $\mathcal{KL}$ (denoted as $f \in \mathcal{KL}$) if it is strictly increasing with respect to the first variable, decreasing with respect to the second variable, and satisfies $f(0, y) = 0$ for any y and $\lim_{y\to\infty} f(x, y) = 0$ for any x. For a subset $\mathcal{W}$ of Euclidean space, the distance $|x|_{\mathcal{W}}$ from x to $\mathcal{W}$ is defined as $\inf_{y\in\mathcal{W}} |x - y|$. Furthermore, $||\phi||_{\mathcal{W}}$ is defined as $\sup_{\substack{(s,k)\in \operatorname{dom} \phi \\ s+k\geq -\Delta-1}} |\phi(s, k)|_{\mathcal{W}}$.

II. PRELIMINARIES AND PROBLEM SETUP

A. TIME-DELAYED SYSTEMS

A system is said to be *time-delayed* when its current state depends on its state at some time in the past. Time-delayed systems arise very often in practice, due to physical limitations on the speed of transport of matter (e.g. in process industry) and transmission of information (e.g. in networked systems).

This work is focused on the hybrid control of first order plants with delay, also called First Order Plus Dead Time (FOPDT) systems. These are linear time invariant systems where the delay is characterized by a single parameter h, such that the first order dynamics of the output is dependent on the input shifted backwards h units in time. A first order plant with delay is represented in the frequency domain by the following transfer function:

$$P(s) = \frac{be^{-sh}}{s + a},$$

with three parameters a, b, h. Despite its simplicity, it is known that many practical processes in industry can be well approximated by such a dynamics [1].

B. THE HI FRAMEWORK

The formalism of Hybrid Dynamical Systems, also called the Hybrid Inclusions (HI) framework, which is developed in [13], is the mathematical basis upon which the developments of this work rest. This subsection includes a brief description of the HI framework for systems with inputs and for systems with memory.

1) THE HI FRAMEWORK FOR SYSTEMS WITH INPUTS

This section briefly describes the formalism of Hybrid Dynamical Systems adapted to systems with inputs [14] as will be used in this work. A *hybrid system with inputs* Σ for a state $\mathbf{x} \in \mathbb{R}^{n_x}$ and input $\mathbf{w} \in W$ for some set $W \subseteq \mathbb{R}^{n_w}$ is given by

$$\Sigma : \begin{cases} \dot{\mathbf{x}} \in f(\mathbf{x}, \mathbf{w}), & \text{if } (\mathbf{x}, \mathbf{w}) \in \mathcal{C}, \\ \mathbf{x}^+ \in g(\mathbf{x}, \mathbf{w}), & \text{if } (\mathbf{x}, \mathbf{w}) \in \mathcal{D}. \end{cases} \tag{1}$$

where $f : \mathbb{R}^{n_x} \rightrightarrows \mathbb{R}^{n_x}$ is the flow map, $g : \mathbb{R}^{n_x} \rightrightarrows \mathbb{R}^{n_x}$ is the jump map (f and g are both set-valued maps in general), and $\mathcal{C}, \mathcal{D} \subseteq \mathbb{R}^{n_x} \times \mathbb{R}^{n_w}$ are the flow and jump sets respectively. Note that as in [14], we do not regard $\mathbf{w}$ as a hybrid signal (as otherwise its domain must be known in advance, which is unrealistic in practice); instead, the space of admissible inputs $\mathbf{w}(t)$ will be taken as the set of *piecewise continuous* functions from $\mathbb{R}_{\geq 0}$ to W. With this reduced set of inputs, the two concepts of solution considered in [14] coincide, and the results for existence of solutions and completeness therein developed easily follow.

2) THE HI FRAMEWORK FOR SYSTEMS WITH MEMORY

This work uses the formalism of Hybrid Dynamical Systems for systems with memory introduced in [15] and further generalizes in [16]. The reader is referred to [15] for a detailed overview of basic definitions and results. A key concept is the size of the memory Δ (a kind of generalization of the delay that may now be continuous or discrete or a combination of both). Here $\mathcal{M}^\Delta$ is the set of hybrid memory arcs *with memory of size* Δ. To simplify the notation,[1] besides the state $\mathbf{x}(t, j)$ at some $(t, j) \in \operatorname{dom} \mathbf{x}$ consider the *distributed* state $\mathbf{x}_{[t,j]} \in \mathcal{M}^\Delta$ given by $\mathbf{x}_{[t,j]}(s, k) = \{\mathbf{x}(s + t, j + k) \in \mathbb{R}^n : (s, k) \in \mathbb{R}_{\leq 0} \times \mathbb{Z}_{\leq 0}, (t + s, j + k) \in \operatorname{dom} \mathbf{x}, s + k \geq -\Delta_{\inf}\}$.

A *hybrid system* $\Sigma^\Delta = (\mathcal{C}, \mathcal{F}, \mathcal{D}, \mathcal{G})$*, with memory of size* Δ, is given by

$$\Sigma^\Delta : \begin{cases} \dot{\mathbf{x}}(t, j) \in \mathcal{F}(\mathbf{x}_{[t,j]}), & \mathbf{x}_{[t,j]} \in \mathcal{C}, \\ \mathbf{x}(t, j + 1) \in \mathcal{G}(\mathbf{x}_{[t,j]}), & \mathbf{x}_{[t,j]} \in \mathcal{D}. \end{cases} \tag{2}$$

[1] Note that in [15], [16], a shift operator $\mathcal{A}^\Delta_{[\cdot,\cdot]}$ is used; here the notation $\mathbf{x}_{[\cdot,\cdot]} = \mathcal{A}^\Delta_{[\cdot,\cdot]}\mathbf{x}$ is employed.

where $\mathcal{F} : \mathcal{M}^\Delta \rightrightarrows \mathbb{R}^n$ is the flow map, $\mathcal{G} : \mathcal{M}^\Delta \rightrightarrows \mathbb{R}^n$ is the jump map, and $\mathcal{C}, \mathcal{D} \subseteq \mathcal{M}^\Delta$ are the flow and jump sets respectively.

When Σ is *well-posed*, for example if it satisfies the *basic hybrid conditions* [16], its solution sets inherit several good structural properties: upper-semicontinuous dependence with respect to initial conditions, robustness against perturbations like measurement noise, and preservation of asymptotic stability under small perturbations (which is referred to as robust stability).

C. RESET CONTROL

A reset controller consists of a LTI controller (the *base controller*) and a mechanism to reset some of its states according to some resetting law [17]. Informally speaking, the *zero-crossing* resetting law enables a jump (reset) when the closed-loop error is zero, while the *variable band* resetting law enables a reset when the absolute value of the closed-loop error reaches a certain threshold.

Following [18], the resetting law will be implemented by using a discrete state $q \in \{-1, 1\}$.

1) A GENERIC RESET CONTROLLER

A reset controller R with state $(\mathbf{x}_r, q) \in \mathcal{O}^{n_r}$, input $e \in \mathbb{R}$, and output $u \in \mathbb{R}$ is given by

$$R : \begin{cases} \dot{\mathbf{x}}_r = A_r \mathbf{x}_r + B_r e, & \text{if } (\mathbf{x}_r, q, e) \in \mathcal{C} \\ \begin{pmatrix} \mathbf{x}_r^+ \\ q^+ \end{pmatrix} = \begin{pmatrix} A_\rho & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \mathbf{x}_r \\ q \end{pmatrix}, & \text{if } (\mathbf{x}_r, q, e) \in \mathcal{D} \\ u = C_r \mathbf{x} + D_r e. \end{cases} \tag{3}$$

where A_r, B_r, C_r, D_r are constant matrices of the appropriate dimensions, and A_ρ is a diagonal matrix that sets to zero the last n_ρ elements of $\mathbf{x}_r$. The flow and jump sets $\mathcal{C}$ and $\mathcal{D}$ are given by (3) and (4), respectively, where

$$\mathcal{C} = \{(\mathbf{x}_r, q, e) \in \mathcal{O}^{n_r} \times \mathbb{R} : q\, S(e) \geq 0\}, \tag{4a}$$
$$\mathcal{D} = \{(\mathbf{x}_r, q, e) \in \mathcal{O}^{n_r} \times \mathbb{R} : q\, S(e) \leq 0\}. \tag{4b}$$

and $S(e)$ is the output of some (possibly nonlinear) transformation applied to the signal e. The *zero-crossing* resetting law corresponds to $S(e) = e$; the more general *variable band* resetting law is obtained for $S(e) = e + \theta \dot{e}$, where $\theta \in \mathbb{R}$ is a design parameter (note that the zero-crossing resetting law is recovered for $\theta = 0$).

2) THE PI+CI CONTROLLER

The *PI+CI controller* [12] is a particular case of reset controller, where $n_r = 2$, $A_r = 0$, $B_r = (1, 1)^\top$, $A_\rho = \left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right)$, $C_r = (\frac{k_P}{T_I}(1 - p_r), \frac{k_P}{T_I} p_r)$ and $D_r = k_P$.

The PI+CI controller constitutes a hybrid extension of the proportional-integral (PI) controller, in which the integral part is replaced with a weighted sum of a linear integrator and a CI. In addition to the proportional gain k_P and the integral time T_I, it has an extra design parameter p_r, called the *reset ratio*, which determines the weight of the CI state in the output. If $p_r = 0$, the linear PI controller's behavior is recovered. Despite its simplicity, the PI+CI controller has been found useful in several practical applications [17], [19].

III. THE RESET-AND-HOLD CONTROLLER

A new hybrid controller, referred to as *reset-and-hold*, inspired by the *distributed state resetting* approach of [10], will be shown to be specially useful for systems with time delays. The main motivation has been to overcome the performance of reset controllers for systems with time delays. This is based on the fact that the performance improvement due to resetting crucially depends on a balance between the after-reset states of both plant and controller. However, the presence of delay in the feedback path destroys that balance, typically producing an undesired undershooting, which limits the potential performance improvement. To avoid this problem, besides resetting the basic idea is to hold the control signal after a jump, for some time interval.

The reset-and-hold controller R_H, with state $(\mathbf{x}_r, q, m, \tau) \in \mathcal{O}^H := \mathcal{O}^{n_r} \times \{0, 1\} \times \mathbb{R}_{\geq 0}$, and input $e \in \mathbb{R}$, is defined as a hybrid system with inputs, given by:

$$R_H : \begin{cases} \begin{pmatrix} \dot{\mathbf{x}}_r \\ \dot{\tau} \end{pmatrix} = \begin{pmatrix} m A_r \mathbf{x}_r + m B_r e \\ 1 \end{pmatrix}, & (\mathbf{x}_r, q, m, \tau, e) \in \mathcal{C}_H \\ \begin{pmatrix} \mathbf{x}_r^+ \\ q^+ \\ m^+ \\ \tau^+ \end{pmatrix} = \begin{pmatrix} A_\rho \mathbf{x}_r \\ (1 - 2m) q \\ 1 - m \\ 0 \end{pmatrix}, & (\mathbf{x}_r, q, m, \tau, e) \in \mathcal{D}_H \end{cases} \tag{5}$$

where the output is $u = C_r \mathbf{x}_r + m D_r e$, the flow set is $\mathcal{C}_H = \mathcal{C}_{H_0} \cup \mathcal{C}_{H_1}$, where

$$\mathcal{C}_{H_0} = \{(\mathbf{x}_r, q, m, \tau, e) \in \mathcal{O}^H \times \mathbb{R} : m = 0, \tau \leq \tau_H\}, \tag{6a}$$
$$\mathcal{C}_{H_1} = \{(\mathbf{x}_r, q, m, \tau, e) \in \mathcal{O}^H \times \mathbb{R} : m = 1, q\, S(e) \geq 0\}, \tag{6b}$$

and the jump set is $\mathcal{D}_H = \mathcal{D}_{H_0} \cup \mathcal{D}_{H_1}$, being

$$\mathcal{D}_{H_0} = \{(\mathbf{x}_r, q, m, \tau, e) \in \mathcal{O}^H \times \mathbb{R} : m = 0, \tau \geq \tau_H\}, \tag{7a}$$
$$\mathcal{D}_{H_1} = \{(\mathbf{x}_r, q, m, \tau, e) \in \mathcal{O}^H \times \mathbb{R} : m = 1, q\, S(e) \geq 0\}. \tag{7b}$$

Note that, in comparison with the reset controller (3), R_H includes a extra discrete state $m \in \{0, 1\}$, that will be used to switch between two operating modes, and a timer τ that will be in charge of regulating the time interval in which the controller output is held constant after every jump due to reset. The two operating modes are:

- $m = 1$ (*resetting mode*). The controller output corresponds to that of the base linear controller (A_r, B_r, C_r, D_r). Jumps are enabled only when the resetting law $q\, S(e) \leq 0$ is triggered: a crossing is detected and thus the sign of q is changed, $\mathbf{x}_r$ is reset, the timer is reset to zero, and m switches to 0 (holding mode).

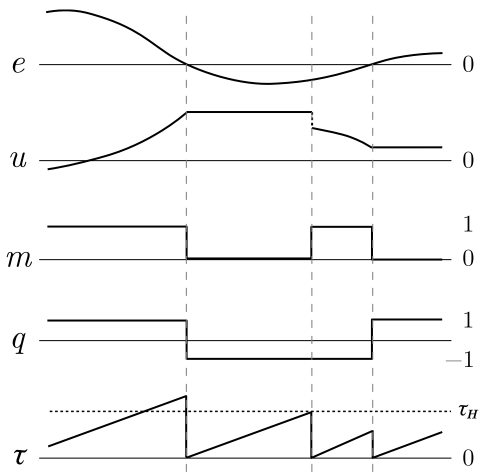


FIGURE 1. Reset-and-hold controller state and output response for a given input *e* (with a zero-crossing resetting law in the resetting mode).

- $m = 0$ (*holding mode*). The state $\mathbf{x}_r$ is kept constant (note that $\dot{\mathbf{x}}_r = 0$ from (5)), and thus, since $u = C_r\mathbf{x}_r + mD_r e = C_r\mathbf{x}_r$, then the control output is also constant in this mode. On the other hand, the timer τ is activated, and when it reaches the value $\tau = \tau_H$ a jump is triggered. After jumping, m switches to 1 (resetting mode), the timer τ is initialized to zero, and the rest of states are kept identical.

Fig. 1 depicts an example of a state/output response for a given controller input, where the two modes are represented. Moreover, note that the reset controller R, as given by (3), can be obtained from (5) simply by making $\tau_H = 0$.

Note that the basic idea behind the reset-and-hold strategy is to temporarily disable the feedback path during the intervals of time $[t_i, t_i + \tau_H]$ after jumps at time t_i, until the effect of reset is able to properly reach a plant with time delay. This strategy is especially useful in cases where resetting actions aim to drive this plant to a stationary state where $e \to 0$, as will be seen in further sections.

A. CONTROL OF SYSTEMS WITH TIME-DELAYS

Consider a linear time-invariant system P with time delay h, defined by the delay-differential equation

$$P : \begin{cases} \dot{\mathbf{x}}_p(t) = A_p\mathbf{x}_p(t) + B_p v(t-h) \\ y = C_p\mathbf{x}_p(t). \end{cases} \tag{8}$$

with state $\mathbf{x} \in \mathbb{R}^{n_p}$, input $v \in \mathbb{R}$ and output $v \in \mathbb{R}$, and the feedback connection between P and a reset-and-hold controller R_H, given by $e = r - y$, where $r \in \mathbb{R}$ is a reference signal, and $v = u + d$, being u the controller output and $d \in \mathbb{R}$ a disturbance signal. The closed-loop state is $\mathbf{z} \in \mathcal{O} := \mathbb{R}^{n_p} \times \mathcal{O}^H$, and is partitioned as $\mathbf{z} = (\mathbf{x}, \mathbf{s})$, where $\mathbf{x} = (\mathbf{x}_p, \mathbf{x}_r)$, and $\mathbf{s} = (q, m, \tau)$. From (5)–(7) and (8), the closed loop system (without exogenous inputs) is given by

$$\begin{aligned} \begin{pmatrix} \dot{\mathbf{x}} \\ \dot{q} \\ \dot{m} \\ \dot{\tau} \end{pmatrix} &= \begin{pmatrix} A_0(m)\mathbf{x} + A_h(m)\mathbf{x}(t-h) \\ 0 \\ 0 \\ 1 \end{pmatrix}, \\ &\quad (\mathbf{z}(t), \mathbf{z}(t-h)) \in \mathcal{C}_0 \\ \begin{pmatrix} \mathbf{x}^+ \\ q^+ \\ m^+ \\ \tau^+ \end{pmatrix} &= \begin{pmatrix} A_R\mathbf{x} \\ (1-2m)q \\ 1-m \\ 0 \end{pmatrix}, \\ &\quad (\mathbf{z}(t), \mathbf{z}(t-h)) \in \mathcal{D}_0 \end{aligned} \tag{9}$$

where the flow and jump sets are given by

$$\begin{aligned} \mathcal{C}_0 &= \{(\mathbf{z}_1, \mathbf{z}_2) \in \mathcal{O}^2 : m_1 = 0, \tau_1 \le \tau_H\} \\ &\cup \{(\mathbf{z}_1, \mathbf{z}_2) \in \mathcal{O}^2 : m_1 = 1, \\ &\qquad qC((I + \theta A_0)\mathbf{x}_1 + \theta A_h\mathbf{x}_2)) \ge 0\} \end{aligned} \tag{10}$$

and

$$\begin{aligned} \mathcal{D}_0 &= \{(\mathbf{z}_1, \mathbf{z}_2) \in \mathcal{O}^2 : m_1 = 0, \tau_1 \ge \tau_H\} \\ &\cup \{(\mathbf{z}_1, \mathbf{z}_2) \in \mathcal{O}^2 : m_1 = 1, \\ &\qquad qC((I + \theta A_0)\mathbf{x}_1 + \theta A_h\mathbf{x}_2)) \le 0\} \end{aligned} \tag{11}$$

respectively, and

$$\begin{aligned} A_0(m) &= \begin{pmatrix} A_p & 0 \\ -mB_rC_p & mA_r \end{pmatrix}, \\ A_h(m) &= \begin{pmatrix} -mB_pD_rC_p & B_pC_r \\ 0 & 0 \end{pmatrix}, \end{aligned} \tag{12}$$

$$A_R = \begin{pmatrix} I & 0 \\ 0 & A_\rho \end{pmatrix}, \quad C = \begin{pmatrix} -C_r & 0 \end{pmatrix}. \tag{13}$$

IV. A HYBRID CONTROL SYSTEM WITH MEMORY

Now, let $\mathbf{z} = (\mathbf{x}, q, m, \tau)$ be a hybrid arc and $\mathbf{z}_{[t,j]} \in \mathcal{M}^\Delta$ a hybrid memory arc. One way of interpreting the closed-loop hybrid system (9)–(13) as a hybrid dynamical system with memory $\Sigma^\Delta = (\mathcal{C}, \mathcal{F}, \mathcal{D}, \mathcal{G})$ is to use the following data (some similar cases are described in [15] and [16]):

$$\begin{aligned} &\mathcal{C} \\ &= \{\varphi \in \mathcal{M}^\Delta : (\varphi(0,0), \varphi(-h, -k_m)) \in \mathcal{C}_0\}, \\ &\mathcal{F}(\mathbf{z}_{[t,j]}) \\ &= \begin{pmatrix} \overline{\text{conv}} \bigcup\limits_{(-h,-k) \in \text{dom}\, \mathbf{x}_{[t,j]}} \{A_0\mathbf{x}(t,j) + A_h\mathbf{x}(t-h, j-k)\} \\ 0 \\ 0 \\ 1 \end{pmatrix}, \\ &\mathcal{D} \\ &= \{\varphi \in \mathcal{M}^\Delta : (\varphi(0,0), \varphi(-h, -k_m)) \in \mathcal{D}_0\}, \\ &\mathcal{G}(\mathbf{z}_{[t,j]}) \\ &= \begin{pmatrix} A_R\mathbf{x}(t,j) \\ (1 - 2m(t,j))q(t,j) \\ 1 - m(t,j) \\ 0 \end{pmatrix}, \end{aligned} \tag{14}$$

where $k_m = \max\{k : (-h, k) \in \text{dom } \varphi\}$. Note that having the time delay h means that the change by flow of $\mathbf{x}$ at $(t, j) \in \text{dom } \mathbf{x}$ depends on both $\mathbf{x}_{[t,j]}(0, 0) = \mathbf{x}(t, j)$ and the value of $\mathbf{x}_{[t,j]}(-h, -k) = \mathbf{x}(t-h, j-k)$, and that due to possibility of multiple instantaneous jumps at $t-h$, there can be more than one k satisfying $(-h, -k) \in \text{dom } \mathbf{x}_{[t,j]}$. The choice to take the convex hull of all those points is related to the fulfilment of regularity conditions to obtain robustness to small variations in the size of the delay [15]. In addition, jumps also depend on $m_{[t,j]}(0, 0) = m(t, j)$ and $q_{[t,j]}(0, 0) = q(t, j)$. Moreover, the matrices A_0 and A_h implicitly depend on $m_{[t,j]}(0, 0) = m(t, j)$ as shown in (12).

A. WELL-POSEDNESS

In this section, the well-posedness of the closed-loop system Σ^Δ, as given by (14), is analyzed. Firstly, since there may be at most two instantaneous consecutive jumps when jumping from $m = 1$ to $m = 0$, when the system immediately jumps again to $m = 0$ and then it is forced to flow during at least τ_H time units, then any hybrid arc which is a solution to Σ^Δ has at most $2h/\tau_H$ jumps in any time interval $[t-h, t]$, for any $t \in \mathbb{R}_{\geq} 0$. Thus, Σ^Δ has a finite memory with size $\Delta = h + 2h/\tau_H + 1$.

Recall that a hybrid system with memory $\Sigma^\Delta = (\mathcal{C}, \mathcal{F}, \mathcal{D}, \mathcal{G})$ is *well-posed* if it satisfies the *basic hybrid conditions*,[2] that is, for any $b, \lambda \in \mathbb{R}_{>0}$:

1) $\mathcal{C} \cap \mathcal{M}^\Delta_{b,\lambda}$ and $\mathcal{D} \cap \mathcal{M}^\Delta_{b,\lambda}$ are closed subsets of $\mathcal{M}^\Delta$.
2) $\mathcal{F}$ is outer semicontinuous relative to $\mathcal{C} \cap \mathcal{M}^\Delta_{b,\lambda}$, locally bounded relative to $\mathcal{C} \cap \mathcal{M}^\Delta_b$, and $\mathcal{F}(\varphi)$ is nonempty and convex for each $\varphi \in \mathcal{C} \cap \mathcal{M}^\Delta_{b,\lambda}$.
3) $\mathcal{G}$ is outer semicontinuous relative to $\mathcal{D} \cap \mathcal{M}^\Delta_{b,\lambda}$, locally bounded relative to $\mathcal{D} \cap \mathcal{M}^\Delta_b$, and $\mathcal{G}(\varphi)$ is nonempty for each $\varphi \in \mathcal{D} \cap \mathcal{M}^\Delta_{b,\lambda}$.

From their definition in (14) and (10)–(11), it is clear that $\mathcal{C}$ and $\mathcal{D}$ are closed sets, and thus the hybrid basic condition #1 easily follows. Regarding condition #2, it directly follows from (14) that that $\mathcal{F}(\varphi)$ is convex and nonempty.

Also local boundedness of $\mathcal{F}$ is straightforward, since hybrid memory arcs $\varphi \in \mathcal{C} \cap \mathcal{M}^\Delta_{b,\lambda}$ are upper bounded by b and thus $\|\mathcal{F}(\varphi)\|^2 \leq \|A_0(\varphi)\varphi + A_h(\varphi)\varphi\|^2 + 1 \leq (\max\{\|A_0(0)\|^2 + \|A_h(0)\|^2, \|A_0(1)\|^2 + \|A_h(1)\|^2\})b^2 + 1$. Moreover, outer semicontinuity of $\mathcal{F}$ is strongly based on the choice of the convex hull in its definition (a formal proof would involve analyzing graphical convergence of hybrid memory arcs and it is not given here). Finally, condition #3 easily follows, for example since $\|\mathcal{G}(\varphi)\|^2 \leq \|A_R\|^2\|\varphi\|^2 + (1 + 4\|\varphi\|^2)\|\varphi\|^2 + (1 + \|\varphi\|^2) \leq (\|A_R\|^2 + 2 + 4b^2)b^2 + 1$, then local boundedness is assured. On the other hand, outer semicontinuity of $\mathcal{G}$ directly follows since $\mathcal{G}$ is single-valued and continuous. As a result, it follows that the proposed hybrid control system $\Sigma^\Delta = (\mathcal{C}, \mathcal{F}, \mathcal{D}, \mathcal{G})$ is well-posed.

[2] Here $\mathcal{M}^\Delta_{b,\lambda}$ is a subspace of $\mathcal{M}^\Delta$ with better compactness properties; informally speaking it consists of hybrid memory arcs whose norm at every point in the domain is upper bounded by $b \in \mathbb{R}_{>0}$, and are Lipschitz continuous in the t-domain with Lipschitz constant $\lambda \in \mathbb{R}_{>0}$ (see [16] for technical details).

B. STABILITY

In this section, stability of the closed loop hybrid system Σ^Δ, given by (14), in investigated. More specifically, a closed set $\mathcal{W}$ is asymptotically stable for the hybrid system Σ^Δ if there exists a candidate Lyapunov-Krasovskii functional $V : \mathcal{M}^\Delta \to \mathbb{R}_{\geq 0}$, $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$ and a continuous positive definite function ρ such that the three following conditions are satisfied:

$$\alpha_1(|\phi(0,0)|_\mathcal{W}) \leq V(\phi) \leq \alpha_2(\|\phi\|_\mathcal{W}) \quad \forall \phi \in \mathcal{C} \cup \mathcal{D} \cup \mathcal{G}^+(\mathcal{D}), \tag{15a}$$

$$\dot{V}(\phi) \leq -\rho(|\phi(0,0)|_\mathcal{W}) \quad \forall \phi \in \mathcal{C}, \tag{15b}$$

$$V(\phi^+_\gamma) - V(\phi) \leq -\rho(|\phi(0,0)|_\mathcal{W}) \quad \forall \phi \in \mathcal{D}, \gamma \in \mathcal{G}(\phi), \tag{15c}$$

where $\dot{V}$ denotes the upper right hand derivative of the functional V, and ϕ^+_γ is the new hybrid arc obtained after a single jump to the value γ; the set of all possible ϕ^+_γ is denoted $\mathcal{G}^+(\mathcal{D})$ (the reader is referred to [15] for precise definitions and technical details).

Here, the stability of $\mathcal{W} = \{\mathbf{0}\} \times \{0, 1\} \times \{-1, 1\} \times \mathbb{R}_{\geq 0}$, which corresponds to the set of all the points $\mathbf{z} = (\mathbf{x}, q, m, \tau)$ such that $\mathbf{x} = 0$, is considered. In the following, delay-dependent stability conditions are obtained. The approach is based on postulating a quadratic Lyapunov-Krasovskii functional and deriving sufficient conditions for stability in the form of linear matrix inequalities. Recall that A_0, A_h, as given by (12), depend on the discrete state $m \in \{0, 1\}$.

(Delay-dependent stability conditions) Consider the reset-and-hold control system Σ^Δ given by (14)*. The set $\mathcal{W}$ is asymptotically stable for Σ^Δ if there exist matrices $P > 0$, $Q > 0$, $X = X^\top$, Y and $Z > 0$ such that the following conditions are satisfied*:

1)
$$\Gamma(m, m') \leq -\varepsilon I, \tag{17}$$
for all combinations of $m, m' \in \{0, 1\}$ and for some $\varepsilon > 0$, where $\Gamma(m, m')$ is given by (16)*, as shown at the bottom of the next page, and*

2)
$$\begin{pmatrix} X & Y \\ Y^\top & Z \end{pmatrix} \geq 0, \tag{18}$$
$$Y(A_R - I) = 0, \tag{19}$$
$$A_R P A_R - P \leq 0. \tag{20}$$

The proof is based on stability results in [20] and [15], and is only sketched here for the sake of brevity. For a solution $\phi = (\mathbf{x}_\phi, \mathbf{s}_\phi)$ to Σ^Δ, consider the Lyapunov-Krasovskii functional

$$V(\phi) = \mathbf{x}_\phi(0,0)^\top P \mathbf{x}_\phi(0,0) + \int_{-h}^{0} \mathbf{x}_\phi(t, u_t)^\top Q \mathbf{x}_\phi(t, u_t) dt$$

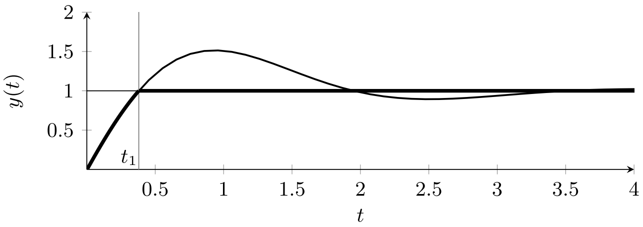


FIGURE 2. Example of a closed-loop flat step response y under a reference change: (thin line) LTI control system, (thick line) reset control system.

$$+\int_{-h}^{0}\int_{t}^{0} f(\phi)(t')^\top Z f(\phi)(t')dt'dt, \qquad (21)$$

where u_t is the value maximizing the norm $|\mathbf{x}_\phi(t,j)|$ among the j such that $(t,j) \in \text{dom}\,\phi$ (this choice is related to the definition of upper right hand derivative), and for any $t \in (-h, 0)$ we define the functional $f(\phi)(t) = A_0(m_\phi(t,u_t))\mathbf{x}_\phi(t,u_t) + A_h(m_\phi(t-h,u_{t-h}))\mathbf{x}_\phi(t-h,u_{t-h})$ (note the difference with $\mathcal{F}(\phi)$).

Note that (15a) easily follows from the fact that V, as given by (21), is positive definite and radially unbounded. Computing the upper right hand derivative of V and using Leibniz's rule, one obtains an expression consisting of a term depending on the states $x_\phi(0,0)$ and $x_\phi(-h,u_{-h})$ plus an integral term. Following [20], Lemma 1 of [21] is applied to replace this integral with another expression in terms of $x_\phi(0,0)$ and $x_\phi(-h,u_{-h})$. This step introduces new matrices X, Y satisfying (18), (19) in the stability conditions. Imposing condition (15b) results in the inequalities (17)–(19), by considering all four possible values for $m(0,0)$ and $m(-h,u_{-h})$. Finally, a straightforward application of (15c) results in the final condition (20).

V. DESIGN OF RESET-AND HOLD PI+CI CONTROLLERS FOR FOPDT SYSTEMS

Here, the design of reset-and-hold controllers for FOPDT (first order plus dead time) systems is investigated. A FOPDT system with state x_p is given by (8), where

$$A_p = -a, \quad B_p = b, \ C_p = 1. \qquad (22)$$

The focus will be on controllers based on the PI+CI (see section II.B.2), and the development of tuning rules, with the aim of obtaining, besides a well-posed and stable closed-loop hybrid system, an improved performance (*flat response*) with respect to LTI control. Both tracking of step references and rejection of step disturbances are considered. The term "flat response" means that the error signal is ideally zeroed out (becomes identically zero) after the first reset instant (Fig. 2). It is known that a PI+CI controller is able to produce a flat response for first order systems without delay [17]; this result was recently extended to MISO plants in [22]. It will be shown that a flat response can also be attained for FOPDT systems (to a very good approximation) using a well-tuned reset-and-hold controller.

The reset-and-hold PI+CI controller with state $(x_I, x_{CI}, q, m, \tau)$ is given by (5), with the controller parameters given in section II.B.2. Note that the design parameters are k_P and T_I (corresponding to the base PI controller), and p_r, τ_H, and θ corresponding to the reset-and-hold strategy. It is assumed that the base PI controller, that is, the parameters k_P and T_I, are designed to produce a fast oscillatory response (note that an oscillatory response occurs whenever the base linear control system has a pair of complex poles in the frequency domain; in terms of the parameters of the plant and controller, this happens whenever the inequality $T_I < 4bk_P/(bk_P+a)^2$ is satisfied. The base controller can be designed in the usual way using any common tuning method, taking into account this constraint). The role of the reset-and-hold strategy will be to reduce the overshoot as much as possible to obtain a flat response without decreasing the initial speed of the response. In the following, the tuning strategy for the parameters p_r, τ_H, and θ is detailed.

A. REFERENCE TRACKING

Consider a step reference change r of amplitude w_{10}, and assume that the error signal crosses zero at the instant $t = t_c$ and the first reset action is produced at $t = t_1$. By direct substitution in (14), the value of $x_p(t,j)$ for $(t,j) \in [0, t_c) \times \{0\}$ is simply obtained from

$$\begin{aligned}\dot{x}_p(t,j) &\\ &= -ax_p(t,j) + bk_P\Big(e(t-h,j)\\ &\quad + \frac{1}{T_I}\big((1-p_r)x_I(t-h,j) + p_r x_{CI}(t-h,j)\big)\Big) \qquad (23)\end{aligned}$$

and

$$x_I(t,j) = x_{CI}(t,j) = \int_0^t e(t',j)\,dt' := x_I(t). \qquad (24)$$

The first design choice is to use a reset band equal to the delay, that is to make $\theta = h$. In this way, the reset action will occur at $t_1 \approx t_c - h$, where a first order approximation $e(t+\delta,j) \approx e(t,j) + \delta\dot{e}(t,j)$, for $t \in [t_c - h, t_c]$ has been used. As a result, it is obtained that $x_I(t_1,1) = x_I(t_c - h, 1) = x_I(t_1)$ and $x_{CI}(t_1,1) = 0$, and directly from (23)–(24) that

$$\dot{x}_p(t_c,1) = -ax_p(t_c,1) + b\frac{k_P}{T_I}(1-p_r)x_I(t_1). \qquad (25)$$

Here, note that the proportional part is zeroed out after a jump under the reset-and-hold strategy.

$$\Gamma(m,m') = \begin{pmatrix} A_0(m)P + PA_0(m)^\top + Y + Y^\top + hA_0(m)^\top ZA_0(m) + Q & PA_h(m')^\top - Y + hA_0(m)^\top ZA_h(m') \\ A_h(m')^\top P - Y^\top + hA_h(m')^\top ZA_0(m) & -Q + hA_h(m')^\top ZA_h(m') \end{pmatrix} \qquad (16)$$

The second design choice is to make $\tau_H = h$. This choice prevents the controller from reacting to spurious input values $e(t, 1)$, for $t \in [t_1, t_c]$, by forcing the controller output to hold its value until the plant has reacted to the effect of resetting. It is then clear that a flat response will be achieved if $\dot{x}_p(t_c, 1) = 0$, since in that way the system reaches a steady state (as the time derivatives $\dot{x}_I(t, 1) = \dot{x}_{CI}(t, 1) = e(t, 1) = 0$ for $t > t_c$).

Finally, the parameter p_r is tuned by making $\dot{x}_p(t_c, 1) = 0$ in (25); the result is

$$p_r = 1 - \frac{aT_I w_{10}}{bk_P x_I(t_1)}, \tag{26}$$

where the fact that $x_p(t_c, 1) = w_{10}$ has been used. At first glance, this tuning rule is formally identical to the corresponding tuning rule for first order systems *without* time delay [17]. However, the underlying approach is very different, since a flat response with traditional resetting is in general unobtainable in the presence of delays.

In summary, the resulting tuning rules for the reference tracking case are

$$(p_r, \theta, \tau_H) = (1 - \frac{aT_I w_{10}}{bk_P x_I(t_1)}, h, h). \tag{27}$$

B. DISTURBANCE REJECTION

Now consider a step disturbance of amplitude w_{20}. In this case, using a similar reasoning to the above section it is obtained that

$$\begin{aligned}\dot{x}_p(t, j) = -ax_p(t, j) + bk_P\Big(e(t-h, j) \\ + \frac{1}{T_I}((1-p_r)x_I(t-h, j) + p_r x_{CI}(t-h, j))\Big) \\ +bd(t-h, j).\end{aligned} \tag{28}$$

Again, we take $\theta = \tau_H = h$. Thus, after the controller jump at $t = t_1 \approx t_c - h$, we have $x_I(t_1, 1) = x_I(t_c - h, 0) = x_I(t_1)$ and $x_{CI}(t_1, 1) = 0$, and these values will be kept constant during an interval of τ_H units of time. Right after the holding time interval, the flow equation reduces to

$$\dot{x}_p(t_c, 1) = -ax_p(t_c, 1) + b\frac{k_P}{T_I}(1-p_r)x_I(t_1) + bw_{20}. \tag{29}$$

Imposing again $\dot{x}_p(t_c, 1) = 0$, the result is now

$$p_r = 1 + \frac{T_I w_{20}}{k_P x_I(t_1)}. \tag{30}$$

where the fact that $x_p(t_1 + h, 1) = 0$ in the disturbance rejection case has been used. In summary, the obtained tuning rules for the disturbance rejection case are

$$(p_r, \theta, \tau_H) = (1 + \frac{T_I w_{20}}{k_P x_I(t_1)}, h, h). \tag{31}$$

C. GUIDELINES FOR PARAMETER TUNING

Note that although in (26) and (30) the amplitudes w_{10} and w_{20} appear explicitly, these values are cancelled since they are also a factor of $x_I(t_1)$ due to the linearity of the base system. As a result, the proposed values for p_r are constant and intrinsic to the hybrid control system, that is, they depend only on the plant and the base PI controller. However, since an explicit computation is $x_I(t_1)$ is hard to obtain, in practice the value of p_r may be simply computed by using the value of the integrator state at the first reset instant.

The reset-and-hold strategy is also applicable in cases where the delay h is not known precisely, but has some associated degree of uncertainty. If it is known that $h \in [h_{\min}, h_{\max}]$, then a simple conservative approach is to take $\tau_H = \theta = h_{\min}$ (and the same reset ratios than in (26) or (30)); this maximizes the time the controller spends in flowing mode, producing a flat response only in the best case, but improving the performance in all cases with respect to more standard reset approaches.

On the other hand, the proposed reset-and-hold strategy may not be suitable in those cases in which the appearance of several disturbances or reference changes in a short time span, lower than the delay h, is expected. This is because the controller outputs are held constant during a fixed time interval, so they will not be able to reject any incoming disturbances until after this interval has passed. In such cases, an additional supervisory mechanism is needed so that holding can be disabled whenever a new disturbance or reference change is detected.

VI. CASE STUDY: CONTROL OF A HEAT EXCHANGER

To demonstrate the capabilities of the proposed tuning rule, a simulated example, consisting of a delayed first-order process model of a heat exchanger in an experimental food processing pilot plant, is considered (see Section 6.1 of [17] for a detailed description). For a given operation point, the plant is given by the transfer function

$$P(s) = 0.49\frac{e^{-139s}}{1 + 106s}, \tag{32}$$

In the following, a PI+CI reset-and-hold controller (as given by (5) with the data of Section II.B.2) will be designed. The base PI controller parameters have been tuned to produce an oscillatory base closed-loop response. The chosen parameters are

$$k_P = 1.3, \quad T_I = 118. \tag{33}$$

Well-posedness of the closed-loop hybrid control system directly follows (see Section IV.A). Two design cases are considered: tracking step references, and rejecting step disturbances. Both sections V.A and V.B will be closely followed.

A. REFERENCE TRACKING

A step reference change of amplitude $w_{10} = 2$ starting at time $t = 30$ is considered. The tuning rule (26) has then been used to determine the reset ratio. Both the instant

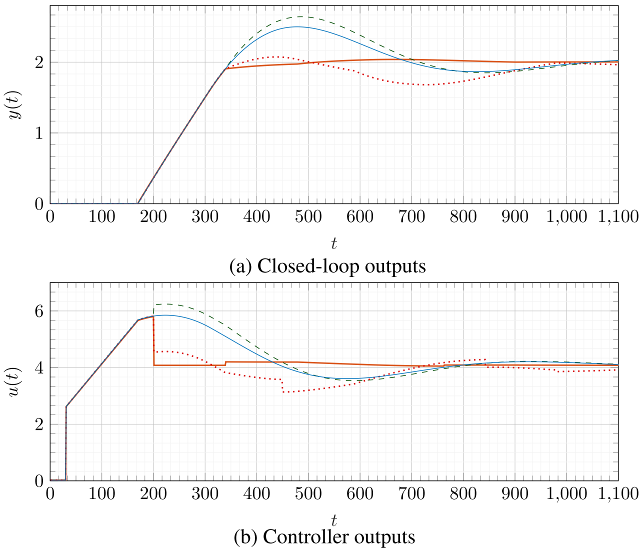


FIGURE 3. Closed-loop step response output y and controller output u for different controllers: (solid/blue) base PI controller; (dashed/green) PI+CI reset controller with $p_r = -0.107$; (dotted/red) PI+CI reset controller with $p_r = 0.350$; (solid/orange) PI+CI reset-and-hold controller.

$t_1 = 199.14$ and the corresponding integrator state $x_I(t_1)$ have been obtained by simulating the step response of the closed-loop base control system. The resulting PI+CI reset-and-hold controller parameters are

$$p_r = -0.107, \quad \theta = \tau_H = 139. \tag{34}$$

Note that a negative value for p_r indicates that the controller output will increase its magnitude after a reset action. Closed-loop stability is analyzed by checking (17)–(20) for the designed p_r value (34) (see Section IV.B), and a solution has been found for the matrices (P, Q, X, Y, Z). As a result, the set $\mathcal{W}$ is asymptotically stable for the hybrid control system.

Fig. 3a shows the step response of the closed-loop hybrid system, together with the response of the base linear system. The controller output of the controller is shown in Fig. 3b. For the purposes of comparison, a PI+CI reset controller is also considered; two different cases will be compared: (i) a case with the same parameters p_r and θ of the reset-and-hold controller, as given by (27), and (ii) a case with value $p_r = 0.350$ (and $\theta = 139$).

As can be observed, in the first PI+CI reset controller case, the response is actually worse than the base linear control system response. This fact is not surprising, since the base linear controller has been specifically designed for a good performance of the reset-and hold controller. The second PI+CI reset controller case corresponds to a better design, and has been obtained by properly tuning the parameter p_r; note that some balance between the overshoot and undershoot of the step response must be attained, due to the fact that some improvement in the overshoot is necessarily paired to an increase in the undershoot, and vice versa. Finally, the PI+CI reset-and-hold controller breaks that overshoot/undershoot balance, by both zeroing out the proportional part and forcing the controller to hold its output constant during a time interval equal to the plant delay. The result is an (almost) flat response as desired.

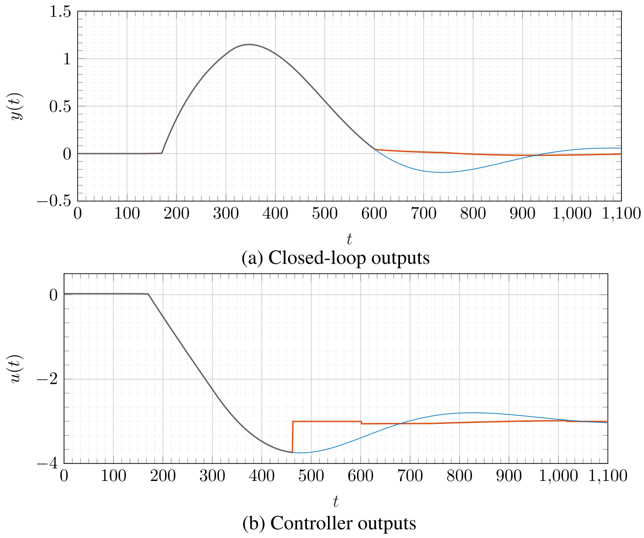


FIGURE 4. Closed-loop output y and controller output u under a step disturbance: (blue) base PI controller; (orange) PI+CI reset-and-hold controller.

B. DISTURBANCE REJECTION

The PI+CI reset-and-hold controller is designed for rejecting a step disturbance of amplitude $w_{20} = 3$, also starting at time $t = 30$. The base PI controller parameters (33) are used. Here, the tuning rule (30) is used to determine the controller's parameter p_r, while both the reset band θ and the time interval τ_H are also set to the plant delay value. As a result,

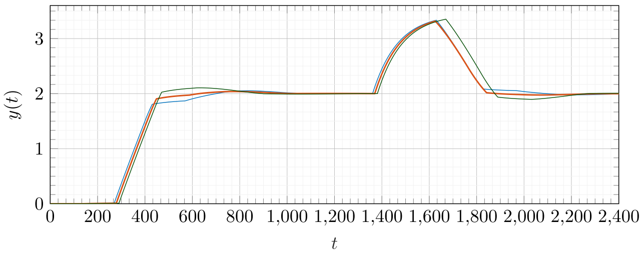
$$p_r = -0.094, \quad \theta = \tau_H = 139. \tag{35}$$

Note that the designed controller parameters are only slightly different to the reference tracking parameters given by (34). Closed-loop stability has been again checked by solving (17)–(20) for some matrices (P, Q, X, Y, Z) and the parameters (31). Fig. 4a shows the closed-loop response with the designed PI+CI reset-and-hold controller, and the with the base linear controller. The corresponding controller outputs are shown in Fig. 4b. As expected, the reset-and-hold controller produces an (almost) flat response after rejecting the step disturbance (note that the step disturbance has an amplitude $w_{20} = 3$).

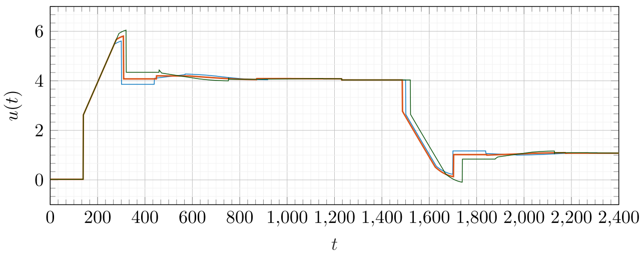
C. ROBUSTNESS ANALYSIS

In this Section, the robustness of the proposed hybrid control system approach with respect to noise and parameter variations is analyzed; two simulations are performed where a combined case of reference tracking and disturbance rejection is considered, and with

- three different values of the delay: $h_- = 130$, $h_0 = 139$ and $h_+ = 150$ (Fig. 5).

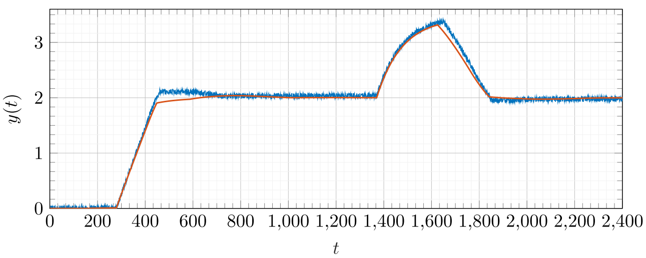


(a) Closed-loop outputs

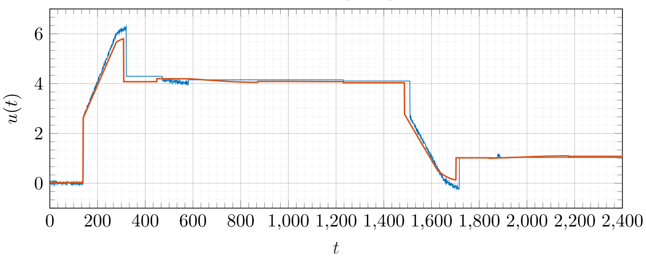


(b) Controller outputs

FIGURE 5. Robustness against delay uncertainty: (orange) nominal delay $h_0 = 139$, (blue) $h_- = 130$, and (green) $h_+ = 150$.



(a) Closed-loop outputs



(b) Controller outputs

FIGURE 6. Robustness against sensor noise: (orange) noiseless case, (blue) noisy case.

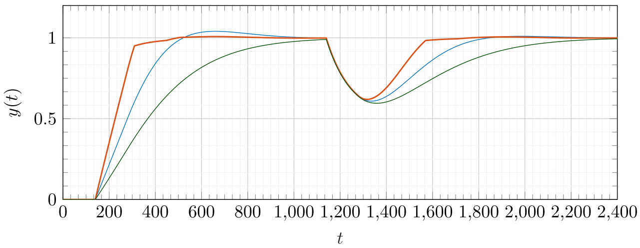
- a pseudo-random sensor noise of amplitude 0.05, and the nominal value $h_0 = 139$ (Fig. 6).

In both cases, the step reference starts at time $t = 130$ and the step disturbance at $t = 1360$. The amplitudes and controller parameters are the same as in both previous analyses, with the reset ratio p_r being changed from (34) to (35) at time $t = 1230$.

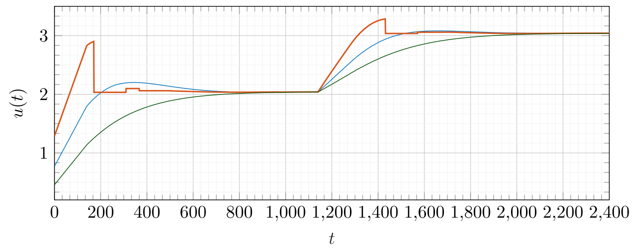
Fig. 5a and Fig. 6a clearly show that the performance of the response is not degraded too much in any case, revealing that the designed closed-loop hybrid system with the PI+CI reset-and-hold controller is robust both to small variations in the delay and to sensor noise.

D. COMPARISON WITH OTHER STRATEGIES

Note that the base PI controller underlying a PI+CI is not necessarily well-tuned, since overshoot is neglected in its design.



(a) Closed-loop outputs



(b) Controller outputs

FIGURE 7. Closed-loop output *y* and controller output *u* under a combined reference change and step disturbance: (blue) PI controller tuned using SIMC; (green) PI controller tuned using AMIGO; (orange) PI+CI reset-and-hold controller.

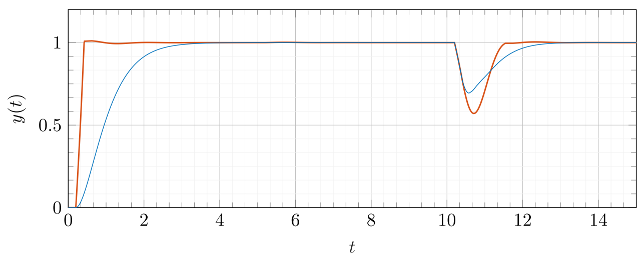


FIGURE 8. Closed-loop step response output *y* for Example 1 in Kumar *et al.* [25]: (orange) PI+CI reset and hold controller; (blue) control strategy in [25].

Thus, to demonstrate the effectiveness of the PI+CI reset-and-hold strategy with respect to linear PI compensation, a comparison will be made to two common tuning methods for PI controllers applicable to FOPDT plants: Skögestad Internal Model Control (SIMC) [23] with the closed loop time constant $\tau_c = h$, and Approximate M_s Constrained Gain Optimization (AMIGO) [24].

The combination of a unit step reference change at $t = 0$ and a negative unit step disturbance at $t = 1100$ is considered. Again the amplitudes and controller parameters for the PI+CI are the same as in the previous cases, with p_r being changed from (34) to (35) at time $t = 1090$. The SIMC rule results in the parameters

$$k_P = 0.778, \quad T_I = 106,$$

for the PI, while the AMIGO method results in

$$k_P = 0.462, \quad T_I = 94.7.$$

The results are shown in Figure 7 and Table 1. As expected, the PI+CI controller achieves better performance indices than its linear counterparts.

TABLE 1. Integrated squared error (ISE), integrated absolute error (IAE), integrated time absolute error (ITAE), and maximum overshoot percentage in reference tracking and disturbance rejection for the comparison example.

	SIMC	MIGO	Reset-&-Hold
IAE (ref.)	301.0	416.7	232.2
ISE (ref.)	234.3	299.8	196.4
ITAE (ref.)	5.498×10^4	11.02×10^4	2.996×10^4
Overshoot (ref.)	4.0%	0.0%	0.7%
IAE (dist.)	142.1	206.0	104.1
ISE (dist.)	40.33	57.98	29.42
ITAE (dist.)	4.386×10^4	8.369×10^4	2.606×10^4
Overshoot (dist.)	39.1%	41.5%	38.1%

TABLE 2. Integrated squared error (ISE), integrated absolute error (IAE), integrated time absolute error (ITAE), and maximum overshoot percentage in reference tracking and disturbance rejection for the comparison example.

	Kumar *et al.*	Reset-and-Hold
IAE (ref.)	0.886	0.326
ISE (ref.)	0.558	0.280
ITAE (ref.)	0.584	0.070
Overshoot (ref.)	0%	1%
IAE (dist.)	0.298	0.308
ISE (dist.)	0.060	0.097
ITAE (dist.)	0.289	0.244
Overshoot (dist.)	30.5%	43.9%

To further showcase the possibilities of the proposed strategy, a comparison is made with a more advanced control method appearing in recent literature. Specifically, we focus on a linear control strategy [25] for plants with delay, based on a modified Smith predictor.

Note that the control strategy in [25] is designed for unstable first order plants with delay (UFOPDT). However, the previous results for the design rules do not make any assumption on the stability of the plant and can be applied in this case, provided the reset-and-hold strategy is slightly modified so that resetting and holding is disabled when the error satisfies $|e(t,j)| \leq \varepsilon$ for some small ε. This modification is made because the output of an unstable plant will in general diverge when the input is constant (holding mode), implying that the original strategy would produce a seesaw-like response in the output instead of converging to the expected value.

Example 1 of [25] is considered (nominal case). The plant's transfer function is

$$P(s) = \frac{e^{-0.2\,s}}{s-1}.$$

First, the parameters for the base PI controller are manually chosen to obtain a fast oscillating base closed-loop response:

$$k_P = 3.75, \quad T_I = 1.25.$$

Next, applying the design rules (26), (30) results in the values

$$\theta = \tau_H = 0.2, \quad p_{r,\mathrm{ref}} = 2.505,\ p_{r,\mathrm{dist}} = -0.12.$$

The closed loop response to a unit step reference change at $t = 0$ plus a negative unit step disturbance at $t = 10$ has been simulated. The results of the comparison are shown in Figure 8 and Table 2. As can be seen, the performance in all metrics is greatly improved in reference tracking without causing any significant overshoot. In contrast, the overshoot in disturbance rejection is degraded, but the settling time is improved, in such a way that most other metrics remain of similar magnitude. Note that the strategy in [25] deals with reference tracking and disturbance rejection using a combination of controllers, where as our proposed setup utilizes only one controller. It is possible that a similar setup using combined PI+CI controllers with the reset-and-hold strategy would achieve a better handling of disturbances; however, this is out of the scope of the current work.

VII. CONCLUSION

A new hybrid controller for systems with time delays is proposed, consisting of a combination of reset and hold strategies. This reset-and-hold controller has been analyzed in detail in the framework of Hybrid Inclusions, equipping the resulting closed-loop hybrid system with good structural properties. Besides well-posedness, which has been shown to be guaranteed for any LTI plant with time delays, stability conditions have been developed. Moreover, for the specific case of a PI+CI reset-and-hold controller and a FOPDT system, a set of design rules have been proposed. An (almost) closed-loop flat response is obtained both in step tracking and in rejecting step disturbances, notably improving the performance of PI+CI reset controllers.

The developments in this work apply only to FOPDT systems. A possible idea for extending the current strategy to deal with more general time-delayed processes, such as second order plus dead time (SOPDT) or integrating first order plus dead time (IFOPDT), is to combine the reset-and-hold strategy with the on-line PI+CI tuning method for second order plants from [19]. In this way, the parameters $p_r(t_k)$, and possibly $\theta(t_k)$, $\tau_H(t_k)$, would become functions of the kth reset instant, computed on-line using a simple quadratic optimization algorithm. This possibility will be explored in future work.

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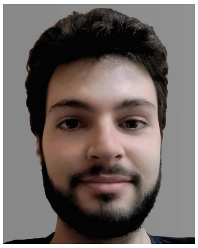

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