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RESEARCH ARTICLE

Finite-Time H_∞ Robust Controller Design for a Class of Singular Discrete-Time Markov Jump Delay Systems With Packet Loss Compensation and Input Saturation

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ABSTRACT This paper is concerned with the finite-time H_∞ robust control problem for a class of discrete-time Markov jump systems with time-varying delays and random packet losses. The phenomenon of packet losses occurs between the plant and the controller, which is characterized by introducing a random variable. Based on the single exponential smoothing method, the prediction of the missing measurement is used as the packet loss compensation when a packet is lost. Then, by employing local sector conditions and an appropriate Lyapunov function, a state feedback controller is designed to guarantee that the resulted closed-loop constrained system is mean-square locally finite-time stabilizable. Furthermore, some sufficient conditions for the solution to this problem are derived in terms of linear matrix inequalities. Finally, two numerical examples are provided to demonstrate the effectiveness of the proposed method.

INDEX TERMS Partially known transition rates, input saturation, singular systems, time-varying delay, packet loss compensation.

I. INTRODUCTION

In many practical systems, such as chemistry, economy, aerospace, etc., due to the influence of various internal and external factors, the system structure and parameters will mutate, and this can be properly described by the Markov jump system model. At the same time, in the actual system, such phenomena as parameter perturbation, actuator saturation, communication delay and communication failure often occur, and these phenomena will bring great difficulties to the design of system control scheme. Therefore, it is very meaningful to study the relevant control problems of Markov jump system with the above practical factors. According to the literature, many outstanding results have been obtained (see [1], [2], [3], [4], [5], [6], [7], [8], [9], [10] and the references therein). Compared with nonsingular systems, the study of singular Markov systems is more general. Due to the existence of singular system matrix, in order to ensure the existence and uniqueness of the solution, it is necessary to ensure the regularity and causality of the system, which adds some difficulties to the solution of related problems. In recent years, lots of attentions have been attracted on singular Markov systems and many results have been proposed, such as stability analysis [11], filtering design [12], controller design [13], [14], [15], [16], [17], [18], [19]. It can be seen from the above literature that the control problem of singular systems is challenging, and the results obtained are generally conservative, especially considering various practical factors. How to deal with these challenging problems while reducing the conservatism of the results is one of the motivations of this paper.

It is worth to note that, the above results are obtained in the sense of infinite time stability. However, in practical engineering systems, it is more valuable to consider the related problems of finite-time control which has attracted the attention of many scholars and lots of results have been reported

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in [20], [21], [22], [23], and [24]. For instance, by the state undecomposed method, the finite-time stable and finite-time control problems of affine nonlinear singular systems subject to actuator saturation was discussed in [25]. The finite-time H_∞ output tracking control problem was addressed for the networked switched systems and a hybrid event-triggered scheme was introduced to reduce the network transmission overload in [26]. On the other hand, packet loss is inevitable in network communication. In the case of packet losses, the control input of the system may not be updated in time that results in the system performance degradation or even instability. Therefore, it is necessary to consider the data packet loss in the research of the above problems. Up to now, there have been numerous works related to this issue [27], [28], [29], [30], [31], [32], [33]. For instance, the dissipativity-based filtering problem for a class of discrete-time Markov jump systems with mode-dependent time-varying delays and random packet losses was studied in [34]. In [35], the optimal output feedback control problem for discrete-time Markov jump linear system with input delay and packet losses in finite horizon was considered. According to the literature, the finite-time control of singular systems is well studied. However, the results mainly focus on the filter design, fuzzy control, output-feedback controller design, and the state feedback controller design is relatively small. Considering the practical factors such as data packet loss, the design of controller has important theoretical significance and application background.

Summarizing the above discussions, this paper is aimed at the finite-time H_∞ controller design for singular discrete-time Markov jump systems with time-varying delay and input saturation, where random packet losses which happens between plant and controller are taken into account. **The main contributions of this paper are twofold**.

i)Compared with [14], various practical factors, such as packet losses and input saturation are considered in this paper and the proposed method is more general.

ii)Compared with [14], by design appropriate Lyapunov-Krasovskii function, the delay-depended result is derived to reduce conservatism in this paper.

Notations. R^n denotes the n-dimensional Euclidean space, and $\varepsilon\{\cdot\}$ is the mathematical expectation. $P > 0$ indicates that P is symmetric and positive-definite, while $P < 0$ implies that P is a symmetric and negative-definite matrix. $Diag\{\cdot\}$ and I represent, respectively, a block-diagonal matrix and an identity matrix with appropriate dimensions. Besides, $*$ refers to symmetry elements of the matrix. $Prob\{\cdot\}$ means the probability. $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ represent, respectively, the minimum and maximum eigenvalue of matrix.

II. MODEL DESCRIPTIONS AND PRELIMINARIES

Consider the following stochastic Markov jump systems (Σ) in the probability space $(\Omega, \mathcal{F}, \mathcal{P})$:

$$\begin{aligned} E(r(k))x(k+1) = {} & (A(r(k)) + \Delta A(r(k)))x(k) \\ & +(A_d(r(k)) + \Delta A_d(r(k)))x(k-d(k)) \\ & +(B(r(k)) + \Delta B(r(k)))sat(u(k)) \\ & +(D(r(k)) + \Delta D(r(k)))w(k), \end{aligned} \tag{1}$$

$$\begin{aligned} z(k) = {} & (C_1(r(k)) + \Delta C_1(r(k)))x(k) \\ & +(C_2(r(k)) + \Delta C_2(r(k)))x(k-d(k)) \\ & +(C_3(r(k)) + \Delta C_3(r(k)))u(k), x(j) \\ & = \eta(j),\ j = -d_M,\ -d_M \\ & +1, \cdots,\ -1,\ 0. \end{aligned} \tag{2}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $z(k) \in \mathbb{R}^q$ is the controlled output, $u(k) \in \mathbb{R}^m$ is the input. $w(k)$ is the external disturbances, $\eta(j)$ are the initial conditions. The positive integer $d(k)$ denotes the time-varying delay satisfying:

$$d_m \le d(k) \le d_M, k \in \mathbf{N}^+, \tag{3}$$

where d_m and d_M are the known positive integers. The parameter $\{r(k)\}$ is a discrete-time Markovian process with right continuous trajectories and taking values from a finite set $S = \{1, 2, \ldots, \mathcal{N}\}$ with transition probabilities given by:

$$\Pr\{r(k+1) = j\,|r(k) = i\} = \pi_{ij}$$

where $\pi_{ij} \ge 0$,and for any $i \in S$

$$\sum_{j=1}^{s} \pi_{ij} = 1. \tag{4}$$

In this paper, the transition rates of the Markov jumping process are partly known. For example, the transition rates matrix is given as the follows:

$$Pr = \begin{bmatrix} \pi_{11} & ? & \pi_{13} & \cdots & \pi_{1n} \\ \pi_{21} & ? & \pi_{23} & \cdots & ? \\ \vdots & \vdots & ? & \ddots & \vdots \\ \pi_{n1} & & \pi_{n3} & \ldots & ? \end{bmatrix}$$

where "?" is the unknown part of the transition rates. For notational clarity, $\forall i S$, the set S^i denotes:

$$S^i = S_k^i \bigcup S_{uk}^i$$

with

$$\begin{aligned} S_k^i &\doteq \{j : \pi_{ij}\ is\ known\ for\ j \in S\}, \\ S_{uk}^i &\doteq \{j : \pi_{ij}\ is\ unknown\ for\ j \in S\}. \end{aligned}$$

This paper supposes that the input of the considered systems is bounded as follows:

$$-u_{0(i)} \le u_{(i)} \le u_{0(i)},\ u_{0(i)} > 0,\ i = 1, \cdots,\ m. \tag{5}$$

For the system matrix, we denote $A_i = A(r(t))$ for each $r(t) = i \in S$, and the other symbols are similarly denoted as $A_i, A_{di}, B_i, D_i, C_{1i}, C_{2i}, C_{3i}$ which are known mode-dependent constant matrices with appropriate dimensions. E_i is a singular constant matrix. $\Delta A_i = MFN_{1i}$, $\Delta A_{di} = MFN_{2i}$, $\Delta B_i = MFN_{3i}$, $\Delta D_i = MFN_{4i}$, $\Delta C_{1i} = MFN_{5i}$, $\Delta C_{2i} = MFN_{6i}$, $\Delta C_{3i} = MFN_{7i}$ are unknown matrices representing norm-bounded parameter uncertainties, and M and N_i are known real constant matrices with appropriate dimensions. The uncertain matrices F satisfies

$$F^T F \leq I. \tag{6}$$

Assumption 1: The varying disturbance $w(k)$ of the considered systems is supposed as the follows:

$$w(k)^T w(k) \leq d, \; d \geq 0. \tag{7}$$

In this paper, we attempt to design a state feedback controller. However, unfortunately, affected by unreliable networks, some state data packets may not be successfully transmitted to the controller. Thus, the single exponential smoothing(SES) method is used to predict x(k). The **forecasting model** is built as

$$\bar{\mathbf{x}}(k+1) = \alpha \mathbf{x}(k) + (1-\alpha)\bar{\mathbf{x}}(k). \tag{8}$$

where $\alpha \in [0, \; 1]$ is the smoothing parameter. Introduce $\beta(k)$ as an indicator function, which is described as

$$\begin{aligned} &\beta(k) = 1, \; \textit{successful transmition};\\ &\beta(k) = 0, \; \textit{otherwise}. \end{aligned} \tag{9}$$

Besides, $\beta(k)$ obeys Bernoulli distribution with

$$Prob\{\beta(k) = 1\} = \varepsilon\{\beta(k) = 1\} = \beta \in [0, \; 1].$$

Then we have the controller based on **hidden Markov mode** as the follows:

$$\begin{aligned} \mathbf{u}(k) = \; &\mathbf{k}_{1\sigma(t)}\{\beta(k)(\alpha\mathbf{x}(k) + (1-\alpha)\bar{\mathbf{x}}(k))\}\\ &+\mathbf{k}_{2\sigma(t)}\{(1-\beta(k))\bar{\mathbf{x}}(k)\} \end{aligned}$$

where $k_{\sigma(t)} \in R^{m\times n}$ and with the emission probability defined as follow:

$$Pr(\sigma(k) = p|r(k) = i) = \lambda_{ip}, \; \sum_{p=1}^{M} \lambda_{ip} = 1. \tag{10}$$

Define $\psi(u(k)) = sat(u(k)) - u(k)$, $\xi(k) = [x(k)^T \; \bar{x(k)}^T]^T$, $\bar{\xi}(k) = [\xi^T(k) \; \xi^T(k-d(k)) \; \psi^T(u(k)) \; w^T(k)]^T$. Then, we have the resulted closed-loop systems as the follows:

$$\begin{aligned} \bar{E}\xi\,(k+1) &= \Pi\bar{\xi}(k) + \bar{\Delta}\bar{\xi}(k),\\ z(k) &= \bar{C}_i\bar{\xi}(k) + \Delta\bar{C}_i\bar{\xi}(k), \end{aligned} \tag{11}$$

where

$$\begin{aligned} \Pi^T &= \begin{bmatrix} \bar{A}_i^T + \bar{K}_p^T\bar{B}_i^T \\ \bar{A}_{di}^T \\ \bar{B}_i^T \\ \bar{D}_i^T \end{bmatrix},\\ \bar{\Delta}^T &= \begin{bmatrix} \bar{N}_{1i}^T + \bar{K}_p^T\bar{N}_{3i}^T \\ \bar{N}_{2i}^T \\ \bar{N}_{3i}^T \\ \bar{N}_{4i}^T \end{bmatrix} F^T M^T,\\ \Delta\bar{C}_i &= MF\left[\bar{N}_5 + N_{7i}\bar{K}_p \; \bar{N}_6\right],\\ \bar{C}_i &= \left[\bar{C}_{1i} + C_{3i}\bar{K}_p \; \bar{C}_{2i}\right], \end{aligned}$$

with

$$\begin{aligned} \bar{A}_i^T &= \begin{bmatrix} A_i^T & \alpha I \\ 0 & (1-\alpha)I \end{bmatrix}, A_{di}^T = \begin{bmatrix} A_{di}^T & 0 \\ 0 & 0 \end{bmatrix},\\ \bar{B}_i^T &= \begin{bmatrix} B_i^T & 0 \end{bmatrix}, \; \bar{D}_i^T = \begin{bmatrix} D_i^T & 0 \end{bmatrix},\\ \bar{N}_{1i}^T &= \begin{bmatrix} N_{1i}^T & 0 \\ 0 & 0 \end{bmatrix}, \; \bar{N}_{2i}^T = \begin{bmatrix} N_{2i}^T & 0 \\ 0 & 0 \end{bmatrix},\\ \bar{N}_{3i}^T &= \begin{bmatrix} N_{3i}^T \; 0 \end{bmatrix}, \; \bar{N}_{4i}^T = \begin{bmatrix} N_{4i}^T & 0 \end{bmatrix},\\ \bar{N}_5 &= \begin{bmatrix} N_{5i} & 0 \end{bmatrix}, \; \bar{N}_6 = \begin{bmatrix} N_{6i} & \mathbf{0} \end{bmatrix},\\ \bar{C}_{1i} &= \begin{bmatrix} C_{1i} & 0 \end{bmatrix}, \; \bar{C}_{2i} = \begin{bmatrix} C_{2i} & \mathbf{0} \end{bmatrix}, \; \bar{E} = \begin{bmatrix} E_i & 0 \\ 0 & I \end{bmatrix},\\ \bar{K}_p &= \left[\beta\alpha k_{1p} \;\; \beta(1-\alpha)k_{1p} + (1-\beta)k_{2p}\right]. \end{aligned}$$

Meanwhile, it's easy to find nonsingular matrices f_i and o_i such that

$$\tilde{E} = f_i\bar{E}_i o_i = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix},$$

Make $\tilde{\xi}(k) = o_i^{-1}\xi(k)$, we can rewrite the system (11) as the follows:

$$\begin{aligned} \tilde{E}\tilde{\xi}\,(k+1) &= \tilde{\Pi}\tilde{\xi}_1(k) + \tilde{\Delta}\tilde{\xi}_1(k),\\ z(k) &= \tilde{C}_i\tilde{\xi}(k) + \Delta\tilde{C}_i\tilde{\xi}(k), \end{aligned} \tag{12}$$

where $\tilde{\xi}_1(k) = [\tilde{\xi}^T(k) \; \tilde{\xi}^T(k-d(k)) \; \psi^T(u(k)) \; w^T(k)]^T$, and

$$\begin{aligned} \tilde{\Pi}^T &= \begin{bmatrix} o_i^T\bar{A}_i^T f_i^T + o_i^T\bar{K}_p^T\bar{B}_i^T f_i^T \\ o_i^T\bar{A}_{di}^T f_i^T \\ \bar{B}_i^T f_i^T \\ \bar{D}_i^T f_i^T \end{bmatrix},\\ \tilde{\Delta}^T &= N^T F^T \bar{M}^T,\\ N^T &= \begin{bmatrix} o_i^T\bar{N}_{1i}^T f_i^T + o_i^T\bar{K}_p^T\bar{N}_{3i}^T f_i^T \\ o_i^T\bar{N}_{2i}^T f_i^T \\ \bar{N}_{3i}^T f_i^T \\ \bar{N}_{4i}^T f_i^T \end{bmatrix},\\ \Delta\tilde{C}_i &= MF\left[\bar{N}_5 o_i + N_{7i}\bar{K}_p o_i \; \bar{N}_6 o_i\right],\\ \tilde{C}_i &= \left[\bar{C}_{1i}o_i + C_{3i}\bar{K}_p o_i \; \bar{C}_{2i}o_i\right],\\ \bar{M}^T &= M^T f_i^T,\\ \tilde{A}_i^T &= o_i^T\bar{A}_i^T f_i^T + o_i^T\bar{K}_p^T\bar{B}_i^T f_i^T\\ &\quad + (o_i^T\bar{N}_{1i}^T f_i^T + o_i^T\bar{K}_p^T\bar{N}_{3i}^T f_i^T)F^T\bar{M}^T. \end{aligned}$$

Before presenting the main results, we give the following lemmas and definitions:

Lemma 1 [25]: For the system (12), the matrix $\bar{K}_p$ and the given matrix $L_i \in \mathbb{R}^{m\times n}$ with appropriate dimension, if $\tilde{\xi}(k)$ is in the set $D(u_o)$, where $D(u_o)$ is defined as follows:

$$\begin{aligned} D(u_o) = \{&\tilde{\xi}(k) \in \mathbb{R}^n; \; -u_{0(k)} \leq (\bar{K}_{p(k)} + L_{i(k)})\tilde{\xi}(k)\\ &\leq u_{0(k)}, \; u_{0(k)} > 0, \; k = 1, \ldots, m\}, \end{aligned}$$

then for any diagonal positive matrix $T \in \mathbb{R}^{m\times m}$, we derive:

$$\psi(u(k))^T T(\psi(u(k)) - L_i\tilde{\xi}(k)) \leq 0.$$

Lemma 2 [12]: For the given symmetric matrix $S \in \mathbb{R}^{(n+m)\times(n+m)}$

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix},$$

where $S_{11} \in \mathbb{R}^{n\times n}$, $S_{12} \in \mathbb{R}^{n\times m}$, $S_{22} \in \mathbb{R}^{m\times m}$, the following conditions are equivalent:

1) $S < 0$.
2) $S_{11} < 0,\ S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$.
3) $S_{22} < 0,\ S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 3 [22]: Let X and Y be any given real matrices of appropriate dimensions. Then, for any scalar $\epsilon > 0$,

$$X^T Y + Y^T X \le \epsilon^{-1} X^T X + \epsilon Y^T Y$$

Definition 1 [20]: For the given constant integer $N > 0$, positive scalar ($c_1,\ c_2$, with $c_1 < c_2$, and mode-dependent positive matrix $\hat{R}_i > 0$, the resulting closed-loop systems (12) is said to be stochastically finite-time bounded stable with respect to ($c_1\ c_2\ N\ \hat{R}_i\ d$), if the following relation holds

$$\begin{aligned} &E\{\xi^T(k_1)E^T\hat{R}_iE\xi(k_1)\} \le c_1 \\ &\quad \Rightarrow E\{\xi^T(k_2)E^T\hat{R}_iE\xi(k_2)\} < c_2 \\ &\quad k_1 \in [-d_M\ 0],\ k_2 \in [0\ N]. \end{aligned} \tag{13}$$

Definition 2 [17]: Regular and causal.
(i) System (12) with w(k)=0 is said to be regular, if $\det\{s\tilde{E} - \tilde{A}_i\} \ne 0$ for all $k \in [0,\ N]$.
(ii) System (12) with w(k)=0 is said to be causal, if $\det\{s\tilde{E} - \tilde{A}_i\} = rank(\tilde{E})$ for all $k \in [0,\ N]$.

III. MAIN RESULTS

In this section, we investigate the design of a state feedback controller which guarantees the locally finite-time stabilizable of the resulted closed-loop system with constant time-varying delay. Some sufficient conditions and the method of designing state feedback controller are given.

Theorem 1: For $\forall\ r(k) = i \in S$ and the constant integers $N > 0,\ 0.5 > \nu > 0.\ \bar{\upsilon} > 0,\ \lambda > 0,\ \lambda_1 > 0$ which are given, the closed-loop Markov jump systems(12) with initial conditions belonging to $\varepsilon(\bar{P}_i,\ 1)$ is said to be locally stochastically finite-time bounded stabilizable with respect to ($c_1\ c_2\ N\ R_i\ d$), if there exists positive constant $\rho > 0$, $\varepsilon_1 > 0$, mode-dependent matrix $\bar{\Omega}_{ip}$, symmetric positive-define matrix J, $\bar{J}_1$, S and diagonal positive definite matrix T_i, and matrix L_i, such that

$$\begin{bmatrix} \Gamma_1 & \bar{N}^T & \Phi_{ijp}^1 & \Phi_{ijp}^2 \\ 0 & -\varepsilon_1 I & 0 & 0 \\ * & * & -\Theta_i & 0 \\ * & * & * & -\Theta_i \end{bmatrix} < 0, \tag{14}$$

$$\begin{bmatrix} \bar{\Omega}_{ip}^T\tilde{E} & \tilde{K}^T \\ * & u_{0(k)}^2 \end{bmatrix} \ge 0_{k=1\to m}, \tag{15}$$

$$\begin{bmatrix} -\bar{J}_1 & \bar{\Omega}_{ip}^T \\ * & -\frac{1}{\lambda_1}I \end{bmatrix} < 0, \tag{16}$$

$$J < \lambda_1 I, \tag{17}$$

$$\varepsilon_1 \tilde{E}^T\bar{M}\bar{M}^T\tilde{E} \le \lambda\bar{\Omega}_{ip}^T\tilde{E}^T, \tag{18}$$

$$\hat{\Omega}_{ip} > \upsilon(\tilde{E}^T\bar{\Omega}_{ip} + \bar{\Omega}_{ip}^T\tilde{E}) + \bar{\upsilon}, \tag{19}$$

$$\Upsilon + \rho\lambda_S Nd < \sigma_p c_2, \tag{20}$$

with

$$\tilde{J}_i = -\bar{\Omega}_{ip}^T\bar{E} + (d_M - d_m + 1)\bar{J}_1$$

$$\tilde{K} = \bar{K}_p o_i\bar{\Omega}_{ip} + L_i\bar{\Omega}_{ip}$$

$$\Gamma_1 = \begin{bmatrix} \tilde{J}_i & 0 & \bar{\Omega}_{ip}^T L_i^T & 0 \\ * & -J & 0 & 0 \\ * & * & -2\bar{T}_i & 0 \\ * & * & * & -\rho S \end{bmatrix},$$

$$\bar{N}^T = \begin{bmatrix} \bar{\Omega}_{ip}^T o_i^T \bar{N}_{1i}^T f_i^T + \check{K}_p^T \bar{N}_{3i}^T f_i^T \\ o_i^T \bar{N}_{2i}^T f_i^T \\ \bar{N}_{3i}^T f_i^T \\ \bar{N}_{4i}^T f_i^T \end{bmatrix},$$

$$\bar{\Pi}^T = \begin{bmatrix} \bar{\Omega}_{ip}^T o_i^T \bar{A}_i^T f_i^T + \check{K}_p^T \bar{B}_i^T f_i^T \\ o_i^T \bar{A}_{di}^T f_i^T \\ \bar{B}_i^T f_i^T \\ \bar{D}_i^T f_i^T \end{bmatrix},$$

where $\Upsilon = \sigma_P + ((d_M - 1) + (d_M + d_m - 2)\frac{(d_M - d_m + 1)}{2})\delta\lambda_J)c_1$, $\sigma_P = max_{i\in S}\sigma_{max}(\bar{P}_i)$, $\sigma_p = min_{i\in S}\sigma_{min}(\bar{P}_i)$, $\lambda_J = max_{i\in S}\sigma_{max}(\bar{J})$, $\lambda_j = min_{i\in S}\sigma_{min}(\bar{J})$, $\lambda_S = \sigma_{max}(S)$, $\bar{J} = R_i^{-1/2}JR_i^{-1/2}$, $\bar{P}_i = R_i^{-1/2}P_iR_i^{-1/2}$, $\check{K}_p = \bar{K}_p o_i\bar{\Omega}_{ip}$, $\bar{T}_i = T_i^{-1}$ with

$$\Phi_{ijp}^1 = (F_{11},\ F_{12},\ \cdots F_{1M}),$$

$$\Phi_{ijp}^2 = (F_{21},\ F_{22},\ \cdots F_{2M}),$$

$$\tilde{\Xi}_{jq} = \upsilon(\bar{E}^T\bar{\Omega}_{jq} + \bar{\Omega}_{jq}^T\bar{E}) + \bar{\upsilon},$$

$$\Xi_j = diag(\tilde{\Xi}_{j1},\ \tilde{\Xi}_{j2},\ \cdots \tilde{\Xi}_{jM}),$$

$$\Theta_i = diag(\Xi_1,\ \Xi_2,\ \cdots,\ \Xi_N),$$

$$F_{1q} = \sqrt{\pi_{ij}\lambda_{jq}}(\bar{N}^T M^T)$$

$$F_{2q} = \sqrt{\pi_{ij}\lambda_{jq}(1+\lambda)}\bar{\Pi}^T$$

$$\bar{I} = \begin{bmatrix} 0 & I \end{bmatrix},\ \tilde{I} = \begin{bmatrix} I \\ 0 \end{bmatrix},\ \Lambda = \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix},$$

$$\hat{\Omega}_{ip} = \bar{\Omega}_{ip} - \bar{I}\bar{\Omega}_{ip}\tilde{I}\Lambda,\ \bar{\Omega}_{ip} = \begin{bmatrix} \bar{\Omega}_{1ip} & 0 \\ \bar{\Omega}_{1ip} & \bar{\Omega}_{1ip} \end{bmatrix},$$

with the following controller gain $\bar{K}_p = \check{K}_p\bar{\Omega}_{ip}^{-1}o_i^{-1}$.

Proof: For each $r(k) = i \in S$, the following Lyapuonv-Krasovkii function is designed for the closed-loop system (12):

$$\begin{aligned} V(\tilde{\xi}(k), i, p) = &\tilde{\xi}(k)^T\tilde{E}^T P_{ip}\tilde{E}\tilde{\xi}(k) \\ &+ \Sigma_{m=k-d(k)}^{k-1}\tilde{\xi}(m)^T J\tilde{\xi}(m) \\ &+ \Sigma_{l=k-d_M+1}^{k-d_m}\Sigma_{m=l}^{k-1}\tilde{\xi}(m)^T J\tilde{\xi}(m) \end{aligned}$$

Denote $\Omega_{ip} = P_{ip}\tilde{E} + HW$, where $\tilde{E}^T H = 0$,, then we have $\tilde{E}^T\Omega_{ip} = \tilde{E}^T\tilde{E}^T\Omega_{ip}\tilde{E}$. Define $\tilde{P}_i = \sum_{j=1}^{N}\sum_{q=1}^{M}\pi_{ij}\lambda_{jq}\tilde{E}^T\Omega_{jq}$ It is easily obtained that

$$\mathbb{L}\Delta V(\tilde{\xi}(k), i, p) = \mathbb{L}[\tilde{\xi}^T(k+1)\tilde{E}^T\tilde{P}_i\tilde{E}\tilde{\xi}(k+1)$$

$$
\begin{aligned}
&-\tilde{\xi}^T(k)\tilde{E}^T\Omega_{ip}\tilde{\xi}(k)\\
&+(d_M-d_m+1)\tilde{\xi}^T(k)J\tilde{\xi}(k)\\
&-\tilde{\xi}^T(k-d(k))J\tilde{\xi}(k-d(k))]
\end{aligned}
$$

By using the lemma 1, it follows that

$$
\begin{aligned}
&\mathbb{L}\Delta V(x(k),i,p)\\
&\le \mathbb{L}[(\tilde{\Pi}\tilde{\xi}_1(k)+\tilde{\Delta}\tilde{\xi}_1(k))^T\tilde{P}_i(\tilde{\Pi}\tilde{\xi}_1(k)+\tilde{\Delta}\tilde{\xi}_1(k))\\
&\quad -\tilde{\xi}^T(k)\tilde{E}^T\Omega_{ip}\tilde{\xi}(k)+(d_M-d_m+1)\tilde{\xi}^T(k)J\tilde{\xi}(k)\\
&\quad -\tilde{\xi}^T(k-d(k))J\tilde{\xi}(k-d(k))]\\
&\quad -2\psi(u(k))^TT_i\psi(u(k))+He(\psi(u(k))^TT_iL_i\tilde{\xi}(k))\\
&=\tilde{\xi}_1^T(k)\tilde{\Pi}^T\tilde{P}_i\tilde{\Pi}\tilde{\xi}_1(k)+\tilde{\xi}_1^T(k)\tilde{\Delta}^T\tilde{P}_i\tilde{\Delta}\tilde{\xi}_1(k)\\
&\quad +He(\tilde{\xi}_1^T(k)\tilde{\Pi}^T\tilde{P}_i\tilde{\Delta}\tilde{\xi}_1(k))-\tilde{\xi}^T(k-d(k))J\tilde{\xi}(k-d(k))\\
&\quad -\tilde{\xi}^T(k)\tilde{E}^T\Omega_{ip}\tilde{\xi}(k)+(d_M-d_m+1)\tilde{\xi}^T(k)J\tilde{\xi}(k)\\
&\quad -2\psi(u(k))^TT_i\psi(u(k))+He(\psi(u(k))^TT_iL_i\tilde{\xi}(k))
\end{aligned}
$$

In this paper, we assume that the matrix $M\in\mathbb{R}^n$. Since that $F^TF\le I$, we have

$$
\begin{aligned}
\tilde{\xi}_1^T(k)\tilde{\Delta}^T\tilde{P}_i\tilde{\Delta}\tilde{\xi}_1(k)&=\tilde{\xi}_1^T(k)N^TF^T\bar{M}^T\tilde{P}_i\bar{M}FN\tilde{\xi}_1(k)\\
&\le \tilde{\xi}_1^T(k)N^T\bar{M}^T\tilde{P}_i\bar{M}N\tilde{\xi}_1(k)
\end{aligned}
$$

Based on lemma 3 and condition (18), one can obtained

$$
\begin{aligned}
&He(\tilde{\xi}_1^T(k)\tilde{\Pi}^T\tilde{\Pi}\tilde{\Delta}\tilde{\xi}_1(k))\\
&\quad\le \frac{1}{\varepsilon_1}\tilde{\xi}_1{}^T(k)N^TF^TFN\tilde{\xi}_1(k)\\
&\qquad +\varepsilon_1\tilde{\xi}_1{}^T(k)\tilde{\Pi}^T\tilde{P}_i\bar{M}\bar{M}^T\tilde{P}_i\tilde{\Pi}\tilde{\xi}_1\\
&\quad\le \frac{1}{\varepsilon_1}\tilde{\xi}_1{}^T(k)N^TF^TFN\tilde{\xi}_1(k)+\lambda\tilde{\xi}_1{}^T(k)\tilde{\Pi}^T\tilde{P}_i\tilde{\Pi}\tilde{\xi}_1
\end{aligned}
$$

From conditions (16)-(17), it is easy to get $\bar{\Omega}_{ip}^TJ\bar{\Omega}_{ip}<\bar{J}_1$. Consider that the condition (19) can be rewritten as the follow

$$
\begin{aligned}
&\hat{\Omega}_{jq}>\upsilon(\tilde{E}^T\bar{\Omega}_{jq}+\bar{\Omega}_{jq}^T\tilde{E})+\bar{\upsilon},\\
&\quad(\upsilon(\tilde{E}^T\bar{\Omega}_{jq}+\bar{\Omega}_{jq}^T\tilde{E})+\bar{\upsilon})^{-1}>\tilde{E}^T\hat{\Omega}_{jq}^{-1}=\tilde{E}^T\Omega_{jq}
\end{aligned}
$$

Define $\bar{\Omega}_{ip}=\Omega_{ip}^{-1}$, and pre- and post multiplying matrix inequality (14) with $diag(\Omega_{ip}^T,\ I,\ T_i,\ I,\ I)$ and $diag(\Omega_{ip},\ I,\ T_i,\ I,\ I)$, by using Schur lemma, one can obtained

$$
V(\tilde{\xi}(k+1))\le V(\tilde{\xi}(k))+\rho w(k)^TSw(k) \tag{21}
$$

From $k=0\rightarrow N$, and based on assumption 1, we derive

$$
\begin{aligned}
V(\tilde{\xi}(k))&\le V(\tilde{\xi}(0))+\rho\lambda_SNd\\
&\le \tilde{\xi}(0)^T\tilde{E}^TP_i\tilde{E}\tilde{\xi}(0)+\Sigma_{i=-d(0)}^{-1}\tilde{\xi}(i)^TJ\tilde{\xi}(i)\\
&\quad +\Sigma_{j=-d_M+1}^{-d_m}\Sigma_{i=j}^{-1}\tilde{\xi}(i)^TJ\tilde{\xi}(i)+\rho\lambda_SNd
\end{aligned}
$$

Sine that $P_i=R_i^{1/2}R_i^{-1/2}P_iR_i^{-1/2}R_i^{1/2}$,
and $\tilde{\xi}(0)^T\tilde{E}^TR_i\tilde{E}\tilde{\xi}(0)\le c_1$, it is easy to find a scalar δ which satisfy $\tilde{\xi}(0)^TR_i\tilde{\xi}(0)\le\delta c_1$, then we derive

$$
V(\tilde{\xi}(k))\le\Upsilon+\rho\lambda_SNd
$$

On the other hand, it is easy to know

$$
\begin{aligned}
V(\tilde{\xi}(k))&\ge\tilde{\xi}(k)^T\tilde{E}^TP_i\tilde{E}\tilde{\xi}(k)\\
&\ge\sigma_p\tilde{\xi}(k)^T\tilde{E}^TR_i\tilde{E}\tilde{\xi}(k)
\end{aligned} \tag{22}
$$

Then, we have

$$
\begin{aligned}
\tilde{\xi}(k)^T\tilde{E}^TR_i\tilde{E}\tilde{\xi}(k)&\le\frac{V(\tilde{\xi}(k))}{\sigma_p}\\
&\le\frac{\Upsilon+\rho\lambda_SNd}{\sigma_p}
\end{aligned} \tag{23}
$$

Condition (20) implies that $E\{\tilde{\xi}^T\tilde{E}^TR_i\tilde{E}\tilde{\xi}(k)\}<c_2$. Define $L_i=[L_{1i}\ L_{2i}]$, and pre- and post multiplying matrix inequality (15) with $diag(\Omega_{ip}^T,\ I)$ and $diag(\Omega_{ip},\ I)$, we derive that $\varepsilon(\tilde{E}^T\Omega_{ip},\ 1)\in D(u(0))$.

Now, we prove that the system (12) is causal and regular in the time interval $[0,\ N]$ with $w(k)=0$. Consider condition (14) in theorem 1 and based on the lemma 2, we derive

$$
(d_M-d_m+1)J-\bar{E}^T\Omega_{ip}\bar{E}+\tilde{A}_i{}^T(\sum_{j=1}^{N}\sum_{q=1}^{M}\bar{\Xi}_{jq}^{-1})\tilde{A}_i<0,
$$

Since that $(d_M-d_m+1)J>0$, we have

$$
-\bar{E}^T\Omega_{ip}\bar{E}+\tilde{A}_i{}^T(\sum_{j=1}^{N}\sum_{q=1}^{M}\bar{\Xi}_{jq}^{-1})\tilde{A}_i<0. \tag{24}
$$

$$
\bar{E}^T\Omega_{ip}\bar{E}\ge 0. \tag{25}
$$

To this end we choose two nonsingular matrices M and N which satisfy the follows

$$
\bar{E}=M\begin{bmatrix} I & 0\\ 0 & 0\end{bmatrix}N. \tag{26}
$$

Write the follows

$$
\check{\Xi}_{jq}=M^T(\sum_{j=1}^{N}\sum_{q=1}^{M}\bar{\Xi}_{jq}^{-1})M=\begin{bmatrix} W_{i1} & W_{i2}\\ W_{i3} & W_{i4}\end{bmatrix}N. \tag{27}
$$

$$
\tilde{A}_i=M\begin{bmatrix} A_{i1} & A_{i2}\\ A_{i3} & A_{i4}\end{bmatrix}N. \tag{28}
$$

Pre- and post-multiplying (24) by N^{-T} and N^{-1}, based on the notations in (25)-(28), one can obtained that

$$
\begin{bmatrix} H_{1i} & H_{2i}\\ H_{2i}^T & H_{3i}\end{bmatrix}<0, \tag{29}
$$

where

$$
\begin{aligned}
H_{3i}&=A_{2i}^T(\sum_{j=1}^{\mathbf{N}}W_{j1})A_{2i}+A_{2i}^T(\sum_{j=1}^{\mathbf{N}}W_{j2})A_{4i}\\
&\qquad A_{4i}^T(\sum_{j=1}^{\mathbf{N}}W_{j2}^T)A_{2i}+A_{4i}^T(\sum_{j=1}^{\mathbf{N}}W_{j3})A_{4i}
\end{aligned}
$$

It is easy to see that (29) implies

$$
H_{3i}<0. \tag{30}
$$

From (30), we have that the matrix A_{4i} is nonsingular for each $j\in\mathcal{S}$. Thus, by Definition 2, it is easy to get that the discrete Markov jump singular systems in (12) is regular and causal.

Theorem 2: For $\forall\ r(k) = i \in S$ and the given constant integer $\varepsilon_2 < \lambda_2$, the closed-loop Markov jump systems(12) with initial conditions belonging to $\varepsilon(\bar{P}_i,\ 1)$ is said to be locally stochastically finite-time H_∞ bounded stabilizable with respect to $(c_1\ c_2\ N\ R_i\ d)$, if there exists positive constant $\gamma > 0$, such that conditions (15)-(20) of theorem 1 holds and

$$\begin{bmatrix} (14) & \tilde{C}^T & \tilde{C}^T & \tilde{N}_7^T M^T & I & \breve{K} \\ * & -\varepsilon_2 I & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -\frac{1}{1+\lambda_2} I & 0 & 0 \\ * & * & * & * & -\bar{Q}_1 & 0 \\ * & * & * & * & * & -\bar{Q}_2 \end{bmatrix} < 0, \tag{31}$$

$$\Upsilon + \gamma^2 Nd < \sigma_p c_2, \tag{32}$$

with

$$\Gamma_1 = \begin{bmatrix} \tilde{J}_i & 0 & \bar{\Omega}_{ip}^T L_i^T & 0 \\ * & -J & 0 & 0 \\ * & * & -2\bar{T}_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\begin{aligned} \tilde{C} &= [\tilde{C}_2\ \ \tilde{C}_1\ \ \mathbf{0}],\ \tilde{N}_7 = [\tilde{N}_6\ \ \tilde{N}_5\ \ \mathbf{0}] \\ \breve{K}_p &= \bar{K}_p o_i \bar{\Omega}_{ip},\ \tilde{C}_1 = \bar{C}_{1i} o_i \bar{\Omega}_{ip} + C_{3i} \breve{K}_p \\ \tilde{C}_2 &= \bar{C}_{2i} o_i,\ \tilde{N}_5 = \bar{N}_5 o_i \bar{\Omega}_{ip} + N_{7i} \breve{K}_p, \\ \tilde{N}_6 &= \bar{N}_6 o_i,\ \bar{Q}_1 = Q_1^{-1}, \bar{Q}_2 = Q_2^{-1}, \end{aligned}$$

and the definitions of other variables and matrices are consistent with Theorem 1 and with the following controller gain $\bar{K}_p = \breve{K}_p \bar{\Omega}_{ip}^{-1} o_i^{-1}$.

Proof: In this case, we consider the H_∞ control problem. It is easy to derive

$$\begin{aligned} z^T(k)z(k) &= \tilde{\xi}^T(k)(\tilde{C}_i^T + \Delta\tilde{C}_i^T)(\tilde{C}_i^T + \Delta\tilde{C}_i^T)^T\tilde{\xi}(k) \\ &= \tilde{\xi}^T(k)\tilde{C}_i^T\tilde{C}_i\tilde{\xi}(k) + \tilde{\xi}^T(k)\Delta\tilde{C}_i^T\Delta\tilde{C}_i\tilde{\xi}(k) \\ &\quad + He\{\tilde{\xi}^T(k)\tilde{C}_i^T\Delta\tilde{C}_i\tilde{\xi}(k)\} \end{aligned}$$

Define $\bar{N}_7 = [\bar{N}_5 o_i + N_{7i}\bar{K}_p o_i\ \ \bar{N}_6 o_i\ \ \mathbf{0}]$, and we assume that the matrix $M \in \mathbb{R}^n$. Since that $F^T F \leq I$, one can derive

$$\begin{aligned} \tilde{\xi}^T(k)\Delta\tilde{C}_i^T\Delta\tilde{C}_i\tilde{\xi}(k) &= \tilde{\xi}^T(k)\bar{N}_7^T F^T M^T M F \bar{N}_7\tilde{\xi}(k) \\ &\leq \tilde{\xi}^T(k)\bar{N}_7^T M^T M \bar{N}_7\tilde{\xi}(k) \end{aligned}$$

Based on Lemma 3 and denote $\varepsilon_2 < \lambda_2$, then we derive

$$\begin{aligned} He\{\tilde{\xi}^T(k)\tilde{C}_i^T\Delta\tilde{C}_i\tilde{\xi}(k)\} &\leq \varepsilon_2\tilde{\xi}^T(k)\bar{N}_7^T M^T M\bar{N}_7\tilde{\xi}(k) \\ &\quad + \frac{1}{\varepsilon_2}\tilde{\xi}^T(k)\tilde{C}_i^T\tilde{C}_i\tilde{\xi}(k) \end{aligned}$$

From condition (31) we derive

$$\begin{aligned} V(\tilde{\xi}(k+1)) - V(\tilde{\xi}(k)) &\leq \gamma^2 w(k)^T w(k) \\ &\quad - \tilde{\xi}(k)^T Q_1\tilde{\xi}(k) - u(k)^T Q_2 u(k) - z^T(k)z(k). \end{aligned}$$

From $k = 0 \rightarrow N$, it is easy to know

$$\begin{aligned} \sum_0^N (x(k)^T Q_1 x(k) + u(k)^T Q_2 u(k)) + \sum_0^N Z(k)^T Z(k) \\ \leq \gamma^2 Nd + V(x(0)) - V(x(N)). \end{aligned} \tag{33}$$

Then we have $\sum_0^N (x(k)^T Q_1 x(k) + u(k)^T Q_2 u(k)) \leq \gamma^2 Nd + V(x(0))$. Meanwhile, under the assumed zero initial condition, one can drive from (33)

$$\sum_0^N Z(k)^T Z(k) \leq \gamma^2 Nd. \tag{34}$$

Then following the similar proof of Theorem 1.

Theorem 3: For $\forall\ r(k) = i \in S$, the closed-loop Markov jump systems(12) with initial conditions belonging to $\varepsilon(\bar{P}_i,\ 1)$ and partially known transition rates is said to be locally stochastically finite-time H_∞ bounded stabilizable with respect to $(c_1\ c_2\ N\ R_i\ d)$, if there exists positive constant $\gamma > 0$, mode-dependent matrix $\bar{\Omega}_{ip}$, symmetric positive-define matrix J and diagonal positive definite matrix T_i, and matrix L_i, such that conditions of theorem 2 holds and

$$\begin{bmatrix} (14) & \tilde{C}^T & \tilde{C}^T & \tilde{N}_7^T M^T & I & \breve{K} \\ * & -\varepsilon_2 I & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -\frac{1}{1+\lambda_2} I & 0 & 0 \\ * & * & * & * & -\bar{Q}_1 & 0 \\ * & * & * & * & * & -\bar{Q}_2 \end{bmatrix} < 0,$$

$$j \in S_k \tag{35}$$

$$\begin{bmatrix} -\bar{\Omega}_{ip}^T\tilde{E} & \Phi_{ijp}^3 & \Phi_{ijp}^3 \\ * & -\Xi_j & 0 \\ * & * & -\Xi_j \end{bmatrix} < 0_{j\in S_{uk}}, \tag{36}$$

with $\bar{\pi} = \Sigma\pi_{ij},\ j \in S_k, \breve{K}_p = \bar{K}_p o_i \bar{\Omega}_{ip}$ and

$$\begin{aligned} \tilde{J}_i &= -\bar{\pi}\bar{\Omega}_{ip}^T\tilde{E} + (d_M - d_m + 1)\bar{J}_1 \\ \Phi_{ijp}^3 &= (\tilde{F}_{11},\ \tilde{F}_{12},\ \cdots \tilde{F}_{1M}), \\ \Phi_{ijp}^4 &= (\tilde{F}_{21},\ \tilde{F}_{22},\ \cdots \tilde{F}_{2M}), \\ \tilde{F}_{1q} &= \sqrt{\lambda_{jq}}(\bar{N}^T M^T), \\ \tilde{F}_{2q} &= \sqrt{\lambda_{jq}(1+\lambda)}\bar{\Pi}^T, \end{aligned}$$

and the definitions of other variables and matrices are consistent with Theorem 1 and 2, and with the following controller gain $\bar{K}_p = \breve{K}_p \bar{\Omega}_{ip}^{-1} o_i^{-1}$.

Proof: Design the same Lyapuonv-Krasovkii function as theorem 1. Consider that $\Sigma_{j\in S}\pi_{ij} = 1$, and define $\tilde{d}_m = d_M - d_m + 1$, it is easily obtained that

$$\begin{aligned} &\mathbb{L}\Delta V(x(k), i, p) \\ &\leq \tilde{\xi}_1^T(k)\tilde{\Pi}^T\tilde{P}_i\tilde{\Pi}\tilde{\xi}_1(k) + \tilde{\xi}_1^T(k)\tilde{\Delta}^T\tilde{P}_i\tilde{\Delta}\tilde{\xi}_1(k) \\ &\quad + He(\tilde{\xi}_1^T(k)\tilde{\Pi}^T\tilde{P}_i\tilde{\Delta}\tilde{\xi}_1(k)) - \tilde{\xi}^T(k - d(k))J\tilde{\xi}(k - d(k)) \\ &\quad - \tilde{\xi}^T(k)\Sigma_{j\in S}\pi_{ij}\tilde{E}^T\Omega_{ip}\tilde{\xi}(k) + \tilde{d}_m\tilde{\xi}^T(k)J\tilde{\xi}(k) \\ &\quad - 2\psi(u(k))^T T_i\psi(u(k)) + He(\psi(u(k))^T T_i L_i\tilde{\xi}(k)) \end{aligned}$$

Then we have

$$\begin{aligned}&\mathbb{L}\Delta V(x(k),i,p)\\&\le \{\tilde{\xi}_1^T(k)\tilde{\Pi}^T\tilde{P}_i\tilde{\Pi}\tilde{\xi}_1(k)+\tilde{\xi}_1^T(k)\tilde{\Delta}^T\tilde{P}_i\tilde{\Delta}\tilde{\xi}_1(k)\\&\quad+He(\tilde{\xi}_1^T(k)\tilde{\Pi}^T\tilde{P}_i\tilde{\Delta}\tilde{\xi}_1(k))-\tilde{\xi}^T(k-d(k))J\tilde{\xi}(k-d(k))\\&\quad-\tilde{\xi}^T(k)\Sigma_{j\in S}\pi_{ij}\tilde{E}^T\Omega_{ip}\tilde{\xi}(k)+(d_M-d_m+1)\tilde{\xi}^T(k)J\tilde{\xi}(k)\\&\quad-2\psi(u(k))^TT_i\psi(u(k))+He(\psi(u(k))^TT_iL_i\tilde{\xi}(k))\}_{j\in S_k}\\&\quad+\{\tilde{\xi}^T(k)\Sigma_{j\in S}\pi_{ij}\tilde{E}^T\Omega_{ip}\tilde{\xi}(k)+\tilde{\xi}_1^T(k)\tilde{\Pi}^T\tilde{P}_i\tilde{\Pi}\tilde{\xi}_1(k)\\&\quad+\tilde{\xi}_1^T(k)\tilde{\Delta}^T\tilde{P}_i\tilde{\Delta}\tilde{\xi}_1(k)+He(\tilde{\xi}_1^T(k)\tilde{\Pi}^T\tilde{P}_i\tilde{\Delta}\tilde{\xi}_1(k))\}_{j\in S_{uk}}\end{aligned}$$

Then following the similar proof of Theorem 2. The proof is completed.

IV. NUMERICAL EXAMPLES

In this section, two numerical examples are provided to demonstrate the effectiveness of the proposed method.

Example 1: In this case, we chose the same parameters of literature [14], and ignore saturation and packet loss. Assume that the data can be transmitted successfully, then we have $\beta = \alpha = 1$. For this numerical example, the initial values are given as the follows: $c_1 = 0.4,\ c_2 = 0.5,\ N = 10,\ R_i = I_3,\ d = 0.05$, and describe the delay as $d(k) = 1$. Given $\bar{\upsilon} = 0.2,\ \upsilon = 0.25$. By using the theorem 3, we derive

$$k_{11} = \begin{bmatrix}1.2371 & -0.8321 & 1.3653\end{bmatrix},\quad k_{12} = \begin{bmatrix}6.0142 & -1.9275 & -2.0721\end{bmatrix}.$$

Remark 1: Figs. 1 is the system jump mode, and *Figs.* 4 is state response of the closed-loop system (12) without packet loss. Compared with literature [14], this paper provides delay-dependent results by constructing appropriate Lyapunov-Krasovskii function to reduce conservatism. From the simulation results, the H_∞ performance $\gamma = 0.7$ is less than the results of literature [14].

Example 2: Consider the uncertain discrete-time singular Markov delay system (1)-(2) with two operation modes described as follows:

$$A_1 = \begin{bmatrix}-1 & 2\\-3 & -2\end{bmatrix},\ A_{d1} = \begin{bmatrix}-0.1 & 0.2\\-0.1 & -0.2\end{bmatrix},$$
$$A_2 = \begin{bmatrix}-2 & -3\\4 & -1\end{bmatrix},\ A_{d2} = \begin{bmatrix}0.2 & 0.1\\-0.1 & 0.4\end{bmatrix},$$
$$B_1 = \begin{bmatrix}0.3\\-0.1\end{bmatrix},\ D_1 = \begin{bmatrix}0.1\\0.1\end{bmatrix},$$
$$B_2 = \begin{bmatrix}0.1\\0.2\end{bmatrix},\ D_2 = \begin{bmatrix}0.1\\0.1\end{bmatrix},\ M = \begin{bmatrix}0.1\\-0.1\end{bmatrix},$$
$$C_{11} = \begin{bmatrix}-1 & -1\end{bmatrix},\ C_{21} = \begin{bmatrix}0.1 & -0.1\end{bmatrix},$$
$$C_{12} = \begin{bmatrix}1 & 1\end{bmatrix},\ C_{22} = \begin{bmatrix}0.2 & 0.1\end{bmatrix},$$
$$C_{31} = \begin{bmatrix}0.2 & -0.3\end{bmatrix},\ E_1 = \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix},$$
$$C_{32} = \begin{bmatrix}0.1 & -0.1\end{bmatrix},\ E_2 = \begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}.$$

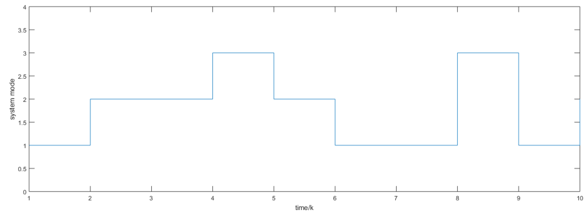

FIGURE 1. System mode of example 1.

with

$$N_{11} = \begin{bmatrix}0.2 & 0.1\\0.2 & 0.1\end{bmatrix},\ N_{21} = \begin{bmatrix}0.6 & 0.2\\0.1 & 0.1\end{bmatrix},$$
$$N_{31} = \begin{bmatrix}0 & 0.2\\0.5 & 0.1\end{bmatrix},\ N_{41} = \begin{bmatrix}0.4 & 0.1\\0.1 & 0.1\end{bmatrix},$$
$$N_{51} = \begin{bmatrix}0.1 & 0.2\\0.3 & 0.1\end{bmatrix},\ N_{61} = \begin{bmatrix}0.3 & 0.1\\0.1 & 0.4\end{bmatrix},$$
$$N_{71} = \begin{bmatrix}0.5 & 0\\0.2 & 0.1\end{bmatrix},\ N_{12} = \begin{bmatrix}0.5 & 0\\0.1 & 0.1\end{bmatrix},$$
$$N_{22} = \begin{bmatrix}0.4 & 0\\0.1 & 0.1\end{bmatrix},\ N_{32} = \begin{bmatrix}0.4 & 0.1\\0.2 & 0.1\end{bmatrix},$$
$$N_{42} = \begin{bmatrix}0.4 & 0\\0.2 & 0.1\end{bmatrix},\ N_{52} = \begin{bmatrix}0.4 & 0.1\\0 & 0.2\end{bmatrix},$$
$$N_{62} = \begin{bmatrix}0.4 & 0\\0.2 & 0.1\end{bmatrix},\ N_{72} = \begin{bmatrix}0.3 & 0.1\\0 & 0.2\end{bmatrix}.$$

For this numerical example, the initial values are given as the follows: $c_1 = 0.3,\ c_2 = 0.5,\ \gamma = 0.8,\ N = 10,\ R_i = I_2,\ d = 1,$, and describe the delay as $d(k) = 1 + 2 \mid cos(k) \mid$, then we have $d_M = 3,\ d_m = 1$,the bounds of the input $sat(u_k) \le 0.05$. The initial values of state are given as follows,

$$\eta(j) = \begin{bmatrix}0.2j + 0.5\\-0.2j - 0.5\end{bmatrix},\ j \in [-d_M,\ 0].$$

The transition rate matrix are given by the follows:

$$\pi_{ij} = \begin{bmatrix}? & ?\\0.4 & 0.6\end{bmatrix},\ \lambda_{ij} = \begin{bmatrix}0.6 & 0.4\\0.5 & 0.5\end{bmatrix}.$$

Given $\bar{\upsilon} = 0.2,\ \upsilon = 0.25,\ \beta = 0.7,\ \alpha = 0.7$. By using the theorem 3, we derive

$$k_{11} = \begin{bmatrix}1.4326 & -0.7934\end{bmatrix},\quad k_{12} = \begin{bmatrix}0.2143 & -0.0326\end{bmatrix},$$
$$k_{21} = \begin{bmatrix}-1.7931 & 0.8542\end{bmatrix},\quad k_{22} = \begin{bmatrix}10.1934 & -0.936\end{bmatrix}.$$

Remark 2: Figs. 2 is the controller jump mode, and *Figs.* 3 is the system jump rates, *Figs.* 5 is state response of the closed-loop system (12) with partly unknown transition rate and packet loss compensation. From the figures provided, the controller we designed guarantees that the resulted closed-loop constrained systems(12) are mean-square locally H_∞ finite-time bounded stabilizable.

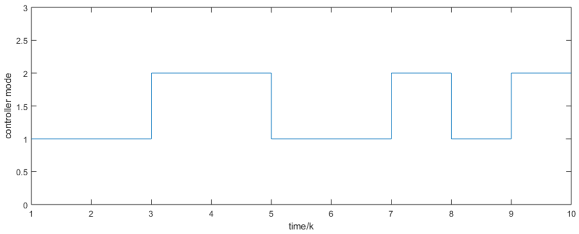

FIGURE 2. Controller mode of example 2.

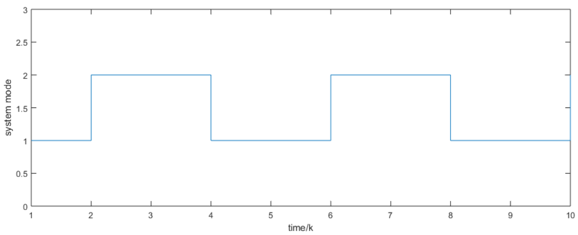

FIGURE 3. System mode of example 2.

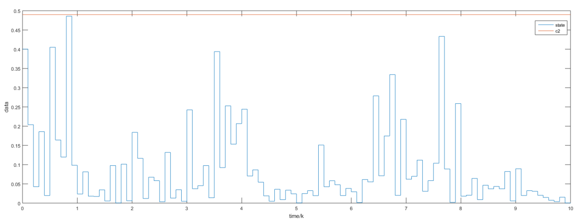

FIGURE 4. $\tilde{\xi}^T(k)\tilde{E}^T R_i \tilde{E}\tilde{\xi}(k)$ of the closed-loop system (12) of example 1.

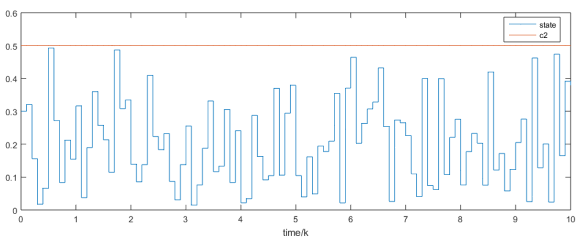

FIGURE 5. $\tilde{\xi}^T(k)\tilde{E}^T R_i \tilde{E}\tilde{\xi}(k)$ of the closed-loop system (12) of example 2.

TABLE 1. The minimum of c_2 for different α and β.

c_2	$\alpha = 1$	$\alpha = 0.7$	$\alpha = 0.5$
$\beta = 1$	0.41	0.45	0.68
$\beta = 0.7$	0.43	0.49	0.71
$\beta = 0.5$	0.55	0.62	0.81

Remark 3: From the Table 1, it is clear that the c_2 arrives at the minimum value 0.41 when $\alpha = \beta = 1$. This is mainly caused by that when $\beta = 1$, none of packet is lost and there is no need to predict the state for the controller.

V. CONCLUSION

The robust finite-time control issue has been investigated for a class of discrete-time singular Markov jump systems with time-varying delays and input saturation, in which random packet losses are considered. Based on the SES method, the prediction value has been taken as the compensation when a packet is lost. By utilizing a novel mode-dependent Lyapunov-Krasovskii functional, the state feed-back controller has been designed to ensure that the considered system is stochastically finite-time stable with a H_∞ performance. Finally, the validity of the proposed approach has been illustrated by two numerical examples. In addition, since the *LMI* method is used to solve the parameters, in order to obtain strict *LMIs*, the adopted mathematical processing method will increase the conservatism. In future work, we will strive to reduce conservatism and try to obtain the value of $\beta(k)$ in real time according to the network environment of the actual system, so as to further improve the controller design method.

STATEMENTS AND DECLARATIONS

CONFLICT OF INTEREST

The authors declare that we have no conflicts of interests about the publication of this article.

AVAILABILITY OF DATA AND MATERIAL

The authors declare that all data generated or analyzed during this study are included in this article.

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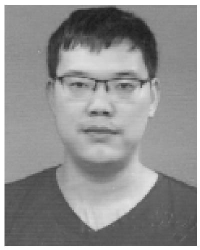

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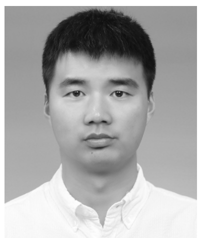

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