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RESEARCH ARTICLE

A Novel Scheme for the Construction of the SCMA Codebook

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ABSTRACT As a code-domain non-orthogonal multiple access technique, sparse code multiple access (SCMA) is considered as a promising technique for future wireless Internet of Things (IoT) networks. The minimum Euclidean distance (MED) and minimum product distance (MPD) have been highlighted as the key performance indicators of the codebooks in additive gaussian white noise (AWGN) and downlink Rayleigh channels respectively. In this paper, based on the mother codebook, a novel codebook design scheme is proposed to achieve better error performance in both AWGN and downlink Rayleigh channel. The problem of constructing the mother codebook is considered as the quadratic assignment problem (QAP), where the Tabu searching algorithm is employed to reduce the complexity of searching for the best permutation result. Then, the rotation matrix is adopted to find the best degrees for the generation of the constellation group, and two algorithms are proposed to assign the obtained constellation in factor graph matrix. Taking the degree optimization and the constellation assignment into joint consideration, an improved unified optimization is further explored to maximize the MED of each user. Besides, a novel polarized modulation scheme is proposed, which places the symbols in the three dimensional (3D) stokes parameters to improve the performance of the system. Finally, simulation results are provided to show the performance of the proposed codebooks, and the comparisons of symbol error performance (SER) in different codebooks are also discussed in detail.

INDEX TERMS Sparse code multiple access (SCMA), codebook design, minimum Euclidean distance (MED), minimum product distance (MPD), three dimensional SCMA codebooks.

I. INTRODUCTION

The integration of 5G terrestrial networks with low earth orbit (LEO) satellites can provide nearly-global coverage and support for the Internet of Things (IoT) [1]. However, with the rapidly growing number of intelligent devices and services, massive devices inevitably cause profound inference, which leads to sharp degradation of system performance [2], [3]. Therefore, traditional orthogonal multiple access (OMA) schemes are not applicable for future communication [3]. In order to meet the demand of IoT communication with a higher spectrum efficiency, non-orthogonal multiple access (NOMA) scheme is viewed as one of the most promising strategies in embracing the future multiple access technology. Existing NOMA schemes can be divided into power-domain NOMA and code-domain NOMA [4], [5]. As a code-domain NOMA scheme, sparse code multiple access (SCMA) has been studied extensively in recent years since its high robustness and capacity [6].

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A. LITERATURE AND MOTIVATION

Presently, research on SCMA focus on three aspects: codebook design [7], [8], [9], [10]; low complexity decoding algorithm [11], [12], [13] and the applications of SCMA technology [14], [15], [16]. While the key factor that affects the error performance of the SCMA lies on the codebook design [17]. The design criteria are important factors that direct how to formulate codebooks with better error performance. In the additive gaussian white noise (AWGN) channel, the minimum Euclidian distance (MED) is widely considered as the key factor affecting the symbol error rate (SER) performance [18], [19], [20], [21]. In contrast, the minimum product

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distance (MPD) of the superimposed codewords is viewed as the key performance indicator in downlink Rayleigh fading channel, [17], [22], [23]. [24]. However, research has shown that the codebook designed by maximizing the MED can also achieve preferable performance in downlink Rayleigh fading channel [18], [24], [25], and it can also be proved by average inequality that maximizing the MED is the upper bound of the minimum product distance [25]. Therefore, maximizing the MED will also help improving the error performance in downlink Rayleigh fading channel.

On the aspect of codebook design, following the initial codebook proposed by *Nikopour* et al [26], extensive research has been conducted to construct the SCMA codebook [17], [18], [19], [23], [25], [27], [28], [29], [30], [31]. Among which, the mother codebook based method has been widely adopted in some approaches [17], [23], [27], [32]. Besides, there are other methods to construct SCMA codebooks, such as the quadratic programming based codebook design [20], and uniquely decomposable constellation group based codebook design [25]. Though numerous methods have been proposed to construct the codebooks, the optimal codebook is still an open problem, and it is unknown how close the current SCMA codebooks are to the optimal ones [33]. Besides, since the design criteria in AWGN channel is different from that in downlink Rayleigh channel, the codebook with better performance in AWGN channel does not achieve satisfactory error performance in downlink Rayleigh channel. Therefore, based on the maximum MED, exploring codebooks with better performance in downlink Rayleigh fading channel is still a promising task. Among existing approaches for the construction of the SCMA codebook, the implements of the mother codebook based method is an attractive scheme since its flexible and simpleness.

For uplink Rayleigh fading channel, the construction of the mother codebook is the key procedure [17], [34], and a good mother codebook can obtain much diversity gains [35]. So far, a lot researches have been conducted to design mother codebook with higher symbol mapping diversity. For example, the interleaving method reported in [10] and [25]. Though the complexity of this approach is low, it cannot guarantee the best diversity gains. Besides, the binary switching algorithm (BSA) [36], symbol exchange algorithm (SEA) [17] and dimensional permutation switching algorithm (DPSA) [29] are also proposed to improve the mapping diversity of the mother codebook. While, the symbol mapping problem in the mother codebook can also be viewed as the quadratic assignment problem (QAP) to maximize the diversity gain [35], which provides another aspect to solve the problem, such as Lagrangian relaxation algorithm [37], genetic algorithm [38] *et al.* Among which, Tabu searching algorithm is a metaheuristics method depending on both theoretical and experimental factors and has a moderate computational complexity than metaheuristics algorithms [39]. Besides, similar to SEA algorithm, Tabu searching algorithm further adopted Tabu table to avoid falling into local optimal solutions. Therefore, Tabu algorithm will be adopted to construct the mother codebook.

Different from the uplink Rayleigh channel, the MED of the superimposed codewords (MED-SC) should be maximized to reduce interference between users. Since the rotation procedure does not change the Euclidean distance between the codewords, which has been widely adopted to generate the multiuser codebooks [10], [27], [32], [40]. However, searching the optimal rotation angles is time-consuming. Although some researchers directly obtained the rotation angle based on the colliding users over each frequency [10], [40], the results show that the optimized rotation angles can achieve better error rate performance. Thus, the rotation matrix is adopted in this paper to reduce the interference between users and increase the MED-SC.

Besides, the limited freedom degree in 2D space blocks the Euclidean distance improvements. Thus, constructing SCMA codebooks with high dimensional freedom can further improve the system's performance. Therefore, based on the proposed scheme, constructing the 3D SCMA codebook is also a promising topic.

B. CONTRIBUTION

As mentioned in above part, SCMA codebook design has been investigated for a long time, and the mother codebook based approaches have been widely deployed in the construction of SCMA codebooks. In this paper, a modified mother codebook based approach is proposed to construct SCMA codebooks. And the proposed design scheme can also be applied in the construction of the 3D SCMA codebook to further improve the error performance without requiring additional energy. However, building 3D constellation requires additional orthogonal parameters, such as frequency [41], polarization [42] *et al.* In this paper, the polarization dimension is firstly adopted to increase the available dimensional information. The main contribution of this work can be summarized as follows.

- Based on the mother codebook, a novel codebook design approach is introduced in this paper. Rather than the permutation method, the symbol mapping diversity in the mother codebook is considered as QAP, where the Tabu searching algorithm is employed. Similar to the SEA algorithm, but with a list of forbidden moves, Tabu searching algorithm can prevent being trapped into local optimal. Then, the rotation matrix is introduced to reduce interference among users. And two assignment algorithms are proposed to assign the optimized constellation sets. Furthermore, a modified optimization is proposed to improve the performance of the codebook in both AWGN and downlink Rayleigh channels. And the detailed reasons are also discussed.
- In order to further improve the system performance of SCMA, the 3D SCMA modulation scheme is firstly proposed where the 3D symbols are mapped into the stokes parameters to increase the MED of each user. And the

detailed system model and the construction procedures of the 3D SCMA codebook are presented. Finally, simulation results are provided showing the promising error performance of the 3D SCMA codebook, especially in high codebook size and downlink Rayleigh fading channel.

The rest of the paper is organized as follows. In Section II, the system model of SCMA and the design criteria are introduced. In Section III, the new proposed approach is introduced in detail. In Section IV, a brief introduction of 3D polarized modulation scheme in SCMA model is firstly presented, followed by the construction of the 3D SCMA codebook. Section V is the simulation results and we conclude the paper in Section VI.

II. SYSTEM DESCRIPTION OF SCMA

A. SYSTEM MODEL OF SCMA

In this paper, we consider downlink SCMA system in Rayleigh fading channel where a base station transmits signals to L users with K frequency resources, and the overloading factor is set as $\zeta = L/K > 1$ to improve the spectrum efficiency. The factor graph matrix determined the relationship between users and resources, which can be written as

$$F = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}. \quad (1)$$

In factor graph matrix F, the row denotes the frequency resources and the column is the serviced users. The nonzero elements in the F represent the adopted frequencies in different user. Here, we only consider the matrix with regular form that the degree of the user nodes is d_v and the resources nodes is d_f.

In SCMA encoder, the transmitted bits data are directly mapped into multidimensional constellation with the pre-assigned codebooks

$$f_l : \mathbb{B}^{log_2 M\times 1} \rightarrow \mathcal{X}_l \quad i.e.,\ \mathbf{x}_l = f_l(\boldsymbol{b}_l), \quad (2)$$

where $\boldsymbol{b}_l \in \mathbb{B}^{log_2 M\times 1}$ denotes the bits data of the l-th user, M represents the codebook size, and $\mathbf{x}_l = [x_{1,l}, \cdots, x_{K,l}]^T$ is the transmitted codewords in the l-th user from the codebook $\mathcal{X}_l$.

The received signal at base station is the product of the channel coefficients and superimposed codewords, which can be written as

$$\mathbf{y} = \mathbf{H}_l \sum_{l=1}^{L} \mathbf{x}_l + \mathbf{z}, \quad (3)$$

where $\mathbf{y} = [y_1, y_1, \cdots, y_K]^T$ is the received signal in the employed frequency resources, $\mathbf{H}_l = diag(h_{1,l}, h_{2,l}, \cdots, h_{K,l})$ is the channel coefficients that has i.i.d Rayleigh distribution among users and resources and $\mathbf{z} = [z_1, z_1, \cdots, z_K]^T$ denotes the noise vector and the variance is N_0.

At the receiver, the message passing algorithm (MPA) is adopted as the detection algorithm, where message exchanges in user and resource nodes

$$I_{k\rightarrow l}(\mathbf{x}_l) = \sum_{\mathbf{x}_i : i\in(Nl/l)} P(\mathbf{y}|\,\mathbf{x},\mathbf{H}) \prod_{i\in(Nl/l)} I_{i\rightarrow k}(\mathbf{x}_i), \quad (4)$$

$$I_{l\rightarrow k}(\mathbf{x}_l) = p(\mathbf{x}_l) \prod_{m\in(Nk/k)} (I_{m\rightarrow l}(\mathbf{x}_l)), \quad (5)$$

where $P(\mathbf{y}|\,\mathbf{x},\mathbf{H})$ denotes the conditional probability density function (PDF) of the received signal. $p(\mathbf{x}_l)$ is the probability of the symbol distribution for the l-th user, Nl/l and Nk/k represents the neighboring node of user l and frequency k respectively.

B. DESIGN CRITERIA FOR SCMA CODEBOOKS

In this subsection, the pairwise error probability (PEP) of each user is analyzed to introduce the codebook design criteria.

Using the Chernoff boundary, the conditional PEP of the received signal is closer to $\mathbf{e}_l$ than $\mathbf{x}_l$ under the channel coefficient $\mathbf{h}_l$ is given by [43]

$$P(\mathbf{x}_l \rightarrow \mathbf{e}_l|\,\mathbf{h}_l) \leq \frac{1}{2}\exp\left(\frac{E_s}{4N_0}\left\|\mathrm{diag}(\mathbf{h}_l)(\mathbf{x}_l - \mathbf{e}_l)\right\|^2\right), \quad (6)$$

where $\mathbf{h}_l = [h_{1,l}, \cdots, h_{k,l}]^T$ is the channel vector for the l-th user, E_s is the power of the transmitted power, and the Euclidean distance of each user can be written as

$$\left\|\mathrm{diag}(\mathbf{h}_l)(\mathbf{x}_l - \mathbf{e}_l)\right\|^2 = \sum_{k=1}^{K}\left|h_{k,l}\sum_{l=1,F_{k,l}\neq 0}^{L}\Delta x_{k,l}\right|^2, \quad (7)$$

where $\Delta x_{k,l} = x_{k,l} - \tilde{x}_{k,l}$ denotes the Euclidean distance of the wrongly decoded symbols. Since the channel coefficient $h_{k,l}$ is a circular complex gaussian random variable (RV), the probability density function (PDF) of the magnitude is Rayleigh distribution [44], which is given by

$$p(|h_{k,l}|) = \frac{|h_{k,l}|}{\sigma^2}\exp\left(-\frac{|h_{k,l}|^2}{2\sigma^2}\right), \quad (8)$$

where σ^2 is the variance of the Rayleigh fading channel. Thus, we can obtain the unconditional PEP of (6) through a simple mathematical integral

$$P(\mathbf{x}_l \rightarrow \mathbf{e}_l|\,\mathbf{h}) \leq \frac{1}{2}\prod_{k=1}^{K}\left(1 + \frac{\sigma^2 E_s}{2N_0}\left|\sum_{l=1,F_{k,l}\neq 0}^{L}\Delta x_{k,l}\right|^2\right)^{-1}. \quad (9)$$

It can be observed that maximizing the product distance of the superimposed codewords dominates the PEP under high signal to noise ratio (SNR). While in low SNR, maximizing the MED in each frequency resource is an important factor for improving the error performance. Furthermore, in high SNR, according to the average inequality [25]

$$\frac{\sum_{k=1}^{K}\left|x_{k,l}-\tilde{x}_{k,l}\right|^2}{K} \geq \sqrt[K]{\prod_{k=1}^{K}\left|x_{k,l}-\tilde{x}_{k,l}\right|^2}, \tag{10}$$

It can be obtained that the MED-SC is the upper bound of the MPD. Thus, maximizing the MED-SC will also help improve the performance in downlink Rayleigh channel.

III. IMPLEMENTS FOR DESIGNING THE SCMA CODEBOOK

A. CONSTRUCTION OF THE MOTHER CODEBOOK

The mother codebook based approach has been investigated for a long time [45]. The brief idea of the method is to maximize the frequency diversity of the mother codebook, then the rotation operation is adopted to generate the multiuser codebooks.

In the first step, we intend to adopt existing benchmark constellations. The remaining procedures focus on optimizing the mapping rules in the mother codebook. The permutation method has been widely applied in constructing the mother codebook [17], [29]. For small constellation size, the optimal solution can be found by extensive search, while as the constellation size grows, exhaustive searching is computationally expensive, which needs $(M!)^{K-1}$ permutations to reach the optimal results. While the symbol mapping problem in the mother codebook can also be viewed as the quadratic assignment problem (QAP) to maximize the diversity gain [35]. Mathematically, the construction of the mother codebook can be written as

$$\mathcal{X}_{MC} = \arg\max_{\pi_k \in \boldsymbol{\Psi}} \sum_{i=1}^{M} \sum_{\substack{j=1 \\ i \neq j}}^{M} \left(\sum_{\substack{k=1 \\ F_{k,l}\neq 0}}^{K} |\boldsymbol{\Delta} x_k|^2 \right)$$
$$s.t. \begin{cases} \boldsymbol{\Delta} x_k = x_k^i - x_k^j \\ x_k^j = \pi_k\left(x_1^i\right) \quad i.e.\ \pi_k : \{1,2,\cdots,M\} \\ \qquad\qquad\qquad\qquad \rightarrow \{M,2,\cdots,1\}, \end{cases} \tag{11}$$

where $\boldsymbol{\psi}$ is the set of all possible permutation combinations, π_k is the permutation operation for the k-th constellation and x_k^i is the i-th transmitted codeword in the k-th constellation sets. The optimization problem (11) aims to maximize the product Euclidean distance of the mother codebook, which can be solved from the aspect of QAP. Among which, Tabu searching algorithm is a metaheuristics method depending on both theoretical and experimental factors and has a moderate computational complexity than metaheuristics algorithms [39]. Similar to SEA algorithm, Tabu searching algorithm further adopted Tabu table to avoid falling into local optimal solutions. Thus, Tabu searching algorithm is adopted for the generation of the k-th constellation. Besides, different initial values are utilized to obtain better optimization result. The detailed Tabu searching algorithm is summarized in **Algorithm 1**.

Algorithm 1 Tabu Searching Algorithm in Constructing the Mother Codebook

Input: Labelling the first constellation of the mother codebook in order. The maximum iteration N. The Tabu table Λ and Tabu length T.
for $i = 1 : N$
  for $m = 1 : M$
    for $j = m + 1 : M$
      check the Tabu table
      if symbol switching is allowed $\Lambda(m, j) = 0$
        exchange the m-th symbol with j-th symbol and calculate the cost function (11).
      else
        skip the symbol switching.
      end if
      if the cost function (11) is maximum through symbol switching
        retain the switch combination
      end if
    end for j
end for m
for $m = 1 : M$
  for $j = m + 1 : M$
    if the switch combination achieves maximum cost function
      update Tabu table $\Lambda(m, j) = \mathrm{T}$.
    else
      update Tabu table $\Lambda(m, j) = \max(\Lambda(m, j) - 1, 0)$.
    end if
  end for j
end for m
end for i
find the best permutation result that has the maximum cost function.

As the labelling result is obtained, the mother codebook can be written as

$$\mathcal{X}_{MC} = \begin{bmatrix} \mathcal{A}_1 & \mathcal{A}_2 \end{bmatrix}^T, \tag{12}$$

where $\mathcal{A}_i$ is the i-th sub-constellation set.

B. GENERATION OF THE CONSTELLATION GROUP

When discarding the null dimensional sparse codebooks in each d_v frequency resources, the remained codebook formed into a constellation group with d_f sub-constellation sets in each frequency resource. And the constellation group can also be used to calculate the MED of users (MED-U). For example, if the first two frequency resources in (1) are adopted, we can obtain the MED of user 1.

Therefore, in this subsection, we aim to generate a constellation group with maximized minimum Euclidean distance. Then, the constellations will be assigned in the factor graph matrix to generate the SCMA codebook. Here, the rotation matrices are introduced to increase the minimum Euclidean distance of the constellation group.

$$\begin{bmatrix} \mathcal{I}_1 \\ \mathcal{I}_2 \end{bmatrix} = \begin{bmatrix} \mathcal{A}_1 R(\theta_1) & \mathcal{A}_1 R(\theta_2) & \mathcal{A}_1 R(\theta_3) \\ \mathcal{A}_2 R(\theta_1) & \mathcal{A}_2 R(\theta_2) & \mathcal{A}_2 R(\theta_3) \end{bmatrix}, \tag{13}$$

where $R(\theta_i)$ is the rotation matrix, and the corresponding rotation angle is θ_i

$$R(\theta_i) = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix}. \tag{14}$$

And the best rotation angle can be found by solving

$$\theta_1, \theta_2, \cdots, \theta_{d_f} = \arg \max_{0 \le \theta_1, \cdots, \theta_{d_f} < \pi} \left(\min \sum_{k=1}^{d_v} \left| d_k - \tilde{d}_k \right|^2 \right)$$
$$s.t. \begin{cases} d_k = \sum_{i=1}^{d_f} x_{k,i} R(\theta_i) \\ \left| x_{k,i} \right|^2 \le E_s, \end{cases} \tag{15}$$

where $\tilde{d}_k$ is the mis-detected superimposed codewords in the k-th frequency and the transmitted symbols are modulated in limited energy E_s. The optimization problem can be solved by 'fmincon' in MATLAB using the interior point method. Meanwhile, we obtain a constellation group with maximized MED. For codebook size $M = 4$, the maximized MED of the constellation group can achieve 1.40.

C. ASSIGN CONSTELLATIONS IN THE FACTOR GRAPH MATRIX

The factor graph matrix maps the relationship between users and frequency resources. In order to achieve the maximum MED-U for all users in SCMA system, the structure of the factor graph matrix should be consistent with (13) that each two frequency resources should employ three users. However, this approach omits the effect of the MPA on the error performance. Since the MPA decoding algorithm is based on tree graph, which assumes the transmitted messages are statistically independent [46]. And the small girth in the factor graph matrix should be avoided [47]. Referring to the methods in LDPC codes, we adopted the progressive edge growth algorithm (PEG) to design the factor graph matrix [48].

$$F = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}. \tag{16}$$

Supposing the obtained factor graph matrix is shown as (16), the constellation group (13) needed to be assigned in (16) to formulate the SCMA codebook. However, it can be noticed that more than d_f users are transmitted in each two frequency resources. Thus, the MED-U may not be the maximum when assigned in factor graph matrix (16). Therefore, two algorithms are proposed to assign the constellation group in the factor graph matrix in order to maximize the MED-U and diversity gains.

In **Algorithm 2**, the frequency diversity of each user is maximized at the cost of the maximum MED-U. While in **Algorithm 3**, we intend to maximize the MED-U of the most users while the frequency diversity gains are not the maximum. The generated codebooks are showed in (17) and (18), as shown at the bottom of the next page, respectively. And we refer the codebook generated by **Algorithm 2** and **Algorithm 3** as "codebook 2" and "codebook 3" respectively.

Algorithm 2 Assigning Codebooks in Factor Graph Matrix With Maximum Frequency Diversity Gains

Input:the constellation group and the factor graph matrix.
Assign the codebook sets $\mathcal{I}_1$ to the first frequency resources.
for $k = 2 : K$
  $n = 1$.
  for $l = 1 : L$
    if $F(k, l) = 1$
      if $\text{sum}(F(1 : k-1, l)) = 0$
        $\mathcal{X}(k, l) = \mathcal{I}_1(n)$.
      elseif $\text{sum}(F(1 : k-1, l)) = 1$
        $\mathcal{X}(k, l) = \mathcal{I}_2(n)$.
      end if
      $n = n + 1$.
    end if
  end for l
end for k
output: the generated SCMA codebook with maximum frequency diversity gains in each user.

Algorithm 3 Assigning Codebooks in Factor Graph Matrix With Maximum MED-U for the Most Users

Input: the constellation group and the factor graph matrix.
Assign the codebook sets $\mathcal{I}_1$ to the first frequency resources.
for $k = 2 : K$
  find the employed users index = find $(F(k, :) = 1)$, check the codebooks $\mathcal{X}(1 : k-1, \text{index})$, and compare the number of used codebooks sets $|\mathcal{I}_1|$ and $|\mathcal{I}_2|$.
    if $|\mathcal{I}_1| \le |\mathcal{I}_2|$
      $\mathcal{X}(k, \text{index}) = \mathcal{I}_1$.
    elseif $|\mathcal{I}_1| > |\mathcal{I}_2|$
      $\mathcal{X}(k, \text{index}) = \mathcal{I}_2$.
    end if
end for k
output: the generated SCMA codebook with maximum MED-U for the most users.

It can be observed that from (17), the SCMA codebook generated by **Algorithm 2** can guarantee the diversity gains of each user, and for the SCMA codebook generated by **Algorithm 3**, the diversity gains and the MED-U of the most users are maximized at the expense of the error performance of user 5 and 6 in codebook (18).

D. THE IMPROVED OPTIMIZATION METHOD

A codebook with larger MED-U and diversity gain has been obtained through above procedures, while the maximum MED-SC does not provide the maximum product distance (MPD) certainly, which means the error performance of the codebook is different between AWGN and Rayleigh channel in some cases. Besides, the procedure of assignment does not guarantee the maximum MED-U for each user. Therefore, an improved method is explored in this section.

The average inequality (10) indicates the MED-SC is the upper bound of the MPD [25], and the equality holds only when the Euclidean distance in each frequency is the same

$$\left|x_{1,l}-\tilde{x}_{1,l}\right|^2=\cdots=\left|x_{k,l}-\tilde{x}_{k,l}\right|^2, \tag{19}$$

Thus, the minimized Euclidean distance in one-dimensional frequency resource should also be increased to maximize the MPD. Besides, the procedures of generating the codebook group and assigning constellations in factor graph are separated, causing the loss of the maximum MED-U. Thus, a jointly designed method is proposed and the optimization problem can be written as

$$\begin{aligned}
\theta_i &= \arg\max_{0\le\theta_i<\pi}\left(\min\left(\mathcal{L}_1,\mathcal{L}_2,\cdots,\mathcal{L}_L\right)\right)\\
s.t.\ d_k &= \sum_{\substack{l=1\\F_{k,l}\neq 0}}^{L} x_{k,l}R\left(\theta_i\right),\quad i=1,2,3\\
\mathcal{L}_l &= \sum_{k\in\Xi_l}\left|d_k-\tilde{d}_k\right|^2\\
&\left|d_k-\tilde{d}_k\right|\ge\varepsilon\\
&\left|x_{k,l}\right|^2\le E_s,
\end{aligned} \tag{20}$$

where Ξ_l is the adopted frequency resources for the l-th user. For the first user in (17), $\Xi_1 = 3, 4$. And ε is the lower bound of the ED in each frequency resource, which aims guarantee the MED in each frequency resource. And the optimization can be solved in several times to obtain better results.

For **Algorithm 2**, the modified optimization improves the loss of MED-U when assigning the constellation sets. While for **Algorithm 3**, the improved optimization further increases the minimized Euclidean distance in one-dimensional resource, as well as the MPD. However, the MED-U of the most users is decreased when the MED-U of the worst users is increased and the diversity gains are not maximum. Therefore, the improved optimization with **Algorithm 2** has better error performance and will be employed to construct the 3D SCMA codebook.

IV. CONSTRUCTION OF THE 3D CODEBOOK

Although the improved method can achieve better error performance, the traditional In-phase/quadrature (I/Q) modulation scheme only provide limited Euclidean distance under the same modulation power. Therefore, exploring high dimensional modulation is important to improve the performance of SCMA system.

The modulation of 3D constellation requires additional orthogonal parameters, such as frequency [41], polarization [42] *et al.* In this paper, the polarization dimension is firstly adopted to increase the available dimensional information. And the 3D symbols are placed on the stokes parameters to increase the MED-U.

A. PRELIMINARIES ON ELECTROMAGNETIC WAVES

The electromagnetic wave can be decomposed into two orthogonal components

$$\mathbf{E}=\begin{pmatrix}E_x\\E_y\end{pmatrix}=\begin{pmatrix}E_{0x}e^{j\varphi_x}\\E_{0y}e^{j\varphi_y}\end{pmatrix}, \tag{21}$$

where E_{0x}, E_{0y} represent the amplitude of each component, and φ_x, φ_y the corresponding phases. Besides, the electromagnetic wave can also be described by stokes parameters

$$\mathbf{S}=\begin{bmatrix}S_0\\S_1\\S_2\\S_3\end{bmatrix}=\begin{bmatrix}E_{0x}^2+E_{0y}^2\\E_{0x}^2-E_{0y}^2\\2E_{0x}E_{0y}\cos\delta\\2E_{0x}E_{0y}\sin\delta\end{bmatrix}, \tag{22}$$

where $\delta=\varphi_x-\varphi_y$, and the degree of the polarization can be calculated

$$\mathcal{P}=\frac{\sqrt{S_1^2+S_2^2+S_3^2}}{S_0},\qquad \begin{matrix}0\le\mathcal{P}\le 1\\ S_0^2\ge S_1^2+S_2^2+S_3^2.\end{matrix} \tag{23}$$

The stokes parameters S_1, S_2 and S_3 constitute a three-dimensional polarization space, where the transmitted information will be mapped. However, stokes parameters only measure the intensities of the polarized wave, while the Jones vector involves the magnitude and the phase of the electromagnetic, which can better describe the propagation of the

$$\mathcal{X}=\begin{bmatrix}0&\mathcal{A}_1R(\theta_1)&\mathcal{A}_1R(\theta_2)&0&\mathcal{A}_1R(\theta_3)&0\\0&\mathcal{A}_2R(\theta_1)&0&\mathcal{A}_1R(\theta_2)&0&\mathcal{A}_1R(\theta_3)\\\mathcal{A}_1R(\theta_1)&0&\mathcal{A}_2R(\theta_2)&0&0&\mathcal{A}_2R(\theta_3)\\\mathcal{A}_2R(\theta_1)&0&0&\mathcal{A}_2R(\theta_2)&\mathcal{A}_2R(\theta_3)&0\end{bmatrix}, \tag{17}$$

$$\mathcal{X}=\begin{bmatrix}0&\mathcal{A}_1R(\theta_1)&\mathcal{A}_1R(\theta_2)&0&\mathcal{A}_1R(\theta_3)&0\\0&\mathcal{A}_2R(\theta_1)&0&\mathcal{A}_2R(\theta_2)&0&\mathcal{A}_2R(\theta_3)\\\mathcal{A}_2R(\theta_1)&0&\mathcal{A}_2R(\theta_2)&0&0&\mathcal{A}_2R(\theta_3)\\\mathcal{A}_1R(\theta_1)&0&0&\mathcal{A}_1R(\theta_2)&\mathcal{A}_1R(\theta_3)&0\end{bmatrix}, \tag{18}$$

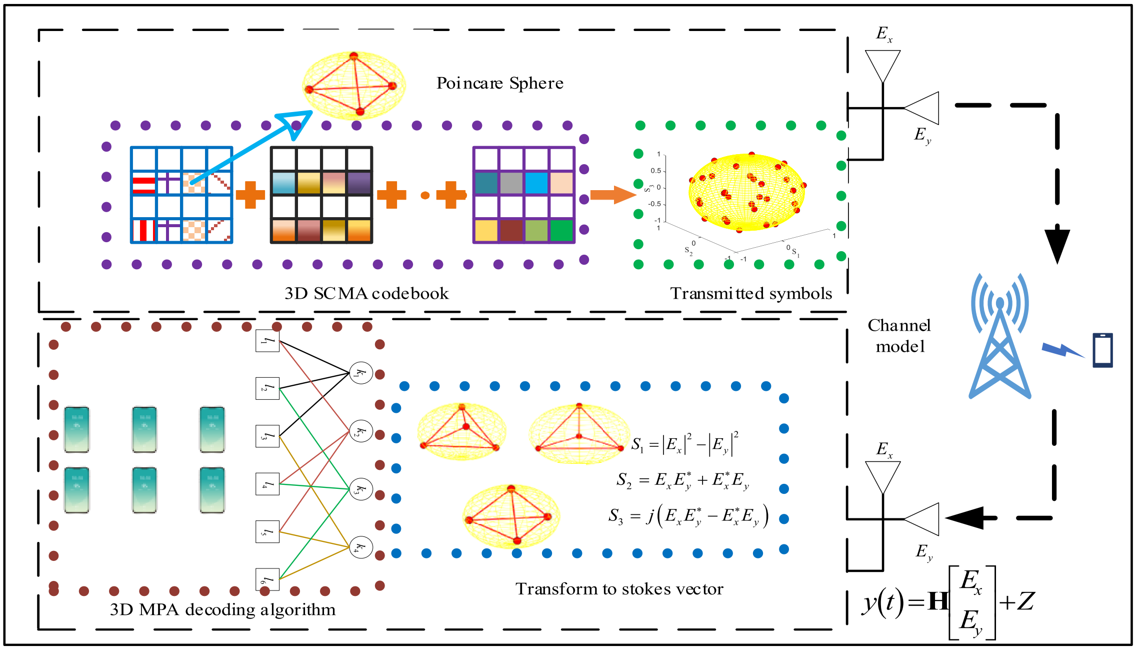


FIGURE 1. The SER performance of codebooks with codebook size M=4.

electromagnetic wave. Therefore, we will express the Jones vector from the stokes parameters [42]

$$\mathbf{E} = \begin{pmatrix} \sqrt{\mathcal{E}} \cos \frac{\vartheta}{2} e^{-j\frac{\phi}{2}} \\ \sqrt{\mathcal{E}} \sin \frac{\vartheta}{2} e^{j\frac{\phi}{2}} \end{pmatrix}, \tag{24}$$

where $\mathcal{E} = S_1^2 + S_2^2 + S_3^2$ is the transmitted energy of the symbol, and the thetas can be obtained by the transformation from cartesian coordinate into sphere coordinate

$$\begin{aligned} S_1 &= \mathcal{E} \cos \vartheta \\ S_2 &= \mathcal{E} \sin \vartheta \cos \phi \\ S_3 &= \mathcal{E} \sin \vartheta \sin \phi. \end{aligned} \tag{25}$$

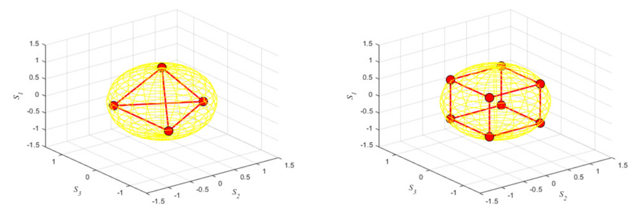
Since the initial phase of the electromagnetic wave does not affect the computation of stokes parameters [42]. For simplicity, the transmitted symbols in polarized electromagnetic wave can be expressed as

$$\mathbf{w} = \begin{pmatrix} E_{0x} e^{j\varphi_x} \\ E_{0y} e^{j\varphi_y} \end{pmatrix} = \begin{pmatrix} \sqrt{\mathcal{E}} \cos \frac{\vartheta}{2} e^{-j\frac{\phi}{2}} \\ \sqrt{\mathcal{E}} \sin \frac{\vartheta}{2} e^{j\frac{\phi}{2}} \end{pmatrix}. \tag{26}$$

B. SYSTEM MODEL OF 3D SCMA

The system model of 3D SCMA transmission model is shown in Fig. 1, where the transmitted vector is the 3D superimposed codewords, which can be calculated as

$$d_k = \sum_{\substack{l=1 \\ F_{k,l} \neq 0}}^{L} x_{k,l}, \quad (d_k, x_{k,l}) \in \mathbb{R}^3. \tag{27}$$



(c) 3D constellation where $M = 4$ (d) 3D constellation where $M = 8$

FIGURE 2. The adopted 3D constellation in stokes parameters.

The transmitted codewords d_k can be expressed in Jones vector, which is written as

$$\mathbf{w}_k = \mathcal{G}(d_k), \quad \mathbf{w}_k \in \mathbb{R}^2, \tag{28}$$

where $\mathcal{G}(\cdot)$ is the mapping function from the stokes vector to Jones vector and $\mathbf{w}_k$ denotes the polarized electromagnetic wave in the k-th frequency. The polarized electromagnetic transmission can be viewed as 2×2 multiple input multiple output (MIMO). Thus, the received signal of the l-th user in the k-th frequency $\mathbf{r}_{k,l}$ can be written as

$$\mathbf{r}_{k,l} = \mathbf{H}_{k,l} \mathbf{w}_k + \mathbf{z}_k, \tag{29}$$

where we have the following definition

- $\mathbf{r}_{k,l} = \left(r_{k,l}^1 \;\; r_{k,l}^2 \right)^T$ is the 2×1 received signal vector in two receive antenna.
- $\mathbf{H}_{k,l} = \begin{bmatrix} h_{k,l}^{1,1} & h_{k,l}^{1,2} \\ h_{k,l}^{2,1} & h_{k,l}^{2,2} \end{bmatrix}$ is the 2×2 channel fading matrix.
- $\mathbf{w}_k = \left(E_{kx} e^{j\varphi_{k,x}}, E_{ky} e^{j\varphi_{k,y}} \right)^T$ represents the Jones vector of the superimposed codewords in two orthogonal components.

TABLE 1. The MED-U and MED-SC in various codebooks.

Codebook size $M=4$	User 1	User 2	User 3	User 4	User 5	User 6	Average MED-U	MED-SC
Deka [19]	0.88	0.88	1.32	1.32	1.08	0.69	**1.03**	**0.94**
Chen downlink [17]	1.10	0.90	1.09	1.40	1.09	1.05	**1.10**	**0.92**
Huang [18]	1.01	1.10	0.55	0.09	0.67	0.18	**0.60**	**1.30**
Li [24]	0.71	0.71	0.68	0.68	1.30	1.29	**0.90**	**1.14**
Codebook 2	1.10	0.94	1.40	0.49	0.98	0.98	**0.98**	**1.04**
Codebook 3	0.56	0.56	1.40	1.40	1.40	1.40	**1.12**	**1.00**
Improve codebook 2	0.84	0.84	0.93	0.93	1.08	1.08	**0.95**	**1.08**
Improve codebook 3	0.81	0.81	1.08	1.08	1.08	1.08	**0.99**	**0.82**
3D codebook	0.93	0.93	1.55	1.55	1.59	1.59	**1.48**	**1.27**

- $\mathbf{z}_{k,l} = \left(z_{k,l}^1 \;\; z_{k,l}^2\right)^T$ denotes the 2×1 i.i.d. gaussian noise vector with noise variance N_0.

While at the receiver of the 3D polarized system, we have the likelihood function of the received signal $\mathbf{r}$, conditioned on the user's transmitted 3D symbols, formulating as

$$p\left(\mathbf{r}|\,\mathbf{H},\mathbf{x}\right) \propto \exp\left(-\frac{1}{N_0}\sum_{k=1}^{K}\left\|\mathbf{r}_{k,l}-\mathbf{H}_{k,l}\mathcal{G}\left(\sum_{l=1}^{L}x_{k,l}\right)\right\|^2\right). \tag{30}$$

And the message update rules are the same with the traditional MPA algorithm, passing belief messages between nodes.

C. CONSTRUCTION OF THE 3D SCMA CODEBOOKS

In reference to the design procedures in above section, the 3D SCMA codebooks can also be obtained. Firstly, two completely polarized $\mathcal{P} = 1$ benchmark constellations are adopted in this paper, which are shown in Fig. 2. For codebook size $M = 4$, the constellation has the equal Euclidean distance between any two points. As for high codebook size $M = 8$, the cube constellation is adopted.

Based on the provided 3D benchmark constellations, the 3D mother codebook can be constructed using the Tabu searching algorithm. While in the procedure of generating the constellation group, the optimization of the rotation degree is more complicated than traditional 2D codebooks since 3D rotation matrix introduces more degrees.

$$R_{S_2}\left(\theta_{S_2}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{S_2} & -\sin\theta_{S_2} \\ 0 & \sin\theta_{S_2} & \cos\theta_{S_2} \end{bmatrix}$$
$$R_{S_3}\left(\theta_{S_3}\right) = \begin{bmatrix} \cos\theta_{S_3} & 0 & \sin\theta_{S_3} \\ 0 & 1 & 0 \\ -\sin\theta_{S_3} & 0 & \cos\theta_{S_3} \end{bmatrix}$$
$$R_{S_1}\left(\theta_{S_1}\right) = \begin{bmatrix} \cos\theta_{S_1} & -\sin\theta_{S_1} & 0 \\ \sin\theta_{S_1} & \cos\theta_{S_1} & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{31}$$

Meanwhile, the joint optimization (20) in 3D SCMA system can be further written as

$$\left(\boldsymbol{\theta}_1,\boldsymbol{\theta}_2,\boldsymbol{\theta}_3\right) = \arg\max_{0\le\theta_{i,s_i}<2\pi}\left(\min\left(\mathcal{L}_1,\mathcal{L}_2,\cdots,\mathcal{L}_L\right)\right)$$
$$s.t.\; d_k = \sum_{\substack{l=1\\F_{k,l}\neq 0}}^{L} x_{k,l}R\left(\theta_{i,s_2}\right)R\left(\theta_{i,s_3}\right)R\left(\theta_{i,s_1}\right),\quad i=1,2,3$$
$$\boldsymbol{\theta}_i = \left(\theta_{i,s_2},\theta_{i,s_3},\theta_{i,s_1}\right)$$
$$\mathcal{L}_l = \sum_{k\in\Xi_l}\left|d_k-\tilde{d}_k\right|^2$$
$$\left|d_k-\tilde{d}_k\right| \ge \varepsilon$$
$$\left|x_{k,l}\right|^2 \le E_s. \tag{32}$$

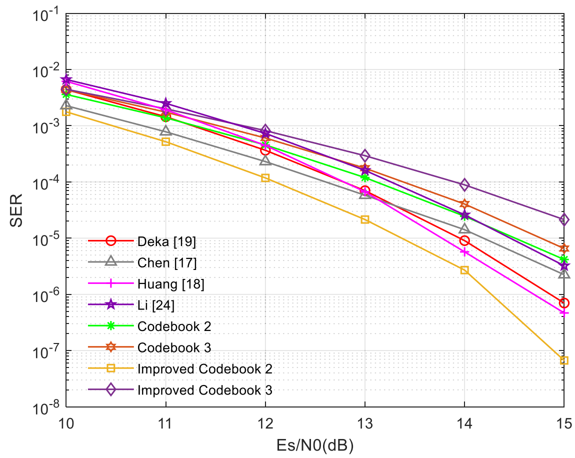


FIGURE 3. The SER performance of the codebooks in AWGN channel where codebook size M=4.

Different from 2D rotation procedures, the best angles in 3D SCMA codebook are searched in $[0, 2\pi)$. And the rotation procedures consist of three dimensions. Thus, searching the 3D best angles are more time-consuming than that in 2D codebooks. Here, we only generate the 3D SCMA codebooks with respect to Algorithm 2.

V. SIMULATION RESULTS

In this section, the simulation results are provided to show the performance of the proposed codebooks in AWGN and downlink Rayliegh fading channel respectively, and the iteration number of MPA is set as 10 times. Besides, Table 1 compares the MED-U and MED-SC in different codebooks where we

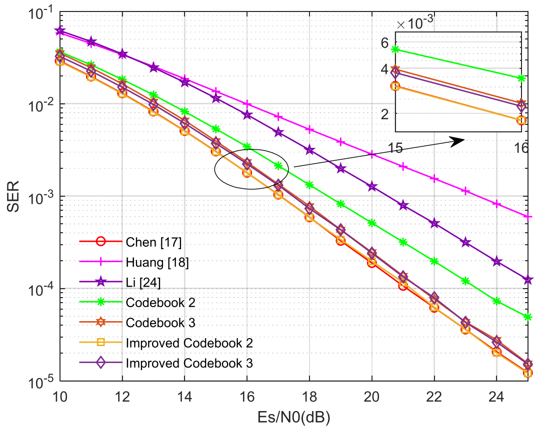


FIGURE 4. The SER performance of the codebooks in downlink rayleigh channel where codebook size M=4.

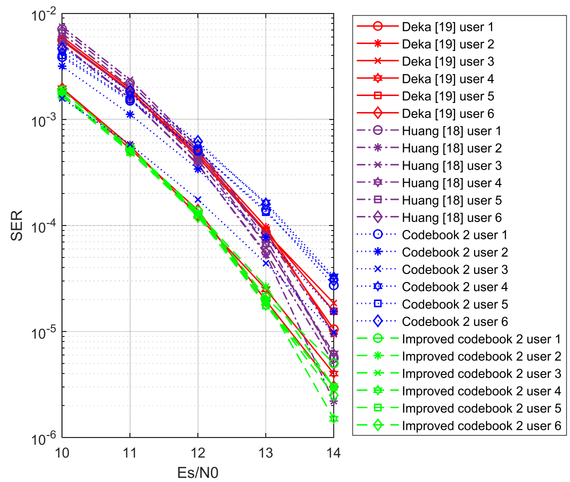


FIGURE 5. The SER performance of each user in various codebooks where codebook size M=4.

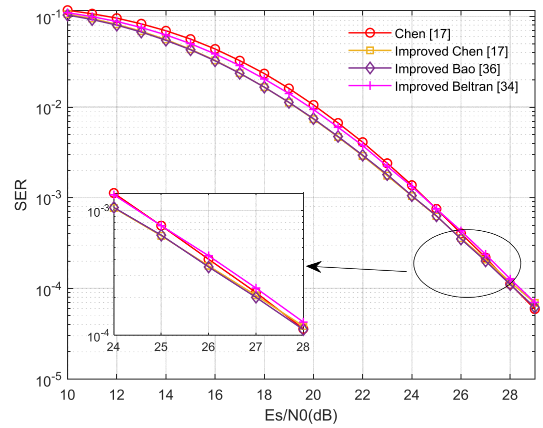


FIGURE 6. The SER performance of the modified mother codebooks in downlink rayleigh channel where codebook size M=8.

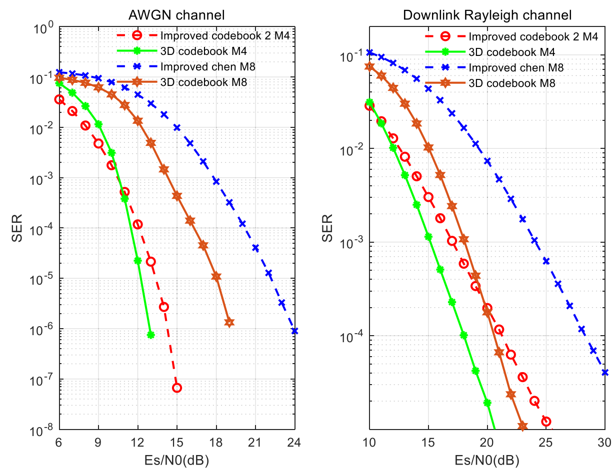


FIGURE 7. The SER performance of the proposed 3D SCMA codebooks.

suppose the transmitted codeword pairs in each two resources consists of three mother codebooks.

In Fig. 3, both codebook 2 and 3 achieve better SER performance in low SNR, while as the increase of SNR, the performance of these two codebooks become worse, which is because of the loss of MED-U and diversity gains when assigning the constellation sets in algorithm 2 and algorithm 3. Thus, the improved optimization is proposed to increase the MED-U of the worst user (from 0.49 to 0.84 in codebook 2 and 0.56 to 0.81 in codebook 3) and the diversity gains. Therefore, significant error performance improvement can be achieved after the improved optimization. And in Table 1, it can be observed that the joint optimization method enlarges the MED-SC of codebook 2, while decreases the MED-SC of codebook 3. Therefore, we can find that the improved codebook 2 has better error performance than the improved codebook 3. Besides, it can be noticed that the improved codebook 2 achieves the best error performance in the provided codebook though the MED-SC is not larger than Huang's codebook (1.08 to 1.30), And we believe that SCMA system can achieve better error performance if the MED-U is maximized to increase the belief of each node since the MPA also affect the performance of the system through the messages circles in the factor graph matrix.

Fig. 4 shows the SER comparison of the proposed codebooks in downlink Rayleigh fading channel. We can observe that the improved codebook generated by Algorithm 2 has nearly the same error performance with codebook [17]. And compared with previous codebook 2, a gain of 2dB can be observed at a SER of 10^{-4} in the improved codebook 2. As for codebook 3, the constellation sets distribution algorithm ensure the MED in one-dimensional resources, and codebook 3 has a larger MPD than codebook 2 (0.28 compared to 0). Thus, codebook 3 outperforms codebook 2 in downlink Rayleigh fading channels. Besides, although the average MED-U of all users in improved codebook 3 is decreased by the modified optimization, it further increases the MED in one-dimensional resources (from 0.28 to 0.40). Therefore, a slight error improvement can be noticed in downlink Rayleigh channel.

Besides, observing from Fig. 3 and Fig. 4, we can find that the improved codebook 2 has better performance in both AWGN and Rayleigh fading channels. Thus, this codebook design scheme and resources assignments method will be further employed in high codebook size. And the improved codebook 2 is provided in **Appendix I**.

In Fig. 5, we provided the performance of each user in the AWGN channel. In Table 1, the MED-U is not maximized for all users in codebook 2, thus, we can find that the fairness of the error performance among users cannot be guaranteed, and the same condition also occurred in Deka's codebook. Thus, improving the MED-U is important for improving the performance of each user. As for Huang's codebook, it can be noticed that the MED-SC is the largest in the provided codebook, while the MED-U varies among different users. And the users with small MED-U influence the nodes' beliefs and error performance of each user. Thus, we can observe that Hunag's codebook does not achieve the best error rate when compared with improved codebook 2.

$$\mathcal{X}_1 = \begin{bmatrix} 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i \\ 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i \\ 0.7071+0.7071i & 0.7071-0.7071i & -0.7071+0.7071i & -0.7071-0.7071i \\ -0.7071-0.7071i & -0.7071+0.7071i & 0.7071+0.7071i & 0.7071-0.7071i \end{bmatrix}$$

$$\mathcal{X}_2 = \begin{bmatrix} 0.7071+0.7071i & 0.7071-0.7071i & -0.7071+0.7071i & -0.7071-0.7071i \\ -0.7071-0.7071i & -0.7071+0.7071i & 0.7071+0.7071i & 0.7071-0.7071i \\ 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i \\ 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i \end{bmatrix}$$

$$\mathcal{X}_3 = \begin{bmatrix} 0.4384+0.8988\mathrm{i} & -0.8988+0.4384\mathrm{i} & -0.4384-0.8988\mathrm{i} & 0.8988-0.4384\mathrm{i} \\ 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i \\ 0.8988-0.4384\mathrm{i} & -0.4384-0.8988\mathrm{i} & 0.4384+0.8988\mathrm{i} & -0.8988+0.4384\mathrm{i} \\ 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i \end{bmatrix}$$

$$\mathcal{X}_4 = \begin{bmatrix} 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i \\ 0.4384+0.8988\mathrm{i} & -0.8988+0.4384\mathrm{i} & -0.4384-0.8988\mathrm{i} & 0.8988-0.4384\mathrm{i} \\ 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i \\ 0.8988-0.4384\mathrm{i} & -0.4384-0.8988\mathrm{i} & 0.4384+0.8988\mathrm{i} & -0.8988+0.4384\mathrm{i} \end{bmatrix}$$

$$\mathcal{X}_5 = \begin{bmatrix} -0.9877+0.1564\mathrm{i} & -0.1564-0.9877\mathrm{i} & 0.9877-0.1564\mathrm{i} & 0.1564+0.9877\mathrm{i} \\ 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i \\ 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i \\ 0.1564+0.9877\mathrm{i} & 0.9877-0.1564\mathrm{i} & -0.9877+0.1564\mathrm{i} & -0.1564-0.9877\mathrm{i} \end{bmatrix}$$

$$\mathcal{X}_6 = \begin{bmatrix} 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i \\ -0.9877+0.1564\mathrm{i} & -0.1564-0.9877\mathrm{i} & 0.9877-0.1564\mathrm{i} & 0.1564+0.9877\mathrm{i} \\ 0.1564+0.9877\mathrm{i} & 0.9877-0.1564\mathrm{i} & -0.9877+0.1564\mathrm{i} & -0.1564-0.9877\mathrm{i} \\ 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i & 0.0000+0.0000i \end{bmatrix}$$

$$\mathcal{X}_1 = \begin{bmatrix} 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j \\ 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j \\ 0.0000+0.0000i+1.0000j & 0.9428+0.0000i-0.3333j & -0.4714-0.8165i-0.3333j & -0.4714+0.8165i-0.3333j \\ 0.9428+0.0000i-0.3333j & 0.0000+0.0000i+1.000j & -0.4714+0.8165i-0.3333j & -0.4714-0.8165i-0.3333j \end{bmatrix}$$

$$\mathcal{X}_2 = \begin{bmatrix} 0.0000+0.0000i+1.0000j & 0.9428+0.0000i-0.3333j & -0.4714-0.8165i-0.3333j & -0.4714+0.8165i-0.3333j \\ 0.9428+0.0000i-0.3333j & 0.0000+0.0000i+1.000j & -0.4714+0.8165i-0.3333j & -0.4714-0.8165i-0.3333j \\ 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j \\ 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j \end{bmatrix}$$

$$\mathcal{X}_3 = \begin{bmatrix} 0.0668+0.6727i+0.7369j & 0.8411-0.5396i-0.0360j & -0.7774-0.6054i+0.1708j & -0.1305+0.4723i-0.8717j \\ 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j \\ 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j \\ 0.8411-0.5396i-0.0360j & 0.0668+0.6727i+0.7369j & -0.1305+0.4723i-0.8717j & -0.7774-0.6054i+0.1708j \end{bmatrix}$$

$$\mathcal{X}_5 = \begin{bmatrix} 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j \\ 0.0668+0.6727i+0.7369j & 0.8411-0.5396i-0.0360j & -0.7774-0.6054i+0.1708j & -0.1305+0.4723i-0.8717j \\ 0.8411-0.5396i-0.0360j & 0.0668+0.6727i+0.7369j & -0.1305+0.4723i-0.8717j & -0.7774-0.6054i+0.1708j \\ 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j \end{bmatrix}$$

$$\mathcal{X}_4 = \begin{bmatrix} -0.4852+0.8097i+0.3301j & 0.5524-0.3825i+0.7406j & -0.6623-0.6827i-0.3087j & 0.5951+0.2555i-0.7620j \\ 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j \\ 0.5524-0.3825i+0.7406j & -0.4852+0.8097i+0.3301j & 0.5951+0.2555i-0.7620j & -0.6623-0.6827i-0.3087j \\ 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j \end{bmatrix}$$

$$\mathcal{X}_6 = \begin{bmatrix} 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j \\ -0.4852+0.8097i+0.3301j & 0.5524-0.3825i+0.7406j & -0.6623-0.6827i-0.3087j & 0.5951+0.2555i-0.7620j \\ 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j & 0.0000+0.0000i+0.0000j \\ 0.5524-0.3825i+0.7406j & -0.4852+0.8097i+0.3301j & 0.5951+0.2555i-0.7620j & -0.6623-0.6827i-0.3087j \end{bmatrix}$$

Besides, the proposed scheme has preferable flexibility which can generate codebooks for the downlink Rayleigh channel from the previous mother codebooks by optimizing (20). Since the uplink Rayleigh codebook omits the procedure of rotation and only provides the mother codebooks. Thus, we can generate the downlink Rayleigh codebook based on the existing uplink Rayleigh codebooks. In Fig. 6, the SER performance is provided. We can notice that the modified codebooks have better performance than Chen's codebook [17], especially in lower SNR.

In addition to applying the proposed scheme in high codebook sizes, it can also be introduced in constructing the 3D SCMA codebook with the aid of 3D rotation matrix. We firstly provided the performance of the 3D SCMA codebooks under the same stokes modulation scheme. The 3D codebook is generated through algorithm 2 and the average MED-U achieves 1.48. In the AWGN channel, as shown in the left hand of Fig. 7, we can find that in low SNR, the 3D codebook has worse error performance due to the extra noise is introduced in Jones's vector when compared with 2D I/Q transmission scheme. However, with the increase of SNR, we can observe that the 3D SCMA codebook outperforms improved codebook 2, providing a 1.2dB gains and 3.8dB gains at SER$=10^{-5}$ when codebook size $M = 4$ and $M = 8$ respectively.

In downlink Rayleigh fading channel, the MED-SC determines the slope of the error probability plots of the system [44], and it can be found that the curve of the 3D codebook drops steeply than that of 2D codebook. Besides, the introduce of the space diversity in the polarized antenna also improve the SER performance, and a gain of around 3.2dB and 7.6dB can be achieved over $M = 4$ and $M = 8$ respectively at SER$=10^{-4}$ in comparison with 2D codebooks.

VI. CONCLUSION

In this paper, we proposed a novel codebook design scheme that achieves better error performance in AWGN and downlink Rayleigh channel. Rather than maximizing the MED-SC, more indicator factors are employed to construct the codebook. Besides, we show the flexibility and compatibility of the proposed scheme. The approach can be well applied in high codebook size with existing mother codebooks and for the construction of 3D SCMA codebook. Finally, simulation results are provided to illustrate the performance of the codebooks. And the codebook generated by Algorithm 2 is discussed in detail, the results show that the codebook can guarantee the SER of each user and achieve better performance. Besides, significant SER gains can be observed in 3D codebooks, which is also a promising scheme for the development of codebook design.

APPENDIX I
THE IMPROVED CODEBOOK OBTAINED BY ALGORITHM 2

$\mathcal{X}_1$–$\mathcal{X}_6$, as shown in the equation at the bottom of the previous page.

APPENDIX II
THE 3D SCMA CODEBOOK GENERATED BY ALGORITHM 2

$\mathcal{X}_1$–$\mathcal{X}_6$, as shown in the equation at the bottom of the previous page.

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