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THEORY

On Online Adaptive Direct Data Driven Control

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ABSTRACT Based on our recent contributions on direct data driven control scheme, this paper continues to do some new research on direct data driven control, paving another way for latter future work on advanced control theory. Firstly, adaptive idea is combined with direct data driven control, one parameter adjustment mechanism is constructed to design the parameterized controller online. Secondly, to show the input-output property for the considered closed loop system, passive analysis is studied to be similar with stability. Thirdly, to validate whether the designed controller is better or not, another safety controller modular is added to achieve the designed or expected control input with the essence of model predictive control. Finally, one simulation example confirms our proposed theories. More generally, this paper studies not only the controller design and passive analysis, but also some online algorithm, such as recursive parameter identification and online subgradient descent algorithm. Furthermore, safety controller modular is firstly introduced in direct data driven control scheme.

INDEX TERMS Adaptive direct data driven control, passive analysis, safety controller, online.

I. INTRODUCTION

Most current techniques for designing open loop or closed loop control systems are based on a good understanding of the considered plant under study and its related environment. But in some special instances, the plant to be controlled is too complex and the basic physical process within it are not fully observable. Control design techniques then need to be combined with an additional system identification process aimed at obtaining a nice understanding of the considered plant. It is thus defined as system identification and control, i.e. identification for control. Roughly speaking, two steps are taken separately. The first step is to apply system identification to identify one mathematical equation for the considered plant, so the plant model is periodically updated on the basis of previous estimates and new measured data-identification and control may be performed concurrently. Then the plant model or mathematical equation is benefit for the latter controller design or other interesting subjects, such as fault detection, target recognition, nonlinear analysis and structural validation etc. This control method based on the identified plant model is named as the model-based control, whose control performance depends on the plant model.

Although system identification could be aimed at determining if the plant to be controlled is linear or nonlinear, finite or infinite dimensional, and has continuous or discrete event dynamics. As the mission of system identification is to identify or construct one mathematical equation for the considered plant through some statistical methods, so as new idea was put forth in these recent years, i.e. can system identification be applied to design the unknown controller directly? It means the observed input-output data sequence are used to obtain the priori knowledge of the unknown controller without any intermediate system identification process, being called as direct data driven control. To describe the more detailed description about direct data driven control, consider one closed loop system structure with unknown plant and unknown controller simultaneously. That traditional model-based control firstly identifies the plant model, the secondly this identified plant model is for the next controller design. But for our considered direct data driven control, the unknown controller is devised from the observed input-output data sequence without the system identification process for that unknown plant. The feasible of this new control strategy is that lots of important and intrinsic information about the

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unknown controller are included in the observed data, then our mission is to abstract these intrinsic information for the unknown controller, while neglecting that unknown plant.

There are vast references about the research on direct data driven control from different points. More specifically, system identification is used to extract the intrinsic principle of the considered system, is the sample size [1]. In case of the number of observations be more exceed this sample size, then the input is persistent excitation, while the identification model satisfies the expected accuracy. From the knowledge of system identification theory, the situation with observed disturbance or noise in the output corresponds to the robust system identification [2], which being also extended to robust optimal control. When using the probabilistic or statistical inference in system identification theory in [3] to measure the asymptotic accuracy about the final identification model. Furthermore in recent years, risk sensitive theory and reinforce learning are all introduced in system theory and advanced control theory [4] and [5], i.e. the risk decision and limitations of policies were considered during the whole process of identification and controller design. Then the final identification system or plant is more realistic then classical theoretical result [6]. From these ongoing subjects about applying risk theory, dynamic programming and probabilistic limitation for system identification and control theory, we are thinking to extend graph theory and topology to system identification. More specifically, the second step-model structure choice is related with graph theory [7], i.e. the chosen model is constructed as one network system, being formulated as graph theory. System identification theory is not only for our considered aircraft system identification, but also for robot system identification in [8], where the detailed identification steps are all similar with each other, and only the considered plants are different. As lots of identification processed are transformed into their corresponding constrain optimization problems, so some existed optimization results can be applied directly, for example, convex optimization [9], scenario optimization [10], and scenario robust control [11], etc. Consider the last step for system identification-model validation, some nice properties are satisfied for the final identification model or designed controller, such as controllability, stochastic chance constraints, robustness and nonlinearity, which are seen in reference [10]. For that nonlinearity in system identification and control, nonlinear identification and nonlinear control are our ongoing work, whose plant and system is nonlinear form, not the simple linear form [12]. Roughly the research on nonlinear identification depends on neural network and other mathematical tools, being used to change the considered nonlinear plant to its approximated linear form, then the existed results about linear identification are all applied directly [13]. In our opinions, this linearized process is not good in practice, as it is the linear form that can not be used to replace the original nonlinear form. Can we find out one direct method to identify or design the nonlinear plant without the above linearized process? This problem is our studying case through topology [14]. Due to the closed relation between system identification theory, generally he step of experiment design concerns determining which physical quantities will be measured, how those quantities will be measured, what the test conditions will be and how the system being studied will be excited [15].

During these recent years, the first author studies this direct data driven control too, for example, the closed relation between system identification and direct data driven control [15], and data driven model predictive control [16]. A new interesting subject about persistently of excitation is studied again in data driven control and model predictive control.Willem's fundamental lemma from [17] gives a data based parametrization of trajectories for one linear time invariant system. Based on this Willem's fundamental lemma, one parametrization of linear closed loop system is derived to pave a way to study important controller design problems [18]. Reference [19] asserts that all trajectories of a linear time invariant system is obtained from a single given one on the condition that a persistently of excitation. One necessary and sufficient condition on the informativity of data is derived for some data driven control and analysis problems [20]. In recent years, more novel ways ar explored to develop direct data driven control, for example, the idea of data driven, mentioned above, is combined with model predictive control to yield a new control strategy-data driven model predictive control. In [21], data driven model predictive control is applied to design the classical PID for a deterministic continuous time system. For the case of switching controllers in some industries, data driven model predictive control is also benefit in regulating the switching rule [22]. Consider the uncertain factors exist in the closed loop situation, one robust data driven model predictive control is proposed to alleviate and suppress the bad effect, coming from these uncertainties [23].To be convenient for the use of data driven model predictive control, some existed softwares are produced for researchers, such as in python package [24] and in its intelligent form to control the heat treatment electric furnace [25].

By the way, in this early year our two new contributions propose new ways for direct data driven control. For example, paper [26] introduces an adaptive idea into direct data driven control to design the forward controller and feedback controller simultaneously. The parameter adjustment law is modified to satisfy Lyapunov stability. Then paper [27] combines model predictive control and direct data driven control to form our proposed direct data driven model reference control. Furthermore, some control performances are studied in that paper, such as stability validation, synthesis analysis and application engineering for flight simulation table, etc. Moreover during the conclusions of these two papers, we point out latter emphasis are concerned on adaptive direct data driven control, robust direct data driven control and learning direct data driven control for linear or nonlinear controller, so this new paper is our ongoing work about our previous contributions on direct data driven control scheme.

Based on above mentioned references and our previous contributions on direct data driven control, this new paper continues the study of adaptive direct data driven control from the point of online way, i.e. online adaptive direct data driven control. More generally, adaptive idea is a technique of applying some system identification technique to obtain a model of the process and its environment from input-output experiments and using this method to design a controller. The parameters of the controller are adjusted during the operation of the plant as the amount of data available for plant identification increases. For conciseness adaptive idea covers a set of techniques which provides a systematic approach for automatic adjustment of unknown controller in real time or online, in order to achieve or to maintain a desired level of control system performance when the parameters of the plant dynamic model are unknown and change in time.

After reviewing the physical principle of direct data driven control, firstly one parameter adjustment mechanism is constructed from the point of adaptive strategy, whose parameters correspond to the parameterized controller. To show the online property, the parameter estimation is derived online recursively to be our considered adaptive direct data driven control. Secondly, passive analysis about this control strategy is considered, as passivity properties or Lyapunov function with two terms are well suited for the stability analysis of feedback systems. The passivity approach is more natural and systematic, but the same results can be obtained by using Lyapunov functions of particular form. The passivity approach concerns input-output properties of systems and the implications of these properties for the case of feedback interconnection. We present a pragmatic approach without formal generalized and proven results concerning the use of the passivity approach for the analysis and the explanation of adaptive direct data driven control. Some conditions about power spectral and strictly positive real transfer function are derived for passive analysis. The above two contributions are related with the principle and stability analysis based on input-output property is the same with the idea of direct data driven control. Thirdly, the designed controller, obtained from our proposed adaptive direct data driven control, may be bad for the whole closed loop performance,so we add another module to change the former controller to be one good or appropriate controller. This added module is safety controller, while guaranteeing the designed controller approach to its desired or expected controller. This safe problem can be solved by virtue of model predictive control, i.e. one constrain optimization problem is established. Consider this constrain optimization problem with the unknown safety controller, online subgradient descent algorithm is proposed to yield one optimal safety controller, and some optimal analysis are also given.

Generally, the main contributions of this new paper are formulated as follows.

(1) Adaptive direct data driven control scheme is proposed, and one parameter adjustment mechanism is constructed.

(2) Passive analysis is studied for the input-output property, being similar to the classical Lyapunov stability.

(3) One additional safety controller module is added in the closed loop system structure, and the detailed process of solving this safety controller is given.

Moreover, online means two aspects:

(1) The unknown controller parameters are identified online within that parameter adjustment mechanism.

(2) Online subgradient descent algorithm is applied to get one optimal safety controller.

Direct data driven control appears based on the classical model based control, it is suited for the data science period. This paper adds one adaptive mechanism and apply the online subgradient descent algorithm, then its computational complexity is increased surely. But in data science times, it is tolerable due to some advanced physical devices.

This paper is organized as follows. In section 2, adaptive direct data driven control scheme is proposed to design the unknown controller for one closed loop system, while no any information of that unknown plant. For one parameterized controller, the parameter adjustment mechanism is constructed with online parameter estimation. Similar to the Lyapunov stability analysis, passive analysis is given in section 3, where some necessary conditions are shown to satisfy this input-output properties, i.e. passive property. To validate whether the designed controller be appropriate, one additional safety controller is added to pass through the designed controller, within the case of an desired or expected controller in section 4. The idea of model predictive control is introduced to solve that safety controller, while combining online subgradient descent algorithm. Section 5 uses one numerical example to illustrate the effectiveness of our considered online adaptive direct data driven control scheme. Finally, section 6 ends the paper with final conclusion and points out the next subject.

II. ADAPTIVE DIRECT DATA DRIVEN CONTROL

Data driven control is developed from the classical model based control,being satisfied for the requirement of data science times. it is well known that in our data science times, lots of data are easily yielded from some advanced physical devices or sensors. The important information about the considered plant or unknown controller are all included in these lots of data, so the mission of data driven control is to extract these important information for the unknown controller from the data. Whatever the important information exist in the data points explicitly or implicity, statistical method, machine learning or other reinforcement learning etc can be applied to extract these useful information as much as possible.

As two system structures exist around our normal life, i.e. open loop or closed loop system structure, here the considered closed loop system structure is plotted in the following Figure 1, due to its more complex than the single open loop system.

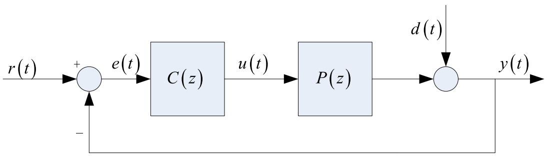

FIGURE 1. Closed loop system structure.

where in Figure 1, $r(t)$ is the external input signal, $y(t)$ is the output signal for the whole closed loop system, $u(t)$ is the input signal for the plant $P(z)$, $e(t) = r(t) - y(t)$ is the error value for controller $C(z)$, $d(t)$ is the external noise, such as while noise, color noise or bounded noise. z is time shift operator. For this unit feedback system, plant $P(z)$ and controller $C(z)$ are all unknown.

From Figure 1, some mathematical relations hold, for example

$$\begin{cases} y(t) = \dfrac{P(z)C(z)}{1+P(z)C(z)}r(t) + \dfrac{1}{1+P(z)C(z)}d(t) \\ u(t) = \dfrac{C(z)}{1+P(z)C(z)}r(t) + \dfrac{C(z)}{1+P(z)C(z)}d(t) \end{cases} \tag{1}$$

The difference between model-based control and direct data driven control in that input-output signal $\{r(t), y(t)\}$ are used for model-based control, but input-output signa $\{u(t), y(t)\}$ for direct data driven control. Here we do not mention the detailed process for model-based control and direct data driven control due to space limitations. Readers can refer to our previously published papers, listed in the Reference part. Roughly, direct data driven control collects those input-output signal $\{u(t), y(t)\}_{t=1}^{N}$ around the unknown controller $P(z)$, where N is the total number of signals, then it holds that.

$$u(t) = C(z)e(t) = C(z)[r(t) - y(t)] \tag{2}$$

so prediction error equation $\varepsilon(t)$ is that.

$$\begin{aligned} \varepsilon(t) &= C^{-1}(z)\{[C_0(z) - C(z)]r(t) \\ &\quad + [C_0(z) - C(z)]y(t)\} + y(t) \\ &= C^{-1}(z)[C_0(z) - C(z)][r(t) - y(t)] \end{aligned} \tag{3}$$

where $C_0(z)$ denotes the true controller, and it does not exist, only for convenience.

Consider one special case with respect to the parameterized controller. It means the unknown controller $C(z)$ is parameterized by one unknown parameter vector θ, then this unknown controller is rewritten as $C(z, \theta)$. Based on this parameterized controller $C(z, \theta)$, above equation (2) and (3) are modified as

$$\begin{aligned} u(t) &= C(z, \theta)[r(t) - y(t)] \\ \varepsilon(t) &= C^{-1}(z, \theta)[C_0(z) - C(z, \theta)][r(t) - y(t)] \end{aligned} \tag{4}$$

Minimizing mathematical operation $E\{\varepsilon^2(t)\}$ leads to the basic function of asymptotic bias distribution for the optimal controller parameter $\hat{\theta}$, i.e.

$$\begin{aligned} \hat{\theta} = \arg\min_{\theta} \int_{-\pi}^{\pi} & |C(z, \theta)|^{-2}[C_0(z) - C(z, \theta)]^2 \\ & \times (\phi_r(w) + \phi_y(w))dw \end{aligned} \tag{5}$$

where $\{\phi_r(w), \phi_y(w)\}$ are power spectral, corresponding to external input $r(t)$ and output signal $y(t)$ respectively.

This ideals case is that $C_0(z) = C(z, \theta)$, i.e. $\exists\theta_0$ such that $C_0(z) = C(z, \theta_0)$, then the above cost function (5) is zero. Consider equation (5) again, it is equal to that.

$$\begin{aligned} \hat{\theta} &= \arg\min_{\theta} J(\theta) = \arg\min_{\theta} \frac{|C_0(z) - C(z, \theta)|^2}{|C(z, \theta)|^2} \\ &= \arg\min_{\theta}[\frac{C_0(z) - C(z, \theta)}{C(z, \theta)}]^2 \\ J(\theta) &= [\frac{C_0(z) - C(z, \theta)}{C(z, \theta)}]^2 \end{aligned} \tag{6}$$

An online recursive algorithm is used to yield the optimal controller parameter through computing some partial derivations.

$$\begin{aligned} \frac{\partial J(\theta)}{\partial \theta} &= \frac{-\frac{\partial C(z,\theta)}{\partial \theta}C(z, \theta) - [C_0(z) - C(z, \theta)]\frac{\partial C(z,\theta)}{\partial \theta}}{C^2(z, \theta)} \\ &= -2[C_0(z)C^{-1}(z, \theta) - 1]C_0(z)C^{-2}(z, \theta)\frac{\partial C(z, \theta)}{\partial \theta} \end{aligned} \tag{7}$$

Making use of above partial derivation, an online recursive algorithm is formulated as follows.

$$\begin{aligned} \hat{\theta}(t+1) &= \hat{\theta}(t) - \frac{\partial J(\theta)}{\partial \theta}|_{\hat{\theta}(t)} \\ \frac{\partial J(\theta)}{\partial \theta}|_{\hat{\theta}(t)} &= -2[C_0(z)C^{-1}(z, \hat{\theta}(t)) - 1] \\ &\quad \times C_0(z)C^{-2}(z, \hat{\theta}(t))\frac{\partial C(z, \theta)}{\partial \theta}|_{\hat{\theta}(t)} \end{aligned} \tag{8}$$

where this online recursive algorithm is similar to the classical gradient algorithm.In equation (8), $\hat{\theta}(t+1)$ and $\hat{\theta}(t)$ are the optimal controller parameter at time instant $t+1$ and t respectively.

Consider one parameterized controller in the closed loop system structure, then the problem of designing controller is transformed into one problem of parameter estimation. This process of parameter estimation can be completed into one adaptive mechanism, being plotted in Figure 2.

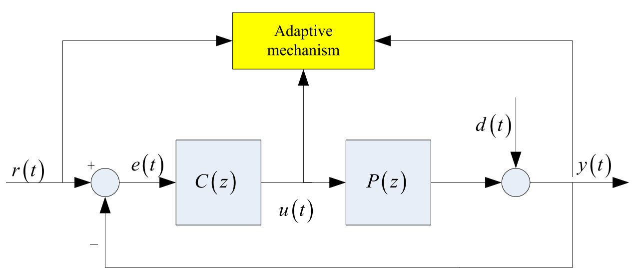

FIGURE 2. Online adaptive mechanism.

where in Figure 2, three kinds of signals $\{r(t), u(t), y(t)\}$ are all sent to that adaptive mechanism, then optimal controller parameter $\hat{\theta}$,generated by online recursively from equation (8), is substituted into that parameterized controller $C(z, \theta)$.

In that online recursive algorithm,i.e. equation (8), we can add one adaption gain $\eta(t)$ to consider the time varying property.

$$\hat{\theta}(t+1) = \hat{\theta}(t) - \eta(t)\frac{\partial J(\theta)}{\partial\theta}|_{\hat{\theta}(t)}$$

where above adaption gain $\eta(t)$ is a time varying factor, but in equation (8) this adaption gain is chosen as 1, i.e. $\eta(t) = 1$. If the forgetting property is considered, then this adaption gain can be chosen as one decreased function, for example, $\eta(t) = \frac{1}{t}$.

Comment: Due to the parameterized controller is considered here, then the controller design is turned to the parameter estimation. To testify whether the designed controller is good, we can check whether the closed loop system satisfies the desired system response or the identified parameters approach their true values. More specifically, assume the true controller parameter is θ_0, then latter we need to check whether the following equity is satisfied, i.e. $\hat{\theta}(t+1) \rightarrow \theta_0, t \rightarrow \infty$. But the case $t \rightarrow \infty$ is one ideal case, as the number of data point is always limit, so we relax this condition as that $\|\hat{\theta}(t+1)\| < 0.5$. It means one approximated controller parameter is used in practice.

III. PASSIVE ANALYSIS

After designing that controller $C(z)$ by our proposed adaptive direct data driven control scheme, stability consideration is needed as unstable system is useless.

A. ANALYSIS RESULT

Consider that plant $P(z)$ in Figure 1, define its input-output product as follows.

$$\eta_1(0,t) = \sum_{i=0}^{t} u(\tau)y(\tau) \tag{9}$$

Similarly define the input-output product for that controller $C(z)$ as that.

$$\eta_2(0,t) = \sum_{i=0}^{t} e(\tau)u(\tau) = \sum_{i=0}^{t}[r(\tau) - y(\tau)]u(\tau) \tag{10}$$

Before to do passive analysis for closed loop system in Figure 1, two definitions are needed.

Definition 1: Plant $P(z)$ is termed passive if

$$\eta_1(0,t) = \sum_{i=0}^{t} u(\tau)y(\tau) \geq -\gamma_1^2, \forall t > 0 \tag{11}$$

i.e. the input-output product between input $u(t)$ and output $y(t)$ satisfies the above inequality.

Definition 2: Controller $C(z)$ is termed passive if

$$\eta_2(0,t) = \sum_{i=0}^{t} e(\tau)u(\tau) \geq -\gamma_2^2, \forall t > 0 \tag{12}$$

Due to the following relations hold.

$$u(t) = C(z)[r(t) - y(t)]$$
$$y(t) = P(z)u(t) + d(t)$$

$$u(t) = C(z)[r(t) - P(z)u(t) - d(t)]$$
$$u(t) = C(z)r(t) - C(z)P(z)u(t) - C(z)d(t)] \tag{13}$$

It holds that

$$u(t) = \frac{C(z)}{1+P(z)C(z)}r(t) + \frac{C(z)}{1+P(z)C(z)}d(t) \tag{14}$$

substituting equation (14) into $\eta_2(0,t)$,we have

$$\eta_2(0,t) = \sum_{i=0}^{t} e(\tau)u(\tau) = \sum_{i=0}^{t}[r(\tau) - y(\tau)]u(\tau)$$
$$= \sum_{i=0}^{t} r(\tau)u(\tau) - \sum_{i=0}^{t} u(\tau)y(\tau) \geq -\gamma_2^2$$

$$\sum_{i=0}^{t} r(\tau)u(\tau) \geq \gamma_2^2 + \sum_{i=0}^{t} u(\tau)y(\tau) \geq -\gamma_1^2 - \gamma_2^2 \tag{15}$$

and

$$\sum_{i=0}^{t} r(\tau)u(\tau) = \sum_{i=0}^{t}\frac{C(z)r^T(\tau)r(\tau)}{1+P(z)C(z)} + \sum_{i=0}^{t}\frac{C(z)r^T(\tau)d(\tau)}{1+P(z)C(z)} \tag{16}$$

Making use of the property that externa, input $r(t)$ is independent of external noise $d(t)$, it holds that

$$\sum_{i=0}^{t} r^T(\tau)d(\tau) = 0 \tag{17}$$

so equation (16) is reduced to

$$\sum_{i=0}^{t} r(\tau)u(\tau) = \sum_{i=0}^{t}\frac{C(z)r^T(\tau)r(\tau)}{1+P(z)C(z)} \geq -\gamma_1^2 - \gamma_2^2$$
$$\frac{C(z)}{1+P(z)C(z)}\phi_r(w) \geq -\gamma_1^2 - \gamma_2^2 \tag{18}$$

i.e.

$$\phi_r(w) \geq \frac{(-\gamma_1^2 - \gamma_2^2)(1+P(z)C(z))}{C(z)} \tag{19}$$

Equation (19) shows input-output power spectral $\phi_r(w)$ must have one lower bound, while guaranteeing passive property.

Generally, let us consider the whole closed loop system with input $r(t)$ and output $y(t)$, the passive property is formulated in Theorem 1.

Theorem 1: Consider the feedback interconnection between plant $P(z)$ and controller $C(z)$, the input-output product must satisfy that.

$$\sum_{i=0}^{t} r(\tau)y(\tau) = \sum_{i=0}^{t} r(\tau)[\frac{P(z)C(z)}{1+P(z)C(z)}r(t)$$
$$+ \frac{1}{1+P(z)C(z)}d(t)]$$
$$= \frac{P(z)C(z)}{1+P(z)C(z)}\phi_r(w)$$
$$\geq \frac{P(z)C(z)}{1+P(z)C(z)}\frac{(-\gamma_1^2 - \gamma_2^2)(1+P(z)C(z))}{C(z)}$$
$$= (-\gamma_1^2 - \gamma_2^2)P(z) \tag{20}$$

The proof of Theorem 1 is very easily only through some basic substitution operations.

B. ONE SPECIAL CASE

When plant $P(z)$ is one special case of linear form, then output $y(t)$ is the output of a case of a linear function, characterized by a linear transformation, we have.

$$y(t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} P(e^{jw})U(e^{jw})e^{jtw}dw$$
$$U(e^{jw}) = \sum u(t)e^{-jtw} \tag{21}$$

The above integrated exists under the assumption that plant $P(z)$ is asymptotically stable.Then we have

$$\eta_1(0,t) = \frac{1}{2\pi}\sum_{t=0}^{\infty}\int_{-\pi}^{\pi} u(t)e^{jtw}P(e^{jw})U(e^{jw})dw \tag{22}$$

Interchanging the sum and the integral, one has

$$\begin{aligned}\eta_1(0,t) &= \frac{1}{2\pi}\sum_{t=0}^{\infty}\int_{-\pi}^{\pi} u(t)e^{jtw}P(e^{jw})U(e^{jw})dw \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\sum_{t=0}^{\infty} u(t)e^{jtw}P(e^{jw})U(e^{jw})dw \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} U(e^{jw})P(e^{jw})U(e^{jw})dw \\ &= \frac{1}{4\pi}\int_{-\pi}^{\pi} U(e^{jw})[P(e^{jw})+P(e^{-jw})]U(e^{jw})dw\end{aligned} \tag{23}$$

To satisfy the above passive property, it must hold that

$$\frac{1}{2}[P(e^{jw})+P(e^{-jw})] = ReP(e^{jw}) > 0 \tag{24}$$

IV. SAFETY CONTROLLER

Observing Figure 2 again, the designed controller $C(z)$ or its parameterized form $C(z,\theta)$ needs to be validated whether it is good or appropriate to make plant $P(z)$ work well. If it does,then this designed controller is accepted, or refuse it. To implement this controller validation process, another modular safe controller is added in closed loop system structure, plotted in Figure 3.

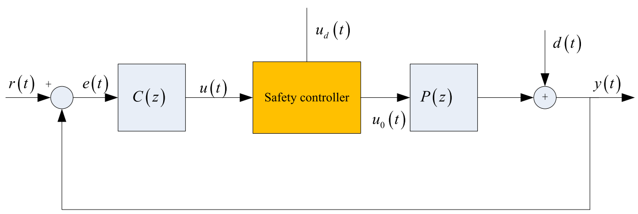

FIGURE 3. Safety controller modular.

where in Figure 3, safety controller modular is added. Its mission is to let the obtained control $u(t)$ approach to one desired or expected control $u_d(t)$. For example, assume plant $P(z)$ be an vehicle or UAV, vehicle must work in a limited range, i.e. some physical variables $y(t)$ are bounded. Then that desired control input $u_d(t)$ is safe, while providing an good input to keep an vehicle work in its limited range. This section gives a way of devising this safety controller modular based on the idea of model predictive control due to the desired control $u_d(t)$.

A. OPTIMALITY ANALYSIS

More precisely, this additional modular wants to guarantee the formal control $u(t)$ approach to its desired control $u_d(t)$, i.e. $u(t) \to u_d(t)$, so the following receding horizon problem is constructed.

$$\min_{\{u(t)\}_{t=1}^{N}} \sum_{t=1}^{N}[u(t)-u_d(t)]^2$$
$$subject\ to\ \begin{bmatrix} H_u(1) & H_u(2) & \cdots & H_u(N)\end{bmatrix}\begin{bmatrix} u(1) \\ u(2) \\ \vdots \\ u(N)\end{bmatrix} \leq h_u \tag{25}$$

define some vectors as follows

$$u = \begin{bmatrix} u(1) \\ u(2) \\ \vdots \\ u(N)\end{bmatrix};\quad u_d = \begin{bmatrix} u_d(1) \\ u_d(2) \\ \vdots \\ u_d(N)\end{bmatrix};$$
$$H_u = \begin{bmatrix} H_u(1)\ H_u(2) \cdots H_u(N)\end{bmatrix} \tag{26}$$

Then that receding horizon problem is reduced to one quadratic programming problem, i.e.

$$\min_u (u-u_d)^T(u-u_d)$$
$$subject\ to\ H_u u \leq h_u \tag{27}$$

where inequality constraint corresponds to the safety region. The following derivation result is used in equation (27).

$$\begin{aligned}&\sum_{t=1}^{N}[u(t)-u_d(t)]^2 \\ &= [u(1)-u_d(1)]^2 \\ &\quad + [u(2)-u_d(2)]^2 + \cdots + [u(N)-u_d(N)]^2 \\ &= \begin{bmatrix} u(1)-u_d(1) & u(2)-u_d(2) & \cdots & u(N)-u_d(N)\end{bmatrix} \\ &\quad \times \begin{bmatrix} u(1)-u_d(1) \\ u(2)-u_d(2) \\ \vdots \\ u(N)-u_d(N)\end{bmatrix} \\ &= (u-u_d)^T(u-u_d) \\ &= u^Tu - 2u^Tu_d + u_d^Tu_d\end{aligned}$$

Consider the quadratic programming problem (27) again, construct its Lagrange function to be that.

$$L(u,\lambda) = (u-u_d)^T(u-u_d) + \lambda(H_u u - h_u) \tag{28}$$

where λ is Lagrange multiply.

Applying the optimality KKT condition to satisfy that.

$$\frac{\partial L(u,\lambda)}{\partial u} = 2[u-u_d] + \lambda H_u = 0$$
$$\lambda(H_u u - h_u) = 0 \tag{29}$$

If Lagrange multiply $\lambda = 0$, then from the first equation of (29) we have $u = u_d$, i.e. perfect matching, so it is an idea case. In case of $\lambda > 0$, then from the second equation of equation (29), we have $H_u u = h_u$, i.e. $u = H_u^{-1} h_u$,where H_u^{-1} denotes inverse matrix operation.After substituting $u = H_u^{-1} h_u$ into $2[u - u_d] + \lambda H_u = 0$, it holds that $\lambda = -2(H_u^{-1} h_u - u_d) H_u^{-1}$. From above optimality analysis, if some constraints or working bounds are imposed on the plant, then the optimal or safety controller is determined on the constraint bound.

B. ONLINE SUBGRADIENT DESCENT ALGORITHM

For that quadratic programming optimization problem (27) with one quadratic cost function and one inequality constrain condition, for convenience to show the online subgradient descent algorithm, we combine (u, λ) as one unknown variable v, i.e.

$$v = [u \ \ \lambda]^T; \quad \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_N \end{bmatrix} \tag{30}$$

Then on the basis of optimization theory, our mission is to solve the optimal optimization variable v_* to guarantee that.

$$v_* = \min_v L(u, \lambda) = \min_v L(v) \tag{31}$$

Online subgradient descent algorithm is the following recurrence.

$$v_{t+1} = \pi_v(v_t - \gamma_t L'(v_t)) \tag{32}$$

where $\gamma_t > 0$ are stepsize, $\pi_v(v)$ is the standard projector on V, where V is one set, i.e. $v \in V$. $L'(v)$ is a subgradient of L at v, i.e.

$$L(w) \geq L(v) + (w - v)^T L'(v) \tag{33}$$

We always assume that $\int V \neq \phi$ and that the subgradients $L'(v)$ reported by the first order oracle at point $v \in V$. As subgradient operations $L'(v)$ are needed in that recursive form (33), here we derive some subgradients as follows.

$$\begin{aligned} \frac{\partial L(u, \lambda)}{\partial u(t)} &= 2[u(t) - u_d(t)] + \lambda_t H_u(t) = 0 \\ \frac{\partial L(u, \lambda)}{\partial \lambda_t} &= H_u(t) u(t) - h_u(t) = 0 \end{aligned}$$

To demonstrate the merit of online subgradient descent algorithm, the following two propositions are given to achieve it.

Proposition 1: After given the original value $v(0)$, the vector $e = v - \pi_v(v)$ forms an accute angle with every vector of the form $w - \pi_v(v)$, $w \in V$,i.e.

$$(v - \pi_v(v))^T (w - \pi_v(v)) \leq 0, \forall w \in V \tag{34}$$

Proof: Let $w \in V$, and $0 \leq t \leq 1$, we have

$$\begin{aligned} \phi(t) &= \|[\pi_v(v) + t(w - \pi_v(v))] - x\|_2^2 \\ &\geq \|\pi_v(v) - v\|_2^2 = \phi(0) \end{aligned} \tag{35}$$

Then

$$0 \leq \phi'(0) = 2[\pi_v(v) - v]^T (w - \pi_v(v)) \tag{36}$$

i.e.

$$(v - \pi_v(v))^T (w - \pi_v(v)) \leq 0 \tag{37}$$

In particular

$$\begin{aligned} \|w - v\|_2^2 &= \|w - \pi_v(v)\|_2^2 + \|\pi_v(v) - v\|_2^2 \\ &\quad + 2(\pi_v(v) - v)^T (w - \pi_v(v)) \\ &\geq \|w - \pi_v(v)\|_2^2 + \|\pi_v(v) - v\|_2^2 \end{aligned} \tag{38}$$

i.e.

$$\|w - \pi_v(v)\|_2^2 \leq \|w - v\|_2^2 - \|\pi_v(v) - v\|_2^2, \forall w \in V \tag{39}$$

Which completes the proof of the Proposition 1.

Proposition 2: For that subgradient descent algorithm, then for every $w \in V$, we have

$$\begin{aligned} \gamma_t (v_t - w)^T L'(v_t) \leq & \ \frac{1}{2} \|v_t - w\|_2^2 \\ & - \frac{1}{2} \|v_{t+1} - w\|_2^2 \\ & + \frac{1}{2} \gamma_t^2 \|L'(v_t)\|_2^2 \end{aligned} \tag{40}$$

Proof: By using above Proposition 1, we have.

$$\begin{aligned} d_{t+1} &= \frac{1}{2} \|v_{t+1} - w\|_2^2 \\ d_t &= \frac{1}{2} \|v_t - w\|_2^2 \end{aligned} \tag{41}$$

Then

$$\begin{aligned} d_{t+1} &\leq \frac{1}{2} \|[v_t - w] - \gamma_t L'(v_t)\|_2^2 \\ &= d_t - \gamma_t (v_t - w)^T L'(v_t) + \frac{1}{2} \|L'(v_t)\|_2^2 \end{aligned} \tag{42}$$

Summing up inequalities over $t = 1, 2, \cdots N$, we get

$$\sum_{t=1}^{N} \gamma_t (L(v_t) - L(w)) \leq d_1 - d_2 + \sum_{t=1}^{N} \frac{1}{2} \gamma_t^2 \|L'(v_t)\|_2^2 \tag{43}$$

Further it holds that

$$d_1 - d_N \leq \max_{w, v \in V} \frac{1}{2} \|w - v\|_2^2 \tag{44}$$

Then we have the following upper bound in equation (43).

$$\begin{aligned} & \max_{t \in [1, N]} L(v_t) - L_* \\ & \leq \frac{\max_{w, v \in V} \frac{1}{2} \|w - v\|_2^2 + \sum_{t=1}^{N} \frac{1}{2} \gamma_t^2 \|L'(v_t)\|_2^2}{\sum_{t=1}^{N} \gamma_t} \end{aligned} \tag{45}$$

The above inequality shows the convergence results for the online subgradient descent algorithm.

V. SIMULATION EXAMPLE

Consider one closed loop system structure in Figure 1, where plant is

$$P(z) = \frac{(z-1.5)(z-0.5)}{z(z-0.3)(z-0.6)} \quad (46)$$

The true controller $C_0(z)$ is one parameterized controller with a sequence of orthogonal basis function, i.e.

$$C_0(z) = \begin{bmatrix} \frac{z^4}{z^4-z} & \frac{z^3}{z^4-z} & \frac{z^2}{z^4-z} & \frac{z}{z^4-z} & \frac{1}{z^4-z} \end{bmatrix} \times \begin{bmatrix} 0.35 \\ 0.24 \\ 0.13 \\ 0 \\ -0.05 \end{bmatrix} \quad (47)$$

Its parameterized form is that.

$$C(z,\theta) = \begin{bmatrix} \frac{z^4}{z^4-z} & \frac{z^3}{z^4-z} & \frac{z^2}{z^4-z} & \frac{z}{z^4-z} & \frac{1}{z^4-z} \end{bmatrix} \times \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix};$$

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix} \quad (48)$$

During the whole simulation process, let that external noise $d(t)$ be one white noise sequence with zero mean and variance 1. Input-output measured data sequence $\{u(t), y(t)\}_{t=1}^{N}$ are collected in the closed loop experiment condition, and the total number $N = 1000$. Before to start our simulation example, one kind of input signal $r(t)$, plotted in Figure 4, is used to excite the whole closed loop system, then some physical devices are placed to collect the observed output signal $y(t)$,being plotted in Figure 5. From Figure 4, the excitation input is chosen as the commonly used square eave signal, which always satisfied the requirement of persistent excitation, as it can excite all the internal property of the considered system. after the square wave signal is applied, then some physical devices or sensors are used to collect the output response, being shown in Figure 5.

The mission of direct data driven control is to extract some information about the parameterized controller from the input-output data sequence $\{u(t), y(t)\}_{t=1}^{N}$. Due to the parameterized controller $C(z,\theta)$ exists, our mission is changed to identify that unknown parameter vector θ, existing in the unknown parameterized controller. The whole parameter identification process is similar to the data fitting problem, whose parameter estimations are given online recursively from equation (8). Before to implement the recursive algorithm, the initial parameter vector is chosen as $\theta(0) = (0.1\ 0.1\ 0.1\ 0.1\ 0.1)^T$. After 50 steps, we terminate the

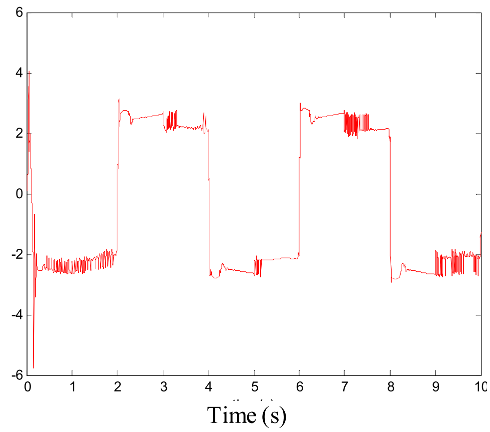

FIGURE 4. The applied input signal.

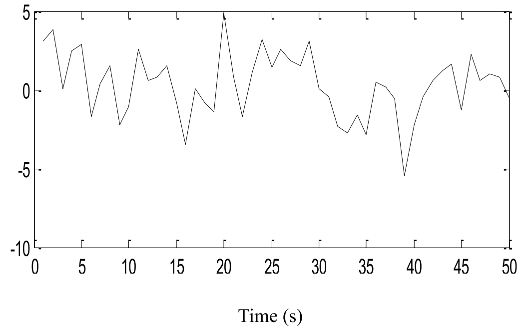

FIGURE 5. The observed output signal.

identification algorithm. The final identification results, i.e. controller parameters are shown in Figure 6.

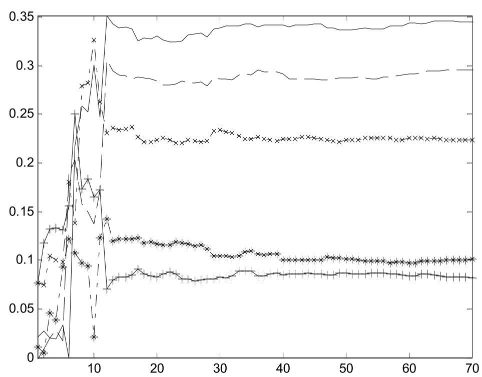

FIGURE 6. Controller parameter estimations.

Comparing the controller parameter curves in Figure 6 and their true controller parameters in equation (47), we see although some deviations in 10 steps, but finally all parameter estimations will converge to their own true values.

To testify the safety controller modular, the designed controller output is $u(t) = C(z,\theta)e(t) = C(z,\theta)[r(t) - y(t)]$. Suddenly, we set one desired or expected controller output be that $u_0(t) = C(z,\eta)e(t) = C(z,\eta)[r(t) - y(t)]$, where the

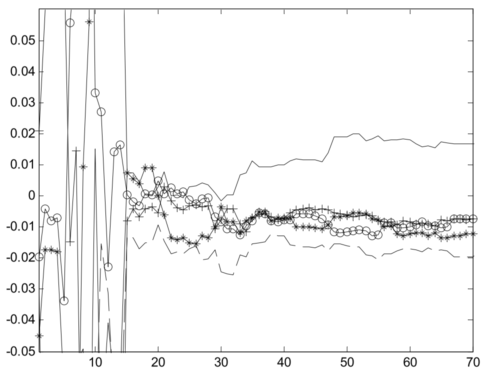

FIGURE 7. Safety controller parameters.

desired controller $C(z, \eta)$ is as follows.

$$C(z,\eta) = \begin{bmatrix} \frac{z^4}{z^4-z} & \frac{z^3}{z^4-z} & \frac{z^2}{z^4-z} & \frac{z}{z^4-z} & \frac{1}{z^4-z} \end{bmatrix} \times \begin{bmatrix} 0.28 \\ 0.12 \\ 0.05 \\ -0.02 \\ -0.06 \end{bmatrix};$$

$$\eta = \begin{bmatrix} 0.28 \\ 0.12 \\ 0.05 \\ -0.02 \\ -0.06 \end{bmatrix} \tag{49}$$

so the task about that safety controller modular is switch the former controller $C(z, \theta)$ to the new controller $C(z, \eta)$. To achieve this sudden switch movement, one receding horizon problem is established to achieve the perfect matching, while guaranteeing the bounded inequality, i.e. $u \in [-5, 5]$. The switch result is seen in Figure 7, where the safety controller parameters coincide with their true values in equation (49).

Observing Figure 6 and 7 again, the horizontal axis is the iterative step, and the vertical axis is the parameter estimation, corresponding to the controller. Firstly in Figure 6, each controller parameter is identified by virtue of our considered online subgradent descent algorithm. Each curve corresponds to each controller parameter. During the 10 iterative steps, the parameter estimations are biased, then after 10 iterative steps, all parameter curves converge to their true values, for example, $\theta_1 \rightarrow 0.35$. Secondly, safety requirement is considered in Figure 7. Specifically, when the safety property is considered, then the controller parameters must be changed automatically with the variety of the safety. This continuous modified process is reformulated as follows.

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix} \rightarrow \begin{bmatrix} 0.35 \\ 0.24 \\ 0.13 \\ 0 \\ -0.05 \end{bmatrix} \rightarrow \eta \rightarrow \begin{bmatrix} 0.28 \\ 0.12 \\ 0.05 \\ -0.02 \\ -0.06 \end{bmatrix}$$

VI. CONCLUSION

In this paper, adaptive direct data driven control is proposed from the theory and application. Adaptation shows one parameter adjustment mechanism and online recursive parameter identification are combined to devise the unknown controller parameter. Another input-output property, i.e. passive property is analyzed to grasp the closed relation between the input and output. Controller validation is proposed as a safety controller modular by virtue of the principle about model predictive control. During this new paper, new theories on direct data driven control are considered to pave a new road for future work, for example, learning direct data driven control and robust adaptive direct data driven control etc. For example, nonlinearity is one important factor in practice, but it is very difficult to analyze nonlinearity directly. Now the commonly used method for analyzing nonlinearity is to linearized that nonlinear system, so one simplified linear system is obtained. Then all existed knowledge about linear system theory can be applied directly. In our opinion, new strategy is needed to considered that nonlinearity directly, being our next research point.

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