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PERSPECTIVE

A Review and Perspective on Neutrosophic Statistical Process Monitoring Methods

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ABSTRACT We review the literature on statistical process monitoring methods based on neutrosophic principles. We question some of the underlying assumptions and raise important questions about these and other neutrosophic statistical methods that need to be addressed before the methodology could be taken seriously.

INDEX TERMS Control chart, exponentially weighted moving average (EWMA) chart, interval arithmetic, interval data, Shewhart control chart.

I. INTRODUCTION

Neutrosophic methods were first proposed by Smarandache [1]. As stated by Khan *et al.* [2], "The idea of neutrosophic sets is a broader platform that expands the notions of the fuzzy and classical sets." Neutrosophic statistics was said by Albassam and Aslam [3] to be a generalization of traditional statistics that is used to analyze uncertain, unclear, vague, and incomplete data. Many neutrosophic statistical methods have been proposed, but in our paper we focus on neutrosophic statistical process monitoring (NSPM) methods. Some of the issues we raise, however, extend to other types of neutrosophic statistical methods, many of which have been proposed, including multiple regression analysis [4], analysis of variance [5], [6], forecasting [7], [8], [9], acceptance sampling plans [10], [11], and cluster analysis [12].

Statistical process monitoring is based on samples of process data collected over time. Historical data are collected in Phase I in order to understand the process variability, to identify opportunities for process improvement, and, if stability is achieved, to fit a statistical model. Important Phase I issues were discussed by Jones-Farmer *et al.* [13]. Data are collected sequentially in Phase II in order to detect changes from the baseline model fitted in Phase I. Only Phase II methods are considered in our paper.

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It is assumed in NSPM that the observed data are interval-valued. Typically, the sample sizes and the control chart design constants are also assumed to be interval-valued. It is often assumed that one is sampling from a particular neutrosophic probability distribution, i.e., a distribution for which the parameter, or parameters, are interval-valued. Many neutrosophic control charts have been proposed. After our review of these methods, we question some of the underlying assumptions and raise important questions about these and other neutrosophic statistical methods that need to be addressed before the methodology could be taken seriously.

Neutrosophic control charts for attribute data have been proposed by Aslam [14], Aslam *et al.* [15], Albassam and Aslam [3], and Aslam *et al.* [16]. These methods are designed for monitoring proportions based on sampling from a binomial distribution where the sample sizes, counts of defective items and possibly the probability of a defective item are all imprecisely known and represented by intervals. Aslam [14] modified the basic neutrosophic approach of Aslam *et al.* [15] to allow repetitive sampling, i.e., additional samples if the neutrosophic control chart statistic at a given time was not outside the neutrosophic control limits or sufficiently close to the neutrosophic centerline. Albassam and Aslam [3] incorporated runs rules into the neutrosophic p-chart while Aslam *et al.* [16] used an exponentially weighted moving average (EWMA) chart approach.

Khan *et al.* [17] proposed a neutrosophic u-chart for monitoring count data along with a neutrosophic p-chart for monitoring proportions. We note, however, that the sample sizes of six or fewer items used in their application of the neutrosophic p-chart were much too low for use of a p-chart to be appropriate. Ercan-Teksen [18] proposed a neutrosophic c-chart while Aslam *et al.* [19], [20] proposed neutrosophic methods for monitoring the COM-Poisson distribution.

With continuous data, one needs to monitor the mean and the variation of the variable of interest. Control charts for monitoring the mean were proposed by Aslam and Khan [21], Aslam [22], and Aslam *et al.* [23]. Aslam [22] modified the neutrosophic approach of Aslam and Khan [21] to allow repetitive sampling. Aslam *et al.* [23] used a moving average chart approach while Aslam *et al.* [24] proposed a neutrosophic cumulative sum (CUSUM) chart. Aslam *et al.* [25] proposed an EWMA chart for monitoring the process mean. Shafqat *et al.* [26] proposed double and triple EWMA methods. Typically it was assumed in these papers that sampling was from the neutrosophic normal distribution. The data, sample sizes, control chart limit multipliers, and target mean were assumed to be known imprecisely and represented by intervals.

Under similar assumptions, methods for monitoring the variance were proposed by Aslam *et al.* [27], Aslam *et al.* [28], Aslam [29], Khan *et al.* [30], [31], and Khan *et al.* [32]. In particular, Aslam *et al.* [28] used an EWMA chart approach whereas Aslam [29] incorporated the repetitive sampling feature. Khan *et al.* [32] incorporated multiple dependent state sampling, i.e., the use of runs rules.

Almarashi and Aslam [33], Shawky *et al.* [34], and Aslam *et al.* [35] developed monitoring methods under the assumption of an underlying neutrosophic gamma distribution while Khan *et al.* [2] assumed an underlying neutrosophic Rayleigh distribution. Aslam *et al.* [36] and Arif *et al.* [37] assumed an underlying neutrosophic Weibull distribution. Neutrosophic Hotelling T^2 control charts based on multivariate data were proposed by Alsam and Arif [38] and Wibawati *et al.* [39].

In Section II we consider a process monitoring example based on the neutrosophic Rayleigh distribution so that readers not familiar with neutrosophic methods can more easily understand the approach and some of our questions about it. This example and other work on NSPM led us to the issues and comments discussed in Section III. Our conclusions follow in Section IV.

II. NSPM EXAMPLE BASED ON RAYLEIGH DISTRIBUTION

Khan *et al.* [2] proposed a method for monitoring with an underlying neutrosophic Rayleigh distribution. The parameter of this distribution is a neutrosophic number. The neutrosophic Rayleigh distribution (RD_N) with imprecision in the scale parameter θ_N was said to have the following characteristics:

$$f(Z,\theta_N) = \frac{z}{\theta_N^2} e^{-\frac{1}{2}(\frac{z}{\theta_N})^2} \tag{1}$$

$$F(Z,\theta_N) = 1 - e^{-\frac{1}{2}(\frac{z}{\theta_N})^2,} \tag{2}$$

where $\theta_N > 0, Z > 0, \theta_N \in [\theta_l, \theta_u], f(Z,\theta_N)$, and $F(Z,\theta_N)$ represent the neutrosophic scale parameter, the neutrosophic density function (PDF_N) and cumulative distribution function (CDF_N) of the RD_N. Based on the neutrosophic version of the Rayleigh distribution, the mean and variance of the neutrosophic random variable Z were given respectively as

$$\mu_N = \theta_N\sqrt{\frac{\pi}{2}} \text{ and } \sigma_N^2 = \theta_N^2(2 - \frac{\pi}{2}). \tag{3}$$

The graphical displays of $f(Z,\theta_N)$, and $F(Z,\theta_N)$ for the neutrosophic Rayleigh random variable Z with imprecise scale parameter $\theta_N = [0.5, 0.75]$ are shown in Fig. 1, which was reproduced from Khan *et al.* [2].

From the general description, it seems that the underlying distribution could be any Rayleigh distribution with a parameter in the interval $[\theta_l, \theta_u]$. In constructing their control chart, Khan *et al.* [2] proposed neutrosophic estimators of the in-control parameter value, an estimator of the parameter at each sampling point, and then derived their neutrosophic distributions. Probability-based control limits were based on an underlying neutrosophic chi-distribution. In the neutrosophic approach, there are intervals for each control limit, on the center line of the control chart and for the plotted statistics. Their method was justified using neutrosophic power curves and neutrosophic average run length (ARL) profiles, where the ARL is the expected number of samples until a control chart signal is given. An ARL curve reproduced from Khan *et al.* [2] is shown in Fig.2 where α is the probability of a false alarm and m represents the sample size.

The neutrosophic Shewhart control chart resulting from the data in their example is shown in Fig. 3. The control limits and chart statistic are represented by intervals at each sampling time.

III. QUESTIONS AND CONCERNS

In our view there are some important issues to be addressed with respect to the neutrosophic methodology. Some of our questions apply to neutrosophic statistical methods in general while other questions and concerns are more focused on NSPM issues. We have the following comments and questions:

A. NEUTROSOPHIC MATHEMATICS

It is imperative to clarify the rules of neutrosophic arithmetic and extensions of the basic neutrosophic distribution theory discussed by Alhabib *et al.* [40]. It is not clear, for example, how to add an indeterminate number of neutrosophic numbers.

According to Zhang *et al.* [41], the neutrosophic rules of arithmetic for interval-valued data are the same as those

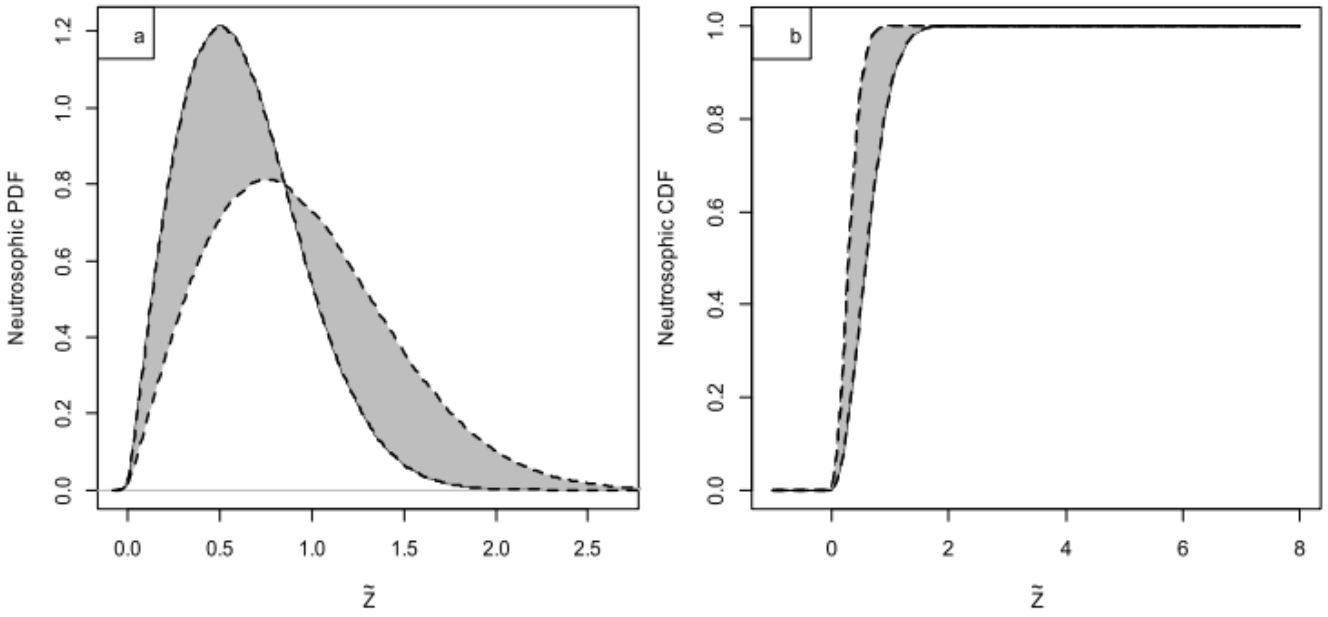

FIGURE 1. Neutrosophic a) pdf and b) cdf for the neutrosophic Rayleigh random variable with imprecise scale parameter $\theta_N = [0.5, 0.75]$.

used in interval arithmetic for putting bounds on rounding errors and measurement errors in mathematical computations. In particular, we have the following definitions from Zhang *et al.* [41]:

> Consider any two interval numbers, $\tilde{a} = [a^L, a^U]$ and $\tilde{b} = [b^L, b^U]$, and then their operations are defined as follows:
>
> 1) $\tilde{a} = \tilde{b} \Leftrightarrow a^L = b^L, a^U = b^U$,
> 2) $\tilde{a} + \tilde{b} = [a^L + b^L, a^U + b^U]$,
> 3) $\tilde{a} - \tilde{b} = [a^L - b^L, a^U - b^U]$,
> 4) $\tilde{a} \times \tilde{b} = [\min(a^L b^L, a^L b^U, a^U b^L, a^U b^U), \max(a^L b^L, a^L b^U, a^U b^L, a^U b^U)]$,
> 5) $k\tilde{a} = [ka^L, ka^U]$, $k > 0$.

Interval arithmetic originated with Young [42]. More details can be found in Moore [43], Moore *et al.* [44] and IEEE Standards Association [45]. Under this framework the endpoints of the calculated intervals represent extremes of what could be calculated given all possible combinations of values in the various input intervals. We note that the neutrosophic sample mean and variance for interval-valued observations proposed by Smarandache [46, pp. 31-33] are calculated differently from the sample mean and variance calculated using interval statistics [47].

The basic rules for arithmetic given by Smarandache [46, pp. 31-33] do not match the rules given by Zhang *et al.* [41]. Smarandache [46] expressed neutrosophic numbers in the form $a + bI$, where a and b are real numbers, and I represents the indeterminacy interval such that $I^2 = I$ and $0 \cdot I = 0$. Thus the interval neutrosophic number $[4, 6]$ could be represented as $4 + 2I$. Smarandache [46] calculated the average of two neutrosophic numbers, say $a + bI$ and $c + dI$, as $(a + c)/2 + [(b + d)/2]I$.

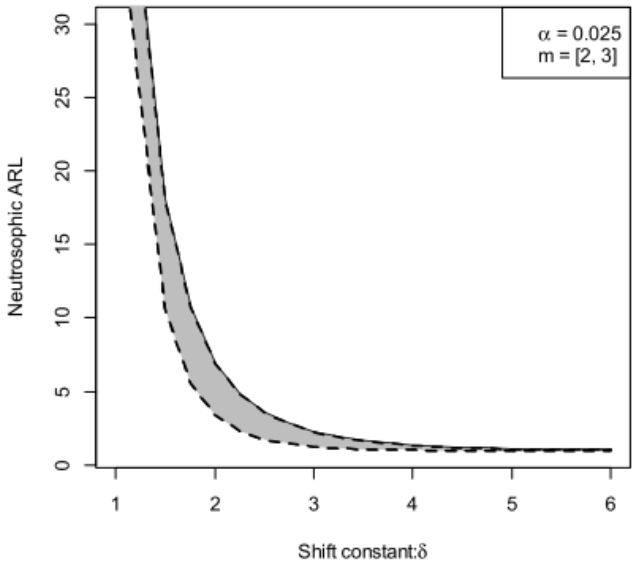

FIGURE 2. A neutrosophic ARL curve.

As an example, consider the two neutrosophic numbers $[4, 6]$ and $[2, 4]$ represented as $4+2I$ and $4-2I$, respectively. Then using the approach of Smarandache [46], the average of these two neutrosophic numbers would be $4 + 0I$, or simply the precise value 4. This result does not seem reasonable. Using the approach of Zhang *et al.* [41] and interval arithmetic, however, the interval for the average would be $[3, 5]$. We consider the interval arithmetic approach to lead to the much more useful and realistic results. As a reviewer pointed out, the neutrosophic approach would also yield an average of $[3, 5]$ under the restriction that the coefficient of I is non-negative.

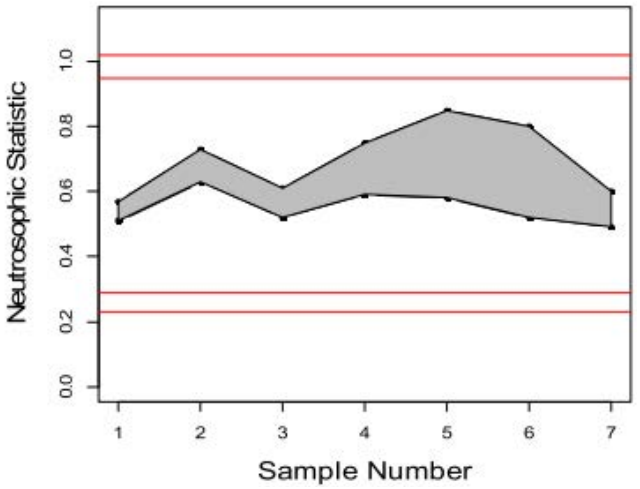

FIGURE 3. A neutrosophic control chart. Reproduced from Khan *et al.* [2].

Since the control limits and the plotted statistic values are all neutrosophic, the rule for declaring the process to be out-of-control needs to be clearly expressed. An out-of-control signal was given by Khan *et al.* [17] when only one limit of the interval representing the neutrosophic control chart statistic fell completely outside a neutrosophic control limit interval. Concern was expressed, however, if just one of the limits for the control chart statistic interval exceeded any control chart limit. To understand the signaling rules and the simulations performed to study neutrosophic chart performance, it is important to clarify the use of inequalities for interval data.

In order to understand neutrosophic mathematics, it would be helpful to see an example of control chart construction worked out in detail, preferably with the computer code being provided.

B. INTERVAL STATISTICAL METHODS

There is an extensive body of literature on statistical methods based on data points represented as intervals. See, for example, Gioia, and Lauro [47], Brito [48], and the references in Zhang and Lin [49]. We note that NSPM researchers do not refer to interval statistical methods despite their very strong similarities with neutrosophic statistical methods. We see value in taking the interval statistical approach in the design and analysis of control charts when the data are interval-valued, but with the sample sizes and the control chart design parameters known precisely. The resulting center lines and control limits would be very conservative, however, given that the endpoints would represent extreme scenarios for the values within the intervals representing the observations.

C. NEUTROSOPHIC SAMPLE SIZES

Having the sample sizes given as imprecise can have a large effect on the resulting widths of the neutrosophic control limit intervals, the neutrosophic power curves, and the neutrosophic ARL curves. Under the neutrosophic framework it is typically assumed that the sample sizes are not known precisely even after the sample is collected. With NSPM methods the level of imprecision in the sample sizes is assumed to be maintained over time. We know of no application in quality control, however, where the sample size would not be known precisely after the sample is collected. The examples involving imprecise sample sizes given in Smarandache [46] all involve attribute data without carefully expressed operational definitions. It seems impossible to have a sample of variables data without knowing the sample size.

Aslam [14] considered the monitoring of attribute data with neutrosophic sample sizes in some cases represented by $[50, 150]$ and $[20, 200]$. This represents an extreme amount of imprecision. Having the sample size be represented by the neutrosophic intervals $[3, 5]$ or $[2, 4]$, as in Khan *et al.* [2], causes their control limits to be 29% and 41% wider at the lower sample size compared to the width at the higher sample size, respectively. The width of control limits tends to be inversely proportional to the square root of the sample size. Note, however, that in all the case study examples involving variables data presented in the neutrosophic literature, the sample sizes are assumed to be known precisely.

The use of neutrosophic sample sizes is made more explicit in the neutrosophic acceptance sampling literature. Aslam [10] considered an example with $n_N = [26, 34]$ and simply chose the sample size to be within this interval. In particular, he selected $n = 28$. In this situation, however, the sample size is known and the neutrosophic approach to the sample size would not be needed. This approach of selecting a sample size to be some value in the interval was also used by Aslam [11]. If the sample size is randomly or arbitrarily selected to be a fixed value within the neutrosophic interval, there is no reason to base the further calculations on the neutrosophic sample size interval or to use the neutrosophic representation of the sample size at all.

Sample sizes in statistical process monitoring applications often vary, especially when monitoring proportions, but the sample sizes are always assumed to be known. There are standard control charting methods for handling variable sample sizes, as described, for example, by Montgomery [50].

D. NEUTROSOPHIC CONTROL CHART CONSTANTS

The control chart limit multiplier with the various neutrosophic Shewhart control charts is always considered to be a neutrosophic number. With the neutrosophic EWMA charts of Aslam *et al.* [16], [25], [28], both the smoothing parameter and the control limit multiplier are neutrosophic numbers. Aslam *et al.* [23] considered the span of a moving average used in a moving average chart to be unknown precisely. Since the EWMA smoothing parameter and the moving average span are both selected by the practitioner, these values would be known precisely. Thus it is unclear why they are represented as neutrosophic numbers. We see no advantage in considering the control limit multiplier to be imprecise. Representing control chart limit multiplier and other control chart design constants as neutrosophic numbers adds

unnecessary imprecision to the control chart limits and the control chart statistics.

E. MEASUREMENT ERROR AND ROUNDING IN SPM

There would always be at least some small amount of imprecision in variables data, such as with weight and dimensional measurements. Variability will always be present in variables or measurement data due to the presence of common cause variability (or background process noise), possible assignable causes, measurement error, rounding, and data recording or transmission errors. Maleki *et al.* [51] provided a review of the literature on the effect of measurement error on process monitoring methods.

The effect of rounding on control chart performance was discussed by Tricker *et al.* [52], [53], Meneces *et al.* [54], [55] and Wheeler [56]. With rounding, the implicitly defined intervals would be of equal length with the analysis typically based on the midpoint of each observed interval. With variables data it would be of interest to compare the performance of neutrosophic control charts for monitoring the mean and variance to that of charts based on using as input the middle value of each data interval.

The monitoring methods proposed by Steiner *et al.* [57], [58] and Steiner [59] are based on the collection of interval data, but under the assumption that there is a fixed set of possible interval endpoints resulting from gauging or rounding.

F. GENERATING RANDOM NEUTROSOPHIC DATA

In order to generate random neutrosophic data, the endpoints of random intervals could be generated from neutrosophic cumulative distribution functions such as the one shown on the right-hand-side of Figure 1 by using the inverse probability integral transformation. It is not as clear, however, how random intervals could be simulated from the neutrosophic normal distribution when the mean and variance are both neutrosophic numbers. One possibility is to use the inverse probability integral transformation for the four resulting cdfs, and picking the interval endpoints as the smallest and largest values of the four values obtained.

G. SOME TECHNICAL POINTS

As a technical point, all of the ARL calculations for the EWMA charts in Aslam *et al.* [25], [28] and Khan *et al.* [60] are incorrect. These authors used the asymptotic normal distribution of the EWMA statistic, but ignored the effect of the control limits. In addition, they ignored the dependence of the EWMA statistics over time to use a geometric distribution for the run length. Similar errors were made in assessing the performance of the belief function method of Aslam *et al.* [35] and Shawky *et al.* [34] since the control chart statistic is based on an equal weighting of all the past data values. As discussed by Knoth *et al.* [61], giving all of the past data values equal weight is inadvisable and leads to poor detection of delayed shifts in the process.

Knoth *et al.* [62] showed that the double and triple EWMA chart approach taken by Shafqat *et al.* [26] should not be used. These charts give much greater weight to some past data values than to recent data values, resulting in poor detection capability for delayed shifts in the process. Khan *et al.* [63] recommended using a moving average statistic as input to an EWMA chart. Knoth *et al.* [62] showed that this added complication of using one control chart statistic as input in to another control chart statistic has weak justification at best.

In the repetitive sampling generalizations proposed by Aslam [14], [22], [29], and any other methods that allow multiple samples at each time point, the fixed sample size chart competitor should have a sample size set equal to the expected sample size under the repetitive sampling. This has not been done, so the ARL comparisons have been biased toward favoring the repetitive sampling method since it is based on more data. The rules for implementing repetitive sampling, however, are not explained clearly.

The multiple dependent state sampling (MDSS) approach of Albassam and Aslam [3], Shawky *et al.* [34], Khan *et al.* [32], [60], and Khan *et al.* [64] is equivalent to the use of standard runs rules. Under the MDSS approaches there is the signal region outside the control limits, an interior region close to the centerline, and intermediate regions in between. If a point falls into the intermediate region, no signal is given provided at least a specified number of the previous i ($i > 1$) points are within the interior region. As pointed out by Woodall *et al.* [65], an equivalent standard runs rule can be expressed in terms of providing an out-of-control signal provided at least a specified number of the most recent points are in the intermediate region. Runs rules have been widely used and studied since the 1950s. See, for example, Champ and Woodall [66].

IV. CONCLUSION

In our view the justification for NSPM methods, as currently proposed, is very weak at best. This is due in part to the lack of explanations of the mathematical details. Other issues need to be addressed. The sample sizes and control chart design constants will be known precisely in practice. Representing them as neutrosophic numbers in NSPM adds unnecessary imprecision to the control chart statistics and the control limits. If only the data themselves are interval-valued, then we see value in obtaining conservative bounds on the control chart statistics and control limits using interval statistical methods.

The neutrosophic arithmetic, distribution theory, and NSPM methods need to be fully explained so that results can be reproduced. Once this is done, we think it would be useful to compare the neutrosophic control chart methods for variables data to standard methods based on the midpoints of the data intervals. It would be interesting to investigate how much of the neutrosophic control limit uncertainty is due solely to the uncertainty in the data values.

Although unstated, it seems that an important goal in many applications should be to increase the amount of information

obtained by reducing the imprecision in the collected data. This would improve the measurement system.

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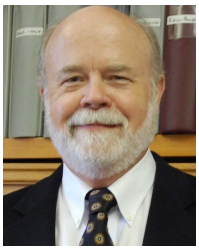

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