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RESEARCH ARTICLE

On Homogeneous Descriptor Systems and Homogeneity-Based Finite-Time Control of Linear Descriptor Systems

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ABSTRACT A new definition of homogeneity for descriptor systems is introduced. The homogeneity property can be verified algebraically, and, as for systems described by ordinary differential equations only, it implies scalability of solutions. A finite-time stabilizing feedback controller is designed for linear descriptor systems. The proposed control contains a homogenizing linear feedback term and a nonlinear homogeneous stabilizing term. The parameters tuning is presented in the form of linear matrix equations and inequalities. Performance of the approach is illustrated by numerical examples.

INDEX TERMS Homogeneity, descriptor systems, finite-time stabilization.

I. INTRODUCTION

The homogeneity property is widely used in automatic control theory for system analysis, control and observer design (see, for example, [2], [3], [4], [5], [6], [7], [8], [9] and references therein). Some of the useful features of homogeneous systems are scalability of trajectories (local properties of homogeneous systems can be translated into global ones); the convergence rate of homogeneous systems can be assessed by its homogeneity degree; robustness with respect to external perturbations and measurement noises; etc. For example, the homogeneity property is widely used for finite-time (ensuring the completion of all transients in a finite time) control design: if a homogeneous system with negative degree is asymptotically stable, then it is finite-time stable (see, for example, [6], [7], [10], [11], etc.).

All mentioned works are focused on the study of systems described by Ordinary Differential Equations (ODEs) only. Descriptor (also referred to as singular or differential-algebraic) systems, in turn, are of great importance in control systems theory. For example, representation of systems in the descriptor form is relevant for a number of applications, e.g., electrical circuits [12], [13], [14], mechanical systems with phase constraints [15], [16], chemistry and biology [17], [18], [19], economics [20], etc. In the present paper a new definition of homogeneity for descriptor systems (we refer to it as $\mathbf{d}_E$-homogeneity) is introduced. As well as for systems in the form of ODEs, the homogeneity property can be verified algebraically, and it implies that a dilation of initial conditions leads to a scaling of trajectories. Necessary and sufficient criterion for a linear descriptor system to be homogeneous with nonzero degree are given.

The finite-time control design problem is addressed in the paper as well. This problem has received much less attention in the literature due to difficulty involved, and as a result, there are few related works (e.g., [21], [22], [23]). Based on the homogeneity property a finite-time stabilizing control is proposed for linear descriptor systems. The control law contains two terms that homogenize (of given negative degree) and stabilize the closed-loop system, respectively. Note that the proposed approach does not require a transition to the ODE form, block decomposition or coordinate transformation, which may be accompanied by computational errors. These advantages and Linear Matrix Inequalities (LMI)-based parameters tuning make the proposed control easier to use in practice.

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With respect to the preliminary results of [1], the key differences are as follows:

- a simplified definition of homogeneity for descriptor systems is given;
- a finite-time control for linear descriptor systems with accelerated convergence is proposed;
- the proofs of all claims are given;
- new examples are considered.

Notation: $\mathbb{R}$ is the field of real numbers; $\mathbb{R}_+ = \{x \in \mathbb{R} : x > 0\}$; $\mathbb{C}$ is the field of complex numbers; $\|\cdot\|$ denotes the Euclidean norm on $\mathbb{R}^n$; I_n denotes the identity matrix of order n; $\lambda(E, A)$ denotes $\lambda(E, A) = \{s | s \in \mathbb{C}, s \text{ is finite}, \det(sE - A) = 0\}$ for $E, A \in \mathbb{R}^{n\times n}$, and $\lambda(A) = \lambda(I_n, A)$ is a spectrum of the matrix A; the eigenvalues of a matrix $G \in \mathbb{R}^{n\times n}$ are denoted by $\lambda_i(G)$, $i = 1, \ldots, n$.

II. PRELIMINARIES

Consider a descriptor system

$$E(x(t))\dot{x}(t) = f(x(t)), \quad x(0) = x_0, \ t \geq 0, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $E : \mathbb{R}^n \to \mathbb{R}^{n\times n}$, $0 < \operatorname{rank} E \leq n$, $f : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous vector field, $f(0) = 0$. It is assumed that the initial condition $x_0 \in \mathbb{R}^n$ is consistent, and the system (1) has a unique solution $\Phi_{x_0}(t)$.

Definition 1 [39]: The system (1) is said to be globally finite-time stable if it is globally asymptotically stable and any solution $\Phi_{x_0}(t)$ reaches the origin in a finite time, i.e., $\Phi_{x_0}(t) = 0$ for all $t \geq T(x_0)$ and $\Phi_{x_0}(t) \neq 0$, $\forall t \in [0, T(x_0))$, $x_0 \neq 0$, where $T : \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$, $T(0) = 0$ is a settling-time function.

Consider a control descriptor system

$$E(x(t))\dot{x}(t) = f(x(t), u(x(t))), \tag{2}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is a control input, $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$, $E : \mathbb{R}^n \to \mathbb{R}^{n\times n}$.

Definition 2 [26]: A control law $u(x)$ is called admissible, if for any consistent initial condition x_0, the system (2) has no impulsive solution. The system (2) is impulse controllable if there exists an admissible control law.

Let us recall some basics on linear descriptor systems. Consider the system (1) in the form

$$E\dot{x}(t) = Ax(t), \tag{3}$$

where $E, A \in \mathbb{R}^{n\times n}$, $\operatorname{rank} E = n_1 \leq n$.

The solution behavior (regularity) of (3) depends on the properties of the pair (E, A).

Definition 3 [12]: The pair (E, A) is said to be regular if $\det(sE - A) \neq 0$ for some $s \in \mathbb{C}$.

The following lemma gives a necessary and sufficient condition of regularity for linear descriptor systems.

Lemma 1 [12]: The pair of matrices (E, A) is regular (the system (3) has a unique solution $\Phi_{x_0}(t)$) iff there exist nonsingular matrices $Q, P \in \mathbb{R}^{n\times n}$ such that

$$QEP = \begin{bmatrix} I_{n_1} & 0 \\ 0 & N \end{bmatrix},$$

$$QAP = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n_2} \end{bmatrix}, \tag{4}$$

where $N \in \mathbb{R}^{n_2\times n_2}$ is a nilpotent matrix, $A_1 \in \mathbb{R}^{n_1\times n_1}$, $n_1 + n_2 = n$.

III. MAIN RESULT

A. HOMOGENEOUS DESCRIPTOR SYSTEMS

Before stating the definition of $\mathbf{d}_E$-homogeneity, let us recall the usual definition of linear geometric (generalized) homogeneity.

Definition 4 [6]: Let $\mathbf{d}(s) : \mathbb{R} \to \mathbb{R}^{n\times n}$ denotes the family of dilations given by $\mathbf{d}(s) := e^{G_{\mathbf{d}} s} = \sum_{i=0}^{+\infty} \frac{s^i G_{\mathbf{d}}^i}{i!}$, where $G_{\mathbf{d}} \in \mathbb{R}^{n\times n}$ is an anti-Hurwitz matrix (i.e. $-G_{\mathbf{d}}$ is Hurwitz) called the generator [27]. Thus,

- a function $h : \mathbb{R}^n \to \mathbb{R}$ is said to be $\mathbf{d}$-homogeneous of degree $\nu \in \mathbb{R}$ if $h(\mathbf{d}(s)x) = e^{\nu s} h(x)$, $\forall x \in \mathbb{R}^n \backslash \{0\}, \forall s \in \mathbb{R}$;
- a vector field $f : \mathbb{R}^n \to \mathbb{R}^n$ is said to be $\mathbf{d}$-homogeneous of degree $\nu \in \mathbb{R}$ if $f(\mathbf{d}(s)x) = e^{\nu s}\mathbf{d}(s) f(x)$, $\forall x \in \mathbb{R}^n \backslash \{0\}, \forall s \in \mathbb{R}$;
- a system $\dot{x} = f(x)$ is said to be $\mathbf{d}$-homogeneous if f is $\mathbf{d}$-homogeneous.

$\mathbf{d}_E$-homogeneity consists in an extension of the $\mathbf{d}$-homogeneity concept for descriptor systems in the form (1).

Definition 5: Let $\mathbf{d}_E(s) : \mathbb{R} \to \mathbb{R}^{n\times n}$ denotes the family of dilations $\mathbf{d}_E(s) := e^{G_{\mathbf{d}} s}$ with anti-Hurwitz generator matrix $G_{\mathbf{d}} \in \mathbb{R}^{n\times n}$. Thus,

- the pair (E, f) is said to be $\mathbf{d}_E$-homogeneous of degree $\nu \in \mathbb{R}$, if for all $x \in \mathbb{R}^n \setminus \{0\}$, $s \in \mathbb{R}$

$$E(\mathbf{d}_E(s)x)\mathbf{d}_E(s) = \Xi(x, s)E(x), \tag{5}$$
$$f(\mathbf{d}_E(s)x) = e^{\nu s}\,\Xi(x, s) f(x), \tag{6}$$

  where $\Xi : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^{n\times n}$ is invertible for all $s \in \mathbb{R}$, $x \in \mathbb{R}^n \setminus \{0\}$;
- the system (1) is said to be $\mathbf{d}_E$-homogeneous if the pair (E, f) is $\mathbf{d}_E$-homogeneous.

Remark 1: In the case $E(x)$ is nonsingular for any $x \in \mathbb{R}^n$ the system (1) is equivalent to $\mathbf{d}$-homogeneous system $\dot{x}(t) = E(x)^{-1} f(x)$ with the dilation $\mathbf{d}(s) = \mathbf{d}_E(s)$.

One of the most important properties of homogeneous systems is the scalability of solutions [2], [5], [28], [29]. The scalability of solutions implies a number of properties useful for qualitative analysis (e.g., local stability implies the global one; the existence of strictly invariant (in forward time) compact set implies asymptotic stability [2], [28], etc.). The following theorem provides an analogous result for $\mathbf{d}_E$-homogeneous descriptor systems.

Theorem 1: Let the system (1) is $\mathbf{d}_E$-homogeneous of degree $\nu \in \mathbb{R}$. If $\Phi_{x_0} : [0, T) \to \mathbb{R}^n$ is a solution to (1), then $\Phi_{\mathbf{d}_E(s)x_0} : \left[0, e^{-\nu s}T\right) \to \mathbb{R}^n$ defined as

$$\Phi_{\mathbf{d}_E(s)x_0}(t) := \mathbf{d}_E(s)\Phi_{x_0}(te^{\nu s}), \quad s \in \mathbb{R} \tag{7}$$

is a solution to (1) with the initial condition $x(0) = \mathbf{d}_E(s)x_0$.

Proof: Since $E(\Phi_{x_0}(t))\frac{d}{dt}\Phi_{x_0}(t) = f(\Phi_{x_0}(t))$, then

$$\begin{aligned} E(\mathbf{d}_E(s)\Phi_{x_0}(t))\frac{d}{dt}\mathbf{d}_E(s)\Phi_{x_0}(t) \\ &= E(\mathbf{d}_E(s)\Phi_{x_0}(t))\mathbf{d}_E(s)\frac{d}{dt}\Phi_{x_0}(t) \\ &= \Xi(\Phi_{x_0}(t), s)E(\Phi_{x_0}(t))\frac{d}{dt}\Phi_{x_0}(t) \\ &= \Xi(\Phi_{x_0}(t), s)f(\Phi_{x_0}(t)) \\ &= e^{-\nu s}f(\mathbf{d}_E(s)\Phi_{x_0}(t)). \end{aligned}$$

Making the change of time $t = e^{\nu s}t_{\text{new}}$, we complete the proof. ■

In order to consider an example of a homogeneous descriptor system let us refer to the paper [30], where the following average-consensus reaching algorithm is proposed

$$\begin{aligned} \dot{x}_i(t) &= u_i, \\ u_i &= \sum_{j=1}^{n} a_{ij}\text{sign}(x_j - x_i)|x_j - x_i|^{\mu}, \end{aligned}$$

for $i = 1, \ldots, n$ agents, and $\mu \in (0, 1)$, $a_{ij} = a_{ji} \geq 0$. Following [30], let us make the change of variables $\delta_i(t) = x^* - x_i(t)$, where x^* is the equilibrium position of the system. Then average-consensus problem will be implied by stability of the following system

$$\begin{aligned} \dot{\delta}_i &= \sum_{j=1}^{n} a_{ij}\text{sign}(\delta_j - \delta_i)|\delta_j - \delta_i|^{\mu}, \quad i = 1, \ldots, n-1, \\ 0 &= \sum_{i=1}^{n} \delta_i(t) \end{aligned} \tag{8}$$

that is $\mathbf{d}_E$-homogeneous of degree $-\mu$ with the generator $G_{\mathbf{d}} = e^s I_n$. Simulation results (Fig. 2) for the system (8) with the structure given in Fig. 1 (the coupling coefficients a_{ij} are 0 or 1 according to the structure) are presented for different initial conditions to show the scalability of the solutions.

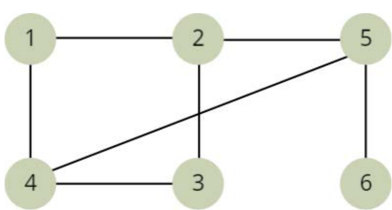

FIGURE 1. Structure of the system (8).

B. LINEAR HOMOGENEOUS DESCRIPTOR SYSTEMS

Now consider the following linear regular descriptor system

$$E\dot{x}(t) = Ax(t), \tag{9}$$

where $x \in \mathbb{R}^n$, $A, E \in \mathbb{R}^{n\times n}$ and $\text{rank}E = n_1 < n$. Let the pair (E, A) is regular, i.e., by Lemma 1 it admits the canonical form

$$\begin{aligned} \dot{x}_1(t) &= A_1 x_1(t), \\ N\dot{x}_2(t) &= x_2(t), \end{aligned} \tag{10}$$

where $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = P^{-1}x(t)$, $x_1(t) \in \mathbb{R}^{n_1}$, $x_2(t) \in \mathbb{R}^{n_2}$.

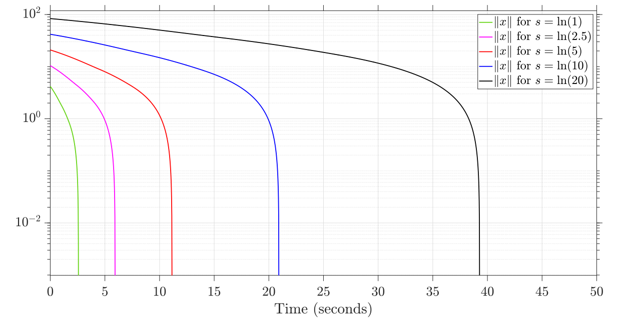

FIGURE 2. The state vector norm for different initial conditions $x(0) = \mathbf{d}_E(s)x_0$, where $x_0 = [1\ 2\ 3\ 4\ 5\ 6]^T$ and s takes the values ln(1), ln(2.5), ln(5), ln(10) and ln(20).

The following result gives the criteria for a linear descriptor system (9) to be $\mathbf{d}_E$- homogeneous of nonzero degree.

Lemma 2: The next statements are equivalent.

(1) The system (9) is $\mathbf{d}_E$-homogeneous of degree $\nu \neq 0$.

(2) The matrix $A_1 \in \mathbb{R}^{n_1\times n_1}$ is nilpotent.

(3) The condition $\lambda(E, A) = 0$ is satisfied.

Proof: **(2)⇒(1)** Due to the pair (E, A) is regular one can rewrite the system (9) in the form

$$Q^{-1}\begin{bmatrix} I_{n_1} & 0 \\ 0 & N \end{bmatrix} P^{-1}\dot{x}(t) = Q^{-1}\begin{bmatrix} A_1 & 0 \\ 0 & I_{n_2} \end{bmatrix} P^{-1}x(t). \tag{11}$$

Assume that the matrix A_1 is nilpotent. According to [10] the equations

$$\begin{aligned} A_1G_1 - G_1A_1 &= \nu A_1, \\ NG_2 - G_2N &= -\nu N \end{aligned} \tag{12}$$

are feasible for any $\nu \neq 0$ and some anti-Hurwitz matrices $G_1 \in \mathbb{R}^{n_1\times n_1}$, $G_2 \in \mathbb{R}^{n_2\times n_2}$, and by [10, Lemma 3]

$$\begin{aligned} A_1 e^{G_1 s} &= e^{\nu s}e^{G_1 s}A_1, \\ Ne^{G_2 s} &= e^{-\nu s}e^{G_2 s}N \end{aligned} \tag{13}$$

for all $s \in \mathbb{R}$. Then choosing the generator in the form $G_{\mathbf{d}} = PGP^{-1}$ and $\Xi(x, s) \equiv \Xi(s) := Q^{-1}\begin{bmatrix} I_{n_1} & 0 \\ 0 & e^{-\nu s}I_{n_2} \end{bmatrix} e^{Gs}Q$ for $G = \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix}$ with the use of (13) we have

$$\begin{aligned} Q^{-1}\begin{bmatrix} I_{n_1} & 0 \\ 0 & N \end{bmatrix} P^{-1}\mathbf{d}_E(s) &= Q^{-1}\begin{bmatrix} I_{n_1} & 0 \\ 0 & N \end{bmatrix} P^{-1}e^{PGP^{-1}s} \\ &= Q^{-1}\begin{bmatrix} I_{n_1} & 0 \\ 0 & N \end{bmatrix} e^{Gs}P^{-1} \\ &= \Xi(s)E, \end{aligned}$$

and

$$\begin{aligned} Q^{-1}\begin{bmatrix} A_1 & 0 \\ 0 & I_{n_2} \end{bmatrix} P^{-1}\mathbf{d}_E(s)\,x &= Q^{-1}\begin{bmatrix} A_1 & 0 \\ 0 & I_{n_2} \end{bmatrix} e^{Gs}P^{-1}x \\ &= e^{\nu s}\Xi(s)Ax, \end{aligned}$$

i.e., (5) and (6) are satisfied.

(1)⇒(2) Let the system (9) is $\mathbf{d}_E$-homogeneous of degree $\nu \neq 0$. The solution of (9) is $\Phi_{x_0}(t) = P\begin{bmatrix} e^{A_1 t} & 0 \\ 0 & 0 \end{bmatrix} P^{-1}x_0$ [12]. According to (7) the system solutions are scalable as follows

$$P\begin{bmatrix} e^{A_1 t} & 0 \\ 0 & 0 \end{bmatrix} P^{-1}\mathbf{d}_E(s)x_0 = \mathbf{d}_E(s)P\begin{bmatrix} e^{A_1 \exp(\nu s)t} & 0 \\ 0 & 0 \end{bmatrix} P^{-1}x_0$$

that corresponds to

$$\begin{bmatrix} e^{A_1 t} & 0 \\ 0 & 0 \end{bmatrix} = e^{P^{-1}G_{\mathbf{d}}Ps}\begin{bmatrix} e^{A_1 \exp(\nu s)t} & 0 \\ 0 & 0 \end{bmatrix} e^{-P^{-1}G_{\mathbf{d}}Ps},$$

i.e.,

$$\lambda\left(\begin{bmatrix} e^{A_1 t} & 0 \\ 0 & 0 \end{bmatrix}\right) = \lambda\left(\begin{bmatrix} e^{A_1 \exp(\nu s)t} & 0 \\ 0 & 0 \end{bmatrix}\right).$$

Due to $\nu \neq 0$ we have that $\lambda(A_1) = 0$ and the matrix A_1 is nilpotent.

(2)⇔(3) Equivalence is straightforward due to

$$\begin{aligned}\lambda(E, A) &= \lambda(QEP, QAP) \\ &= \lambda\left(\begin{bmatrix} I_{n_1} & 0 \\ 0 & N \end{bmatrix}, \begin{bmatrix} A_1 & 0 \\ 0 & I_{n_2} \end{bmatrix}\right) \\ &= \lambda(A_1) \\ &= 0.\end{aligned}$$

■

Remark 2: Note that if the system (9) is $\mathbf{d}_E$-homogeneous with the generator matrix $G_{\mathbf{d}}$, then the corresponding canonical form (10) is $\tilde{\mathbf{d}}_E$-homogeneous with the generator $\tilde{G}_{\mathbf{d}} = P^{-1}G_{\mathbf{d}}P$.

C. FINITE-TIME CONTROL OF LINEAR DESCRIPTOR SYSTEMS

In this section we propose a finite-time control design method for linear descriptor systems based on $\mathbf{d}_E$-homogeneity.

Firstly, let us introduce the notions of $\mathbf{d}_E$-homogenization and $\mathbf{d}_E$-homogeneous stabilization.

Definition 6: The descriptor control system (2) is

- $\mathbf{d}_E$-homogenizable with degree $\nu \in \mathbb{R}$ if there exists a feedback control $u(x)$ such that the system (2) is $\mathbf{d}_E$-homogeneous of degree ν;
- $\mathbf{d}_E$-homogeneously stabilizable with degree $\nu \in \mathbb{R}$ if there exists a feedback control $u(x)$ such that the closed-loop system is $\mathbf{d}_E$-homogeneous of degree ν and globally asymptotically stable.

Consider the linear descriptor system

$$E\dot{x}(t) = Ax(t) + Bu(t), \tag{14}$$

where $x \in \mathbb{R}^n$ is the measurable state vector, $u \in \mathbb{R}^m$ is the vector of control inputs, $E, A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ (the pair (E, A) is regular, the triplet (E, A, B) is completely controllable, rank B $= m \le n$).

The main goal is to propose a constructive (i.e., equipped with reliable tuning rules) $\mathbf{d}_E$-homogenizing and stabilizing in a finite time control for the system (14).

The following lemma gives the necessary and sufficient condition of $\mathbf{d}_E$-homogenization of the system (14) via linear feedback control.

Lemma 3: The system (14) is $\mathbf{d}_E$-homogenizable via linear feedback control $u = K_E x$, $K_E \in \mathbb{R}^{m\times n}$ if and only if K_E is such that $\lambda(E, (A + BK_E)) = 0$.

Proof: The proof is straightforward consequence of Lemma 2. ■

According to [31] there exists an invertible matrix $X \in \mathbb{R}^{n\times n}$ such that $X^T E = E^T X \geq 0$ and $x^T X^T Ex = 0$ iff $Ex = 0$. Let $\|x\|_{X^T E} = \sqrt{x^T X^T Ex}$. For $\mathcal{G} := \{x \in \mathbb{R}^n \setminus \{0\} | Ex \neq 0\}$ define an implicitly defined function $\|\cdot\|_{\mathbf{d}_E} : \mathcal{G} \cup \{0\} \to \mathbb{R}_+ \cup \{0\}$ as $\|x\|_{\mathbf{d}_E} = e^{s_x}$ for $x \neq 0$, where $s_x \in \mathbb{R}$ such that $\|\mathbf{d}_E(-s_x)x\|_{X^T E} = 1$ and $\|0\|_{\mathbf{d}_E} = 0$. Note that $\|\mathbf{d}_E(s)x\|_{\mathbf{d}_E} = e^s\|x\|_{\mathbf{d}_E}$ and

$$\|\mathbf{d}_E(-\ln\|x\|_{\mathbf{d}_E})x\|_{X^T E} = 1. \tag{15}$$

Note that for $E = I_n$ the defined function $\|\cdot\|_{\mathbf{d}_E}$ becomes the canonical $\mathbf{d}$-homogeneous norm as in [11].

The following theorem proposes a $\mathbf{d}_E$-homogeneity based stabilizing control for the system (14).

Theorem 2: Let the control be chosen in the form

$$u(x) = K_E x + \|x\|_{\mathbf{d}_E} K\mathbf{d}_E(-\ln\|x\|_{\mathbf{d}_E})x, \tag{16}$$

where

- $K_E \in \mathbb{R}^{m\times n}$ is chosen such that $\lambda(E, (A + BK_E)) = 0$;
- for some $\nu \in [-1, 0)$ the system of matrix equations and inequalities

$$EM = LE, \tag{17}$$
$$(A + BK_E)M = (L + \nu I_n)(A + BK_E), \tag{18}$$
$$(L + (\nu - 1)I_n)B = 0, \tag{19}$$
$$M + M^T + 2aI_n > 0 \tag{20}$$

  is feasible for some $M, L \in \mathbb{R}^{n\times n}$, $a \in \mathbb{R}$;
- $K \in \mathbb{R}^{m\times n}$, $\beta \in \mathbb{R}_+$ are chosen such that

$$R^T E^T = ER \geq 0, \tag{21}$$
$$\begin{bmatrix} E(M + aI_n)R + R^T(M^T + aI_n)E^T & R^T E^T \\ ER & \Gamma \end{bmatrix} \geq 0, \tag{22}$$
$$\Gamma > 0, \tag{23}$$
$$(A+BK_E)R+R^T(A+BK_E)^T + BY + Y^T B^T \leq -\beta((L + aI_n)ER + R^T E^T(L^T + aI_n)) \tag{24}$$

  for some $R, \Gamma \in \mathbb{R}^{n\times n}$, $Y \in \mathbb{R}^{m\times n}$ with $K = YX$, $X = R^{-1}$ and $G_{\mathbf{d}} = M + aI_n$.

Then the closed-loop system (14), (16) is impulse controllable and finite-time stable with

$$T(x_0) \leq -\frac{1}{\beta\nu}\|x_0\|_{\mathbf{d}_E}^{-\nu}. \tag{25}$$

Proof: **I.** Firstly, let us show that the closed-loop system (14), (16) is $\mathbf{d}_E$-homogeneous.

Consider the linear part of the closed-loop system. The pair $(E, (A + BK_E)x)$ is $\mathbf{d}_E$-homogeneous of degree ν with

$G_{\mathbf{d}} = M + aI_n$, where $a \in \mathbb{R}$ is sufficiently big for the generator to be anti-Hurwiz. Indeed, choosing

$$\Xi(s) = Q^{-1}\begin{bmatrix} I_{n_1} & 0 \\ 0 & e^{-\nu s}I_{n_2}\end{bmatrix} e^{Gs}Q, \quad G = \begin{bmatrix} G_1 & 0 \\ 0 & G_2\end{bmatrix},$$

$G_1 \in \mathbb{R}^{n_1\times n_1}$, $G_2 \in \mathbb{R}^{n_2\times n_2}$ (according to the proof of Lemma 2) one can show that (5), (6) are equivalent to

$$\begin{aligned} Ee^{G_{\mathbf{d}}s} &= e^{\bar{L}s}E, \\ (A+BK_E)e^{G_{\mathbf{d}}s} &= e^{(\bar{L}+\nu I_n)s}(A+BK_E), \quad \forall s \in \mathbb{R} \end{aligned} \tag{26}$$

for $\bar{L} = Q^{-1}(G+\nu_0)Q$, $\nu_0 = \begin{bmatrix} 0 & 0 \\ 0 & -\nu I_{n_2}\end{bmatrix}$. Rewriting this expressions as

$$\begin{aligned} E\sum_{i=0}^{+\infty}\frac{s^iG_{\mathbf{d}}^i}{i!} &= \sum_{i=0}^{+\infty}\frac{s^i\bar{L}^i}{i!}E, \\ (A+BK_E)\sum_{i=0}^{+\infty}\frac{s^iG_{\mathbf{d}}^i}{i!} &= \sum_{i=0}^{+\infty}\frac{s^i(\bar{L}+\nu I_n)^i}{i!}(A+BK_E), \end{aligned}$$

and combining the terms of the same power we obtain that the sufficient condition for the pair $(E,(A+BK_E)x)$ to be $\mathbf{d}_E$-homogeneous is

$$\begin{aligned} E{G_{\mathbf{d}}}^i &= \bar{L}^iE, \\ (A+BK_E){G_{\mathbf{d}}}^i &= (\bar{L}+\nu I_n)^i(A+BK_E) \end{aligned}$$

for any nonnegative integer i. Similarly to [10, Lemma 3] it is easy to show that this sufficient condition is satisfied if

$$\begin{aligned} EG_{\mathbf{d}} &= \bar{L}E, \\ (A+BK_E)G_{\mathbf{d}} &= (\bar{L}+\nu I_n)(A+BK_E). \end{aligned} \tag{27}$$

The condition (27) is satisfied if the equations (17), (18) are satisfied, where $G_{\mathbf{d}} = M + aI_n$, $\bar{L} = L + aI_n$ and $a \in \mathbb{R}$ is sufficiently big for the generator to be anti-Hurwiz according to (20).

The pair $(E, \|x\|_{\mathbf{d}_E}BK\mathbf{d}_E(-\ln\|x\|_{\mathbf{d}_E})x)$ is also $\mathbf{d}_E$-homogeneous of the degree ν with the same generator $G_{\mathbf{d}}$ due to

$$\begin{aligned} &\|\mathbf{d}_E(s)x\|_{\mathbf{d}_E}BK\mathbf{d}_E(-\ln\|\mathbf{d}_E(s)x\|_{\mathbf{d}_E})\mathbf{d}_E(s)x \\ &\quad = Q^{-1}\begin{bmatrix} e^{\nu s}I_{n_1} & 0 \\ 0 & I_{n_2}\end{bmatrix} e^{Gs}Q\|x\|_{\mathbf{d}_E}BK\mathbf{d}_E(-\ln\|x\|_{\mathbf{d}_E})x \\ &\quad = e^{\nu s}\Xi(s)\|x\|_{\mathbf{d}_E}BK\mathbf{d}_E(-\ln\|x\|_{\mathbf{d}_E})x. \end{aligned}$$

Indeed, on the one hand we have

$$\begin{aligned} &\|\mathbf{d}_E(s)x\|_{\mathbf{d}_E}BK\mathbf{d}_E(-\ln\|\mathbf{d}_E(s)x\|_{\mathbf{d}_E})\mathbf{d}_E(s)x \\ &\quad = e^s\|x\|_{\mathbf{d}_E}BK\mathbf{d}_E(-\ln(e^s\|x\|_{\mathbf{d}_E}))\mathbf{d}_E(s)x \\ &\quad = e^s\|x\|_{\mathbf{d}_E}BK\mathbf{d}_E(-\ln\|x\|_{\mathbf{d}_E})x, \end{aligned}$$

and on the other hand one can show that

$$\begin{aligned} &e^{\nu s}\Xi(s)\|x\|_{\mathbf{d}_E}BK\mathbf{d}_E(-\ln\|x\|_{\mathbf{d}_E})x \\ &\quad = Q^{-1}\begin{bmatrix} e^{\nu s}I_{n_1} & 0 \\ 0 & I_{n_2}\end{bmatrix} e^{Gs}Q\|x\|_{\mathbf{d}_E}BK\mathbf{d}_E(-\ln\|x\|_{\mathbf{d}_E})x \\ &\quad = e^{(L+\nu I_n)s}\|x\|_{\mathbf{d}_E}BK\mathbf{d}_E(-\ln\|x\|_{\mathbf{d}_E})x \\ &\quad = e^s\|x\|_{\mathbf{d}_E}BK\mathbf{d}_E(-\ln\|x\|_{\mathbf{d}_E})x, \end{aligned} \tag{28}$$

where the equality

$$e^{(L+\nu I_n)s}B = \sum_0^{+\infty}\frac{(L+\nu I_n)^is^i}{i!}B = e^sB$$

was used taking into account (19).

II. Due to the homogeneity property, on $\mathbb{R}^n \setminus \{0\}$ we have

$$\begin{aligned} &E\mathbf{d}_E(-\ln\|x\|_{\mathbf{d}_E})\dot{x} \\ &\quad = \|x\|_{\mathbf{d}_E}^{\nu}(A+BK_E+BK)\mathbf{d}_E(-\ln\|x\|_{\mathbf{d}_E})x. \end{aligned}$$

According to [32] the inequalities (21), (24) imply that the pair $(E,(A+BK_E+BK))$ is impulse free, i.e.,

$$\begin{aligned} \tilde{Q}E\tilde{P} &= \begin{bmatrix} I_{n_1} & 0 \\ 0 & 0\end{bmatrix}, \\ \tilde{Q}(A+BK_E+BK)\tilde{P} &= \begin{bmatrix} \tilde{A} & 0 \\ 0 & I_{n_2}\end{bmatrix} \end{aligned}$$

for some $\tilde{A} \in \mathbb{R}^{n_1\times n_1}$ and invertible $\tilde{Q}, \tilde{P} \in \mathbb{R}^{n\times n}$. Then the control (16) is admissible (i.e., the closed-loop system is impulse free).

III. Since the system is impulse free, then for $x_0 \in \mathcal{G}$ (for $Ex_0 = 0$ the solution is trivial $\Phi_{x_0}(t) = 0$) we can consider the implicitly defined Lyapunov candidate function in the form $V(x) := \|x\|_{\mathbf{d}_E}$ that is positive and radially unbounded on the set $\mathcal{G}$ and $V(0) = 0$.

Since $\|x\|_{\mathbf{d}_E} = e^s$: $\|\mathbf{d}_E(-s)x\|_{X^TE} = 1$,

$$\begin{aligned} &\frac{\partial\|\mathbf{d}_E(-s)x\|_{X^TE}}{\partial s} \\ &\quad = -\frac{\partial\|z\|_{X^TE}}{\partial z}\Big|_{z=\mathbf{d}_E(-s)x}G_{\mathbf{d}}\mathbf{d}_E(-s)x \\ &\quad = -\|\mathbf{d}_E(-s)x\|_{X^TE}^{-1}x^T\mathbf{d}_E^T(-s)X^TEG_{\mathbf{d}}\mathbf{d}_E(-s)x, \\ &\frac{\partial\|\mathbf{d}_E(-s)x\|_{X^TE}}{\partial x} \\ &\quad = \|\mathbf{d}_E(-s)x\|_{X^TE}^{-1}x^T\mathbf{d}_E^T(-s)X^TE\mathbf{d}_E(-s), \end{aligned}$$

then implying $\frac{\partial s}{\partial x} = -\left[\frac{\partial\|\mathbf{d}_E(-s)x\|_E}{\partial s}\right]^{-1}\frac{\partial\|\mathbf{d}_E(-s)x\|_E}{\partial x}$ (by means of Implicit Function Theorem [33]) we obtain

$$\begin{aligned} \dot{V} &= \frac{\partial}{\partial t}\|x\|_{\mathbf{d}_E} = \frac{\partial}{\partial x}\|x\|_{\mathbf{d}_E}\dot{x} = e^s\frac{\partial s}{\partial x}\Big|_{s=\ln\|x\|_{\mathbf{d}_E}}\dot{x} \\ &= \|x\|_{\mathbf{d}_E}\Upsilon z^TX^TE\mathbf{d}_E(-\ln\|x\|_{\mathbf{d}_E})\dot{x} \\ &= \|x\|_{\mathbf{d}_E}\Upsilon z^TX^TQ^{-1}\begin{bmatrix} I_{n_1} & 0 \\ 0 & \|x\|_{\mathbf{d}_E}^{\nu}I_{n_2}\end{bmatrix}\chi E\dot{x} \\ &= \|x\|_{\mathbf{d}_E}\Upsilon z^TX^TQ^{-1}\begin{bmatrix} I_{n_1} & 0 \\ 0 & \|x\|_{\mathbf{d}_E}^{\nu}I_{n_2}\end{bmatrix}\chi \\ &\quad \times\left(Ax + BK_Ex + \|x\|_{\mathbf{d}_E}BK\mathbf{d}_E(-\ln\|x\|_{\mathbf{d}_E})x\right) \\ &= \|x\|_{\mathbf{d}_E}^{1+\nu}\Upsilon z^TX^T(A+BK_E+BK)z, \end{aligned} \tag{29}$$

where $\Upsilon = \left(z^TX^TEG_{\mathbf{d}}z\right)^{-1}$, $\chi = e^{-G\ln\|x\|_{\mathbf{d}_E}}$, $z = \mathbf{d}_E(-\ln\|x\|_{\mathbf{d}_E})x$.

Since $G_{\mathbf{d}} = M + aI_n$ then (22), (23) imply

$$X^T EG_{\mathbf{d}} + G_{\mathbf{d}}^T E^T X \geq E^T \Gamma^{-1} E$$

for $R = X^{-1}$ and some $\Gamma > 0$. Due to (5) we have $x^T \mathbf{d}_E(-\ln \|x\|_{\mathbf{d}_E})^T X^T EG_{\mathbf{d}} \mathbf{d}_E(-\ln \|x\|_{\mathbf{d}_E})x > 0$ for all $x \in \mathcal{G}$.

Returning to (29) and taking into account that for $K = YR^{-1}$ the inequality (24) implies

$$X^T(A + BK_E + BK) + (A + BK_E + BK)^T X \leq -\beta(X^T EG_{\mathbf{d}} + G_{\mathbf{d}}^T E^T X),$$

we derive

$$\dot{V} \leq -\beta V^{1+\nu}.$$

Finally, applying the comparison lemma the last inequality guarantees (see, e.g., [24]) the system (14), (16) is finite-time stable and

$$T(x_0) \leq -\frac{1}{\beta\nu} V_0^{-\nu},$$

where $V_0 = V(x_0)$. ■

Note, that if $\nu \to 0$ the proposed control (16) becomes a linear one.

Remark 3: The presented control can be considered as an extension of the results in [10] for finite-time stabilization of linear ODEs. Indeed, in the case the matrix E is nonsingular (see Remark 1) the control (16) coincides with the given in [10].

Remark 4: The feedback matrix K_E can be chosen using pole placement methods.

Note that in the control (16) the linear term $K_E x$ homogenizes the closed-loop system with the negative degree ν, while the role of the term $\|x\|_{\mathbf{d}_E} K\mathbf{d}_E(-\ln \|x\|_{\mathbf{d}_E})x$ is to stabilize the system.

Remark 5: The function $\|x\|_{\mathbf{d}_E}$ is defined implicitly by (15). In order to realize the control (16) and find an appropriate value of $\|x_i\|_{\mathbf{d}_E}$ at the time instant t_i the following simple numerical procedure can be used (see, e.g., [34]):

Algorithm 1 [34]

```
INITIALIZATION: V_0 = 1; a = V_min; b = 1;
STEP:
  If x_i^T d_E(-ln b)^T X^T E d_E(-ln b) x_i > 1 then
          a = b; b = 2b;
  elseif x_i^T d_E(-ln a)^T X^T E d_E(-ln a) x_i < 1 then
          b = a; a = max{a/2, V_min};
  else
      c = (a+b)/2
     If x_i^T d_E(-ln c)^T X^T E d_E(-ln c) x_i < 1 then
          b = c;
        else a = max{V_min, c};
   endif;
  endif;
  V_i = b;
```

If STEP is applied recurrently many times to the same vector x_i then it allows to localize the unique positive root of the equation $\|\mathbf{d}_E(-\ln \|x_i\|_{\mathbf{d}_E})x_i\|_{X^T E} = 1$.

In practice, there are often time restrictions on transients. The parameters ν, β affect convergence time and allow the upper bound of the settling time function (25) to be adjusted (e.g., the larger β, the smaller bound of $T(x_0)$). However, for some parameters (e.g., for sufficiently big β), the inequalities (21)-(24) may become unfeasible. In this case, if the control (16) calculated via Theorem 2 provides settling time estimation (25) greater than the desired transient time, the convergence rate of the proposed control can be accelerated via time rescaling. To accelerate the convergence rate of systems in the form of ODEs with homogeneous control, one can refer to the results [35], [36], [37]. The following corollary provides an extension of Theorem 2 in order to accelerate the convergence rate.

Corollary 1: Let all conditions of Theorem 2 be satisfied and matrix equations

$$\begin{aligned} EM_2 &= L_2 E, \\ (A + BK_E)M_2 &= (L_2 + I_n)(A + BK_E), \\ L_2 B &= 0 \\ M + M^T + 2a_2 I_n &> 0 \end{aligned} \tag{30}$$

be feasible for some $M_2, L_2 \in \mathbb{R}^{n\times n}$, $a_2 \in \mathbb{R}$. For $\lambda > 1$ and $\Lambda = e^{-M_2 \ln\lambda}$ the control

$$u_\lambda(x) = K_E x + \lambda \|\Lambda x\|_{\mathbf{d}_E} K\mathbf{d}_E(-\ln \|\Lambda x\|_{\mathbf{d}_E})\Lambda x \tag{31}$$

stabilizes system (14) in finite time with

$$T(x_0) \leq -\frac{1}{\beta\nu}\|\Lambda x_0\|_{\mathbf{d}_E}^{-\nu}\lambda^{-1}. \tag{32}$$

Proof: Analogously to the part **I** of the proof of Theorem 2 it is easy to show that $Ee^{M_2 s} = e^{L_2 s}E$, $(A+BK_E)e^{M_2 s} = e^{(L_2+I_n)s}(A + BK_E)$ and $e^{(L_2+I_n)s}B = e^s B$, $\forall s \in \mathbb{R}$. Consider the system (14) with the control $u_\lambda(x)$:

$$\begin{aligned} E\dot{x} &= Ax + Bu_\lambda(x) \\ &= (A + BK_E)\Lambda^{-1}\Lambda x \\ &\quad + \lambda B \|\Lambda x\|_{\mathbf{d}_E} K\mathbf{d}_E(-\ln \|\Lambda x\|_{\mathbf{d}_E})\Lambda x \\ &= \lambda e^{L_2 \ln\lambda}(A + BK_E)\Lambda x \\ &\quad + \lambda e^{L_2 \ln\lambda} B \|\Lambda x\|_{\mathbf{d}_E} K\mathbf{d}_E(-\ln \|\Lambda x\|_{\mathbf{d}_E})\Lambda x. \end{aligned}$$

Taking into account $e^{-L_2 \ln\lambda}E = Ee^{-M_2 \ln\lambda}$, for $z = \Lambda x$, we obtain

$$E\dot{z} = \lambda[Az + Bu(z)]. \tag{33}$$

The system (33) is finite-time stable since $\Phi_{z_0}^\lambda(t) = \Phi_{z_0}(\lambda t)$, where $\Phi_{z_0}^\lambda(t)$ is a solution of (33) and $\Phi_{z_0}(\lambda t)$ is a solution of the system $E\dot{z} = Az + bu(z)$. Then, the settling-time function is bounded by

$$T_z(z_0) \leq -\frac{1}{\beta\nu}\|z_0\|_{\mathbf{d}_E}^{-\nu}\lambda^{-1}$$

$$= -\frac{1}{\beta\nu}\|\Lambda x_0\|_{\mathbf{d}_E}^{-\nu}\lambda^{-1}.$$

■

Remark 6: Note that the change of a control law in some cases may lead to a change of the initial conditions x_0 associated with the static part and inputs of a descriptor system (system may have solutions only for a special set of initial values consistent with inputs [40]). In this case, due to $E\mathbf{d}_E(-\ln\|x_0\|_{\mathbf{d}_E})x_0 = Q^{-1}\begin{bmatrix} e^{-G_1 \ln\|x_0\|_{\mathbf{d}_E}} & 0 \\ 0 & 0 \end{bmatrix} P^{-1}x_0$, this change does not affect on the value of $\|x_0\|_{\mathbf{d}_E}$ and the control (31) provides an accelerated convergence according to (32).

IV. EXAMPLE

Consider the approximated linear descriptor submarine model with a dive control system, presented in [38], where

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 & 1 & -5\times 10^{-3} \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -5\times 10^{-3} \end{bmatrix},$$
$$B = \begin{bmatrix} 1 & 0 & 1 & 2 \end{bmatrix}^T.$$

The state vector variables $x \in \mathbb{R}^4$ correspond to the depth of immersion, vertical velocity, vertical acceleration and self-rotation velocity.

According to Theorem 2, by means of (17)-(24) solution, the following parameters of the control algorithm (16) with $\nu = -0.5$, $\beta = 0.1$ were obtained:

$$K_E = \begin{bmatrix} 0 & 0 & 0 & 1/200 \end{bmatrix},$$
$$K = \begin{bmatrix} 77.8306 & -44.2391 & 95.5784 & -7.0789 \end{bmatrix},$$
$$G_{\mathbf{d}} = \begin{bmatrix} 2.5 & 0 & 0 & -1 \\ 0 & 1 & 0 & -0.0025 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1.5 \end{bmatrix},$$
$$X = \begin{bmatrix} 0.0007 & 0 & 0 & -0.0004 \\ 0 & 0 & 0.0003 & 0 \\ -0.0124 & 0.0072 & -0.0159 & 0.0008 \\ -0.0004 & 0 & 0 & 0.0004 \end{bmatrix}.$$

The numerical simulation of the closed-loop system has been done for $x_0 = [-3\ \ 4.4\ \ 0\ \ -4]^T$. To find values of $\|\cdot\|_{\mathbf{d}_E}$ Algorithm 1 was used. The results of simulation are shown in Fig. 3, Fig. 4. The results for $\|x\|_{\mathbf{d}_E}$ (Fig. 4) are shown with the use of the logarithmic scale in order to demonstrate finite-time convergence rate. The settling time estimate is $T(x_0) \leq 9.16$ according to (25).

Fig. 5 and Fig. 6 demonstrate plots of the state vector in the presence of additive disturbances $d(t) = \begin{bmatrix} 0 & \frac{1}{4}\sin(2t)) & \frac{1}{4}\sin(t)) & 0 \end{bmatrix}^T$ (in the system under consideration, disturbances can be caused by the presence of compression of the hull, changes in the density of seawater, sea currents, etc.) and measurement band limited noise, respectively. A detailed study of the presented control algorithms on robustness analysis with respect to uncertainties, disturbances, extension of these results on a wider class of systems and experimental approbation goes beyond the scope of this paper providing the subjects for a future research.

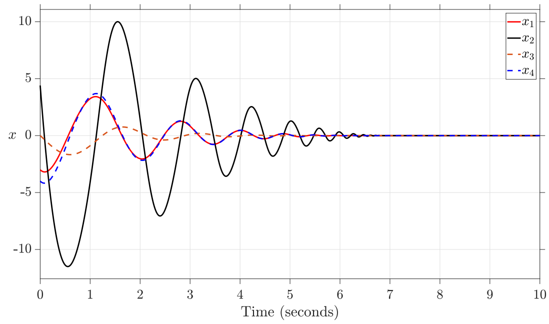

FIGURE 3. **Transients of the state vector with the finite-time control (16).**

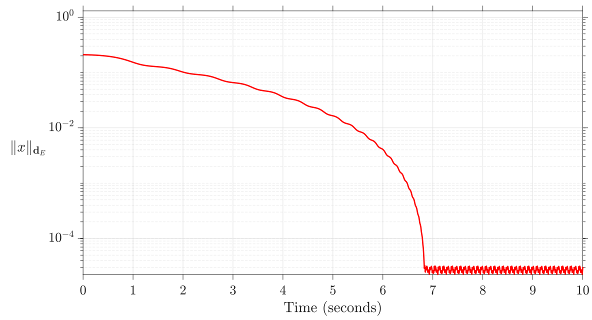

FIGURE 4. **Function $\|x\|_{\mathbf{d}_E}$ versus time.**

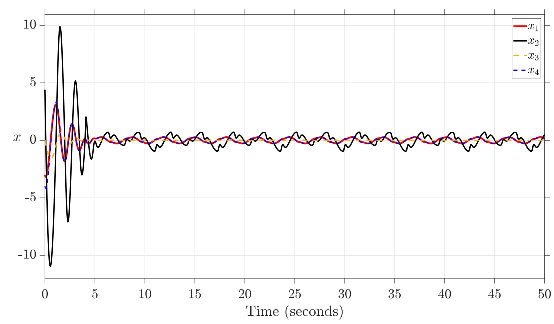

FIGURE 5. **Transients of the state vector in the presence of additive disturbances.**

In order to provide faster convergence the control (31) has been used. The simulation results for $\lambda = 2$, $M_2 = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 4 & 0 & 0.005 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ are shown in Fig. 7 for $x_0 = [-3\ \ 202.7\ \ 0\ \ -4]^T$ (the initial conditions are changed in accordance with Remark 6). The settling time estimate is $T(x_0) \leq 4.15$ according to (32).

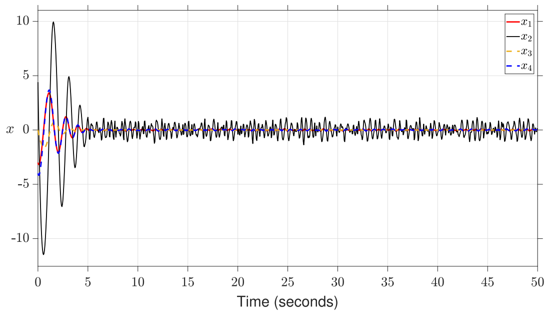

FIGURE 6. Transients of the state vector with measurement band limited noise of power 10^{-4}.

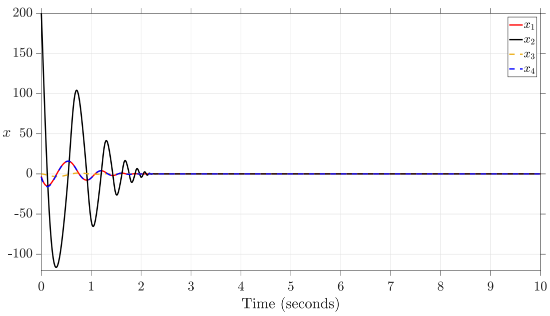

FIGURE 7. Transients of the state vector with the accelerated nonlinear control algorithm (31).

V. CONCLUSION

In this paper, the concept of $\mathbf{d}_E$-homogeneity was introduced for descriptor systems. The homogeneity property can be verified algebraically, and as for the ODE case it implies that a dilation of initial conditions leads to a scaling of trajectories. The homogenizing and stabilizing in a finite-time control is proposed. The parameters tuning procedure is in the form of linear matrix equations and inequalities solution.

The directions of future research include the following: robustness (e.g., Input-to-State Stability) analysis of $\mathbf{d}_E$-homogeneous systems, introduction of $\mathbf{d}_E$-homogeneous approximations, $\mathbf{d}_E$-homogeneous observers and control design, etc. In particular, the proposed approach can be further used to develop existing control algorithms for distributed systems with constraints (e.g., [41]) in order to provide fast convergence.

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REFERENCES

[1] D. Konovalov, K. Zimenko, A. Belov, and H. Wang, "Homogeneity-based finite-time stabilization of linear descriptor systems," in *Proc. 60th IEEE Conf. Decis. Control (CDC)*, Dec. 2021, pp. 3901–3905.
[2] V. Zubov, "On systems of ordinary differential equations with generalized homogeneous right-hand sides," *Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika*, no. 1, pp. 80–88, 1958.
[3] E. Bernuau, A. Polyakov, D. Efimov, and W. Perruquetti, "Verification of ISS, iISS and IOSS properties applying weighted homogeneity," *Syst. Control Lett.*, vol. 62, no. 12, pp. 1159–1167, 2013.
[4] K. Zimenko, D. Efimov, A. Polyakov, and W. Perruquetti, "A note on delay robustness for homogeneous systems with negative degree," *Automatica*, vol. 79, pp. 178–184, May 2017.
[5] M. Kawski, "Geometric homogeneity and stabilization," *IFAC Proc. Volumes*, vol. 28, pp. 147–152, Jun. 1995.
[6] A. Polyakov, *Generalized Homogeneity in Systems and Control* (Communications and Control Engineering). Cham, Switzerland: Springer, 2020.
[7] A. Bacciotti and L. Rosier, *Liapunov Functions and Stability in Control Theory*. Berlin, Germany: Springer, 2005.
[8] E. Cruz-Zavala, T. Sanchez, E. Nuño, and J. Moreno, "Lyapunov-based finite-time control of robot manipulators," *Int. J. Robust Nonlinear Control*, vol. 31, pp. 3090–3114, May 2021.
[9] K. Zimenko, A. Polyakov, D. Efimov, and W. Perruquetti, "On simple scheme of finite/fixed-time control design," *Int. J. Control*, vol. 93, pp. 1353–1361, Jun. 2020.
[10] K. Zimenko, A. Polyakov, D. Efimov, and W. Perruquetti, "Robust feedback stabilization of linear MIMO systems using generalized homogenization," *IEEE Trans. Autom. Control*, vol. 65, no. 12, pp. 5429–5436, Dec. 2020.
[11] A. Polyakov, "Sliding mode control design using canonical homogeneous norm," *Int. J. Robust Nonlinear Control*, vol. 29, pp. 682–701, Feb. 2019.
[12] L. Dai, Ed. *Singular Control Systems*. Berlin, Germany: Springer, 1989.
[13] T. Reis, "An infinite dimensional descriptor system model for electrical circuits with transmission lines," in *Proc. 16th Int. Symp. Math. Theory Netw. Syst.*, Leuven, Belgium, 2004, pp. 1–7.
[14] T. Kaczorek and K. Rogowski, "Descriptor linear electrical circuits and their properties," in *Fractional Linear Systems and Electrical Circuits*. Berlin, Germany: Springer, 2015, pp. 81–115.
[15] M. Géradin and A. Cardona, *Flexible Multibody Dynamics: A Finite Element Approach*. Hoboken, NJ, USA: Wiley, 2001.
[16] L. Petzold, "Differential/algebraic equations are not ODE's," *SIAM J. Sci. Stat. Comput.*, vol. 3, pp. 367–384, Sep. 1982.
[17] I. Bauer, H. Bock, S. Körkel, and J. Schlöder, "Numerical methods for optimum experimental design in DAE systems," *J. Comput. Appl. Math.*, vol. 120, no. 1–2, pp. 1–25, Aug. 2000.
[18] L. Biegler, J. Damiano, and G. Blau, "Nonlinear parameter estimation: A case study comparison," *AIChE J.*, vol.32, pp. 29–45, Jan. 1986.
[19] A. Kumar and P. Daoutidis, *Control of Nonlinear Differential Algebraic Equation Systems With Applications to Chemical Processes*. Boca Raton, FL, USA: CRC Press, 1999.
[20] D. Luenberger and A. Arbel, "Singular dynamic Leontief systems," *Econometrica, J. Econ. Soc.*, vol. 48, no. 4, pp. 991–995, May 1977.
[21] L. Sun, G. Feng, and Y. Wang, "Finite-time stabilization and H_∞ control for a class of nonlinear Hamiltonian descriptor systems with application to affine nonlinear descriptor systems," *Automatica*, vol. 50, pp. 2090–2097, Aug. 2014.
[22] C. Zheng and J. Cao, "Finite-time synchronization of singular hybrid coupled networks," *J. Appl. Math.*, vol. 2013, pp. 1–8, Jan. 2013.
[23] X. Mo, H. Niu, and Q. Lan, "Finite-time stabilization for a class of nonlinear differential-algebraic systems subject to disturbance," *Discrete Dyn. Nature Soc.*, vol. 2017, Feb. 2017, Art. no. 9730312.
[24] S. Bhat and D. Bernstein, "Finite-time stability of continuous autonomous systems," *SIAM J. Control Optim.*, vol. 38, no. 3, pp. 751–766, 2000.
[25] Y. Orlov, "Finite time stability and robust control synthesis of uncertain switched systems," *SIAM J. Control Optim.*, vol. 43, no. 4, pp. 1253–1271, Jan. 2004.
[26] H. Xu and K. Mizukami, "Hamilton-Jacobi equation for descriptor systems," *Syst. Control Lett.*, vol. 21, pp. 321–327, Oct. 1993.
[27] A. Pazy, *Semigroups of Linear Operators and Applications to Partial Differential Equations*. New York, NY, USA: Springer-Verlag, 1983.
[28] L. Rosier, "Homogeneous Lyapunov function for homogeneous continuous vector field," *Syst. Control Lett.*, vol. 19, pp. 467–473, Dec. 1992.
[29] S. Bhat and D. Bernstein, "Geometric homogeneity with applications to finite-time stability," *Math. Control, Signals Syst.*, vol. 17, pp. 101–127, May 2005.
[30] S. E. Parsegov, A. E. Polyakov, and P. S. Shcherbakov, "Fixed-time consensus algorithm for multi-agent systems with integrator dynamics," *IFAC Proc. Volumes*, vol. 46, no. 27, pp. 110–115, 2013.
[31] S. Xu and J. Lam, *Robust Control and Filtering of Singular Systems*. Berlin, Germany: Springer-Verlag, 2006.
[32] H. Wang, C. Yung, and F. Chang, *H_∞ Control for Nonlinear Descriptor Systems*. Berlin, Germany: Springer, 2006.
[33] R. Courant, F. John, A. A. Blank, and A. Solomon, *Introduction to Calculus and Analysis*, vol. 1. New York, NY, USA: Interscience, 1965.

[34] A. Polyakov, D. Efimov, and W. Perruquetti, "Finite-time stabilization using implicit Lyapunov function technique," *IFAC Proc. Volumes*, vol. 46, no. 23, pp. 140–145, 2013.

[35] A. Levant, "Finite-time stabilization of uncertain MIMO systems," in *Proc. 53rd IEEE Conf. Decis. Control*, Dec. 2014, pp. 4753–4758.

[36] D. Efimov, A. Levant, A. Polyakov, and W. Perruquetti, "Supervisory acceleration of convergence for homogeneous systems," *Int. J. Control*, vol. 91, pp. 2524–2534, Apr. 2018.

[37] K. Zimenko, A. Polyakov, and D. Efimov, "On finite-time robust stabilization via nonlinear state feedback," *Int. J. Robust Nonlinear Control*, vol. 28, pp. 4951–4965, Aug. 2018.

[38] Y. Liu, C. Gao, B. Meng, and X. Cong, "Dynamic soft variable structure control for singular systems," in *Proc. 30th Chin. Control Conf.*, 2011, pp. 2572–2577.

[39] N. Shopa, D. Konovalov, A. Kremlev, and K. Zimenko, "Finite-time stability analysis for nonlinear descriptor systems," in *Proc. Int. Conf. Informat. Control, Autom. Robot. (ICINCO)*, 2022, pp. 1–6.

[40] G. R. Duan, *Analysis and Design of Descriptor Linear Systems*, vol. 23. New York, NY, USA: Springer, 2010.

[41] L. An and G. Yang, "Distributed optimal coordination for heterogeneous linear multi-agent systems," *IEEE Trans. Autom. Control*, early access, Dec. 7, 2021, doi: 10.1109/TAC.2021.3133269.

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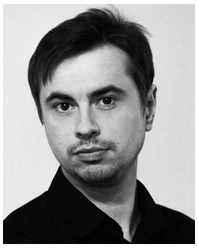

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