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RESEARCH ARTICLE

Stability Analysis of Time-Delay Systems via a Delay-Derivative-Partitioning Approach

WEI WANG, MI-HAN LIU, HONG-BING ZENG, AND GANG CHEN
School of Electrical and Information Engineering, Hunan University of Technology, Zhuzhou 412007, China

Corresponding author: Gang Chen (chengang@hut.edu.cn)

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ABSTRACT This paper is devoted to the study of delay-dependent stability of time-varying delay systems. A delay-derivative-partitioning approach is proposed. By constructing an augmented Lyapunov functional that contains two delay-product-dependent terms and using the delay-derivative-partitioning approach, an improved stability condition is established. The derived condition is described as a set of linear matrix inequalities, which can be readily implemented. Finally, four examples are carried out so as to attest the effect and merit of the presented approach.

INDEX TERMS Stability analysis, time-delay system, time-varying delay, Lyapunov functional, linear matrix inequality.

I. INTRODUCTION

Time-delay system received substantial attention due to its wide application in the modeling of biological, ecological and engineering systems [1], [2], [3], [4], [5]. Stability is a fundamental issue as it is a prerequisite for a system to operate normally. Therefore, considerable effort has been devoted to stability analysis of time-delay systems during the past decades [6], [7], [8], [9], [10], [11], [12]. The stability conditions can be classified into two categories: delay-independent conditions and delay-dependent conditions. Delay-independent conditions are usually very conservative as it does not take the size of delay into account. Therefore, more attention is focused on the problem of delay-dependent stability during the last two decades.

The main purpose of the delay-dependent stability analysis is to determine the stability-preserving region concerning time delay. Lyapunov method is prevalent to investigate the stability of time-delay systems, especially systems with time-varying delay. Based on Lyapunov method to analyze stability, the choice of Lyapunov functional is crucial to the reduction of conservativeness of the derived stability conditions. Various methods were developed for the reduction of conservativeness in view of the construction of Lyapunov functional, such as augmented Lyapunov functional [13], multiple-integral based Lyapunov functional [14], delay-decomposing Lyapunov functional [15], matrix-refined-function-based Lyapunov functional [16], and delay-product-type Lyapunov functional [17].

In the derivation of a delay-dependent condition, one of the main difficulties is to deal with the integral quadratic term, $\int_{t-h}^{t}\dot{x}^T(s)U\dot{x}(s)\mathrm{d}s$. There are three main methods proposed to cope with the integral quadratic term: model transformation approach [18], free-weighting matrix approach [20] and integral inequality approach [19]. To decrease the gap of between the integral quadratic term and its lower bound, many inequalities have been developed, such as Wirtinger-based inequality [21], auxiliary function-based integral inequalities [22], free-matrix-based integral inequalities [23], [31], and Bessel-Legendre inequality [24]. For systems with time-varying, reciprocal convex terms inevitably arise when adopting the Bessel-Legendre inequality to bound the integral quadratic term. Several reciprocal convex inequalities are developed to cope with the reciprocal convex terms (see [25], [26], [27], [28], [29], [30]). Among these inequalities, the generalized reciprocal convex inequality in [30] includes other inequalities as special cases and has the least conservativeness in comparison with others. However, the conservativeness cannot be completely eliminated in bounding the reciprocal convex terms. By contrast, these reciprocal convex

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terms do not appear when the integral quadratic terms are bounded by using the free-matrix-based integral inequalities presented in [31]. Therefore, based on the same Lyapunov functional, stability conditions derived by using the free-matrix-based integral inequalities have the least conservativeness in comparison with using other inequalities.

It is observed that most of the literatures are focused on the construction of Lyapunov functional and decrease the gap of bounding the derivative of Lyapunov functional, while the information about the delay derivative has not been fully considered. This paper revisits the stability problem of systems with time-varying delay. The contributions of the paper are summarized as follows. Firstly, a delay-derivative-partitioning approach is originally proposed. By constructing an improved Lyapunov functional and dividing the delay derivative interval into two parts, a sufficient condition is derived. Furthermore, this condition is extended to robust stability analysis of time-varying delay system with uncertain system parameters. Finally, four illustrative examples including one-area load frequency control system are carried out to attest the effect and the advantage of the proposed method.

Notation: Throughout the paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ and $\mathbb{S}^n(\mathbb{S}^n_+)$ are the sets of $n \times m$ real matrices and $n \times n$ real (positive definite) symmetric matrices, respectively; $diag\{\cdots\}$ refers to a block-diagonal matrix; Q^T and Q^{-1} stand for, respectively, the transpose and the inverse of the matrix Q; 0 represents zero matrix with appropriate dimensions; symmetric terms in a symmetric matrix are represented by the symbol '$*$'; and $\mathrm{Sym}\{U\} = U + U^T$.

II. PRELIMINARIES

Consider the following system

$$\begin{cases} \dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}x(t - r(t)) \\ x(t) = \phi(t), \quad t \in [-h, 0] \end{cases} \tag{1}$$

where $\mathcal{A}, \mathcal{B} \in \mathbb{R}^{n\times n}$ are system matrices, $x(t) \in \mathbb{R}^n$ and $\phi(t)$ are the state vector and its initial condition, $r(t)$ is a continuous time-varying function satisfying

$$0 < r(t) < h, \quad \mu_1 \le \dot{r}(t) \le \mu_2 \tag{2}$$

Before presenting our main results, the following lemma is introduced, which is indispensable to derive the main results.

Lemma 1 [31]: Let ω: $[\alpha, \beta] \to \mathbb{R}^n$ be a differentiable function. Then, for given matrices $U \in \mathbb{S}^n_+$, $W_1, W_2, W_3 \in \mathbb{R}^{m\times n}$ and a vector $\theta \in \mathbb{R}^m$, the following inequality holds,

$$-\int_\alpha^\beta \dot{\omega}^T(s)U\dot{\omega}(s)\mathrm{d}s \le \Upsilon \tag{3}$$

where

$$\Upsilon = \sum_{i=1}^{3}\left\{\frac{\beta-\alpha}{2i-1}\theta^T W_i U^{-1} W_i^T \theta + \mathrm{Sym}\{\theta^T W_i \rho_i \bar{\theta}\}\right\},$$

$$\rho_1 = \tilde{e}_1 - \tilde{e}_2,\ \rho_2 = \tilde{e}_1 + \tilde{e}_2 - 2\tilde{e}_3,$$

$$\rho_3 = \tilde{e}_1 - \tilde{e}_2 + 6\tilde{e}_3 - 12\tilde{e}_4,\ \nu = \beta - \alpha,$$

$$\bar{\theta} = col\left\{\omega(\beta), \omega(\alpha), \frac{1}{\nu}\int_\alpha^\beta \omega(s)\mathrm{d}s, \frac{1}{\nu^2}\int_\alpha^\beta\int_s^\beta \omega(u)\mathrm{d}u\mathrm{d}s\right\},$$

$$\tilde{e}_j = \left[\, 0_{n\times(j-1)n}\ I_n\ 0_{n\times(4-j)n} \,\right], j = 1, 2, 3, 4.$$

III. MAIN RESULTS

This section will present several stability conditions for the considered system. For brevity, we define the following notations.

$$\begin{aligned}
\bar{r}(t) &= h - r(t),\\
\nu_1(t) &= \int_{t-r(t)}^{t} x(s)\mathrm{d}s,\ \nu_2(t) = \int_{t-h}^{t-r(t)} x(s)\mathrm{d}s,\\
\nu_3(t) &= \frac{1}{r(t)}\int_{t-r(t)}^{t}\int_\theta^t x(s)\mathrm{d}s\mathrm{d}\theta,\\
\nu_4(t) &= \frac{1}{\bar{r}(t)}\int_{t-h}^{t-r(t)}\int_\theta^{t-r(t)} x(s)\mathrm{d}s\mathrm{d}\theta,\\
\nu_5(t,s) &= \int_s^t x(\alpha)\mathrm{d}\alpha,\\
\chi(t) &= \left[\, \chi_1^T(t)\ \ \chi_2^T(t)\ \ \chi_3^T(t)\ \ \chi_4^T(t) \,\right]^T,\\
\chi_1(t) &= \left[\, x^T(t)\ \ x^T(t-r(t))\ \ x^T(t-h) \,\right]^T,\\
\chi_2(t) &= \left[\, \dot{x}^T(t-r(t))\ \ \dot{x}^T(t-h) \,\right]^T,\\
\chi_3(t) &= \left[\, \frac{1}{r(t)}\nu_1^T(t)\ \ \frac{1}{\bar{r}(t)}\nu_2^T(t)\ \ \frac{1}{r(t)}\nu_3^T(t)\ \ \frac{1}{\bar{r}(t)}\nu_4^T(t) \,\right]^T,\\
\chi_4(t) &= \left[\, \nu_1^T(t)\ \ \nu_2^T(t)\ \ \nu_3^T(t)\ \ \nu_4^T(t)\ \ \dot{x}^T(t) \,\right]^T,\\
e_i &= \left[0_{n\times(i-1)n}\ I_n\ 0_{n\times(14-i)n}\right], \quad i = 1, 2, \cdots, 14,\\
\varrho_1(t) &= \left[\, \chi_1^T(t)\ \ \nu_1^T(t)\ \ \nu_2^T(t)\ \ \nu_3^T(t)\ \ \nu_4^T(t) \,\right]^T,\\
\varrho_2(t,s) &= \left[\chi_1^T(t)\ \ \nu_1^T(t)\ \ \nu_2^T(t)\ \ \nu_5^T(t,s)\ \ \dot{x}^T(s)\ \ x^T(s)\right]^T,\\
\varrho_3(t) &= \left[e_1^T\ \ e_2^T\ \ e_6^T\ \ e_8^T\ e_{10}^T\ e_{11}^T\ \ e_{12}^T\right]^T\chi(t),\\
\varrho_4(t) &= \left[e_1^T\ \ e_2^T\ e_7^T\ \ e_9^T\ \ e_{10}^T\ \ e_{11}^T\ \ e_{13}^T\right]^T\chi(t).
\end{aligned}$$

Based on a delay-derivative-partitioning approach, the following stability criterion is obtained.

Theorem 1: For given scalars h, μ_1 and μ_2, suppose that there exist $W, Y_1, Y_2 \in \mathbb{S}^{7n}_+$, $Q_1, Q_2 \in \mathbb{S}^{8n}_+$, $X_1, X_2 \in \mathbb{S}^n_+$, $N_{11}, N_{12}, N_{21}, N_{22} \in \mathbb{R}^{14n\times 3n}$ and $S_{11}, S_{12}, S_{13}, S_{14}, S_{21}, S_{22}, S_{23}, S_{24}, U \in \mathbb{R}^{14n\times n}$ such that LMIs (4) and (5) are satisfied for $r(t) \in [0, h]$,

$$\begin{bmatrix} \Phi(r(t),\dot{r}(t)) & \sqrt{\bar{r}(t)}N_{12} & \sqrt{r(t)}N_{11} \\ * & -\hat{X}_1 & 0 \\ * & * & -\hat{X}_2 \end{bmatrix}_{\dot{r}(t)\in[\mu_1,\frac{\mu_1+\mu_2}{2}]} < 0 \tag{4}$$

$$\begin{bmatrix} \bar{\Phi}(r(t),\dot{r}(t)) & \sqrt{\bar{r}(t)}N_{22} & \sqrt{r(t)}N_{21} \\ * & -\hat{X}_1 & 0 \\ * & * & -\hat{X}_2 \end{bmatrix}_{\dot{r}(t)\in[\frac{\mu_1+\mu_2}{2},\mu_2]} < 0 \tag{5}$$

where

$$\Phi(r(t), \dot r(t)) = \Xi_1 + \Xi_2 + \Xi_3,$$
$$\bar\Phi(r(t), \dot r(t)) = \Xi_1 + \Xi_2 + \bar\Xi_3,$$
$$\begin{aligned}\Xi_1 &= Sym\{\Pi_1^T W\lambda_1 + \Pi_5^T Q_1\lambda_2 + \Pi_6^T Q_2\lambda_2\} \\ &\quad + \Pi_2^T Q_1\Pi_2 - (1-\dot r(t))\Pi_3^T(Q_1 - Q_2)\Pi_3 \\ &\quad - \Pi_4^T Q_2\Pi_4 + U(\mathcal{A}e_1 + \mathcal{B}e_2 - e_{14}) \\ &\quad + he_{14}^T X_2 e_{14} + (1-\dot r(t))\bar r(t)e_4^T(X_1 - X_2)e_4,\\ \Xi_2 &= Sym\{\Pi_7^T Y_1\lambda_3 + \Pi_8^T Y_2\lambda_4\} + \dot r(t)\Pi_7^T Y_1\Pi_7 \\ &\quad - \dot r(t)\Pi_8^T Y_2\Pi_8,\\ \Xi_3 &= Sym\{S_{11}(r(t)e_6 - e_{10}) + S_{12}(\bar r(t)e_7 - e_{11}) \\ &\quad + S_{13}(r(t)e_8 - e_{12}) + S_{14}(\bar r(t)e_9 - e_{13}) \\ &\quad + N_{11}M_1 + N_{12}M_2\},\\ \bar\Xi_3 &= Sym\{S_{21}(r(t)e_6 - e_{10}) + S_{22}(\bar r(t)e_7 - e_{11}) \\ &\quad + S_{23}(r(t)e_8 - e_{12}) + S_{24}(\bar r(t)e_9 - e_{13}) \\ &\quad + N_{21}M_1 + N_{22}M_2\},\end{aligned}$$

with

$$\begin{aligned}
\Pi_1 &= \begin{bmatrix} e_1^T & e_2^T & e_3^T & e_{10}^T & e_{11}^T & e_{12}^T & e_{13}^T \end{bmatrix}^T,\\
\Pi_2 &= \begin{bmatrix} e_1^T & e_2^T & e_3^T & e_{10}^T & e_{11}^T & 0 & e_{14}^T & e_1^T \end{bmatrix}^T,\\
\Pi_3 &= \begin{bmatrix} e_1^T & e_2^T & e_3^T & e_{10}^T & e_{11}^T & e_{10}^T & e_4^T & e_2^T \end{bmatrix}^T,\\
\Pi_4 &= \begin{bmatrix} e_1^T & e_2^T & e_3^T & e_{10}^T & e_{11}^T & e_{10}^T + e_{11}^T & e_5^T & e_3^T \end{bmatrix}^T,\\
\Pi_5 &= \begin{bmatrix} r(t)e_1^T & r(t)e_2^T & r(t)e_3^T & r(t)e_{10}^T & \pi_{5a}^T \end{bmatrix}^T,\\
\pi_{5a} &= \begin{bmatrix} r(t)e_{11}^T & r(t)e_{12}^T & e_1^T - e_2^T & e_{10}^T \end{bmatrix}^T,\\
\Pi_6 &= \begin{bmatrix} \bar r(t)e_1^T & \bar r(t)e_2^T & \bar r(t)e_3^T & \bar r(t)e_{10}^T & \pi_{6a}^T \end{bmatrix}^T,\\
\pi_{6a} &= \begin{bmatrix} \bar r(t)e_{11}^T & \bar r(t)(e_{10}+e_{13})^T & e_2^T - e_3^T & e_{11}^T \end{bmatrix}^T,\\
\Pi_7 &= \begin{bmatrix} e_1^T & e_2^T & e_6^T & e_8^T & e_{10}^T & e_{11}^T & e_{12}^T \end{bmatrix}^T,\\
\Pi_8 &= \begin{bmatrix} e_1^T & e_2^T & e_7^T & e_9^T & e_{10}^T & e_{11}^T & e_{13}^T \end{bmatrix}^T,\\
\lambda_1 &= \begin{bmatrix} e_{14}^T & (1-\dot r(t))e_4^T & e_5^T & \tau_1^T & \tau_2^T & \tau_3^T & \tau_4^T \end{bmatrix}^T,\\
\lambda_2 &= \begin{bmatrix} e_{14}^T & (1-\dot r(t))e_4 & e_5 & \tau_1^T & \tau_2^T & e_1^T & 0 & 0 \end{bmatrix}^T,\\
\lambda_3 &= \begin{bmatrix} \tau_5^T & \tau_6^T & \tau_7^T & r(t)\tau_1^T & r(t)\tau_2^T & r(t)\tau_3^T \end{bmatrix}^T,\\
\lambda_4 &= \begin{bmatrix} \tau_8^T & \tau_9^T & \tau_{10}^T & \bar r(t)\tau_1^T & \bar r(t)\tau_2^T & \bar r(t)\tau_4^T \end{bmatrix}^T,\\
\tau_1 &= e_1 - (1-\dot r(t))e_2, \quad \tau_2 = (1-\dot r(t))e_2 - e_3,\\
\tau_3 &= e_1 - (1-\dot r(t))e_6 - \dot r(t)e_8,\\
\tau_4 &= (1-\dot r(t))e_2 - e_7 + \dot r(t)e_9,\\
\tau_5 &= \begin{bmatrix} r(t)e_{14}^T & r(t)(1-\dot r(t))e_4^T \end{bmatrix}^T,\\
\tau_6 &= e_1 - (1-\dot r(t))e_2 - \dot r(t)e_6,\\
\tau_7 &= e_1 - (1-\dot r(t))e_6 - 2\dot r(t)e_8,\\
\tau_8 &= \begin{bmatrix} \bar r(t)e_{14}^T & \bar r(t)(1-\dot r(t))e_4^T \end{bmatrix}^T,\\
\tau_9 &= (1-\dot r(t))e_2 - e_3 + \dot r(t)e_7,\\
\tau_{10} &= (1-\dot r(t))e_2 - e_7 + 2\dot r(t)e_9,\\
\hat X_i &= diag(X_i, 3X_i, 5X_i) \quad i = 1, 2,
\end{aligned}$$

$$M_1 = \begin{bmatrix} e_1 - e_2 \\ e_1 + e_2 - 2e_6 \\ e_1 - e_2 + 6e_6 - 12e_8 \end{bmatrix},$$
$$M_2 = \begin{bmatrix} e_2 - e_3 \\ e_2 + e_3 - 2e_7 \\ e_2 - e_3 + 6e_7 - 12e_9 \end{bmatrix}.$$

Then, system (1) with the delay subject to (2) is asymptotically stable.

Proof: Choose the Lyapunov functional candidate as:

$$V(t) = V_1(t) + V_2(t) \tag{6}$$

where

$$\begin{aligned}
V_1(t) &= \varrho_1^T(t)W\varrho_1(t) + \int_{t-r(t)}^{t} \varrho_2^{\mathrm T}(t,s)Q_1\varrho_2(t,s)\mathrm{d}s \\
&\quad + \int_{t-h}^{t-r(t)} \varrho_2^{\mathrm T}(t,s)Q_2\varrho_2(t,s)\mathrm{d}s \\
&\quad + \int_{t-h}^{t-r(t)} (h-t+s)\dot x^T(s)X_1\dot x(s)\mathrm{d}s \\
&\quad + \int_{t-r(t)}^{t} (h-t+s)\dot x^T(s)X_2\dot x(s)\mathrm{d}s,\\
V_2(t) &= r(t)\varrho_3^T(t)Y_1\varrho_3(t) + \bar r(t)\varrho_4^T(t)Y_2\varrho_4(t).
\end{aligned}$$

Taking the derivative of $V(t)$ along the trajectories of system (1) yields

$$\begin{aligned}
\dot V_1(t) &= 2\varrho_1^T(t)W\dot\varrho_1(t) + \varrho_2^T(t,t)Q_1\varrho_2(t,t) \\
&\quad - (1-\dot r(t))\varrho_2^{\mathrm T}(t,t-r(t))Q_1\varrho_2(t,t-r(t)) \\
&\quad + 2\int_{t-r(t)}^{t} \varrho_2^{\mathrm T}(t,s)Q_1\frac{\partial\varrho_2(t,s)}{\partial t}\mathrm{d}s \\
&\quad + (1-\dot r(t))\varrho_2^{\mathrm T}(t,t-r(t))Q_2\varrho_2(t,t-r(t)) \\
&\quad - \varrho_2^T(t,t-h)Q_2\varrho_2(t,t-h) \\
&\quad + 2\int_{t-h}^{t-r(t)} \varrho_2^{\mathrm T}(t,s)Q_2\frac{\partial\varrho_2(t,s)}{\partial t}\mathrm{d}s \\
&\quad + (1-\dot r(t))\bar r(t)\dot x^T(t-r(t))X_1\dot x(t-r(t)) \\
&\quad - (1-\dot r(t))\bar r(t)\dot x^T(t-r(t))X_2\dot x(t-r(t)) \\
&\quad + h\dot x^T(t)X_2\dot x(t) - \int_{t-h}^{t-r(t)} \dot x^T(s)X_1\dot x(s)\mathrm{d}s \\
&\quad - \int_{t-r(t)}^{t} \dot x^T(s)X_2\dot x(s)\mathrm{d}s
\end{aligned} \tag{7}$$

$$\begin{aligned}
\dot V_2(t) &= \dot r(t)\varrho_3^T(t)Y_1\varrho_3(t) + 2r(t)\varrho_3^T(t)Y_1\dot\varrho_3(t) \\
&\quad - \dot r(t)\varrho_4^T(t)Y_2\varrho_4(t) + 2\bar r(t)\varrho_4^T(t)Y_2\dot\varrho_4(t)
\end{aligned} \tag{8}$$

For brevity, the delay derivative interval $[\mu_1, \mu_2]$ is equally divided into two subintervals $[\mu_1, \mu_a]$ and $[\mu_a, \mu_2]$, where $\mu_a = \frac{\mu_1+\mu_2}{2}$. For $\dot r(t) \in [\mu_1, \mu_a]$, it follows from Lemma 1 that

$$\begin{aligned}
&-\int_{t-h}^{t-r(t)} \dot x^T(s)X_1\dot x(s)\mathrm{d}s - \int_{t-r(t)}^{t} \dot x^T(s)X_2\dot x(s)\mathrm{d}s \\
&\quad \le \chi^T(t)[Sym\{N_{11}M_1 + N_{12}M_2\} + \Omega(r(t))]\chi(t)
\end{aligned} \tag{9}$$

with $\Omega(r(t)) = r(t)N_{11}\hat X_2^{-1}N_{11}^T + \bar r(t)N_{12}\hat X_1^{-1}N_{12}^T$.

It is observed from $\chi(t)$ that

$$2\chi^T(t)S_{11}(r(t)e_6 - e_{10})\chi(t) = 0 \quad (10)$$

$$2\chi^T(t)S_{12}(\bar{r}(t)e_7 - e_{11})\chi(t) = 0 \quad (11)$$

$$2\chi^T(t)S_{13}(r(t)e_8 - e_{12})\chi(t) = 0 \quad (12)$$

$$2\chi^T(t)S_{14}(\bar{r}(t)e_9 - e_{13})\chi(t) = 0 \quad (13)$$

$$2\chi^T(t)U(\mathcal{A}e_1 + \mathcal{B}e_2 - e_{14})\chi(t) = 0 \quad (14)$$

for any matrices $S_{11}, S_{12}, S_{13}, S_{14}, U \in \mathbb{R}^{14n\times n}$.

Adding the left-hand sides of (10)-(13) to $\dot{V}(t)$ and using (9), we get

$$\dot{V}(t) \leq \chi^T(t)(\Phi(r(t), \dot{r}(t)) + \Omega(r(t)))\chi(t) \quad (15)$$

where $\Phi(r(t), \dot{r}(t))$ is defined in Theorem 1.

If $\Phi(r(t), \dot{r}(t)) + \Omega(r(t)) < 0$ for $r(t) \in [0, h]$, which is equivalent to LMI (4) in the sense of Schur complement, then $\dot{V}(t) < 0$.

For $\dot{r}(t) \in [\mu_a, \mu_2]$, replacing N_{1i} with $N_{2i} (i = 1, 2)$ in (9), and S_{1j} with $S_{2j} (j = 1, 2, 3, 4)$ in (10)-(13), we get

$$\dot{V}(t) \leq \chi^T(t)(\bar{\Phi}(r(t), \dot{r}(t)) + \bar{\Omega}(r(t)))\chi(t) \quad (16)$$

with $\bar{\Omega}(r(t)) = r(t)N_{21}\hat{X}_2^{-1}N_{21}^T + \bar{r}(t)N_{22}\hat{X}_1^{-1}N_{22}^T$. Similarly, if $\bar{\Phi}(r(t), \dot{r}(t)) + \bar{\Omega}(r(t)) < 0$ for $r(t) \in [0, h]$, which is equivalent to LMI (5), then $\dot{V}(t) < 0$. Based on Lyapunov stability theory, the system (1) is asymptotically stable. This completes the proof.

Remark 1: In the derivation of Theorem 1, four vectors, $\nu_1(t)$, $\nu_2(t)$, $\nu_3(t)$ and $\nu_4(t)$ are introduced in $\chi(t)$ to avoid the emergence of quadratic terms of $r(t)$. Noted that $r(t)e_6\chi(t) = e_{10}\chi(t)$, $\bar{r}(t)e_7\chi(t) = e_{11}\chi(t)$, $r(t)e_8\chi(t) = e_{12}\chi(t)$ and $\bar{r}(t)e_9\chi(t) = e_{13}\chi(t)$, the equations (10)-(13) are introduced, which contribute to the conservativeness reduction.

Remark 2: Two delay-product-dependent terms, $r(t)\varrho_3^T(t)Y_1\varrho_3(t)$ and $\bar{r}(t)\varrho_4^T(t)Y_2\varrho_4(t)$, are added in the LKF (6) to reduce conservativeness. On the other hand, as the Lyapunov matrices, Y_1 and Y_2, are dependent on $r(t)$, it make the LKF (6) capture more information about the variation of the delay.

Remark 3: Inspired by [32], a delay-derivative- partitioning approach is proposed. Different form existing literature [33], [34], [35], [36], [37], [38], [39], [40], [41], [42], by dividing the interval of delay derivative into two subintervals, two groups of free matrices are employed for the delay derivative to be on different subintervals. Therefore, the derived condition may be less conservative. The conservativeness can be further reduced by increasing the number of subintervals to be divided.

To show the effect of the proposed delay-derivative-partitioning approach, the following stability condition without using the delay-derivative-partitioning approach is presented, which is straightforward obtained by setting $N_{21} = N_{11}, N_{22} = N_{12}, S_{21} = S_{11}, S_{22} = S_{12}, S_{23} = S_{13}$, and $S_{24} = S_{14}$ in Theorem 1.

Corollary 1: For given scalars h, μ_1 and μ_2, suppose that there exist $W, Y_1, Y_2 \in \mathbb{S}_+^{7n}$, $Q_1, Q_2 \in \mathbb{S}_+^{8n}$, $X_1, X_2 \in \mathbb{S}_+^n$, $N_{11}, N_{12} \in \mathbb{R}^{14n\times 3n}$, and $S_{11}, S_{12}, S_{13}, S_{14}, U \in \mathbb{R}^{14n\times n}$, such that LMI (17) is satisfied for $r(t) \in [0, h]$ and $\dot{r}(t) \in [\mu_1, \mu_2]$,

$$\begin{bmatrix} \Phi(r(t), \dot{r}(t)) & \sqrt{\bar{r}(t)}N_{12} & \sqrt{r(t)}N_{11} \\ * & -\hat{X}_1 & 0 \\ * & * & -\hat{X}_2 \end{bmatrix} < 0 \quad (17)$$

where $\Phi(r(t), \dot{r}(t)), \hat{X}_1, \hat{X}_2$ are defined in Theorem 1. Then, system (1) with the delay subject to (2) is asymptotically stable.

In addition, the condition in Theorem 1 can be readily extended to uncertain systems that system parameters $\mathcal{A}$ and $\mathcal{B}$ subject to convex polynomial constraints, which are represented as

$$[\mathcal{A}\ \mathcal{B}] \in \Omega,\ \Omega := \left\{ [\mathcal{A}(\xi)\ \mathcal{B}(\xi)] = \sum_{j=1}^{p} \xi_j [\mathcal{A}_j\ \mathcal{B}_j], \right. \left. \sum_{j=1}^{p} \xi_j = 1,\ \xi_j \geq 0 \right\} \quad (18)$$

where $\mathcal{A}_j$, $\mathcal{B}_j$ $(j = 1, \cdots, p)$ are constant matrices with appropriate dimensions and ξ_j $(j = 1, \cdots, p)$ are time-invariant uncertainties.

For system (1) with uncertain parameters satisfied with (18), the following robust stability criterion is presented.

Theorem 2: For given scalars h, μ_1 and μ_2, suppose that there exist $W_j, Y_{1j}, Y_{2j} \in \mathbb{S}_+^{7n}$, $Q_{1j}, Q_{2j} \in \mathbb{S}_+^{8n}$, $X_{1j}, X_{2j} \in \mathbb{S}_+^n$, $N_{11j}, N_{12j}, N_{21j}, N_{22j} \in \mathbb{R}^{14n\times 3n}$, and $S_{11j}, S_{12j}, S_{13j}, S_{14j}, S_{21j}, S_{22j}, S_{23j}, S_{24j}, U \in \mathbb{R}^{14n\times n}$ such that LMIs (19) and (20) are satisfied for $r(t) \in [0, h]$ and $j = 1, \cdots, p$,

$$\begin{bmatrix} \Phi_{11}^{(j)}(r(t), \dot{r}(t)) & \Phi_{12}^{(j)}(r(t)) \\ * & \Phi_{22}^{(j)} \end{bmatrix}_{\dot{r}(t)\in[\mu_1, \frac{\mu_1+\mu_2}{2}]} < 0 \quad (19)$$

$$\begin{bmatrix} \bar{\Phi}_{11}^{(j)}(r(t), \dot{r}(t)) & \bar{\Phi}_{12}^{(j)}(r(t)) \\ * & \Phi_{22}^{(j)} \end{bmatrix}_{\dot{r}(t)\in[\frac{\mu_1+\mu_2}{2}, \mu_2]} < 0 \quad (20)$$

where

$$\Phi_{11}^{(j)}(r(t), \dot{r}(t)) = \Xi_1^{(j)} + \Xi_2^{(j)} + \Xi_3^{(j)},$$

$$\bar{\Phi}_{11}^{(j)}(r(t), \dot{r}(t)) = \Xi_1^{(j)} + \Xi_2^{(j)} + \bar{\Xi}_3^{(j)},$$

$$\Phi_{12}^{(j)}(r(t)) = \left[\sqrt{\bar{r}(t)}N_{12j}\ \ \sqrt{r(t)}N_{11j}\right],$$

$$\bar{\Phi}_{12}^{(j)}(r(t)) = \left[\sqrt{\bar{r}(t)}N_{22j}\ \ \sqrt{r(t)}N_{21j}\right],$$

$$\Phi_{22}^{(j)} = -diag\{X_{1j}, 3X_{1j}, 5X_{1j}, X_{2j}, 3X_{2j}, 5X_{2j}\}$$

with

$$\begin{aligned}\Xi_1^{(j)} = &\ \mathrm{Sym}\{\Pi_1^T W_j \lambda_1 + \Pi_5^T Q_{1j}\lambda_2 + \Pi_6^T Q_{1j}\lambda_2\} \\ &+ \Pi_2^T Q_{1j}\Pi_2 - (1 - \dot{r}(t))\Pi_3^T(Q_{1j} - Q_{2j})\Pi_3 \\ &- \Pi_4^T Q_{2j}\Pi_4 + U(\mathcal{A}e_1 + \mathcal{B}e_2 - e_{14}) + he_{14}^T X_{2j}e_{14} \\ &+ (1 - \dot{r}(t))\bar{r}(t)e_4^T(X_{1j} - X_{2j})e_4,\end{aligned}$$

$$\begin{aligned}\Xi_2^{(j)} = &\ \mathrm{Sym}\{\Pi_7^T Y_{1j}\lambda_3 + \Pi_8^T Y_{2j}\lambda_4\} + \dot{r}(t)\Pi_7^T Y_{1j}\Pi_7 \\ &- \dot{r}(t)\Pi_8^T Y_{2j}\Pi_8,\end{aligned}$$

TABLE 1. System matrices for given examples.

	$\mathcal{A}$	$\mathcal{B}$
Example 1	$\begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$
Example 2	$\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}$

$$\begin{aligned}\Xi_3^{(j)} &= Sym\{S_{11j}(r(t)e_6 - e_{10}) + S_{12j}(\bar{r}(t)e_7 - e_{11}) \\ &\quad + S_{13j}(r(t)e_8 - e_{12}) + S_{14j}(\bar{r}(t)e_9 - e_{13}) \\ &\quad + N_{11j}M_1 + N_{12j}M_2\}, \\ \bar{\Xi}_3^{(j)} &= Sym\{S_{21j}(r(t)e_6 - e_{10}) + S_{22j}(\bar{r}(t)e_7 - e_{11}) \\ &\quad + S_{23j}(r(t)e_8 - e_{12}) + S_{24j}(\bar{r}(t)e_9 - e_{13}) \\ &\quad + N_{21j}M_1 + N_{22j}M_2\},\end{aligned}$$

and $\Pi_i (i = 1, 2, \cdots, 8), \lambda_1, \lambda_2, \lambda_3, \lambda_4, M_1,$ and M_2 are defined in Theorem 1.

Proof: Choose the Lyapunov functional candidate as:

$$\hat{V}(t) = \hat{V}_1(t) + \hat{V}_2(t) \tag{21}$$

where

$$\begin{aligned}\hat{V}_1(t) &= \sum_{j=1}^{p} \varrho_1^T(t)\xi_j W_j \varrho_1(t) \\ &\quad + \sum_{j=1}^{p} \int_{t-r(t)}^{t} \varrho_2^{\mathrm{T}}(t,s)\xi_j Q_{1j}\varrho_2(t,s)\mathrm{d}s \\ &\quad + \sum_{j=1}^{p} \int_{t-h}^{t-r(t)} \varrho_2^{\mathrm{T}}(t,s)\xi_j Q_{2j}\varrho_2(t,s)\mathrm{d}s \\ &\quad + \sum_{j=1}^{p} \int_{t-h}^{t-r(t)} (h-t+s)\dot{x}^T(s)\xi_j X_{1j}\dot{x}(s)\mathrm{d}s \\ &\quad + \sum_{j=1}^{p} \int_{t-r(t)}^{t} (h-t+s)\dot{x}^T(s)\xi_j X_{2j}\dot{x}(s)\mathrm{d}s, \\ \hat{V}_2(t) &= \sum_{j=1}^{p} r(t)\varrho_3^T(t)\xi_j Y_{1j}\varrho_3(t) + \sum_{j=1}^{p} \bar{r}(t)\varrho_4^T(t)\xi_j Y_{2j}\varrho_4(t).\end{aligned}$$

By employing a similar procedure with the proof of Theorem 1, Theorem 2 can be readily obtained. For brevity, the details are omitted.

IV. NUMERICAL EXAMPLES

In this section, four examples are provided to verify the superiority and the effectiveness of the proposed method. Firstly, consider system (1) without uncertainties. The system matrices are listed in Table 1.

These two examples are ordinarily adopted in previous literature for the purpose of comparing the conservativeness of different stability conditions. Given $\mu_1 = -\mu$ and $\mu_2 = \mu$, Table 2 and Table 3 list the allowable upper bounds (AUBs) of delay obtained by the presented stability conditions, respectively, for Example 1 and Example 2. As shown in Tables 2 and 3, the AUBs obtained by Theorem 1 are larger than others in the existing literature. In comparison with the AUBs obtained by Theorem 1 and Corollary 1, it is observed that the results obtained by Theorem 1 are superior to that by Corollary 1. It indicates that the delay-derivative-partitioning approach presented in this paper is effective to reduce the conservativeness of the derived conditions.

TABLE 2. Allowable upper bounds of delay (Example 1).

Methods	μ=0.1	μ=0.5	μ=0.8
[32]	4.832	3.122	2.676
[26]	4.714	2.608	2.375
[28]	4.910	3.233	2.789
[31]	4.921	3.221	2.792
[33]	4.93	3.09	2.66
[34]	4.936	3.273	2.848
[37]	4.945	3.314	2.882
[38]	5.026	3.428	2.997
[40]	4.943	3.322	2.899
[39]	4.966	3.395	2.983
[41]	5.021	3.493	3.068
[42]	5.078	3.481	3.005
Theorem 1	5.097	3.549	3.147
Corollary 1	5.058	3.522	3.117

TABLE 3. Allowable upper bounds of delay (Example 2).

Methods	μ=0.1	μ=0.2	μ=0.5	μ=0.8
[16]	7.167	4.517	2.415	1.838
[35]	7.190	4.527	2.447	1.856
[28]	7.230	4.556	2.509	1.940
[36]	7.263	4.591	2.575	2.011
[31]	7.308	4.670	2.664	2.072
[40]	7.412	4.797	2.735	2.114
[39]	7.572	4.947	2.801	2.137
[41]	7.682	4.997	2.814	2.149
[42]	7.685	4.985	2.806	2.148
Theorem 1	7.730	5.034	2.841	2.176
Corollary 1	7.708	5.015	2.831	2.167

TABLE 4. Consider system with uncertainties ($|\rho| \leq 0.035$).

	$\mathcal{A}$	$\mathcal{B}$
Example 3	$\begin{bmatrix} 0 & -0.12+12\rho \\ 1 & -0.465-\rho \end{bmatrix}$	$\begin{bmatrix} -0.1 & -0.35 \\ 0 & 0.3 \end{bmatrix}$

TABLE 5. Allowable upper bounds of delay (Example 3).

Methods	μ=0	μ=0.1	μ=0.5	μ=0.9
[43]	0.863	0.786	0.465	0.454
Theorem 2	0.896	0.858	0.770	0.719

Next, consider system (1) with uncertain parameters. The system parameters are given in Table 4, which can be represented as (18) with

$$\begin{aligned}\mathcal{A}_1 &= \begin{bmatrix} 0 & -0.12+12\rho_m \\ 1 & -0.465-\rho_m \end{bmatrix}, \quad \mathcal{A}_2 = \begin{bmatrix} 0 & -0.12-12\rho_m \\ 1 & -0.465+\rho_m \end{bmatrix} \\ \mathcal{B}_1 &= \mathcal{B}_2 = \begin{bmatrix} -0.1 & -0.35 \\ 0 & 0.3 \end{bmatrix}, \quad \rho_m = 0.035.\end{aligned}$$

Setting $\mu_2 = -\mu_1 = \mu$, for various μ, Table 5 lists the AUBs of delay obtained by Theorem 2, along with the results given in [43], which verifies the effectiveness of the proposed method.

Example 4: Consider the one-area load frequency control system presented in [44], which is modeled as system (1) with

$$x(t) = \begin{bmatrix} \Delta f & \Delta P_m & \Delta P_v & \int ACE \end{bmatrix}^T,$$

TABLE 6. Allowable upper bounds of delay (Example 4).

Methods	μ=0	μ=0.1	μ=0.5	μ=0.9
[44]	6.86	–	–	4.76
[45]	7.55	7.38	7.09	6.98
Theorem 1	7.57	7.48	7.27	7.15

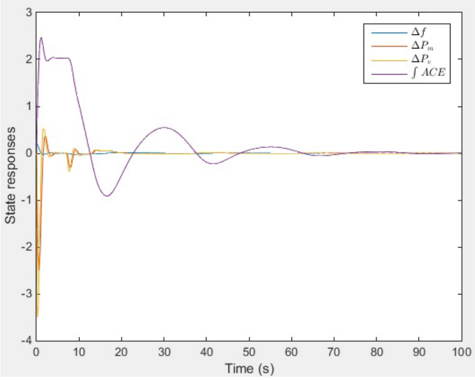

FIGURE 1. State response of systems (1) with the parameters in Example 4.

$$\mathcal{A} = \begin{bmatrix} -\frac{D}{M} & \frac{1}{M} & 0 & 0 \\ 0 & -\frac{1}{T_T} & \frac{1}{T_T} & 0 \\ -\frac{1}{T_G R} & 0 & -\frac{1}{T_G} & 0 \\ \beta & 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{B} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{\beta K_P}{T_G} & 0 & 0 & -\frac{K_I}{T_G} \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

where Δf, ΔP_m, ΔP_v and $\int ACE$ are the deviation of frequency, the mechanical output of generator, the valve position, and the integral of the Area Control Error (ACE), respectively. $D = 1.0$ and $M = 10$ are, respectively, the generator damping coefficient and the moment of inertia of the generator, $T_G = 0.1$ and $T_T = 0.3$ are the time constants of the governor and the turbine, respectively, $K_P = 0.05$ and $K_I = 0.2$ are the gains of PI controller, $R = 0.05$ is the speed drop and $\beta = 21$ is the frequency bias factor. Setting $\mu_2 = -\mu_1 = \mu$, for various μ, the AUBs of delay obtained by Theorem 1 are summarized in Table 6. It is shown in the table that the proposed method outperforms [44], [45]. Choosing $r(t) = 5.27 + 2sin(0.25t)$, which satisfies (2) with $h = 7.24$ and $\mu = 0.5$, the state response of the system with $x(0) = \begin{bmatrix} 1 & 0.1 & 0.5 & 0 \end{bmatrix}^T$ is depicted in Figure 1. It verifies the effectiveness of the proposed method.

V. CONCLUSION

This paper concerns the stability of time-varying delay systems. A delay-derivative-partitioning approach has been proposed. Based on the delay-derivative-partitioning approach and by constructing an augmented Lyapunov functional that includes two delay-product terms, improved stability criteria are obtained. As two groups of free matrices are employed for the delay derivative on different intervals, which relax the condition and hence reduce the conservativeness of the obtained results. Four examples are finally provided to illustrate the effectiveness of the proposed method.

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WEI WANG received the B.Sc. degree in electrical engineering from the Tianjin University of Technology and Education, Tianjin, China, in 2003, and the M.Sc. degree in electrical engineering from Hunan University, Changsha, China, in 2019. From April 2016 to February 2017, she has worked as a Research Associate with the Department of Mathematics and Statistics, Curtin University, Perth, WA, Australia. She is currently an Associate Professor with the Department of Electrical and Information Engineering, Hunan University of Technology, China. Her current research interests include neural networks, sampled-data control, and time-delay systems.

MI-HAN LIU received the B.Sc. degree in electrical engineering and automation from Northeast Electric Power University, Jilin, China, in 2020. He is currently pursuing the M.S. degree in energy and power engineering with the Hunan University of Technology, Zhuzhou, China. His current research interests include time-delay systems and its applications.

HONG-BING ZENG received the B.Sc. degree in electrical engineering from the Tianjin University of Technology and Education, Tianjin, China, in 2003, the M.Sc. degree in computer science from the Central South University of Forestry, Changsha, China, in 2006, and the Ph.D. degree in control science and engineering from Central South University, Changsha, in 2012. From 2013 to 2014, he was a Postdoctoral Research Associate with the Department of Electrical Engineering, Yeungnam University, Kyongsan, South Korea. Since 2003, he has been with the Department of Electrical and Information Engineering, Hunan University of Technology, Zhuzhou, China, where he is currently a Professor in automatic control engineering. His current research interests include time-delay systems, neural networks, and networked control systems.

GANG CHEN received the M.Sc. degree in chemical process machinery from Guangxi University, Nanning, China, in 2006, and the Ph.D. degree in computer application technology from Central South University, Changsha, in 2012. He is currently an Associate Professor with the School of Electrical and Information Engineering, Hunan University of Technology, Zhuzhou, China. His research interests include time-delay systems, networked control systems, and robust control.