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LMI-Based Composite Nonlinear Feedback Tracker for Uncertain Nonlinear Systems With Time Delay and Input Saturation

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ABSTRACT This study proposes a composite nonlinear feedback approach for the robust tracking control problem of uncertain nonlinear systems with input saturation, Lipschitz nonlinear functions, multivariable time-delays, and disturbances. The composite nonlinear feedback technique includes two components: a linear feedback portion constructed in such a way that it changes the damping ratio so as to speed up the system's response. A nonlinear feedback controller is designed to further increment the damping ratio in a way that it ensures the tracking whilst reducing the overshoot created by the linear portion. By creating a suitable Lyapunov functional and by using the linear matrix inequality (LMI) approach, the LMI conditions are determined to guarantee system stability and obtain the required design parameters. The performance of the proposed approach is assessed using a simulation study of a two-dimensional system along with a Chua's circuit system. The advantages of the proposed approach are its less-restrictive assumptions, improved transient performance, and steady-state precision.

INDEX TERMS Composite nonlinear feedback, input saturation, linear matrix inequality, robust tracking, time delay.

I. INTRODUCTION

Dynamic systems can be subjected to time-delays and input saturation; problems that can potentially lead to system instability and malfunctions [1], [2], [3], [4], [5], [6], [7], [8]. Time-varying systems are widely encountered in aerospace and military applications such as airplanes, satellites, missiles and rockets. External disturbances are also other factors that may disrupt the desired performance of these systems [9]. In these systems, since the tracking of the input command is of particular importance, the ability of the controller to track the input command and remove the effects of perturbations from the controlled system is critical [10]. Input saturation

is a constraint commonly found in practical systems that do not allow the control input to exceed the specified control bounds. Ignoring the input saturation constraint in the control design stage can lead to undesirable behavior and potential instability of the closed-loop system. Input saturation and time delay frequently appear simultaneously in control systems [11], [12]. Various approaches have been devised in the literature to mitigate these problem, such as adaptive neural tracking control [13], robust tracking control [14], [15], H_∞ output tracking control [16], observer-based adaptive fuzzy tracking control [17], quantized state feedback [18], and robust H_∞ control [19].

Stabilization and tracking control are critical for systems requiring robust and optimal performance such electronics, chemical processes, helicopter flight control systems.

In addition to ensuring system stability and satisfactory performance, attributes of an ideal controller are a simple structure, high-speed calculation, and robustness against uncertainties and unmodelled dynamics. Fast response and overshoot features are also considered as important criteria in tracking problems. However, the fast response often results in high frequency, which is unwanted in many applications. This can be solved via the composite nonlinear feedback (CNF) method [20], [21], [22], [23], [24].

The CNF approach is an effective robust control method for improving the tracking performance of uncertain nonlinear systems with input saturation. A CNF control method was proposed in [25] for second-order linear systems subject to input saturation. The main features of the CNF are its quick response, robustness against uncertainties, and disturbances, high transient performance, and small overshoot [26], [27], [28]. A CNF procedure is used in [29] for the semi-global stability of discrete-time singular linear systems in the presence of saturation. However, the system nonlinearities and disturbances have not been considered in that approach. An integral sliding mode-based CNF approach was proposed in [30] for linear descriptor systems without uncertainties and external disturbances. A composite nonlinear feedback procedure was proposed in [31] for the robust tracking of systems under uncertain parameters and time delays; but, this study did not consider nonlinearities. In [32], a design process for creating the CNF control is expanded to the transient performance in the tracking problems for the switched linear systems with input saturation; however, the parameter uncertainties and time delays have not been applied to the system. In [33], a CNF control method was proposed for the tracking control problem of singular linear systems with input saturation; however, uncertainties and nonlinearities were not investigated. In [34], a CNF controller was applied for the tracking control problem of strict-feedback nonlinear systems without considering time delays nor nonlinearities. A sliding mode-based adaptive composite nonlinear feedback controller was studied in [35] for nonlinear systems; however, uncertainties, time delays, and saturations were not considered.

A major problem in control design is the conflict between high efficiency and transient response. The CNF method is a practical and impressive procedure, which is used to improve the uncertain nonlinear system performance and overcome the barriers of transient performance [36], [37], [38]. In recent papers, there is more enthusiasm in applying this method for a variety of systems in comparison with other methods. For a specific type of car suspension system, the CNF procedure is applied to diminish the chattering effects [39], [40]. The robust CNF procedure is expressed in [41] to enhance quick and exact set-point tracking for damaged linear systems. In [42], the CNF procedure with a nonlinear term has been studied in terms of smooth and quick regulation with the uncertainty, external disturbances and input saturation rendezvous for the spacecrafts regardless of the nonlinearities and time delays. The CNF control procedure was studied for a

category of linear/nonlinear systems with parallel distributed recovery via sliding mode control method in [43]. In [44], a quick and precise robust path-following control approximate has been conducted for a fully-actuated marine surface vessel with external disturbances. In [45], the quick and precise chaos synchronization of uncertain chaotic systems with Lipschitz nonlinear terms and disturbances have been investigated.

An active front-steering control that combines CNF with a disturbance observer was proposed in [46], to obtain a fast damping rate tracking response and yield robustness to external disturbances. An adaptive nonlinear gain-based CNF controller was proposed in [47] to optimize the system dynamic efficiency. However, disturbances, uncertainties, and time delays were not considered in the design. A CNF technique was proposed in [48] for the tracking problem of a class of single-input single-output nonlinear systems subject to input saturation. The CNF control problem based on the event-triggered strategy was investigated in [49], for saturated systems. In [50], the adaptive tracking control problem of uncertain large-scale nonlinear time-delayed systems in the presence of input saturations is studied. In [51], the boundedness property of the robust tracking CNF controller has been studied for time-delay uncertain systems with input saturation. In [52], a hybrid controller design for a quarter car is developed. A design that combines active disturbance rejection control with a fuzzy control approach was proposed in [53].

For the intermittent control, its control signal is updated in a continuous manner on control time intervals. To overcome the limitation, time-triggered intermittent control is considered in [54]. In [55], the time-triggered intermittent control is proposed to examine the exponential synchronization issue of chaotic Lur'e systems. In [56], a control approach is suggested for the robust stabilization of the inertial wheel inverted pendulum, with norm-bounded parametric uncertainties and both motion constraints and actuator saturation. The robust stabilization of a class of continuous-time nonlinear systems via an affine state-feedback control law using the linear matrix inequality approach is studied in [57]. In [58], an enhanced composite nonlinear feedback technique using adaptive control is developed for a nonlinear delayed system subject to input saturation and exogenous disturbances. In [59], the precise tracking problem for electrostatic micromirror systems with disturbances and input saturation is studied. A composite nonlinear feedback-based adaptive integral sliding mode controller with a reaching law for fast and accurate control of a servo position system subject to external disturbance is presented in [60]. In [61], an L_2-L_∞/H_∞ optimization control issue is studied for a family of nonlinear plants by Takagi-Sugeno (T-S) fuzzy approach with actuator failure. In [62], the problem of resilient event-trigger-based security controller design is investigated for nonlinear networked control systems described by interval type-2 fuzzy models subject to non-periodic denial of service attacks. In [63], an adaptive performance guaranteed tracking

control problem is studied for multiagent systems with power integrators and measurement sensitivity. To the best of the authors knowledge no work has investigated CNF-based control designs for systems with the simultaneous presence of uncertainties, input saturation, time delay, and Lipschitz disturbances.

This paper presents an LMI-based composite nonlinear feedback control method for nonlinear systems subject to time delay, uncertainties, external disturbances, and input saturation. Its main contributions are as follows:

- A CNF control design that ensures the system's robustness against nonlinearities, parametric uncertainties, and external disturbances whilst guaranteeing transient performance.
- Compared to the existing results, the proposed method can be applied to a wider class of uncertain systems.

The rest of the paper is organized as follows: In Section II, the problem formulation and required assumptions are presented. In Section III, the original theoretical outcomes are expanded and proper choice of the nonlinear function in CNF is studied. Simulation results are provided in Section IV. Conclusions are lastly drawn in Section V.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following nonlinear system with time delay, input saturation, uncertainties and disturbances:

$$
\begin{aligned}
\dot{x}(t) &= f\left(x(t), x\left(t-(\tau)_1(t)\right), \ldots, x\left(t-\tau_N(t)\right)\right) \\
&\quad + (A + \Delta A(r(t))) x(t) \\
&\quad + \sum_{i=1}^{N}\left(A_{d_i} + \Delta A_{d_i}(v(t))\right) x\left(t-\tau_i(t)\right) \\
&\quad + B sat(u(t)) + W(q(t)), \quad (y(t) = Cx(t), \quad (1)
\end{aligned}
$$

where $t \in [t_0, \infty)$, $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ denotes the control signal, A, A_{d_i}, B, and C signify matrices of proper dimensions, matrices $\Delta A(.)$, $\Delta A_{d_i}(.)$, $i = 1, \cdots, N$ represent the uncertainties, $q(t)$, $r(t)$, $v(t)$ are uncertain scalar functions, $W(q(t))$ is disturbance, f is an uncertain nonlinear function and $\tau_i \in R^+$ is the time-delay. The saturation function is described by

$$
sat(u(t)) = \begin{bmatrix} sat(u_1(t)) \\ sat(u_2(t)) \\ \vdots \\ sat(u_m(t)) \end{bmatrix}
$$
$$
sat(u_i(t)) = sign(u_i(t)) min(|u_i(t)|, \overline{u}_i(t)) \quad (2)
$$

where $\overline{u}_i(t)$ is the maximum value of the ith control input.

The main control purpose is to synthesize a CNF law so that the output $y(t)$ can follow the reference output $y_m(t)$, as quickly and smoothly as possible. The reference model is represented by:

$$
\begin{aligned}
\dot{x_m}(t) &= A_m x_m(t) \\
&\quad + f_m\left(x_m(t), x_m\left(t-\tau_1(t)\right), \ldots, x_m\left(t-\tau_N(t)\right)\right) \\
y_m(t) &= C_m x_m(t), \quad (3)
\end{aligned}
$$

where A_m and C_m are constant matrices, $x_m(t) \in R^{n_m}$ represents the state vector of the reference model with $\|x_m(t)\| \leq M$, where M is a positive scalar. The reference model is selected so that there exist two matrices $G \in R^{n \times n_m}$ and $H \in R^{m \times n_m}$ satisfying:

$$
\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} G \\ H \end{bmatrix} = \begin{bmatrix} GA_m \\ C_m \end{bmatrix} \quad (4)
$$

Assumption 1: There are matrices with continuous boundary conditions $N(.)$, $N_{d_i}(.)$, L_{d_i}, and $\tilde{W}(.)$ so that

$$
\begin{aligned}
\Delta A(r(t)) &= BN(r(t)), \\
\Delta A_{d_i}(v(t)) &= BN_{d_i}(v(t)), \quad i = 1, \ldots, N \\
W(q(t)) &= \tilde{W}(q(t)) \\
A_{d_i} &= BL_{d_i}, \quad i = 1, \ldots, N \quad (5)
\end{aligned}
$$

which the bounds of the uncertainties are provided by

$$
\begin{aligned}
\rho_r &= sup\, \|N(r(t))\|, \\
\rho_{vi} &= sup\, \left\|N_{d_i}(v(t))\right\|, \quad i = 1, \cdots, N \\
\rho_q &= sup\, \|\tilde{W}(q(t))\|, \quad (6)
\end{aligned}
$$

Remark 1: Assumption 1 illustrates that the uncertain nonlinear system with time-delay and input saturation (1) contains a special structure which is commonly denominated by a matching condition for parametric uncertainties; a standard assumption in robust control problems. According to the matching conditions, all external disturbances, and uncertainties must be controlled in the control vector space, thus limiting the structure of the system. The boundary conditions are continuous and bounded matrix functions, meaning that the arguments of the matrix are bounded and continuous functions.

Assumption 2: The nonlinear function f is uncertain and Lipschitz for all $x(t) \in R^n$ and $x_m(t) \in R^n$ so that [64], [65]:

$$
\begin{aligned}
&\|f\left(x(t), x\left(t-\tau_1(t)\right), \ldots, x\left(t-\tau_N(t)\right)\right) \\
&\quad - Gf_m\left(x_m(t), x_m\left(t-\tau_1(t)\right), \ldots, x_m\left(t-\tau_N(t)\right)\right)\| \\
&\leq (N+1) \left(\|L_0\left(x(t) - Gx_m(t)\right)\| \right. \\
&\quad + \|L_1(x(t-\tau_1(t)) - Gx_m\left(t-\tau_1(t)\right))\| \\
&\quad + \cdots + \|L_N(x(t-\tau_N(t)) - Gx_m\left(t-\tau_N(t)\right))\|\right) \quad (7)
\end{aligned}
$$

where $L_i \in R^{n \times n}$, $i = 0, 1, \ldots, N$ are constant matrices. In other words, it is stated as

$$
\begin{aligned}
&(f\left(x(t), x\left(t-\tau_1(t)\right), \ldots, x\left(t-\tau_N(t)\right)\right) \\
&\quad - Gf_m\left(x_m(t), x_m\left(t-\tau_1(t)\right), \ldots, x_m(t-\tau_N(t))\right))^T \\
&\quad \times (f\left(x(t), x\left(t-\tau_1(t)\right), \ldots, x(t-\tau_N(t))\right) \\
&\quad - Gf_m\left(x_m(t), x_m(t-\tau_1(t)), \ldots, x_m\left(t-\tau_N(t)\right)\right)) \\
&\leq (N+1)((x(t) - Gx_m(t))^T L_0 L_0^T (x(t) - Gx_m(t)) \\
&\quad + (x(t-\tau_1(t)) - Gx_m\left(t-\tau_1(t)\right))^T L_1^T L_1(x(t-\tau_1(t)) \\
&\quad - Gx_m\left(t-\tau_1(t)\right)) \\
&\quad + \cdots + (x(t-\tau_N(t)) - Gx_m\left(t-\tau_N(t)\right))^T \\
&\quad \times L_N^T L_N(x(t-\tau_N(t)) - Gx_m\left(t-\tau_N(t)\right))) \quad (8)
\end{aligned}
$$

The tracking error $e(t)$ and auxiliary vector tilde $\tilde{x}(t)$ are defined by:

$$e(t) = y(t) - y_m(t), \qquad (9)$$

$$\tilde{x}(t) = x(t) - Gx_m(t), \qquad (10)$$

where G is obtained using (4). From (4), (9), and (10), we have:

$$\begin{aligned}
\|e(t)\| &= \|y(t) - y_m(t)\| \\
&= \|Cx(t) - C_m x_m(t)\| \\
&= \|Cx(t) - CGx_m(t)\| \\
&= \|C\tilde{x}(t)\| \\
&\leq \|C\| \, \|\tilde{x}(t)\|, \qquad (11)
\end{aligned}$$

Since $\|C\|$ is bounded, then $\|\tilde{x}(t)\| \to 0$ implies $\|e(t)\| \to 0$. Hence, to prove the perfect tracking one has to show that $\|\tilde{x}(t)\|$ is convergent.

III. MAIN RESULTS

The CNF control design method is suggested for the tracking control of uncertain nonlinear systems with time delay and input saturation. The aim of the tracking controller design is to minimize the tracking error. To this end, define the linear control part of the system (1) as follows [66]:

$$u_L(t) = Kx(t) + (H - KG)x_m(t), \qquad (12)$$

where G and H are obtained from (4), and K denotes a gain matrix that is specified in the LMI form. Then define the nonlinear function as:

$$u_N(t) = \psi(\tilde{x}(t))B^T P\tilde{x}(t), \qquad (13)$$

where P denotes a positive-symmetric matrix, and $\psi(\tilde{x}(t))$ is a selection of non-positive Lipschitz function in tilde $\tilde{x}(t)$, that is applied to regulate the damping ratio of the system as the output converges to the reference to diminish the overshoot created by the linear portion. The CNF controller is the sum of the linear and nonlinear parts as follows:

$$\begin{aligned}
u(t) &= u_L(t) + u_N(t) \\
&= Kx(t) + (H - KG)x_m(t) + \psi(\tilde{x}(t))B^T P\tilde{x}(t). \qquad (14)
\end{aligned}$$

From (1), (3), (4), (10), and (14), it can be obtained

$$\begin{aligned}
&\dot{\tilde{x}}(t) \\
&= f\left(x(\mathrm{t}), x(t - \tau_1(t)), \ldots, x(t - \tau_N(t))\right) \\
&\quad - Gf_m\left(x_m(t), x_m(t - \tau_1(t)), \ldots, x_m(t - \tau_N(t))\right) \\
&\quad + (A + \Delta A(r(t)) + BK)\tilde{x}(t) \\
&\quad + \Delta A(r(t))Gx_m(t) + Bsat(u(t)) + W(q(t)) - B(K\tilde{x}(t)
\end{aligned}$$

$$\begin{aligned}
&\quad + Hx_m(t)) + \sum_{i=1}^{N} \left(A_{d_i} + \Delta A_{d_i}(v(t))\right)(\tilde{x}(t - \tau_i) \\
&\quad + Gx_m(t - \tau_i)) \\
&= f\left(x(t), x(t - \tau_1(t)), \ldots, x(t - \tau_N(t))\right) \\
&\quad - Gf_m\left(x_m(t), x_m(t - \tau_1(t)), \ldots, x_m(t - \tau_N(t))\right) \\
&\quad + (A + BN + BK)\tilde{x}(t) + \sum_{i=1}^{N} \left(A_{d_i} + BN_{d_i}\right)\tilde{x}(t - \tau_i(t)) \\
&\quad + Bw + g(r, s, q, v, x_m) \qquad (15)
\end{aligned}$$

where

$$\begin{aligned}
&g(r, s, q, v, x_m) \\
&= \Delta A(r(t))Gx_m(t) \\
&\quad + \sum_{i=1}^{N} (A_{d_i} + \Delta A_{d_i}(v(t)))Gx_m(t - \tau_i) + W(q(t)) \qquad (16)
\end{aligned}$$

and

$$w = sat(K\tilde{x} + Hx_m + \psi(\tilde{x})B^T P\tilde{x}) - K\tilde{x} - Hx_m. \qquad (17)$$

By applying Assumption 1, Eq. (16) is stated as

$$g(r, s, q, v, x_m) = BF(r, s, q, v, x_m) \qquad (18)$$

where

$$\begin{aligned}
&F(r, s, q, v, x_m) \\
&= N(r(t))Gx_m(t) \\
&\quad + \sum_{i=1}^{N} (L_{d_i} + N_{d_i}(v(t)))Gx_m(t - \tau_i) + \tilde{W}(q(t)). \qquad (19)
\end{aligned}$$

Then, from (6) and (19), and determining the bounds $\rho = \sup \|F(r, s, v, q, x_m)\|$ and $\|x_m(t)\| \leq M$, the following inequality is obtained

$$\rho \leq \rho_r \|G\| M + \sum_{i=1}^{N} (\|L_{d_i}\| + \rho_{vi}) \|G\| M + \rho_q. \qquad (20)$$

The nonlinear function $\psi(\tilde{x}(t))$ in (13) is defined as in (21), shown at the bottom of the page, where $\sigma(\tilde{x}(t)) \in R^+$ is any positive uniform continuous bounded function with

$$\lim_{t \to \infty} \int_{t_0}^{t} \sigma(\tilde{x}(\tau))d\tau \leq \sigma_0, \qquad (22)$$

where σ_0 is a bounded constant.

The liberty in choosing a nonlinear expression $\sigma(\tilde{x}(t))$ causes the control law to be adjusted and the performance is improved as the system output converges to the reference input. The nonlinear expression $\sigma(\tilde{x}(t))$ applies to the following features

(1) Because $\sigma(\tilde{x}(t))$ a function of $\|\tilde{x}(t)\|$, the following equation is established:

$$\sigma(\tilde{x}(t)) = \sigma(-\tilde{x}(t)) \geq 0. \qquad (23)$$

$$\psi(\tilde{x}(t)) = -\frac{\left(\rho + \rho_r \|\tilde{x}(t)\| + \sum_{i=1}^{N} \rho_{vi} \|\tilde{x}(t - \tau_i)\|\right)^2}{\|B^T P\tilde{x}(t)\|\left(\rho + \rho_r \|\tilde{x}(t)\| + \sum_{i=1}^{N} \rho_{vi} \|\tilde{x}(t - \tau_i)\|\right) + \sigma(\tilde{x}(t))}. \qquad (21)$$

(2) If the system output is not close to the reference output, the function $\sigma(\tilde{x}(t))$ becomes larger, as a result, the function shrinks and the efficacy of the nonlinear portion of the CNF law reduces.

(3) If the system output converges the reference output, $\sigma(\tilde{x}(t))$ function becomes very small and reaches its lowest value, thus increasing the $|\psi(\tilde{x}(t))|$ value and therefore, the efficacy of the nonlinear controller will be evident.

Since the choice of the nonlinear function $\sigma(\tilde{x}(t))$ is free so it can be expressed in different ways. To compatible alteration of the tracking aim, a nonlinear function is defined as follows:

$$\sigma(\tilde{x}(t)) = \beta e^{\frac{-\alpha_0 \alpha}{\|y(t) - y_m(t)\|}}, \tag{24}$$

where

$$\alpha_0 = \begin{cases} \|y(t_0) - y_m(t)\|, & y(t_0) \neq y_m(t) \\ 1, & y(t_0) = y_m(t) \end{cases} \tag{25}$$

The nonlinear expression $\psi(\tilde{x}(t))$ converges from the primary value $\beta e^{-\alpha}$ to the steady state value 0, thus the function $\|y(t_0) - y_m(t)\|$ converges to zero. From (25), we conclude that the parameter α_0 changes for different tracking purposes $y_m(t)$ so the primary value of the nonlinear function is not dependent on $y_m(t)$.

Theorem 1: Consider the system (15) with $f(0, 0, \ldots, 0) = 0$ and the CNF law of (14) satisfying assumptions 1-2. Also, assume that the time delays $\tau_i(t)$ are bounded by scalares ζ_i holding $|\dot{\tau}_i(t)| \leq \zeta_i$. For any $\delta \in (0, 1)$, the following properties hold:

$$|K_i \tilde{x}| \leq (1 - \delta)\overline{u}_i, \quad i = 1, \ldots, m \tag{26}$$

$$|H_i x_m| \leq \delta \overline{u}_i, \quad i = 1, \ldots, m \tag{27}$$

If there exist matrices $S_i > 0, i = 1, \cdots, N, Q = Q^T > 0$, and Y with suitable dimensions so that, (28), as shown at the bottom of the page, where $\Sigma = AQ + QA^T + BY + Y^T B^T + \sum_{i=1}^{N} S_i$ and by applying $\beta_i = \gamma_i^{-1}$, for $i = 0, 1, \ldots, N$ and $P = Q^{-1}$ in (28), the CNF control (14) guarantees that the system output $y(t)$ follows the reference output $y_m(t)$ as a result the tracking error $e(t)$ would be ultimately bounded. Then, the control feedback gain K is obtained by $K = YQ^{-1}$.

Proof: To prove the sustainability of the system, we construct the Lyapunov candidate functional as

$$V(\tilde{x}, t) = \tilde{x}^T(t)P\tilde{x}(t) + \sum_{i=1}^{N} \int_{t-\tau_i}^{t} \tilde{x}^T(s)R_i\tilde{x}(s)ds, \tag{29}$$

where P and $R_i, i = 1, \cdots, N$ are the weighting matrices that is characterized by LMI. By deriving (29) along the directions of the system in (15) outcomes

$$\begin{aligned}
&\dot{V}(\tilde{x}, t) \\
&= \dot{\tilde{x}}^T(t)P\tilde{x}(t) + \tilde{x}^T(t)P\dot{\tilde{x}}(t) \\
&\quad + \sum_{i=1}^{N} \tilde{x}^T(t)R_i\tilde{x}(t) \\
&\quad - \sum_{i=1}^{N} (1 - \zeta_i)\tilde{x}^T(t - \tau_i(t))R_i\tilde{x}(t - \tau_i(t)) \\
&= \tilde{x}^T(t)[(A + BK)^T P + \\
&\quad \times (PBN)^T + PBN + P(A + BK)]\tilde{x}(t) \\
&\quad + F^T B^T P\tilde{x}(t) + \tilde{x}^T(t)PBF + (f(x(\text{t}), x(t - \tau_1(t)) \\
&\quad , \ldots, x(t - \tau_N(t))) - \text{Gf}_m(x_m(t), x_m(t - \tau_1(t)) \\
&\quad , \ldots, x_m(t - \tau_N(t))))^T P\tilde{x}(t) \\
&\quad + \tilde{x}^T(t)P(f(x(t), x(t - \tau_1(t)), \ldots, x(t - \tau_N(t))) - \\
&\quad \times G_m(x_m(t), x_m(t - \tau_1(t)), \ldots, x_m(t - \tau_N(t)))) \\
&\quad + (B\sum_{i=1}^{N} N_{d_i}\tilde{x}(t - \tau_i(t)))^T P\tilde{x} \\
&\quad + \tilde{x}^T PB \sum_{i=1}^{N} N_{d_i}\tilde{x}(t - \tau_i(t)) \\
&\quad + \sum_{i=1}^{N} A_{d_i}\tilde{x}^T(t - \tau_i(t))P\tilde{x}(t) \\
&\quad + \tilde{x}^T(t)P \sum_{i=1}^{N} A_{d_i}\tilde{x}(t - \tau_i(t)) \\
&\quad + 2\tilde{x}^T(t)PBw + \sum_{i=1}^{N} \tilde{x}^T(t)R_i\tilde{x}(t) \\
&\quad - \sum_{i=1}^{N} (1 - \zeta_i)\tilde{x}^T(t - \tau_i(t))R_i\tilde{x}(t - \tau_i(t)) \tag{30}
\end{aligned}$$

According to Assumption 1, it can obtain that:

$$\begin{aligned}
&\dot{V}(\tilde{x}, t) \\
&\leq \tilde{x}^T(t)[PA + A^T P + PBK + K^T B^T P]\tilde{x}(t) \\
&\quad + \tilde{x}^T(t)P \sum_{i=1}^{N} A_{d_i}\tilde{x}(t - \tau_i(t)) \\
&\quad + (\sum_{i=1}^{N} A_{d_i}\tilde{x}^T(t - \tau_i(t)))^T P\tilde{x}(t) \\
&\quad - \sum_{i=1}^{N} (1 - \zeta_i)\tilde{x}^T(t - \tau_i(t))R_i\tilde{x}(t - \tau_i(t)) \\
&\quad + 2\rho\|B^T P\tilde{x}(t)\|
\end{aligned}$$

$$\begin{bmatrix}
\Sigma & A_{d1}Q & \cdots & A_{dN}Q & \beta_0 I & 0 & 0 & 0 & \sqrt{N+1}QL_0^T & 0 & 0 & 0 \\
* & -(1-\zeta_1)S_1 & 0 & 0 & 0 & \beta_1 I & 0 & 0 & 0 & \sqrt{N+1}QL_1^T & 0 & 0 \\
* & * & \ddots & 0 & 0 & 0 & \ddots & 0 & 0 & 0 & \ddots & 0 \\
* & * & * & -(1-\zeta_N)S_N & 0 & 0 & 0 & \beta_N I & 0 & 0 & 0 & \sqrt{N+1}QL_N^T \\
* & * & * & * & -\beta_0 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\beta_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \ddots & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\beta_N I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\beta_0 I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\beta_1 I & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & \ddots & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -\beta_N I
\end{bmatrix} < 0 \tag{28}$$

$$\begin{aligned}
&+ 2\rho_r ||\tilde{x}(t)|| ||B^T P\tilde{x}(t)|| + (f(x(t), x(t - \tau_1(t)), \\
&\ldots, x(t - \tau_N(t))) - Gf_m(x_m(t), x_m(t - \tau_1(t)), \\
&\ldots, x_m(t - \tau_N(t))))^T P\tilde{x}(t) \\
&+ \tilde{x}^T(t)P(f(x(t), x(t - \tau_1(t)), \ldots, x(t - \tau_N(t))) \\
&- Gf_m(x_m(t), x_m(t - \tau_1(t)), \ldots, x_m(t - \tau_N(t)))) \\
&+ 2||B^T P\tilde{x}(t)|| \sum_{i=1}^{N} \rho_{v_i} \tilde{x}(t - \tau_i(t)) + 2\tilde{x}^T(t)PBw \\
&+ \sum_{i=1}^{N} \tilde{x}^T(t)R_i\tilde{x}(t)
\end{aligned} \quad (31)$$

The following equation holds for all $\gamma_i, \ i = 0, 1, \ldots, N$

$$\begin{aligned}
&(f(x(t), x(t - \tau_1(t)), \ldots, x(t - \tau_N(t))) \\
&- Gf_m(x_m(t), x_m(t - \tau_1(t)) \\
&, \ldots, x_m(t - \tau_N(t))))^T P\tilde{x}(t) \\
&+ \tilde{x}^T(t)P(f(x(t), x(t - \tau_1(t)) \\
&, \ldots, x(t - \tau_N(t))) - Gf_m(x_m(t), x_m(t - \tau_1(t)) \\
&, \ldots, x_m(t - \tau_N(t)))) \le \frac{1}{\gamma}\tilde{x}^T(t)P^2\tilde{x}(t) \\
&+ \gamma(f(x(t), x(t - \tau_1(t)), \ldots, x(t - \tau_N(t))) \\
&- Gf_m(x_m(t), x_m(t - \tau_1(t)), \ldots, x_m(t - \tau_N(t))))^T \\
&\times (f(x(t), x(t - \tau_1(t)) \\
&, \ldots, x(t - \tau_N(t))) - Gf_m(x_m(t), x_m(t - \tau_1(t)) \\
&, \ldots, x_m(t - \tau_N(t)))) \\
\le &\ \frac{1}{\gamma}\tilde{x}^T(t)P^2\tilde{x}(t) \\
&+ \sqrt{N+1}(\gamma_0\tilde{x}^T(t)L_0^T L_0\tilde{x}(t) \\
&+ \gamma_1\tilde{x}^T(t - \tau_1(t))L_1^T L_1\tilde{x}(t - \tau_1(t)) \\
&+ \cdots + \gamma_N\tilde{x}^T(t - \tau_N(t))^T L_N^T L_N\tilde{x}(t - \tau_N(t)))
\end{aligned} \quad (32)$$

In the following, we investigate four states of the saturation function.

Case 1: In this case, all input channels are higher than the upper bound, that is, $u > \overline{u_i}$ for this situation, we have

$$K_i\tilde{x} + H_ix_m + \psi(\tilde{x})B_i^T P\tilde{x} \ge \overline{u_i}. \quad (33)$$

Given inequality (26) and (27) we obtain

$$\begin{aligned}
K_i\tilde{x} + H_ix_m &\le |K_i\tilde{x} + H_ix_m| \\
&\le |K_i\tilde{x}| + |H_ix_m| \\
&\le (1 - \delta)\bar{u}_i + \delta\bar{u}_i \\
&\le \bar{u}_i
\end{aligned} \quad (34)$$

for all $\tilde{x} \in X_\delta$, where X_δ signifies an invariant set of dynamics. Therefore, all paths from the inside of X_δ will approach the reference, and hence

$$\begin{aligned}
w_i &= sat(K_i\tilde{x} + H_ix_m + \psi(\tilde{x})B_i^T P\tilde{x}) - K_i\tilde{x} - H_ix_m \\
&= \overline{u}_i - K_i\tilde{x} - H_ix_m \ge 0.
\end{aligned} \quad (35)$$

From (33), we obtain

$$\psi(\tilde{x})B_i^T P\tilde{x} \ge \overline{u}_i - K_i\tilde{x} - H_ix_m \ge 0. \quad (36)$$

since $\psi(\tilde{x}(t))$ is a nonpositive expression, it results:

$$B_i^T P\tilde{x} = \tilde{x}^T PB_i \le 0, \quad (37)$$

Case 2: In this case, all input channels are assumed to be smaller than their lower bound, that is, $u < -\overline{u_i}$ then, we have:

$$K_i\tilde{x} + H_ix_m + \psi(\tilde{x})B_i^T P\tilde{x} \le -\bar{u}_i. \quad (38)$$

Similarly, we obtain:

$$K_i\tilde{x} + H_ix_m \ge -\bar{u}_i, \quad (39)$$

therefore, we have

$$\begin{aligned}
w_i &= sat\left(K_i\tilde{x} + H_ix_m + \psi(\tilde{x})B_i^T P\tilde{x}\right) - K_i\tilde{x} - H_ix_m \\
&= -\bar{u}_i - K_i\tilde{x} - H_ix_m \le 0,
\end{aligned} \quad (40)$$

which yields that

$$\psi(\tilde{x})B_i^T P\tilde{x} \le -\bar{u}_i - (K_i\tilde{x} + H_ix_m) \le 0, \quad (41)$$

and

$$B_i^T P\tilde{x} = \tilde{x}^T PB_i \ge 0. \quad (42)$$

Case 3: In this case, we will have the combination of modes 1 and 2 which means that some of the control inputs are saturated and some are not. In order to represent the unsaturated factors, we have:

$$\tilde{x}^T PB_iw_i = \psi(\tilde{x})\tilde{x}^T PB_iB_i^T p\tilde{x} \le 0, \quad (43)$$

and for the saturated factors exceeding their upper bounds, from $w_i \ge 0$ and $\tilde{x}^T PB_i \le 0$, we have

$$\tilde{x}^T PB_iw_i \le 0. \quad (44)$$

Eventually, for the unsaturated factors exceeding their lower bounds, from $w_i \le 0$ and $\tilde{x}^T PB_i \ge 0$, we get:

$$\tilde{x}^T PB_iw_i \le 0. \quad (45)$$

Case 4: If all input factors are not saturated, that is, $|u| \le \overline{u_i}$. So, from (17), it gives

$$w = \psi(\tilde{x})B^T P\tilde{x}, \quad (46)$$

where substituting (46) into (31) yields

$$\begin{aligned}
\dot{V}(\tilde{x}, t) \le &\ \Psi^T Q_1 \Psi + 2(\rho + \rho_r ||\tilde{x}(t)|| \\
&+ \sum_{i=1}^{N} \rho_{v_i} ||\tilde{x}(t - \tau_i(t))||)||B^T P\tilde{x}|| \\
&+ 2\tilde{x}^T(t)PB\psi(\tilde{x})B^T P\tilde{x}
\end{aligned} \quad (47)$$

where

$$\Psi = \left[\tilde{x}(t) \ \tilde{x}(t - \tau_1(t)) \ \ldots \ \tilde{x}(t - \tau_N(t))\right]^T$$

and, (48), as shown at the bottom of the next page, where $\Lambda = PA + A^T P + PBK + K^T B^T P + \sum_{i=1}^{N} R_i + \sqrt{N+1}\gamma_0 L_0^T L_0$. Substituting (21) into (47), we achieve

$$\dot{V}(\tilde{x}, t) \le \Psi^T Q_1 \Psi + 2\frac{\bar{\alpha}||B^T P\tilde{x}||\sigma(\tilde{x}(t))}{\bar{\alpha}||B^T P\tilde{x}|| + \sigma(\tilde{x}(t))} \quad (49)$$

where $\bar{\alpha} = \sup\left(\rho + \rho_r ||\tilde{x}(t)|| + \sum_{i=1}^{N} \rho_{v_i} ||\tilde{x}(t - \tau_i(t))||\right)$.

Considering the following fact:

$$0 \leq \frac{\sigma(\tilde{x}(t))\theta}{\sigma(\tilde{x}(t)) + \theta} \leq \sigma(\tilde{x}(t)), \quad \forall \sigma(\tilde{x}(t)) > 0, \theta > 0 \quad (50)$$

From (49) and (50), we have

$$\dot{V}(\tilde{x}, t) \leq \Psi^T Q_1 \Psi + 2\sigma(\tilde{x}(t)) \quad (51)$$

Then, it follows that

$$\dot{V}(\tilde{x}, t) \leq \Psi^T Q_1 \Psi + 2\bar{\sigma}, \quad \forall \tilde{x} \in X_\delta \quad (52)$$

since $\sigma(\tilde{x}(t)) \leq \bar{\sigma}$, and overline ($\sigma$ is a bounded constant. Based on the equation (52), all of the responses of the system are bounded, and also according to inequality (22), we conclude that tilde $\tilde{x}(t) \longrightarrow 0$.

$$\lim_{t \to \infty} x(t) = Gx_m, \quad (53)$$

$$\lim_{t \to \infty} u(t)$$
$$= K \lim_{t \to \infty} \tilde{x}(t) + Hx_m$$
$$+ \lim_{t \to \infty} \psi(\tilde{x})B^T P\tilde{x}(t) = Hx_m, \quad (54)$$

$$\lim_{t \to \infty} y(t)$$
$$= C \lim_{t \to \infty} \tilde{x}(t) = CGx_m = C_m x_m = y_m(t). \quad (55)$$

so, the auxiliary vector tilde ($x(t)$) of the system (15) tends uniformly to zero and thereby it follows from (11) that the tracking error $e(t)$ reduces asymptotically to zero.

The condition dot $\dot{V}(\tilde{x}, t) < 0$ holds if there exists a scalar γ so that $Q_1 < 0$. In order to satisfy the inequality (48) with the form of LMIs assuming $Q = P^{-1}$, $K = YQ^{-1}$, $S_i = QR_iQ$, and pre-and post-multiplying (48) by diag $(Q, \ldots, Q, \gamma_0^{-1}I, \gamma_1^{-1}I, \ldots, \gamma_N^{-1}I)$ obtained as in (56), shown at the bottom of the page, where $M = AQ + QA^T + BY + Y^T B^T + \sum_{i=1}^{N} S_i + \sqrt{N+1}\gamma_0 QL_0^T L_0 Q$. By applying the Schur complement, the following inequality is obtained as in (57), shown at the bottom of the page, where $\Sigma = AQ + QA^T + BY + Y^T B^T + \sum_{i=1}^{N} S_i$ and defining $\beta_i = \gamma_i^{-1}$ for $i = 0, 1, \ldots, N$, LMI (28) is achieved. Thus, completing the proof.

Note that, the proposed robust tracking controller by CNF improves the transient efficiency and steady-state precision at the same time. Obviously, the CNF control law leads to a linear controller when the nonlinear portion tends to zero. As a result, the added nonlinear expression allows modifying the linear control law to recover system transient performance and the error converges to zero. Selection of the nonlinear feedback part is important because the

$$Q_1 = \begin{bmatrix} \Lambda & PA_{d1} & \cdots & PA_{dN} & P & 0 & 0 & 0 \\ * & -(1-\zeta_1)R_1 + \sqrt{N+1}\gamma_1 L_1^T L_1 & 0 & 0 & 0 & P & 0 & 0 \\ * & * & \ddots & 0 & 0 & 0 & P & 0 \\ * & * & * & -(1-\zeta_N)R_N + \sqrt{N+1}\gamma_N L_N^T L_N & 0 & 0 & 0 & P \\ * & * & * & * & -\gamma_0 I & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma_1 I & 0 & 0 \\ * & * & * & * & * & * & \ddots & 0 \\ * & * & * & * & * & * & * & -\gamma_N I \end{bmatrix} \quad (48)$$

$$\begin{bmatrix} M & A_{d1}Q & \cdots & A_{dN}Q & \gamma_0^{-1}I & 0 & 0 & 0 \\ * & -(1-\zeta_1)S_1 + \sqrt{N+1}\gamma_1 QL_1^T L_1 Q & 0 & 0 & 0 & \gamma_1^{-1}I & 0 & 0 \\ * & * & \ddots & 0 & 0 & 0 & \ddots & 0 \\ * & * & * & -(1-\zeta_N)S_N + \sqrt{N+1}\gamma_N QL_N^T L_N Q & 0 & 0 & 0 & \gamma_N^{-1}I \\ * & * & * & * & -\gamma_0^{-1}I & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma_1^{-1}I & 0 & 0 \\ * & * & * & * & * & * & \ddots & 0 \\ * & * & * & * & * & * & * & -\gamma_N^{-1}I \end{bmatrix} < 0 \quad (56)$$

$$\begin{bmatrix} \Sigma & A_{d1}Q & \cdots & A_{dN}Q & \gamma_0^{-1}I & 0 & 0 & 0 & 0 & \sqrt{N+1}QL_0^T & 0 & 0 & 0 \\ * & -(1-\zeta_1)S_1 & 0 & 0 & 0 & \gamma_1^{-1}I & 0 & 0 & 0 & 0 & \sqrt{N+1}QL_1^T & 0 & 0 \\ * & * & \ddots & 0 & 0 & 0 & \ddots & 0 & 0 & 0 & 0 & \ddots & 0 \\ * & * & * & -(1-\zeta_N)S_N & 0 & 0 & 0 & \gamma_N^{-1}I & \gamma_N^{-1}I & 0 & 0 & 0 & \sqrt{N+1}QL_N^T \\ * & * & * & * & -\gamma_0^{-1}I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma_1^{-1}I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \ddots & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\gamma_N^{-1}I & -\gamma_N^{-1}I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\gamma_0^{-1}I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\gamma_1^{-1}I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \ddots & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -\gamma_N^{-1}I \end{bmatrix} < 0 \quad (57)$$

nonlinear part affects the performance of various parts of the system. In this work, we tried to select a nonlinear function so that the system achieves optimum transient and steady state performance. The extra nonlinear component aims to help the controller further improve the system's performance and ensure its robustness to external disturbances and parametric uncertainties.

IV. SIMULATION RESULTS

The performance and efficiency of the proposed approach are assessed using two different examples. The first example considers a two-dimensional system with input saturation, time delay in the presence of perturbations, and uncertainties. A comparison with the methods proposed in [51] and [31] is also carried out. The second example considers the uncertain Chua's circuit system with nonlinearity and compares the performance of the proposed method to that of the methods proposed in [51] and [31].

Example 1: The unstable nonlinear system with time delays and disturbance are considered as:

$$\dot{x}(t) = \begin{bmatrix} 0.5 \cos x_1(t) - 0.5 \\ 0.5 \sin x_2(t) \end{bmatrix} + \left(\begin{bmatrix} 1 & 1.5 \\ 0.3 & -2 \end{bmatrix} \right.$$
$$+ \begin{bmatrix} 0 & 0 \\ r_1(t) & r_2(t) \end{bmatrix} \right) x(t)$$
$$+ \sum_{i=1}^{2} [A_{di} + \Delta A_{di}(v(t))] x(t - \tau_i)$$
$$+ \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} q(t).$$
$$y_t = [1 \ 1]x(t) \tag{58}$$

where A_{di}, $i = 1, 2$ are constant parameters, $r_1(t)$, $r_2(t)$ and $\Delta A_{di}(v(t))$, $i = 1, 2$ are the uncertain parameters and $q(t)$ is the disturbance. The disturbance and uncertain bounds are as follows:

$$(|r_1(t)| \leq 0.5, |r_2(t)| \leq 1, |q(t)| \leq 0.5, |v(t| \leq 1.5) \text{ and}$$
$$\Delta A_{d1}(v(t)) = \begin{bmatrix} v_1(t) & 0 \\ 0 & v_2(t) \end{bmatrix}, \Delta A_{d2}(v(t)) = \begin{bmatrix} v_1(t) & 0 \\ v_2(t) & 0 \end{bmatrix}.$$

Then, from (5) and (58), we have $\tilde{W}(q(t)) = q(t)$, $N(r(t)) = [r_1(t)r_2(t)]$
The parameter of reference model is given by

$$A_m = \begin{bmatrix} -10 & 0 \\ 0 & -10 \end{bmatrix}$$
$$C_m = \begin{bmatrix} -2.6 & -1.3 \end{bmatrix} \tag{59}$$

Using (4), the G and H can be determined as $G = \begin{bmatrix} -2.206 & -1.183 \\ -0.393 & -0.116 \end{bmatrix}$ and $H = \begin{bmatrix} 2.486 & -1.047 \end{bmatrix}$. For simulation use, take $r_1(t) = 0.5 \sin(3t)$, $r_2(t) = \sin(3t)$, $v_1(t) = \sin(2t)$, $v_2(t) = 1 + 0.5 \sin(2t)$, $q(t) = 0.5 \cos(5t)$, $A_{d1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $A_{d2} = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$. The constant quantities are considered as $\alpha = 0.126$,

$$\beta = 1.12, \rho = 0.5, \rho_r = 1.12, \rho_{v_1} = \rho_{v_2} = 1.5.$$
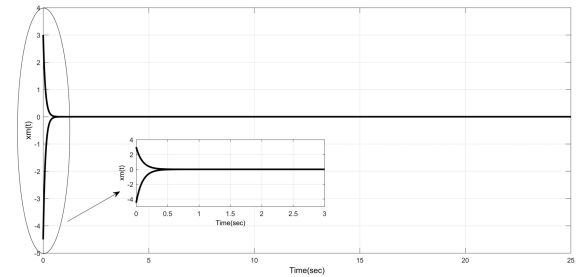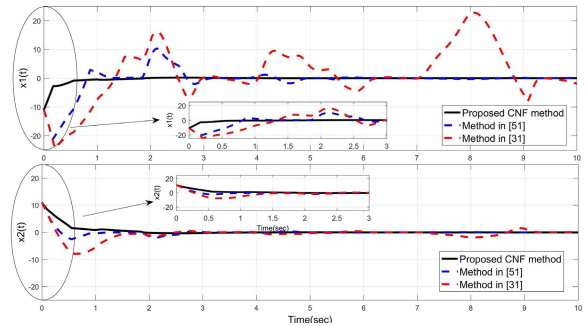

FIGURE 1. State vector of the reference model.

FIGURE 2. Trajectory of the system states.

The initial values values are considered as

$$x(0) = [-11 \quad 11]^T, \ x_m(0) = [-4.5 \quad 3]^T \ and$$

$\tau_1 = 0.5\sin(\pi t) + 1$, $\tau_2 = -\cos(2t) + 1$. The Lipschitzian matrix is specified by

$$L_0 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

The solutions of the LMI (28) are determined using the LMI toolbox in *MATLAB*® software as

$$P = 10^{-4} \begin{bmatrix} 0.0018 & -0.0119 \\ -0.0119 & 0.1224 \end{bmatrix}$$
$$K = \begin{bmatrix} -28.6102 & -15.5797 \end{bmatrix}$$
$$R_1 = 10^{-4} \begin{bmatrix} 0.0139 & 0.0196 \\ 0.0196 & 0.2240 \end{bmatrix}$$
$$R_2 = 10^{-5} \begin{bmatrix} 0.1507 & 0.0672 \\ 0.0672 & 0.2014 \end{bmatrix}$$
$$S_1 = 10^8 \begin{bmatrix} 4.9164 & 0.5316 \\ 0.5316 & 0.0584 \end{bmatrix}$$
$$S_2 = 10^8 \begin{bmatrix} 4.1048 & 0.4105 \\ 0.4105 & 0.0412 \end{bmatrix}$$

Figure 1 shows the state vector of the reference model. Figure 2 displays a diagram of state vector paths. The trajectories of the tracking error are illustrated in Figure 3 and the nonlinear state-feedback controller is depicted in Figure 4. Note that all the state paths approach the origin. So, the simulation results show that the system is resistant to time
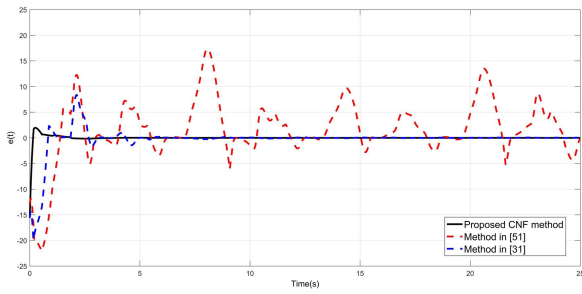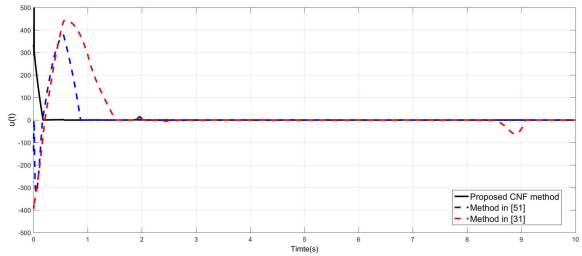
FIGURE 3. The tracking errors.

FIGURE 4. Control input.

delays and disturbances, and also the proposed controller displays a good convergence efficiency.

The proposed approach exhibits high tracking accuracy and optimum and stable performance compared to the two other methods.

Example 2: Consider the uncertain Chua's circuit system with nonlinearity considered by [67]:

$$\dot{x}(t) = \begin{bmatrix} f_1(x(t)) \\ 0 \\ 0 \end{bmatrix} + \left(\begin{bmatrix} -10 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -\frac{100}{7} & 0 \end{bmatrix} \right.$$

$$+ \left. \begin{bmatrix} 0.2\sin(t) & 0.3\cos(2t) & 0.1\cos(t) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) x(t)$$

$$+ \sum_{i=1}^{2} [A_{di} + \Delta A_{di}(v(t))] x(t - \tau_i)$$

$$+ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} q(t).$$

$$y(t) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x(t). \tag{60}$$

where $f_1(x(t)) = -5x_1^2(t) + 11.42\,x_1(t) + 2.14\,(|\,x_1(t) + 1| - |x_1(t) - 1|)$ and $A_{di}, i = 1, 2$ are fixed parameters, $r_1(t), r_2(t)$ and $\Delta A_{di}(v(t)), i = 1, 2$ are the uncertain parameters and $q(t)$ is the disturbance, and

$$\Delta A_{d1}(v(t)) = \begin{bmatrix} 0.1\cos(t) & 0.2\sin(t) & 0.2\sin(t) \\ 0 & 0.2\sin(t) & 0 \\ 0 & 0 & 0.2\sin(t) \end{bmatrix},$$

$$\Delta A_{d2}(v(t)) = \begin{bmatrix} 0.1\sin(t) & -0.2\cos(t) & 0.3\sin(t) \\ 0 & 0.3\sin(t) & 0 \\ 0 & 0 & 0.3\sin(t) \end{bmatrix}. \text{ The}$$

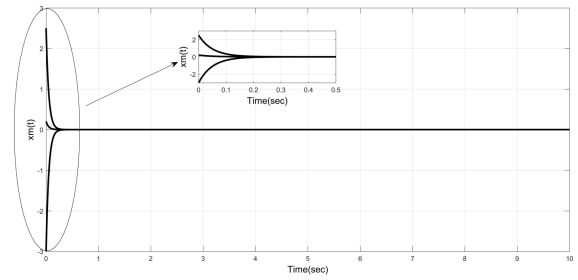**FIGURE 5.** State vector of the reference model.

The parameter of reference model is given by

$$A_m = \begin{bmatrix} -20 & 0 & 0 \\ 0 & -20 & 0 \\ 0 & 0 & -20 \end{bmatrix}$$

$$C_m = \begin{bmatrix} -5.2 & -2.6 & 0 \end{bmatrix}. \tag{61}$$

Using (4), the G and H can be achieved as $G = \begin{bmatrix} -5.2 & -2.6 & 0 \\ 0.27 & 0.13 & 0 \\ 0.19 & 0.09 & 0 \end{bmatrix}$ and $H = \begin{bmatrix} 49.3 & 24.7 & 0 \end{bmatrix}$. Assuming $q(t) = 0.05\cos(0.25\,t) + 1$

$$A_{d1} = \begin{bmatrix} -1 & -1 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -1 & -1 & 0.1 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}.$$

The constant parameters are considered as: $\alpha = 0.126, \beta = 0.12, \rho = 0.5, \rho_r = 0.3742, \rho_{v_1} = 0.3, \rho_{v_2} = 0.3742$. The initial values are specified as:

$$x(0) = [0.65\ 0\ 0]^T, \quad x_m(0) = [-2\ 1.5\ 0.2]^T \text{ and}$$
$$\tau_1 = \tau_2 = 0.1.$$

The solutions of LMI (28) are determined by the LMI toolbox in *MATLAB*® software as

$$P = 10^{-7} \begin{bmatrix} 0.2023 & 0.2255 & -0.0291 \\ 0.2255 & 0.6197 & -0.0508 \\ -0.0291 & -0.0508 & 0.0331 \end{bmatrix},$$

$$K = \begin{bmatrix} -11.1072 & -34.4317 & 2.4647 \end{bmatrix}$$

$$R_1 = 10^{-6} \begin{bmatrix} 0.1291 & 0.1438 & -0.0207 \\ 0.1438 & 0.1977 & -0.0247 \\ -0.0207 & -0.0247 & 0.0045 \end{bmatrix},$$

$$R_2 = 10^{-6} \begin{bmatrix} 0.1335 & 0.1488 & -0.0213 \\ 0.1488 & 0.1874 & -0.0250 \\ -0.0213 & -0.0250 & 0.0046 \end{bmatrix}$$

$$S_1 = 10^8 \begin{bmatrix} 3.4429 & -0.3206 & -0.2625 \\ -0.3206 & 0.2817 & 0.0883 \\ -0.2625 & 0.0883 & 1.4204 \end{bmatrix},$$

$$S_2 = 10^8 \begin{bmatrix} 3.4093 & -0.1881 & -0.2051 \\ -0.1881 & 0.1596 & 0.0486 \\ -0.2051 & 0.0486 & 1.4425 \end{bmatrix}$$
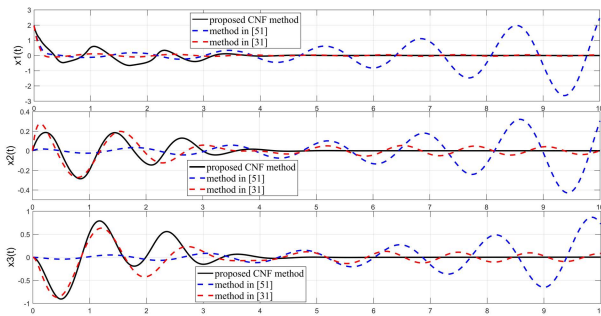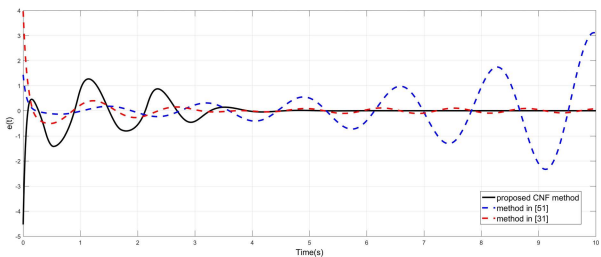
FIGURE 6. Trajectory of the system states.

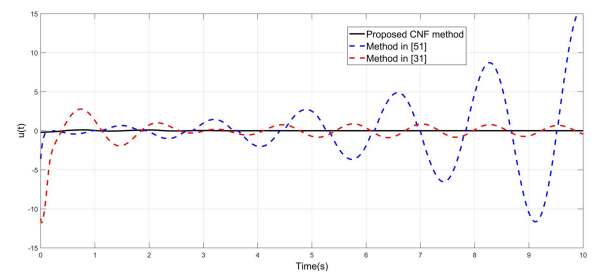**FIGURE 7.** The tracking errors.

FIGURE 8. Control input.

The simulation results are illustrated in Figs. 5 through 8. Fig. 5 shows the dynamics of the reference model. Fig. 6 illustrates the state estimation. It is clearly shown that the estimation error reduces and converges to the reference. Fig. 7 demonstrates the tracking error. Fig. 8 displays the control input with reasonable and appropriate values. The above results confirm the desirable performance and feasibility of the proposed approach.

The obtained outcomes verified the superior performance of the suggested control approach compared to [51] and [31].

V. CONCLUSION

In this paper, we proposed a robust tracking control procedure based on the CNF technique for uncertain nonlinear systems with time delay and input saturation. The proposed control approach provides optimal performance, robustness, and stability despite external disturbances, time delays, and saturations. The LMI technique guaranteed the asymptotic conditions for the tracking controllers and also proved the

stability of the system and convergence of the tracking errors to the origin. Implementation of the proposed approach to a two-dimensional system and the Chua's circuit system confirmed its superior performance and robustness to external disturbances and parametric uncertainties. Addressing the design problem of disturbance observer for nonlinear systems under input saturation using the CNF method can be the topic for future investigations.

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