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RESEARCH ARTICLE

Reliability-Box of Systems Under Model Parameter Uncertainty Based on Evidential Variable and Evidential Network

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ABSTRACT Advanced engineering systems possess a large number of components with complicated failure dependencies. To accurately assess the system reliability, the degradation models of components should be known in advance and the model parameters should be accurately estimated via a large quantity of historical time-to-failure data. In real-world situations, due to limited data, lack of knowledge, and vague judgments from experts, components' degradation model parameters are, however, inevitably encountered with epistemic uncertainty and oftentimes quantified as evidential variables. In this article, upper and lower bounds of system reliability, termed as reliability-box, are estimated when components' degradation model parameters are elicited from experts and quantified by evidential variables. In the first place, the constrained optimization model is leveraged to assess the reliability-box of each component by giving the evidential variable of the component's degradation model parameters. Next, based on the system structure, the evidential network of the system is constructed to propagate the epistemic uncertainty from the component level to the system level. Therefore, the focal elements of the evidential variable of system reliability, i.e., the system reliability bounds, can be assessed via the belief and plausibility functions to the mass function of the leaf node of the evidential network. The effectiveness of the proposed methods is demonstrated by a rolling system in the chip cutting detection module.

INDEX TERMS Reliability-box, epistemic uncertainty, evidential variable, evidential network, Dempster-Shafer theory, chip cutting detection system.

I. INTRODUCTION

Reliability modelling and assessment are important activities in the lifecycle management of advanced engineered systems. Modern industrial applications, such as transportation systems and manufacturing systems, are increasingly complex and sophisticated due to functional integration [1], [2]. Oftentimes, the systems contain a hierarchical structure that can be decomposed into a large number of modules with each module consists of a set of components [3]. For instance, the commercial aero-engine mainly consisting of six constituents, i.e., engine fans, low-pressure compressor, high-pressure compressor, combustion chamber, high-pressure turbine, and

low-pressure turbine, while the engine fans, for example, contain a lot of blades, bearings, and gears. The increasing complexity of engineered systems brings various challenges to system reliability modelling and assessment. The first challenge is the complicated failure dependency among components, such as common cause failure modes, propagated failure modes, isolation failure modes, and cascading failure modes [4]. The second challenge is the rise of epistemic uncertainty associated with both the system reliability models and degradation model parameters [5]. The epistemic uncertainty of system reliability models refers to the failure dependency of components is not deterministic, and several candidates of system reliability models can be used to construct the system reliability function [6], [7]. The epistemic uncertainty of degradation model parameters arises when

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the time-to-failure data of the studied systems/components is limited and the estimated degradation model parameters are uncertain. Sometimes, especially for newly developed systems with no time-to-failure data, experts are invited to express their judgements on the degradation model parameters, while the elicited information/knowledge may contain epistemic uncertainty [8], [9], [10].

In general, the aleatory uncertainty, due to the input inherent randomness, can be represented as probability models [5], while the epistemic uncertainty, arising from inadequate knowledge, subjectivity, ambiguity, fragmentary or dubious information, is mainly quantified by non-probabilistic models, such as interval theory, fuzzy theory, and Dempster-Shafer theory [11]. The aleatory uncertainty is oftentimes related to observable quantities [12] and is considered as irreducible, such as the temperature and flow rates. In contrast, the epistemic uncertainty is oftentimes related to unobservable quantities [12], such as degradation model parameters, and can be reduced, even eliminated, in the sense that new knowledge has been acquired. The probability theory is well suited to quantify the aleatory uncertainty. However, accurately estimating the parameter(s) of the probability distribution needs a large population of historical data which is impossible for modern engineered systems. Alternatively, some non-probabilistic measures are frequently utilized for epistemic uncertainty quantification of the model parameters when the historical data is limited. The interval set theory [13] is a well-known method to quantify the epistemic uncertainty via using the lower and upper bounds of the model parameters. However, the interval set theory can only quantify the epistemic uncertainty while aleatory and epistemic uncertainties always coexist and couple together in engineering practices [4]. For example, in manufacturing systems, the length and thickness of the critical structures are stochastic because of manufacturing deviations and measurement errors while the estimation of the stochastic model parameters is imprecise due to limited data and data uncertainty [14].

Dempster-Shafer theory has been widely implemented in engineering practices due to its strong capability of quantifying both aleatory and epistemic uncertainties. Dempster-Shafer theory is a unified framework that can model probability information by assigning zero masses to the non-singleton sets, and model logical statements via using categorical mass functions [15], [16]. In the literature, there are plenty of works on studying reliability modelling, reliability assessment, importance measures, reliability optimization, and dynamic reliability assessment under Dempster-Shafer theory framework. Simon and Webber [17] are pioneers to leverage Dempster-Shafer theory to quantify the epistemic uncertainty associated with the failure rates of components' lifetime distribution, and they designed an evidential network (EN) to propagate the epistemic uncertainty from the component level to the system level. Dersteracke and Sallak [18] developed the belief universal generating function (BUGF) method for multi-state systems (MSSs) under epistemic uncertainty of components'

states. Mi *et al.* [3] developed another type of EN model by constructing the conditional belief table and conditional plausibility table to respectively calculate the system's belief reliability and plausible reliability. Recently, Mi *et al.* [4] developed some special EN models that can handle both epistemic uncertainty of components' states and common cause failure (CCF). Xiahou *et al.* [19], [20], [21] have published a series of works on system reliability modelling, redundancy allocation, and remaining useful life (RUL) prediction by using Dempster-Shafer theory to fuse multiple-source imprecise information. Qiu *et al.* [6], [22] extended the component assignment problem (CAP) under Dempster-Shafer theory framework. CAP is a special reliability optimization problem aiming at maximizing system reliability by assigning the appropriate positions for components with different reliability functions. It should be noted that all these reliability modelling and optimization methods under Dempster-Shafer theory framework were conducted by using the interval-to-mass transformation method. That is to say, all the above works assumed that the component reliability, component states probability, or the components' degradation model parameters are interval values. The interval-valued data are, then, converted into a discrete Dempster-Shafer structure by assuming the component states or system states as the frame of discernment (FoD).

The evidential variable is another measure to quantify the epistemic uncertainty of components' degradation model parameters under the continuous Dempster-Shafer structure. In contrast to the interval set theory, the evidential variable additionally assigns each possible interval value with a specific mass function. The merits of using the evidential variable to model the epistemic uncertainty are twofold: 1) the evidential variable carries more information than the interval set theory, whereas it requires fewer data to be constructed than the probability theory; 2) the evidential variable is also suitable for experts to express their knowledge with uncertainty. The experts are only needed to judge how many interval values and their specific bounds with each has its probability. Compared with the discrete Dempster-Shafer structure, the evidential variable is a more generalized framework to model continuous random variables with epistemic uncertainty. When the lower and upper bounds of each focal element of the evidential variable are identical, the evidential variable degenerates to the discrete Dempster-Shafer structure [23], [24].

In the literature, only a few research attempts have been made on studying the reliability assessment using the evidential variable. Liu and Wang [25] studied the reliability estimation of engineering systems via two types of reliability-related data, i.e., the lifetime testing data and degradation testing data. The stochastic degradation process of components was characterized by the Wiener process and the associated parameters were described as evidential variables. Thereby, the Dempster rule of combination was used to fuse the mass functions constructed from lifetime testing data and degradation testing data. Liu *et al.* [26] further extended the method

in [25] by fusing more sources of information, including the accelerated degradation test (ADT) data, the accelerated life test (ALT) data, and expert knowledge. However, it should be noted that these methods based on evidential variables were only suitable for continuous deteriorating systems where the system was considered as a whole and cannot be broken down into subsystems and components.

In this article, we aim to assess the multi-component systems' reliability when the components' degradation model parameters contained epistemic uncertainty. We use the evidential variable to quantify the epistemic uncertainty associated with the components' degradation model parameters. Experts are invited to express their judgments on the intervals and the mass function of the evidential variables. A constrained optimization model is constructed to assess the evidential variable of the components' reliability. On the other hand, in order to assess the system reliability, we construct the evidential network of multi-component systems and use it to propagate the epistemic uncertainty from the component level to the system level. Therefore, the evidential variable of the system reliability is assessed. Finally, the system reliability-box is constructed by using the belief and plausibility functions on the evidential variable of the system reliability. It bears noting that Simon and Bicking [24] have proposed the system reliability assessment based on the p-box and evidential network. The p-box variable was transformed into the evidential variable, and the evidential network was utilized to assess the focal elements of the system reliability. In our work, we directly use the evidential variable to model the epistemic uncertainty of the component degradation model parameters, and utilize the constrained optimization model to assess the component reliability-box. To sum up, the unique contributions of this work lie in three aspects:

1) We assess the system reliability-box under the evidential variable framework to consider the epistemic uncertainty associated with the components' degradation model parameters.

2) We use the evidential variable to quantify the epistemic uncertainty of the components' degradation model parameters, and a constrained optimization model is formulated to assess the component reliability-box.

3) We construct the EN of multi-component systems, and the system reliability-box is assessed by leveraging the belief and plausibility functions.

The remainder of this article is rolled out as follows. Section II gives the theoretical backgrounds of the Dempster-Shafer theory and the evidential variable. The components' reliability-box is assessed by a constrained optimization model in Section III. Section IV presents the EN, and the system reliability box is assessed based on the belief and plausibility function. A real-world engineering system, i.e., a rolling system in the chip cutting detection module, is presented to demonstrate the effectiveness of the proposed method in Section V. Section VI concludes this article and provides a set of future works.

II. DEMPSTER-SHAFER THEORY

The Dempster-Shafer theory, also called evidence theory or belief function theory, was originated by A. Dempster and his Ph.D. student G. Shafer to model aleatory and epistemic uncertainties by using the set theory and probability theory. The probability theory can represent aleatory uncertainty well, whereas it was argued by many authors that it cannot represent lack of information/knowledge, small sample, imprecise data, and expertise with uncertainty [15]. Dempster-Shafer theory includes the classic probability theory and set theory as special cases. Moreover, the fuzzy set theory is also a type of special case of the Dempster-Shafer theory when the focal sets are nested. In addition, the probability-box theory is also included in the Dempster-Shafer theory with continuous evidential variables. Therefore, the Dempster-Shafer theory can be considered as a generalized framework to represent aleatory and epistemic uncertainties, also called hybrid uncertainty, under various real-world situations.

Basic belief assignment (BBA) is the basic building block of the Dempster-Shafer theory. A BBA is also called mass function $m(A)$ that maps the power set of the framework of discernment (FoD) Ω to the interval $[0, 1]$. Generally, the mass function has the following two axioms:

$$\sum_{A \subset \Omega} m(A) = 1, \quad (1)$$

$$m(\emptyset) = 0. \quad (2)$$

Element A with that $m(A) > 0$ is called a focal element. Some special cases of mass function are defined as follows. Bayesian mass is defined as the mass function with the focal elements are all singletons. Fuzzy sets are those mass functions with nested focal elements. Categorical mass function means the unity mass is assigned to only one focal element, i.e., $m(A) = 1$ where $A \subset \Omega$. Simple mass function means only two subsets can be the focal elements and one of them is the FoD.

For the mass function $m(A)$, two functions, namely the belief and plausibility functions, can be defined as:

$$Bel(A) = \sum_{B \subset A} m(B), \quad (3)$$

$$Pl(A) = \sum_{B \cap A \neq \emptyset} m(B). \quad (4)$$

Belief function $Bel(A)$ measures the least commitment of hypothesis A . Plausibility function $Pl(A)$ measure the most plausibility of hypothesis A . these two measures have the inequality $Bel(A) \leq Pl(A)$, which means what is certain is plausible. Moreover, these two functions are dual functions that $Bel(A) = 1 - Pl(\bar{A})$, where \bar{A} is the complement of set A in the FoD Ω . Furthermore, the quantity of $Pl(A) - Bel(A)$ oftentimes represents the epistemic uncertainty value of set A .

Up to here, the mass function, belief function, and the plausibility function are defined over discrete spaces, that is, all

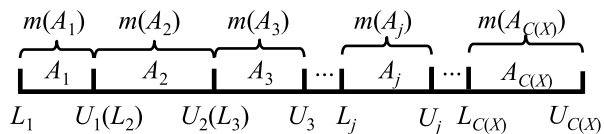


FIGURE 1. An illustration of the evidential variable X with non-overlapping focal elements.

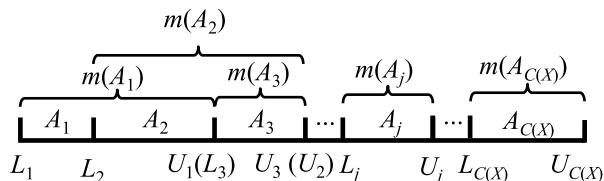


FIGURE 2. An illustration of the evidential variable X with overlapping focal elements.

the possible values of subset A are discrete. Another type of mass function is defined over the continuous space \mathbb{R} . Such a mass function is called the evidential variable in the Dempster-Shafer theory. Details of the definition of evidential variables are given below.

The evidential variable is defined as the mass function whose focal elements are all closed intervals. In general, the evidential variable X has $C(X)$ number of closed intervals A_j ($j = 1, 2, \dots, C(X)$) with each A_j has its associated mass function $m(A_j)$ and $\sum_j m(A_j) = 1$. For each focal element, we have $A_j = [L_j, U_j]$, where L_j and U_j are the lower and upper bound of interval A_j , respectively. An illustration of the evidential variable X with nonoverlapping focal elements is given in Fig. 1. Another type of evidential variable is that with overlapping focal elements, as illustrated in Fig. 2. The evidential variable is used to quantify the epistemic uncertainty associated with the degradation model parameters. For instance, if the lifetime of a component follows the exponential distribution, where the failure rate of the exponential distribution is quantified by an evidential variable as $\langle [1.6, 1.8], 0.3 \rangle$; $\langle [1.8, 2.0], 0.3 \rangle$; $\langle [2.0, 2.2], 0.4 \rangle$. The evidential variable represents that the expert believes that the failure rate was located in the interval $[1.6, 1.8]$ with a mass function of 0.3, in the interval $[1.8, 2.0]$ with a mass function of 0.3, while in the interval $[2.0, 2.2]$ with a mass of 0.4.

The relation among the evidential variable, random variable, and the interval variable is depicted in Fig. 3. It can be seen that the informativeness of an evidential variable is between those of a random variable and an interval variable. The random variable is more informative than the evidential variable, while the evidential variable is more informative than the interval variable. Moreover, the interval variable is a special evidential variable with only one focal element with a mass of unity. The evidential variable can be converted into a random variable with the probability density function (PDF) f_X can be created via the uniformity approach:

$$f_X(x) = \sum_{j=1}^{C(A)} \delta_j(x)m(A_j)/(U_j - L_j), \quad (5)$$

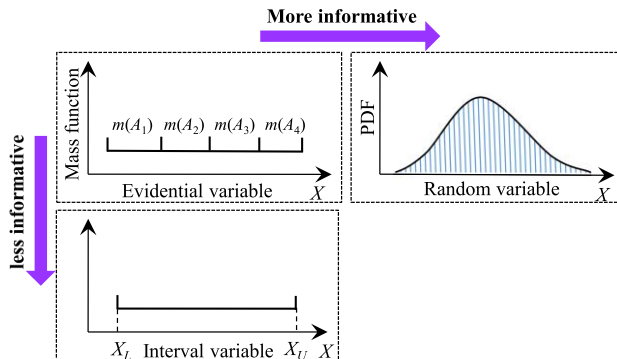


FIGURE 3. Comparison among the evidential variable, random variable, and the interval variable.

where $\delta_j(x) = 1$ if $x \in A_j$ and 0 otherwise. It can be seen that the uniformity approach is to assume a uniform distribution over each focal element according to its mass function. Note that the transformation of the evidential variable to the random variable is conducted by transforming the epistemic uncertainty associated with the focal elements to the aleatory uncertainty of the focal elements. In (5), the intervals of contained focal elements are nonoverlapped. For the case that the intervals overlapping, the authors in [27] have proposed a least committed method to convert the evidential variable to a random variable by accumulating the mass functions of the overlapping parts.

The evidential variable is difficult to be visualized and is not intuitively understood by engineers. In fact, the probability-box (p-box) is oftentimes used to visualize the evidential variable by converting the mass functions into the lower and upper bounds of the cumulative distribution function (CDF). In general, a p-box can be represented by a pair of CDFs $[F(x), \bar{F}(x)]$ where $F(x) < \bar{F}(x)$ for all $x \in R$. In this article, the p-boxes are all discrete, that is, the lower and upper bounds of CDFs are both stair-wise functions. For example, assuming that the mean time to failure (MTTF) of a component is elicited from an expert. The expert states that the probabilities of the MTTF of this component is lower than 6, 12, and 24 hours, and the corresponding statements are:

$$\begin{aligned} 0 &\leq \Pr\{0 \leq MTTF \leq 10\} \leq 0.2; \\ 0.2 &\leq \Pr\{0 \leq MTTF \leq 20\} \leq 0.4; \\ 0.6 &\leq \Pr\{0 \leq MTTF \leq 30\} \leq 0.8. \end{aligned}$$

The expert also provides that the MTTF of this component is no more than 35 hours, and is no less than 5 hours, therefore, we have $F(35) = \bar{F}(35) = 1$ and $F(5) = \bar{F}(5) = 0$. The above statements correspond to a discrete p-box which is depicted in Fig. 4. The p-box illustrated in Fig. 4 can be converted into an evidential variable with mass function on interval-valued focal elements. Figure 5 gives the corresponding evidential variable of the p-box in Fig. 4. Mathematically, given a general p-box $[F(x), \bar{F}(x)]$, we have the any value α

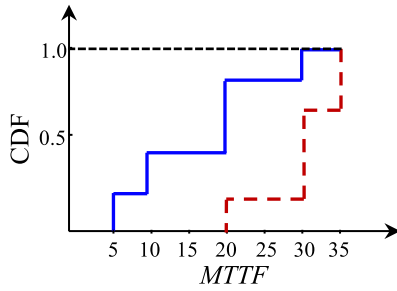


FIGURE 4. The p-box provided by the expert.

of focal element $[\underline{F}^{-1}(\alpha), \bar{F}^{-1}(\alpha)]$ with the mass function of:

$$m([\underline{F}^{-1}(\alpha), \bar{F}^{-1}(\alpha)]) = L(\{x | \underline{F}^{-1}(x) = \underline{F}^{-1}(\alpha), \bar{F}^{-1}(x) = \bar{F}^{-1}(\alpha)\}), \quad (6)$$

where $L(\{x | \underline{F}^{-1}(x) = \underline{F}^{-1}(\alpha), \bar{F}^{-1}(x) = \bar{F}^{-1}(\alpha)\})$ is the length of the interval of which the upper and lower bounds of inverse CDFs are exactly the same. Note that, an evidential variable can also be converted into a p-box $[\underline{F}(\alpha), \bar{F}(\alpha)]$ that:

$$\bar{F}(\alpha) = \sum_{[x] \in \Omega, \bar{x} \leq \alpha} m([x]) = Pl((-\infty, x]), \quad (7)$$

$$\underline{F}(\alpha) = \sum_{[x] \in \Omega, \bar{x} \leq \alpha} m([x]) = Bel((-\infty, x]), \quad (8)$$

where $Bel((-\infty, x])$ and $Pl((-\infty, x])$ are the belief and plausible CDFs, respectively.

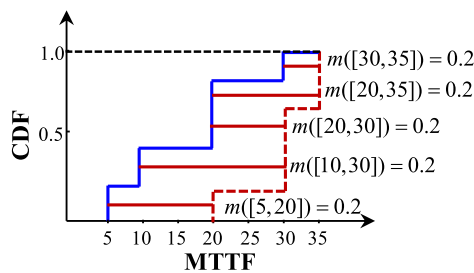


FIGURE 5. The corresponding evidential variable of the p-box in Fig. 4.

III. RELIABILITY-BOX OF COMPONENTS

In this section, we aim to assess the reliability-box of components when the components' degradation model parameters are represented by evidential variables. In engineering practice, expert knowledge is oftentimes used to determine the parameters of the components' lifetime distribution when the time-to-failure data are limited. As the parameters associated with the components' degradation models are continuous variables, we use the evidential variable to quantify the epistemic uncertainty of the expert knowledge on the possible values of the parameters of components' lifetime distributions. In general, the lifetime of components can be characterized by some specific distributions, such as exponential distribution and Weibull distribution. The component reliability is, therefore, a function of the lifetime distribution parameters θ

and time t . We use $R_C(t) = \mathcal{L}(\theta, t)$ to describe that the component reliability is calculated by the parameters θ and time t with a specific probability law \mathcal{L} . Suppose that the evidential variable of parameter θ is described as $m([\theta_j] = m([\theta_{jL}, \theta_{jU}])$, where $j = 1, 2, \dots, C(\theta)$. The purpose is to calculate the evidential variable of the component reliability, denoted as $m([R_{C,j}(t)]) = m([R_{C,jL}(t), R_{C,jU}(t)])$ ($j = 1, 2, \dots, C(R_C(t))$), via the evidential variable of the parameter θ and time t . For the evidential variable of the component reliability, we have to determine three quantities in $m([R_{C,i}(t)])$. The first quantity is the mass function associated with each focal element of the component reliability, i.e., $m([R_{C,i}(t)])$, which is exactly the same as $m([\theta_j])$ because we do not have any operations on the mass functions of the component reliability. The second term is the lower bound of the component reliability of each focal element, i.e., $R_{C,jL}(t)$. In general, $R_{C,jL}(t)$ can be determined by a constrained optimization, which is mathematically written as:

$$R_{C,jL}(t) = \min(R_C(t) = \mathcal{L}(\theta, t)) \quad \text{s.t. } \theta_{jL} \leq \theta \leq \theta_{jU}. \quad (9)$$

The third quantity is $R_{C,jU}(t)$, which can be also determined by a constrained optimization as:

$$R_{C,jU}(t) = \max(R_C(t) = \mathcal{L}(\theta, t)) \quad \text{s.t. } \theta_{jL} \leq \theta \leq \theta_{jU}. \quad (10)$$

For instance, if the lifetime of the component follows the exponential distribution with the parameter θ , where θ is the MTTF of the component. Thereby, the component reliability function can be described as: $R_C(t) = \exp(-\theta^{-1}t)$. Then, the evidential variable of the component reliability can be formulated as:

$$m([R_{C,j}(t)]) = m([R_{C,jL}(t), R_{C,jU}(t)]) = m([\exp(-(\theta_{jL})^{-1}t), \exp(-(\theta_{jU})^{-1}t)]). \quad (11)$$

For example, considering the evidential variable of the MTTF given in Fig. 4 at a given time instant $t = 10$ hours. We can calculate the evidential variable of the component reliability as following.

$$\begin{aligned} m([5, 20]) = 0.2 &\rightarrow m([R_{C,1}(10)]) \\ &= m([0.135, 0.607]) = 0.2; \\ m([10, 30]) = 0.2 &\rightarrow m([R_{C,2}(10)]) \\ &= m([0.368, 0.717]) = 0.2; \\ m([20, 30]) = 0.2 &\rightarrow m([R_{C,3}(10)]) \\ &= m([0.607, 0.717]) = 0.2; \\ m([20, 35]) = 0.2 &\rightarrow m([R_{C,4}(10)]) \\ &= m([0.607, 0.752]) = 0.2; \\ m([30, 35]) = 0.2 &\rightarrow m([R_{C,5}(10)]) \\ &= m([0.717, 0.752]) = 0.2. \end{aligned}$$

The above evidential variable of the component reliability can be converted into a p-box via (7) and (8), and the

corresponding discrete p-box of the component reliability is visualized in Fig. 6.

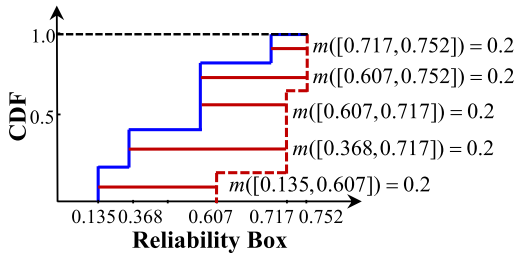


FIGURE 6. The corresponding evidential variable of the component reliability given the evidential variable of the MTTF of the component in Fig.5.

If the lifetime of the component obeys the Weibull distribution with the scale parameter and shape parameter are quantified as evidential variables, the evidential variable of the component reliability can be also constructed. First, the reliability function of the component with Weibull distribution can be described as:

$$R_C(t) = \exp \left[- (t/\theta)^\beta \right], \quad (12)$$

where θ is the scale parameter, and β is the shape parameter. If both the scale parameter and shape parameter are evidential variables, the evidential variable of the component reliability can be formulated as $m([R_{C,j}(t)]) = m([R_{C,jL}(t), R_{C,jU}(t)])$, where $[R_{C,jL}(t), R_{C,jU}(t)]$ can be identified by the following optimization model:

$$\begin{aligned} [R_{C,jL}(t), R_{C,jU}(t)] = \min/\max (R_C(t) = \exp \left[- (t/\theta)^\beta \right]) \\ \text{s.t.} \quad \begin{cases} \theta_{jL} \leq \theta \leq \theta_{jU} \\ \beta_{jL} \leq \beta \leq \beta_{jU} \end{cases} \end{aligned} \quad (13)$$

Note that, if there are more than one parameter are evidential variables for the lifetime distribution, such as the Weibull distribution, the mass function and the number of focal elements of the evidential variable of component reliability should be granulated as:

$$m([R_{C,j}(t)]) = \prod_{j \in N_\theta} m([\theta_j]), \quad (14)$$

and

$$C(R_C(t)) = \prod_{j \in N_\theta} C(\theta_j), \quad (15)$$

where N_θ is the number of parameters in the studied lifetime distribution. For instance, for the two-parameter Weibull distribution, the evidential variable of the shape parameter and scale parameter are $m(\theta) = \{m([200, 250]) = 0.5, m([250, 300]) = 0.5\}$, $m(\beta) = \{m([1, 1.5]) = 0.5, m([1.5, 2]) = 0.5\}$. Hence, there are four focal elements of the evidential variable of the component reliability with each having a mass function of 0.25. if $t = 100$ hours, the

evidential variable of the component reliability is given as following:

$$\begin{aligned} m([R_{C,1}(10)]) &= m([0.9512, 0.9920]) = 0.25; \\ m([R_{C,2}(10)]) &= m([0.9889, 0.9984]) = 0.25; \\ m([R_{C,3}(10)]) &= m([0.9608, 0.9939]) = 0.25; \\ m([R_{C,4}(10)]) &= m([0.9920, 0.9989]) = 0.25. \end{aligned}$$

The above evidential variable of the component reliability can be converted into a p-box via (7) and (8), and the corresponding discrete p-box of the component reliability is visualized in Fig. 7.

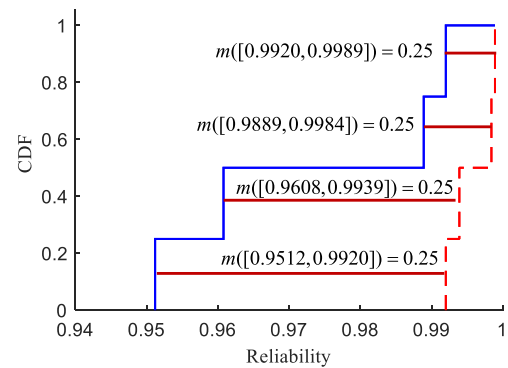


FIGURE 7. The evidential variable of the component reliability with Weibull distribution.

IV. RELIABILITY-BOX OF THE SYSTEM BY EVIDENTIAL NETWORK

In this section, the evidential variable of the system reliability is assessed by given the evidential variables of component reliabilities and the system structure. We separately assess the mass functions and the focal elements of the evidential variable of the system reliability. First, the focal elements of the evidential variable of the system reliability, i.e., the lower and upper bounds of the system reliability is assessed by the EN model. Then, the mass functions associated with the focal elements of the evidential variable of system reliability is assessed by the mass functions of the evidential variables of component reliabilities.

A. EVIDENTIAL NETWORK MODELS

The main purpose of assessing the evidential variable of the system reliability is to assess the focal elements and its associated mass function. The focal elements of the evidential variable of system reliability contains the lower and upper bounds of the system reliability. In general, given the system structure function $\varphi_S(\cdot)$, the system reliability bounds, denoted by $[R_{S,jL}(t), R_{S,jU}(t)]$ can be assessed by an optimization model [8]:

$$\begin{aligned} [R_{S,jL}(t), R_{S,jU}(t)] \\ = \min/\max R_S(t) = \varphi_S(R_{C1}(t), \dots, R_{CM}(t)) \end{aligned}$$

$$s.t. \begin{cases} R_{C1,jL}(t) \leq R_{C1}(t) \leq R_{C1,jU}(t) \\ R_{C2,jL}(t) \leq R_{C2}(t) \leq R_{C2,jU}(t) \\ \vdots \\ R_{CM,jL}(t) \leq R_{CM}(t) \leq R_{CM,jU}(t) \\ R_{Cm,jL}(t) + F_{Cm,jU}(t) = 1, \quad m = 1, 2, \dots, M \\ R_{Cm,jU}(t) + F_{Cm,jL}(t) = 1, \quad m = 1, 2, \dots, M \end{cases}, \quad (16)$$

where $R_{Cm}(t)$ ($m = 1, 2, \dots, M$) is the component reliability function and M is the number of components in the system; $R_{Cm,jL}(t)$ and $R_{Cm,jU}(t)$ are the lower and upper bounds of the component reliability, respectively; $F_{Cm,jL}(t)$ and $F_{Cm,jU}(t)$ are the lower and upper bounds of the component failure probability, respectively. In fact, resolving (16) can be computational tedious when the number of components M is large enough. In the literature, the EN model is an effective tool to calculate the system reliability bound by given the mass functions of the component reliabilities. In our work, we introduce the EN model to assess the focal elements of the evidential variable of the system reliability, i.e., the system reliability bound, in a computationally efficient manner.

EN is initialized by Simon and Webber to represent and propagate epistemic uncertainty of components' states [17]. In general, EN can be viewed as a graphic representation of mass function relationships. In contrast to the Bayesian networks (BNs), The EN model has the same directed acyclic graph (DAG). However, the EN represents the state dependency among components, subsystems, and the system via the conditional belief mass functions (CBMFs), rather than the conditional probability functions in BNs. If the mass functions of the EN are Bayesian masses, the EN model degenerates to the traditional BNs. Therefore, due to the strong modeling and inference capabilities, the EN model have been extensively implemented for system reliability modelling and assessment of multi-component systems to cope with the hybrid uncertainty of component states [28], [29].

For an EN model, the DAG consists of $M + N + 1$ nodes, denoted as $\{X_1, X_2, \dots, X_M, S_1, S_2, \dots, S_N, S\}$, and Z directed edges, where node X_i ($i = 1, 2, \dots, M$) represents the i th component; node S_i ($i = 1, 2, \dots, N$) represents the i th subsystem; node S represents the entire system. Each root node X_i ($i = 1, 2, \dots, M$) represents a variable that contains all the singletons and non-singletons of the i th component under the Dempster-Shafer theory. A directed edge from node X_j to node S_i is leveraged to represent the failure dependency between the components and subsystems. Here, node S_i is the child node of X_j , whereas X_j is a parent node of S_i , denoted as $\mathbf{pa}(S_i) = X_j$. In ENs, CBFMs provide the quantitative measure of the failure dependencies among components, subsystems, and system, which are formed as a conditional belief mass table (CBMT). For ENs, nodes S_i and S both have CBMTs as they are child nodes of components, denoted as $m(S_i|\mathbf{pa}(S_i))$ and $m(S|\mathbf{pa}(S))$, respectively. For the root node, i.e., node X_i ($i = 1, 2, \dots, M$), they do not own parent nodes, and therefore, we should define the marginal mass

distributions of these nodes. In fact, the marginal mass distribution of components has been calculated in Section III. To compute the marginal mass distribution of the child nodes, i.e., nodes S_i ($i = 1, 2, \dots, N$) and node S , in an EN model, we have to decompose the multi-component system into a combination of series and parallel structures [24].

The components in a system are connected with a series-parallel structure. Without loss of generality, we study two types of systems, i.e., series system and parallel system, and any sophisticated system can be decomposed into these two types of systems. Firstly, considering two components connected in series, components $C1$ and $C2$ are assumed to have only two states, i.e., either perfectly functioning $\{1\}$ or completely failed $\{0\}$. However, due to the epistemic uncertainty regarding the components' states, an uncertain state $\{0, 1\}$ is introduced to represent the epistemic uncertainty associated with the component reliability. Thus, the FoD of the two components is $\Omega = \{\{0\}, \{1\}\}$. Based on the structure function of series systems, the fault tree of a series system can be constructed by an "OR" gate. To quantify the dependency between the parent nodes and child node, the CBMT [30], [31] of the EN model for the "OR" gate is tabulated in Table 1. Based on the CBMT of the OR gate, the mass function of the child node can be computed.

TABLE 1. CBMT of "OR" gate.

X_1	X_2	$m(S_i \mathbf{pa}(S_i))$		
		$\{0\}$	$\{1\}$	$\{0,1\}$
$\{0\}$	$\{0\}$	1	0	0
$\{0\}$	$\{1\}$	1	0	0
$\{0\}$	$\{0,1\}$	1	0	0
$\{1\}$	$\{0\}$	1	0	0
$\{1\}$	$\{1\}$	0	1	0
$\{1\}$	$\{0,1\}$	0	0	1
$\{0,1\}$	$\{0\}$	1	0	0
$\{0,1\}$	$\{1\}$	0	0	1
$\{0,1\}$	$\{0,1\}$	0	0	1

If the two components, i.e., components $C1$ and $C2$, are connected in parallel, the fault tree of the parallel system can be modeled by an "AND" gate. In the same manner, the "AND" gate can be converted to an EN model. The corresponding CBMT can be constructed based on the structure function of parallel systems, and it is tabulated in Table 2 [30], [31].

Based on the CBMTs of the series and parallel systems, the mass function of the subsystem S_i can be assessed by the inference algorithm as:

$$m(S_i) = \sum_{\mathbf{pa}(S_i)} \left(m(S_i|\mathbf{pa}(S_i)) \times \prod_{\mathbf{pa}(S_i)} m(\mathbf{pa}(S_i)) \right), \quad (17)$$

TABLE 2. CBMT of “AND” gate.

X_1	X_2	$m(S_i \mathbf{pa}(S_i))$		
		{0}	{1}	{0,1}
{0}	{0}	1	0	0
{0}	{1}	0	1	0
{0}	{0,1}	0	0	1
{1}	{0}	0	1	0
{1}	{1}	0	1	0
{1}	{0}	0	1	0
{0,1}	{0}	0	0	1
{0,1}	{1}	0	1	0
{0,1}	{0,1}	0	0	1

where $m(\mathbf{pa}(S_i))$ is the mass functions of the parent nodes of S_i , i.e., the mass function of components, and $m(S_i|\mathbf{pa}(S_i))$ is the CBMT of node S_i . If nodes S_i is the child node of a series-connected system, then, $m(S_i|\mathbf{pa}(S_i))$ corresponds to Table 1. If nodes S_i is the child node of a parallel-connected system, $m(S_i|\mathbf{pa}(S_i))$ corresponds to Table 2.

For the node S , its mass function can also be inferred by the inference algorithm as:

$$m(S) = \sum_{\mathbf{pa}(S)} \left(m(S|\mathbf{pa}(S)) \times \prod_{\mathbf{pa}(S)} m(\mathbf{pa}(S)) \right), \quad (18)$$

where $m(\mathbf{pa}(S))$ is the mass functions of node S , i.e., the mass function of subsystems, and $m(S|\mathbf{pa}(S))$ is the CBMT of node S , and it corresponds to one of Tables 1 and 2. Based on the mass function of the leaf nodes, the system reliability bounds can be assessed by using the belief and plausibility function. Specifically, the lower bounds of the system reliability can be assessed by:

$$Bel_{S,jL}(t) = \sum_{i=\{1\}} m(S(t) = i), \quad (19)$$

where as the upper bound of the system reliability, denoted by $Pl_{S,jU}(t)$, can be calculated by:

$$Pl_{S,jU}(t) = \sum_{i=\{1\} \text{ or } i=\{0,1\}} m(S(t) = i). \quad (20)$$

For the focal elements of the evidential variable of the system reliability, i.e., $[R_{S,jL}(t), R_{S,jU}(t)]$, it should be calculated by the focal elements of the evidential variable of component reliability, i.e., $R_{Cm,jL}(t)$ and $R_{Cm,jU}(t)$. First, for each component m , the mass distribution can be inferred as:

$$\begin{aligned} m_{Cm}(t) &= \{F_{Cm,jL}(t), R_{Cm,jL}(t), R_{Cm,jU}(t) - R_{Cm,jL}(t)\} \\ &= \{1 - R_{Cm,jU}(t), R_{Cm,jL}(t), R_{Cm,jU}(t) - R_{Cm,jL}(t)\}, \end{aligned} \quad (21)$$

where $m_{Cm}(t)$ represents the mass distribution of the component reliability at time instant t . Based on the EN model, the

system reliability bounds can be found via inputting the mass distributions of all components, and the focal elements of the evidential variable of system reliability $[R_{S,jL}(t), R_{S,jU}(t)]$ can be assessed by:

$$[R_{S,jL}(t), R_{S,jU}(t)] = [Bel_{S,jL}(t), Pl_{S,jU}(t)]. \quad (22)$$

Thereby, when input all the combinations of all component focal elements of its evidential variables, all focal elements of the evidential variable of the system reliability can be assessed.

B. MASS FUNCTION OF THE SYSTEM RELIABILITY

The EN model only assesses the focal elements of the evidential variable of the system reliability, while the mass function of the evidential variable of the system reliability still remains unsolved. In this section, the mass function of the evidential variable of the system reliability is computed by all the mass functions of the evidential variable of component reliability as following:

$$m([R_{S,j}(t)]) = \prod_{m \in M} m([R_{Cm,j}(t)]). \quad (23)$$

Note that, the number of focal elements of the evidential variable of system reliability is the product of the numbers of focal elements in the evidential variable of component reliability., i.e.,

$$C(R_S(t)) = \prod_{m \in M} C(R_{Cm}(t)), \quad (24)$$

where $C(R_{Cm}(t))$ is the number of focal elements of the evidential variable of the m th component and $C(R_S(t))$ is the number of focal elements of the evidential variable of system reliability. Based on (22) and (23), the evidential variable of system reliability can be assessed.

V. AN ILLUSTRATIVE EXAMPLE

In chip manufacturing, the chip cutting and testing processes are key procedures after the silicon wafer has been manufactured. In general, these two processes are done via a chip cutting detection module in the assembly factory [32]. The chip cutting detection module mainly consists of four subsystems, i.e., the cutting system, driving system, testing system, and the rolling system, as shown in Fig.8. The chip cutting detection module has only one assembly line. In the assembly line, each chip is transmitted by magnetic belt, and the belt is driven by stepper motor to ensure a reasonable space between chips. In the chip cutting detection module, the chip is detected by the testing system first, and the chip functioning error is detected in this process. The defective chip will be discarded and the non-defective products will be sent to the cutting system. The cutting system is compatible with a material roll within 60mm and uses diagonal blade cutting to ensure the cutting quality. All material rolls with chips are driven by the driving system and rolling system with stepper motors.

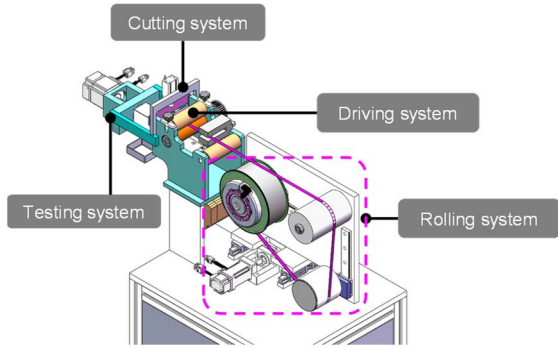


FIGURE 8. The chip cutting detection module.

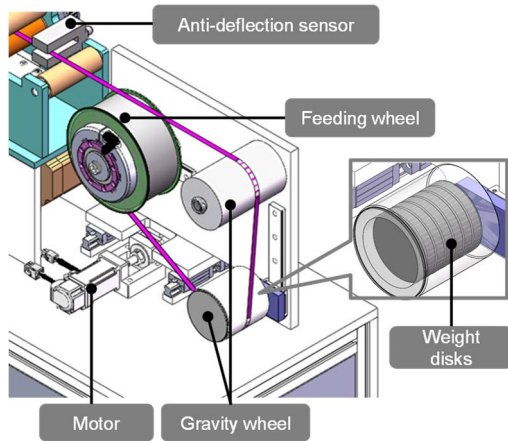


FIGURE 9. The rolling system of the chip cutting detection module.

The rolling system, as delineated in Fig.9, is equipped with an anti-deflection sensor, feeding wheel, motor, gravity wheel, and weight disks. The anti-deflection sensor can adjust the coil position in real time by adjusting the motor to prevent the deviation of chips. Gravity wheel is used to tighten the material belt. Weight disks can be loaded on the gravity wheel, and the weight can be increased or decreased according to the width of the material roll. The rolling system is most susceptible to degradation and its reliability is critical to the chip quality. However, the chip manufacturing factory does not have enough time-to failure data of the rolling system, and they invited some experts to elicit their knowledge on the reliability-related information of the components in the rolling system.

The rolling system mainly consists of five types of components, and it has two gravity wheels. The ant-deflection sensor is an electronic device, and its lifetime follows the exponential distribution. The gravity wheel, feeding wheel, and the weight disks are relatively mature products and their lifetime also follows the exponential distribution. The two gravity wheels are the same components with the same degradation law. The motor is a mechanical system with a Weibull distributed lifetime. The experts are invited to express their knowledge of the components' degradation model parameters. However, due to lack of knowledge, vague judgements

TABLE 3. The evidential variable of component degradation model parameters.

Units	Parameters	Evidential variables
AD	Exponential (λ)	$\langle [2.0 \times 10^{-3}, 2.2 \times 10^{-3}], 1 \rangle$
FW	Exponential (λ)	$\langle [1.5 \times 10^{-3}, 2.0 \times 10^{-3}], 0.5 \rangle;$ $\langle [2.0 \times 10^{-3}, 2.5 \times 10^{-3}], 0.5 \rangle$
MO	Weibull (θ)	$\langle [1500, 1600], 0.3 \rangle;$ $\langle [1600, 1800], 0.7 \rangle$
GW	Weibull (β)	$\langle [1.5, 1.8], 1 \rangle$
GW	Exponential (λ)	$\langle [2.5 \times 10^{-3}, 3 \times 10^{-3}], 1 \rangle$
WD	Exponential (λ)	$\langle [1.8 \times 10^{-3}, 2.1 \times 10^{-3}], 1 \rangle$

of experts, the elicited data contain epistemic uncertainty and they are quantified as evidential variables as given in Table 3. In Table 3, AD, FW, MO, GW, and WD, respectively, represent the anti-deflection sensor, feeding wheel, motor, gravity wheel, and weight disks. As given in Table 3, the units AD, GW, and WD are all exponentially distributed components with interval-valued failure rates (In this case, the evidential variable degenerates to the interval variable with only one focal element). The evidential variables of units FW and MO have two focal elements and their associated mass functions. The evidential variables of all components' degradation model parameters are illustrated in Fig. 10.

Given the evidential variables of components' degradation model parameters, the evidential variable of the component reliability can be assessed via (9), (10), (14), and (15), and the results at the given time instant $t = 100$ hours are presented in Fig. 11. As shown in Fig. 11, the evidential variables of units AD, GW, and WD have only one focal element because the evidential variables of the failure rates of these components are only interval-valued. For units FW and MO, the evidential variables of the component reliability have two focal elements with the mass function are exactly the same as these of the evidential variables of components' degradation model parameters. All the evidential variables of components' reliability at time instant $t = 100$ hours are given as follows:

$$\begin{aligned}
 m([R_{AD,1}(100)]) &= m([0.8025, 0.8187]) = 1; \\
 m([R_{FW,1}(100)]) &= m([0.7788, 0.8187]) = 0.5; \\
 m([R_{FW,2}(100)]) &= m([0.8187, 0.8607]) = 0.5; \\
 m([R_{MO,1}(100)]) &= m([0.9829, 0.9932]) = 0.3; \\
 m([R_{MO,2}(100)]) &= m([0.9845, 0.9945]) = 0.7; \\
 m([R_{GW,1}(100)]) &= m([0.7408, 0.7788]) = 1; \\
 m([R_{WD,1}(100)]) &= m([0.8106, 0.8353]) = 1.
 \end{aligned}$$

To assess the reliability-box of the rolling system, the EN model should be constructed first. The EN model of the rolling system is depicted in Fig. 12. As shown in Fig.12, X_1 and X_2 represent the two gravity wheels. X_3 , X_4 , X_5 , and X_6 represent the anti-deflection sensor, feeding wheel, motor, and weight disks, respectively. In accordance with the

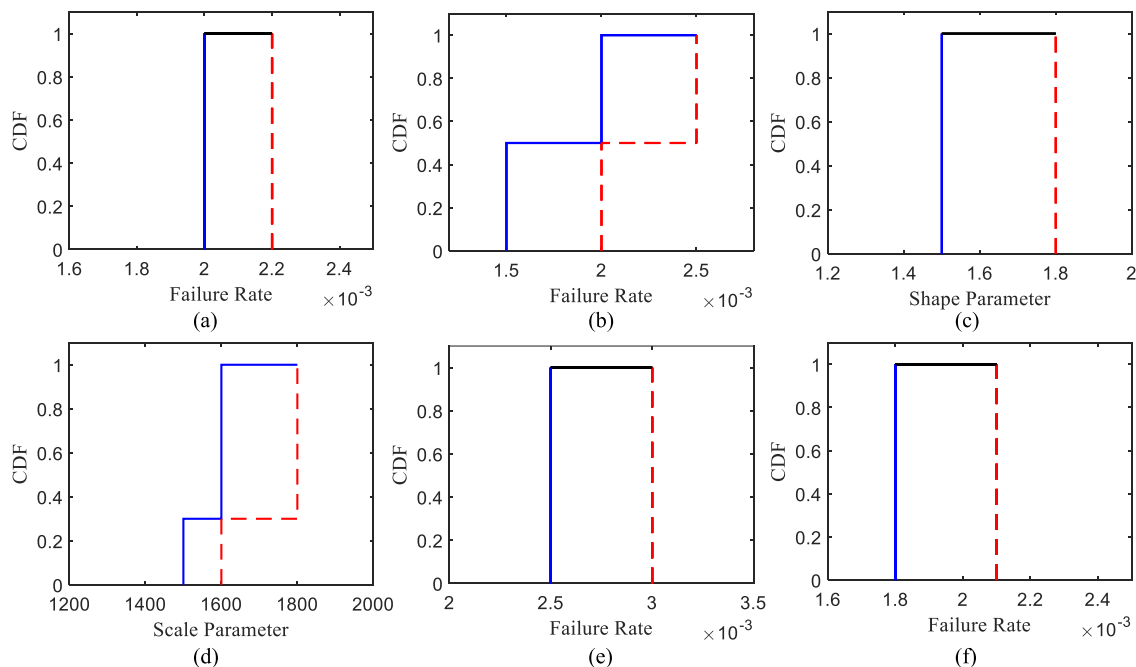


FIGURE 10. The evidential variables of all components' degradation model parameters: (a) failure rate of Unit AD; (b) failure rate of Unit FW; (c) Shape parameter of Unit MO; (d) Scale parameter of Unit MO; (e) Failure rate of Unit GW; (f) Failure rate of Unit WD.

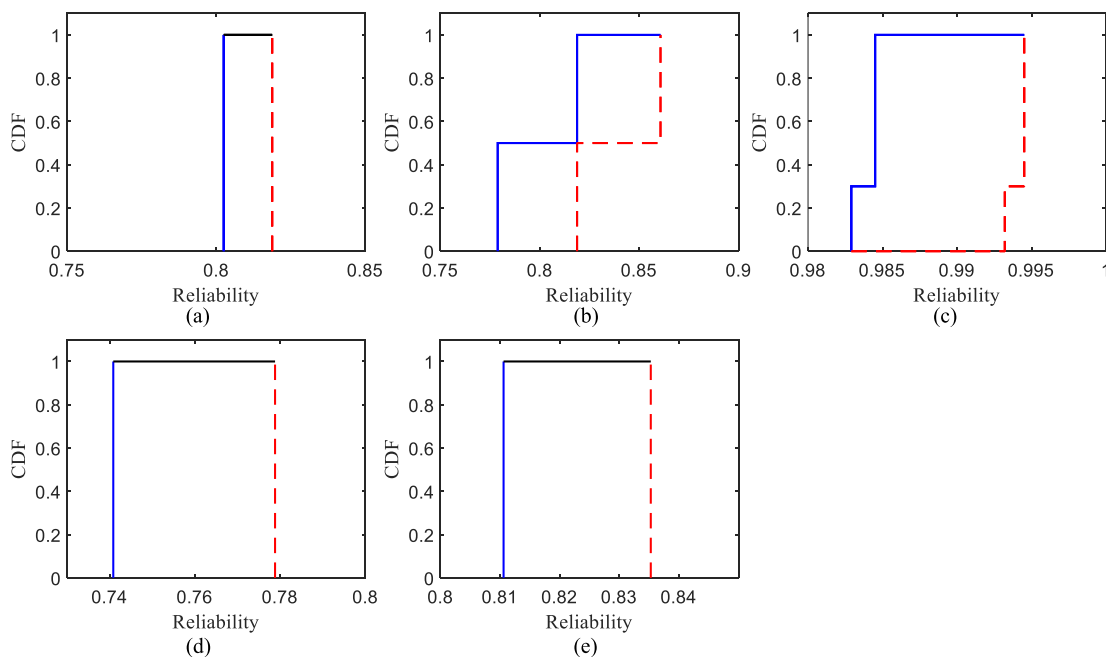


FIGURE 11. The evidential variables of all components' reliabilities at time instant $t = 100$ hours: (a) Unit AD; (b) Unit FW; (c) Unit MO; (d) Unit GW; (e) Unit WD.

functional dependency of the rolling system, the two gravity wheels are connected in parallel, and the failure dependency is described by an AND gate. The gravity wheels are connected with the anti-deflection sensor, feeding wheel, motor, and weight disks in a series configuration, the OR gate is used to manifest the failure dependency among these components.

$S_1, S_2, S_3,$ and S_4 represent the subsystems, and node S represents the entire system. All the subsystems are connected in series, and the failure dependency is described as an OR gate.

Based on the EN model and evidential variable of the component reliability, the mass functions of the root nodes, i.e., the mass functions of the component state can be assessed via

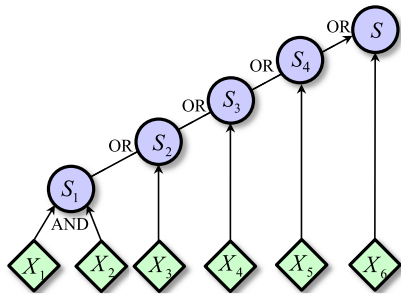


FIGURE 12. The EN model of the rolling system in the chip cutting detection module.

the interval-to-mass transformation:

$$\begin{aligned}
 [R_{AD,1L}(100), R_{AD,1U}(100)] &= [0.8025, 0.8187] \\
 &\rightarrow \{0.1813, 0.8025, 0.0162\} \\
 [R_{FW,1L}(100), R_{FW,1U}(100)] &= [0.7788, 0.8187] \\
 &\rightarrow \{0.1813, 0.7788, 0.0399\} \\
 [R_{FW,2L}(100), R_{FW,2U}(100)] &= [0.8187, 0.8607] \\
 &\rightarrow \{0.1393, 0.8187, 0.0420\} \\
 [R_{MO,1L}(100), R_{MO,1U}(100)] &= [0.9829, 0.9932] \\
 &\rightarrow \{0.0068, 0.9829, 0.0103\} \\
 [R_{MO,2L}(100), R_{MO,2U}(100)] &= [0.9845, 0.9945] \\
 &\rightarrow \{0.0055, 0.9845, 0.0100\} \\
 [R_{GW,1L}(100), R_{GW,1U}(100)] &= [0.7408, 0.7788] \\
 &\rightarrow \{0.2212, 0.7408, 0.0380\} \\
 [R_{WD,1L}(100), R_{WD,1U}(100)] &= [0.8106, 0.8353] \\
 &\rightarrow \{0.1647, 0.8106, 0.0247\}
 \end{aligned}$$

The system reliability bounds can be computed by (19) and (20), and the evidential variable of the system reliability can be assessed by (22), (23), and (24). For instance, given the focal elements and mass functions of the component reliability at time instant $t = 100$ hours, the evidential variable of the system reliability is given as follows:

$$\begin{aligned}
 m([R_{S,1}(100)]) &= m([0.4645, 0.5289]) = 0.15; \\
 m([R_{S,2}(100)]) &= m([0.4653, 0.5296]) = 0.35; \\
 m([R_{S,3}(100)]) &= m([0.4882, 0.5560]) = 0.15; \\
 m([R_{S,4}(100)]) &= m([0.4891, 0.5567]) = 0.35.
 \end{aligned}$$

The evidential variable of the system reliability can be visualized by the evidential variable to p-box transformation as given in (7) and (8), the result is depicted in Fig. 13. It is shown that the reliability-box of the rolling system of the chip cutting detection system has 4 focal elements, with each has an interval-valued system reliability. As given in Fig. 13, the rolling system has the lowest reliability value of 0.4645, whereas the highest reliability value is 0.5567 at time instant $t = 100$ hours.

The computational time of the proposed method can be compared with that of computing the focal elements of

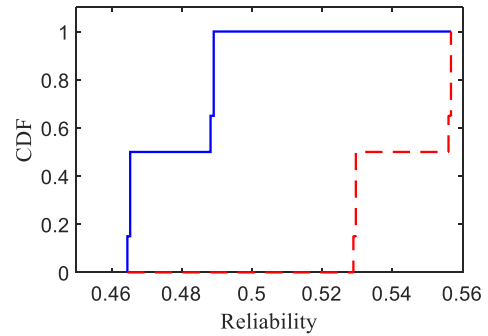


FIGURE 13. The reliability-box of the rolling system at time instant $t = 100$ hours.

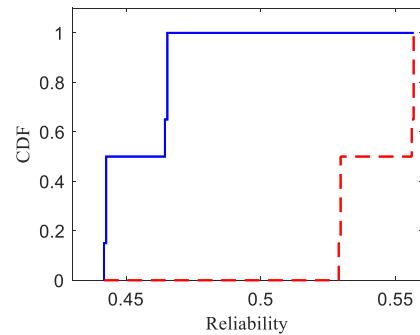


FIGURE 14. The reliability-box of the rolling system at time instant $t = 100$ hours with overlapping focal elements.

system reliability by the constrained optimization model, i.e., (16). In this case study, the computational time of the proposed method is 1.3948 seconds, while 40.2361 seconds for the constrained optimization model. It shows that the proposed evidential network model is more computationally efficient than the constrained optimization model.

If the focal elements of the evidential variable of FW is overlapping, say $\langle [1.5 \times 10^{-3}, 2.5 \times 10^{-3}], 0.5 \rangle; \langle [2.0 \times 10^{-3}, 3 \times 10^{-3}], 0.5 \rangle$, the corresponding reliability-box of the system can also be assessed, and the result is given in Fig.14. As shown in Fig.14, the upper bound CDF of the system reliability is the same as that in Fig.13, while the lowest reliability of the system is 0.4418.

VI. CONCLUSION AND FUTURE WORK

This article presented the reliability assessment of hierarchical systems under the epistemic uncertainty associated with the components' degradation model parameters. The evidential variable, represented by a set of possible intervals with its associated mass functions, was leveraged to quantify the epistemic uncertainty of the components' degradation model parameters. First, a constrained optimization model was constructed to assess the evidential variable of component reliability given the evidential variables of the degradation model parameters. Then, we introduce the EN model to construct the causal dependency between the system reliability and component reliability, and the reliability bound of the system

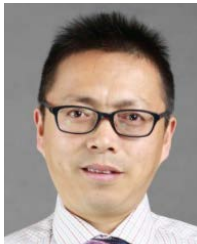
reliability was calculated by utilizing the belief and plausibility functions on the leaf nodes of the EN model. Finally, the mass function associated with each focal element of the system reliability, i.e., the system reliability bounds, was assessed by using the mass function of the evidential variable of the component reliability. The final system reliability-box was constructed via the evidential variable to p-box transformation. A rolling system in the chip cutting detection module was exemplified to demonstrate the effectiveness of the proposed methods. The results showed that the proposed method can accurately assess the system reliability-box with the given evidential variables of components' degradation model parameters.

In this article, the evidential variable was only used to quantify the epistemic uncertainty of components' degradation model parameters, it can be also used to quantify other types of reliability-related information/knowledge, such as the MTTF of components, components' failure probability, and sojourning time of components' states. However, how to integrate all these types of information under the evidential variable framework still remains unsolved. Therefore, one of our future works is to assess the system reliability-box by fusing the multiple sources of information/knowledge under the evidential variable framework. Furthermore, the studied system in our work is binary-state, our future work is to extend the proposed method under the multi-state systems [33], [34], [35]. The curse of dimension of the number of focal elements should be carefully treated in the context of multi-state systems. Some approximate algorithms [36], [37] can be used to approximate the mass functions and the focal elements of the evidential variables, and therefore, the system reliability-box can be assessed in a computationally efficient fashion. Lastly, the reliability-related decision-makings, such as inspections and maintenance planning under the evidential variable framework, are still worth exploring. Note that as we have the lower and upper bounds of the system reliability results, robust optimization is a promising way to address decision-makings under epistemic uncertainty.

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