IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received 13 August 2022, accepted 5 September 2022, date of publication 12 September 2022, date of current version 19 September 2022.
Digital Object Identifier 10.1109/ACCESS.2022.3205728

RESEARCH ARTICLE

A Novel Multiobjective Game IDEA Cross-Efficiency Method Based on Boolean Possibility Degree for Ranking Biomass Materials With Interval Data

NARONG WICHAPA AND SUPHAN SODSOON
Department of Industrial Engineering, Faculty of Engineering and Industrial Technology, Kalasin University, Kalasin 46000, Thailand
Corresponding author: Narong Wichapa (narong.wi@ksu.ac.th)

This work was supported by Kalasin University.

ABSTRACT The concept of processing biomass materials into charcoal briquettes is a viable solution for every developing nation's energy crisis. However, the important properties of each biomass material must first be considered to find suitable biomass materials for processing into charcoal briquettes. Sometimes these qualities are measured with imprecise values, making it exceedingly challenging to rank biomass materials' decision-making units (DMUs). This problem is one of the interval data envelopment analysis (IDEA) ranking issues that make it difficult to calculate and rank all DMUs. In this paper, the concepts of the Game IDEA cross-efficiency method and Boolean possibility degree were utilized to solve the IDEA ranking problems. Unlike existing IDEA ranking models, a new multi-objective Game IDEA cross-efficiency (MO-G-IDEA-CE) method was used to obtain the Game interval cross-efficiency (GICE) scores of each DMU simultaneously. After that, the Boolean possibility degree was used to transform GICE scores into crisp values for ranking all DMUs. Three numerical examples, including a simple numerical example of China's primary schools and seven biomass materials problems, are provided to demonstrate and validate the effectiveness of the proposed model. For the case study of seven biomass materials, after the Spearman correlation test, the correlation coefficients (r_s) for the proposed method and Wang's method, and Wu *et al.*'s method are calculated as $r_s = 1.000$ and 0.964, respectively. In addition, it is worth noting that the proposed MO-G-IDEA-CE method has a very high correlation with the other ranking methods for all three numerical examples.

INDEX TERMS Multi-objective game cross-efficiency method, Boolean possibility degree, biomass, interval data envelopment analysis.

I. INTRODUCTION

Energy is essential to every nation's sustainable development and quality of life. Urbanization and industrialization in modern cities are connected to high energy demand. Currently, fossil fuels are employed to supply the growing need for energy. However, the availability of fossil fuels continues to decline. There is a significant increase in the price of conventional fuels, and the combustion of these fuels generates air pollution, such as unburned carbon, oxides of nitrogen, and sulfur. Therefore, an alternative fuel is required to address these issues. Renewable energy sources are one of the most effective solutions to these issues. Biomass energy is one of the cheapest and most accessible kinds of renewable energy that may be produced from decomposing organic waste. Biomass is a fuel obtained from organic waste products. It is a renewable and sustainable energy source that can produce electricity or other forms of energy, like heat energy.

Forest debris, scrap lumber, manure, some crops, and agricultural waste leftovers are all examples that can be used to create biomass fuels. As biomass is abundantly available, renewable, and environmentally beneficial, it is

The associate editor coordinating the review of this manuscript and approving it for publication was Jamshid Aghaei.

gaining increasing attention [1]. The concept of processing biomass materials into charcoal briquettes is one good idea for resolving the energy shortage problem and is found almost everywhere, particularly in agricultural nations. Agricultural resources can be used to manufacture biomass briquettes for cooking and heating in impoverished nations. Thailand is one of the farming countries in Southeast Asia with an abundance of unexploited biomass [2], [3]. Several policies and regulations are implemented to encourage and promote the research sector's pursuit of renewable energy sources. The government encourages and supports processing agricultural waste into charcoal briquettes for domestic cooking as one of its most critical issues. To find suitable materials for processing into charcoal briquettes, it is necessary first to analyze the essential properties of each category of agricultural waste.

The choice of agricultural biomass influences the quality of the manufactured fuel briquettes. In contrast, the selection of residues for briquette production depends on their properties (low moisture, low ash content, high calorific value, high density, and medium fixed carbon) [4], [5]. Therefore, these properties must be considered when selecting biomass sources for manufacturing charcoal briquettes. This is, therefore, one of the multi-criteria decision-making problems (MCDM problems) that all of these criteria must be considered concurrently. Furthermore, selecting suitable biomass materials requires identifying effective and reliable methods to measure and rank the biomass materials, which will provide helpful information for further use. In addition, if the values of biomass attributes are ambiguous or imprecise, measuring and ranking biomass materials becomes more complicated.

The two most well-known operations research and management science approaches are data envelopment analysis (DEA) and MCDM. These two approaches are interrelated and can be used to solve MCDM problems [6]. Hwang and Yoon [7] classified the MCDM processes as Multiple Attribute Decision-making (MADM) and Multiple Objective Decision-making (MODM).

MADM is utilized for assessing discrete variables. Experts participate in the initial phase of the process by assigning weights to the criteria for evaluating alternatives. MODM enables the acquisition of a continuous collection of solutions for two or more criteria, known as the Pareto front. Many methods can be used for MADM problems. Nevertheless, the classic MADM approaches can be divided into distinct groups based on their shared characteristics [8]: (I) Scoring methods such as Simple additive weighting (SAW) and Complex proportional assessment (COPRAS); (II) Distance-based approaches such as Goal programming (GP), Compromise programming (CP), Technique for order of preference by similarity to ideal solution (TOPSIS), Multi-criteria optimization and compromise solution (VIKOR) and Data envelopment analysis (DEA); (III) Pairwise comparison methods such as Analytic hierarchy process (AHP), Analytic network process (ANP) and Measuring Attractiveness by a Categorical Based Evaluation Technique (MACBETH); (IV) Outranking methods such as the Preference ranking organization method for enrichment of evaluations (PROMETHEE) and Elimination and choice expressing reality (ELECTRE); (V) Utility/Evaluate methods such as Multi-attribute utility theory (MAUT) and Multi-attribute value theory (MAVT); and (VI) others such as Quality function development (QFD). DEA is a distance-based approach that is widely used in solving MADM problems. In the decision matrix, criteria can be viewed as inputs and outputs for DEA, and alternatives can be considered decision-making units (DMUs). The DEA can be regarded as one of the MCDM tools because it can be used to generate optimal weights of each criterion for ranking alternatives/DMUs. Recently, the DEA is still being used as an MCDM tool in various fields, such as the case of irrigation management [9], the supplier's selection [10], and the solar PV power plant site selection [11]. According to the relevant literature review [12], despite the wide range of applications of the DEA concept in renewable energy applications, there has been no research using the multi-objective Game interval data envelopment analysis (MO-G-IDEA-CE) approach for evaluating charcoal briquettes.

DEA is a popular mathematical method used to measure the performance of a set of DMUs with multiple inputs and outputs. The relative efficiency of each DMU can be obtained by calculating the ratio of the weighted sum of outputs to the weighted sum of inputs. If a DMU has a relative efficiency score of 1, it is defined as efficient. Otherwise, it is specified as inefficient. Over the past several decades, various forms of DEAs, the Charnes-Cooper-Rhodes (CCR) model [13] and Banker-Charnes-Cooper (BCC) model [14], have been used in a wide range of fields, such as banking [15], engineering [16], education [17], agriculture [18], and corporate administration [19]. The main advantages of DEA are that it does not require any possible assumptions related to the structure of the production function, and the values of inputs and outputs can have different measurement units [20], [21], [22]. The traditional DEAs can estimate the relative efficiencies of DMUs with precise values of inputs and outputs. If the values of the inputs or outputs of DMUs are imprecise, such as interval data, the existing DEAs fail to measure the performance of the DMUs. Hence, many researchers [23], [24], [25] have offered various Interval Data Envelopment Analysis models (IDEA models) to solve this weak point. Cooper *et al.* [26] first offered the IDEA model to measure the performance of a set of DMUs with inaccurate data. Subsequently, this theoretical approach has contributed to further development by a group of scholars. Despotis and Smirlis [27] converted the DEA-CCR model into the IDEA –CCR model to solve DMUs with interval data, and outcomes were obtained as the lower and upper values of efficiency scores. Entani *et al.* [28] offered a pair of IDEA models, called the optimistic IDEA and pessimistic IDEA models, to solve IDEA ranking problems for DMUs with interval data of inputs and outputs. However, Wang *et al.* [29] noted that Despotis and Smirlis' model [27] employed two different production frontiers to calculate the efficiencies of DMUs, which may result in the incomparability of DMU efficiencies.

To address this issue, Wang *et al.* [29] suggested new IDEA models based on a common frontier to determine each DMU's interval efficiency. A minimax regret-based technique was used to rank the interval efficiencies of each DMU.

Wang *et al.* [30] presented a virtual anti-ideal DMU into an IDEA model to combine optimistic and pessimistic viewpoints. Later, Azizi and Jahed [31] pointed out a disadvantage of Wang's model [28] when determining the range of interval efficiency of each DMU, and they offered a pair of improved IDEA models to overcome this disadvantage. Toloo *et al.* [32] developed the pessimistic IDEA and optimistic IDEA models for identifying the specific states of each imprecise dual role factor. Sun *et al.* [33] offered alternative IDEA models with common weights for evaluating and ranking all DMUs. Wu *et al.* [34] presented an IDEA cross-efficiency model based on secondary goals and a new TOPSIS to evaluate and rank DMUs with interval data. Wang *et al.* [35] proposed a cross-efficiency IDEA model based on entropy for assessing and ranking DMUs with interval data. In this model, interval cross-efficiency values are firstly generated. After that, an entropy formulation is used to obtain the criteria weights of interval efficiency. Finally, the relative Euclidean distance from the positive solution is used to rank all DMUs.

Although there are several DEA cross-efficiency methods, the Game cross-efficiency method presented by Liang *et al.* [36] is one of the most popular and effective techniques for solving DEA ranking problems. Liang *et al.* [36] demonstrated the existence of equivalence between the value of the game cross-efficiency and Nash equilibrium for the game with a specific continuous concave payoff. This approach can produce a unique efficiency value in a pair-wise game between competing DMUs, without affecting the efficiency of other DMUs. Based on the concept of game cross efficiency, this method is widely accepted and utilized in numerous applications, such as the supplier selection problem [37], urban public infrastructure investment [38], survey of ecological efficiency of the area [39], energy efficiency [40] and land utilization efficiency [41]. Therefore, this research is worth extending the concepts of a cross-efficiency method to the Game IDEA cross-efficiency concepts for finding other effective ways to rank charcoal briquettes with interval data.

There are many ranking methods for interval numbers. However, the possibility degree method is a popular method for ranking them. The principle of the possibility degree method can be described as follows. Let a and b be two interval numbers; the possibility degree of $a \geq b$ is defined as $p\ (a \geq b)$. The higher value of $p\ (a \geq b)$ means that the possibility degree of a over b is a greater value [42]. Nakahara *et al.* [43] and Nakahara [44] first developed the possibility degree, and the formula was utilized to tackle a fuzzy mathematical model. Facchinetti *et al.* [45] proposed the alternative possibility degree formula for comparing two fuzzy triangular numbers. Wang *et al.* [46] offered new possibility degree formulae for generating weights from interval comparison matrices. Li *et al.* [47] proposed a new possibility degree formula that is easy to use but powerful. They also presented the Boolean matrix to overcome previous studies' disadvantages for ranking interval numbers. Based on this ranking method [47], this paper should use the possibility degree formula and Boolean matrix to rank DMUs with interval efficiency.

The main contributions of this research are in the following ways:

1) Based on the idea of Wang *et al.* [29], we formulated a new multi-objective interval data envelopment analysis CCR model (MO-IDEA-CCR) for measuring the interval efficiencies of DMUs with interval data.

2) Based on the idea of Liang *et al.* [36], we formulate a new multi-objective Game IDEA cross-efficiency method (MO-G-IDEA-CE) to evaluate the Game interval cross-efficiency (GICE) scores of each DMU.

3) Based on the idea of Li *et al.* [47], we also apply a Boolean possibility degree formula, a combination of the possibility degree formula and Boolean matrix, to transform the lower and upper values of GICE scores into Boolean possibility degree scores for ranking all DMUs, which is simple but effective.

4) We apply the proposed method to a real case of the fuel briquette problem; this will be extremely valuable for study in this field in most, especially farming, nations.

The rest of this paper is structured as follows: In Section II, we first offer existing IDEA-CCR models, IDEA cross-efficiency method, and IDEA cross-efficiency method based on secondary goals. Next, Section III provides a new solution for measuring and ranking DMUs with interval data. Then three examples, a simple numerical example, China's primary schools, and a biomass materials problem, are provided to illustrate the idea proposed in Section IV. Finally, Section V is the conclusion.

II. BACKGROUND

A. IDEA-CCR MODELS

Consider a set of n observed DMUs to be measured, DMUs: $\{DMU_j\ /\ j = 1, 2, 3, \ldots, n\}$. Each DMU_j consumes m different inputs and produces s different outputs, denoted as y_{rj} and x_{ij}, respectively. Due to the ambiguity, only their bounding intervals $[x^l_{ij}, x^u_{ij}]$ and $[y^l_{rj}, y^u_{rj}]$, with $x^l_{ij} > 0$ and $y^l_{rj} > 0$, are identified. To solve the IDEA problem, Wang *et al.* [29] offered two linear programming models to obtain the bounded interval efficiency $[E^l_{kk}, E^u_{kk}]$, as follows:

$$\begin{aligned} &Max\ E^l_{kk} = \sum_{r=1}^{s} u_{rk} \cdot y^l_{rk} \\ &s.t.\ \sum_{r=1}^{s} u_{rk} \cdot y^u_{rk} - \sum_{i=1}^{m} v_{ik} \cdot x^l_{ij} \leq 0, \quad \forall j,\ j = 1, 2, \ldots, n \\ &\sum_{i=1}^{m} v_{ik} \cdot x^u_{ik} = 1, \quad \forall k,\ k = 1, 2, \ldots, n \\ &v_{ik},\ u_{rk} \geq 0, \quad \forall i, \forall r, \forall k \end{aligned} \tag{1}$$

$$
\begin{aligned}
&Max\ E_{kk}^{u} = \sum_{r=1}^{s} u_{rk} \cdot y_{rk}^{u} \\
&s.t.\ \sum_{r=1}^{s} u_{rk} \cdot y_{rk}^{u} - \sum_{i=1}^{m} v_{ik} \cdot x_{ij}^{l} \leq 0, \quad \forall j,\ j = 1,\ 2,\ \ldots,\ n \\
&\quad \sum_{i=1}^{m} v_{ik} \cdot x_{ik}^{l} = 1 \\
&\quad v_{ik},\ u_{rk} \geq 0, \quad \forall i,\ \forall r,\ \forall k
\end{aligned} \tag{2}
$$

In Equations (1) and (2), DMU_k is to be measured. Let v_{ik} and u_{rk} be the weights of the input i and output r, respectively. Then, E_{kk}^{l} and E_{kk}^{u} are the lower and upper efficiencies for each DMU_k, respectively. In the above two models, it is clear that DMU_k can be defined as an efficient DMU if its optimal solution is $E_{kk}^{u} = 1$, or it is inefficient if $E_{kk}^{u} < 1$.

B. IDEA CROSS-EFFICIENCY METHODS

After solving the IDEA-CCR models in Equation (1) and Equation (2), let u_{rd}^{l*} and u_{rd}^{u*} be the lower and upper bound of the optimal output weights for a specific DMU_k, respectively. If v_{id}^{l*} and v_{id}^{u*} are the lower and upper bound of optimal input weights for a particular DMU_k, respectively, then the small cross-efficiency scores of each DMU_j peer-evaluated by DMU_k, are provided by

$$
E_{kj}^{l} = \frac{\sum_{r=1}^{s} u_{rk}^{l*} y_{rj}^{l}}{\sum_{i=1}^{m} v_{ik}^{l*} x_{ij}^{u}}, \quad k, j = 1, 2, \ldots, n \tag{3}
$$

As a result, the average cross-efficiency (ACE^{l}) score of DMU_j is defined as

$$
\bar{E}_{j}^{l} = \frac{1}{n} \sum_{k=1}^{n} E_{kj}^{l}, \quad k,\ j = 1, 2, \ldots, n \tag{4}
$$

Similarly, the values of large cross-efficiency can be defined as

$$
E_{kj}^{u} = \frac{\sum_{r=1}^{s} u_{rk}^{u*} y_{rj}^{u}}{\sum_{i=1}^{m} v_{ik}^{u*} x_{ij}^{l}}, \quad k, j = 1, 2, \ldots, n \tag{5}
$$

As a result, from Sexton *et al.* [38], the ACE^{u} of DMU_j can be defined as

$$
E_{kj}^{u} = \frac{\sum_{r=1}^{s} u_{rk}^{u*} y_{rj}^{u}}{\sum_{i=1}^{m} v_{ik}^{u*} x_{ij}^{l}}, \quad k, j = 1, 2, \ldots, n \tag{6}
$$

After computing all cross-efficiency scores, an interval cross-efficiency matrix (ICEM) can be generated according to Table 1. It is noted that the elements on the main diagonal are self-assessed limits that can be computed using IDEA-CCR models, Equation (1) and Equation (2).

TABLE 1. An interval cross-efficiency matrix (ICEM).

DMU_k	Target DMU_j			
	1	2	⋯	n
1	$[E_{11}^{l}, E_{11}^{u}]$	$[E_{12}^{l}, E_{12}^{u}]$	⋯	$[E_{1n}^{l}, E_{1n}^{u}]$
2	$[E_{21}^{l}, E_{21}^{u}]$	$[E_{22}^{l}, E_{22}^{u}]$	⋯	$[E_{2n}^{l}, E_{2n}^{u}]$
3	$[E_{31}^{l}, E_{31}^{u}]$	$[E_{32}^{l}, E_{32}^{u}]$	⋯	$[E_{3n}^{l}, E_{3n}^{u}]$
⋮	⋮	⋮	⋮	⋮
n	$[E_{n1}^{l}, E_{n1}^{u}]$	$[E_{n2}^{l}, E_{n2}^{u}]$	⋯	$[E_{nn}^{l}, E_{nn}^{u}]$

C. IDEA CROSS-EFFICIENCY METHOD BASED ON SECONDARY GOALS

To enhance its efficiency ratio, the DMU under assessment considers the inputs and outputs of a few favorable DMUs, while ignoring the rest. In addition, optimal weights determined with models (1) and (2) are not typically unique. As a result, the calculating software may give varying ideal weights, rendering the cross-efficiency scores arbitrary. To address this deficiency, an interval cross-efficiency evaluation method is employed. DEA's cross-efficiency process uses peer evaluation rather than self-evaluation. It can define the cross-efficiency ratings of DMUs based on their interval [29]. Some choices of weights in the traditional cross-efficiency approach may result in a lower cross-efficiency for some DMUs and a higher cross-efficiency for others. To alleviate the ambiguity, a secondary goal function is introduced. Model (7), proposed by Wu *et al.* [34], can calculate the values of small cross-efficiency for interval data.

$$
\begin{aligned}
&Max\ E_{kk}^{l} = \sum_{r=1}^{s} u_{rk}^{l} \cdot y_{rk}^{l} \\
&s.t.\ \sum_{r=1}^{s} u_{rk}^{l} \cdot y_{rk}^{u} - \sum_{i=1}^{m} v_{ik}^{l} \cdot x_{ij}^{l} \leq 0, \quad \forall j,\ j = 1,\ 2,\ \ldots,\ n \\
&\quad \sum_{i=1}^{m} v_{ik}^{l} \cdot x_{ik}^{u} = 1, \quad \forall k,\ k = 1,\ 2,\ \ldots,\ n \\
&\quad \sum_{r=1}^{s} u_{rk}^{l} \cdot y_{rk}^{l} = E_{kk}^{l} \sum_{i=1}^{m} v_{ik}^{l} \cdot x_{ij}^{u}, \quad \forall j,\ j = 1,\ 2,\ \ldots,\ n \\
&\quad v_{ik}^{l},\ u_{ik}^{l} \geq 0, \quad \forall i,\ \forall r,\ \forall k
\end{aligned} \tag{7}
$$

Similarly, the values of large cross-efficiency can be calculated using model (8).

$$
\begin{aligned}
&Max\ E_{kk}^{u} = \sum_{r=1}^{s} u_{rk}^{u} \cdot y_{rk}^{u} \\
&s.t.\ \sum_{r=1}^{s} u_{rk}^{u} \cdot y_{rk}^{u} - \sum_{i=1}^{m} v_{ik}^{u} \cdot x_{ij}^{l} \leq 0, \quad \forall j,\ j = 1,\ 2,\ \ldots,\ n \\
&\quad \sum_{i=1}^{m} v_{ik}^{u} \cdot x_{ik}^{l} = 1, \quad \forall k,\ k = 1,\ 2,\ \ldots,\ n \\
&\quad \sum_{r=1}^{s} u_{rk}^{u} \cdot y_{rk}^{u} = E_{kk}^{u} \sum_{i=1}^{m} v_{ik}^{u} \cdot x_{ij}^{l}, \quad \forall j,\ j = 1,\ 2,\ \ldots,\ n \\
&\quad v_{ik}^{u},\ u_{ik}^{u} \geq 0, \quad \forall i,\ \forall r,\ \forall k
\end{aligned} \tag{8}
$$

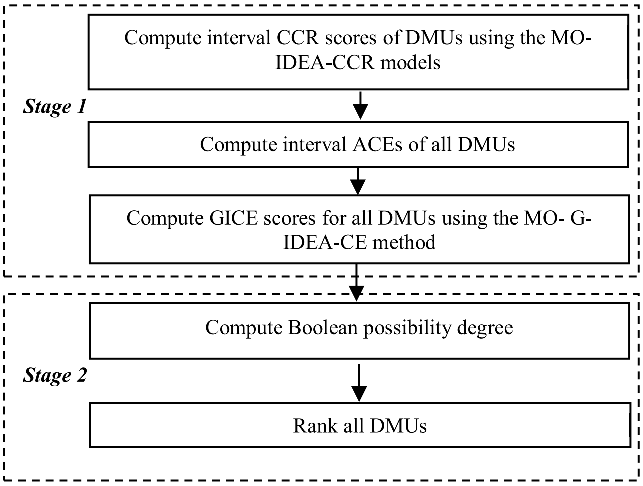

FIGURE 1. The proposed framework.

After solving Equation (7) to Equation (8), ACE scores of each DMU can be generated according to Equations (3) to (6).

III. THE PROPOSED METHOD

This section offers a new multi-objective Game DEA-CE method (MO-G-IDEA-CE method) based on the possibility degree of the Gibbs interval entropy model for ranking a group of homogeneous DMUs with interval data. The framework proposed in this paper appears in Fig.1.

A. MULTI-OBJECTIVE IDEA-CCR MODEL

In this section, we provide the multi-objective IDEA-CCR model (MO-IDEA-CCR model) based on the combination of Equation (1) and Equation (2), with added constraints of $v_{ik}^{u} \geq v_{ik}^{l}$, $u_{rk}^{u} \geq u_{rk}^{l}$, $\forall i, \forall r, \forall k$, for obtaining interval CCR scores, $\left[E_{kk}^{l}, E_{kk}^{u}\right]$, as shown in Equation (9).

$$\begin{aligned}
Max &= E_{kk}^{l} + E_{kk}^{u} \\
&= \sum_{r=1}^{s} u_{rk}^{l} \cdot y_{rk}^{l} + \sum_{r=1}^{s} u_{rk}^{u} \cdot y_{rk}^{u} \\
s.t.\ & \sum_{r=1}^{s} u_{rk}^{l} \cdot y_{rk}^{u} - \sum_{i=1}^{m} v_{ik}^{l} \cdot x_{ij}^{l} \leq 0, \quad \forall j,\ j = 1, 2, \ldots, n \\
& \sum_{r=1}^{s} u_{rk}^{u} \cdot y_{rk}^{u} - \sum_{i=1}^{m} v_{ik}^{u} \cdot x_{ij}^{l} \leq 0, \quad \forall j,\ j = 1, 2, \ldots, n \\
& \sum_{i=1}^{m} v_{ik}^{l} \cdot x_{ik}^{u} = 1, \quad \forall k,\ k = 1, 2, \ldots, n \\
& \sum_{i=1}^{m} v_{ik}^{u} \cdot x_{ik}^{l} = 1, \quad \forall k,\ k = 1, 2, \ldots, n \\
& v_{ik}^{u} \geq v_{ik}^{l}, \quad u_{rk}^{u} \geq u_{rk}^{l}, \ \forall i,\ \forall r, \forall k \\
& v_{ik}^{l}, u_{rk}^{l}, v_{ik}^{u}, u_{rk}^{u} \geq 0, \quad \forall i,\ \forall r,\ \forall k
\end{aligned} \tag{9}$$

B. MULTI-OBJECTIVE GAME IDEA CROSS-EFFICIENCY METHOD

In this section, we formulate the multi-objective Game IDEA cross-efficiency method (MO-G-IDEA) for ranking all DMUs with interval data. Based on the concept of a traditional Game cross-efficiency method presented by Liang *et al.* [36], the MO-G-IDEA method can be defined as

$$\begin{aligned}
Max &= E_{kk}^{l} + E_{kk}^{u} \\
&= \sum_{r=1}^{s} u_{rk}^{l} \cdot y_{rk}^{l} + \sum_{r=1}^{s} u_{rk}^{u} \cdot y_{rk}^{u} \\
s.t.\ & \sum_{r=1}^{s} u_{rk}^{l} \cdot y_{rk}^{u} - \sum_{i=1}^{m} v_{ik}^{l} \cdot x_{ij}^{l} \leq 0, \\
& \sum_{r=1}^{s} u_{rk}^{u} \cdot y_{rk}^{u} - \sum_{i=1}^{m} v_{ik}^{u} \cdot x_{ij}^{l} \leq 0, \quad \forall j,\ j = 1, 2, \ldots, n \\
& \sum_{i=1}^{m} v_{ik}^{l} \cdot x_{ik}^{u} = 1, \\
& \sum_{i=1}^{m} v_{ik}^{u} \cdot x_{ik}^{l} = 1, \quad \forall k,\ k = 1, 2, \ldots, n \\
& \alpha_{d}^{l} \times \sum_{i=1}^{m} v_{ij}^{ld} x_{id}^{u} - \sum_{r=1}^{s} u_{rj}^{ld} y_{rd}^{l} \leq 0 \\
& \alpha_{d}^{u} \times \sum_{i=1}^{m} v_{ij}^{ud} x_{id}^{l} - \sum_{r=1}^{s} u_{rj}^{ud} y_{rd}^{u} \leq 0 \\
& v_{ik}^{u} \geq v_{ik}^{l}, \quad u_{rk}^{u} \geq u_{rk}^{l}, \ \forall i,\ \forall r,\ \forall k \\
& v_{ik}^{l}, u_{rk}^{l}, v_{ik}^{u}, u_{rk}^{u} \geq 0, \quad \forall i,\ \forall r,\ \forall k
\end{aligned} \tag{10}$$

If $\left[\alpha_{j}^{l}, \alpha_{j}^{u}\right] = \left[\frac{1}{n}\sum_{d=1}^{n}\sum_{r=1}^{s} u_{rj}^{ld} y_{rj}^{l}, \frac{1}{n}\sum_{d=1}^{n}\sum_{r=1}^{s} u_{rj}^{ud} y_{rj}^{u}\right]$, then $\left[\alpha_{j}^{l}, \alpha_{j}^{u}\right]$ is called the Game interval cross-efficiency (GICE) score of DMU$_j$ ($j = 1, 2, \ldots, n$). Based on Equation (10), the iteration algorithm leading to Nash-equilibrium is:

Step I: To obtain the GICE scores, the MO-IDEA-CCR model in Equation (9) must be computed first. For each DMU$_j$, let $t = 1$ and $\left[\alpha_{d}^{l}, \alpha_{d}^{u}\right] = \left[\alpha_{d}^{l1}, \alpha_{d}^{u1}\right] = \left[\bar{E}_{d}^{l}, \bar{E}_{d}^{u}\right]$.

Step II: Solve Equation (10). Let

$$\left[\alpha_{j}^{l2}, \alpha_{j}^{u2}\right] = \left[\frac{1}{n}\sum_{d=1}^{n}\sum_{r=1}^{s} u_{rj}^{ld^*}(\alpha_{d}^{l1}) y_{rj}^{l}, \frac{1}{n}\sum_{d=1}^{n}\sum_{r=1}^{s} u_{rj}^{ud^*}(\alpha_{d}^{u1}) y_{rj}^{u}\right]$$

or

$$\left[\alpha_{j}^{lt+1}, \alpha_{j}^{ut+1}\right] = \left[\frac{1}{n}\sum_{d=1}^{n}\sum_{r=1}^{s} u_{rj}^{ld^*}(\alpha_{d}^{lt+1}) y_{rj}^{l}, \frac{1}{n}\sum_{d=1}^{n}\sum_{r=1}^{s} u_{rj}^{ud^*}(\alpha_{d}^{ut+1}) y_{rj}^{u}\right]$$

where $\left[u_{rj}^{ld^*}(\alpha_d^{lt}), u_{rj}^{ud^*}(\alpha_d^{ut})\right]$ represents an optimal solution of $\left[u_{rj}^{ld}, u_{rj}^{ud}\right]$ when $\left[\alpha_d^l, \alpha_d^u\right] = \left[\alpha_d^{lt}, \alpha_d^{ut}\right]$.

Step III: If $\left|\alpha_j^{lt+1} - \alpha_j^{lt}\right| \geq \varepsilon$ and $\left|\alpha_j^{ut+1} - \alpha_j^{ut}\right| \geq \varepsilon$ for some j, where ε is a specified small positive value, then let $\left[\alpha_d^l, \alpha_d^u\right] = \left[\alpha_j^{lt+1}, \alpha_j^{ut+1}\right]$ and go to step II. If $\left|\alpha_j^{lt+1} - \alpha_j^{lt}\right| < \varepsilon$ and $\left|\alpha_j^{ut+1} - \alpha_j^{ut}\right| < \varepsilon$ for all j, then stop. $\left[\alpha_j^{lt+1}, \alpha_j^{ut+1}\right]$ is the GICE score given to DMU$_j$.

C. BOOLEAN POSSIBILITY DEGREE

Based on the solutions of the MO-G-IDEA method, GICE scores of all DMUs can be utilized for ranking all DMUs using the Boolean possibility degree. Li *et al.* [47] offered a ranking method of interval numbers based on the Boolean matrix. A higher Boolean possibility degree value means a better DMU ranking. The possibility degree formula can be described as follows. Let $\otimes G_1 = \left[x_1^l, x_1^u\right]$ and $\otimes G_2 = \left[x_2^l, x_2^u\right]$. Then the possibility degree of $\otimes G_1 \geq \otimes G_2$ is determined by:

$$p(\otimes G_1 \geq \otimes G_2) = 0.50\left[1 + \frac{(x_1^u - x_2^u) + (x_1^l - x_2^l)}{\left|(x_1^u - x_2^u)\right| + \left|(x_1^l - x_2^l)\right| + l_{\otimes G_1 \otimes G_2}}\right] \tag{11}$$

where

$$l_{\otimes G_1 \otimes G_2} = \min\left\{\begin{array}{l}\max\left\{x_1^u - \max\left\{x_1^l, x_2^l\right\}, 0\right\}, \\ \max\left\{x_2^u - \max\left\{x_1^l, x_2^l\right\}, 0\right\}\end{array}\right\},$$

$l_{\otimes G_1 \otimes G_2}$ is the length of $\otimes G_1 \cap \otimes G_2$. If $\otimes G_1 \cap \otimes G_2 = \phi$, then $l_{\otimes G_1 \otimes G_2} = 0$. In the following, the Boolean possibility degree was proposed to rank a set of grey numbers. For grey numbers $\otimes G_1, \otimes G_2, \ldots, \otimes G_n$, the ranking algorithm can be described as follows.

Comparing any two grey numbers$\otimes G_i$, $\otimes G_j$, we generate $p_{ij} = p(\otimes G_i \geq \otimes G_j)$ and establish the possibility degree matrix $P = (p_{ij})_{n\times n}$, $i, j = 1, 2, .., n$. Establish the Boolean matrix $Q = (q_{ij})_{n\times n}$, where

$$q_{ij} = \begin{cases} 1, & p_{ij} \geq 0.50 \\ 0, & p_{ij} < 0.50 \end{cases}$$

Q is the ranking matrix of the grey numbers. Let $\lambda_i' = \sum_{j=1}^{n} q_{ij}$, we have the ranking vector $\lambda' = (\lambda_1', \lambda_2', \ldots, \lambda_n')$.

Rank the grey numbers based on the value λ_i'; a higher value λ_i' is better.

D. CHARCOAL BRIQUETTE MANUFACTURING PROCESS

This study aims to make compressed charcoal from seven biomass resources, including Bagasse, Incense reed, Water hyacinth, Rice husk, Coconut shell, Sawdust, and Sensitive plant, in the form of 100 percent weight charcoal briquettes. The manufacture of charcoal briquettes can be broken down into six steps: **Step I:** Collection of biomass materials. Agricultural biomass is collected, sorted, diced into smaller pieces, and then dried in the sun. **Step II:** Biomass carbonization. The collected biomass materials are burned in an oil drum. After burning, the carbonized biomass materials must be collected and weighed. **Step III:** Preparation of binder. The binding substance strengthens the charcoal briquettes. For every 10 kilograms of complete carbonized charcoal powder, combine 0.5 to 0.6 kilograms of starch or cassava flour with 5 to 10 liters of water to create a binder. **Step IV:** Mixing. Ensure that the binder is uniformly distributed throughout the carbonized charcoal's particles. It will enhance the adhesion of the charcoal and produce uniform briquettes. **Step V:** Briquetting. A briquetting machine is used to turn the charcoal mixture into charcoal briquettes. To manufacture briquettes of the same size, pour the mixture immediately into the briquetting machine. **Step VI:** Drying. Each charcoal briquette is air-dried outdoors. For the briquette quality test, important properties (moisture content, ash content, heating value, and fixed carbon) are analyzed using ASTM D3173, ASTM D3174, ASTM D5865, and ASTM D3172, in that order. These properties can be considered inputs and outputs of each charcoal briquette/DMU in terms of DEA. The selection of suitable biomass materials from agricultural products for processing into fuel briquettes is a complicated decision-making problem due to the multiple interval qualities that must be considered simultaneously. Consequently, the proposed approach is utilized to evaluate each biomass material.

TABLE 2. The data set of the simple numerical example.

DMU$_j$	Inputs		Outputs		Interval CCR
	$\otimes x_1$	$\otimes x_2$	$\otimes y_1$	$\otimes y_2$	
1	[1,2]	[2,3]	[23,24]	[22,24]	**[0.6389,1.0000]**
2	[2,3]	[3,4]	[20,22]	[20,21]	**[0.4167,0.6111]**
3	[3,4]	[5,6]	[18,21]	[19,19]	**[0.2500,0.3500]**
4	[3,4]	[5,7]	[16,17]	[15,18]	**[0.1905,0.2881]**
5	[3,5]	[5,7]	[14,17]	[13,15]	**[0.1667,0.2833]**
6	[4,5]	[6,7]	[10,15]	[10,14]	**[0.1190,0.2083]**

IV. NUMERICAL EXAMPLES

A. THE SIMPLE NUMERICAL EXAMPLE

Wu *et al.* [34] proposed a simple numerical example. There are six DMUs with two inputs and two outputs. The data set of the simple numerical example is shown in Table 2.

The calculation procedure is as follows. Firstly, by solving Equation (9) (MO-IDEA-CCR model), the interval CCR scores are obtained as in the last column of Table 2. After that, the optimal weights of inputs and outputs are utilized to calculate the interval ACE scores of each DMU, according to Equations (3) to (6). As a result, the interval ACE scores for each DMU are achieved, as shown in Table 3.

TABLE 3. Interval cross-efficiency scores for the simple numerical example.

DMU_j	Target DMU 1	2	3	4	5	6	Interval ACE
1	**[0.6389, 1.0000]**	[0.6389, 1.0000]	[0.6389, 1.0000]	[0.6389, 1.0000]	[0.6389, 1.0000]	[0.6389, 1.0000]	**[0.6389, 1.0000]**
2	[0.4167, 0.6111]	**[0.4167, 0.6111]**	[0.4167, 0.6111]	[0.4167, 0.6032]	[0.4167, 0.6111]	[0.4167, 0.6111]	**[0.4167, 0.6098]**
3	[0.2500, 0.3500]	[0.2500, 0.3500]	**[0.2500, 0.3500]**	[0.2500, 0.3405]	[0.2500, 0.3500]	[0.2500, 0.3500]	**[0.2500, 0.3484]**
4	[0.1905, 0.2833]	[0.1905, 0.2833]	[0.1905, 0.2833]	**[0.1905, 0.2881]**	[0.1905, 0.2833]	[0.1905, 0.2833]	**[0.1905, 0.2841]**
5	[0.1667, 0.2833]	[0.1667, 0.2833]	[0.1667, 0.2833]	[0.1667, 0.2738]	**[0.1667, 0.2833]**	[0.1667, 0.2833]	**[0.1667, 0.2817]**
6	[0.1190, 0.2083]	[0.1190, 0.2083]	[0.1190, 0.2083]	[0.1190, 0.2044]	[0.1190, 0.2083]	**[0.1190, 0.2083]**	**[0.1190, 0.2077]**

TABLE 4. The GICE scores of the simple numerical example.

DMU_j	Iteration 1	2	3	4	5	6	7*
1	[0.6343, 1.0000]	[0.6381, 1.0000]	[0.6362, 1.0000]	[0.6365, 1.0000]	[0.6363, 1.0000]	[0.6364, 1.0000]	**[0.6364, 1.0000]**
2	[0.4167, 0.6065]	[0.4167, 0.6103]	[0.4167, 0.6091]	[0.4167, 0.6095]	[0.4167, 0.6093]	[0.4167, 0.6094]	**[0.4167, 0.6094]**
3	[0.2523, 0.3444]	[0.2581, 0.3491]	[0.2571, 0.3475]	[0.2576, 0.3480]	[0.2575, 0.3479]	[0.2576, 0.3479]	**[0.2576, 0.3479]**
4	[0.1885, 0.2861]	[0.1901, 0.2907]	[0.1893, 0.2892]	[0.1895, 0.2897]	[0.1894, 0.2895]	[0.1894, 0.2896]	**[0.1894, 0.2896]**
5	[0.1647, 0.2778]	[0.1663, 0.2824]	[0.1655, 0.2809]	[0.1656, 0.2814]	[0.1656, 0.2812]	[0.1656, 0.2813]	**[0.1656, 0.2812]**
6	[0.1190, 0.2060]	[0.1190, 0.2079]	[0.1190, 0.2073]	[0.1190, 0.2075]	[0.1190, 0.2074]	[0.1190, 0.2075]	**[0.1190, 0.2075]**

Table 3 shows that the main diagonal elements are self-assessed limits computed using Equation (9) (MO-IDEA-CCR models). In this paper, the interval ACE score of arbitrary strategy is set as the initial solution for iteration 1, while ε is set to 0.001. Then, using Equation (10) through three ranking steps of the MO-G-IDEA-CE method in Section B, the GICE scores for all iterations are shown in Table 4.

Table 4 shows that the final GICE scores for all DMUs were achieved at Iteration 7. After obtaining the GICE scores of all DMUs, the possibility degree formula, Equation (11), was used to generate the possibility degree matrix, $P = (p_{ij})_{6\times 6}$. For example, the possibility degree score when $\otimes DMU_1 \geq \otimes DMU_2$ is

$$p(\otimes G_1 \geq \otimes G_2) = 0.50\left[1 + \frac{(x_1^u - x_2^u) + (x_1^l - x_2^l)}{\left|(x_1^u - x_2^u)\right| + \left|(x_1^l - x_2^l)\right| + l_{\otimes G_1 \otimes G_2}}\right]$$

$$p(\otimes DMU_1 \geq \otimes DMU_2) = 0.50\left[1 + \frac{(1.0000 - 0.6094) + (0.6364 - 0.4167)}{|(1.0000 - 0.6094)| + |(0.6364 - 0.4167)| + 0}\right] = 0.50\,[1 + 1] = 1.$$

Details of P are shown in Table 5.

TABLE 5. The possibility degree matrix (*P*) for the simple numerical example.

i	*j* = 1	2	3	4	5	6
1	0.500	1.000	1.000	1.000	1.000	1.000
2	0.000	0.500	1.000	1.000	1.000	1.000
3	0.000	0.000	0.500	0.899	0.935	1.000
4	0.000	0.000	0.109	0.500	0.630	0.947
5	0.000	0.000	0.065	0.370	0.500	0.871
6	0.000	0.000	0.000	0.053	0.129	0.500

TABLE 6. The Boolean matrix (q) for the simple numerical example.

i	*j* = 1	2	3	4	5	6	λ_i'	Rank
1	1	1	1	1	1	1	6	1
2	0	1	1	1	1	1	5	2
3	0	0	1	1	1	1	4	3
4	0	0	0	1	1	1	3	4
5	0	0	0	0	1	1	2	5
6	0	0	0	0	0	1	1	6

TABLE 7. Ranking comparisons of the proposed method and the other techniques for the simple numerical example.

DMU_j	IDEA-CCR [29]			Wu *et al.* [34]		Proposed method		
	Eff.	MRA	Rank	RED	Rank	GICE	λ_i'	Rank
1	[0.6389, 1.0000]	0.0000	1	6	1	[0.6364, 1.0000]	6	1
2	[0.4167, 0.6111]	0.5833	2	5	2	[0.4167, 0.6094]	5	2
3	[0.2639, 0.3500]	0.7361	3	4	3	[0.2576, 0.3479]	4	3
4	[0.1905, 0.3000]	0.8095	4	3	4	[0.1894, 0.2896]	3	4
5	[0.1667, 0.2833]	0.8333	5	2	5	[0.1656, 0.2812]	2	5
6	[0.1190, 0.2083]	0.8810	6	1	6	[0.1190, 0.2075]	1	6

After obtaining Table 5, the possibility degree matrix (P) was transformed into the Boolean matrix (Q). If the possibility degree score of DMU_i vs DMU_j (p_{ij}) ≥ 0.50, the Boolean value of DMU_i vs DMU_j (q_{ij}) $= 1$. Otherwise, $q_{ij} = 0$. Details of Q are shown in Table 6.

After that, the ranking vector (λ_i'), $\lambda_i' = \sum_{j=1}^{n} q_{ij}$, was calculated. For example, $\lambda_1' = 1 + 1 + 1 + 1 + 1 + 1 = 6$. After obtaining the λ', $\lambda' = \{6, 5, 4, 3, 2, 1\}$, DMUs were ranked in descending order following the value of the higher value of λ_i', so we have $DMU_1 \geq DMU_2 \geq DMU_3 \geq DMU_4 \geq DMU_5 \geq DMU_6$. In addition, ranking comparisons of the proposed method and the other methods for all DMUs, are shown in Table 7.

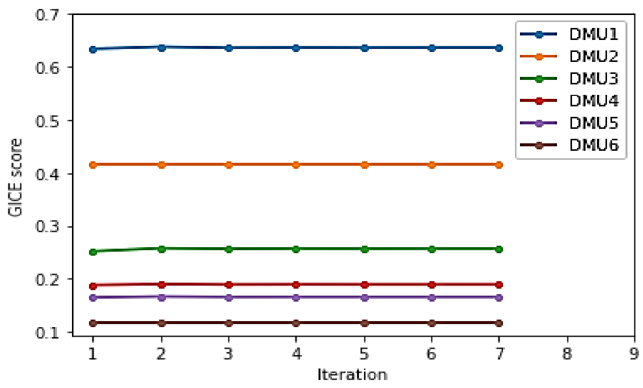

FIGURE 2. The Convergence procedure of the MO-G-IDEA-CE method for lower values of GICE scores.

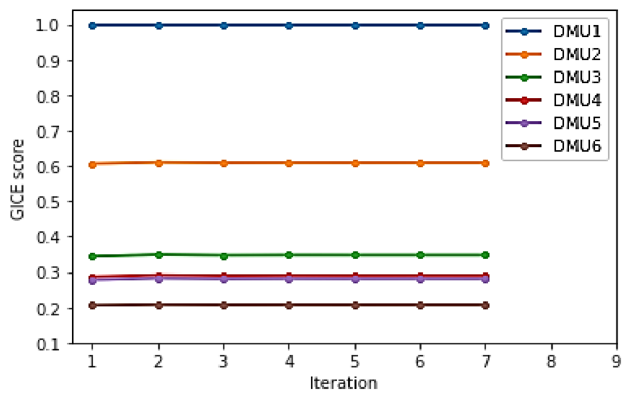

FIGURE 3. The Convergence procedure of the MO-G-IDEA-CE method for upper values of GICE scores.

Table 7 shows that the proposed method was compared with the IDEA-CCR model [29] and Wu *et al.*'s method [34]. Wang's model [29] proposed the minimax regret approach (MRA) for IDEA ranking problems. The lower value of the smallest maximum loss of efficiency means better ranking. Wu *et al.*'s method [34] used the distance model based on TOPSIS to generate the relative Euclidean distance (RED) from the positive solution. The lower value of the RED means better rankings. It is noticeable that all methods have the same ranking. After the Spearman correlation test, the correlation coefficients (r_s) for the proposed method and IDEA-CCR model [29] and Wu *et al.*'s method [34] are calculated as $r_s = 1.000$ and 1.000, respectively.

In addition, we demonstrate the Convergence procedure of the MO-G-IDEA-CE approach, which utilizes the relevant efficiency score from an arbitrary strategy. We set $\varepsilon = 0.001$, and Fig. 2 and Fig. 3 demonstrate that after seven iterations, all DMUs generated by the proposed method attain constant GICE scores, which means the optimal solutions are the Nash equilibrium points, as demonstrated in [36].

B. CHINA'S PRIMARY SCHOOLS

Wang *et al.* [35] proposed the data set of China's primary schools listed in Table 8. There are twenty-five primary schools (DMUs) with five inputs and one output. The number of staff ($\otimes x_1$), building area ($\otimes x_2$), copies of books ($\otimes x_3$), fixed assets ($\otimes x_4$), and budget ($\otimes x_5$) are inputs. The output variable ($\otimes y_1$) is the number of students in each school.

The calculation procedure is the same as in Section A. Firstly, by solving Equation (9), the interval CCR scores are obtained as in the last column of Table 8. After that, the interval ACE scores of each DMU are obtained according to Equations (3) to (6). As a result, the interval ACE scores for all DMUs are achieved as in the second column of Table 9.

TABLE 8. The data set of China's primary schools.

DMU_j	Inputs					Outputs	Interval CCR
	$\otimes x_1$	$\otimes x_2$	$\otimes x_3$	$\otimes x_4$	$\otimes x_5$	$\otimes y_1$	
1	[47, 53]	3964	8947	3.54	9.26	[313, 360]	[0.5038, 0.5794]
2	[39, 40]	965	4247	2.04	3.41	[102, 110]	[0.4119, 0.4442]
3	[65, 70]	2222	8543	2.23	12.07	[263, 300]	[0.5118, 0.5838]
4	[43, 54]	2316	7560	2.42	5.7	[261, 274]	[0.5503, 0.5777]
5	[47, 49]	3362	11,035	1.23	5.9	[292, 312]	[0.6296, 0.6727]
6	[49, 59]	3273	6120	5.61	8.53	[261, 289]	[0.5647, 0.6253]
7	[30, 36]	1534	7439	2.55	5.73	[256, 270]	[0.6373, 0.6721]
8	[45, 57]	1130	4043	2.25	10.07	[73, 81]	[0.2949, 0.3272]
9	[38, 45]	2278	7306	1.51	7.6	[293, 311]	[0.6630, 0.7037]
10	[104, 124]	7321	25,218	16.91	15.73	[1129, 1195]	[0.7240, 0.7663]
11	[92, 110]	6218	11,552	10.86	13.95	[410, 455]	[0.4688, 0.5202]
12	[38, 40]	1878	4155	3.89	6.43	[191, 202]	[0.6485, 0.6858]
13	[42, 46]	2649	6986	1.41	6.22	[242, 263]	[0.5748, 0.6247]
14	[39, 50]	2402	8623	2.18	7.25	[264, 341]	[0.5004, 0.6463]
15	[55, 57]	2359	7200	5.06	8.57	[221, 264]	[0.4801, 0.5736]
16	[30, 39]	1328	6260	1.87	5.68	[179, 227]	[0.5147, 0.6527]
17	[132, 137]	11,922	53,840	8.28	20.07	[2672, 3122]	[0.8559, 1.0000]
18	[59, 62]	3552	11,674	6.76	8.2	[417, 505]	[0.5704, 0.6908]
19	[17, 19]	1666	3926	2.98	2.83	[125, 147]	[0.4592, 0.5400]
20	[173, 180]	23,200	40,000	23.09	25.18	[3066, 3122]	[0.9821, 1.0000]
21	[73, 74]	3271	21,484	2.34	10.9	[360, 386]	[0.4203, 0.4506]
22	[59, 72]	4301	10,300	2.26	10.14	[290, 363]	[0.4608, 0.5768]
23	[99, 112]	21,175	47,060	7.34	14.35	[1995, 2317]	[0.8610, 1.0000]
24	[35, 41]	1410	13,803	1.65	5.37	[212, 230]	[0.5742, 0.6229]
25	[65, 105]	30,705	22,000	38.3	15.99	[1252, 1276]	[0.6950, 0.9152]

TABLE 9. The GICE scores of China's primary schools.

DMU_j	Iteration						
	1	2	3	4	5	6	7*
1	[0.4416, 0.5086]	[0.4615, 0.5709]	[0.4750, 0.5760]	[0.4804, 0.5761]	[0.4822, 0.5762]	[0.4828, 0.5762]	**[0.4829, 0.5762]**
2	[0.3692, 0.3976]	[0.3901, 0.4390]	[0.3937, 0.4408]	[0.3962, 0.4411]	[0.3973, 0.4413]	[0.3977, 0.4413]	**[0.3978, 0.4414]**
3	[0.4670, 0.5325]	[0.4725, 0.5647]	[0.4804, 0.5706]	[0.4845, 0.5718]	[0.4858, 0.5727]	[0.4862, 0.5729]	**[0.4864, 0.5730]**
4	[0.4987, 0.5250]	[0.4964, 0.5587]	[0.5122, 0.5635]	[0.5181, 0.5645]	[0.5199, 0.5655]	[0.5204, 0.5658]	**[0.5206, 0.5659]**
5	[0.4172, 0.4461]	[0.5971, 0.6605]	[0.6211, 0.6727]	[0.6227, 0.6727]	[0.6227, 0.6727]	[0.6227, 0.6727]	**[0.6227, 0.6727]**
6	[0.4652, 0.5159]	[0.5156, 0.6034]	[0.5307, 0.6149]	[0.5370, 0.6156]	[0.5396, 0.6158]	[0.5405, 0.6159]	**[0.5408, 0.6160]**
7	[0.5615, 0.5939]	[0.6069, 0.6657]	[0.6128, 0.6703]	[0.6162, 0.6706]	[0.6178, 0.6707]	[0.6183, 0.6708]	**[0.6184, 0.6708]**
8	[0.2522, 0.2802]	[0.2578, 0.3160]	[0.2644, 0.3219]	[0.2676, 0.3228]	[0.2689, 0.3231]	[0.2694, 0.3232]	**[0.2695, 0.3233]**
9	[0.5897, 0.6273]	[0.6180, 0.6883]	[0.6315, 0.6960]	[0.6352, 0.6972]	[0.6364, 0.6977]	[0.6368, 0.6978]	**[0.6370, 0.6979]**
10	[0.6349, 0.6742]	[0.6552, 0.7305]	[0.6726, 0.7416]	[0.6791, 0.7445]	[0.6813, 0.7459]	[0.6821, 0.7464]	**[0.6823, 0.7466]**
11	[0.3866, 0.4296]	[0.4285, 0.5039]	[0.4411, 0.5123]	[0.4463, 0.5128]	[0.4485, 0.5130]	[0.4492, 0.5130]	**[0.4494, 0.5130]**
12	[0.5315, 0.5614]	[0.5631, 0.6552]	[0.5844, 0.6695]	[0.5952, 0.6685]	[0.5985, 0.6692]	[0.5996, 0.6694]	**[0.6000, 0.6695]**
13	[0.4817, 0.5237]	[0.5291, 0.6137]	[0.5452, 0.6204]	[0.5484, 0.6213]	[0.5494, 0.6217]	[0.5497, 0.6218]	**[0.5498, 0.6218]**
14	[0.4622, 0.5993]	[0.4640, 0.6281]	[0.4723, 0.6352]	[0.4756, 0.6372]	[0.4768, 0.6380]	[0.4772, 0.6382]	**[0.4773, 0.6383]**
15	[0.4080, 0.4868]	[0.4020, 0.5630]	[0.4215, 0.5678]	[0.4282, 0.5686]	[0.4309, 0.5689]	[0.4318, 0.5691]	**[0.4321, 0.5691]**
16	[0.4628, 0.5890]	[0.4898, 0.6490]	[0.4939, 0.6518]	[0.4967, 0.6521]	[0.4981, 0.6522]	[0.4985, 0.6522]	**[0.4986, 0.6523]**
17	[0.8532, 1.0000]	[0.8508, 1.0000]	[0.8508, 1.0000]	[0.8508, 1.0000]	[0.8508, 1.0000]	[0.8508, 1.0000]	**[0.8508, 1.0000]**
18	[0.5016, 0.6076]	[0.4945, 0.6727]	[0.5102, 0.6818]	[0.5204, 0.6798]	[0.5230, 0.6805]	[0.5239, 0.6808]	**[0.5241, 0.6809]**
19	[0.3940, 0.4644]	[0.4189, 0.5206]	[0.4314, 0.5259]	[0.4366, 0.5259]	[0.4379, 0.5267]	[0.4383, 0.5270]	**[0.4384, 0.5271]**
20	[0.8741, 0.8908]	[0.9700, 0.9949]	[0.9790, 1.0000]	[0.9810, 1.0000]	[0.9813, 1.0000]	[0.9813, 1.0000]	**[0.9813, 1.0000]**
21	[0.3234, 0.3471]	[0.4103, 0.4461]	[0.4155, 0.4498]	[0.4158, 0.4499]	[0.4158, 0.4500]	[0.4159, 0.4500]	**[0.4159, 0.4500]**
22	[0.3766, 0.4725]	[0.4202, 0.5679]	[0.4339, 0.5740]	[0.4377, 0.5741]	[0.4387, 0.5744]	[0.4390, 0.5745]	**[0.4391, 0.5746]**
23	[0.6007, 0.7018]	[0.8433, 0.9926]	[0.8516, 1.0000]	[0.8519, 1.0000]	[0.8519, 1.0000]	[0.8519, 1.0000]	**[0.8519, 1.0000]**
24	[0.3362, 0.3660]	[0.5355, 0.6055]	[0.5497, 0.6201]	[0.5524, 0.6211]	[0.5535, 0.6215]	[0.5538, 0.6216]	**[0.5539, 0.6217]**
25	[0.4042, 0.4202]	[0.6766, 0.8854]	[0.6917, 0.9124]	[0.6934, 0.9129]	[0.6938, 0.9129]	[0.6939, 0.9129]	**[0.6940, 0.9129]**

After obtaining the ACE scores of each DMU, the interval ACE score of arbitrary strategy is set as the initial solution for iteration 1, while ε is set as 0.001. Finally, using Equation (10) through three ranking steps of the MO-G-IDEA-CE method in Section B, the GICE scores for all iterations are shown in Table 9.

Table 9 shows that the final GICE scores for all DMUs were achieved at Iteration 7. After obtaining the GICE scores of all DMUs, the possibility degree formula, Equation (11), was used to generate the possibility degree matrix, $P = (p_{ij})_{25\times25}$. Details of the possibility degree matrix (P) are shown in Table 10.

TABLE 10. The possibility degree matrix (*P*) for the primary schools.

i	j				
	1	2	3	4	5
1	0.5000	1.0000	0.4987	0.3535	0.0000
2	0.0000	0.5000	0.0000	0.0000	0.0000
3	0.5013	1.0000	0.5000	0.3437	0.0000
4	0.6465	1.0000	0.6563	0.5000	0.0000
5	1.000	1.000	1.000	1.000	0.500
6	0.867	1.000	0.876	0.868	0.000
7	1.000	1.000	1.000	1.000	0.442
8	0.000	0.000	0.000	0.000	0.000
9	1.000	1.000	1.000	1.000	0.762
10	1.000	1.000	1.000	1.000	1.000
11	0.119	1.000	0.108	0.000	0.000
12	1.000	1.000	1.000	1.000	0.321
13	0.905	1.000	0.914	0.921	0.000
14	0.675	1.000	0.675	0.590	0.040
15	0.299	0.973	0.293	0.189	0.000
16	0.771	1.000	0.776	0.710	0.085
17	1.000	1.000	1.000	1.000	1.000
18	0.868	1.000	0.874	0.870	0.211
19	0.160	0.989	0.151	0.025	0.000
20	1.000	1.000	1.000	1.000	1.000
21	0.000	0.756	0.000	0.000	0.000
22	0.334	0.994	0.331	0.231	0.000
23	1.000	1.000	1.000	1.000	1.000
24	0.920	1.000	0.929	0.941	0.000
25	1.000	1.000	1.000	1.000	1.000

TABLE 10. *(Continued.)* The possibility degree matrix (*P*) for the primary schools.

i	j				
	6	7	8	9	10
1	0.1332	0.0000	1.0000	0.0000	0.0000
2	0.0000	0.0000	1.0000	0.0000	0.0000
3	0.1245	0.0000	1.0000	0.0000	0.0000
4	0.1316	0.0000	1.0000	0.0000	0.0000
5	1.000	0.558	1.000	0.238	0.000
6	0.500	0.000	1.000	0.000	0.000
7	1.000	0.500	1.000	0.213	0.000
8	0.000	0.000	0.500	0.000	0.000
9	1.000	0.787	1.000	0.500	0.071
10	1.000	1.000	1.000	0.929	0.500
11	0.000	0.000	1.000	0.000	0.000
12	0.938	0.361	1.000	0.166	0.000
13	0.592	0.014	1.000	0.000	0.000
14	0.372	0.051	1.000	0.003	0.000
15	0.077	0.000	1.000	0.000	0.000
16	0.481	0.098	1.000	0.038	0.000
17	1.000	1.000	1.000	1.000	1.000
18	0.654	0.231	1.000	0.126	0.000
19	0.000	0.000	1.000	0.000	0.000
20	1.000	1.000	1.000	1.000	1.000
21	0.000	0.000	1.000	0.000	0.000
22	0.096	0.000	1.000	0.000	0.000
23	1.000	1.000	1.000	1.000	1.000
24	0.617	0.014	1.000	0.000	0.000
25	1.000	1.000	1.000	0.993	0.886

TABLE 10. ***(Continued.)*** **The possibility degree matrix (*P*) for the primary schools.**

i	j 11	12	13	14	15
1	0.8813	0.0000	0.0952	0.3247	0.7011
2	0.0000	0.0000	0.0000	0.0000	0.0271
3	0.8921	0.0000	0.0858	0.3254	0.7065
4	1.0000	0.0000	0.0795	0.4096	0.8112
5	1.000	0.679	1.000	0.960	1.000
6	1.000	0.062	0.408	0.628	0.923
7	1.000	0.639	0.986	0.949	1.000
8	0.000	0.000	0.000	0.000	0.000
9	1.000	0.834	1.000	0.997	1.000
10	1.000	1.000	1.000	1.000	1.000
11	0.500	0.000	0.000	0.095	0.359
12	1.000	0.500	0.909	0.900	1.000
13	1.000	0.091	0.500	0.674	0.949
14	0.905	0.100	0.326	0.500	0.777
15	0.641	0.000	0.051	0.223	0.500
16	0.964	0.153	0.433	0.601	0.840
17	1.000	1.000	1.000	1.000	1.000
18	1.000	0.294	0.607	0.720	0.910
19	0.517	0.000	0.000	0.124	0.370
20	1.000	1.000	1.000	1.000	1.000
21	0.003	0.000	0.000	0.000	0.058
22	0.689	0.000	0.068	0.244	0.544
23	1.000	1.000	1.000	1.000	1.000
24	1.000	0.094	0.528	0.687	0.960
25	1.000	1.000	1.000	1.000	1.000

TABLE 10. ***(Continued.)*** **The possibility degree matrix (*P*) for the primary schools.**

i	j 16	17	18	19	20
1	0.2291	0.0000	0.1315	0.8400	0.0000
2	0.0000	0.0000	0.0000	0.0114	0.0000
3	0.2243	0.0000	0.1257	0.8489	0.0000
4	0.2905	0.0000	0.1302	0.9748	0.0000
5	0.915	0.000	0.789	1.000	0.000
6	0.519	0.000	0.346	1.000	0.000
7	0.902	0.000	0.769	1.000	0.000
8	0.000	0.000	0.000	0.000	0.000
9	0.962	0.000	0.874	1.000	0.000
10	1.000	0.000	1.000	1.000	0.000
11	0.036	0.000	0.000	0.483	0.000
12	0.847	0.000	0.706	1.000	0.000
13	0.567	0.000	0.393	1.000	0.000
14	0.399	0.000	0.280	0.876	0.000
15	0.160	0.000	0.090	0.630	0.000
16	0.500	0.000	0.352	0.934	0.000
17	1.000	0.500	1.000	1.000	0.063
18	0.648	0.000	0.500	0.994	0.000
19	0.066	0.000	0.006	0.500	0.000
20	1.000	0.937	1.000	1.000	0.500
21	0.000	0.000	0.000	0.052	0.000
22	0.178	0.000	0.104	0.677	0.000
23	1.000	0.504	1.000	1.000	0.063
24	0.581	0.000	0.406	1.000	0.000
25	1.000	0.102	1.000	1.000	0.000

TABLE 10. ***(Continued.)*** **The possibility degree matrix (*P*) for the primary schools.**

i	j 21	22	23	24	25
1	1.0000	0.6660	0.0000	0.0802	0.0000
2	0.2440	0.0065	0.0000	0.0000	0.0000
3	1.0000	0.6689	0.0000	0.0705	0.0000
4	1.0000	0.7688	0.0000	0.0590	0.0000
5	1.000	1.000	0.000	1.000	0.000
6	1.000	0.904	0.000	0.383	0.000
7	1.000	1.000	0.000	0.986	0.000
8	0.000	0.000	0.000	0.000	0.000
9	1.000	1.000	0.000	1.000	0.007
10	1.000	1.000	0.000	1.000	0.114
11	0.997	0.311	0.000	0.000	0.000
12	1.000	1.000	0.000	0.906	0.000
13	1.000	0.932	0.000	0.472	0.000
14	1.000	0.756	0.000	0.313	0.000
15	0.942	0.456	0.000	0.040	0.000
16	1.000	0.822	0.000	0.419	0.000
17	1.000	1.000	0.496	1.000	0.898
18	1.000	0.896	0.000	0.594	0.000
19	0.948	0.323	0.000	0.000	0.000
20	1.000	1.000	0.937	1.000	1.000
21	0.500	0.034	0.000	0.000	0.000
22	0.966	0.500	0.000	0.056	0.000
23	1.000	1.000	0.500	1.000	0.900
24	1.000	0.944	0.000	0.500	0.000
25	1.000	1.000	0.100	1.000	0.500

TABLE 11. **The Boolean matrix (q) for China's primary schools.**

i	j 1	2	3	4	5
1	1	1	0	0	0
2	0	1	0	0	0
3	1	1	1	0	0
4	1	1	1	1	0
5	1	1	1	1	1
6	1	1	1	1	0
7	1	1	1	1	0
8	0	0	0	0	0
9	1	1	1	1	1
10	1	1	1	1	1
11	0	1	0	0	0
12	1	1	1	1	0
13	1	1	1	1	0
14	1	1	1	1	0
15	0	1	0	0	0
16	1	1	1	1	0
17	1	1	1	1	1
18	1	1	1	1	0
19	0	1	0	0	0
20	1	1	1	1	1
21	0	1	0	0	0
22	0	1	0	0	0
23	1	1	1	1	1
24	1	1	1	1	0
25	1	1	1	1	1

After obtaining Table 10, the possibility degree matrix (P) was transformed into the Boolean matrix (Q). If the possibility degree score of DMU$_i$ vs DMU$_j$ (p_{ij}) $\geq$ 0.50, the Boolean value of DMU$_i$ vs DMU$_j$ (q_{ij}) = 1. Otherwise, $q_{ij} = 0$. Details of Q are shown in Table 11.

TABLE 11. ***(Continued.)*** **The Boolean matrix (q) for China's primary schools.**

i	j				
	6	7	8	9	10
1	0	0	1	0	0
2	0	0	1	0	0
3	0	0	1	0	0
4	0	0	1	0	0
5	1	1	1	0	0
6	1	0	1	0	0
7	1	1	1	0	0
8	0	0	1	0	0
9	1	1	1	1	0
10	1	1	1	1	1
11	0	0	1	0	0
12	1	0	1	0	0
13	1	0	1	0	0
14	0	0	1	0	0
15	0	0	1	0	0
16	0	0	1	0	0
17	1	1	1	1	1
18	1	0	1	0	0
19	0	0	1	0	0
20	1	1	1	1	1
21	0	0	1	0	0
22	0	0	1	0	0
23	1	1	1	1	1
24	1	0	1	0	0
25	1	1	1	1	1

TABLE 11. ***(Continued.)*** **The Boolean matrix (q) for China's primary schools.**

i	j				
	11	12	13	14	15
1	1	0	0	0	1
2	0	0	0	0	0
3	1	0	0	0	1
4	1	0	0	0	1
5	1	1	1	1	1
6	1	0	0	1	1
7	1	1	1	1	1
8	0	0	0	0	0
9	1	1	1	1	1
10	1	1	1	1	1
11	1	0	0	0	0
12	1	1	1	1	1
13	1	0	1	1	1
14	1	0	0	1	1
15	1	0	0	0	1
16	1	0	0	1	1
17	1	1	1	1	1
18	1	0	1	1	1
19	1	0	0	0	0
20	1	1	1	1	1
21	0	0	0	0	0
22	1	0	0	0	1
23	1	1	1	1	1
24	1	0	1	1	1
25	1	1	1	1	1

TABLE 11. ***(Continued.)*** **The Boolean matrix (q) for China's primary schools.**

i	j				
	16	17	18	19	20
1	0	0	0	1	0
2	0	0	0	0	0
3	0	0	0	1	0
4	0	0	0	1	0
5	1	0	1	1	0
6	1	0	0	1	0
7	1	0	1	1	0
8	0	0	0	0	0
9	1	0	1	1	0
10	1	0	1	1	0
11	0	0	0	0	0
12	1	0	1	1	0
13	1	0	0	1	0
14	0	0	0	1	0
15	0	0	0	1	0
16	1	0	0	1	0
17	1	1	1	1	0
18	1	0	1	1	0
19	0	0	0	1	0
20	1	1	1	1	1
21	0	0	0	0	0
22	0	0	0	1	0
23	1	1	1	1	0
24	1	0	0	1	0
25	1	0	1	1	0

TABLE 11. ***(Continued.)*** **The Boolean matrix (q) for China's primary schools.**

i	j						
	1	2	3	4	5	λ_i'	Rank
1	1	1	0	0	0	8	18
2	0	0	0	0	0	2	24
3	1	1	0	0	0	9	17
4	1	1	0	0	0	10	16
5	1	1	0	1	0	19	7
6	1	1	0	0	0	13	13
7	1	1	0	1	0	18	8
8	0	0	0	0	0	1	25
9	1	1	0	1	0	20	6
10	1	1	0	1	0	21	5
11	1	0	0	0	0	4	22
12	1	1	0	1	0	17	9
13	1	1	0	0	0	14	12
14	1	1	0	0	0	11	15
15	1	0	0	0	0	6	20
16	1	1	0	0	0	12	14
17	1	1	0	1	1	23	3
18	1	1	0	1	0	16	10
19	1	0	0	0	0	5	21
20	1	1	1	1	1	25	1
21	1	0	0	0	0	3	23
22	1	1	0	0	0	7	19
23	1	1	1	1	1	24	2
24	1	1	0	1	0	15	11
25	1	1	0	1	1	22	4

TABLE 12. Ranking comparisons of the proposed method and the other methods for China's primary schools.

DMU_j	Wang *et al.*'s method [35]		Proposed method		
	RED	Rank	GICE	$\lambda_i^{'}$	Rank
1	0.3445	18	[0.4829,.5762]	8	18
2	0.5383	24	[0.3978,0.4414]	2	24
3	0.3348	17	[0.4864,0.5730]	9	17
4	0.3097	16	[0.5206,0.5659]	10	16
5	0.2000	9	[0.6227,0.6727]	19	7
6	0.2671	12	[0.5408,0.6160]	13	13
7	0.1926	8	[0.6184,0.6708]	18	8
8	0.7852	25	[0.2695,0.3233]	1	25
9	0.1612	6	[0.6370,0.6979]	20	6
10	0.1030	5	[0.6823,0.7466]	21	5
11	0.4191	22	[0.4494,0.5130]	4	22
12	0.1783	7	[0.6000,0.6695]	17	9
13	0.2610	11	[0.5498,0.6218]	14	12
14	0.3031	15	[0.4773,0.6383]	11	15
15	0.3683	19	[0.4321,0.5691]	6	20
16	0.2880	14	[0.4986,0.6523]	12	14
17	0.0135	3	[0.8508,1.0000]	23	3
18	0.2250	10	[0.5241,0.6809]	16	10
19	0.4118	21	[0.4384,0.5271]	5	21
20	0.0000	1	[0.9813,1.0000]	25	1
21	0.5298	23	[0.4159,0.4500]	3	23
22	0.3839	20	[0.4391,0.5746]	7	19
23	0.0125	2	[0.8519,1.0000]	24	2
24	0.2681	13	[0.5539,0.6217]	15	11
25	0.1009	4	[0.6940,0.9129]	22	4

After that, the ranking vector (λ_i') was calculated. As a result, ranking comparisons of the proposed method and the other methods for all DMUs, are shown in Table 12.

Table 12 shows that the proposed method was compared with Wang *et al.*'s [35]. In the Wang *et al.* method [35], the distance model based on entropy and TOPSIS was used to generate the relative Euclidean distance (RED) from the positive solution. A lower value of the RED means better rankings. After the Spearman correlation test, the r_s for the proposed method and Wang *et al.*'s method [35] were determined to be $r_s = 0.994$ (Sig. = 0.000). It is worth noting that the proposed ranking method has a very high correlation with Wang *et al.*'s method [35]; see the correlation test presented in Fig. 4.

In addition, we demonstrate the Convergence procedure of the MO-G-IDEA-CE approach, which utilizes the relevant efficiency score from an arbitrary strategy. We set $\varepsilon = 0.001$, and Fig. 5 and Fig. 6 demonstrate that after seven iterations,

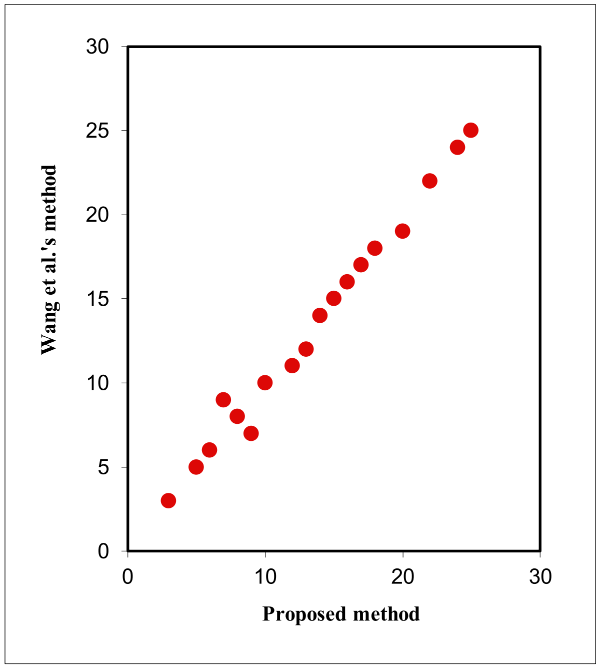

FIGURE 4. The correlation test for the proposed method and the Wang *et al.*'s method.

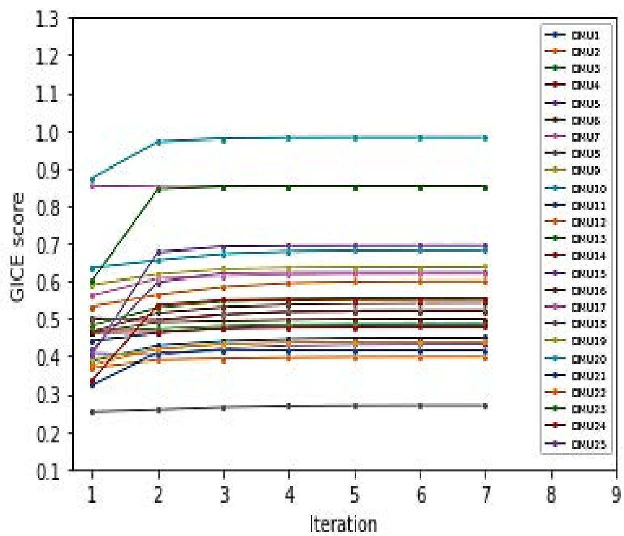

FIGURE 5. The Convergence procedure of the MO-G-IDEA-CE method for lower values of GICE scores.

all DMUs generated by the proposed method attain constant GICE scores, which means the optimal solutions are the Nash equilibrium points, as demonstrated in [36].

C. APPLICATION OF SEVEN BIOMASS MATERIALS

Thailand is an agricultural-based economy with various agricultural residue resources that can be used for manufacturing charcoal briquettes. In developing countries, biomass from farm residues can be transformed into charcoal briquettes to

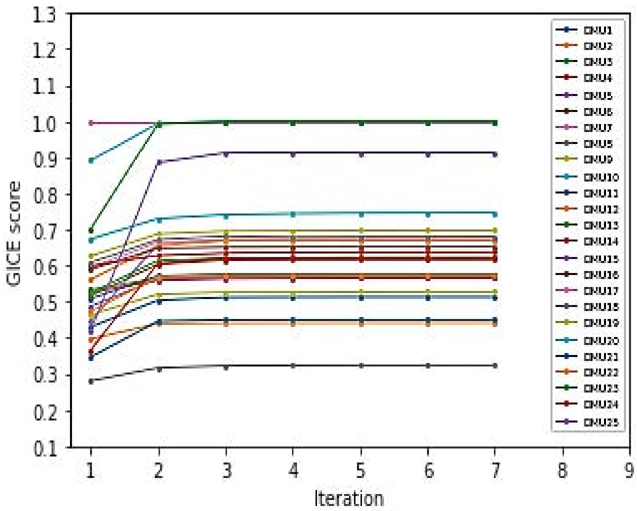

FIGURE 6. **The Convergence procedure of the MO-G-IDEA-CE method for upper values of GICE scores.**

FIGURE 7. **A sample biomass charcoal briquette.**

replace fossil fuels. The appearance of a biomass charcoal briquette is shown in Fig. 7.

The moisture content (MC), ash content (AC), heating value (HV), and fixed carbon (FC) are essential properties of agricultural residues for manufacturing charcoal briquettes. In the IDEA model, biomass residues can be viewed as DMUs. The moisture and ash content can be viewed as inputs because, according to IDEA principles, a lower value is better. The heating value and fixed carbon can be defined as outputs because the higher value, the better. The characteristics of input data of charcoal briquettes are shown in Table 13.

The properties of the seven biomass charcoal briquettes are shown in Table 14, including seven agricultural residues (DMUs) with interval data of inputs and outputs. Let moisture content (%) and ash content (%) be input 1 ($\otimes x_1$) and input 2 ($\otimes x_2$), respectively. The heating value (kcal/kg) and fixed carbon (%) are output 1 ($\otimes y_1$) and output 2 ($\otimes y_2$), respectively. The DMU_1, DMU_2, DMU_3, DMU_4, DMU_5, DMU_6, and DMU_7 are Bagasse, Incense reed, Water hyacinth, Rice husk, Coconut shell, Sawdust, and Sensitive plant, respectively.

TABLE 13. **The characteristics of input data of charcoal briquettes.**

Data	Characteristics
Moisture content (%) ($\otimes x_1$)	The biomass's moisture content can substantially affect its combustion characteristics. During combustion, the moisture in biomass absorbs heat from the burning fuel to form vapor due to the heat of vaporization, reducing the heating value of the used fuel significantly. The incomplete combustion of volatile substances and the deposition of unburned carbon around stoves, vessels, and pans make them challenging to clean. In addition, high levels of humidity can make ignition difficult. Therefore, the combustion of a fuel with such high moisture content will produce numerous incomplete combustion products. In the view of DEA, this property is considered an input because the lower the moisture content, the better the fuel properties.
Ash content (%) ($\otimes x_2$)	Ashes are the incombustible component of biomass, and the greater the fuel's ash content, the lower its calorific value. Ashes are known to cause problems in combustion systems, mainly due to the formation of slag and deposition on the surface of metals, plus their tendency to increase the corrosion rate of the system's metal components. Therefore, in the view of DEA, this property is considered an input because the lower the ash content, the better the fuel properties.
Heating value (kcal/kg) ($\otimes y_1$)	The calorific value (or heating value) is the standard unit of measurement for a fuel's energy content. It is the heat produced when one unit of fuel is completely burned. Because of the DEA's view, this property is considered an output because the higher the heating value, the better the fuel properties.
Fixed carbon (%) ($\otimes y_2$)	The percentage of fixed carbon is typically determined by the difference between the percentages of the total biomass's other quantities, such as moisture, volatile matter, and ash content. Essentially, the fixed carbon of a fuel is the proportion of carbon available for char combustion following the removal of all volatile matter from the biomass. This is not the same as the total amount of carbon in the fuel (the ultimate carbon) because a significant amount is also released as hydrocarbons from volatile matter. Fixed carbon provides a substantial indication of the fraction of char remaining after the phase of volatilization. These carbons will combine with oxygen to generate heat. In the view of DEA, this property is considered an output because the higher the heating value, the better the fuel properties.

TABLE 14. The data set of the biomass charcoal briquettes.

DMU_j	Inputs		Outputs		Interval CCR
	$\otimes x_1$	$\otimes x_2$	$\otimes y_1$	$\otimes y_2$	
1	[6.18, 6.68)	[8.51, 8.92]	[4457, 4465]	[17.35, 18.22]	**[0.5833, 0.6317]**
2	[5.91, 6.42)	[24.29, 24.74]	[3244, 3255]	[14.18, 15.01]	**[0.4418, 0.4815]**
3	[6.48, 7.00]	[25.34, 25.82]	[3137, 3151]	[14.42, 15.24]	**[0.3918, 0.4251]**
4	[7.29, 7.77]	[20.67, 21.18]	[3875, 3893]	[16.96, 17.76]	**[0.4360, 0.4669]**
5	[6.67, 7.19]	[3.05, 3.59]	[6757, 6764]	[72.27, 73.25]	**[0.9239, 1.0000]**
6	[4.27, 4.79]	[1.09, 1.56]	[4870, 4884]	[26.96, 28.16]	**[0.8889, 1.0000]**
7	[9.93, 10.49]	[3.50, 4.00]	[4368, 4385]	[24.31, 25.19]	**[0.3640, 0.3861]**

TABLE 15. Interval cross-efficiency scores for biomass charcoal briquettes.

DMU_j	Target DMU							Interval ACE
	1	2	3	4	5	6	7	
1	[0.5833, 0.6317]	[0.5833, 0.6317]	[0.5833, 0.6317]	[0.5833, 0.6317]	[0.5557, 0.5130]	[0.5833, 0.6228]	[0.5833, 0.6317]	[0.5794, 0.6134]
2	[0.4418, 0.4815]	[0.4418, 0.4815]	[0.4418, 0.4815]	[0.4418, 0.4815]	[0.4262, 0.2958]	[0.4418, 0.4769]	[0.4418, 0.4815]	[0.4395, 0.4543]
3	[0.3918, 0.4251]	[0.3918, 0.4251]	[0.3918, 0.4251]	[0.3918, 0.4251]	[0.3802, 0.2682]	[0.3918, 0.4219]	[0.3918, 0.4251]	[0.3902, 0.4022]
4	[0.4360, 0.4669]	[0.4360, 0.4669]	[0.4360, 0.4669]	[0.4360, 0.4669]	[0.4207, 0.3253]	[0.4360, 0.4622]	[0.4360, 0.4669]	[0.4338, 0.4460]
5	[0.8216, 0.8866]	[0.8216, 0.8866]	[0.8216, 0.8866]	[0.8216, 0.8866]	[0.9239, 1.0000]	[0.8216, 0.9238]	[0.8216, 0.8866]	[0.8362, 0.9081]
6	[0.8889, 1.0000]	[0.8889, 1.0000]	[0.8889, 1.0000]	[0.8889, 1.0000]	[0.8837, 1.0000]	[0.8889, 1.0000]	[0.8889, 1.0000]	[0.8881, 1.0000]
7	[0.3640, 0.3861]	[0.3640, 0.3861]	[0.3640, 0.3861]	[0.3640, 0.3861]	[0.3622, 0.3804]	[0.3640, 0.3860]	[0.3640, 0.3861]	[0.3638, 0.3852]

The calculation procedure is the same as in Section A. By solving the MO-IDEA-CCR model; the interval CCR scores were obtained as in the last column of Table 14. After getting the optimal weights of inputs and outputs from the MO-IDEA-CCR model, the interval ACE scores were calculated according to Equations (3) to (6). As a result, the interval ACE scores achieved for each DMU are shown in Table 15.

After obtaining the interval ACE scores for each DMU, the GICE scores were calculated using the MO-G-IDEA-CE method. As a result, GICE scores were achieved, as shown in Table 16.

Table 16 shows that the final GICE scores for all DMUs were achieved at Iteration 10. After obtaining the GICE scores of all DMUs, the possibility degree formula, Equation (11), was used to generate the possibility degree matrix, $P = (p_{ij})_{7\times 7}$, as shown in Table 17.

After obtaining Table 17, the Boolean possibility degree matrix (Q) was generated using Equation (11). Details of Q are shown in Table 18.

TABLE 16. The GICE scores of biomass charcoal briquettes.

DMU_j	Iteration				
	1	2	3	4	5
1	[0.5794, 0.6134]	[0.5811, 0.6250]	[0.5812, 0.6295]	[0.5812, 0.6306]	[0.5812, 0.6307]
2	[0.4395, 0.4543]	[0.4389, 0.4711]	[0.4390, 0.4783]	[0.4389, 0.4811]	[0.4388, 0.4815]
3	[0.3902, 0.4022]	[0.3894, 0.4168]	[0.3894, 0.4228]	[0.3893, 0.4249]	[0.3892, 0.4251]
4	[0.4338, 0.4460]	[0.4336, 0.4584]	[0.4336, 0.4642]	[0.4333, 0.4666]	[0.4331, 0.4669]
5	[0.8362, 0.9081]	[0.9098, 0.9886]	[0.9204, 1.0000]	[0.9229, 1.0000]	[0.9232, 1.0000]
6	[0.8881, 1.0000]	[0.8879, 1.0000]	[0.8879, 1.0000]	[0.8879, 1.0000]	[0.8878, 1.0000]
7	[0.3638, 0.3852]	[0.3635, 0.3858]	[0.3635, 0.3860]	[0.3635, 0.3860]	[0.3635, 0.3860]

TABLE 16. *(Continued.)* The GICE scores of biomass charcoal briquettes.

DMU_j	Iteration				
	6	7	8	9	**10***
1	[0.5811, 0.6307]	[0.5811, 0.6307]	[0.5811, 0.6307]	[0.5811, 0.6306]	**[0.5811, 0.6306]**
2	[0.4388, 0.4815]	[0.4388, 0.4815]	[0.4387, 0.4815]	[0.4387, 0.4815]	**[0.4387, 0.4815]**
3	[0.3892, 0.4251]	[0.3891, 0.4251]	[0.3891, 0.4251]	[0.3891, 0.4251]	**[0.3891, 0.4251]**
4	[0.4331, 0.4669]	[0.4330, 0.4669]	[0.4330, 0.4669]	[0.4330, 0.4669]	**[0.4330, 0.4669]**
5	[0.9233, 1.0000]	[0.9233, 1.0000]	[0.9233, 1.0000]	[0.9233, 1.0000]	**[0.9233, 1.0000]**
6	[0.8878, 1.0000]	[0.8878, 1.0000]	[0.8878, 1.0000]	[0.8878, 1.0000]	**[0.8878, 1.0000]**
7	[0.3634, 0.3860]	[0.3634, 0.3860]	[0.3634, 0.3860]	[0.3634, 0.3860]	**[0.3634, 0.3860]**

TABLE 17. The possibility degree matrix (*P*) for the biomass charcoal briquettes.

i	j							
	1	2		3	4	5	6	7
1	0.50	1.00	1.00	1.00	0.00	0.00	1.00	0.50
2	0.00	0.50	1.00	0.71	0.00	0.00	1.00	0.00
3	0.00	0.00	0.50	0.00	0.00	0.00	1.00	0.00
4	0.00	0.29	1.00	0.50	0.00	0.00	1.00	0.00
5	1.00	1.00	1.00	1.00	0.50	0.66	1.00	1.00
6	1.00	1.00	1.00	1.00	0.34	0.50	1.00	1.00
7	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.00

After obtaining the λ', $\lambda' = \{5, 4, 2, 3, 7, 6, 1\}$, ranking DMUs in descending order in accordance with the higher value of λ'_i, we have $DMU_5 \geq DMU_6 \geq DMU_1 \geq DMU_2 \geq DMU_4 \geq DMU_3 \geq DMU_7$. In addition, Wang *et al.*'s method [29] and Wu *et al.*'s method [34] were used to solving this problem for ranking comparisons. The ranking comparisons

TABLE 18. **The Boolean matrix (q) for the biomass charcoal briquettes.**

i	j								
	1	2	3	4	5	6	7	λ_i'	Rank
1	1	1	1	1	0	0	1	5	3
2	0	1	1	1	0	0	1	4	4
3	0	0	1	0	0	0	1	2	6
4	0	0	1	1	0	0	1	3	5
5	1	1	1	1	1	1	1	7	1
6	1	1	1	1	0	1	1	6	2
7	0	0	0	0	0	0	1	1	7

TABLE 19. **The ranking comparisons of the proposed method and the other techniques for the biomass charcoal briquettes.**

DMU_j	IDEA-CCR [29]			Wu *et al.* [34]		Proposed method		
	Eff.	MRA	Rank	RED	Rank	GICE	λ_i'	Rank
1	[0.5833, 0.6317]	0.4167	3	0.0484	3	[0.5811, 0.6306]	5	3
2	[0.4418, 0.4815]	0.5582	4	0.0880	4	[0.4387, 0.4815]	4	4
3	[0.3918, 0.4251]	0.6082	6	0.1056	6	[0.3891, 0.4251]	2	6
4	[0.4360, 0.4669]	0.5640	5	0.0913	5	[0.4330, 0.4669]	3	5
5	[0.9239, 1.0000]	0.0761	1	0.0062	2	[0.9233, 1.0000]	7	1
6	[0.8889, 1.0000]	0.1111	2	0.0027	1	[0.8878, 1.0000]	6	2
7	[0.3640, 0.3861]	0.6360	7	0.1170	7	[0.3634, 0.3860]	1	7

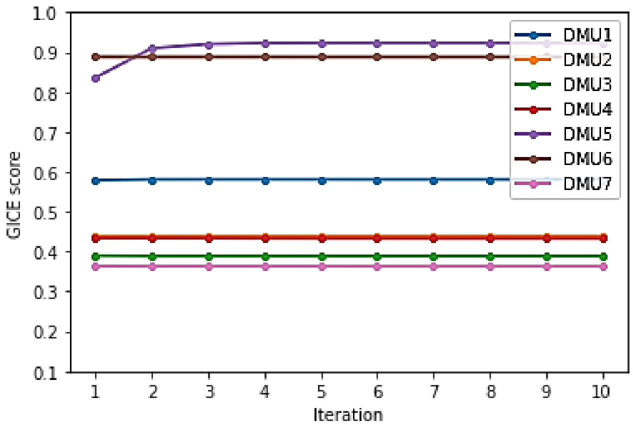

FIGURE 8. **The Convergence procedure of the MO-G-IDEA-CE method for lower values of GICE scores.**

of the proposed method and the other techniques for all DMUs are shown in Table 19.

After the Spearman correlation test, the r_s for the proposed MO-G-IDEA-CE method and the IDEA-CCR model [29] and Wu *et al.*'s method [34] are evaluated as $r_s = 1.000$ and 0.964, respectively.

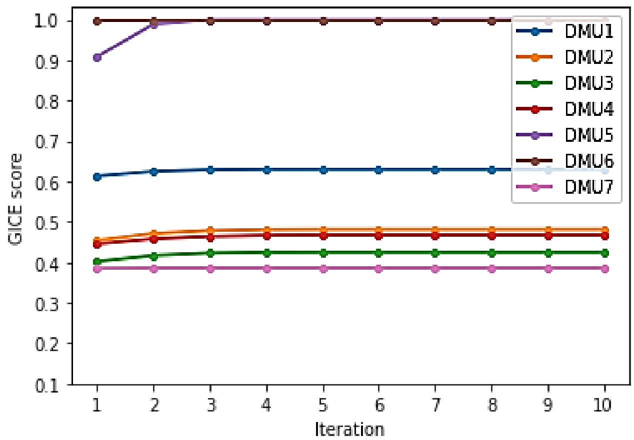

FIGURE 9. **The Convergence procedure of the MO-G-IDEA-CE method for upper values of GICE scores.**

Fig. 8 and 9 demonstrate that after ten iterations, all DMUs generated by the proposed method attain constant GICE scores, which means the optimal solutions are the Nash equilibrium points, as demonstrated elsewhere [36].

V. CONCLUSION

In developing countries, agricultural residuals can be used to make biomass charcoal briquettes for cooking and heating. This idea is one good idea to solve the energy shortage problem of almost every agricultural country. However, the important properties of each biomass material must first be considered to find suitable biomass for processing into charcoal briquettes. Sometimes these qualities are measured with imprecise values, making it exceedingly challenging to rank biomass materials (DMUs). To solve this problem, this paper offers the new MO-G-IDEA-CE method based on the Boolean possibility degree to tackle the IDEA ranking problems, including seven biomass materials with interval properties and a simple numerical example. Unlike the existing IDEA models, the proposed models can be used to generate the lower and upper bounds of interval efficiencies for all DMUs simultaneously. The optimal weights of inputs and outputs from the proposed models satisfy the entire IDEA, but the optimal weights of inputs and outputs from the existing models do not. Through three examples, we find that the proposed method has a very high correlation with the other methods and provides a new direction for IDEA ranking problems based on the ideas of the traditional Game cross-efficiency method and the Boolean possibility degree. In particular, for the case study of seven biomass materials, after the Spearman correlation test, the correlation coefficients (r_s) for the proposed method and Wang's method, and Wu *et al.*'s method are calculated as $r_s = 1.000$ and 0.964, respectively.

Although three numerical examples have illustrated our method's advantages, potential, and applications, the limitation of the proposed method is that using it for solving more significant IDEA problems or other IDEA problems with various data may be more challenging to calculate the equilibrium point, or it may take a more significant number of calculation iterations. However, for future work, we believe the proposed method can be extended or adapted to tackle other complicated IDEA problems in real-world situations. In addition, it is hard to develop the proposed method with the fuzzy cross-efficiency evaluation method to measure DMUs with fuzzy or missing data, but this direction is worth further investigation.

REFERENCES

[1] Z. Guo, J. Wu, Y. Zhang, F. Wang, Y. Guo, K. Chen, and H. Liu, "Characteristics of biomass charcoal briquettes and pollutant emission reduction for sulfur and nitrogen during combustion," *Fuel*, vol. 272, Jul. 2020, Art. no. 117632.

[2] N. Wichapa, P. Khokhajaikiat, and K. Chaiphet, "Aggregating the results of benevolent and aggressive models by the CRITIC method for ranking of decision-making units: A case study on seven biomass fuel briquettes generated from agricultural waste," *Decis. Sci. Lett.*, vol. 10, no. 1, pp. 79–92, 2021.

[3] K. Promdee, J. Chanvidhwatanakit, S. Satitkune, C. Boonmee, T. Kawichai, S. Jarernprasert, and T. Vitidsant, "Characterization of carbon materials and differences from activated carbon particle (ACP) and coal briquettes product (CBP) derived from coconut shell via rotary kiln," *Renew. Sustain. Energy Rev.*, vol. 75, pp. 1175–1186, Aug. 2017.

[4] B. Falemara, V. Joshua, O. Aina, and R. Nuhu, "Performance evaluation of the physical and combustion properties of briquettes produced from agro-wastes and wood residues," *Recycling*, vol. 3, no. 3, p. 37, Aug. 2018.

[5] M. Njenga, J. K. Gitau, M. Iiyama, R. Jamnadassa, Y. Mahmoud, and N. Karanja, "Innovative biomass cooking approaches for sub-Saharan Africa," *Afr. J. Food, Agricult., Nutrition Develop.*, vol. 19, no. 1, pp. 14066–14087, Oct. 2019.

[6] T. J. Stewart, "Relationships between data envelopment analysis and multicriteria decision analysis," *J. Oper. Res. Soc.*, vol. 47, no. 5, pp. 654–665, May 1996.

[7] C. L. Hwang and K. Yoon, *Multiple Attribute Decision Making: Methods and Applications* (Lecture Notes in Economics and Mathematical Systems), vol. 186, 1st ed. Berlin, Germany: Springer, 1981, pp. 58–191.

[8] V. Penadés-Plà, T. García-Segura, J. Martí, and V. Yepes, "A review of multi-criteria decision-making methods applied to the sustainable bridge design," *Sustainability*, vol. 8, no. 12, p. 1295, Dec. 2016.

[9] B. Yılmaz and M. Yurdusev, "Use of data envelopment analysis as a multi criteria decision tool—A case of irrigation management," *Math. Comput. Appl.*, vol. 16, no. 3, pp. 669–679, Dec. 2011.

[10] K. F. Čiković, I. Martinčević, and J. Lozić, "Application of data envelopment analysis (DEA) in the selection of sustainable suppliers: A review and bibliometric analysis," *Sustainability*, vol. 14, no. 11, p. 6672, May 2022.

[11] C.-N. Wang, T.-T. Dang, N.-A.-T. Nguyen, and J.-W. Wang, "A combined data envelopment analysis (DEA) and grey based multiple criteria decision making (G-MCDM) for solar PV power plants site selection: A case study in Vietnam," *Energy Rep.*, vol. 8, pp. 1124–1142, Nov. 2022.

[12] F. S. M. Chachuli, N. A. Ludin, S. Mat, and K. Sopian, "Renewable energy performance evaluation studies using the data envelopment analysis (DEA): A systematic review," *J. Renew. Sustain. Energy*, vol. 12, no. 6, Nov. 2020, Art. no. 062701.

[13] A. Charnes, W. W. Cooper, and E. Rhodes, "Measuring the efficiency of decision making units," *Eur. J. Oper. Res.*, vol. 2, no. 6, pp. 429–444, Nov. 1978.

[14] R. D. Banker, A. Charnes, and W. W. Cooper, "Some models for estimating technical and scale inefficiencies in data envelopment analysis," *Manag. Sci.*, vol. 30, no. 9, pp. 1078–1092, Sep. 1984.

[15] R. Tamatam, P. Dutta, G. Dutta, and S. Lessmann, "Efficiency analysis of Indian banking industry over the period 2008–2017 using data envelopment analysis," *Benchmarking, Int. J.*, vol. 26, no. 8, pp. 2417–2442, Sep. 2019.

[16] Y. Zhang and H. Hu, "Data envelopment analysis based efficiency measurement of engineering change controlling for infrastructure construction under integrated project delivery mode," *IET Intell. Transp. Syst.*, vol. 14, no. 11, pp. 1433–1439, Nov. 2020.

[17] S. R. Agha, I. Kuhail, N. A. Nabi, M. Salem, and A. Ghanim, "Assessment of academic departments efficiency using data envelopment analysis," *J. Ind. Eng. Manag.*, vol. 4, no. 2, pp. 1–26, Jul. 2011.

[18] E. Kelly, L. Shalloo, U. Geary, A. Kinsella, and M. Wallace, "Application of data envelopment analysis to measure technical efficiency on a sample of Irish dairy farms," *Irish J. Agric. Food Res.*, vol. 51, no. 1, pp. 63–77, Jun. 2012.

[19] A. V. Oghani, F. M. Khiyabani, and N. F.-H. Farahmand, "Data envelopment analysis technique to measure the management ability: Evidence from Iran cement industry," *Cogent Bus. Manag.*, vol. 7, no. 1, Jan. 2020, Art. no. 1801960.

[20] Y. Fan, B. Bai, Q. Qiao, P. Kang, Y. Zhang, and J. Guo, "Study on eco-efficiency of industrial parks in China based on data envelopment analysis," *J. Environ. Manage.*, vol. 192, pp. 107–115, May 2017.

[21] C.-N. Wang, X.-T. Nguyen, and Y.-H. Wang, "Automobile industry strategic alliance partner selection: The application of a hybrid DEA and grey theory model," *Sustainability*, vol. 8, no. 2, p. 173, Feb. 2016.

[22] N. Wichapa, A. Lawong, and M. Donmuen, "Ranking DMUs using a novel combination method for integrating the results of relative closeness benevolent and relative closeness aggressive models," *Int. J. Data Netw. Sci.*, vol. 5, no. 3, pp. 401–416, 2021.

[23] R. Mo, H. Huang, and L. Yang, "An interval efficiency measurement in DEA when considering undesirable outputs," *Complexity*, vol. 2020, Nov. 2020, Art. no. 7161628.

[24] M. Baghery, S. Yousefi, and M. J. Rezaee, "Risk measurement and prioritization of auto parts manufacturing processes based on process failure analysis, interval data envelopment analysis and grey relational analysis," *J. Intell. Manuf.*, vol. 29, no. 8, pp. 1803–1825, Dec. 2018.

[25] A. Torres-Ruiz and A. R. Ravindran, "Use of interval data envelopment analysis, goal programming and dynamic eco-efficiency assessment for sustainable supplier management," *Comput. Ind. Eng.*, vol. 131, pp. 211–226, May 2019.

[26] W. W. Cooper, K. S. Park, and G. Yu, "IDEA and AR-IDEA: Models for dealing with imprecise data in DEA," *Manag. Sci.*, vol. 45, no. 4, pp. 597–607, Apr. 1999.

[27] D. K. Despotisa and Y. G. Smirlis, "Data envelopment analysis with imprecise data," *Eur. J. Oper. Res.*, vol. 140, no. 1, pp. 24–36, Jul. 2002.

[28] T. Entani, Y. Maeda, and H. Tanaka, "Dual models of interval DEA and its extension to interval data," *Eur. J. Oper. Res.*, vol. 136, no. 1, pp. 32–45, Jan. 2002.

[29] Y. M. Wang, R. Greatbanks, and J. B. Yang, "Interval efficiency assessment using data envelopment analysis," *Fuzzy Sets Syst.*, vol. 153, no. 3, pp. 347–370, Aug. 2005.

[30] N.-S. Wang, R.-H. Yi, and W. Wang, "Evaluating the performances of decision-making units based on interval efficiencies," *J. Comput. Appl. Math.*, vol. 216, no. 2, pp. 328–343, Jul. 2008.

[31] H. Azizi and R. Jahed, "Improved data envelopment analysis models for evaluating interval efficiencies of decision-making units," *Comput. Ind. Eng.*, vol. 61, no. 3, pp. 897–901, Oct. 2011.

[32] M. Toloo, E. Keshavarz, and A. Hatami-Marbini, "Dual-role factors for imprecise data envelopment analysis," *Omega*, vol. 77, pp. 15–31, Jun. 2018.

[33] J. Sun, Y. Miao, J. Wu, L. Cui, and R. Zhong, "Improved interval DEA models with common weight," *Kybernetika*, vol. 50, no. 5, pp. 774–785, Dec. 2014.

[34] J. Wu, J. Sun, M. Song, and L. Liang, "A ranking method for DMUs with interval data based on DEA cross-efficiency evaluation and TOPSIS," *J. Syst. Sci. Syst. Eng.*, vol. 22, no. 2, pp. 191–201, Jun. 2013.

[35] L. Wang, L. Li, and N. Hong, "Entropy cross-efficiency model for decision making units with interval data," *Entropy*, vol. 18, no. 10, p. 358, Oct. 2016.

[36] L. Liang, J. Wu, W. D. Cook, and J. Zhu, "The DEA game cross-efficiency model and its Nash equilibrium," *Oper. Res.*, vol. 56, no. 5, pp. 1278–1288, Mar. 2008.

[37] R. Ma, L. Yao, M. Jin, and P. Ren, "The DEA game cross-efficiency model for supplier selection problem under competition," *Appl. Math. Inf. Sci.*, vol. 8, no. 2, pp. 811–818, Mar. 2014.

[38] Y. Sun, H. Huang, and C. Zhou, "DEA game cross-efficiency model to urban public infrastructure investment comprehensive efficiency of China," *Math. Problems Eng.*, vol. 2016, pp. 1–10, Mar. 2016.

[39] F. Dong, Y. Zhang, and X. Zhang, "Applying a data envelopment analysis game cross-efficiency model to examining regional ecological efficiency: Evidence from China," *J. Cleaner Prod.*, vol. 267, Sep. 2020, Art. no. 122031.
[40] L. Wang, X. Song, and X. Song, "Research on the measurement and spatial–temporal difference analysis of energy efficiency in China's construction industry based on a game cross-efficiency model," *J. Cleaner Prod.*, vol. 278, Jan. 2021, Art. no. 123918.
[41] X.-Q. Zou, M. Guo, W.-C. Zou, Q.-Q. Song, C. Tian, and Y. Ma, "The new model of game cross efficiency DEA with index groups and an application to land utilization efficiency," *IEEE Access*, vol. 10, pp. 3608–3616, 2022, doi: 10.1109/ACCESS.2021.3139084.
[42] Z. S. Xu and Q. L. Da, "The uncertain OWA operator," *Int. J. Intell. Syst.*, vol. 17, no. 6, pp. 569–575, Apr. 2002.
[43] Y. Nakahara, M. Sasaki, and M. Gen, "On the linear programming problems with interval coefficients," *Comput. Ind. Eng.*, vol. 23, nos. 1–4, pp. 301–304, Nov. 1992.
[44] Y. Nakahara, "User oriented ranking criteria and its application to fuzzy mathematical programming problems," *Fuzzy Sets Syst.*, vol. 94, no. 3, pp. 275–286, Mar. 1998.
[45] G. Facchinetti, R. G. Ricci, and S. Muzzioli, "Note on ranking fuzzy triangular numbers," *Int. J. Intell. Syst.*, vol. 13, no. 7, pp. 613–622, Jul. 1998.
[46] Y.-M. Wang, J.-B. Yang, and D.-L. Xu, "A two-stage logarithmic goal programming method for generating weights from interval comparison matrices," *Fuzzy Sets Syst.*, vol. 152, no. 3, pp. 475–498, Jun. 2005.
[47] D. Li, W. Zeng, and Q. Yin, "Novel ranking method of interval numbers based on the Boolean matrix," *Soft Comput.*, vol. 22, no. 12, pp. 4113–4122, Jun. 2018.

NARONG WICHAPA received the B.Eng., M.Eng., and Ph.D. degrees in industrial engineering from Khon Kaen University. He is currently an Assistant Professor with the Department of Industrial Engineering, Kalasin University. His research interests include multi-criteria decision making, mathematical optimization, data envelopment analysis, heuristics and meta-heuristics, and operations research.

SUPHAN SODSOON received the Ph.D. degree in industrial engineering from Ubon Ratchathani University. He is currently an Associate Professor with the Department of Industrial Engineering, Kalasin University. His research interests include heuristics and meta-heuristics, and operations research.