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RESEARCH ARTICLE

Fast Speed Convergent Stability of T-S Fuzzy Sliding-Mode Control and Disturbance Observer for a Secure Communication of Chaos-Based System

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ABSTRACT This article presents the fast convergent stability of disturbance observer (DO) and sliding mode control (SMC) for a secure communication of fractional-order chaotic-based system. First, the fractional-order is remodeled into a Takagi-Sugeno fuzzy (TSF) system with the aim of softening the calculations of observer and controller design. Second, the master and slave systems (MSSs) were synchronized by the fast convergent stability (FCS) sliding mode control with double phases of the same stability condition. Third, the disturbance observer was newly proposed for estimating the disturbance and uncertainty of the secure communication system (SCS). Fourth, the stability of the proposed method was archived via the Lyapunov condition. The MATLAB simulation with support of FOMCON tool box was used to validate the correction of the proposed control theory. The obtained results such as small tracking errors and small settling-times were used to confirm that the proposed theory is good at rejecting perturbations and used control method is good at synchronizing the chaotic systems.

INDEX TERMS Disturbance observer, sliding-mode control, Takagi-Sugeno fuzzy, master and slave systems, fast convergent stability, secure communication system.

I. INTRODUCTION

The secure communication of chaos-based system (CBS) is an attractive topics in applied control area. The background of the secure communication of CBS is synchronization [1]. The discussions of secure communication system (SCS) of CBS based on internet communication can be found in [2] and [3]. The secure communications of the CBS of electronic circuits can be found in [4] and [5]. The synchronization control methods of electronic circuits can be found in [6], [7], [8], and [9]. The synchronization of network system can be found in [10], [11], and [12]. Some neural networks are used for chaos and secure communication, such as brain imitating neural network [13], dual function link brain emotional [14] and TSK Brain emotional learnings controller [15].

SCS of the CBS can be applied for image encryption [16], [17], for satellite image [17], and for multiple image and video encryption [18]. To suppress the disturbance and uncertainty of the SCS of CBS, the mathematical model of the chaotic systems should be remodeled by the Takagi-Sugeno fuzzy (TSF) systems. The synchronization control design and disturbance observer (DO) design are then simpler the in original format.

The concept of TSF modeling can be found in [19]. TSF consists of sub-linear systems and fuzzy memberships. The applications of TSF modeling in control can be found in

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[20], [21], [22], and [23]. In [24] the H_∞ was designed for T-S fuzzy system. The energy-to-peak filtering for T-S fuzzy system can be found in [25]. To the best of our knowledge, the number of discussion of DO for such a synchronization of CBS based on the TSF control is limited. There are improvement of nonlinear DO [1], [3], [4], and [6]. This paper used the basic concept of the previous published paper of the new DO of inversed model-based [26], [27], [28] to design a new fast DO for the SCS of CBS. There are some first publications of the new DO methods.

DO is an estimation method for estimating the unknown information. Due to the reason of cost, structure, working space. The DO is now more and more applied in industrial equipment. As well known, the nonlinear DO can be found in [29]. The applications of the nonlinear DO can be found in [30] and [31]. Sometimes, the DO needs the information of disturbance such as a fixed-format [32]. This paper aim to provide a DO without the conjunction of the first derivative and the fixed format of disturbance. The proposed DO with the fast reaching law of previous SMC with an improvement of adaptive gains to avoid the chattering phenomenon. The DO can work with the background of SMC. Which was used to synchronize the MSSs of SCS of CBS.

SMC is a nonlinear control method with existence of equivalent and switching control values. These values are used to stabilize and force the state of system onto the predefined surface. The basic concept of SMC can be found in [33] and [34]. The mixed-delays and uncertainty problems were investigated by using robust SMC [35]. The main problem of SMC is chattering, which comes from the switching control. To avoid the inversed effects of chattering, the boundary layer thickness of SMC can be found in [36]. In this paper, the fast reaching law is used with the adaptive law to ovoid the chattering. The basic mathematical model of the fast reaching law can be found in [37]. The SMC in this paper is aim to force the states of MSSs close to each other. The MSSs in this paper are fractional-order (FO). The mathematical models of MSSs are reused from [4] and [38]. The FO calculus operations can be found in [39].

Motivations of this paper are came from the previous papers. The proposed DO in [1] is complicated. In [3], the DO is depended on the SMC. In [5], disturbance rejection of SMC was not mentioned. The number discussions of complicated disturbance and uncertainty is limited. Especially, the free condition of the first derivative disturbance for SCS is also limited number of investigations. Combinations of SMC and DO, the contributions of this paper are listed as follows:

1. The MSSs were synchronized with an improvement of fast reaching law SMC. The double phases with the same reaching condition were proposed for synchronizing two different chaotic systems. The adaptive laws were designed to avoid the chattering value. The proof of improvement of SMC was provided in this paper.
2. A new DO was proposed for rejecting both disturbance and parameter variations of SCS. Two case studies were used for illustrating the performances of proposed DO with disturbances and fully disturbances and uncertainties. In both cases, the performances were good for rejection ability.
3. The mathematical proof was provided precisely together with the simulation in MATLAB software to show the correction and effectiveness of proposed method on the SCS of CBS.

The organization of this paper is as follows: First, the introduction of the basic related control method is presented. Second, the preliminary mathematics are presented. There are mathematical of fractional-order (FO) of Chen chaotic system together with some FO calculus operations and primary concept of fast convergent speed SMC and proposed DO are all represented. Third, the proposed control methods for SCS of CBS are presented. Fourth, an illustrative example is given to show the effectiveness and correction of the proposed theories. Final, the conclusion and future work will be given to conclude the contributions of the work and draw the future direction of our research.

Notes: $sign(\mathbf{x}) = \left[sign(\mathbf{x}_1),\ sign(\mathbf{x}_2),\ ,sign(\mathbf{x}_n) \right]^T$ where $\mathbf{x} = \left[\mathbf{x}_1,\ \mathbf{x}_2,\ ,\mathbf{x}_n \right]^T$. $sign(\mathbf{x}_i) = \frac{\mathbf{x}_i}{|\mathbf{x}_i|}$. $sign(\mathbf{x}_i) = 1$ if $\mathbf{x}_i > 0$, $sign(\mathbf{x}_i) = -1$ if $\mathbf{x}_i < 0$, and $sign(\mathbf{x}_i) = 0$ if $\mathbf{x}_i = 0$. $I^{m\times m}$ is the identity matrix. $\boldsymbol{A}^T$ is transposed matrix of $\boldsymbol{A}$.

II. MATHEMATICAL MODEL OF CHAOTIC SYSTEM AND SOME PRELIMINARY MATHEMATICS

In this section, the preliminary of FO, TSF, fast convergent SMC, and proposed DO are all presented. First, the concepts of FO operation is presented. Second, the concept of TSF is presented. Third, the basic concept of fast convergent SMC is represented. Final, the basic operation of the proposed DO for SCS is shown.

A. FO CALCULUS

Definition 1 [39]: The Euler's Gamma function.

The Gamma function is

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-\alpha} dt \tag{1}$$

Herein, the α and t are the order and the time of the the operation.

Definition 2 [39]: Fractional function derivatives and its integrals.

$$D_t^\alpha = \begin{cases} \dfrac{d^\alpha}{dt^\alpha}, & \alpha > 0 \\ 1, & \alpha = 0 \\ \displaystyle\int_a^t (d\tau)^{-\alpha}, & \alpha < 0 \end{cases} \tag{2}$$

Definition 3 [39]: The Caputo fractional derivative.

$$D_t^\alpha h(t) = \frac{1}{\Gamma(l-\alpha)} \int_a^t \frac{f^\alpha(\tau)}{(t-\tau)^{\alpha-l+1}} d\tau \tag{3}$$

where $l - 1 < \alpha < l$.

Definition 4 [39]: Stability of the FO system.
Consider

$$D_t^{\alpha} X = h(X) \tag{4}$$

where $X = [X_1, \dots X_l]^T$ *and* $h(X) = [h_1(X), \dots h_l(X)]^T$ *are the state and* the functions of the system. Where $0 < \alpha < 1$ *is the FO*. System (*4*) stable if

$$|\arg(eig(J))| > \alpha \frac{\pi}{2} \tag{5}$$

where $J = \partial h(X)/\partial x$. Some properties of FO calculus are as below.

Property 1: If $\alpha = 0$ the operation is then

$$D_t^0 h(X) = h(X) \tag{6}$$

Property 2: Caputo operation with property of linearization

$$D_t^n(g(X) + h(X)) = D_t^n g(X) + D_t^n h(X) \tag{7}$$

Property 3: The product property.

$$D_t^{n+m} h(X) = D_t^n D_t^m h(X) \tag{8}$$

B. TSF MODELING

Definition 5: TSF modeling [19].
By considering system

$$\begin{cases} \dot{X} = g(X, u)X + h(X, u)u \\ y = l(X, u)X \end{cases} \tag{9}$$

where X and y are state and output vectors, respectively. g, h, and l are the smooth functions. The condition $X_j \in [X_{\min}, X_{\max}]$, where $j = 1, \dots, p$. The weighting function s are

$$\begin{cases} n_0^j(\cdot) = \dfrac{X_{\max} - X_j(\cdot)}{X_{\max} - X_{\min}} \\ n_1^j(\cdot) = 1 - n_0^j(\cdot) \end{cases} \tag{10}$$

The fuzzy membership is

$$\varphi_i(X) = \prod_{j=1}^{p} \varphi_{ij}(X_i) \tag{11}$$

where $\varphi_{ij}(X_i)$ is either $n_0^j(\cdot)$ or $n_1^j(\cdot)$. System (9) can be

$$\begin{cases} \dot{X} = \sum_{i=1}^{m} \varphi_i(X_j)(A_i X + B_i u) \\ y = \sum_{i=1}^{m} \varphi_i(X_j) C_i X \end{cases} \tag{12}$$

System (12) is called TSF system. m is number of fuzzy rules.

C. TSF MODELING

Consider the reaching law as follows:

$$\dot{s} = -\frac{\gamma}{\sigma_N + \kappa \exp(-\alpha_N |s|)} sat(\frac{s}{\varepsilon_0 + \int |e_x|}) \tag{13}$$

where s is surface of SMC, $\gamma > 0, 0 < \sigma_N < 1, \alpha_N > 0$, and $\kappa > 0$. The Lyapunov is selected as follows:

$$V(s) = \frac{1}{2} s^2 \tag{14}$$

Taking derivative for Eq. (14) have

$$\begin{aligned} \dot{V}(s) &= s\dot{s} \\ &= s(-\frac{\gamma}{\sigma_N + \kappa \exp(-\alpha_N |s|)} sat(\frac{s}{\varepsilon_0 + \int |e_x|})) \end{aligned} \tag{15}$$

Case 1: $|s| > \varepsilon_0 + \int |e_x|$

According to the [37], the settling-time of Eq. (15) can be calculated by solving Eq. (22).

$$\dot{s}[\sigma_N + \kappa \exp(-\alpha_N |s|)] = -\kappa sign(s) \tag{16}$$

Integrating Eq. (22) respect to the time from zero to $T_{\max}$, where $T_{\max}$ is maximum value of time, where the stability is obtained.

$$\int_{s(0)}^{s(T_{\max})} \frac{1}{sign(s)} [\sigma_N + \kappa \exp(-\alpha_N |s|)] ds = \int_0^{T_{\max}} -\gamma \tag{17}$$

If $s \geq 0$, we have

$$\sigma_N s|_{s(0)}^{s(T_{\max})} + \int_{s(0)}^{s(T_{\max})} \kappa \exp(-\alpha_N |s|) ds = -\gamma t|_0^{T_{\max}} \tag{18}$$

or

$$T_{\max} = \frac{\sigma_N s(0)}{\gamma} - \frac{1}{\gamma} \int_{s(0)}^{s(T_{\max})} \kappa \exp(-\alpha_N |s|) ds \tag{19}$$

or

$$T_{\max} = \frac{\sigma_N s(0)}{\gamma} + \frac{1}{\gamma \alpha_N} \kappa (1 - \exp(-\alpha_N s(0))) \tag{20}$$

If $s < 0$, Eq. (17) becomes

$$T_{\max} = -\frac{\sigma_N s(0)}{\gamma} + \frac{1}{\gamma \alpha_N} \kappa (1 - \exp(\alpha_N s(0))) \tag{21}$$

or

$$T_{\max} = \frac{\sigma_N |s(0)|}{\gamma} + \frac{1}{\gamma \alpha_N} \kappa (1 - \exp(-\alpha_N |s(0)|)) \tag{22}$$

We have $1 - \exp(-\alpha_N |s(0)|) < 0$, then

$$T_{\max} < \frac{\sigma_N |s(0)|}{\gamma} \tag{23}$$

The settling-time of reaching law in Eq. (13) better than the conventional SMC with reaching law as below.

$$\dot{s} = -\kappa sign(s) \tag{24}$$

Case 2: $|s| \leq \varepsilon_0 + \int |e_x|$

We have

$$\dot{s}[\sigma_N + \kappa \exp(-\alpha_N |s|)] < -\gamma \tag{25}$$

Integrating both sides Eq. (25) yields

$$\int_{s(0)}^{s(T_{\max})} [\sigma_N + \kappa \exp(-\alpha_N |s|)]ds = \int_0^{T_{\max}} -\gamma \tag{26}$$

or

$$T_{\max} = \frac{\sigma_N s(0)}{\gamma} - \frac{1}{\gamma} \int_{s(0)}^{s(T_{\max})} \kappa \exp(-\alpha_N |s|)ds \tag{27}$$

If $s \geq 0$, Eq. (27) can be

$$T_{\max} = \frac{\sigma_N s(0)}{\gamma} - \frac{\kappa}{\gamma\alpha_N}(1 - \exp(-\alpha_N s(0))) \tag{28}$$

Maximum value of settling-time of Eq. (28) is

$$T_{\max} < \frac{\sigma_N}{\gamma} \tag{29}$$

If $s < 0$, Eq. (32) can be written as follows:

$$T_{\max} = \frac{\sigma_N s(0)}{\gamma} - \frac{\kappa}{\gamma\alpha_N}(1 - \exp(\alpha_N s(0))) \tag{30}$$

or

$$T_{\max} < \frac{1}{\gamma\alpha_N}\kappa(1 - \exp(-|\alpha_N s(0)|)) \tag{31}$$

The development of fast reaching law of SMC is better than the conventional SMC. Then, the reaching law of this section is used for constructing the synchronization and DO. In this paper, the mathematical model of previous paper [4] is reused. The master system is

$$\frac{D^\alpha}{dt}X_m(t) = \sum_{i=1}^{2} \omega(X_{mi}(t)A_iX_m(t) + Cl_m(t)) \tag{32}$$

where $l_m(t) = \left[\, l_{xm}(t) \;\; l_{ym}(t) \;\; l_{zm}(t) \,\right]^T$ is master' disturbance and uncertainty vector. The parameters of (32) are as follows:

$$C = I^{3\times3}, \quad A_1 = \begin{bmatrix} -35 & 35 & 0 \\ -7 & 28 & -100 \\ 0 & 25 & -3 \end{bmatrix} \text{ and } A_2 = \begin{bmatrix} -35 & 35 & 0 \\ -7 & 28 & 100 \\ 0 & -25 & -3 \end{bmatrix}.$$

The slave system is

$$\frac{D^\alpha}{dt}X_s(t) = \sum_{i=1}^{2} \omega(X_{si}(t)A_iX_s(t) + Bu_s(t) + Cl_s(t)) \tag{33}$$

where $l_s(t) = \left[\, l_{xs}(t) \;\; l_{ys}(t) \;\; l_{zs}(t) \,\right]^T$ is disturbance and uncertainty vector of slave system. $B = I^{3\times3}$.

Assumption 1: The disturbances and uncertainties of the MSSs need to be bounded as follows: $|l_{xm}(t)| \leq L_{m1}$, $|l_{ym}(t)| \leq L_{m2}$, $|l_{zm}(t)| \leq L_{m3}$, $|l_{xs}(t)| \leq L_{s1}$, $|l_{ys}(t)| \leq L_{s2}$, and $|l_{zs}(t)| \leq L_{s3}$, where $L_{m1}, L_{m2}, L_{m3}, L_{s1}, L_{s2}$, and L_{s3} are positively defined.

The proposed DO can be found below.

Remark 1: The parameters of master and slave systems were an example of remodel the original chaotic system to the TSF system. These value depends on the rescaled states of chaotic system.

D. PROPOSED DO

We have

$$\frac{D^\alpha}{dt}X_m(t) - \frac{D^\alpha}{dt}X_s(t) = \sum_{i=1}^{2} \omega(X_{mi}(t)A_iX_m(t) + Cl_m(t)) - \sum_{i=1}^{2} \omega(X_{si}(t)A_iX_s(t) + Bu_s(t) + Cl_s(t)) \tag{34}$$

or

$$\sum_{i=1}^{2} \omega(X_{is}(t))A_iX_s(t) + Bu_s(t) - \sum_{i=1}^{2} \omega(X_{im}(t))A_iX_m(t) = \frac{D^\alpha}{dt^{\alpha-1}}X_s(t) - Cl_s(t) - (\frac{D^\alpha}{dt}X_m(t) - Cl_m(t)) \tag{35}$$

or

$$Cl(t) = \sum_{i=1}^{2} \omega(X_{is}(t))A_iX_s(t) + Bu_s(t) - \sum_{i=1}^{2} \omega(X_{im}(t))A_iX_m(t) \tag{36}$$

According to the previous paper [38], if $\frac{D^\alpha}{dt^\alpha}X_s(t) \to \frac{D^\alpha}{dt^\alpha}X_m(t)$ or $X_s(t) \to X_m(t).l(t) = [\, l_x(t),\ \ l_y(t),\ \ l_z(t)\,]^T$. In this paper the proposed DO is shown in Theorem 1 below.

Theorem 1: If DO for the SCS in Eq. (34) is proposed as follows:

$$\hat{l}_j(t) = (C_{jj})\,[\sum_{i=1}^{2} \omega(X_{ijs}(t))A_{ij}X_s(t) + B_{ij}u_s(t) - \sum_{i=1}^{2} \omega(X_{ijm}(t))A_{ij}X_m(t)] + \int \frac{\gamma_{lj}}{\sigma_{Nlj} + \kappa_{lj} exp(-\alpha_{Nlj}\left|\tilde{l}_j(t)\right|)} sat(\frac{\tilde{l}_j(t)}{\varepsilon_{0lj} + \int \left|\tilde{l}_j(t)\right|}) \tag{37}$$

The disturbance tracking error is then calculated as follows:

$$T_{\tilde{l}_j \max} < \frac{\sigma_{Nlj}\left|\tilde{l}_j(0)\right|}{\gamma_{lj}} \tag{38}$$

if

$$\left|\tilde{l}_j\right| > \varepsilon_{0lj} + \int \left|\tilde{l}_j(t)\right| \tag{39}$$

and

$$T_{\tilde{l}j\max} < \frac{1}{\gamma_{\tilde{l}j}\alpha_{N\tilde{l}j}}\kappa_{\tilde{l}j}(1 - \exp(-\left|\alpha_{N\tilde{l}j}\tilde{l}(0)\right|)) \tag{40}$$

when

$$\left|\tilde{l}_j\right| \le \varepsilon_{0lj} + \int \left|\tilde{l}_j(t)\right| \tag{41}$$

Proof: The disturbance error is

$$\tilde{l}_j(t) = (C_{jj}) \sum_{i=1}^{2} \omega(X_{ijs}(t))A_{ij}X_{sj}(t) + B_{ij}u_{sj}(t) - \sum_{i=1}^{2} \omega(X_{ijm}(t))A_{ij}X_{mj}(t) - \hat{l}_j(t) \tag{42}$$

Taking the first derivative for both sides of Eq. (42) yields

$$\dot{\tilde{l}}_j(t) = \dot{l}_j(t) + \frac{\gamma_{lj}}{\sigma_{Nlj} + \kappa_{lj}exp(-\alpha_{Nlj}\left|\tilde{l}_j(t)\right|)} sat(\frac{\tilde{l}_j(t)}{\varepsilon_{0lj} + \int \left|\tilde{l}_j(t)\right|}) \tag{43}$$

or

$$\dot{\tilde{l}}_j(t) = -\frac{\gamma_{lj}}{\sigma_{Nlj} + \kappa_{lj}exp(-\alpha_{Nj}\left|\tilde{l}_j(t)\right|)} sat(\frac{\tilde{l}_j(t)}{\varepsilon_{0lj} + \int \left|\tilde{l}_j(t)\right|}) \tag{44}$$

The stablity of the proposed DO is same with the reaching law in Eq. (13).

This completes proof of Theorem 1:

III. PROPOSED APPROACH

A. SMC FOR SCS

The surface of SMC for each axis is designed as below.

$$s_j = \frac{D^{\alpha-1}}{dt^{\alpha-1}}e_j + \frac{D^{\alpha-2}}{dt^{\alpha-2}}[\frac{\gamma_{ej}}{\sigma_{Nej} + \kappa\ \exp(-\alpha_{N_{ej}}\left|e_j\right|)} sat(\frac{e_j}{\varepsilon_{0ej} + \int \left|e_j\right|})] \tag{45}$$

where $j = 1 \div 3$, taking derivative for Eq. (45) have

$$\dot{s}_j = \frac{D^{\alpha}}{dt^{\alpha}}e_j + \frac{D^{\alpha-1}}{dt^{\alpha-1}}[\frac{\gamma_{ej}}{\sigma_{Nej} + \kappa ejexp(-\alpha_{N_{ej}}\left|e_j\right|)} sat(\frac{e_j}{\varepsilon_{0ej} + \int \left|e_j\right|})] \tag{46}$$

By ignoring the effection of disturbance and considering $\dot{s}_j = 0$,. Solving Eq. (46) by

$$0 = \frac{D^{\alpha}}{dt^{\alpha}}e_j + \frac{D^{\alpha-1}}{dt^{\alpha-1}}[\frac{\gamma_{ej}}{\sigma_{Nej} + \kappa ejexp(-\alpha_{N_{ej}}\left|e_j\right|)} sat(\frac{e_j}{\varepsilon_{0ej} + \int \left|e_j\right|})] \tag{47}$$

or

$$\sum_{i=1}^{2} \omega(X_{im}(t))A_iX_m(t) - \sum_{i=1}^{2} \omega(X_{is}(t))A_iX_s(t) - Bu_s(t) + \frac{D^{\alpha-1}}{dt^{\alpha-1}}[\frac{\gamma_{ej}}{\sigma_{Nej} + \kappa ejexp(-\alpha_{N_{ej}}\left|e_j\right|)} sat(\frac{e_j}{\varepsilon_{0ej} + \int \left|e_j\right|})] = 0 \tag{48}$$

The equivalnent control is then calculated

$$B_{j,j}u_{eqsj} = \sum_{i=1}^{2} \omega(\Omega_{ijm})A_{ij}X_m - \sum_{i=1}^{2} \omega(\Omega_{ijs})A_{ij}X_s + \frac{D^{\alpha-1}}{dt^{\alpha-1}}[\frac{\gamma_{ej}}{\sigma_{Nej} + \kappa ejexp(-\alpha_{N_{ej}}\left|e_j\right|)} sat(\frac{e_j}{\varepsilon_{0ej} + \int \left|e_j\right|})] \tag{49}$$

The reaching law is

$$B_{j,j}u_{sws_j}(t) = \frac{\gamma_{sj}}{\sigma_{Nsj} + \kappa_{sj}exp(-\alpha_{Nsj}\left|s_{sj}\right|)} sat(\frac{s_j}{\varepsilon_{0sj} + \int \left|e_{xj}\right|}) \tag{50}$$

The proposed DO for SCS can be found in Eq. (37).

B. STABILITY ANALYSIS

The Lyapunov condition is selected as follows:

$$V_j(t) = \frac{1}{2}s_j^2 + \frac{1}{2}\tilde{l}_j^2 \tag{51}$$

Taking the derivative for Eq. (51) have

$$\dot{V}_j(t) = s_j\dot{s}_j + \tilde{l}_j\dot{\tilde{l}}_j \tag{52}$$

By using Eqs. (37) and (45-50) to solve Eq. (52) yields

$$\begin{aligned}\dot{V}_j &= s_j\dot{s}_j + \tilde{l}_j\dot{\tilde{l}}_j \\ &= -s_j[\frac{\gamma_{sj}}{\sigma_{Nsj} + \kappa_{sj}exp(-\alpha_{Nsj}\left|s_{sj}\right|)} sat(\frac{s_j}{\varepsilon_{0sj} + \int \left|e_{xj}\right|})] \\ &\quad - \tilde{l}_j(\frac{\gamma_j}{\sigma_{Nj} + \kappa_j exp(-\alpha_{Nj}\left|\tilde{l}_j(t)\right|)} sat(\frac{\tilde{l}_j(t)}{\varepsilon_{0j} + \int \left|\tilde{l}_j(t)\right|})) \\ &< 0\end{aligned} \tag{53}$$

This completes the proof.

Therefore, the proposed DO for three axes are as below. First, DO for x-axis is

$$\begin{aligned}C_{1,1}\hat{l}_x(t) = [&\sum_{i=1}^{2} \omega(X_{ijs}(t))A_{11}X_s(t) + B_{11}u_s(t) \\ &- \sum_{i=1}^{2} \omega(X_{ijm}(t))A_{11}X_m(t)] \\ &+ \int \frac{\gamma_{l1}}{\sigma_{Nl1} + \kappa_{lj}exp(-\alpha_{Nl1}\left|\tilde{l}_1(t)\right|)} \\ &\times sat(\frac{\tilde{l}_1(t)}{\varepsilon_{0l1} + \int \left|\tilde{l}_1(t)\right|})\end{aligned} \tag{54}$$

DO for y-axis is

$$C_{2,2}\hat{l}_y(t) = [\sum_{i=1}^{2} \omega(X_{ijs}(t))A_{i,2}X_s(t) + B_2u_s(t)$$

$$- \sum_{i=1}^{2} \omega(X_{ijm}(t))A_{i,2}X_m(t)] + \int \frac{\gamma_{l2}}{\sigma_{Nl2} + \kappa_{l2} exp(-\alpha_{Nl2} \left|\tilde{l}_2(t)\right|)} \times sat(\frac{\tilde{l}_2(t)}{\varepsilon_{0l2} + \int \left|\tilde{l}_2(t)\right|}) \tag{55}$$

DO for z-axis is

$$C_{3,3}\hat{l}_z(t) = [\sum_{i=1}^{2} \omega(X_{ijs}(t))A_{31}X_s(t) + B_{31}u_s(t) - \sum_{i=1}^{2} \omega(X_{ijm}(t))A_{31}X_m(t)] + \int \frac{\gamma_{l3}}{\sigma_{Nl3} + \kappa_{l3} exp(-\alpha_{Nl3} \left|\tilde{l}_3(t)\right|)} \times sat(\frac{\tilde{l}_3(t)}{\varepsilon_{0l3} + \int \left|\tilde{l}_3(t)\right|}) \tag{56}$$

The equivalent control value for x-axis is

$$B_{1,1}u_{eqs1} = \sum_{i=1}^{2} \omega(\Omega_{ijm})A_{11}X_m - \sum_{i=1}^{2} \omega(\Omega_{ijs})A_{11}X_s + \frac{D^{\alpha-1}}{dt^{\alpha-1}}[\frac{\gamma_{e1}}{\sigma_{Ne1} + \kappa e1 exp(-\alpha_{N_{e1}} |e_1|)} sat(\frac{e_1}{\varepsilon_{0e1} + \int |e_1|})] \tag{57}$$

The reaching law is

$$B_{1,1}u_{sws1}(t) = \frac{\gamma_{s1}}{\sigma_{Ns1} + \kappa_{s1} exp(-\alpha_{Ns1} |s_{s1}|)} sat(\frac{s_1}{\varepsilon_{0s1} + \int |e_{x1}|}) \tag{58}$$

The equivalent control value for y-axis is

$$B_{1,2}u_{eqs2} = \sum_{i=1}^{2} \omega(\Omega_{ijm})A_{12}X_m - \sum_{i=1}^{2} \omega(\Omega_{ijs})A_{12}X_s + \frac{D^{\alpha-1}}{dt^{\alpha-1}}[\frac{\gamma_{e2}}{\sigma_{Ne2} + \kappa e2 exp(-\alpha_{N_{e2}} |e_2|)} sat(\frac{e_2}{\varepsilon_{0e2} + \int |e_2|})] \tag{59}$$

The reaching law is

$$B_{1,2}u_{sws2}(t) = \frac{\gamma_{s2}}{\sigma_{Ns2} + \kappa_{s2} exp(-\alpha_{Ns2} |s_{s2}|)} sat(\frac{s_2}{\varepsilon_{0s2} + \int |e_{x2}|}) \tag{60}$$

The equivalent control value for z-axis is

$$B_{1,3}u_{eqs3} = \sum_{i=1}^{2} \omega(\Omega_{ijm})A_{13}X_m - \sum_{i=1}^{2} \omega(\Omega_{ijs})A_{13}X_s$$

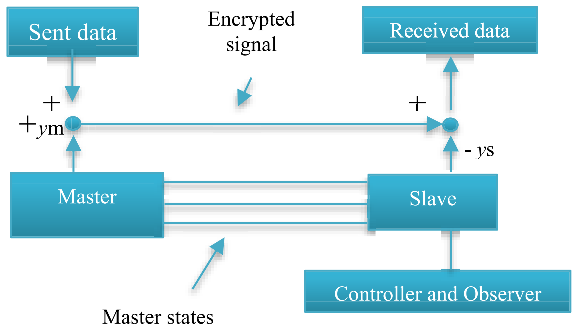

FIGURE 1. SCS of CBS.

$$+ \frac{D^{\alpha-1}}{dt^{\alpha-1}}[\frac{\gamma_{e3}}{\sigma_{Ne3} + \kappa e3 exp(-\alpha_{N_{e3}} |e_3|)} sat(\frac{e_3}{\varepsilon_{0e3} + \int |e_3|})] \tag{61}$$

The reaching law is

$$B_{1,3}u_{sws3}(t) = \frac{\gamma_{s3}}{\sigma_{Ns3} + \kappa_{s3} exp(-\alpha_{Ns3} |s_{s3}|)} sat(\frac{s_3}{\varepsilon_{0s3} + \int |e_{x3}|}) \tag{62}$$

Remark 2: $A_{i,j}$ is element at row i[th] and colum j[th] of maxtrix A. A_{ij} is row i with colum from 1 to j.

IV. AN ILLUSTRATIVE EXAMPLE

This paper used the MATLAB simulation to validate the correction and power of the proposed control method. There are two cases of studies of parameter variations and both disturbance and parameter variations are considered. The structure of SCS of CBS is shown in Figure 1 below.

Remark 3: The order of FO chaotic system was selected $\alpha = 0.98$.

In Figure 1, the basic communication method of CBS is shown with the function of secure the data. First, the states of MSSs need to be precisely tracked each other. Second, any changes need to be suppressed such as disturbance, uncertainty or sometimes the attack from hacker or destroyer. The parameters of control system are as follows:

The parameters of sliding surface were designed as follows: $\sigma_{Ne1} = 0.75, \kappa_{e1} = 10, \alpha_{N_{e1}} = 10, \gamma_{e1} = 5, \varepsilon_{0e1} = 0.1, \sigma_{Ne2} = 0.8, \kappa_{e2} = 2, \alpha_{N_{e2}} = 20, \gamma_{e2} = 15, \varepsilon_{0e2} = 0.15, \sigma_{Ne3} = 0.85, \kappa_{e3} = 2, \alpha_{N_{e3}} = 15, \gamma_{e3} = 3$, and $\varepsilon_{0e3} = 0.2$.

The parameters of reaching law are as follows: $\sigma_{Ns1} = 0.9, \kappa_{s1} = 15, \alpha_{N_{s1}} = 15, \gamma_{s1} = 500, \varepsilon_{0s1} = 0.15, \sigma_{Ns2} = 0.85, \kappa_{s2} = 15, \alpha_{N_{s2}} = 15, \gamma_{s2} = 500, \varepsilon_{0e2} = 0.15, \sigma_{Ns3} = 0.95, \kappa_{s3} = 20, \alpha_{N_{s3}} = 20, \gamma_{s3} = 1 = 500$, and $\varepsilon_{0s3} = 0.2$.

The parameters of proposed DO are as follows: $\sigma_{Nl1} = 0.8, \kappa_{l1} = 0.15, \alpha_{N_{l1}} = 0.15, \gamma_{l1} = 3750, \varepsilon_{0l1} = 20, \sigma_{Nl2} = 0.85, \kappa_{l2} = 1.5, \alpha_{N_{l2}} = 1.5, \gamma_{l2} = 3750, \varepsilon_{0l2} = 10, \sigma_{Nl3} = 0.9, \kappa_{l3} = 1.5, \alpha_{N_{l3}} = 0.15, \gamma_{l3} = 5000$, and $\varepsilon_{0l3} = 20$.

This section shows two cases study. First the disturbances on public channels are considered.

Remark 4: Both cases are used the same control gains.

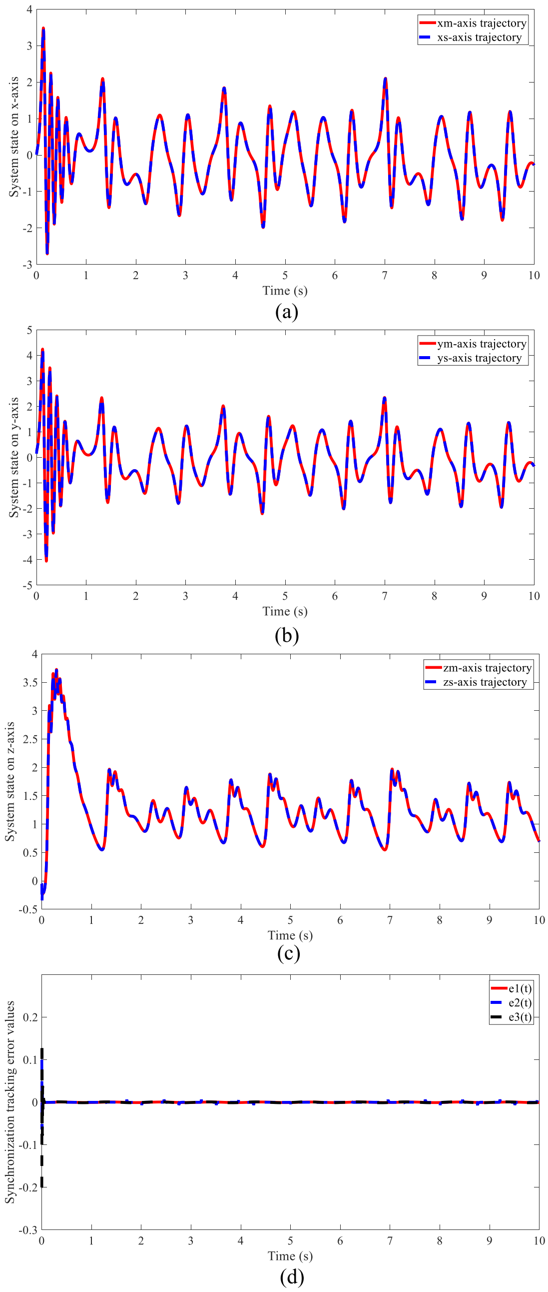

FIGURE 2. States of MSSs: (a), states on x-axis, (b) states on y-axis, (c) states on z-axis, and (d) tracking errors.

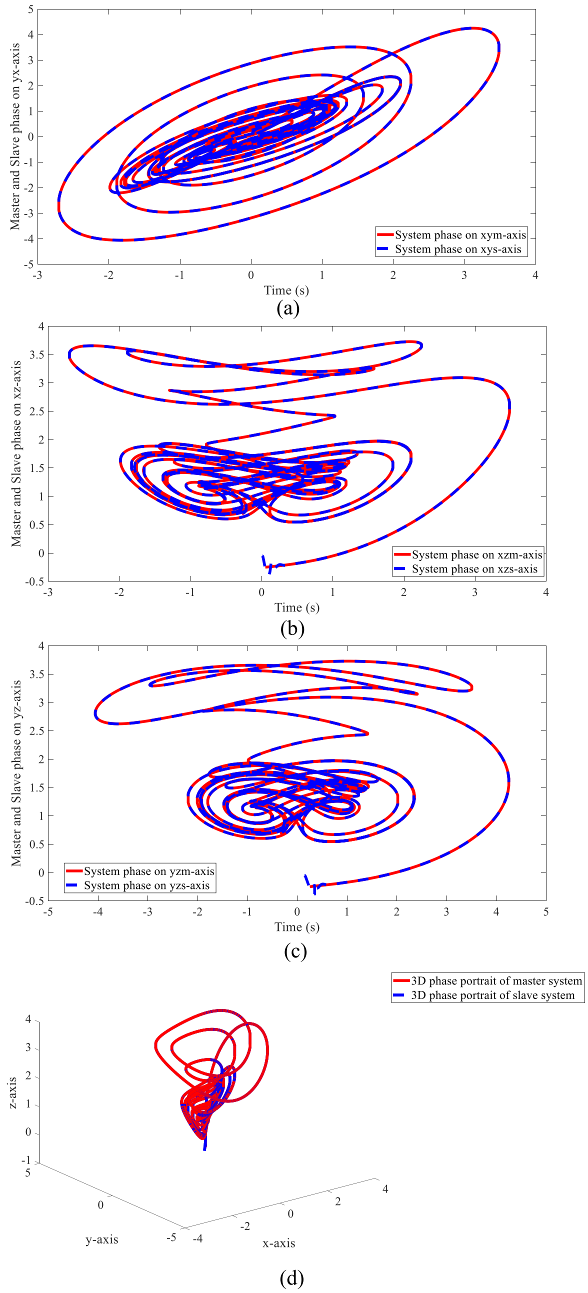

FIGURE 3. Phases of MSSs: (a), phases on x-axis, (b) phases on y-axis, (c) phases on z-axis, and (d) phases in 3D space.

Case 1: Effectiveness of the proposed DO on SCS with disturbances on public channels.

The tested disturbances are $l_1 = 5\sin(2\pi t)$, the information of l_2 are amplitude is 2, Period is 0.75 second, Pulse Width is 60%, and phase delay is 1.25 second. $l_3 = 2.5\sin(2\pi t) + 2.5\cos(2\pi t)$, for x-, y-, and z- axes, respectively. The performances of the control system for SCS of CBS are shown in Figures 2 to 7 below. First, Figure 2 is used to show the performances of synchronization control.

The synchronization is obtained in the very short times. $T_{ex\max} < 0.03$ (s), $T_{ey\max} < 0.035$ (s) and $T_{ez\max} < 0.05$ (s). The tracking error values are $e_x(t) < (-6.75,\ 6.75) \times 10^{-4}$, $e_y(t) < (-1.04,\ 1.04) \times 10^{-4}$, and $e_z(t) < (-6.5,\ 6.5) \times 10^{-3}$. The maximum values of tracking errors are $e_{x\max} \sim 0.026$, $e_{y\max} \sim 0.065$, and $e_{z\max} \sim 0.14$. The phases of MSSs are mostly identical to each other and same format with the original chaotic system. The performance of phase portraits are as in Figure 3 below.

The phases of MSSs are mostly identical, which is used to perform that the synchronization control is good at complicated disturbances such as sine, square and cosine waves. The performances of the proposed DO in the fist 10 seconds can be found in Figure 4 below.

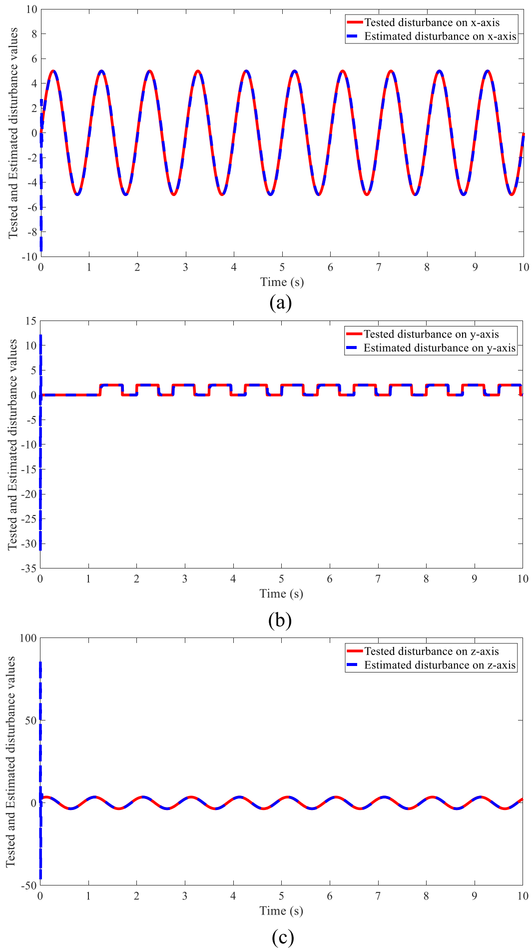

FIGURE 4. **DO performances: (a), tested disturbances on x-axis, (b) tested disturbances on y-axis, and (c) tested disturbances on z-axis.**

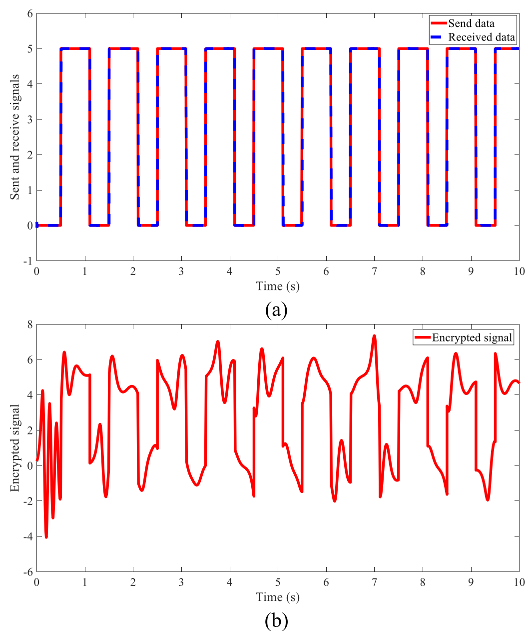

FIGURE 5. **Secured data: (a) sent and received data and (b) encrypted data.**

As Shown in Figure 4, the DO is good at rejecting disturbances at sine, cosine, and square formats. The tested disturbance was mostly rejected by proposed DO. The performances of DO effects to the outcome of SCS. The sent and received data are shown in Figure 5 below.

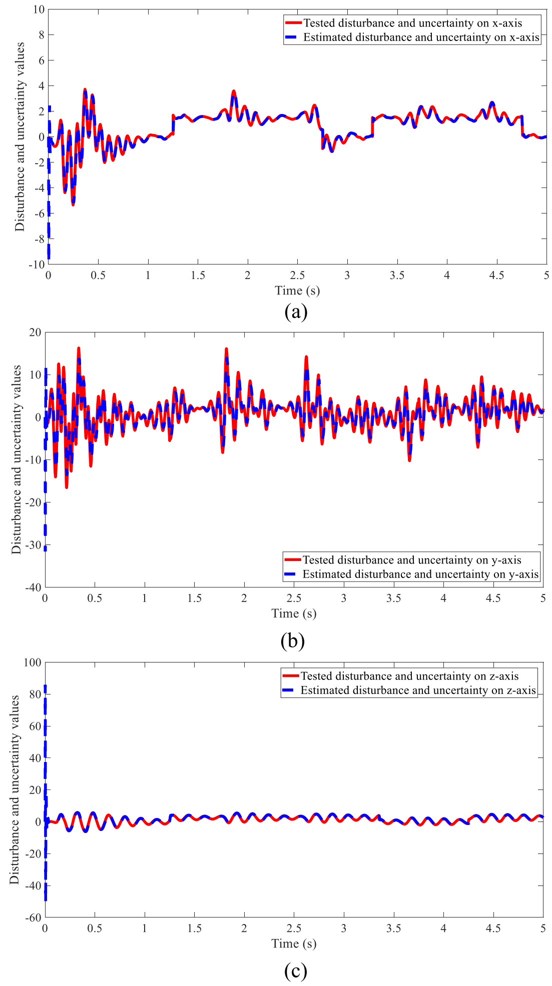

FIGURE 6. **DO performances with fully disturbance and uncertaity: (a), tested perturbations on x-axis, (b) tested perturbations on y-axis, and (c) tested perturbations on z-axis.**

The sent and received data were mostly tracked each other, which is used to confirm that the proposed method is good in synchronization the chaotic system. The secure communication was successfully archived.

Case 2: Effectiveness of the proposed DO on SCS with fully disturbances on public channels and uncertainties.

In this section, the information of disturbance on x-axis are as follows: amplitude is 1.5, Period is 2 second, Pulse Width is 75%, and phase delay is 1.25 second, the information of disturbance on y-axis are as follows: amplitude is 2, Period is 2.5 second, Pulse Width is 60%, and phase delay is 1.25 second, and the information of disturbance on z-axis are as follows: amplitude is 2.5, Period is 3 second, Pulse Width is 70%, and phase delay is 1.25 second. The uncertainties of master are add as follows: $\Delta a_m = 1.5\sin(10\pi t)$, $\Delta b_m = 1.5\cos(12.5\pi t)$, and $\Delta c_m = 2.5\sin(25\pi t)$. The uncertainties of TSF are then calculated as follows: as shown in the

$$\Delta A_{1m} = \begin{bmatrix} -1.5sin(10\pi t) & 1.5sin(10\pi t) & 0 \\ -1.5sin(10\pi t) + 2.5sin(25\pi t) & 2.5sin(25\pi t) & 0 \\ 0 & 0 & -1.5cos(12.5\pi t) \end{bmatrix},$$

$$\Delta A_{2m} = \begin{bmatrix} -1.5sin(10\pi t) & 1.5sin(10\pi t) & 0 \\ -1.5sin(10\pi t) + 2.5sin(25\pi t) & 2.5sin(25\pi t) & 0 \\ 0 & 0 & -1.5cos(12.5\pi t) \end{bmatrix},$$

TABLE 1. Comparison of this work to previous paper [5].

Factors	This paper	Paper [5]
Maximum settling-time	$T_{ez}\max < 0.05$ (second)	>1 second
Maximum tracking error	$e_{z}\max \sim 0.14.$	> 5
Disturbance rejection	Yes	No

equation at the top of the page, and as shown in the equation at the top of the page.

The performances of proposed DO method with fully disturbance and uncertainties in the first 5 seconds are shown in Figure 6 below.

Remark 5: This section shown the performance of DO with disturbance rejection ability only.

The comparisons of this work to previous paper are shown in Table 1 below.

V. CONCLUSION

This paper proposed a new DO for the synchronization of the CBS with an application of SCS. Two cases of disturbances on public channels and fully disturbances and uncertainties were considered and tested. Both cases, the perturbations were rejected mostly. The proposed DO can reject the perturbations in very fast time. Especially, the disturbance information was not needed such as the information of first derivative or disturbance formats. The proposed DO can be applied in other control system such as motor control, power electronics, etc. This is open a new gate of design the DO for SCS of CBS with the inversed model based. Furthermore, the improved reaching law was provided with the reduction chattering function and fast convergent speed. The improvement of reaching law was successfully applied to design the SMC and DO. Otherwise, many types of disturbance was tested on the SCS with a good result. The novelty of this paper is the new DO. In the future work, the improvement of DO will be considered for the FO system with fully considerations of disturbance, uncertainty, and time-delay.

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