

## RESEARCH ARTICLE

# Intersection Outlet Saturation Flow Rate (OSFR) Considering Game Behavior Within Intersection

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**ABSTRACT** Most congestion occurs at intersections. Increasing the intersection capacity is crucial to alleviate the congestion. Saturation flow rate (SFR) is the basis for capacity calculation. SFR depends on many factors, and the guidelines sketched in the highway capacity manual can be applied. Most SFR are defined at the stop-line. The vehicles depart from the stop-line, enter the intersection, and exit from the outlet of the intersection. In a congestion scenario, vehicles would compete with each other before leaving the intersection and even get stuck within the intersection. The flow rate at the outlet thus decreased, and as a result, the theoretical capacity based on conventional SFR cannot be achieved. The manuscript defines a concept called OSFR (outlet saturation flow rate) to reflect the influence of the game behavior at the outlet and proposes a model that generates the OSFR. The model divides the vehicle's movement into three parts: departure from the stop-line, drive along its trajectory, and finally exit from the outlet with game behavior with other drivers. A game model for outlet lane-choosing is proposed. Based on empirical data, the stop-line headway distribution model, speed model, and payoff function involved in the model were calibrated. The results show that the proposed model can generate real-world OSFR distributions. This model can effectively describe the invisible interaction between vehicles and can be used to get the outlet headway distribution in the case of mismatched lanes number.

**INDEX TERMS** Headway, speed model, game theory, intersections.

## I. INTRODUCTION

Intersection plays a vital role in the urban road system. Most congestions originate from intersections. The entire road network's operational efficiency is determined by the intersectional traffic efficiency [1]. Therefore, the correct assessment of intersection capacity is of great significance for traffic planning, operational assessment, and intersection management [2].

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According to the Highway Capacity Manual, the basic method for calculating intersection capacity is based on the

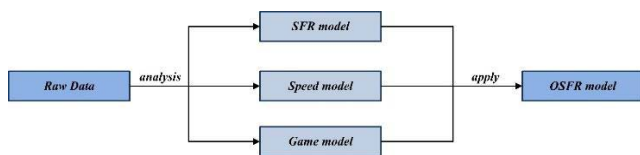
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saturation flow rate [3]. The traditional way to calculate the saturation flow rate is the headway method. However, the departure headways are unstable for many reasons, including driver behavior, vehicle performance, etc [4]. In order to describe the instability of the headway, a log-normal distribution model was introduced to model the headway with reasonable accuracy [5]. Jin *et al.* further found that departure headways followed a position-dependent log-normal distribution [6].

The headways above are defined at the stop-line. After the onset of green, vehicles depart from the stop-line, drive through the intersection, and exit from the intersection outlet. During rush hours, the vehicles may compete at the intersection outlet, and thus some vehicles get stuck within the intersection, and finally, the queue influence the headways at the stop-line. The theoretical capacity using the SFR at

the stop-line cannot be achieved, and the actual capacity is overestimated. As a result, the intersection planning and control optimization performances may be compromised.

It is clear that SFR (saturation flow rate) defined at the stop-line cannot reflect this mechanism. As the vehicle entering the intersection must exit from the corresponding outlet lanes, the SFR defined at outlet lanes would be more accurate than that defined at the stop-line during competing scenarios. Thus we propose a concept, OSFR (outlet saturation flow rate), to describe the operational efficiency at the intersection outlet. The OSFR is influenced by the competing behavior among vehicles, such as in the case in **Figure 2**. In this paper, the game theory method is used to model the competing behavior. The advantage of game theory over traditional rule-based models is that it can reflect dynamic interactions among drivers [7]. In recent years, game theory has been successfully used in the human-like decision-making of autonomous vehicles [8], [9], [10] and lane-changing behavior modeling in a connected environment [7], [11]. The proposed OSFR model divides the vehicle movements into three stages: (1) According to a certain SFR model, the vehicles enter the intersection from the stop-line; (2) After entering the intersection, the vehicles will drive along their trajectory based on certain speed profiles; (3) Before the vehicles exit the intersection, the vehicles need to play games with other vehicles to get the opportunity to exit, which means some vehicles will slow down. The overall model framework is shown in Fig. 1.



**FIGURE 1.** The overall framework of the OSFR model.

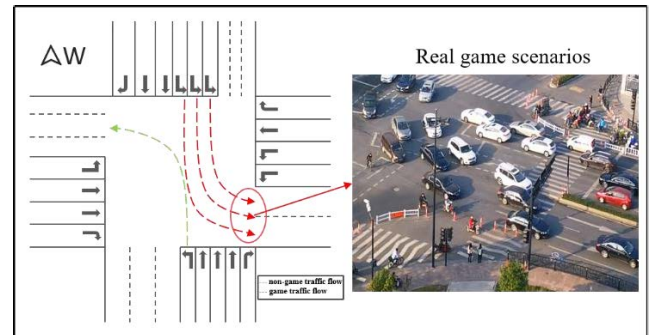
The structure of this paper is organized as follows. Section 2 describes the data source and the pre-processing methods. Section 3 presents the models, including stop-line headway distribution, speed curve model, and the game model. Section 4 establishes a simulation framework using the model developed in section 3 to obtain the OSFR. Section 5 gives the summary of this paper.

## II. GUIDELINES FOR MANUSCRIPT PREPARATION

### A. DATA ACQUISITION AND PROCESSING PROCESS

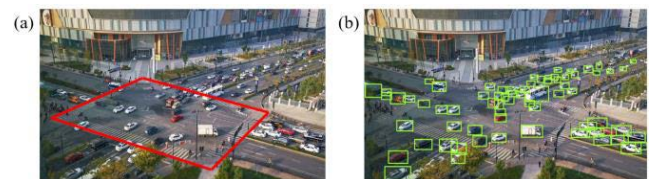
The vehicle trajectory dataset used in this paper was collected by a camera from an intersection (YuHangTang RD & Jiang-Dun RD) in Hangzhou, China, on October 30, 2020. The recording time is from 7:30 AM to 8:30 AM (morning peak). The main reason for choosing this intersection is that for the left-turn vehicles at the west entrance, the number of incoming lanes is greater than the number of exit lanes(outlet lanes). As such, the three incoming movements (each incoming lane corresponds to a movement) compete with each other for the two outlet lanes. While the incoming lanes number equals the

outlet lanes number for left-turn flow at the east approach, there is no competing behavior. In this paper, we define the left-turn traffic flow at the east approach as non-game traffic flow and the left-turn traffic flow at the west approach as game traffic flow. The following analysis will also focus on these two kinds of traffic flow. The channelization status of the intersection is shown in Fig. 2.



**FIGURE 2.** The channelization of the intersection and the trajectory of the two kinds of traffic flow.

The original video of the vehicle trajectory was taken from a high position on the 11 floors of a building near the intersection. The video resolution is 1080P (1920\*1080) and 30 frames per second. Through the image recognition method based on deep learning, the trajectory data of the vehicle is obtained, and the recognition result is shown in Fig. 3.



**FIGURE 3.** (a) The original video and the area within the red rectangle is the study area. (b) The recognition result.

The output trajectory of the vehicles is pixel-based and not meter-based. We use the homogeneous coordinate transformation method to convert the image coordinates to real-world coordinates (i.e., meter-based) [12]. The transformation formula is as follows:

$$s\mathbf{m} = \mathbf{pM} \tag{1}$$

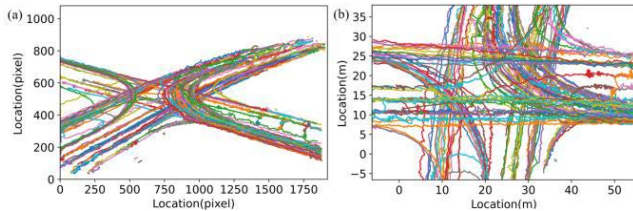
where  $s$  is a non-zero scale factor;  $\mathbf{M}$  is a three-dimensional world homogeneous coordinate,  $\mathbf{M} = [X \ Y \ Z \ 1]^T$ ;  $\mathbf{m}$  is a two-dimensional image homogeneous coordinate,  $\mathbf{m} = [u \ v \ 1]^T$ ;  $\mathbf{p}$  is the mapping transformation matrix.

Since the three-dimensional world coordinates are not considered in this paper, the movement of vehicles at the intersection is regarded as a two-dimension trajectory. Therefore, it can be assumed that the vehicles movement plane is located on the plane of the world coordinate system  $Z=0$ , and the

conversion equation can be rewritten as:

$$s \times \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 \\ p_4 & p_5 & p_6 \\ p_7 & p_8 & p_9 \end{bmatrix} \times \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} \quad (2)$$

After the conversion, the unit of the vehicle trajectory data is changed from pixels to meters, and the trajectory error caused by the shooting angle is greatly reduced, making it possible to calculate the speed. The conversion result is shown in Fig. 4.



**FIGURE 4.** (a) The original vehicle trajectory in pixel. (b) The converted trajectory of the vehicle in the intersection (the red area in Figure 3(a)).

After transformation, the speed can be calculated from the distance of the trajectory points and the corresponding time stamps differences. However, due to the random factors (the image capturing error, the vehicle identification error, etc.), the speed will be very large or even negative [13]. To overcome this, we smooth the data using a similar method in [13]. The average of the vehicle’s coordinates within a predefined time horizon is regarded as the vehicle’s current position. In other similar studies, Wan *et al.* chose 1 second as the time horizon [13]. In this article, we select 5 frames (0.17s), 15 frames (0.5s), and 30 frames (1s) as the time horizon to compare the results after smooth. Acceleration is used to determine the best horizon. Under normal driving conditions, the range of vehicle acceleration should be from  $-6\text{m/s}^2$  to  $5\text{m/s}^2$  [14], [15], [16]. The influence of different smooth horizons is shown in Fig. 5. The error rates (percentage of acceleration outliers) of 5, 15 and 30 frames are 61.4%, 1.0%

and 0.1%, respectively. Since the speed curve is too smooth when the 30-frames range is selected, the driver’s decision is difficult to be recognized, 15 frames are selected as the final time range.

**B. STATISTICS**

After the above processing, the statistics of game traffic flow and non-game traffic flow can be obtained. The results are shown in Table 1 and Table 2, respectively (trajectory time refers to the time the vehicle spends in the intersection). The result shows that the average speed of the game traffic flow is slower than that of the non-game traffic flow, and it takes more time to spend in the intersection. This shows that due to the game between vehicles, the operating states of non-game traffic flow and game traffic flow at intersections are quite different. In the following, this difference will be compared in detail, and a model will be established to analyze the cause.

**TABLE 1.** The statistics of game traffic flow.

Field	Sample size	Statistics			
		Mean	SD	Min	Max
Speed (km/h)	562	23.83	9.73	5.49	39.78
Trajectory length (m)	562	47.44	4.01	40.12	54.89
Trajectory time (s)	562	14.32	2.60	11.34	19.36

**TABLE 2.** The statistics of non-game traffic flow.

Field	Sample size	Statistics			
		Mean	SD	Min	Max
Speed (km/h)	303	29.48	9.99	10.23	45.77
Trajectory length (m)	303	43.86	3.45	40.11	57.86
Trajectory time (s)	303	5.12	0.77	2.97	6.93

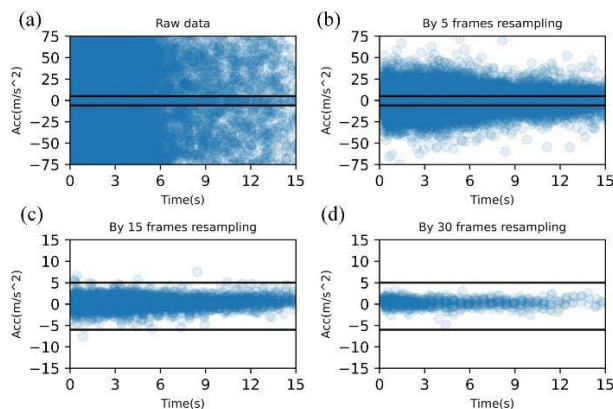
**III. MODEL**

**A. MODEL FRAMEWORK**

As aforementioned, the trajectory of a vehicle inside an intersection can be divided into three parts: entering, accelerating, and exiting. According to a certain headway distribution (SFR model), vehicles enter the intersection from the stop-line. After that, the vehicles will drive along their trajectory based on certain speed profiles. When the vehicle is ready to exit the intersection (enter the outlet), the vehicle needs to play games with other vehicles (if there are any) to get the opportunity to exit. The models that describe the above three parts will be presented in this section.

**B. SFR MODEL AT STOPLINE**

Many models have been used to describe the distribution of headway, such as Gaussian distribution [17], Shifted



**FIGURE 5.** The acceleration distribution scatter plots of raw and each resampling data (time refers to the driving time of vehicles in the intersection).

exponential distribution [18], and Gamma distribution [19]. In this section, we follow Jin’s approach [6] and use a log-normal distribution to fit both the departure headway and the empirical outlet headway, and the result is shown in Fig. 6. The probability density function of the log-normal distribution:

$$f(x, \mu, \sigma) = \begin{cases} \frac{1}{x\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2\sigma^2}(\ln x - \mu)^2\right], & x > 0 \\ 0, & x \leq 0 \end{cases} \quad (3)$$

where  $x$  is the continuous random variable with a positive value;  $\mu$  is the expected value;  $\sigma$  is the standard deviation.

The estimated parameters of log-normal distribution and corresponding Kolmogorov-Smirnov hypothesis testing (K-S test) results are given in Table 3.

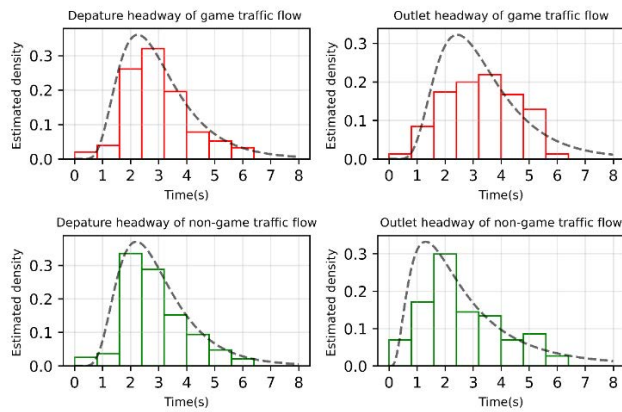


FIGURE 6. The distribution of headway and the fitting results of the log-normal distribution.

TABLE 3. The estimated parameters of different log-normal distributions.

Type	Flow	Sample size	$\mu$	$\sigma$	Mean
departure	game	562	1.0113	0.4442	3.0344
outlet	game	562	1.0991	0.4580	3.3329
departure	non-game	303	1.0117	0.3709	2.9461
outlet	non-game	303	0.8090	0.4569	2.4928

It can be observed that (1) the departure headways are very alike. Although the departure headway is greater for the game scenario, the difference is trivial; (2) the outlet headways of the game scenario increase substantially by about 33.7% (calculated as  $(3.33-2.49)/2.49$ ). This is caused by the game behavior of vehicles in the intersection. Such influences are never reported in the literature. The specific game behavior modeling and how the model affects the headway will be described below.

### C. SPEED CURVE MODEL

Traditionally it is assumed that vehicles depart from the stop-line and accelerate until the speed limit is reached. It is not true when the game behavior of vehicles exists, as the vehicles that fail in the competition need to decelerate to give way to other vehicles. The speed profile of the game traffic flow and the non-game traffic flow thus are quite different. This conclusion can be confirmed in Fig. 7. Fig. 6-a and b are speed curves for the non-game scenario, while Fig. 7-c and d are speed curves for the game scenario. In Fig. 7, the first few cars before the stop line in the same phase are selected for comparison. For non-game vehicles, their speed tends to increase, and they can pass the intersection in less than 8 seconds. For the game vehicles, their speed is very unstable, and most of the vehicles experience more than one deceleration process.

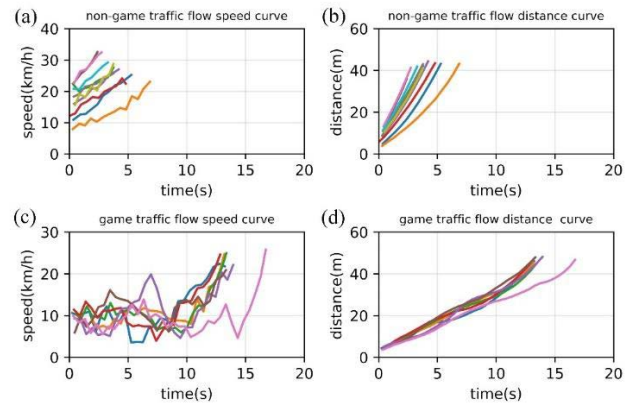


FIGURE 7. The speed and distance curve of the two kinds of traffic flow ( $t=0$  is when the vehicle leaves the stop line).

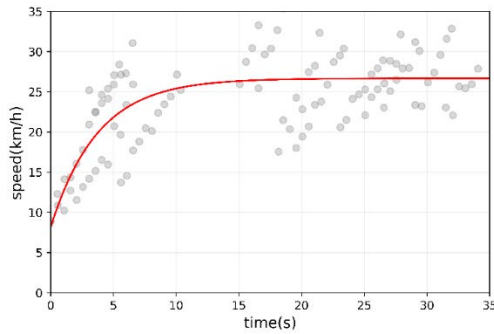
By analyzing the data of 18 intersections in Melbourne and Sydney, Akcelik obtained a queue discharge speed model for signalized intersections [20]. From this model, the relationship between speed and the time since the start of green can be established:

$$v = v_m[1 - e^{-m(t-t_0)}] \quad (4)$$

where  $v$  is the queue discharge speed (km/h) at time  $t$ ;  $v_m$  is the maximum queue discharge speed;  $t$  is the time since the start of green (s);  $m$  and  $t_0$  are the parameters to be calibrated.

The advantage of this model is that it uses time since the green start as the independent variable rather than the time since the vehicles leave the stop line. In this way, the speed of all vehicles can be considered in a single model. We employ this model and fit using the speed data and get the following result (the result is shown in Fig. 8):

$$v = 26.671 \times [1 - e^{-0.268(t+1.36)}] \quad (5)$$



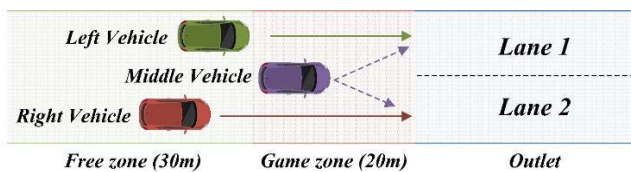
**FIGURE 8.** The speed model of non-game vehicles at the intersection ( $t=0$  is the time when the green start).

**D. GAME BEHAVIOR MODELING AT THE INTERSECTION OUTLET**

1) GAME ENVIRONMENT SETTING

Applying game theory to the microscopic traffic flow analysis is not new [7], [8], [9], [10], [11], [21]. Some researchers use game theory to analyze how drivers make decisions when changing lanes [7]. The problem studied in this paper is more like lane-choosing, which is similar to lane-changing or merging. Fig. 9 shows the lane-choosing scenario of the game traffic flow. The movements of the vehicles are divided into three zones: free zone, game zone, and outlet.

In the free zone, the vehicle can accelerate freely without the influence of competition behavior from other movements while obeying the speed profiles in Eq 5. In the game zone, the vehicle needs to play with each other when competing for the outlet lane. Note that the game may only occur when the vehicles are close enough, or equivalently more than one vehicle is presented simultaneously. For example, if the middle vehicle is in front and far away from the left and right vehicle, the middle vehicle can go straight into the outlet without playing games.



**FIGURE 9.** The schematic of the lane-choosing of the game traffic flow.

2) GAME MODEL FORMULATION

Different types of drivers have different driving habits, which are related to their gender, age, and personality [22], [23], [24]. In this research, three types of drivers are considered: polite, neutral, and rude. For polite drivers, they tend to give way to competing vehicles. The reasonable explanation is that these drivers want to minimize the possibility of collision and are not in a hurry. For rude drivers, they tend to be aggressive. For neutral drivers, they have no obvious inclinations and will make judgments based on specific circumstances.

The following assumptions are made:

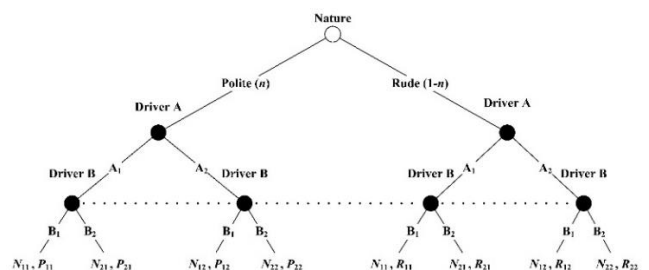
- The middle vehicle is called the ego vehicle, and the left or the right vehicle is called the target vehicle;
- The ego vehicle will only play with one target vehicle at a time;
- The drivers are aware of each other’s speed and distance.

In this paper, the target vehicle does not know whether the driver in the ego vehicle is rude or polite. So the lane-choosing behavior is modeled as a two-person non-zero-sum non-cooperative game under incomplete information. The payoff matrix is shown in Table 4. In this study, all types of drivers have the same action set (give way or don’t give way to other drivers, or equivalently, accelerate and decelerate). The difference is that different types of drivers will have different payoffs under the same action. For example,  $P_{11}$  will be less than  $R_{11}$  because rude drivers are less worried about the impact of a collision than polite drivers.

**TABLE 4.** The payoff matrix corresponding to the different strategic combinations of the two drivers ( $N, P,$  and  $R$  correspond to the payoff for neutral, polite, and rude driver, respectively).

Action	Ego vehicle	
	A <sub>1</sub> (Accelerate)	A <sub>2</sub> (Decelerate)
Target vehicle		
B <sub>1</sub> (Accelerate)	( $N_{11}$ or $P_{11}$ or $R_{11}$ , $N_{11}$ or $P_{11}$ or $R_{11}$ )	( $N_{12}$ or $P_{12}$ or $R_{12}$ , $N_{12}$ or $P_{12}$ or $R_{12}$ )
B <sub>2</sub> (Decelerate)	( $N_{21}$ or $P_{21}$ or $R_{21}$ , $N_{21}$ or $P_{21}$ or $R_{21}$ )	( $N_{22}$ or $P_{22}$ or $R_{22}$ , $N_{22}$ or $P_{22}$ or $R_{22}$ )

Because the game of incomplete information is difficult to solve, we convert the game of incomplete information into the game of imperfect information by the Hysanyi transformation [25]. This approach introduces a virtual participant, “nature”. Nature moves first and determines the type of ego driver. And every driver knows the specific distribution of the probability. As the extensive form of the game was very complex, we made a schematic diagram under a simplified scenario in Fig. 9 (assume that the driver in the target vehicle is only neutral, while the driver in the ego vehicle is a mixture of rude and polite).



**FIGURE 10.** Lane-choosing game in extensive form.

3) PAYOFF FUNCTION

In the game of lane-changing, Alireza [7] constructs the payoff function based on the speed changes before and after the

lane change and the acceleration required to avoid collisions. In this study, in addition to the speed and distance benefits of the vehicle, the payment function will also consider the psychological benefits of different types of drivers. Rude drivers will get greater psychological benefits when accelerating (more inclined to accelerate). The payoff function formulas for different driver types are as follows.

For neutral drivers:

$$N_{11} = a_{brake} + D/d + b_{acc}^n \quad (6)$$

$$N_{12} = a_{acc} + D/d + b_{acc}^n \quad (7)$$

$$N_{21} = a_{dec} - D/d + b_{dec}^n \quad (8)$$

$$N_{22} = a_{dec} - D/d + b_{dec}^n \quad (9)$$

where  $a_{brake}$ : Both drivers choose to accelerate, the deceleration corresponding to the sudden braking of the vehicle in order to avoid a collision;  $a_{acc}$ : When the other driver chooses to decelerate, the acceleration that the vehicle can obtain;  $a_{dec}$ : Deceleration when the vehicle actively chooses to decelerate;  $D$ : The total distance of the intersection;  $d$ : The distance between the vehicle and the lane entrance;  $b_{acc}^n, b_{dec}^n$ : Psychological benefits of neutral drivers when they choose to accelerate and decelerate, respectively.

For polite drivers:

$$P_{11} = a_{brake} + D/d + b_{acc}^p \quad (10)$$

$$P_{12} = a_{acc} + D/d + b_{acc}^p \quad (11)$$

$$P_{21} = a_{dec} - D/d + b_{dec}^p \quad (12)$$

$$P_{22} = a_{dec} - D/d + b_{dec}^p \quad (13)$$

where  $b_{acc}^p, b_{dec}^p$ : Psychological benefits of polite drivers when they choose to accelerate and decelerate, respectively.

For rude drivers:

$$R_{11} = a_{brake} + D/d + b_{acc}^r \quad (14)$$

$$R_{12} = a_{acc} + D/d + b_{acc}^r \quad (15)$$

$$R_{21} = a_{dec} - D/d + b_{dec}^r \quad (16)$$

$$R_{22} = a_{dec} - D/d + b_{dec}^r \quad (17)$$

where  $b_{acc}^r, b_{dec}^r$ : Psychological benefits of rude drivers when they choose to accelerate and decelerate, respectively.

By analyzing the data, the values of the above parameters are obtained. Under general circumstances,  $a_{brake} = -5 \text{ m/s}^2$ ,  $a_{acc} = 2 \text{ m/s}^2$  and  $a_{dec} = -2 \text{ m/s}^2$ . The same type of drivers is assumed to have the same psychological benefits,  $b_{acc}^n = b_{dec}^n = 0$ ,  $b_{acc}^p = 6$ ,  $b_{dec}^p = -2$ ,  $b_{acc}^r = -1$  and  $b_{dec}^r = 2$ . Note that the setting of these values has taken into account the range of data values.

#### 4) NASH EQUILIBRIUM SOLUTION

In this section, the details of solving Nash equilibrium are described. The first step is to decompose the incomplete information game into three complete information games, that is, to play with the neutral, polite, and rude drivers, respectively. For the complete information game, the driver's

strategy can be uniquely determined.

$$s(action, type) = \begin{cases} acc & \text{if } u_{acc}^{type} \geq u_{dec}^{type} \\ dec & \text{else.} \end{cases} \quad (18)$$

where  $n_1, n_2$ , and  $n_3$  are the proportions of neutral, polite, and rude drivers, respectively ( $n_1 + n_2 + n_3 = 1$ ).

After that, by comparing the expected payoff of acceleration and deceleration, the driver's action can be determined:

$$E(u_{acc}^{type}) = n_1 u_{s(acc,neutral)}^{type} + n_2 u_{s(acc,polite)}^{type} + n_3 u_{s(acc,rude)}^{type} \quad (19)$$

$$E(u_{dec}^{type}) = n_1 u_{s(dec,neutral)}^{type} + n_2 u_{s(dec,polite)}^{type} + n_3 u_{s(dec,rude)}^{type} \quad (20)$$

where  $n_1, n_2$ , and  $n_3$  are the proportions of neutral, polite, and rude drivers, respectively ( $n_1 + n_2 + n_3 = 1$ ).

If  $E(u_{acc}^{type}) > E(u_{dec}^{type})$ , then the driver will choose to accelerate. Otherwise, it will choose to decelerate (when equal, it is assumed that the vehicle will accelerate). Since the game is symmetric, the actions of the other driver can be determined in the same way. In fact, Nash equilibrium is directly related to the distribution of driver types. Under different proportions of drivers, there will be different Nash equilibriums.

## IV. SIMULATION

### A. MODEL FRAMEWORK

To investigate the OSFR under the influence of game behavior within the intersection, we resort to the simulation method. The simulation uses the models above and gets the moment when vehicles depart from the outlet, given the vehicles that enter the intersection from the stop-line. Once the vehicle exiting moments are obtained, the OSFR can be calculated. The simulation framework is shown in Fig. 11 (the corresponding game scenario is shown in Fig. 9). First of all, vehicles are generated at the stop line, and the

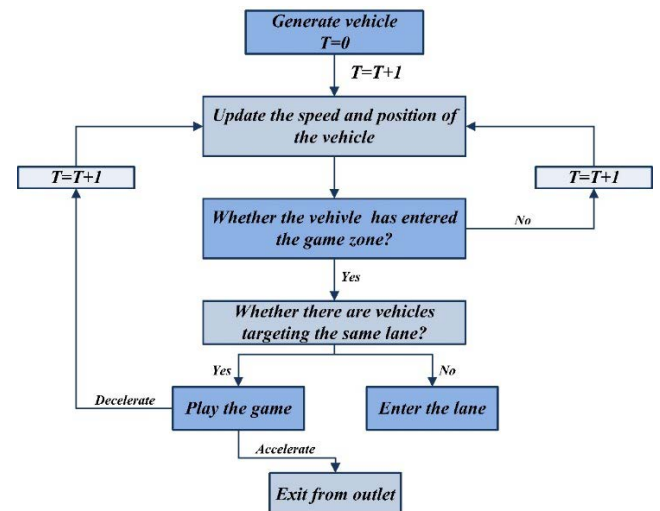


FIGURE 11. The simulation framework of the game model between vehicles (the simulation step size is 0.3s).

**Algorithm 1** Proposed Game Theory-Based OSFR Model

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input: Time when the vehicles leave the stop line, departure_set; Speed model,  $v(t)$ ;
        Driver type ratio,  $n_1, n_2, n_3$ ; Game model, game; Number of simulated vehicles,  $n$ ;
output: Time when the vehicles enter the outlet line, outlet_set;
t = 0, i = 0;
while  $\text{len}(\text{outlet\_set}) < n$  do
    t = t + 1;
    update vehicle.distance =  $\int v(t)dt$ ;
    if departure_set[i] > t then
        | i = i + 1;
        | generate vehicle[i];
    end
    if vehicle.distance > 30 and vehicle[id1].target_lane == vehicle[id2].target_lane then
        | game(vehicle[id1], vehicle[id2]);
        | if vehicle[id1].action == acc and vehicle[id1].action == acc then
            | | update vehicle[id1].speed = vehicle[id1].speed - 2.5m/s;
            | | update vehicle[id2].speed = vehicle[id2].speed - 2.5m/s;
        | end
        | if vehicle[id1].action == dec and vehicle[id1].action == dec then
            | | update vehicle[id1].speed = vehicle[id1].speed - 0.5m/s;
            | | update vehicle[id2].speed = vehicle[id2].speed - 0.5m/s;
        | end
        | if vehicle[id1].action == acc and vehicle[id1].action == dec then
            | | update vehicle[id1].speed = vehicle[id1].speed + 1m/s;
            | | update vehicle[id2].speed = vehicle[id2].speed - 0.5m/s;
        | end
    end
    if vehicle[id].distance > 50 then
        | delete vehicle[id];
        | update outlet_set.append(t);
    end
end

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departure headways distribution of vehicles follows the log-normal distribution. After that, the vehicles drive according to the proposed speed model. When a vehicle enters the game zone, if there are other vehicles in the game zone that choose the same target lane, then the game needs to be played. Otherwise, it will exit from the outlet lane directly.

The details of the implementation of the proposed game theory-based OSFR model are illustrated in Algorithm 1. Note that the distribution of vehicle headway in the simulation follows the log-normal distribution obtained by fitting in Section 3.2. And the details of the speed model and game model are illustrated in Section 3.2 and Section 3.4, respectively.

## B. SIMULATION RESULTS

In the simulation, we assume that 20 cars are generated in each cycle, and 10 cycles are simulated. As there are three incoming lanes, we have 600 vehicles. After all of these vehicles leave the intersection, we gather the headways that the vehicles leave through the outlet (or equivalently, the two outgoing lanes of the intersection). As there are three types of drivers, we repeat the simulation 9 times to accommodate different types of ratios. We set three levels of maximum

ratio for each type of driver, i.e., 100%, 80%, and 60%. The remaining ratios are assigned equally to the other two types of drivers. The simulation results are shown in Fig. 12. It is shown that the distribution of outlet headway is different under different proportions of types of drivers. The results show that when there is only one type of driver in the road network, the headway does not increase significantly (the first row in Fig. 12). This is because, in this situation, the driver has an accurate expectation of what the other driver is going to do and will not brake sharply. When the flow is a mixture of different types of drivers, the OSFR deteriorates. The minimal headway is the case when all drivers are of rude type, as each vehicle wants to exit the intersection as early as possible; the maximal headway is the case when major vehicles (80%) are rude drivers, and its mean headway is 3.29 sec.

In essence, the reason for the increase in headway is that both drivers choose to accelerate, which causes the driver to brake sharply in order to avoid a collision. This extreme situation is mainly related to two factors. One is that drivers misjudge the intentions of other drivers (the driver thinks the other driver will decelerate, so he chooses to accelerate, but the other driver also chooses to accelerate). The other is that rude drivers choose to accelerate when they should

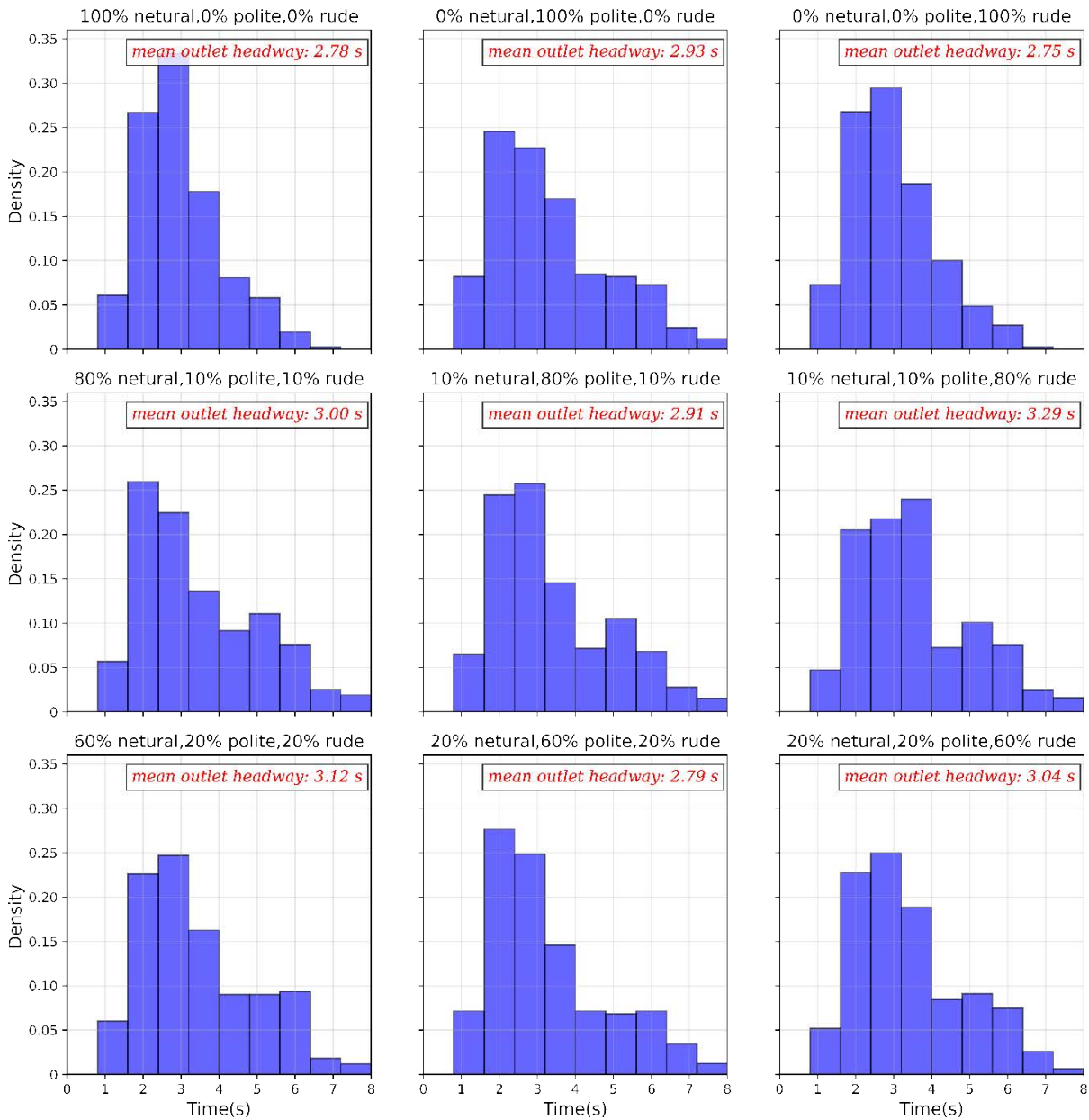


FIGURE 12. The simulation results of outlet headway under different driver proportions.

yield. In Fig. 13, the log-normal distribution is used to fit the headway time under simulation scenarios and the real scene, respectively. The results show that the closest approximation to the real situation is the scenario in which rude drivers accounted for 80%.

In order to further analyze the influence of the game model on the traffic flow at the intersection, we compare the outlet headway of different vehicles number under the same driver proportion (10% neutral, 10% polite, 80% rude). The vehicle's number can be interpreted as the duration of the green phase, as a longer green duration would feed more vehicles into the intersection. The result is shown in Fig. 14.

With the increase in the number of simulated vehicles, the mean outlet headway increases, and the growth rate increases first and then stabilizes. When the number of simulation vehicles is more than 35, the mean outlet headway is basically stable at 3.43s. The reason for the increase in headway is that with the increase of vehicles, the game between vehicles becomes more and more frequent, and the overall speed will slow down as a result of failure in the competition. As the vehicle's number increase to a certain level, each vehicle would experience a game, and thus the outlet headway becomes stable, which can be interpreted as the saturated headway at the outlet (OSFR).



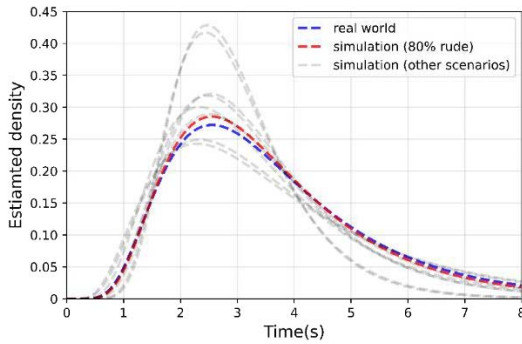


FIGURE 13. Comparison of headway distribution between real-world and simulation scenarios.

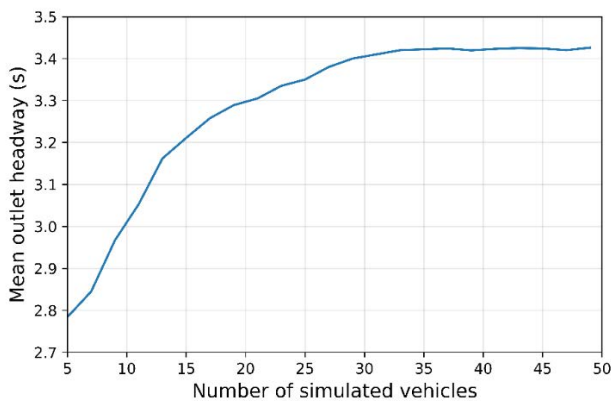


FIGURE 14. The simulation results of outlet headway under different numbers of simulated vehicles.

### C. APPLICATION IN CALCULATING SFR ADJUSTMENT FACTOR

Based on the above conclusions, the formula for calculating intersection capacity can be modified. According to HCM 2010 [3], the capacity of each lane group at the intersection can be calculated as follow:

$$c_i = s_i \frac{g_i}{C} \quad (21)$$

where  $c_i$ : Capacity of lane group  $i$  (veh/h);  $s_i$ : Saturation flow rate for lane group  $i$  (veh/h);  $g_i$ : Effective green time for lane group  $i$  (s);  $C$ : Intersection cycle length (s).

And the saturation flow rate  $s_i$  is related to the basic saturation flow rate and the adjustment factors:

$$s_i = s_o f_w f_{HV} f_g f_p f_{bb} f_a f_{LU} f_{LT} f_{RT} f_{Lpb} f_{Rpb} \quad (22)$$

where  $s_o$ : Basic saturation flow rate (veh/h);  $f$ : The adjustment factors (the specific definition can refer to HCM 2010, which will not be repeated in this paper).

Note that the saturation flow rate is calculated from the departure headway, which is suitable for most intersections. As shown in Fig. 6, when there is no game between vehicles, departure headway is greater than outlet headway. In this case, the above calculation can be applied. However, when the

number of incoming lanes is greater than the number of exit lanes, the effective saturation flow rate should be determined by the outlet headway rather than the departure headway based on the above results. Therefore, for intersections with mismatched lanes, the saturation flow rate calculation formula needs to be adjusted:

$$s_i = s_o f_w f_{HV} f_g f_p f_{bb} f_a f_{LU} f_{LT} f_{RT} f_{Lpb} f_{Rpb} f_{out} \quad (23)$$

where  $f_{out}$ : The adjustment factor caused by the game behavior between vehicles.

$$f_{out} = \begin{cases} 1 & \text{if } n_{in} \leq n_{out} \\ \bar{h}_{in} / \bar{h}_{out} & \text{else} \end{cases} \quad (24)$$

where  $n_{in}$ : the number of incoming lanes;  $n_{out}$ : the number of exit lanes;  $\bar{h}_{in}$ : Mean departure headway(s);  $\bar{h}_{out}$ : Mean outlet headway(s), which can be calculated by the model present in this paper ( $\bar{h}_{in} < \bar{h}_{out}$ ).

Taking the game and non-game traffic flow in this article as examples, for game traffic flow  $n_{in} = 3 > n_{out} = 2$ , so  $f_{out} = \bar{h}_{in} / \bar{h}_{out} = 3.03 / 3.29 = 0.92$ . For non-game traffic flow  $n_{in} = 1 < n_{out} = 3$ , so  $f_{out} = 1$ .

### V. CONCLUSION

Solving the congestion that occurs in the intersection is a challenging thing. The prerequisite to solving this problem is that we have to understand how the vehicles operate in the intersection. The existing results show that scholars focus on the study of SFR but ignore the OSFR. The game behavior of vehicles in the intersection will affect the operation of the traffic flow.

Based on empirical data, this paper proposes an OSFR model to explain the impact of game behavior between vehicles on SFR. And define different types of drivers to make the model more in line with the real scene. The experimental results show that the game behavior between vehicles will increase the headway. And under different proportions of drivers, there will be different effects. These results help to re-understand how the congestion in the intersection is generated and how to calculate the OSFR when the number of entrances and exits does not match. Besides, it can be used to adjust the calculation method of intersection capacity.

Due to the lack of information such as heading Angle, accelerator aperture, and specific size of the vehicle, the model presented in this paper does not consider the vehicle dynamics model. In future work, the vehicle dynamics model can be introduced to make the game model more accurate. Furthermore, it needs to be realized that drivers in different regions have different driving habits, and the payoff function needs to be redefined when the model is applied elsewhere.

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