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RESEARCH ARTICLE

Symbiotic Learning Grey Wolf Optimizer for Engineering and Power Flow Optimization Problems

AALA KALANANDA VAMSI KRISHNA REDDY
AND KOMANAPALLI VENKATA LAKSHMI NARAYANA, (Senior Member, IEEE)
School of Electrical Engineering, Vellore Institute of Technology, Vellore, Tamil Nadu 632014, India
Corresponding author: Komanapalli Venkata Lakshmi Narayana (vlnarayana.k@vit.ac.in)

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ABSTRACT This article presents a symbiotic learning-based Grey Wolf Optimizer (SL-GWO) formulated through the introduction of symbiotic hunting and learning strategies to achieve a better trade-off between exploration and exploitation while standing immune to *the curse of dimensionality*. The proposed method improves the performance of the algorithm to effectively handle problems with larger dimensions while avoiding local entrapment, accelerates convergence, and improves the precision and accuracy of exploitation. SL-GWO's symbiotic hunting strategies provide a major overhaul to the exiting hierarchical hunting through population sub-grouping into attacking hunters and experienced hunters with individually crafted dynamic adaptive tuning. The hunting mechanisms are implemented through the inclusion of random omega wolves from the wolfpack thereby reducing the algorithm's excessive dependence on the three dominant wolves and enhancing the population diversity. SL-GWO is tested and validated through a series of benchmarking, engineering and real-world optimization problems and compared against the standard version of GWO, eight of its latest and state-of-the-art variants and five modern meta-heuristics. Different testing scenarios are considered to analyze and evaluate the performance of the proposed method such as the effect of *dimensionality* (CEC2018 benchmarking suite), convergence speeds, avoidance of local entrapment (CEC2019 benchmarking suite) and constrained optimization problems (four standard engineering problems). Furthermore, two power flow problems namely, the optimal power flow (13 cases for IEEE 30 and 57-bus system) and optimal reactive power dispatch (8 cases for IEEE 30 and 57-bus system) from the recent literature are investigated. The proposed method performed competitively compared to all its competitors with statistically significant performance while requiring lower computational times. The performance for the standard engineering problems and the power flow problems was excellent with good accuracy of the solutions and the least standard deviation rates.

INDEX TERMS Symbiotic learning grey wolf optimizer (SL-GWO), grey wolf optimizer (GWO), benchmark functions, CEC 2019 benchmarking, optimal power flow problems, optimal reactive power flow problems.

I. INTRODUCTION

Recent multi-disciplinary research and real-world scenarios have shown that optimization using meta-heuristics is a powerful tool for problem resolution and resource management. For its simplicity and usefulness in tackling complicated issues, optimization has been accepted and promoted by researchers and specialists alike. "Heuristic" means "random" or "creative" search procedure used to find the optimal/best solution combination to maximize or minimize the desired system characteristics [1]. No previous knowledge of the system, or in mathematical jargon, no gradient

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information of the mathematical objective, is required to determine the best set of solutions. In addition to tackling both single-objective and multi-objective problems, meta-heuristic optimization algorithms have the advantage of being easier to implement, robust and require lower computational resources. The fact that meta-heuristics are stochastic processes with a degree of unpredictability connected with their optimality has compelled many academics and specialists to enhance them to find the best balance of exploration and exploitation for the considered problem [2]. To recapitulate, meta-heuristic optimization techniques have improved several aspects of real-world problems such as resource management, control, operation, allocation, division of workforce, performance efficiency, speed of computation, error reduction, etc. [3].

A. DEMERITS WITH SWARM-BASED NATURE-INSPIRED TECHNIQUES

In order for the swarm-based nature-inspired optimization algorithms to work effectively, their limitations must be addressed. There is no guarantee that the intelligently crafted search techniques can solve all optimization problems and a near-perfect optimization approach has eluded experts for years despite their diligent crafting. The development of better/improved variations of nature-inspired algorithms best suited to the complexity of problem being dealt with has recently become prominent through various methodologies. At the same time, more articles on developing better/improved variations addressing the following issues have surged over the last decade.

1) CURSE OF DIMENSIONALITY

The deterioration in the performance (stagnation of fitness or sluggish convergence characteristics) of an optimization technique with the increasing number of problem dimensions is ascribed as "*the curse of dimensionality*", coined by Richard E. Bellman [4]. The manifold reasons are that there could be several possibilities of every decision variable for each combination of values and the fitness of all such possibilities must be computed within a limited number of function evaluations (NFEs) resulting in solutions very far from the global optimum.

2) EXPLORATION VS. EXPLOITATION

Achieving the perfect balance of exploration and exploitation is the most common issue with a multitude of swarm-intelligent, nature-inspired meta-heuristic optimization techniques. The balance of exploration (global search) and exploitation (local search) is intrinsic to the swarm intelligent optimization algorithms to deliver precise and diverse solutions across all decision variables. The algorithm must be devised to comprehend the condition to explore further or enhance current solutions such that an optimal trade-off between exploration (diversification) and exploitation (intensification) is achieved [2], [5].

3) TUNING REQUISITES

Furthermore, the requirement to tune several of parameters (called "*algorithm-specific tuning parameters*") to extract the best possible performance is often tedious and time consuming, and improper or inappropriate tuning has often been the main reason for algorithms' failure.

4) NO FREE LUNCH THEOREM

Algorithms that work well in unconstrained instances may not deliver exceptional optimality for multi-constrained tasks, and vice versa. It is also possible that algorithms designed for extensive global exploration may not be effective for local search. "*No free lunch theory*" [6], states that no meta-heuristic can perform optimally for every optimization problem and that the perfect algorithm cannot be realized that performs well for various scenarios.

B. IMPROVEMENT TECHNIQUES

Several improved/upgraded meta-heuristics have been developed over time and have gained prominence due to their improved performance in terms of optimality, consistency, and robustness [7].

The introduction and empirical development of unique techniques/operators that expand the exploration to newer places within the solution space while balancing the exploitation/local search is the norm to overcome the limitations with the standard paradigms. The process of "*improving*" or "*enhancing*" or "*modifying*" a meta-heuristics is extensively researched concerning the algorithmic structure for the considered optimization task [8]. The limitations of the existing tuning settings are analyzed and newer tuning strategies to tackle premature convergence and enhance population diversity are also considered to improve the performance. The improved versions are devised to surpass the standard version of the meta-heuristic at global and local search ensuring greater diversity in the population pool while converging steadily and quickly to the global optimal solution and dealing with multiple constraints effectively.

Another methodology approached is the hybridization of two existing meta-heuristics wherein a solid meta-heuristic immune to the negative aspects of the parent algorithms and benefitting from the reliable aspects is worked up [9]. Hybridization promotes the collective collaboration of both the meta-heuristics since the advantageous aspects are consolidated to such an extent that the different search mechanisms help each other in dynamically exploring while exploiting the search space [10]. Hybridization of the swarm-intelligence-based algorithms with the evolutionary techniques from Genetic Algorithm (GA) [11], [12] and DE (Differential Evolution) [13] has been experimented with several times to realize stronger synergistic hybrids. Besides incorporating the evolutionary strategies, numerous other combinatorial algorithms have been proposed in the past and the present scenarios, *e.g.* The hybridization of a swarm-based meta-heuristic with other swarm or nature-based/

physics-based/ metaphor-based algorithms has been actualized on various occasions.

In short, improvement and hybridization techniques are approached as a suitable choice to focus on ruling out the multitude of problems related to tuning frameworks wherein effective methodologies from other contemporary/well-known algorithms are picked and systematically developed to preclude the disadvantages.

C. RELATED WORKS

In recent times, the need to develop improved optimizers to provide the best performance in terms of optimality, accuracy, convergence speeds, limited computational complexity and times has been extensively studied through various complex CEC (Congress on Evolutionary Computation) benchmarking suites [14], [15], [16], [17] and their efficacy at solving real-world multi-constrained, complex, non-linear problems in various domains of engineering have been demonstrated through numerous publications [18], [19]. In addition to these, the number of novel optimization algorithms inspired by the various forces of nature has quadrupled in the last five years and has provided multiple research avenues for several researchers across the globe to develop improved variants of these standard algorithms to extract the best performance with limited computational budgets. A brief survey of some of the latest state-of-the-art improved optimizers is provided below.

Latest improvements in the evolutionary algorithms include the integration of special mutation schemes and population selection mechanisms to combat early entrapment issues. These include (i) An improved Differential evolution (DE) with orthogonal array-based initialization and a novel selection strategy with an ensemble strategy for parameter adaptation named OLSHADE-CS [20]. The performance evaluation with CEC2017 and CEC2020 test suites found OLSHADE-CS to be highly competitive and significantly better and demonstrated better optimality and quicker convergence rates to the global optimum. (ii) An effective multi-trial vector-based DE algorithm (MTDE) combining different search strategies in a sub-population environment incorporating a winner-based distribution policy and life-time archive system has been realized in [21]. Validated against 29 test functions from the CEC2018 test suite and four engineering design problems, MTDE outperformed several advanced variants of DE and recent nature-inspired algorithms.

The domain of swarm-intelligence with nature-inspired meta-heuristics and human-behaviour inspired meta- heuristics witnessed multiple advanced versions of popular meta-heuristics including the Whale Optimization Algorithm (WOA) [22], Chimp Optimization Algorithm (ChOA) [23], Grey Wolf Optimizer (GWO) [24], Social Group Optimization (SGO) [25] etc. The developments include (i) A novel gaze cues learning-based grey wolf optimizer (GGWO) to enhance the exploitation ability and local optima avoidance through reduction of the high selective pressure and low diversification of the standard GWO is developed by Nadimi-Shahraki *et al.* [26]. The integration of two novel search strategies aided GGWO to deliver the best optimality with accelerated convergence for the CEC2018 test suite, four engineering problems and three cases of power flow problems outperforming several other swarm-based meta-heuristics and five variants of GWO. (ii) An enhanced chimp optimization (EChOA) algorithm integrating a disruptive polynomial mutation based-initialization and Spearman's rank correlation coefficient based ranking system is developed at [27] to combat local entrapment. Validated on 12 classical and 15 CEC2017 benchmark functions followed by three engineering design problems and training multilayer perceptron EChOA delivered competitive results. (iii) An Effective Whale Optimization Algorithm (EWOA) to solve optimal power flow problems (Standard IEEE 6-bus, IEEE 14-bus, IEEE 30-bus, and IEEE 118-bus test systems) through the integration of Levy motion and Brownian motion into the standard search mechanism of WOA to combat *the curse of dimensionality* and maintain a proper balance between the exploration and exploitation is developed in [28]. EWOA outperformed the competitor algorithms and delivered solutions with improved optimality for the several cases of OPF investigated. (iv) An improved Harris Hawks Optimization (IHHO) algorithm through the simplification of the search strategies from a six-step decision mechanism to a four-step system is realized in [29]. Benchmarked using the CEC2019 test suite and a three-dimensional bin packing problem (3D-BPP) dataset with 320 samples, IHHO delivered statistically significant results. (v) A multi-strategy ensemble social group optimization algorithm (ME-SGO) to enhance the population diversity from complex landscapes through the integration of distance-based strategy adaption and success-based parameter adaption is proposed in [30]. The benchmarking through CEC2019 test suite demonstrated ME-SGO's robustness to entrapment and its application to four problems on the optimization of energy management in electric vehicles yielded improved results compared to its competitors. (vi) A component-based framework for the automatic design of Particle Swarm Optimization (PSO) Algorithms known as PSO-X embodies a large number of algorithm components developed over more than 25 years of research of PSO into a unified framework to determine the best possible configuration is developed at [31]. Benchmarking tests with over 50 test functions from the various CEC test suites showcased the efficiency of PSO-X at adapting to deliver the best possible solution quality in terms of accuracy, optimality and convergence speeds.

D. IMPROVING GWO

For a more adaptable and efficient approach, the current study proposes a "*Symbiotic learning-based grey wolf optimizer*" (SL-GWO). We chose GWO over other optimization methods for the following reasons. (1) The literature survey shows that GWO outperforms other paradigms in several multi-disciplinary applications. (2) Its simplicity allows it to be implemented in any programming language and used for a variety of optimization issues. (3) GWO could be experimented with for a possibly robust variant of GWO for the many-sided research pathways, as seen by the numerous articles where its performance has been considerably enhanced by application-specific upgrades or hybridization. (4) To increase the accuracy, population diversity and reduce the vulnerability to "*the curse of dimensionality*", GWO's tuning has been studied extensively. (5) Complex and constrained optimization problems with a higher dimensional count may demand further adaptations to the computational framework and dynamic tweaking leading to a larger search gradation. To maximize the algorithm's performance, a variety of population selection and update procedures could be further investigated.

In this regard, the population improvisation strategy and the selection strategies are kept simple and the proposed method is scrutinized and an extensive comparative analysis with the other state-of-the-art optimizers, including the latest meta-heuristics and the variants of GWO is performed to assess the competence and efficacy of the proposed method.

The improvement process through the symbiotic learning strategy proposed is computationally modest and with simpler sorting techniques to limit the computational times and the complexity associated with its execution. The proposed method intends to improve the intensification versus diversification balance while standing insusceptible to *the curse of dimensionality* and guaranteeing sped up convergence and quicker execution times and has been tested extensively to evaluate its performance across all standards. Exclusive of the static tuning parameters that often limit the scope of the search mechanism, the proposed method appends a versatile learning technique promoting elitism to tweak the search process to augment the quality of the solutions. A comprehensive description of the structure and working of the proposed method is given in the impending sections.

E. ORGANIZATION OF THE MANUSCRIPT

The manuscript is partitioned as described below. Section II discusses the working of GWO, followed by a comprehensive literature survey of its variants and a discussion of its merits and demerits. Section III focusses on the development of symbiotic hunting and learning strategies to improve GWO and a comprehensive analysis of its various attributes. The benchmarking analysis through 29 scalable CEC2018 benchmark functions, CEC2019 test suite and four standard engineering problems is given in Section IV. A comparative analysis of the proposed method and other competitive algorithms for the optimal power flow problem (13 cases for IEEE 30 and IEEE 57 bus systems) and optimal reactive power dispatch problem (8 cases for IEEE 30 and IEEE 57 bus systems) through a combination of linear incremental penalty function and archive-based constraint correction approaches is presented in Section V. The merits and demerits of SL-GWO are discussed Section VI. Section VII concludes the manuscript and discusses the future scope of the proposed method.

II. GREY WOLF OPTIMIZER

GWO is a nature-inspired, swarm-based metaheuristic optimization method inspired by grey wolves' leadership structure and hunting mechanism (*Canis lupus*). GWO was developed in 2014 by Seyedali Mirjalili, Seyed Mohammad Mirjalili and Andrew Lewis [24]. GWO is unique in that it organizes grey wolves into alpha, beta, delta, and omega groups and explores and exploits the search area. The GWO tuning requirements are the population size, iteration count, and an optional control vector. Exploration and exploitation are balanced through a linear reduction of the control vector from 2 to 0 over the hunting. Its excellent performance for both unconstrained and constrained, single and multi-objective optimization with improved optimality and fast convergence has attracted academics and practitioners from diverse domains.

A. WORKING OF GWO

To comprehend GWO, the understanding of the mathematical modelling of wolf social hierarchy is essential. It is the dominant wolves (alpha wolves) who govern the group's functioning and are responsible for decision-making and management. The second order is made up of beta wolves that serve as subordinates and advisors to the other wolf orders. The third category is the omega wolves, who are typically used as a scapegoat or babysitters. The delta wolves are the group's scouts, sentinels, elders, hunters, and caregivers. The delta wolves rule the omegas but follow the betas and alphas, establishing a middle ground between the two. GWO's fundamental activity is communal foraging based on the social hierarchy. Figure 1 depicts the social dominant hierarchical system of the grey wolves.

In GWO, the optimal solution is designated as alpha, the second-optimal solution as beta, and the third-optimal solution as delta. The latter group is referred to as the omegas. The following is a thorough discussion of the many parts of GWO's mathematical modelling.

1) ENCIRCLING THE PREY

The initial phase of GWO is dedicated to determining the prey's location. Initially assuming that the prey's position is unknown, the algorithm traverses the search space with the assumption that it is situated near the optimal solution. Once they have determined the position of the prey, they encircle it as part of the hunting process. Grey wolves search the region surrounding the site of prey to find more suitable solutions.

Eq. (2.1) and Eq. (2.2) represent the mathematical model for prey encirclement in GWO.

$$\overrightarrow{X_G}(t+1) = \overrightarrow{X_p(t)} - \overrightarrow{A} \times \overrightarrow{d} \tag{2.1}$$

$$\overrightarrow{d} = \left| \overrightarrow{B} \times \overrightarrow{X_p(t)} - \overrightarrow{X_G(t)} \right| \tag{2.2}$$

where, $\overrightarrow{X_G}$ is the position of the grey wolf, $\overrightarrow{A}$ and $\overrightarrow{B}$ are coefficient vectors, t is the present iteration, $\overrightarrow{X_p(t)}$ is the position of the prey, || is the modulus operator to determine

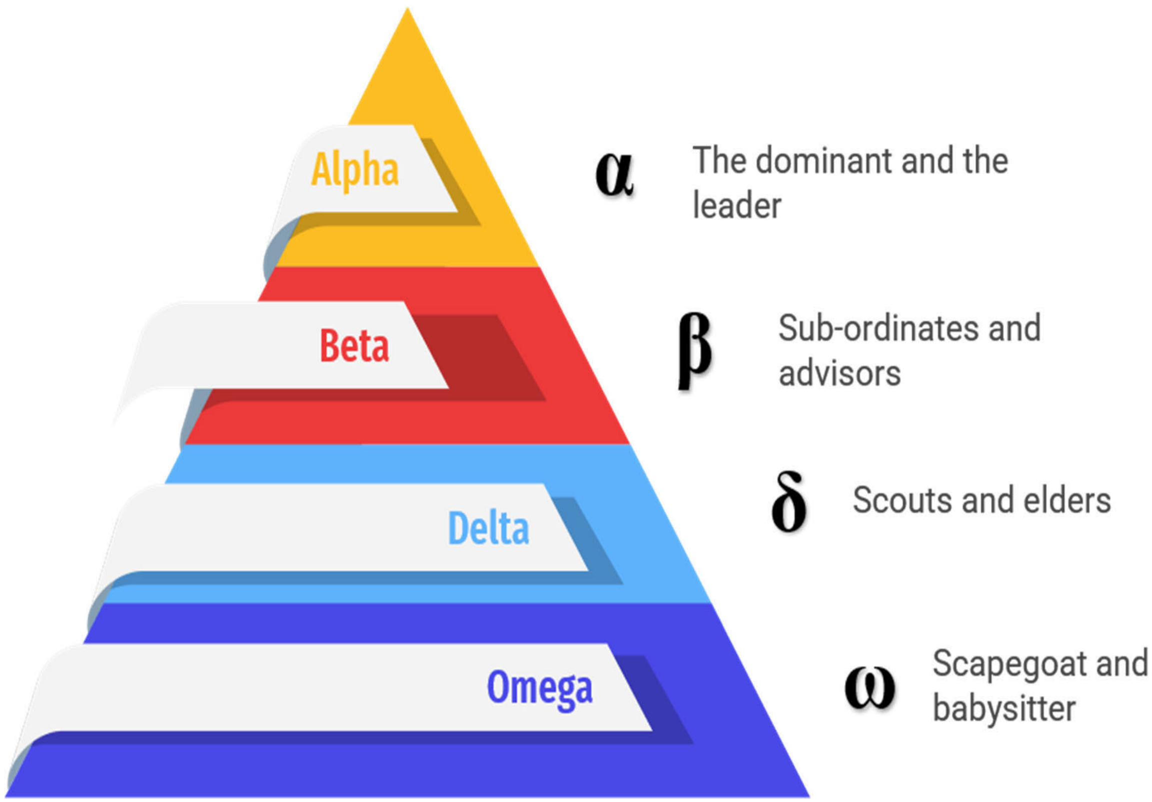

FIGURE 1. Social dominant hierarchy of the grey wolves.

the absolute value and '×' represents multiplication in an element-to-element manner.

Eq. (2.3) and Eq. (2.4) define the mathematical formulation of the vectors of co-efficients $\overrightarrow{A}$ and $\overrightarrow{B}$.

$$\overrightarrow{A} = 2\overrightarrow{a} \times \overrightarrow{r_1} - \overrightarrow{a} \tag{2.3}$$

$$\overrightarrow{B} = 2 \times \overrightarrow{r_2} \tag{2.4}$$

where, $\overrightarrow{a}$ is the control vector whose value tends to linearly decrease from an initial value of 2 to a final value of 0 over the course of iterations and $\overrightarrow{r}$ denotes a random vector in [0, 1].

2) HUNTING

Once the prey's position is determined, the alpha initiates the hunt. Supported by the beta, delta, and, on rare instances, the omega, the locations of the omegas are repositioned as dictated by the alpha, beta, and delta positions. The top three solutions obtained are archived in the hierarchical dominance of the wolves in order to further predict the location of prey and direct the omegas in updating their locations around it in upcoming generations.

Eq. (2.5) specifies the distances between the current grey wolf and the three dominant wolves and Eq. (2.6) specifies the positions derived from their distances.

$$\begin{aligned} \overrightarrow{d_\alpha} &= \left|\overrightarrow{B_1} \times \overrightarrow{X_\alpha} - \overrightarrow{X_G}\right| \\ \overrightarrow{d_\beta} &= \left|\overrightarrow{B_2} \times \overrightarrow{X_\beta} - \overrightarrow{X_G}\right| \\ \overrightarrow{d_\delta} &= \left|\overrightarrow{B_2} \times \overrightarrow{X_\delta} - \overrightarrow{X_G}\right| \end{aligned} \tag{2.5}$$

$$\begin{aligned} \overrightarrow{X_1} &= \overrightarrow{P_\alpha} - \overrightarrow{A_1} \times \left(\overrightarrow{d_\alpha}\right) \\ \overrightarrow{X_2} &= \overrightarrow{P_\beta} - \overrightarrow{A_2} \times \left(\overrightarrow{d_\beta}\right) \\ \overrightarrow{X_3} &= \overrightarrow{X_\delta} - \overrightarrow{A_3} \times \left(\overrightarrow{d_\delta}\right) \end{aligned} \tag{2.6}$$

Finally, the position of the grey wolf is given by Eq. (2.7).

$$\overrightarrow{X_G}(t+1) = \left[\frac{\overrightarrow{X_1} + \overrightarrow{X_2} + \overrightarrow{X_3}}{3}\right] \tag{2.7}$$

where, $\overrightarrow{X_G}$ is the position of the grey wolf, $\overrightarrow{X_\alpha}$, $\overrightarrow{X_\beta}$ and $\overrightarrow{X_\delta}$ represent the positions of the alpha, beta and delta wolves, $\overrightarrow{A}$ and $\overrightarrow{B}$ are the co-efficient vectors.

a: MERITS AND DEMERITS OF GWO

Although GWO is effective in a variety of applications, it has flaws such as a lack of population diversity, local entrapment, premature convergence, and a weak exploitation mechanism, to mention a few. The shortcomings of GWO have been highlighted in several review publications [32], [33], [34], [35], [36], [37] and there has been a greater focus on improving GWO to obtain a dependable and robust variant. This section highlights the advantages and disadvantages of GWO based on numerous review and research publications that have deployed it for optimization.

The merits of GWO include: (i) GWO follows Straightforward and simple optimization approach. In comparison to previous swarm-based optimizers, GWO is easy to construct on any programming platform. (ii) Apart from the population size and iteration count, no further algorithm-specific tuning parameters are required for the majority of optimization problems. (iii) In order to extract the maximum performance out of GWO, its tuning can be further experimented with for both continuous and discrete search environments. Binary GWO variations have been reported in the literature for feature selection and data categorization. (iv) Since GWO does not integrate any population sorting approach, it is quicker with lower computational complexity than the majority of meta-heuristics. Additionally, GWO has a quicker convergence rate than the conventional optimization paradigms and a majority of current meta-heuristics. (v) The strategy of spending the first half of iterations on exploration and the second half on exploitation is an intriguing and effective one for most problems. While one may argue that this is not the most cutting-edge technique for achieving a healthy balance of diversification and intensification, this area is open for proposals and deserves greater investigation. (vi) As GWO continues to explore the search space for a better answer, its reliance on the initial population is minimal. GWO's social order promotes dominant wolves to exert influence over omegas, ensuring that global and local searches are given equal priority for simpler unimodal and multi-modal landscapes. (vii) The ability to improve and enhance GWO in order to get a more robust variant has always been widely exploited and a cursory examination of the literature reveals a diverse range of enhancement strategies applied to GWO to boost its optimization capabilities. Despite the fact that it was published seven years ago, its reputation as a robust optimizer remains unmatched. (viii) Synergistic approaches may be developed by hybridization of the algorithm with the additional swarm and evolutionary meta-heuristics. Since its inception in 2014, a tremendous lot of research has been poured into developing a strong hybrid version of GWO.

The demerits include: (i) *The curse of dimensionality* has plagued GWO in several benchmark and real-world applications. Due to the selection and population updation techniques, performance has declined in situations with numerous complex problems with large number of decision variables. Many researchers have extended research at the problem of the algorithm's incapacity to deal with numerous dimensions since it may not reposition all of the wolves strategically. (ii) Convergence speeds are slower than those of other algorithms with sorting techniques for certain benchmarking and real-world applications. (iii) Despite its superior performance in comparison to the classical paradigms, the system of splitting iterations into exploratory and exploitative intensification phases does not guarantee that the majority of the search space has been covered or that the conflicting aspects of exploration and exploitation have been perfectly balanced. (iv) Complex and multi-modal search landscapes have proven a challenge since the algorithm is more prone to local entrapment, resulting in premature convergence. (v) As exploitation intensifies, the wolves' location narrows, and they may not travel far apart from each other beyond the initial stages of exploration, resulting in premature convergence. (vi) The wolfpack's greater reliance on the three dominant wolves localizes the population towards the end of iterations, making local trapping unavoidable. GWO incorporates no adaptive ways to escape entrapment if it were to happen at earlier levels.

B. LITERATURE REVIEW OF GWO

GWO, published in 2014 is categorized under the nature-inspired meta-heuristic optimization techniques in the domain of "*swarm-intelligence*". It has been regarded as a prominent and powerful stochastic optimizer with a myriad of applications in various domains with an envisioned status. Since its publication, it has been cited over 6500 times based on the data from the *SCOPUS*® database and *CrossRef*. The growth in the citations has been overwhelming over the last 5 years with over 2000 citations in the year 2021 alone. The surge in citations on an annual basis is provided in Figure 2.

The *Web of Science*® database ranks the Electrical and Electronics Engineering application of GWO over Computer Science and artificial intelligence applications with over 350 and 300 publications each. The list contains Computer Science and interdisciplinary applications followed by Energy fuels and Telecommunications.

In terms of applications relating to GWO, the *SCOPUS*® database holds over 1800 publications followed by 1500 publications indexed in the *Web of Science*® database and over 200 publications from the other databases. The growth in the publications deploying GWO to numerous applications has been on the rise since 2017. As far as the recent developments are concerned, over 1000 documents and 800 documents have been indexed in the SCOPUS and Web of Science databases respectively in 2020. The advancement in the publication growth incorporating GWO is shown in Figure 2.

As per the *SCOPUS*® database, the areas of application of GWO include Computer Science, Mathematics, Engineering, Energy, Materials Science, Physics and Astronomy, Environmental Science, Bio-engineering, Decision Sciences, Business, Management and Accounting, Chemical Engineering etc. to name a few. Computer Science and Engineering emerged as the leaders in the deployment of GWO with over 1200 and 1000 documents respectively.

As far as the territories incorporating GWO in their research is concerned, India and China hold the most publications accounting for over 400 and 300 publications respectively. This is followed by Iran, Egypt and Malaysia.

C. VARIANTS OF GWO

The literature survey of GWO unveils multitudinous variants each aimed at countering the limitations of the parent algorithm. Most of the variants are aimed at the commonly

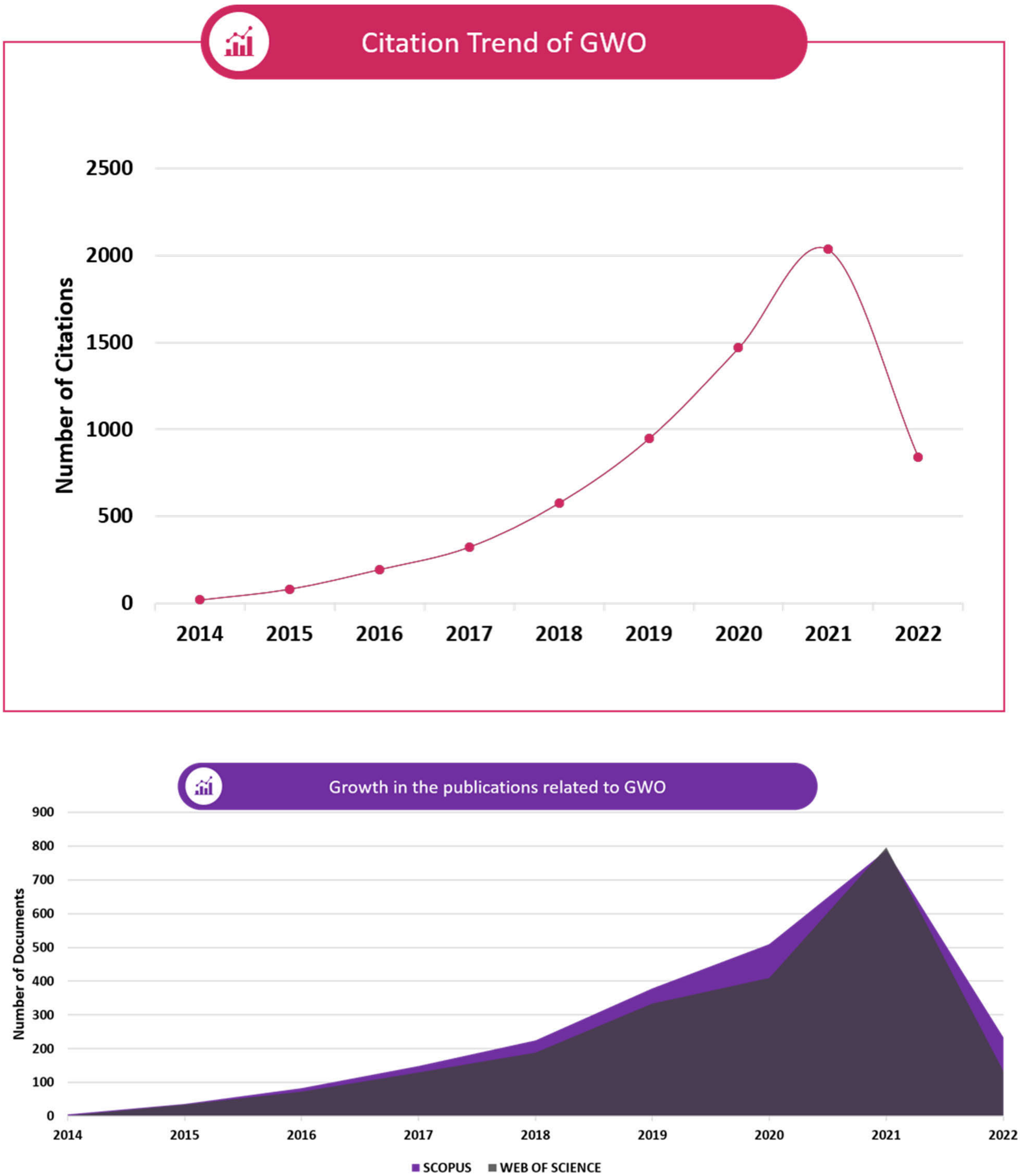

FIGURE 2. Annual citation count and the growth in the number of publications related to GWO over the last eight years.

tackled problem of balancing the exploration and exploitation of GWO through novel search operators or through the use of exiting techniques like hybridization or combining them with other evolutionary, swarm-based meta-heuristics, particle flight trajectory control through Levy Flights etc. The other limitations such as the susceptibility to "*the curse of dimensionality*", and slower convergence are dealt with through the introduction of special tuning parameters, chaos theory, population selection and function evaluation strategies, sorting and ranking mechanism, and population re-initialization, etc. The proposed variants are evaluated for their efficacy and efficiency through standard benchmarking functions from the various CEC session, hybrid test functions, composition functions, constrained standard engineering problems with statistical tests, computational times and convergence graphs to elucidate the performance prowess. Additionally, various real-world optimization problems are tackled through these proposed variants and compared aptly with the other techniques to validate the superiority in the performance. The variants shown in bold face have been chosen for the comparative analysis with SL-GWO in the present work.

1) VARIANTS INCORPORATING CHAOS THEORY AND CHAOTIC MAPS

The incorporation of chaos theory into meta-heuristics is done through various chaotic maps and has gained prominence over the years on account of its ergodicity, easy integration and non-redundancy. In most meta-heuristics, chaos theory is primarily aimed to initialize the population with a good diversity to cover larger and diversified areas in the search space. The advantage of chaotic maps is that it speeds up the exploration of the search space while encouraging good population interaction. The computational speeds are at a higher pace in contrast to the other probability distribution techniques such as the uniform distribution. The other notable property is their dynamic nature to generate the random number which has often been excellent for an exhaustive exploration aimed at attaining the global best solution. The integration of chaos theory with GWO has been researched extensively and has over 60 documents and publications in the literature. Table 1 presents a few of the most cited and state-of-the-art variants of GWO incorporating chaos theory.

2) IMPROVED/ENHANCED VARIANTS

Improvements to meta-heuristics have been one of the most extensively sought-after research avenues with countless variants of several meta-heuristics being published year after year. The improvement/enhancement techniques are proposed to counteract the limitations of the meta-heuristics, expand the exploration reachability, synergize the exploration and exploitation, dynamically tune the tuning criteria with respect to the problem's landscape etc. As a result, the improved meta-heuristic has a good population variety, is capable of circumnavigating the constrained areas with the search space, quicker convergence and lowered susceptibility to *the curse of dimensionality*. Several improvement techniques such as Levy flights, Gaussian/random walks, opposition-based learning methods, sorting and re-initialization have been incorporated into GWO as reported in the literature. With over 300 publications related to the improved and enhanced variants of GWO, most of them have been aimed at delivering a good equilibrium of the exploration and exploitation and validated through various benchmarking standards. Table 2 presents a few of the most cited and state-of-the-art improved/enhanced variants of GWO.

3) MODIFIED AND LEARNING-BASED VARIANTS

Modified and learning-based variants examine and scrutinize the algorithmic structure and alter the search measures with the incorporation of correction mechanisms, adaptive tuning systems, elitism, and so forth with a definitive objective of amplifying the exploratory and exploitative potential across all the norms. A large number of the modified and learning variants restructure and incorporate special soft computing techniques to tackle the problem at hand with the aim of boosting the performance across multiple standards with various test cases and benchmarking to validate that the modified variant is robust and reliable. The modified variations of GWO studied here are pointed toward enhancing the exploratory search gradation while standing immune to *the curse of dimensionality*. Over 200 publications have been found in teg literature survey bearing the title modified/learning-based variants of GWO across various domains and disciplines. Table 3 presents a few of the most cited and state-of-the-art modified and learning-based variants of GWO.

4) HYBRIDIZED/COMBINATORIAL VARIANTS

Hybridization/combinatorial variants of GWO with the existing swarm and evolutionary algorithms have been an ongoing trend since the publication of GWO with over 300 publications. The combinatorial variants operate in synergy combining the best aspects of both their parent algorithms with robust and consistent performance across all standards and have considerable performance improvement for the conflicting cases of exploration versus exploitation. Table 4 presents a few of the most cited and state-of-the-art hybridized/combinatorial variants of GWO.

III. PROPOSED WORK: SYMBIOTIC LEARNING - GREY WOLF OPTIMIZER (SL-GWO)

After a thorough investigation of GWO, its various state-of-the-art versions, review papers, and publications on GWO and its applications, this work proposes a symbiotic learning GWO. The enhanced algorithm, referred as SL-GWO, is developed to overcome the numerous shortcomings of GWO, including susceptibility to *the curse of dimensionality* and local entrapment, as well as to achieve an optimum balance between global (exploration) and local search (exploitation).

A. MOTIVATION

The literature survey of GWO and its variants from the previous section provides an overview of the different aspects of the standard GWO algorithm that require improvements. Considering that there have been numerous publications aimed at improving the performance of GWO [57], [58], [29], [60] in terms of optimality and convergence speeds for the standard benchmark functions, only a handful of studies have aimed at enhancing GWO for shifted and rotated complex and dynamic landscapes including IGWO from [43], GGWO

TABLE 1. Tabulation of the chaos theory/chaotic map integrated variants of GWO from the recent literature.

S.No	Authors and Year	Description of the variant of GWO	Optimization problem tackled	Outcome
1.	**S. Arora *et al.* in 2017** [38]	**Chaotic grey wolf optimization algorithm (CGWO) to improve the global convergence speeds is proposed incorporating ten chaotic sequences.**	**A set of 5 constrained standard engineering problems namely, Tension/Compression spring design problem, Gear train design problem, Welded beam design problem, Pressure vessel design problem and Closed coil helical spring design problem are opted for the validation.**	**The proposed method had the best outcomes in terms of the best, mean and standard devotion with faster convergence rates.**
2.	**M. He *et al.* in 2018** [39]	**An improved chaotic Gray Wolf Optimization (ACGWO) incorporating Logistic Chaotic map and a nonlinear convergence factor to maintain the balance between global search and local search, a nonlinear convergence factor was introduced.**	**Bioactive assay and hyphenated chromatography detection for complex supercritical CO_2 extract from *Chaihu Shugan San* using an experimental design approach are tackled.**	**In a comparative analysis between GWO, CGWO, PSO and ACGWO, the accuracy and stability of the RSM analysis by ACGWO were the best.**
3.	R.A. Ibrahim *et al.* in 2018 [40]	A chaotic opposition-based grey-wolf optimization algorithm based on differential evolution and disruption operator (COGWO2D) to improve the exploration and exploitation capabilities is proposed.	Benchmarking functions from the CEC2005 and CEC2014 suites and galaxy image classification for feature selection are investigated.	The proposed method had superior solutions with better optimal solutions and expedited convergence rates.
4.	C. Lu *et al.* in 2020 [41]	A chaotic-based grey wolf optimizer to strengthen the performance of GWO with three different chaotic strategies comprising eleven various chaotic map functions is proposed.	A set of 23 benchmark functions with 8 unimodal and 15 multi-modal functions and 25 benchmark functions from the CEC2005 suite followed by 4 standard engineering problems are chosen.	The third chaotic strategy with the Chebyshev map turned out to be the most efficient in terms of optimality and convergence.
5.	Li *et al.* in 2020 [42]	An Improved Grey Wolf Optimizer (IGWO) incorporating the Kent chaotic algorithm for population initialization and population diversity enhancement alongside an adaptive adjustment strategy is proposed.	The optimal distributed cooperative task allocation strategy for multi-robot task allocation (MRTA) with the K-means clustering algorithm for clustering of the task target points, and 16 international classical test functions are investigated.	IGWO delivered a reasonable task allocation strategy with higher accuracy and faster convergence.

TABLE 2. Tabulation of the improved/enhanced versions of GWO from the recent literature.

S.No	Authors and Year	Description of the variant of GWO	Optimization problem tackled	Outcome
1.	**M.H. Nadimi-Shahraki *et al.* in 2020** [43]	**An Improved Grey Wolf Optimizer (I-GWO) incorporating a novel movement strategy referred to as dimension learning-based hunting (DLH) search to encourage neighbourhood information sharing is proposed.**	**A set of 29 benchmark functions from the CEC 2017 suite and four standard engineering problems (welded beam design, pressure vessel design, optimal power flow problem for IEEE 30, 118 bus systems is investigated.**	**I-GWO exhibited superior exploration and exploitation with better optimality with faster convergence characteristics**
2.	**Kaiping Luo in 2019** [44]	**An Enhanced grey wolf optimizer (EGWO) with a model for dynamically estimating the location of the prey by lowering the search bias towards the origin of the coordinate system through a more realistic model is proposed to mimic the leadership hierarchy and group hunting mechanism of grey wolves is proposed.**	**A set of 30 benchmark functions from the CEC 2017 suite and two standard engineering problems (tension/compression spring design, pressure vessel design) are chosen.**	**EGWO outperformed GWO and two other variants in terms of the convergence speed and the quality of solutions found.**
3.	W. Long *et al.* in 2017 [45]	An exploration-enhanced grey wolf optimizer (EEGWO) based on a new position-update equation and non-linear control parameter strategy to balance the exploration and exploitation is proposed.	A set of 23 benchmark functions and 4 standard engineering problems (tension/compression coil spring design problem, three-bar truss design problem, pressure vessel design problem and welded beam design problem) are chosen to validate the proposed technique.	EEGWO remained immune to "*the curse of dimensionality*" with accelerated convergence for the benchmarking and produced solutions with greater accuracy for the engineering problems.
4.	Z. Teng *et al.* in 2018 [46]	An improved hybrid grey wolf optimization algorithm (PSO_GWO) combining the non-linear control technique from GWO with Tent chaotic sequence for population initialization and PSO based position updating system is developed.	A set of 18 benchmark functions comprising unimodal and multi-modal functions are considered to validate the performance in terms of the mean fitness and standard deviation.	Better optimality and faster convergence with similar computational times to that of the standard GWO were the outcomes.
5.	A. Kavesh *et al.* in 2018 [47]	An improved grey wolf optimizer (IGWO) to enhance the adaptability of the algorithm while defining a few tuneable parameters to enhance the search capabilities is proposed.	The optimal design of truss structures (25-bar spatial truss, 72-bar spatial truss, 200-bar planar truss and 942-bar spatial truss tower) is investigated.	The proposed technique generated solutions leading to lighter truss structures while maintaining a relatively similar rate of convergence to that of the standard GWO.

from [26], RGWO from [61] with higher dimensionality (up to 50D). Although a few studies including HBBO-GWO [53], MEGWO [48], IGWO [43] GGWO [26] have analyzed their performance with the recent CEC test suites, performance analysis with multiple benchmarking suites with different requirements and computational budgets have not been made.

TABLE 3. Tabulation of the modified and learning-based variants of GWO from the recent literature.

S.No	Authors and Year	Description of the variant of GWO	Optimization problem tackled	Outcome
1.	**Q. Tu *et al.* in 2018** [48]	**Multi-strategy ensemble GWO (MEGWO) incorporating adaptive cooperative learning and global best lead strategies to improve intensification with diversified population is developed.**	**CEC2014 test suite with 30 test functions and feature selection for 12 datasets are investigated.**	**Robust performance with good consistency was observed with MEGWO compared to its competitors.**
2.	X. Zhang *et al.* in 2019 [49]	An improved GWO based on random opposition learning (RSMGWO) to improve the optimization performance and avoid local entrapment is proposed.	A set of 19 high dimensional benchmarking functions and the training of multi-layer perceptron (MLP) for cancer identification are considered for the validation of results.	The proposed method was efficient at accuracy and convergence speeds while boasting lower computational times.
3.	**S. Dhargupta *et al.* in 2020** [50]	**Selective Opposition based GWO (SOGWO) to promote the exploratory characteristics and ensure accelerated convergence through Spearman's correlation coefficient-based learning is proposed.**	**A set of 23 benchmark functions (7 Uni-modal functions, 6 Multi-modal functions and 10 Multi-modal functions with fixed dimensions) are chosen and validated through significance analysis.**	**SOGWO had a good immunity towards "*the curse of dimensionality*" and exhibited faster convergence for most benchmarking cases.**
4.	**M. S. Nasrabadi *et al.* in 2016** [51]	**A Parallel GWO with Opposition based learning with subgrouping to enhance population diversity and promote convergence rates is proposed.**	**A set of 20 benchmark functions is opted and compared against the standard GWO, Cat swarm optimization (CSO), PSO, Gravitational search algorithm (GSA).**	**The proposed method was faster and presented better optimal outcomes in terms of lower standard deviation and better mean values.**
5.	A.A. Heidari *et al.* in 2017 [52]	An efficient modified GWO with Lévy flight (LGWO) and greedy selection strategies with modified hunting phases to improve the efficacy of the search process is proposed.	A set of 29 unconstrained benchmarking problems, 30 artificial and 14 real-world problems from CEC2011 and CEC 2014 suites were chosen along with significance analysis to validate the results.	LGWO had a better global outreach with solutions approximating the global optimum with statistically significant results and lower standard deviation.

TABLE 4. Tabulation of the combinatorial/hybridized versions of GWO from the recent literature.

S.No	Authors and Year	Description of the variant of GWO	Optimization problem tackled	Outcome
1.	X. Zhang *et al.* in 2018 [53]	A hybrid algorithm linking Biogeography-Based Optimization (BBO) and GWO (HBBOG) with an integrated differential mutation operation to improve local and global search abilities is proposed.	30 benchmarking functions from the CEC2014 test set are chosen and compared against the variants of DE, BBO, GWO, ABC, PSO etc. followed by its application to 9 feature selection datasets.	HBBOG delivered a robust and statistically significant performance in terms of optimality and high accuracy with lower computational times for the feature section.
2.	Xinming Zhang *et al.* in 2020 [54]	A Hybrid Particle Swarm and GWO (HGWOP) embedding a novel differential perturbation strategy in conjunction with a stochastic mean example learning strategy to avoid local entrapment and encourage deeper exploitation is proposed.	Benchmarking using the complex functions from CEC2013, CEC2015 test sets are followed by its application to K-means clustering optimization is carried out.	HGWOP showcased better efficiency at reaching the global optimum for the benchmarking tests and exhibited relatively better performance for the clustering datasets compared to the other variants of PSOI and GWO.
3.	N. Singh *et al.* in 2017 [55]	A hybrid GWO – Sine Cosine Algorithm (SCA) known as HGWOSCA combining the exploitation of GWO and exploratory abilities of SCA while aiming to improve its convergence speed is proposed.	A set of 22 benchmark functions, 5 bio-medical datasets and one sine dataset for feature selection in a comparative analysis involving variants of GWO, PSO, SCA etc.	HGWOSCA excelled in terms of solution convergence for the benchmarking and had the highest classification rates for the feature selection datasets in the competitive analysis.
4.	**J. S. Wang *et al.* in 2019** [56]	**An Improved GWO (IGWO) based on DE and elimination mechanism to facilitate faster convergence and increased optimization accuracy incorporating the "*survival of the fittest*" (SOF) principle is proposed.**	**A set of 12 benchmark functions (unimodal and multi-modal) are chosen to validate the performance of the proposed method.**	**A fast convergence was noted for the 12 functions with optimal solutions obtained averaging closer to the global optimum compared to the other algorithms in comparison.**
5.	F. A. Senel *et al.* in 2019 [57]	A hybrid PSO–GWO algorithm incorporating the exploitation system from PSO and exploration strengths from GWO to improve the overall search efficiency is proposed.	A set of 5 benchmark functions and three real-world problems (parameter estimation for frequency-modulated sound waves, process flow sheeting problem, and leather nesting problem) are chosen.	The proposed method achieved better optimality in the solution quality for both benchmarking and real-world problems.

In this regard, the present work provides a comprehensive performance overview using two of the latest benchmarking standards i.e., the CEC2018 test suite with 10, 30 and 50 function dimensions with limited computational budgets followed by the CEC2019 test suite to validate the accuracy and precision with higher computational budgets.

The second motivating factor is the realization of a novel hunting methodology inspired from the social hierarchy of the grey wolves from the standard GWO. The proposed SL-GWO incorporates population restructuring and sub—grouping to promote diversity and takes advantage of the greedy selection system to promote elitism. Furthermore, SL-GWO adopts two diverse set of control mechanisms to guide and tune the parameters to achieve a better balance of exploration and exploitation. Despite the existence of numerous advanced variants of GWO with emphasis on greedy selection such as GGWO, SOGWO, IGWO, IGWO-DE etc., efforts at modifying the hurting mechanism of the grey wolves have been confined to appending the newer strategies over the existing hunting mechanism of grey wolves resulting in higher computational complexities.

The third motivating factor is the lack of extensive comparisons with previous studies aimed at improving GWO. Several studies have demonstrated the improved performance of their novel GWO variants by comparing them with classical paradigms such as Particle Swarm Optimizer (PSO) [62], Krill herd (KH) [63] etc. The current study provides a comprehensive comparative analysis by comparing the performance of SL-GWO with eight state-of-the-art variants of GWO, the standard GWO algorithm and five recent modern swarm-intelligent nature-inspired optimization algorithms.

B. IMPLEMENTATION

The symbiotic hunting and learning processes in SL-GWO are a major overhaul to the hierarchical hunting strategy from the canonical GWO to improve the quality of solutions, expand the solution space and ensure a better balance of exploration and exploitation. First and foremost, the total population of grey wolves is divided into two sub-groups i.e., the *attacking hunters* and *experienced hunters*. In order to preserve the social hierarchical dominance, both the groups are guided by the same alpha, beta and delta wolves which are selected based on their fitness levels from the two sub-groups. The key difference between the two sub-group of wolves lies in their hunting style and learning methodology.

The hunting mechanism for the two sub-group of wolves, i.e., the attacking hunters and experienced hunters is specified as follows. The hunting mechanisms are kept the same for each sub-group, however, their control strategies are different. The two sub-groups are obtained by the following rule specified by Eq. (3.1) initially.

$$P_i = \begin{cases} \textit{Attacking Hunters} & \textit{if } i \leq round(N_{p/2}) \\ \textit{Experienced Hunters} & \textit{otherwise} \end{cases} \tag{3.1}$$

where, i denotes the current population index, P_i denotes the current member of the wolfpack and N_p denotes the total population count or the size of the wolfpack.

Each grey wolf has a learning curve associated with itself that helps it to learn from the various symbiotic hunting strategies. This learning rate is called the symbiosis rate (S_r) and it dictates the reliance of the grey wolf to hunt as per the symbiotic hunting strategies or follow its previous lead. In order to enhance the diversity of the newer solutions obtained on a dimensional basis, S_r is associated with each problem dimension and the new positions are obtained as shown in Eq. (3.2).

$$P_{d,i}^{(t+1)} = \begin{cases} X_{d,i}^{(t+1)} & \textit{if } rand(d) \leq S_r \\ P_{d,i}^{(t)} & \textit{if } rand(d) > S_r \end{cases} \tag{3.2}$$

where, d is the current dimension and $d = 1, 2, \ldots D$, D denotes the total number of problem dimensions, *rand* is a random number in [0, 1] generated through uniform distribution, *rand(d)* denotes a matrix of random numbers with the size *(1,d)*, t is the current iteration, $t = 1, 2 \ldots T$ and T denotes the total number iterations.

The value of S_r is dynamically varied in the range of 0 to 1 for the two sub-groups. The attacking group has a higher affinity to follow the leaders through the various symbiotic hunting strategies and hence has a higher value of S_r. However, since higher S_r may not be beneficial at all times as exploiting across all dimensions may result in premature convergence, a probability of 50% is chosen to dynamically vary the S_r and is specified per Eq. (3.3).

$$S_{r(i)} = \begin{cases} rand & \textit{if } rand \leq 0.5 \\ ((0.99 - 0.5) \times rand) + 0.5 & \textit{otherwise} \end{cases} \tag{3.3}$$

The experienced hunters adopt a cautious approach to follow the leaders and their primary job is to preserve elitism and diversity through their experience which is simply the measure of success and failures associated with their individual symbiosis rates (S_r). The values of S_r that have been proven successful at generating fitter new grey wolves are retained and in cases of failure to produce an elite individual, the older values are incremented or decremented randomly by a value of 0.1. This is specified by Eq. (3.4).

$$S_{r(i)} = \begin{cases} S_{r(i)}^{Old} & \textit{if } f(new) < f(old) \\ S_{r(i)}^{Old} \pm 0.1 & \textit{otherwise} \end{cases} \tag{3.4}$$

where, $S_{r(i)}^{Old}$ is the symbiosis rate retained from the precious iteration for the current member of the wolfpack, $f(new)$ denotes the fitnees value of the new grey wolf and $f(old)$ correpsinds to the fitness valies of the wolf from the previous iteraration.

The symbiotic hunting strategies form the core of SL-GWO as the hunting strategies require the collaboration of the three dominant wolves and other randomly chosen omega wolves to determine new positions that help the current members of the wolf-pack advance their hunt. Four symbiotic hunting strategies are designed and these four strategies are the same for both the population sub-groups. Among the four symbiotic hunting strategies, the first two strategies are the key to improving the hunt as both of them incorporate the alpha, beta and delta wolves to guide the hunt. The first strategy is an extension of the hierarchical hunting scheme from the standard GWO. In this strategy, the hunting is led by alpha, beta and delta wolves with two random sub-ordinates i.e., the two random omega wolves accompany the alpha, beta and delta wolves. Furthermore, the enhance diversity, the three least fitter wolves are added as a supplementary support with lesser emphasis on their contribution. The first hunting strategy is described by Eq. (3.5).

$$X^{First} = \frac{H_\alpha + H_\beta + H_\delta}{3} \tag{3.5}$$

where, X^{First} is the solution vector obtained from the first symbiotic hunting strategy, H_α, H_β and H_δ are the position vectors obtained based on the guidance of the alpha, beta and delta wolves given by Eq. (3.6), Eq. (3.7) and Eq. (3.8) respectively.

$$H_\alpha = \overrightarrow{P_\alpha} + D_1 \times \left(\overrightarrow{P_{\omega_{r1}}} - \overrightarrow{P_{\omega_{r2}}}\right) + \left[L \times D_l \times \left(\overrightarrow{P_{\omega_{r1}}} - \overrightarrow{P_{W1}}\right)\right] \tag{3.6}$$

$$H_\beta = \overrightarrow{P_\beta} + D_1 \times \left(\overrightarrow{P_{\omega_{r3}}} - \overrightarrow{P_{\omega_{r4}}}\right) + \left[L \times D_l \times \left(\overrightarrow{P_{\omega_{r3}}} - \overrightarrow{P_{W2}}\right)\right] \tag{3.7}$$

$$H_\delta = \overrightarrow{P_\delta} + D_1 \times \left(\overrightarrow{P_{\omega_{r5}}} - \overrightarrow{P_{\omega_{r6}}}\right) + \left[L \times D_l \times \left(\overrightarrow{P_{\omega_{r5}}} - \overrightarrow{P_{W3}}\right)\right] \tag{3.8}$$

where, $\overrightarrow{P_\alpha}$, $\overrightarrow{P_\beta}$ and $\overrightarrow{P_\delta}$ are the positions of the three dominant wolves, $\overrightarrow{P_{\omega_r}}$ denotes a random omega wolf chosen from the current population, D_1 is the first dynamic hunting distance in the range 0 and 1, D_l is the limiting distance to the worst performing wolves in the wolfpack, L is the limiting factor to control the influence of the worst performing wolves on the current hunt. Its value is set to 0.01 for the attacking hunters and 0 for the experienced hunters. The limiting distance is linearly decreased in the range 0.5 to 0 and is specified by Eq. (3.9).

$$D_l = \left(0.5 - t \times \left(\frac{0.5}{T}\right)\right) + 0.05 \tag{3.9}$$

The second symbiotic strategy is implemented as described by Eq. (3.10).

$$X^{Second} = \overrightarrow{P_G}(t) + \left[D_1.\overrightarrow{\Delta}_1 - D_2.\overrightarrow{\Delta}_2\right] \tag{3.10}$$

where,

$$\overrightarrow{\Delta}_1 = \overrightarrow{P_{\omega(r1)}} - \overrightarrow{P_\alpha}$$
$$\overrightarrow{\Delta}_2 = \overrightarrow{P_{gw}}(t) + \left(\overrightarrow{P_\beta} + \overrightarrow{P_\delta}\right)$$

where, X^{Second} is the solution vector obtained from the second symbiotic hunting strategy, $\overrightarrow{P_G}(t)$ is the current position of the grey wolf, $\overrightarrow{P_{\omega(r1)}}$ refers to a random omega wolf from the wolfpack, $\overrightarrow{P_\alpha}$, $\overrightarrow{P_\beta}$ and $\overrightarrow{P_\delta}$ represent the positions of the alpha, beta and delta wolves obtained from the standard GWO procedure, $\overrightarrow{r}$ is a random vector in [0, 1], $\overrightarrow{\Delta}_1$ is the difference vector between $\overrightarrow{P_{\omega(r1)}}$ and $\overrightarrow{P_\alpha}$, $\overrightarrow{\Delta}_2$ is the vector to represent the combined positions of $\overrightarrow{P_G}$, $\overrightarrow{P_\beta}$ and $\overrightarrow{P_\delta}$.

The second symbiotic strategy is the most important and the key to maintaining a proper balance of exploration and exploitation. This strategy dynamically changes from the exploration phase to the exploitation phase based on the position of the alpha, beta and delta wolves. In the initial stages of exploration, the distance between the three wolves would be greater resulting in better exploration of the search space. Since one random wolf from the omegas is also chosen to determine the next position of the wolf, the complete dependence on the alpha, beta and delta wolves is lowered. The inclusion of the current position of the wolf from the current population process ensures that the position update is aimed at improving its optimality in the neighborhood as it explores closer to that position for a superior solution. This system encourages random omega wolves to learn from the dominating alpha and the wolf from the two sub-groups and compete with beta and delta wolves to further improve their positions towards a better optimal solution.

The third and the fourth symbiotic learning strategies are described below and are chosen alternatively.

The third symbiotic learning strategy is given by Eq. (3.11).

$$X^{Third} = \overrightarrow{X_\alpha} + D_1 \times \left[\overrightarrow{Z_1}\right] \tag{3.11}$$

where,

$$\overrightarrow{Z_1} = \overrightarrow{X_{\omega(r2)}} - \overrightarrow{X_{\omega(r3)}}$$

The fourth symbiotic learning strategy is given by Eq. (3.12).

$$X^{Fourth} = \overrightarrow{X_\alpha} + \left[D_1 \times \left[\overrightarrow{Z_2}\right] + D_2 \times \left[\overrightarrow{Z_3}\right]\right] \tag{3.12}$$

where,

$$\overrightarrow{Z_2} = \overrightarrow{X_{\omega(r4)}} - \overrightarrow{X_{\omega(r5)}}$$

and

$$\overrightarrow{Z_3} = \overrightarrow{X_{\omega(r6)}} - \overrightarrow{X_{\omega(r7)}}$$

where, X^{third} is the solution vector obtained from the third symbiotic hunting strategy, X^{Fourth} is the solution vector obtained from the fourth symbiotic hunting strategy, $\overrightarrow{Z_1}$, $\overrightarrow{Z_2}$ and $\overrightarrow{Z_3}$ are the difference vectors between any two randomly chosen omega wolves.

The inclusion of two random omega wolves and four random omega wolves in the second and third strategies is to ensure that the algorithm is prevented from being trapped at a local optimum point in the early stages of its exploration. As different omegas are chosen for each of the two strategies, the population diversity is enhanced.

The hunting distances D_1 and D_2 are crucial at attaining the right balance of exploration and exploitation. Very higher values of D_1 and D_2 i.e., D_1, $D_2 > 1.5$ can result in the solution dimensions exceeding the search boundaries and extremely smaller value in the range less than 0.5 may force exploitation at all times. Hence to overcome this limitation, the values of D_1 and D_2 are set to dynamically update their values in the range 0 to 1. The values of D_1 and D_2 for the attacking hunters are designed to complement each other. D_1

is linearly decremented from 1 to 0 with a lower tolerance limit of 0.05 while D_2 varies sinusoidal in the range [-0.5, 0.5] with a periodicity of $10/\pi$. These variations are essential to explore a larger search space and exploit systematically with respect to the progression of iterations. These are given by Eq. (3.13) and Eq. (3.14) respectively.

$$D_1 = \left(1 - t \times \left(\frac{1}{T}\right)\right) + \tau \tag{3.13}$$

$$D_2 = 0.5 \times \sin\left(\frac{10}{\pi \times t}\right) \tag{3.14}$$

where, τ is a tolerance limit and is set to 0.05.

The hunting distances D_1 and D_2 in for the experienced hunters are based on their successful generation of elite new wolves and the same method utilized to adapt S_r is once again adopted here. This is specified by Eq. (3.15).

$$D_{1_{(i)}} = D_{2_{(i)}} = \begin{cases} D_{1_{(i)}}^{Old} & if\ f(new) < f(old) \\ D_{1_{(i)}}^{Old} \pm 0.1 & otherwise \end{cases} \tag{3.15}$$

Finally, in every iteration population is updated based through a random choice of one of the four symbiotic hunting strategies as per Eq. (3.16).

$$X_i^{(t+1)} = \begin{cases} X^{First} \\ \quad if\ r_1 < 0.5 \\ \begin{cases} X^{Second} & if\ r_2 > 0.5 \\ \begin{cases} X^{Third} & if\ r_3 > 0.5 \\ X^{Fourth} & otherwise \end{cases} & otherwise \end{cases} \\ \quad otherwise \end{cases} \tag{3.16}$$

where, r_1, r_2 and r_3 are three random numbers in the interval [0, 1] generated through uniform distribution.

One the new solution is generated by one of the four symbiotic hunting strategies, its individual dimensions are checked and those exceeding the search boundaries are randomly re-initialized within the given lower and upper bounds. This is given by Eq. (3.17) and Eq. (3.18) respectively.

$$X_{i,d} = \begin{cases} X_{i,d} & if\ X_{i,d} > lb \\ X_{i,d} + rand \times (ub_d - lb_d) & otherwise \end{cases} \tag{3.17}$$

$$X_{i,d} = \begin{cases} X_{i,d} & if\ X_{i,d} < ub \\ X_{i,d} - rand \times (ub_d - lb_d) & otherwise \end{cases} \tag{3.18}$$

The final step is the fitness evaluations of all the newer population members. The greedy selection technique is opted for the symbiotic learning phase to update the population pool with superior solutions. The greedy selection allows for the survival of population members from the symbiotic learning strategies with better fitness compared to the older ones. The survival of the fittest strategy is followed to select the fitter population members and discard the rest. In the case of inferior solutions, the old positions are retained as given by Eq. (3.19).

$$\overrightarrow{X_i}\ (t+1) = \begin{cases} \overrightarrow{X_i}\ (t+1) & iff\ \left(\overrightarrow{X_{t(t+1)}}\right) < f\left(\overrightarrow{X_{i(t)}}\right) \\ \overrightarrow{X_i}\ (t) & otherwise \end{cases} \tag{3.19}$$

where, $f\left(\overrightarrow{X_{t(t+1)}}\right)$ is the new fitness score of the decision variables obtained by the symbiotic hunting strategy and $f\left(\overrightarrow{X_{i(t)}}\right)$ fitness score of the old decision variables obtained from the previous iteration.

1) EXPLORATION AND EXPLOITATION

SL-GWO achieves a good balance of exploration and exploitation benefitting from the dynamic nature of the two control parameters namely, Symbiosis rate (S_r) and Hunting distance (D_1 and D_2) respectively. S_r, in particular, is crucial of the two as it helps diversify the population on a dimensional basis. The variation of S_r is kept diverse for the two population sub-groups, i.e., the first sub-group with the attacking hunters work with a higher value of Sr allowing them to explore the newer areas in the search space generated by the different symbiotic hunting strategies. This accounts for an aggressive attacking approach and allows more grey wolves to quickly explore and exploit the promising areas specified by the three dominant wolves. At the same time, the higher values of S_r promote convergence capabilities of the algorithm as more grey wolves follow the three dominant wolves to exploit portions of the search space readily. The hunting distances D_1 and D_2 in the first sub-group help in controlling the exploration distance and aids the smoother transition of exploration to exploitation. To be specific, D_1's linear decremental strategy from 1 to 0 can be quite beneficial to force the exploration within the search space limits during the initial half of the search space and prevent early cases of entrapment since the hunting distance from the three dominant wolves is higher. On the other hand, the sinusoidal bursts of D_2 aids the systematic control of diversification and intensification cycles, encouraging a controlled movement within the search space.

2) IMPORTANCE OF THE SYMBIOTIC HUNITING STRATEGIES

SL-GWO's incorporation of multiple symbiotic hinting strategies is one of its strongholds at achieving an improved performance in terms of optimality and convergence. Despite the increased effort to code these multiple strategies, numerous multi-strategy ensemble optimization algorithms have demonstrated the necessity to incorporate multiple strategies in a systematic yet synergetic configuration with distinctive control schemes used to guide their search. Multi-population ensemble differential evolution (MPEDE) [64], Multi-strategy ensemble grey wolf optimizer (MEGWO) [48], Multi-strategy ensemble artificial bee colony optimization (ME-ABC) [65], Multi-strategy ensemble social group optimization (ME-SGO) [66] etc. from the literature prove that incorporating multiple search strategies can offer numerous performance benefits as their individual effectiveness at either fending off entrapment or accelerating convergence can be beneficial rather than the total dependence on one strategy alone. In SL-GWO, the first and second symbiotic hunting strategies form the basis of achieving a better exploratory reach and encouraging diversification while emphasizing local search equally. The incorporation of multiple random omega wolves to help advance the hunt as additional subordinates to the alpha, beta and delta wolves forms a positive reinforcement to ensure that the search space around each wolf expand during exploration to newer areas and slowly contracts during exploitation as the wolves start moving close to each other. This strategy works effectively at preventing the collapse of search space particularly if the three dominant wolves originate from the same local region of the search space which has been one of the primary reasons for entrapment in the canonical GWO and its variants directly employing the hierarchical hunting. Furthermore, the third and fourth hunting strategies are carefully designed to follow the alpha wolf while incorporating random omega wolves from the population to ensure that the search process is rotationally invariant for search landscapes with several translations. Another benefit of the third and fourth strategies is that they help accelerate the convergence towards the alpha wolf such the resultant position is can explore and exploit around the best possible solution without any interactions with the beta and omega wolves. This can aid the wolves to improve the exploitation around the alpha wolf increasing the accuracy and precision of the best solution while lowering the redundancy associated with exploiting the inferior beta and delta wolves.

The system of selecting the hunting strategy is biased towards the first and second hunting schemes as the enhancement of diversity is emphasized throughout the search process. The system of selecting one out of the four hunting strategies corresponds to lower computational complexity when compared to other variants of GWO wherein the canonical GWO hunting is followed by advanced hunting schemes which are often implemented one after the other resulting in the population updation and selection process to occur twice in a single-iteration.

3) POPULATION SELECTION STRATEGY

SL-GWO relies on the greedy selection or the greedy algorithm [67] to update its population pool such that newer fitter wolves always replace their older counterparts and in case the newer wolf is inferior in fitness, the older wolf retains its position in the pool. This selection system is quite opposite to the population selection from the canonical GWO wherein the newer wolves always replace the older solutions irrespective of their fitness. Conventionally referred to as the *Mu, Lambda* selection [68], [69], [70], [71] with *Mu* denoting the parent population pool and *Lambda* being the children population pool, the *Lambda* population pool always replaces the *Mu* population pool such that the recent solutions are always included. The major disadvantage with the conventional *Mu, Lambda* selection is that personal best information of individual wolves is lost and the newer wolves are forced to occupy positions with no fitness improvements over time. Additionally, the need to include every newer member of the population after the fitness evaluation process adds to the complexity. Finally, the inclusion of every recent member is not effective at advancing the hunt of every individual grey wolf to gradually improve over time and all the problems coupled with the lack of adaptive and robust control and diversifying schemes can trigger the avalanche of stagnation leading to entrapment and ultimately premature convergence. Therefore, to counter these ill-effects associated with the conventional selection process, the greedy selection from SL-GWO provides a robust mechanism to select the best solutions while advancing their personal best fitness levels at all times. The greedy selection in SL-GWO works in synergy with a population sorting mechanism to sort the population of the two sub-groups such that the three best solutions are assigned as the alpha, beta and delta wolves rather than comparing the individual fitness levels of the individual wolves from the standard GWO. The major benefit is that no inferior solutions make it to the population pool preventing the advancement of the individual grey wolves and the second being the re-organization of the population pools based on the sorted population. Furthermore, the re-organization system allocates the top 50 percent of the wolves to the attacking hunters to explore at a faster pace and the remaining 50 percent to the wolves to the experienced hunting group to ensure that diversification is preserved at all times.

4) TIME COMPLEXITY AND COMPUTATIONAL COMPLEXITY

In SL-GWO, the position update system occurs once, followed by the sorting of all wolves from the two sub-groups after evaluating the wolves' fitness in the previous iteration to select the alpha, beta, and delta wolves. This is followed by the parameter adaption, which occurs as a result of any of the symbiotic hunting processes. As a result, SL-GWO performs only one fitness assessment (SFEs) of each population member every iteration. The computational complexity of distinct phases for an iterative count of T iterations with a population size of N and each having a D number of decision variables/dimensions is as follows. In addition to the total computing complexity of the fitness sorting process through quick sort, which is $O(N \log N)$, the symbiotic hunting technique has an computational cost of $O(T \times (N \times D))$ followed by $O(N \times T)$ for the greedy selection of all the new positions. To summarize, the overall computational complexity is $O(ND + N \log N \times (T \times (N \times D))))$.

The time complexity of SL-GWO is measured considering its total run time i.e., 't_{total}' for one independent run. It is shown in Eq. (3.20).

$$t_{total} = t_1 \times O_1 + t_2 \times O_2 + \ldots\ldots t_N \times O_N \quad (3.20)$$

where,

$t_1, t_2 \ldots\ldots t_N$ are the computational times needed by GWO to complete the various operations $O_1, O_2 \ldots\ldots O_N$ for N number of wolves. The various operations and the time requirements are presented in Table 5.

Therefore, based on analysis from Table 5, the time complexity of SL-GWO is $O(N)$.

The pseudocode of SL-GWO is given below.

TABLE 5. The time complexity of SL-GWO algorithm.

Phase	Time	Total Time required for N number of wolves	Time Complexity
Random Initialization of wolves	t_1	$t_1 \times N$	$O(N)$
Fitness evaluation of the initialized population	t_2	$t_2 \times N$	$O(N)$
Sorting the wolves based on their fitness	t_3	$t_3 \times N$	$O(N)$
Parameter adaption for Sr and D1	t_4	$t_4 \times N$	$O(N)$
Symbiotic hunting phase	t_5	$t_5 \times N$	$O(N)$
Fitness evaluation for the greedy selection	t_6	$t_6 \times N$	$O(N)$

Algorithm 1 SL-GWO

1. **Initialize** the positions of the grey wolves through randomization
2. **Evaluate** the fitness of every wolf
3. **Assign** the position of the grey with the best solution to $\overrightarrow{X_\alpha}$
   Assign the position of the grey with the second-best solution to $\overrightarrow{X_\beta}$
   Assign the position of the grey with the third-best solution to $\overrightarrow{X_\delta}$
4. **Divide** the wolfpack into two groups using Eq. (3.1)
5. **Increment** iterations until t< maximum iterations
6. **loop** for all wolves in the first sub-group
7. **Determine** S_r using Eq. (3.3)
8. **Determine** D_1 and D_2 using Eq. (3.13) and Eq. (3.14)
9. **Calculate** the position of new grey wolf using one of the four symbiotic hunting strategies using Eq. (3.16)
10. **Update** the position of the current wolf using Eq (3.2)
11. **Repair** the solutions exceeding the search boundaries using Eq. (3.17) and Eq. (3.18).
12. **Evaluate** the fitness of all the wolves
13. **Perform** population selection using greedy selection, Eq. (3.19)
14. **Update** the population of the first sub-group
15. **end** for
16. **loop** for all wolves in the second sub-group
17. **Determine** S_r using Eq. (3.4)
18. **Determine** D_1 and D_2 using Eq. (3.15)
19. **Calculate** the position of new grey wolf using one of the four symbiotic hunting strategies using Eq. (3.16)
20. **Update** the position of the current wolf using Eq (3.2)
21. **Repair** the solutions exceeding the search boundaries using Eq. (3.17) and Eq. (3.18).
22. **Evaluate** the fitness of all the wolves
23. **Perform** population selection using greedy selection, Eq. (3.19)
24. **Update** the population of the first sub-group
25. **end** for
26. **Update** $\overrightarrow{X_\alpha}$, $\overrightarrow{X_\beta}$ and $\overrightarrow{X_\delta}$
27. **Return** $\overrightarrow{X_\alpha}$ and its corresponding fitness
28. **end** for
29. **Check** for termination criteria

IV. RESULTS AND DISCUSSIONS

The proposed algorithm's performance will be evaluated using a variety of benchmark functions, constrained engineering problems, and real-world optimization problems from the domain of power systems. Tests include 29 scalable benchmark functions from the CEC2018 benchmarking suite (dimensions set to 10, 30 and 50), to verify the algorithm's immunity to *the curse of dimensionality* and convergence characteristics. This is followed by the 10 fixed-dimensional functions from the CEC2019 benchmark functions to evaluate the algorithm's ability to prevent local entrapment and premature convergence. Four common engineering problems (pressure vessel design, welded beam design, tension/compression spring design, and 10-bar truss design optimization) are used to validate the performance under constrained conditions. SL-GWO and other competing algorithms are then applied to 13 distinct case studies of optimum power flow for IEEE 30 and IEEE 57-bus systems followed by the optimal reactive power dispatch problem IEEE 30 and 57-bus systems for 8 different case studies.

The flowchart of SL-GWO is given by Figure 3 and Figure 4 represents the flowchart for the symbiotic hunting and parameter adaption in SL-GWO.

All experimentation evaluated for this study are conducted on an Ultrabook running Microsoft Windows 10® Pro (Version 20H2 - OS Build 19042.867) and equipped with 16 Gigabytes of DDR3 RAM and a quad-core Intel(R) Core (TM) i7-4700MQ CPU running at 2.40GHz. MATLAB R2020a is used to code all of the methods for the comparative analysis.

A. DESCRIPTION OF THE PERFORMANCE EVALUATION CRITERIA

The performance assessment criteria for all fourteen algorithms, including SL-GWO, across the two benchmarking suites (CEC2018 benchmarking suite and CEC2019 benchmarking suite) are as follows. (1) The average (mean) and standard deviation values for each algorithm are calculated for 30 independent runs. (2) The first statical test, Wilcoxon's rank-sum test, is used to compare SL-GWO against the other algorithms at a 0.05 level of significance. The "+" symbol is used to indicate that the other algorithms perform better than SL-GWO, the "≈" symbol indicates that the other algorithms performed similarly to SL-GWO, and the "−" symbol indicates that the other algorithms perform poorly relative to SL-GWO. (3) The second statistical test, a ranking test using non-parametric Friedman's test, is used to determine the best-performing algorithms. (4) Additionally, the mean absolute errors (MAE) are analyzed to determine the difference between the global optimal solution and the best solution achieved using each technique. (5) Convergence

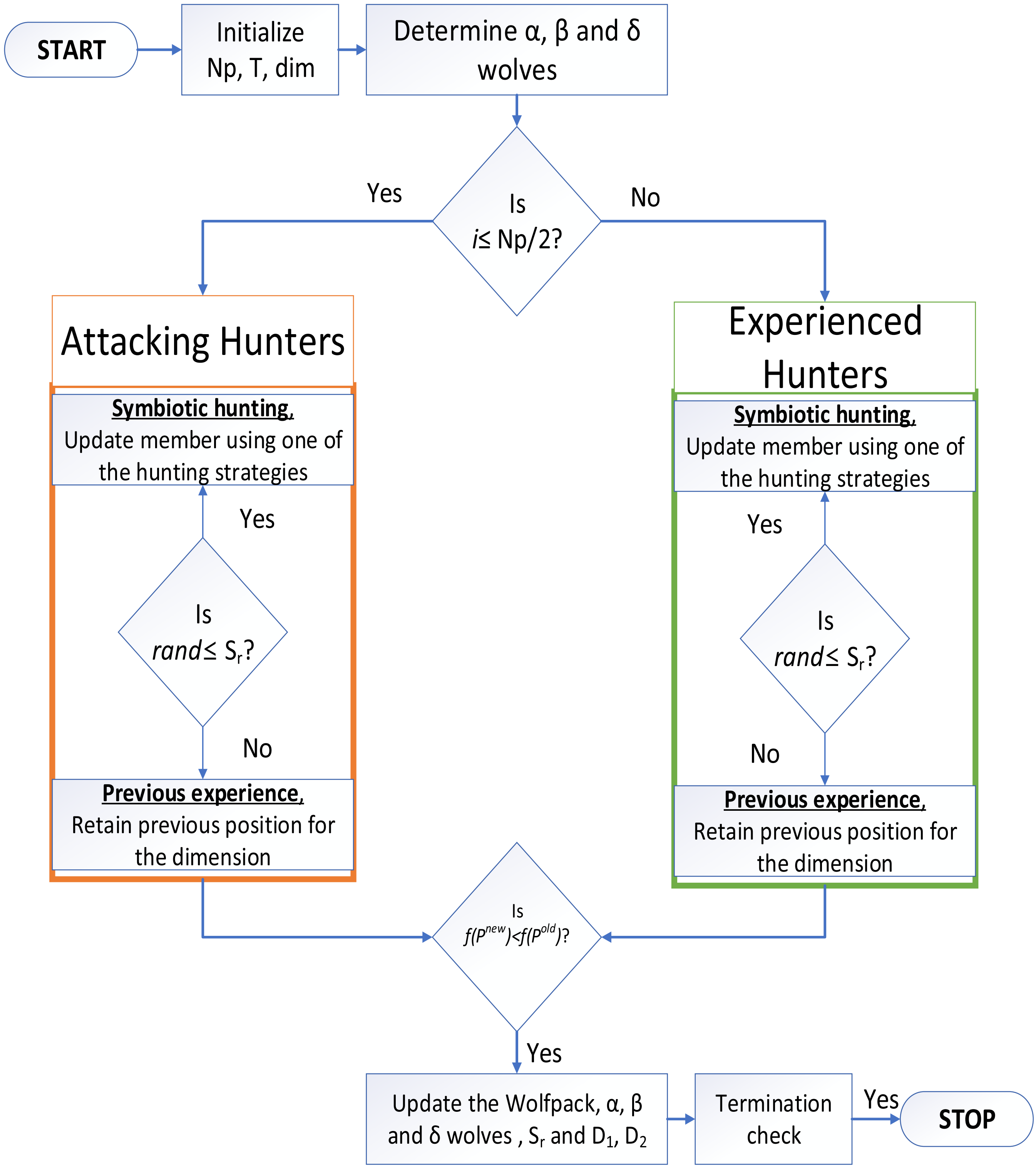

FIGURE 3. Flowchart of SL-GWO.

graphs for the CEC2018 test suite with 50 dimensions, as well as acceleration rates for fixed dimensional benchmarking functions, are included to demonstrate the proposed method's convergence properties.

B. ALGORITHMS IN THE BENCHMARKING FRAMEWORK

The performance of SL-GWO is compared and validated against the standard GWO algorithm from 2014 and eight of its latest state-of-the-art variants whose description is

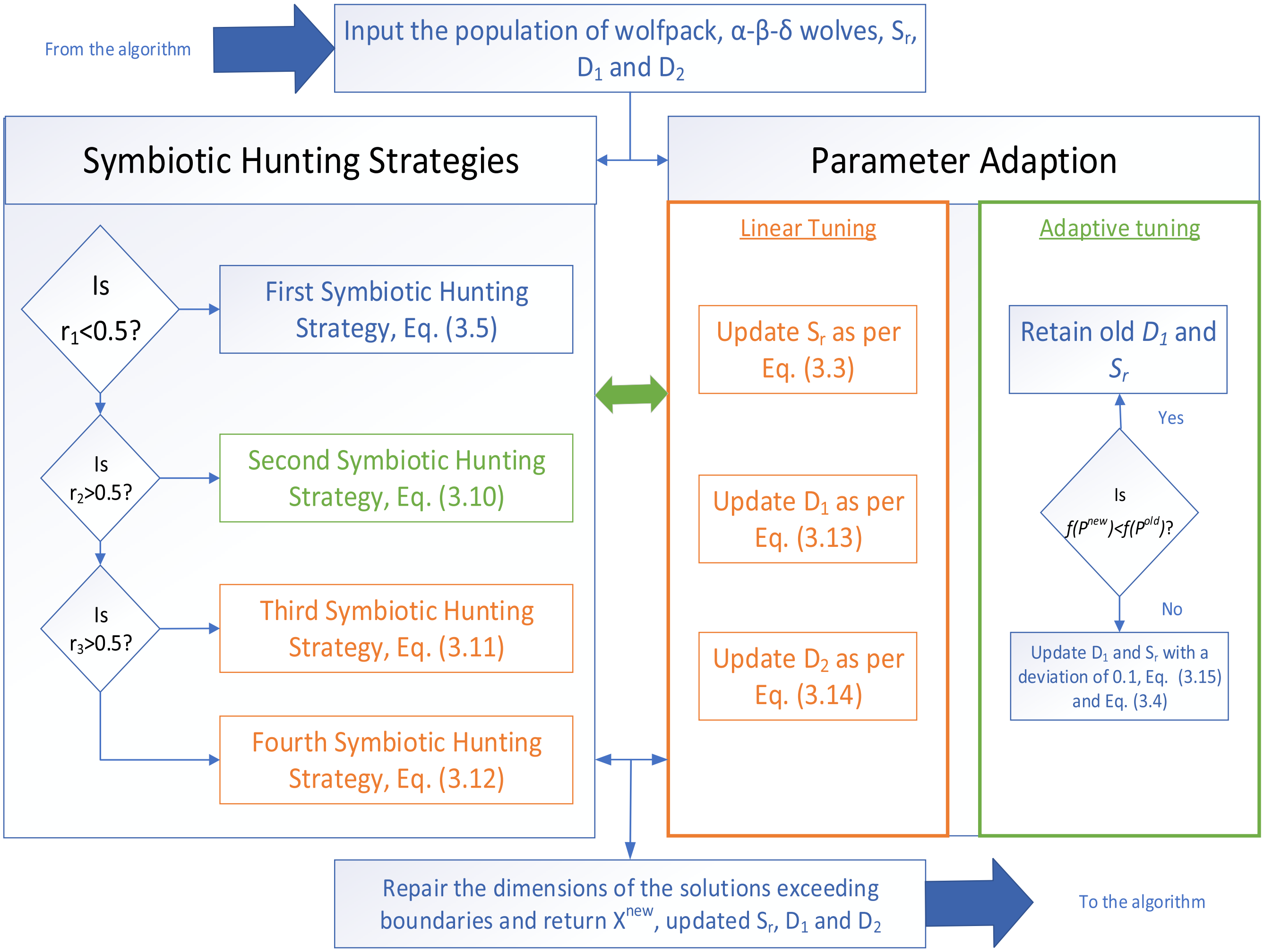

FIGURE 4. Flowchart depicting the symbiotic hunting and parameter adaption process in SL-GWO.

provided in Table 6. To ensure diversity in the comparative analysis, at least one variants from four categorizations are chosen with the latest variants (from 2016 to 2022) considered based on their popularity and performance during the literature survey.

Additionally, five modern meta-heuristics including Harris Hawk optimization (**HHO**) [72] from 2019, Slime Mould optimization Algorithm (**SMA**) [73] from 2020, Gorrila Troops Optimizer (**GTO**) [74] from 2021, Whale Optimization Algorithm (**WOA**) [22] from 2016 and Chimp Optimization Algorithm (**ChOA**) [23] from 2020 are chosen for the comparative analysis in the current work for the following reasons. (1) A few of the prominent and state-of-the-art algorithms from 2015 to 2020 are covered through the selection of HHO, GTO, SMA, WOA and ChOA. (2) The control mechanism of exploration and exploitation in SMA, HHO and WOA is similar to the ones in GWO. (3) These five techniques have similar computational complexities to that of GWO. (4). The group hunting and the control mechanism in ChOA are identical to the social hierarchical system in GWO. (5) SOA incorporates two different phases for exploration and exploitation similar to the system in SL-GWO. (6) The tuning requisites were minimal for the selected five meta-heuristics compared to other contemporary nature-inspired meta-heuristics.

Furthermore, a few other modern algorithms including SSA (Slap Swam Algorithm) [75], SOA (Seagull optimization algorithm) [76], MFO (Moth flame optimizer) [77] have been included for the comparison with the standard engineering problems and power flow optimization problems.

C. TUNING SETTINGS OF THE ALGORITHMS

The algorithm-specific parametric tuning for the competitor algorithms is based on their respective publications and these settings have not been modified for the entire benchmarking and power flow optimization problems. A detailed description of the tuning parameters and their ranges have been provided in Table 7.

TABLE 6. Description of the variants of GWO used in the comparative analysis.

Categorization	Name of the variant	Authors	Year	Reference
Chaotic Map Variants	**CGWO** (Chaotic Grey Wolf Optimization algorithm)	M. Kohli *et al.*	2017	[38]
	ACGWO* (Improved Chaotic Gray Wolf Optimization)	M. He *et al.*	2018	[39]
Improved / Hybridized Variants	**I-GWO** (Improved Grey Wolf Optimizer)	M.H. Nadimi-Shahraki *et al.*	2020	[43]
	IGWO-DE (Improved Grey Wolf Optimizer based on Differential Evolution and Elimination Mechanism)	J. S. Wang *et al.*	2019	[56]
	AGWO (Grey wolf optimizer based on Aquila exploration method)	D. Ma et al.	2022	[78]
Modified / Enhanced Variants	**EGWO** (Enhanced Grey Wolf Optimizer)	Kaiping Luo	2019	[44]
	MEGWO (Multi-strategy Ensemble Grey Wolf Optimizer)	Q.Tu *et al.*	2018	[48]
Opposition based-learning Variants	**SOGWO** (Selective Opposition based Grey Wolf Optimization)	S. Dhargupta *et al.*	2020	[50]
	P-ObGWO* (Parallel Grey Wolf Optimizer combined with Opposition based learning)	M.S. Nasrabadi *et al.*	2016	[51]

* indicates that the selected algorithms were used for comparative analysis in constrained engineering problems and real-world problems

D. PERFORMANCE ANALYSIS WITH CEC2018 BENCHAMRKING SUITE (TEST FOR THE CURSE OF DIMENSIONALITY)

The validation of the proposed algorithm's performance at handling problems with a larger dimension count and evaluate its immunity towards *the curse of dimensionality* is done through a set of 29 scalable/*n*-dimensional benchmark functions with the number of dimensions set to 10, 30 and 50 from CEC (Congress on Evolutionary Computation) 2018 test suite. This testing process aids in establishing the performance of the algorithms towards an increased number of dimensions and tracking the performance deterioration concerning the problem dimensions. The description of the CEC2018 test functions (categorization, problem dimensions, the search ranges and the global best score) and the fixed dimensional benchmark functions are provided in Table 8.

The NFEs are set based on the rules of CEC2018 test suite from [79]. The maximum NFEs are set as per the rule

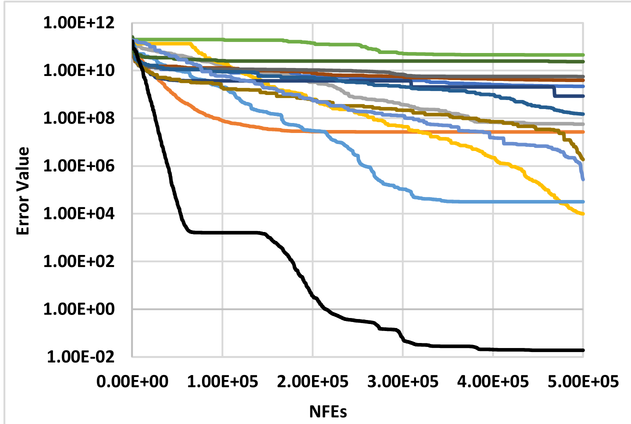

FIGURE 5. Convergence graph for function F1 with 50D.

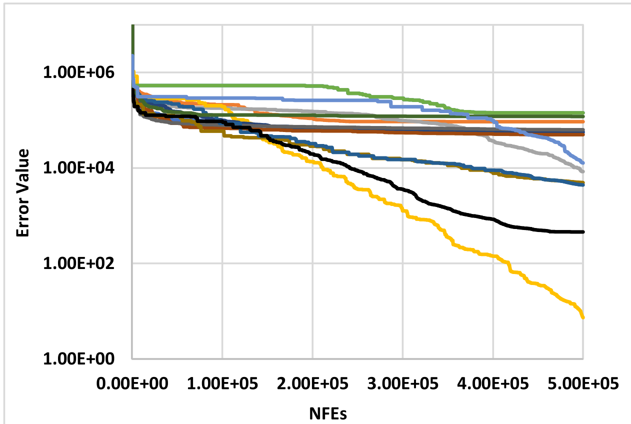

FIGURE 6. Convergence graph for function F3 with 50D.

10000*D (Maximum FES for 10D = 100000; for 30D = 300000; for 50D = 500000) and the population size is set as per the rule 5*D (Maximum population size for 10D = 50; for 30D = 150; for 50D = 250) with the maximum NFEs being the stopping criteria.

The benchmarking results (mean and standard deviation) are shown in Table 9 for the 10-dimensional case followed by Table10 for the 30-dimensional case and Table 11 for the 50-dimensional case. The results of Wilcoxon's rank-sum test are shown in Table 12 followed by the results of Friedman's non-parametrical test shown in Table 13, the mean absolute errors (MAE) for all the fifteen algorithms are given in Table 14. The acceleration rates comparing SL-GWO with the competitor algorithms for the 10, 30 and 50 dimensional cases are shown in Table 15, Table 16 and Table 17 respectively. Furthermore, the average computational times are shown in Table 18. The statistical results comparing SL-GWO with the recent variant of GWO (GGWO) for the CEC2018 test suite are provided in Table 19. The convergence curves for the 29 benchmark functions (50 dimensions) are shown in Figure 5 to Figure 34.

The performance of SL-GWO stands out for the CEC2018 benchmarking suite in terms of optimality and lower deviation. In table 9, SL-GWO emerged as the best-performing

TABLE 7. Description of the algorithm-specific tuning parameters for all the algorithms used in the comparative analysis.

Algorithm	Tuning / Algorithm-specific Parameters	Value
GWO	Control Vector ($\vec{a}$) to balance exploration and exploitation phases	Follows a linearly decrementing nature from an initial value of 2 to a final value of 0 over the progression of iterations.
MFO*	Convergence Constant (*r*)	Linearly decreased from -1 to -2 over the course of iterations
	Number of flames (*flame no*)	$round\left[N - t \times \frac{N-1}{T}\right]$
SOA*	Frequency control variable (f_c)	2
	Frequency of employing variable (*A*)	linearly decreased from f_c to 0 over the course of iterations
WOA	Control Vector ($\vec{a}$) to balance exploration and exploitation phases	linearly decreased from 2 to 0 over the course of iterations
	Coefficient Vector ($\vec{A}$)	Randomized in the interval [-1, 1]
SSA*	Control Coefficient (*c1*)	Set based on an exponential decreasing expression considering the current and maximum iteration count.
SMA	$\overrightarrow{vc}$	oscillates between $[-1,1]$ and tends to zero eventually.
	$\overrightarrow{vb}$	oscillates randomly between $[-a, a]$ and tends to zero eventually where *a* is set based on the iteration count.
HHO	Ability to get food (β)	6
	Scaling constant (*c*)	2
ChOA	Chaotic Vector (*m*)	Tent chaotic map
	Control Vector (*f*)	reduced non-linearly from 2.5 to 0 through the iteration process
GTO	*p*	0.03
	Beta	3
	w	0.8
ACGWO*	Non-linear convergence factor / Improved convergence factor (*a*)	Set between 0 to 2 based on $a = 2 - 2 \times \left(\frac{1}{e-1} \times \left(e^{\frac{t}{T}} - 1\right)\right)$ where *e* represents the natural logarithm
	Chaotic Map	Logistic Map
CGWO	Control Vector ($\vec{a}$) to balance exploration and exploitation phases	Follows a linearly decrementing nature from an initial value of 2 to a final value of 0 over the progression of iterations.
	Chaotic Map	Chebyshev map
I-GWO	Control Vector ($\vec{a}$) to balance exploration and exploitation phases	Follows a linearly decrementing nature from an initial value of 2 to a final value of 0 over the progression of iterations.
	Radius (R_i)	Euclidean distance
IGWO-DE	Control Vector ($\vec{a}$) to balance exploration and exploitation phases	Follows a linearly decrementing nature from an initial value of 2 to a final value of 0 over the progression of iterations.
	Crossover Probability (*CR*)	0.8
	Dynamic Scaling Factor (*F*)	$F = f_{min} + (f_{max} - f_{min}) \times \left[\frac{T-(t-1)}{T}\right]$ where f_{max} and f_{min} are the maximum and minimum values of the scaling factor (2 and 0)
	Elimination Factor (*R*)	random integer between $N/(2 \times \varepsilon)$ and N/ε through $R = [\frac{N}{\varepsilon}, \frac{N}{0.75} \times \varepsilon]$ where ε is scale factor of wolves updating
MEGWO	Scale factor (ρ)	Set between *0.5* and 0.1^2 based on $\rho \sim N(0.5, 0.1^2)$
	Global-best guidance rate (GR)	0.8
	Dispersion rate (DR)	$DR_{max} = 0.4$, $DR_{min} = 0$
	SR	$SR_{max} = 1$, $SR_{min} = 0.6$
	Control Vector ($\vec{a}$) to balance exploration and exploitation phases	Follows a linearly decrementing nature from an initial value of 2 to a final value of 0 over the progression of iterations.
SOGWO	Spearman's correlation coefficient (r_R)	Set between 0 and 1 through $r_R = 1 - \left[\frac{6 \times \sum_i (d_i)^2}{D \times (D^2-1)}\right]$
	Control Vector ($\vec{a}$) to balance exploration and exploitation phases	Follows a linearly decrementing nature from an initial value of 2 to a final value of 0 over the progression of iterations.
	Threshold variable	decreased linearly with iteration from $\vec{a}$
EGWO	Simulated stochastic error (ϵ)	$\epsilon(t) \sim N(0, \sigma(t))$ with $\sigma(t) = 1 - \frac{t}{T}$ (linear setting)
	ω	$\omega_\alpha = 0.5 \times \left[1 - \frac{f_a}{f_a+f_\beta+f_\delta}\right]$, $\omega_\beta = 0.5 \times \left[1 - \frac{f_\beta}{f_a+f_\beta+f_\delta}\right]$ and $\omega_\delta = 0.5 \times \left[1 - \frac{f_\delta}{f_a+f_\beta+f_\delta}\right]$
P-ObGWO*	Control Vector ($\vec{a}$) to balance exploration and exploitation phases	Follows a linearly decrementing nature from an initial value of 2 to a final value of 0 over the progression of iterations.
	Sub Groups for parallel computation	2
AGWO	Control Vector ($\vec{a}$) to balance exploration and exploitation phases	Follows a linearly decrementing nature from an initial value of 2 to a final value of 0 over the progression of iterations.
	Balance factor (B)	0.8
SL-GWO	Hunting distances (D1 and D2)	Dynamically varied through self-adaption for experienced group and D_1 follows a linearly decrementing nature from an initial value of 1 to a final value of 0 over the progression of iterations while D_2 follows sinusoidal variations in the range -0.5 to 0.5 with a periodicity of 0.5 for the attacking group
	Symbiosis rate (Sr)	Self-adapted in the range [0,1] for the experienced group and randomized in 0.1 to 0.9 for the attacking group

* indicates that the selected algorithms were used for comparative analysis in constrained engineering problems and real-world problems

algorithm for 24 out of the 29 functions with 10 dimensions. Competitive performances were observed by MEGWO and IGWO for a few composition functions, but SL-GWO managed to outperform all the competitor algorithms for the majority of the test functions by a greater margin. On increasing the dimensions to 30, the performance of IGWO was more competitive for functions including F8, F9, F16, F19, F20 and F27 as seen in table 10. IGWO's neighborhood hunting strategies proved successful at improving the convergence to the global optimum for these functions.

TABLE 8. Description of the 29 test functions from the CEC2018 benchmarking suite.

Categorization	Function Number	Function	Dimensions	Search Range	f min
Unimodal Functions	F1	Shifted and Rotated Bent Cigar Function	10,30 and 50	[-100, 100]	100
	F3	Shifted and Rotated Zakharov's Function	10,30 and 50	[-100, 100]	300
Simple Multimodal Functions	F4	Shifted and Rotated Rosenbrock's Function	10,30 and 50	[-100, 100]	400
	F5	Shifted and Rotated Rastrigin's Function	10,30 and 50	[-100, 100]	500
	F6	Shifted and Rotated Expanded Scaffer's F6 Function	10,30 and 50	[-100, 100]	600
	F7	Shifted and Rotated Lunacek Bi_Rastrigin Function	10,30 and 50	[-100, 100]	700
	F8	Shifted and Rotated Non-Continuous Rastrigin's Function	10,30 and 50	[-100, 100]	800
	F9	Shifted and Rotated Levy Function	10,30 and 50	[-100, 100]	900
	F10	Shifted and Rotated Schwefel's Function	10,30 and 50	[-100, 100]	1000
Hybrid Functions	F11	Hybrid Function 1 (N=3)	10,30 and 50	[-100, 100]	1100
	F12	Hybrid Function 2 (N=3)	10,30 and 50	[-100, 100]	1200
	F13	Hybrid Function 3 (N=3)	10,30 and 50	[-100, 100]	1300
	F14	Hybrid Function 4 (N=4)	10,30 and 50	[-100, 100]	1400
	F15	Hybrid Function 5 (N=4)	10,30 and 50	[-100, 100]	1500
	F16	Hybrid Function 6 (N=4)	10,30 and 50	[-100, 100]	1600
	F17	Hybrid Function 7 (N=5)	10,30 and 50	[-100, 100]	1700
	F18	Hybrid Function 8 (N=5)	10,30 and 50	[-100, 100]	1800
	F19	Hybrid Function 9 (N=5)	10,30 and 50	[-100, 100]	1900
	F20	Hybrid Function 10 (N=6)	10,30 and 50	[-100, 100]	2000
Composition Functions	F21	Composition Function 1 (N=3)	10,30 and 50	[-100, 100]	2100
	F22	Composition Function 2 (N=3)	10,30 and 50	[-100, 100]	2200
	F23	Composition Function 3 (N=4)	10,30 and 50	[-100, 100]	2300
	F24	Composition Function 4 (N=4)	10,30 and 50	[-100, 100]	2400
	F25	Composition Function 5 (N=5)	10,30 and 50	[-100, 100]	2500
	F26	Composition Function 6 (N=5)	10,30 and 50	[-100, 100]	2600
	F27	Composition Function 7 (N=6)	10,30 and 50	[-100, 100]	2700
	F28	Composition Function 8 (N=6)	10,30 and 50	[-100, 100]	2800
	F29	Composition Function 9 (N=3)	10,30 and 50	[-100, 100]	2900
	F30	Composition Function 10 (N=3)	10,30 and 50	[-100, 100]	3000

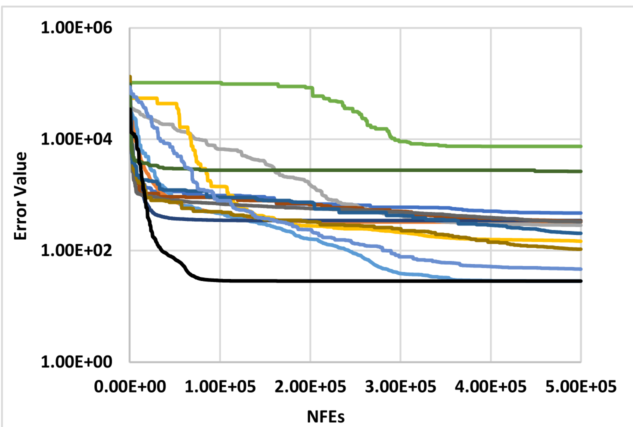

FIGURE 7. Convergence graph for function F4 with 50D.

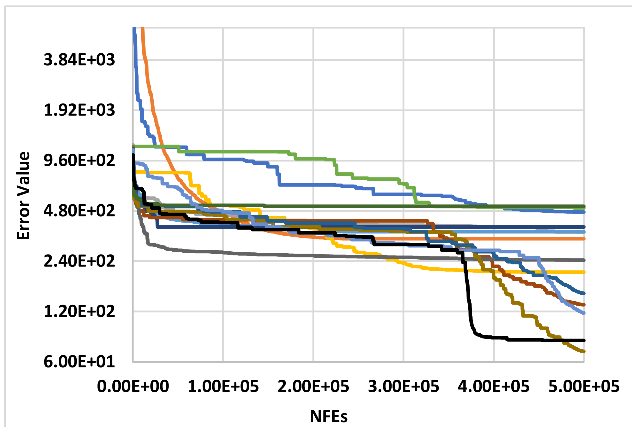

FIGURE 8. Convergence graph for function F5 with 50D.

SL-GWO's performance is on par with IGWO for the aforementioned functions and it outperforms IGWO for 19 out of the 29 functions. On further increasing the dimensions to 50 (Table 11), the performance of most of the competitor algorithms degraded by a larger margin with SL-GWO being the best performing algorithm with a higher immunity towards the curse of dimensionality. The standard GWO algorithm and the five modern meta-heuristics including WOA, HHO, SMA, GTO and ChOA quickly fell victim to entrapment with the increasing problem dimensions. One of the key reasons for the domination of SL-GWO has been its diversified population management system and emphasis on improving the solution quality through multiple symbiotic learning strategies that direct the grey wolves to adapt to the problem landscape. These adaptive control measures to guide the population movement with diversity-enhancing learning systems are absent in most modern meta-heuristics and these algorithms are often tested on simpler standard benchmarking functions with static landscapes where they are known to be the most competitive. Dynamic search landscapes from the CEC2018 suite help assess the quality of exploration and exploitation with the ever-present complexity of random translations to the landscape in the form of shifting and rotating that tend to trap poorly designed optimizers with a higher affinity to exploit the local zones. SL-GWO's performance is consistent through the testing with the increase in the number

TABLE 9. Comparison of mean and standard deviation (std.) of the fourteen algorithms for the CEC2018 benchmarking suite with 10 dimensions.

		GWO	WOA	HHO	SMA	GTO	ChOA	EGWO	SOGWO	CGWO	IGWO	IGWO-DE	AGWO	MEGWO	SL-GWO
F1	Mean	4.44E+06	1.81E+05	3.75E+05	7.81E+03	1.54E+03	1.50E+09	2.59E+03	1.40E+05	8.57E+07	1.06E+04	4.58E+04	2.24E+08	3.70E-10	**3.93E-11**
	Std.	1.03E+07	2.58E+05	1.51E+05	4.68E+03	1.97E+03	1.48E+09	3.40E+03	3.20E+05	1.83E+08	4.90E+03	7.65E+04	2.23E+08	4.37E-10	**1.51E-10**
F3	Mean	1.13E+03	2.47E+02	1.68E+00	2.62E-04	1.10E-12	1.45E+03	1.36E-13	5.43E+02	7.99E+03	2.86E-05	6.19E+01	4.31E+03	7.58E-12	**1.17E-13**
	Std.	1.54E+03	1.51E+02	7.36E-01	1.81E-04	3.19E-12	6.07E+02	8.27E-14	8.32E+02	5.40E+03	2.27E-05	5.17E+01	2.69E+03	2.00E-12	**1.59E-13**
F4	Mean	1.72E+01	2.01E+01	2.31E+01	5.20E+00	6.18E-01	1.44E+02	4.65E-01	1.28E+01	2.33E+01	9.16E-01	1.44E+01	4.18E+01	2.23E-04	**1.39E-10**
	Std.	2.28E+01	3.57E+01	3.21E+01	1.03E+00	5.81E-01	1.33E+02	5.24E-01	1.85E+01	2.44E+01	4.66E-01	1.93E+01	2.81E+01	3.01E-04	**2.80E-10**
F5	Mean	1.71E+01	4.87E+01	4.60E+01	1.69E+01	2.68E+01	4.46E+01	2.31E+01	1.53E+01	2.06E+01	8.46E+00	1.76E+01	3.11E+01	7.30E+00	**5.98E+00**
	Std.	6.18E+00	1.62E+01	1.61E+01	4.82E+00	9.11E+00	1.04E+01	1.79E+01	8.25E+00	8.86E+00	5.80E+00	9.54E+00	9.32E+00	2.54E+00	**2.50E+00**
F6	Mean	1.16E+00	2.92E+01	3.44E+01	4.82E-02	8.31E+00	2.39E+01	1.53E+00	2.49E-01	7.30E+00	5.68E-05	2.02E-01	1.40E+01	5.05E-03	**4.62E-05**
	Std.	2.62E+00	1.05E+01	1.18E+01	2.52E-02	4.24E+00	8.42E+00	3.94E+00	4.11E-01	6.11E+00	5.32E-05	2.21E-01	5.60E+00	1.54E-03	**9.91E-05**
F7	Mean	2.93E+01	7.13E+01	7.84E+01	2.04E+01	5.40E+01	8.54E+01	3.96E+01	2.67E+01	3.72E+01	1.95E+01	3.76E+01	6.05E+01	1.79E+01	**1.50E+01**
	Std.	9.12E+00	2.39E+01	2.29E+01	6.72E+00	1.69E+01	1.76E+01	1.59E+01	7.56E+00	1.44E+01	5.15E+00	1.08E+01	1.09E+01	2.99E+00	**2.63E+00**
F8	Mean	1.30E+01	4.41E+01	3.13E+01	1.30E+01	2.54E+01	3.65E+01	2.93E+01	1.10E+01	1.89E+01	6.53E+00	1.19E+01	2.59E+01	7.90E+00	**5.12E+00**
	Std.	5.34E+00	1.56E+01	1.14E+01	4.02E+00	9.03E+00	7.14E+00	2.03E+01	3.67E+00	7.82E+00	3.06E+00	5.95E+00	5.40E+00	2.54E+00	**2.35E+00**
F9	Mean	4.94E+00	4.09E+02	5.37E+02	5.01E-04	8.19E+01	3.40E+02	5.87E+00	7.77E+00	7.02E+01	7.58E-15	2.60E+00	1.33E+02	3.79E-14	**0.00E+00**
	Std.	8.40E+00	2.72E+02	2.64E+02	2.97E-04	1.04E+02	2.27E+02	1.96E+01	1.36E+01	8.93E+01	2.94E-14	5.10E+00	1.60E+02	9.28E-14	**0.00E+00**
F10	Mean	6.11E+02	9.86E+02	9.33E+02	5.40E+02	9.65E+02	1.72E+03	1.57E+03	5.48E+02	9.49E+02	1.84E+02	5.09E+02	1.09E+03	4.08E+02	**1.68E+02**
	Std.	3.25E+02	3.10E+02	3.41E+02	1.72E+02	3.28E+02	2.24E+02	5.74E+02	3.16E+02	3.88E+02	2.87E+02	3.75E+02	2.69E+02	1.82E+02	**1.11E+02**
F11	Mean	2.51E+01	9.57E+01	5.49E+01	1.08E+01	2.68E+01	2.25E+02	1.93E+01	1.99E+01	6.13E+01	3.29E+00	1.79E+01	7.15E+01	1.96E+00	**1.80E+00**
	Std.	1.42E+01	8.64E+01	2.25E+01	7.30E+00	2.16E+01	9.64E+01	2.47E+01	1.01E+01	5.53E+01	2.41E+00	9.64E+00	3.92E+01	9.95E-01	**1.90E+00**
F12	Mean	9.89E+05	1.83E+06	1.42E+06	3.85E+04	5.72E+03	3.80E+06	2.20E+04	4.04E+05	1.52E+06	1.85E+04	1.04E+06	7.71E+05	1.37E+02	**1.34E+02**
	Std.	1.03E+06	3.56E+06	1.14E+06	1.99E+04	9.46E+03	2.79E+06	1.74E+04	8.26E+05	1.42E+06	1.72E+04	1.29E+06	8.13E+05	7.11E+01	**2.94E+01**
F13	Mean	8.11E+03	2.35E+04	1.62E+04	8.68E+03	2.09E+02	1.58E+04	3.02E+03	9.46E+03	1.21E+04	5.45E+02	9.03E+03	7.25E+03	8.44E+00	**2.87E+00**
	Std.	5.45E+03	1.58E+04	1.17E+04	1.24E+04	2.11E+02	7.06E+03	4.25E+03	6.29E+03	7.46E+03	3.00E+02	6.15E+03	3.78E+03	1.81E+00	**2.50E+00**
F14	Mean	8.24E+02	1.16E+02	1.94E+02	3.51E+01	4.84E+01	2.80E+03	3.73E+03	9.53E+02	1.58E+03	4.17E+01	9.79E+01	2.10E+03	1.89E+01	**2.09E+00**
	Std.	1.54E+03	2.59E+01	1.99E+02	1.17E+01	2.36E+01	1.68E+03	4.61E+03	1.55E+03	1.88E+03	6.23E+00	2.14E+01	1.81E+03	4.31E+00	**1.36E+00**
F15	Mean	1.02E+03	3.03E+03	3.14E+03	2.38E+01	2.71E+01	2.62E+03	6.20E+03	1.82E+03	4.85E+03	1.96E+01	2.96E+02	2.92E+03	1.78E+00	**8.39E-01**
	Std.	1.57E+03	3.02E+03	1.94E+03	1.79E+01	2.62E+01	1.88E+03	6.65E+03	1.95E+03	4.81E+03	8.33E+00	6.16E+02	1.37E+03	8.67E-01	**6.38E-01**
F16	Mean	7.21E+01	2.55E+02	3.21E+02	1.02E+02	5.98E+01	2.85E+02	1.01E+02	1.11E+02	2.36E+02	2.64E+00	7.42E+01	2.54E+02	2.79E+00	**7.68E-01**
	Std.	7.26E+01	8.25E+01	1.56E+02	1.10E+02	6.28E+01	1.11E+02	1.05E+02	9.97E+01	1.14E+02	7.52E-01	8.13E+01	1.71E+02	1.11E+00	**3.54E-01**
F17	Mean	4.14E+01	8.94E+01	6.98E+01	3.41E+01	5.40E+01	6.39E+01	7.79E+01	6.51E+01	7.83E+01	2.12E+01	5.17E+01	7.73E+01	2.37E+01	**1.41E+00**
	Std.	1.91E+01	3.42E+01	2.75E+01	3.30E+01	2.00E+01	1.75E+01	4.05E+01	4.10E+01	4.54E+01	1.12E+01	1.81E+01	2.60E+01	7.07E+00	**6.01E-01**
F18	Mean	2.45E+04	1.30E+04	1.66E+04	2.24E+04	9.15E+01	3.11E+04	1.40E+04	2.46E+04	1.46E+04	3.56E+03	2.67E+04	1.82E+04	1.93E+01	**6.41E-01**
	Std.	1.48E+04	1.13E+04	1.27E+04	1.48E+04	4.50E+01	1.11E+04	1.19E+04	1.46E+04	1.25E+04	3.66E+03	1.49E+04	1.10E+04	3.86E+00	**4.96E-01**
F19	Mean	3.44E+03	2.59E+04	9.92E+03	2.32E+03	3.69E+01	1.50E+04	1.49E+04	3.59E+03	4.69E+03	1.81E+01	5.10E+02	2.11E+04	1.97E+00	**3.46E-02**
	Std.	5.35E+03	3.07E+04	8.88E+03	3.98E+03	3.66E+01	4.39E+03	1.16E+04	5.39E+03	5.90E+03	5.91E+00	1.72E+03	6.57E+04	3.59E-01	**3.47E-02**
F20	Mean	6.77E+01	1.23E+02	1.65E+02	2.50E+01	6.43E+01	1.98E+02	1.20E+02	4.20E+01	9.86E+01	1.28E+01	6.73E+01	1.29E+02	1.08E+01	**4.53E-01**
	Std.	5.28E+01	5.83E+01	7.20E+01	5.82E+00	4.65E+01	7.41E+01	1.38E+02	3.14E+01	6.07E+01	1.11E+01	7.58E+01	5.65E+01	9.71E+00	**5.88E-01**
F21	Mean	2.15E+02	2.14E+02	2.43E+02	2.13E+02	1.02E+02	1.85E+02	2.02E+02	2.00E+02	2.13E+02	1.94E+02	2.15E+02	2.32E+02	**1.00E+02**	1.19E+02
	Std.	4.32E+00	5.48E+01	5.89E+01	3.15E+01	1.55E+00	6.89E+01	5.43E+01	3.88E+01	3.16E+01	3.86E+01	9.26E+00	5.10E+00	**1.30E-06**	3.87E+01
F22	Mean	1.09E+02	1.12E+02	1.15E+02	1.41E+02	1.06E+02	1.13E+03	9.48E+01	1.06E+02	1.30E+02	9.59E+01	1.06E+02	2.83E+02	**7.12E+01**	9.43E+01
	Std.	7.82E+00	5.14E+00	3.65E+00	1.70E+02	3.22E+00	7.35E+02	3.10E+01	4.29E+00	3.75E+01	2.70E+01	2.98E+00	4.23E+02	**4.41E+01**	2.61E+01
F23	Mean	3.19E+02	3.42E+02	3.72E+02	3.22E+02	3.25E+02	3.51E+02	3.24E+02	3.15E+02	3.28E+02	3.07E+02	3.17E+02	3.55E+02	3.10E+02	**3.07E+02**
	Std.	1.05E+01	1.87E+01	2.07E+01	9.34E+00	1.27E+01	4.20E+00	1.73E+01	5.31E+00	1.20E+01	1.93E+00	9.94E+00	1.82E+01	3.49E+00	**1.78E+00**
F24	Mean	3.39E+02	3.67E+02	4.18E+02	3.54E+02	2.76E+02	3.81E+02	3.59E+02	3.43E+02	3.54E+02	3.35E+02	3.51E+02	3.71E+02	2.91E+02	**2.58E+02**
	Std.	1.51E+01	7.25E+01	1.03E+02	9.18E+00	1.17E+02	1.13E+01	1.75E+01	1.05E+01	1.41E+01	7.27E+00	1.44E+01	6.14E+01	9.91E+01	**1.16E+02**
F25	Mean	4.28E+02	4.44E+02	4.31E+02	4.25E+02	4.37E+02	4.78E+02	4.46E+02	4.38E+02	4.58E+02	**4.04E+02**	4.27E+02	4.43E+02	4.25E+02	4.17E+02
	Std.	1.64E+01	2.00E+01	3.76E+01	2.69E+01	2.01E+01	4.73E+01	1.30E+01	1.44E+01	2.85E+01	**1.60E+01**	2.24E+01	2.64E+01	8.90E+01	2.28E+01
F26	Mean	3.30E+02	6.90E+02	6.89E+02	4.64E+02	4.06E+02	1.21E+03	4.95E+02	4.75E+02	6.57E+02	3.97E+02	5.17E+02	8.87E+02	**2.65E+02**	2.93E+02
	Std.	4.05E+01	5.38E+02	5.53E+02	3.39E+02	2.03E+02	4.39E+02	3.75E+02	3.31E+02	4.17E+02	3.21E+02	3.60E+02	4.69E+02	**9.26E+01**	2.58E+01
F27	Mean	3.97E+02	4.26E+02	4.49E+02	3.91E+02	3.94E+02	3.99E+02	3.96E+02	3.94E+02	4.03E+02	3.89E+02	3.92E+02	4.26E+02	3.91E+02	**3.79E+02**
	Std.	8.49E+00	4.34E+01	4.99E+01	1.51E+00	4.14E+00	4.30E+00	4.34E+00	1.76E+00	1.01E+01	2.35E-01	1.46E+00	2.16E+01	4.50E-01	**2.10E+00**
F28	Mean	5.60E+02	6.18E+02	6.80E+02	4.03E+02	4.99E+02	4.41E+02	5.75E+02	5.23E+02	6.07E+02	**3.81E+02**	5.78E+02	5.81E+02	4.55E+02	3.99E+02
	Std.	9.07E+01	1.50E+02	1.50E+02	1.51E+02	1.42E+02	7.81E+00	9.13E+01	1.11E+02	1.39E+02	**1.39E+02**	7.79E+01	1.53E+02	4.85E+01	9.52E+01
F29	Mean	2.78E+02	4.48E+02	4.62E+02	3.09E+02	3.12E+02	3.97E+02	3.42E+02	2.86E+02	3.26E+02	2.48E+02	2.89E+02	3.55E+02	2.47E+02	**2.47E+02**
	Std.	4.14E+01	1.05E+02	9.59E+01	6.42E+01	6.21E+01	5.36E+01	7.30E+01	4.08E+01	3.82E+01	9.07E+00	4.11E+01	6.03E+01	1.21E+01	**7.19E+00**
F30	Mean	5.86E+05	5.76E+05	3.93E+05	1.12E+05	1.56E+06	1.54E+06	2.75E+05	3.42E+05	2.41E+06	1.27E+05	1.73E+04	1.01E+06	4.15E+02	**4.06E+02**
	Std.	8.71E+05	5.04E+05	4.50E+05	2.87E+05	1.71E+06	1.46E+06	3.98E+05	6.47E+05	2.56E+06	2.87E+05	8.46E+03	1.50E+06	8.04E-01	**2.65E+02**
Result	W (+)	29	29	29	29	29	29	29	29	29	23	29	29	21	
	L (-)	0	0	0	0	0	0	0	0	0	2	0	0	3	
	T (≈)	0	0	0	0	0	0	0	0	0	4	0	0	5	

of problem dimensions having no effect on the efficiency of the algorithm as seen in Tables 9, 10 and 11. The other algorithms' performance dwindled over the increase in the dimensions with the standard GWO being the most hard-hit followed by EGWO and CGWO. It is safe to conclude that SL-GWO is quite efficient at handling problems with larger dimensions and is immune to *the curse of dimensionality*. The greedy search mechanism to prioritize the best solutions for the next generations could be traced back to the success of SL-GWO. Another reason could be traced back to the presence of a large number of tuning parameters and no adaptive control strategy for the fourteen chosen algorithms, WOA, GWO, HHO, SMA, I-GWO required no additional tuning parameter settings while IGWO-DE, MEGWO, SOGWO,

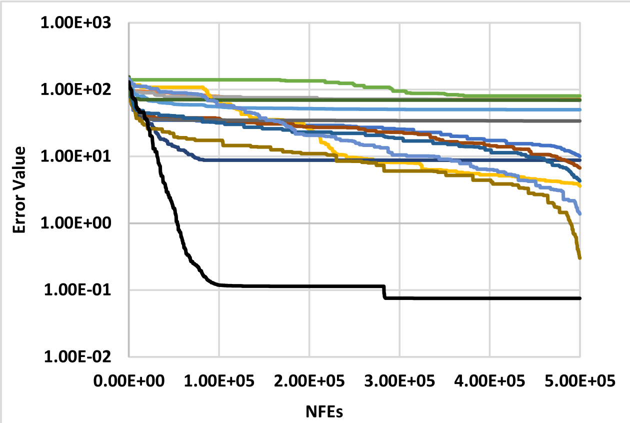

FIGURE 9. Convergence graph for function F6 with 50D.

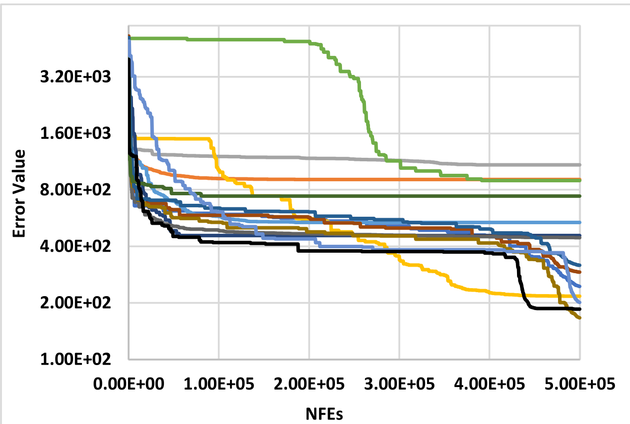

FIGURE 10. Convergence graph for function F7 with 50D.

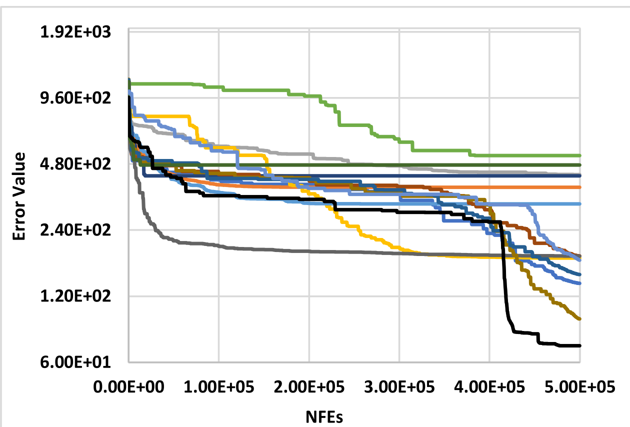

FIGURE 11. Convergence graph for function F8 with 50D.

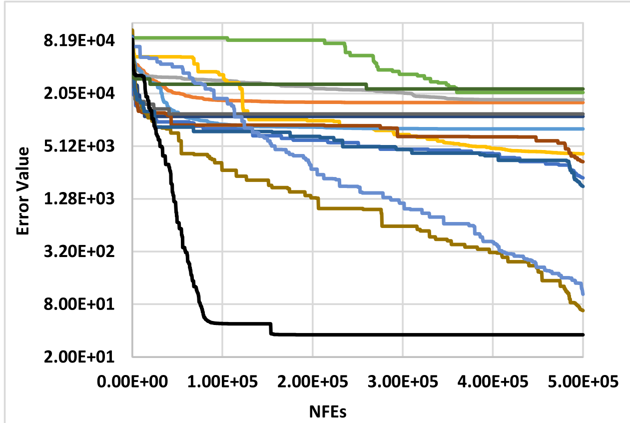

FIGURE 12. Convergence graph for function F9 with 50D.

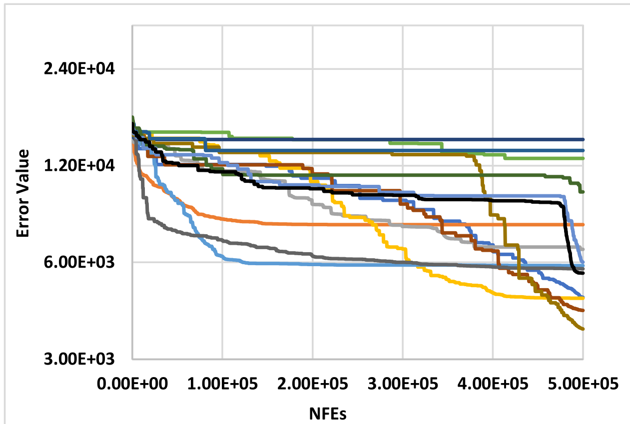

FIGURE 13. Convergence graph for function F10 with 50D.

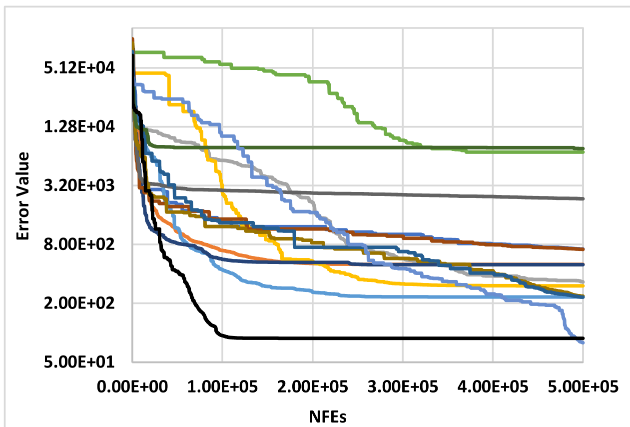

FIGURE 14. Convergence graph for function F11 with 50D.

EGWO, and ChOA required the tuning of special algorithm-specific parameters (2 to 4 parameters) whose values have been set based on their corresponding publications. Although the empirical setting favoured performance for problems with a lower number of dimensions as seen in [22], [72], [73], [74], [80], the same performance was not reflected for the larger dimensional problems. One particular reason for this is to do with the formulation of the solution set wherein every dimension/decision variable has not achieved the global best solution leading to an imbalance in the optimization and thereby producing highly non-optimal solutions.

TABLE 10. Comparison of mean and standard deviation (std.) of the fourteen algorithms for the CEC2018 benchmarking suite with 30 dimensions.

		GWO	WOA	HHO	SMA	GTO	ChOA	EGWO	SOGWO	CGWO	IGWO	IGWO-DE	AGWO	MEGWO	SL-GWO
F1	Mean	7.29E+08	2.23E+06	1.10E+07	7.99E+03	4.19E+03	2.32E+10	2.54E+08	5.71E+08	2.02E+09	1.07E+05	5.02E+08	5.99E+09	2.87E+03	**7.43E-05**
	Std.	4.97E+08	2.18E+06	2.18E+06	7.37E+03	4.93E+03	5.13E+09	7.49E+08	4.27E+08	1.15E+09	1.29E+05	4.65E+08	1.49E+09	1.24E+03	**2.23E-04**
F3	Mean	2.32E+04	1.69E+05	5.28E+03	2.52E-01	**3.30E-04**	4.49E+04	1.64E+04	3.13E+04	4.46E+04	1.11E+03	3.33E+03	5.18E+04	2.46E+02	6.77E-01
	Std.	7.52E+03	8.73E+04	2.12E+03	1.90E-01	**5.49E-04**	4.75E+03	1.60E+04	8.33E+03	1.19E+04	1.22E+03	1.85E+03	1.05E+04	1.61E+02	8.05E-01
F4	Mean	1.46E+02	1.43E+02	1.31E+02	8.87E+01	7.53E+01	3.59E+03	1.03E+02	1.34E+02	2.05E+02	9.22E+01	1.21E+02	5.07E+02	7.25E+01	**6.12E+01**
	Std.	2.46E+01	3.71E+01	3.27E+01	1.26E+00	2.56E+01	2.39E+03	5.50E+01	3.62E+01	1.24E+02	9.16E+00	1.93E+01	2.66E+02	1.76E+00	**3.15E+01**
F5	Mean	7.87E+01	2.68E+02	2.21E+02	7.34E+01	1.71E+02	3.00E+02	1.12E+02	7.74E+01	1.11E+02	5.67E+01	8.04E+01	2.42E+02	6.17E+01	**4.47E+01**
	Std.	2.21E+01	5.37E+01	3.91E+01	1.67E+01	2.75E+01	2.71E+01	8.75E+01	1.35E+01	2.32E+01	4.54E+01	4.79E+01	1.90E+01	2.07E+01	**1.25E+01**
F6	Mean	3.54E+00	6.68E+01	5.91E+01	6.10E-01	3.81E+01	5.63E+01	3.86E+00	3.59E+00	2.12E+01	1.56E-01	1.68E+00	4.88E+01	3.73E-01	**5.31E-03**
	Std.	1.77E+00	9.95E+00	6.25E+00	2.75E-01	8.61E+00	6.72E+00	9.79E+00	1.29E+00	7.86E+00	4.03E-02	1.17E+00	7.56E+00	1.24E-01	**4.55E-03**
F7	Mean	1.17E+02	5.23E+02	5.55E+02	1.18E+02	3.28E+02	4.68E+02	2.03E+02	1.40E+02	1.86E+02	9.86E+01	1.24E+02	3.51E+02	1.14E+02	**7.86E+01**
	Std.	3.46E+01	9.34E+01	6.83E+01	2.86E+01	7.53E+01	5.15E+01	1.22E+02	4.18E+01	4.58E+01	4.05E+01	5.39E+01	3.41E+01	3.29E+01	**7.86E+00**
F8	Mean	7.74E+01	2.20E+02	1.60E+02	8.41E+01	1.51E+02	2.27E+02	1.76E+02	8.43E+01	9.75E+01	**3.32E+01**	6.42E+01	1.82E+02	6.95E+01	6.24E+01
	Std.	1.35E+01	5.30E+01	3.12E+01	2.47E+01	3.72E+01	1.96E+01	8.60E+01	2.96E+01	2.42E+01	**6.22E+00**	1.31E+01	2.64E+01	2.65E+01	1.95E+01
F9	Mean	3.26E+02	6.73E+03	5.44E+03	1.25E+03	2.69E+03	5.20E+03	1.40E+03	4.47E+02	1.50E+03	**2.73E-01**	2.96E+02	3.88E+03	2.75E+00	3.23E+00
	Std.	2.21E+02	2.50E+03	7.55E+02	1.72E+03	1.11E+03	1.49E+03	2.16E+03	4.16E+02	7.86E+02	**1.81E-01**	2.90E+02	1.02E+03	2.38E+00	2.28E+00
F10	Mean	3.14E+03	4.85E+03	4.58E+03	3.53E+03	4.31E+03	7.03E+03	7.24E+03	**3.04E+03**	3.79E+03	4.70E+03	6.40E+03	5.46E+03	4.17E+03	3.73E+03
	Std.	1.17E+03	6.66E+02	5.76E+02	5.86E+02	8.24E+02	3.04E+02	1.83E+03	**6.21E+02**	6.21E+02	2.41E+03	1.77E+03	9.51E+02	1.17E+03	9.43E+02
F11	Mean	2.23E+02	3.65E+02	1.43E+02	1.10E+02	1.17E+02	2.27E+03	3.58E+02	3.49E+02	1.68E+03	4.38E+01	1.53E+02	1.45E+03	5.06E+01	**3.62E+01**
	Std.	7.36E+01	1.80E+02	4.81E+01	5.75E+01	3.49E+01	9.62E+02	7.91E+02	1.94E+02	1.58E+03	2.67E+01	4.46E+01	1.22E+03	4.28E+00	**1.59E+01**
F12	Mean	3.03E+07	3.34E+07	1.51E+07	1.89E+06	4.95E+04	3.68E+09	4.58E+05	8.01E+07	5.07E+07	8.49E+05	1.39E+07	2.79E+08	2.77E+04	**9.91E+03**
	Std.	3.62E+07	2.87E+07	9.45E+06	1.49E+06	3.24E+04	1.73E+09	7.33E+05	1.09E+08	6.46E+07	6.73E+05	1.30E+07	1.24E+08	1.02E+04	**8.79E+03**
F13	Mean	1.12E+05	1.41E+05	2.90E+05	2.23E+04	1.28E+04	6.14E+08	3.87E+04	1.40E+06	8.63E+04	9.35E+04	8.86E+04	1.89E+07	1.34E+03	**9.26E+01**
	Std.	1.30E+05	1.05E+05	1.14E+05	2.49E+04	1.91E+04	1.40E+09	4.17E+04	5.16E+06	4.97E+04	5.69E+04	6.43E+04	4.26E+07	4.21E+02	**4.29E+01**
F14	Mean	2.33E+05	1.23E+06	3.85E+04	3.70E+04	3.60E+02	2.92E+05	2.55E+04	7.53E+04	1.93E+05	3.06E+03	3.83E+04	6.47E+05	9.00E+01	**4.76E+01**
	Std.	4.35E+05	1.23E+06	4.18E+04	2.02E+04	2.51E+02	4.25E+05	3.51E+04	1.14E+05	4.49E+05	4.46E+03	3.61E+04	6.43E+05	6.91E+00	**9.69E+00**
F15	Mean	1.73E+05	7.44E+04	5.88E+04	2.42E+04	2.41E+03	5.91E+06	2.25E+04	1.55E+05	2.32E+05	9.73E+03	5.15E+04	7.90E+05	1.70E+02	**5.81E+01**
	Std.	5.48E+05	5.22E+04	3.72E+04	1.37E+04	3.50E+03	2.77E+06	1.70E+04	4.25E+05	6.54E+05	7.34E+03	2.43E+04	8.23E+05	4.70E+01	**3.36E+01**
F16	Mean	8.07E+02	1.80E+03	1.50E+03	8.16E+02	1.12E+03	1.92E+03	1.30E+03	6.24E+02	1.04E+03	**2.94E+02**	6.34E+02	1.35E+03	7.45E+02	3.23E+02
	Std.	1.66E+02	6.06E+02	2.80E+02	2.18E+02	2.56E+02	4.07E+02	6.61E+02	2.61E+02	3.18E+02	**2.95E+02**	2.79E+02	3.06E+02	3.39E+02	2.28E+02
F17	Mean	2.75E+02	7.54E+02	7.50E+02	5.24E+02	6.23E+02	8.28E+02	4.51E+02	2.96E+02	3.86E+02	1.02E+02	1.96E+02	5.90E+02	2.93E+02	**8.43E+01**
	Std.	1.77E+02	2.48E+02	2.23E+02	1.54E+02	2.38E+02	1.79E+02	3.16E+02	7.78E+01	1.71E+02	8.40E+01	8.54E+01	2.56E+02	1.84E+02	**3.65E+01**
F18	Mean	4.79E+05	1.41E+06	9.10E+05	4.02E+05	1.77E+04	1.28E+06	2.83E+05	5.93E+05	1.79E+06	1.53E+05	7.48E+05	1.54E+06	3.01E+02	**2.97E+02**
	Std.	5.74E+05	1.33E+06	1.30E+06	4.17E+05	1.27E+04	5.09E+05	2.53E+05	8.66E+05	1.91E+06	1.08E+05	6.48E+05	1.62E+06	7.53E+01	**1.40E+02**
F19	Mean	2.99E+05	2.86E+06	2.66E+05	2.72E+04	5.59E+02	1.09E+08	1.84E+04	3.91E+05	8.76E+05	5.76E+03	7.48E+05	5.29E+06	4.68E+01	**2.72E+01**
	Std.	3.47E+05	2.47E+06	2.39E+05	2.17E+04	5.87E+02	1.62E+08	1.63E+04	5.15E+05	7.66E+05	8.50E+03	2.14E+06	1.08E+07	3.76E+00	**7.66E+00**
F20	Mean	3.34E+02	6.15E+02	7.27E+02	3.61E+02	4.61E+02	8.88E+02	7.18E+02	2.69E+02	5.19E+02	**1.57E+02**	3.24E+02	6.04E+02	2.22E+02	1.58E+02
	Std.	9.65E+01	1.99E+02	1.72E+02	1.90E+02	1.38E+02	1.83E+02	4.68E+02	7.99E+01	1.93E+02	**5.87E+01**	2.18E+02	2.11E+02	1.21E+02	6.79E+01
F21	Mean	2.80E+02	4.59E+02	4.40E+02	2.92E+02	3.22E+02	4.69E+02	3.67E+02	2.74E+02	2.88E+02	**2.30E+02**	2.70E+02	4.12E+02	2.66E+02	2.46E+02
	Std.	1.97E+01	5.34E+01	4.74E+01	1.94E+01	3.05E+01	3.16E+01	7.64E+01	1.51E+01	2.15E+01	**1.31E+01**	3.46E+01	3.82E+01	1.60E+01	1.19E+01
F22	Mean	1.50E+03	4.41E+03	3.80E+03	3.53E+03	6.32E+02	6.98E+03	4.93E+03	3.03E+03	2.72E+03	5.68E+02	6.56E+03	3.50E+03	2.41E+03	**1.00E+02**
	Std.	1.43E+03	1.89E+03	2.15E+03	5.68E+02	1.39E+03	2.40E+02	3.47E+03	2.07E+03	1.98E+03	1.40E+03	2.04E+03	2.51E+03	2.70E+03	**1.91E-11**
F23	Mean	4.27E+02	7.47E+02	8.10E+02	4.34E+02	5.82E+02	7.23E+02	4.75E+02	4.37E+02	5.25E+02	4.06E+02	4.56E+02	6.59E+02	4.16E+02	**3.92E+02**
	Std.	3.00E+01	8.35E+01	1.03E+02	1.97E+01	9.28E+01	6.19E+01	8.26E+01	4.15E+01	4.83E+01	4.84E+01	6.97E+01	3.68E+01	2.83E+01	**1.27E+01**
F24	Mean	5.30E+02	7.69E+02	9.82E+02	5.48E+02	6.20E+02	8.05E+02	5.22E+02	5.23E+02	5.61E+02	4.57E+02	5.12E+02	7.68E+02	4.94E+02	**4.45E+02**
	Std.	5.89E+01	8.41E+01	1.24E+02	2.99E+01	7.75E+01	4.00E+01	6.71E+01	6.46E+01	3.91E+01	2.97E+01	6.81E+01	4.27E+01	2.72E+01	**1.38E+01**
F25	Mean	4.48E+02	4.55E+02	4.13E+02	3.87E+02	4.07E+02	1.67E+03	4.29E+02	4.59E+02	5.10E+02	3.87E+02	4.27E+02	6.14E+02	3.96E+02	**3.83E+02**
	Std.	2.29E+01	3.95E+01	1.95E+01	3.45E-01	2.17E+01	5.09E+02	6.51E+01	1.87E+01	5.07E+01	1.25E+00	1.45E+01	4.00E+01	1.08E+00	**6.57E+00**
F26	Mean	1.77E+03	5.16E+03	4.57E+03	1.96E+03	2.04E+03	3.71E+03	2.11E+03	1.75E+03	2.44E+03	1.19E+03	1.96E+03	3.80E+03	1.23E+03	**1.06E+03**
	Std.	1.96E+02	6.52E+02	1.86E+03	2.07E+02	1.34E+03	2.85E+02	6.71E+02	2.08E+02	4.72E+02	3.08E+02	7.11E+02	5.99E+02	4.52E+02	**5.36E+02**
F27	Mean	5.34E+02	6.60E+02	6.39E+02	5.14E+02	5.50E+02	7.70E+02	5.26E+02	5.26E+02	6.02E+02	**4.98E+02**	5.27E+02	6.45E+02	5.00E+02	5.00E+02
	Std.	1.37E+01	6.69E+01	6.55E+01	1.35E+01	2.11E+01	6.49E+01	1.40E+01	8.72E+00	4.12E+01	**8.25E+00**	8.76E+00	5.87E+01	1.59E-04	2.42E-04
F28	Mean	5.38E+02	4.93E+02	4.64E+02	4.41E+02	4.04E+02	1.48E+03	5.04E+02	5.42E+02	7.20E+02	4.14E+02	4.96E+02	8.28E+02	4.95E+02	**3.55E+02**
	Std.	6.65E+01	2.67E+01	3.28E+01	5.34E+01	4.40E+01	4.11E+02	7.64E+01	5.82E+01	1.43E+02	7.15E+00	4.89E+01	1.21E+02	5.87E+00	**4.84E+01**
F29	Mean	7.19E+02	1.91E+03	1.42E+03	7.89E+02	1.05E+03	1.54E+03	8.12E+02	7.50E+02	1.24E+03	5.20E+02	7.65E+02	1.54E+03	5.34E+02	**4.90E+02**
	Std.	1.14E+02	5.16E+02	4.35E+02	1.70E+02	2.30E+02	2.14E+02	2.67E+02	1.15E+02	2.74E+02	6.86E+01	1.15E+02	2.60E+02	1.64E+02	**7.01E+01**
F30	Mean	3.92E+06	1.19E+07	1.52E+06	1.25E+04	5.83E+03	3.31E+07	2.54E+04	4.03E+06	6.77E+06	6.96E+04	4.26E+06	9.34E+06	2.49E+04	**1.93E+03**
	Std.	3.84E+06	5.88E+06	1.38E+06	5.21E+03	2.68E+03	1.22E+07	2.43E+04	3.86E+06	5.42E+06	3.84E+04	2.94E+06	5.16E+06	6.58E+00	**2.71E+03**
Result	W (+)	29	29	29	29	25	29	29	28	29	19	29	29	23	
	L (-)	0	0	0	0	1	0	0	1	0	7	0	0	2	
	T (≈)	0	0	0	0	3	0	0	0	0	3	0	0	4	

Tables 12, 13 and 14 rank the algorithms based on the statistical tests including the Wilcoxon's rank sum test at a significance of 0.05 followed by the non-parametric Friedman's test and the mean absolute error rates. In table A4, the p-values indicate the statistical significance of SL-GWO's results compared to its competitors. MEGWO was the only notable exception for the 10-dimensional case and the disparity grew for the 30D and 50D cases. SL-GWO ranked first in the Friedman's ranking with the least mean absolute errors for all the three cases of increasing problem dimensions.

The acceleration rates from Tables 15, 16 and 17 for the 10D, 30D and 50D cases provide an overview of the convergence capabilities of SL-GWO compared to the others.

TABLE 11. Comparison of mean and standard deviation (std.) of the fourteen algorithms for the CEC2018 benchmarking suite with 50 dimensions.

		GWO	WOA	HHO	SMA	GTO	ChOA	EGWO	SOGWO	CGWO	IGWO	IGWO-DE	AGWO	MEGWO	SL-GWO
F1	Mean	2.81E+09	1.46E+07	4.51E+07	1.46E+04	5.18E+03	4.70E+10	2.07E+09	2.82E+09	8.36E+09	2.10E+06	1.37E+09	2.28E+10	3.26E+05	**7.84E-02**
	Std.	2.04E+09	8.18E+06	9.61E+06	1.20E+04	9.73E+03	3.89E+09	2.58E+09	1.31E+09	5.41E+09	7.18E+05	8.80E+08	2.78E+09	1.31E+05	**1.83E-01**
F3	Mean	6.54E+04	4.50E+04	1.25E+04	**8.58E+00**	3.67E+01	1.26E+05	7.39E+04	6.41E+04	8.97E+04	7.77E+03	1.17E+04	1.25E+05	1.42E+04	1.70E+02
	Std.	1.39E+04	1.96E+04	3.11E+03	**4.56E+00**	2.97E+01	1.15E+04	4.90E+04	1.58E+04	1.39E+04	3.44E+03	4.75E+03	6.77E+03	4.49E+03	1.40E+02
F4	Mean	3.18E+02	2.95E+02	2.61E+02	1.59E+02	1.43E+02	9.14E+03	3.71E+02	4.39E+02	8.16E+02	1.58E+02	2.52E+02	3.51E+03	4.60E+02	**9.94E+01**
	Std.	1.15E+02	6.14E+01	7.00E+01	3.12E+01	5.61E+01	1.39E+03	2.46E+02	2.50E+02	9.10E+02	4.59E+01	5.42E+01	7.30E+02	6.12E-01	**3.42E+01**
F5	Mean	1.58E+02	4.14E+02	3.61E+02	1.84E+02	3.30E+02	5.31E+02	2.66E+02	1.68E+02	2.19E+02	1.16E+02	2.38E+02	4.61E+02	1.43E+02	**9.41E+01**
	Std.	2.65E+01	7.72E+01	3.19E+01	2.93E+01	2.83E+01	3.91E+01	1.52E+02	3.06E+01	2.31E+01	9.62E+01	1.36E+02	3.84E+01	5.59E+01	**1.03E+01**
F6	Mean	8.52E+00	7.29E+01	7.29E+01	2.25E+00	5.32E+01	7.22E+01	4.27E+00	6.32E+00	3.59E+01	5.07E-01	3.75E+00	6.72E+01	1.51E+00	**8.80E-02**
	Std.	3.65E+00	8.14E+00	3.41E+00	9.75E-01	7.56E+00	5.91E+00	3.47E+00	1.65E+00	8.20E+00	1.90E-01	1.36E+00	4.49E+00	3.78E-01	**5.82E-02**
F7	Mean	2.93E+02	9.73E+02	1.13E+03	2.37E+02	7.50E+02	9.20E+02	4.88E+02	2.85E+02	3.80E+02	2.52E+02	3.51E+02	7.41E+02	2.44E+02	**1.64E+02**
	Std.	7.89E+01	1.22E+02	7.81E+01	3.63E+01	1.70E+02	6.76E+01	1.22E+02	3.30E+01	6.09E+01	7.91E+01	1.14E+02	6.67E+01	8.11E+01	**1.73E+01**
F8	Mean	1.71E+02	4.61E+02	3.84E+02	1.63E+02	3.07E+02	4.95E+02	3.97E+02	1.75E+02	2.29E+02	1.16E+02	1.63E+02	4.75E+02	1.52E+02	**9.38E+01**
	Std.	2.68E+01	9.75E+01	2.93E+01	5.02E+01	6.07E+01	2.87E+01	1.52E+02	2.64E+01	3.75E+01	1.94E+01	7.56E+01	1.83E+01	5.67E+01	**2.41E+01**
F9	Mean	2.88E+03	1.72E+04	1.75E+04	6.32E+03	8.61E+03	2.56E+04	4.84E+03	2.59E+03	9.24E+03	6.13E+01	2.21E+03	2.29E+04	1.95E+02	**3.24E+01**
	Std.	1.89E+03	4.48E+03	1.64E+03	4.35E+03	2.82E+03	3.39E+03	4.08E+03	1.74E+03	3.78E+03	2.63E+01	2.14E+03	3.70E+03	1.45E+02	**2.96E+01**
F10	Mean	**5.14E+03**	8.75E+03	7.89E+03	5.37E+03	7.69E+03	1.31E+04	1.13E+04	5.52E+03	7.17E+03	8.55E+03	9.93E+03	1.11E+04	9.58E+03	5.86E+03
	Std.	**7.26E+02**	8.39E+02	8.66E+02	9.54E+02	1.33E+03	5.79E+02	4.24E+03	7.93E+02	5.67E+02	4.54E+03	4.51E+03	1.01E+03	2.19E+03	4.86E+02
F11	Mean	1.58E+03	5.28E+02	3.36E+02	2.04E+02	2.07E+02	8.63E+03	5.99E+02	7.37E+02	4.25E+03	1.96E+02	3.53E+02	5.18E+03	1.79E+02	**9.19E+01**
	Std.	1.14E+03	8.33E+01	7.45E+01	6.12E+01	4.63E+01	9.61E+02	3.59E+02	5.82E+02	2.57E+03	4.46E+01	1.09E+02	1.86E+03	1.50E+01	**2.99E+01**
F12	Mean	2.81E+08	1.54E+08	3.85E+07	8.02E+06	8.92E+05	1.98E+10	4.40E+08	1.91E+08	1.60E+09	1.18E+07	8.18E+07	3.46E+09	1.84E+06	**5.94E+04**
	Std.	2.77E+08	6.76E+07	1.27E+07	5.22E+06	6.76E+05	6.22E+09	4.59E+08	1.89E+08	2.94E+09	5.53E+06	5.71E+07	1.69E+09	4.98E+05	**4.99E+04**
F13	Mean	2.62E+07	2.54E+05	1.72E+06	3.11E+04	1.47E+04	9.99E+09	3.56E+04	4.69E+07	2.23E+08	1.59E+05	2.84E+07	2.59E+08	6.09E+04	**1.76E+03**
	Std.	7.24E+07	2.25E+05	9.91E+05	1.06E+04	1.34E+04	6.89E+09	2.95E+04	7.31E+07	2.66E+08	6.86E+04	7.17E+07	4.58E+07	2.75E+04	**2.14E+03**
F14	Mean	2.33E+05	7.27E+05	3.89E+05	1.52E+05	1.32E+03	1.30E+06	1.18E+05	3.17E+05	8.06E+05	3.71E+04	3.96E+05	1.49E+06	4.41E+02	**1.36E+02**
	Std.	3.19E+05	7.84E+05	2.93E+05	1.14E+05	1.01E+03	3.43E+05	1.47E+05	3.23E+05	1.18E+06	2.19E+04	1.87E+05	1.25E+06	7.09E+01	**6.40E+01**
F15	Mean	3.75E+06	1.14E+05	3.28E+05	2.65E+04	1.34E+04	1.06E+08	1.72E+04	7.25E+06	1.15E+07	4.56E+04	8.07E+05	2.93E+07	6.47E+03	**1.82E+02**
	Std.	7.77E+06	7.87E+04	1.15E+05	7.20E+03	6.90E+03	4.46E+07	1.19E+04	1.92E+07	2.62E+07	2.83E+04	1.56E+06	3.04E+07	2.84E+03	**4.67E+01**
F16	Mean	1.23E+03	3.60E+03	2.62E+03	1.67E+03	1.85E+03	3.42E+03	1.49E+03	1.01E+03	1.65E+03	**5.85E+02**	1.21E+03	2.37E+03	2.11E+03	1.15E+03
	Std.	2.68E+02	8.23E+02	7.30E+02	4.48E+02	4.00E+02	5.38E+02	2.14E+02	2.19E+02	4.56E+02	**2.47E+02**	6.31E+02	3.64E+02	6.40E+02	1.92E+02
F17	Mean	9.35E+02	2.09E+03	2.03E+03	1.26E+03	1.67E+03	2.35E+03	1.59E+03	7.65E+02	1.39E+03	**4.85E+02**	1.15E+03	1.75E+03	8.69E+02	7.86E+02
	Std.	2.62E+02	3.80E+02	4.60E+02	1.89E+02	2.89E+02	4.95E+02	9.03E+02	2.23E+02	3.17E+02	**3.51E+02**	5.80E+02	3.38E+02	2.25E+02	2.65E+02
F18	Mean	2.33E+06	5.13E+06	2.37E+06	9.17E+05	3.72E+04	7.25E+06	1.18E+06	4.24E+06	8.17E+06	2.57E+05	1.41E+06	5.25E+06	9.56E+03	**4.75E+03**
	Std.	1.76E+06	2.70E+06	1.69E+06	5.50E+05	1.73E+04	1.60E+06	2.49E+06	4.10E+06	1.02E+07	1.01E+05	8.41E+05	5.98E+06	2.76E+03	**1.44E+03**
F19	Mean	7.17E+05	2.36E+06	2.84E+05	1.39E+04	8.74E+03	9.78E+07	3.15E+04	1.39E+06	1.94E+06	2.02E+04	9.97E+05	3.72E+07	4.60E+02	**4.78E+01**
	Std.	4.89E+05	2.40E+06	1.83E+05	1.86E+04	9.33E+03	2.35E+07	3.16E+04	1.87E+06	7.12E+05	1.52E+04	3.90E+05	7.33E+07	1.27E+02	**1.77E+01**
F20	Mean	6.42E+02	1.78E+03	1.38E+03	9.30E+02	9.66E+02	1.90E+03	2.16E+03	7.54E+02	9.53E+02	**2.97E+02**	7.95E+02	1.08E+03	1.06E+03	4.14E+02
	Std.	2.43E+02	2.88E+02	2.21E+02	3.41E+02	3.13E+02	1.90E+02	6.69E+02	2.91E+02	2.21E+02	**2.33E+02**	5.66E+02	2.42E+02	2.55E+02	1.01E+02
F21	Mean	3.65E+02	7.24E+02	7.48E+02	3.88E+02	5.04E+02	7.83E+02	4.77E+02	3.55E+02	4.23E+02	3.01E+02	3.27E+02	6.97E+02	3.46E+02	**2.92E+02**
	Std.	3.69E+01	5.90E+01	7.86E+01	2.00E+01	5.00E+01	4.83E+01	1.68E+02	3.23E+01	4.89E+01	2.19E+01	3.76E+01	3.66E+01	5.61E+01	**1.66E+01**
F22	Mean	6.35E+03	9.83E+03	8.68E+03	**5.44E+03**	7.92E+03	1.43E+04	1.42E+04	6.60E+03	7.02E+03	1.03E+04	1.23E+04	1.18E+04	9.29E+03	5.45E+03
	Std.	8.49E+02	1.36E+03	6.90E+02	**7.80E+02**	3.32E+03	4.60E+02	1.43E+03	2.74E+03	9.99E+02	4.52E+03	3.36E+03	9.09E+02	1.93E+03	3.93E+03
F23	Mean	6.05E+02	1.32E+03	1.29E+03	6.25E+02	8.65E+02	1.20E+03	6.70E+02	5.98E+02	7.30E+02	5.59E+02	5.91E+02	1.13E+03	**5.10E+02**	5.16E+02
	Std.	4.88E+01	1.79E+02	1.96E+02	4.18E+01	1.25E+02	9.78E+01	1.44E+02	3.46E+01	4.69E+01	1.08E+02	1.10E+02	4.05E+01	**1.64E+01**	1.97E+01
F24	Mean	6.99E+02	1.16E+03	1.73E+03	7.03E+02	9.21E+02	1.32E+03	7.12E+02	6.51E+02	8.21E+02	6.00E+02	6.86E+02	1.25E+03	6.67E+02	**5.60E+02**
	Std.	1.24E+02	1.45E+02	1.78E+02	3.96E+01	9.72E+01	7.05E+01	1.39E+02	3.73E+01	5.13E+01	7.61E+01	1.23E+02	4.00E+01	6.76E+01	**2.08E+01**
F25	Mean	8.05E+02	6.42E+02	6.44E+02	**5.30E+02**	5.79E+02	4.92E+03	6.77E+02	7.20E+02	1.29E+03	5.58E+02	6.35E+02	2.15E+03	5.31E+02	5.68E+02
	Std.	2.12E+02	3.79E+01	2.06E+01	**2.83E+01**	2.46E+01	1.85E+03	1.57E+02	1.15E+02	3.08E+02	3.15E+01	6.04E+01	3.71E+02	4.00E-02	3.57E+01
F26	Mean	2.99E+03	1.01E+04	8.36E+03	2.95E+03	3.24E+03	7.88E+03	3.85E+03	2.81E+03	4.29E+03	1.97E+03	2.65E+03	8.73E+03	1.82E+03	**9.70E+02**
	Std.	3.69E+02	2.19E+03	9.62E+02	3.79E+02	3.18E+03	5.56E+02	1.44E+03	4.18E+02	3.51E+02	2.66E+02	4.07E+02	6.86E+02	4.04E+02	**8.66E+02**
F27	Mean	7.40E+02	1.46E+03	1.21E+03	6.74E+02	9.25E+02	1.60E+03	6.34E+02	7.63E+02	1.23E+03	5.22E+02	6.85E+02	1.48E+03	5.20E+02	**5.09E+02**
	Std.	4.87E+01	3.27E+02	1.52E+02	9.59E+01	1.22E+02	1.51E+02	5.75E+01	6.94E+01	1.66E+02	1.25E+01	4.77E+01	1.56E+02	8.10E-05	**2.76E+01**
F28	Mean	9.74E+02	6.21E+02	5.77E+02	4.95E+02	5.10E+02	3.29E+03	1.41E+03	9.29E+02	1.64E+03	4.97E+02	7.47E+02	2.67E+03	4.96E+02	**4.95E+02**
	Std.	1.89E+02	6.47E+01	5.83E+01	2.16E+01	1.40E+01	4.75E+02	1.19E+03	1.98E+02	5.12E+02	1.99E+01	1.05E+02	4.76E+02	8.97E+00	**2.05E+01**
F29	Mean	1.30E+03	4.64E+03	2.55E+03	1.41E+03	2.38E+03	4.00E+03	1.15E+03	1.29E+03	2.16E+03	6.59E+02	1.21E+03	3.22E+03	1.24E+03	**5.11E+02**
	Std.	2.54E+02	1.31E+03	5.57E+02	2.27E+02	5.07E+02	3.82E+02	6.35E+02	2.86E+02	3.06E+02	1.93E+02	1.77E+02	5.09E+02	3.91E+02	**1.47E+02**
F30	Mean	6.86E+07	8.75E+07	1.73E+07	1.73E+06	7.32E+05	3.37E+08	1.57E+06	5.93E+07	9.59E+07	3.48E+06	4.81E+07	2.22E+08	5.71E+04	**2.35E+04**
	Std.	2.76E+07	3.75E+07	3.28E+06	4.56E+05	1.06E+05	6.85E+07	6.60E+05	2.44E+07	2.78E+07	1.07E+06	1.40E+07	5.39E+07	1.64E+03	**5.32E+04**
Result	W (+)	28	29	29	23	28	29	29	29	29	22	29	29	24	
	L (-)	1	0	0	3	1	0	0	0	0	4	0	0	2	
	T (≈)	0	0	0	3	0	0	0	0	0	3	0	0	3	

It is quite evident that the SL-GWO is quicker than all the competitor algorithms for all the functions with the exception of one or two functions. The acceleration rates are indicative of the balance of exploration and exploitation achieved by SL-GWO through its multi-populational management and control parameters to efficiently adapt the diversification and intensification rates to achieve the best possible convergence to the global optimum within the given computational budget. The quicker convergence without the risk of entrapment in SL-GWO is made possible through the symbiotic learning schemes i.e., the linear and adaptive controls for the hunting distance and symbiosis rate. The two sub-population each with its diversified control schemes operate independently to explore and exploit around the three leader wolves and

TABLE 12. Results of the Wilcoxon's rank sum test comparing SL-GWO with the ten modern meta-heuristics for the CEC2018 benchmarking suite.

	10D				30D				50D			
SL-GWO vs.	R+	R-	*p* value	Result	R+	R-	*p* value	Result	R+	R-	*p* value	Result
GWO	435	0	1.08E-04	+	434	1	2.74E-04	+	434	1	8.75E-04	+
WOA	435	0	2.18E-05	+	435	0	2.45E-06	+	435	0	6.86E-05	+
HHO	435	0	2.03E-05	+	435	0	6.03E-06	+	435	0	1.22E-04	+
SMA	435	0	2.83E-03	+	432	3	8.75E-04	+	420	15	1.34E-02	+
GTO	434	1	1.03E-02	+	434	1	6.23E-04	+	434	1	5.37E-03	+
ChOA	435	0	2.27E-06	+	435	0	5.51E-07	+	435	0	4.83E-06	+
EGWO	435	0	4.95E-04	+	435	0	2.03E-05	+	435	0	5.24E-04	+
SOGWO	435	0	9.49E-05	+	434	1	1.78E-04	+	429	6	1.15E-03	+
CGWO	435	0	2.34E-05	+	435	0	1.54E-05	+	435	0	1.39E-04	+
IGWO	432	3	2.83E-02	+	414	21	1.18E-02	+	425	10	3.44E-02	+
IGWO-DE	435	0	6.60E-04	+	435	0	3.48E-04	+	435	0	1.51E-03	+
AGWO	435	0	1.16E-05	+	435	0	1.43E-06	+	435	0	1.54E-05	+
MEGWO	390	45	4.92E-02	≈	432	3	3.88E-02	+	432	3	2.95E-02	+

TABLE 13. Ranking the algorithms based on Friedman's rank for the CEC2018 benchmarking suite.

	10D		30D		50D	
Rank	Algorithm	Friedman's rank	Algorithm	Friedman's rank	Algorithm	Friedman's rank
1.	SL-GWO	2.2060	SL-GWO	1.7252	SL-GWO	1.9973
2.	MEGWO	2.9847	MEGWO	3.2732	IGWO	4.4207
3.	IGWO	4.0761	IGWO	3.7934	MEGWO	4.6622
4.	SMA	6.6816	SMA	6.2284	SMA	5.3155
5.	IGWO-DE	7.0040	IGWO-DE	6.9988	GTO	6.6964
6.	GTO	7.2752	GTO	7.1754	GWO	7.2186
7.	GWO	7.5113	GWO	7.9595	IGWO-DE	7.4159
8.	SOGWO	8.0951	SOGWO	8.1224	SOGWO	7.4387
9.	EGWO	9.1775	EGWO	8.6543	EGWO	8.1724
10.	CGWO	10.8992	HHO	10.0012	CGWO	10.2781
11.	WOA	11.6578	CGWO	10.0129	HHO	10.3407
12.	AGWO	11.7666	WOA	12.4144	WOA	10.6144
13.	HHO	11.9607	AGWO	12.5681	AGWO	12.4627
14.	ChOA	12.1921	ChOA	13.9201	ChOA	14.4330

TABLE 14. Ranking the algorithms based on Mean Absolute Errors for the CEC2018 benchmarking suite.

	10D		30D		50D	
Rank	Algorithm	Mean Absolute Error	Algorithm	Mean Absolute Error	Algorithm	Mean Absolute Error
1.	SL-GWO	1.12E+02	SL-GWO	7.22E+02	SL-GWO	3.74E+03
2.	MEGWO	1.23E+02	MEGWO	2.42E+03	GTO	6.00E+04
3.	IGWO	5.63E+03	GTO	3.80E+03	MEGWO	8.09E+04
4.	SMA	6.76E+03	IGWO	4.50E+04	SMA	3.77E+05
5.	EGWO	1.19E+04	SMA	8.41E+04	IGWO	6.18E+05
6.	SOGWO	3.22E+04	HHO	1.01E+06	HHO	3.65E+06
7.	IGWO-DE	3.94E+04	WOA	1.84E+06	WOA	9.14E+06
8.	GTO	5.42E+04	EGWO	8.82E+06	IGWO-DE	5.31E+07
9.	HHO	7.71E+04	IGWO-DE	1.80E+07	EGWO	8.67E+07
10.	WOA	9.19E+04	SOGWO	2.27E+07	SOGWO	1.08E+08
11.	GWO	2.09E+05	GWO	2.64E+07	GWO	1.10E+08
12.	CGWO	3.10E+06	CGWO	7.16E+07	CGWO	3.54E+08
13.	AGWO	7.80E+06	AGWO	2.18E+08	AGWO	9.25E+08
14.	ChOA	5.19E+07	ChOA	9.54E+08	ChOA	2.67E+09

complement each other. The advantage in SL-GWO stems from the fact that either the linear control or adaptive control of sub-population can be beneficial at times when the latter proves ineffectual. This coupled with their dynamic nature of ever-changing values can help evade entrapment in cases of stagnation.

The computational times in Table 18 indicate that SL-GWO's times are higher compared to the standard GWO and a few of its variants such as MEGWO, EGWO and CGWO. However, SL-GWO's times are competitive compared to SOGWO, AGWO and ChOA. SL-GWO's higher computational times are a result of its population sub-grouping which requires the initial population to be sorted into two individual groups that operate one after the other on a single core CPU workload. SL-GWO's diversified control and adaptive tuning add additional CPU times over the existing workload. Furthermore, SL-GWO repairs the individual solution dimensions that exceed the lower or upper obtained through re-initializing them within the problem search bounds for both the sub-groups. This repair system can add to the existing CPU burden while most other variants of GWO rely on the much simpler yet ineffective corner bounding. Despite the effectiveness of corner bounding for static landscapes, the process of forcing the solutions to the corners of the search space can result in entrapment in dynamic landscapes with local optimum points at the

TABLE 15. Comparison of the acceleration rates of SL-GWO with the thirteen competitor algorithms for the CEC2018 benchmarking suite (10 dimensions).

SL-GWO Vs.	GWO	WOA	HHO	SMA	GTO	ChOA	EGWO	SOGWO	CGWO	IGWO	IGWO-DE	AGWO	MEGWO
F1	8.86E-17 (∞)	2.17E-15 (∞)	1.05E-15 (∞)	5.03E-14 (∞)	2.55E-13 (∞)	2.62E-19 (∞)	1.52E-13 (∞)	2.81E-15 (∞)	4.59E-18 (∞)	3.71E-14 (∞)	8.59E-15 (∞)	1.76E-18 (∞)	1.06E+00 (≈)
F3	1.04E-16 (∞)	4.76E-16 (∞)	6.99E-14 (∞)	4.48E-10 (∞)	1.07E-01 (+)	8.10E-17 (∞)	8.64E-01 (+)	2.16E-16 (∞)	1.47E-17 (∞)	4.11E-09 (∞)	1.90E-15 (∞)	2.73E-17 (∞)	1.55E+01 (-)
F4	8.10E-12 (∞)	6.93E-12 (∞)	6.03E-12 (∞)	2.68E-11 (∞)	2.25E-10 (∞)	9.67E-13 (∞)	3.00E-10 (∞)	1.09E-11 (∞)	5.98E-12 (∞)	1.52E-10 (∞)	9.67E-12 (∞)	3.33E-12 (∞)	6.25E-07 (∞)
F5	3.50E-01 (+)	1.23E-01 (+)	1.30E-01 (+)	3.54E-01 (+)	2.23E-01 (+)	1.34E-01 (+)	2.59E-01 (+)	3.91E-01 (+)	2.90E-01 (+)	7.07E-01 (+)	3.40E-01 (+)	1.92E-01 (+)	8.19E-01 (+)
F6	3.98E-05 (∞)	1.58E-06 (∞)	1.34E-06 (∞)	9.58E-04 (∞)	5.56E-06 (∞)	1.93E-06 (∞)	3.02E-05 (∞)	1.85E-04 (∞)	6.33E-06 (∞)	8.13E-01 (+)	2.29E-04 (∞)	3.30E-06 (∞)	9.15E-03 (+)
F7	5.13E-01 (+)	2.11E-01 (+)	1.92E-01 (+)	7.36E-01 (+)	2.78E-01 (+)	1.76E-01 (+)	3.79E-01 (+)	5.63E-01 (+)	4.04E-01 (+)	7.70E-01 (+)	3.99E-01 (+)	2.48E-01 (+)	8.39E-01 (+)
F8	3.94E-01 (+)	1.16E-01 (+)	1.64E-01 (+)	3.94E-01 (+)	2.02E-01 (+)	1.40E-01 (+)	1.75E-01 (+)	4.65E-01 (+)	2.71E-01 (+)	7.84E-01 (+)	4.30E-01 (+)	1.98E-01 (+)	6.48E-01 (+)
F9	2.02E-31 (∞)	2.44E-33 (∞)	1.86E-33 (∞)	2.00E-27 (∞)	1.22E-32 (∞)	2.94E-33 (∞)	1.70E-31 (∞)	1.29E-31 (∞)	1.42E-32 (∞)	1.32E-16 (∞)	3.85E-31 (∞)	7.52E-33 (∞)	2.64E-17 (∞)
F10	2.75E-01 (+)	1.70E-01 (+)	1.80E-01 (+)	3.11E-01 (+)	1.74E-01 (+)	9.76E-02 (+)	1.07E-01 (+)	3.06E-01 (+)	1.77E-01 (+)	9.12E-01 (+)	3.30E-01 (+)	1.54E-01 (+)	4.11E-01 (+)
F11	7.17E-02 (+)	1.88E-02 (+)	3.28E-02 (+)	1.67E-01 (+)	6.71E-02 (+)	8.00E-03 (+)	9.32E-02 (+)	9.04E-02 (+)	2.93E-02 (+)	5.47E-01 (+)	1.00E-01 (+)	2.52E-02 (+)	1.23E+00 (≈)
F12	1.35E-04 (∞)	7.31E-05 (∞)	9.43E-05 (∞)	3.48E-03 (+)	2.34E-02 (+)	3.52E-05 (∞)	6.08E-03 (+)	3.31E-04 (∞)	8.81E-05 (∞)	7.23E-03 (+)	1.29E-04 (∞)	1.74E-04 (∞)	9.77E-01 (+)
F13	3.54E-04 (∞)	1.22E-04 (∞)	1.77E-04 (∞)	3.31E-04 (∞)	1.37E-02 (+)	1.82E-04 (∞)	9.51E-04 (∞)	3.03E-04 (∞)	2.37E-04 (∞)	5.27E-03 (+)	3.18E-04 (∞)	3.96E-04 (∞)	3.40E-01 (+)
F14	2.54E-03 (+)	1.81E-02 (+)	1.08E-02 (+)	5.97E-02 (+)	4.33E-02 (+)	7.48E-04 (∞)	5.61E-04 (∞)	2.20E-03 (+)	1.33E-03 (+)	5.02E-02 (+)	2.14E-02 (+)	9.97E-04 (∞)	1.11E-01 (+)
F15	8.23E-04 (∞)	2.77E-04 (∞)	2.67E-04 (∞)	3.53E-02 (+)	3.10E-02 (+)	3.20E-04 (∞)	1.35E-04 (∞)	4.61E-04 (∞)	1.73E-04 (∞)	4.28E-02 (+)	2.84E-03 (+)	2.87E-04 (∞)	4.71E-01 (+)
F16	1.06E-02 (+)	3.01E-03 (+)	2.39E-03 (+)	7.53E-03 (+)	1.28E-02 (+)	2.69E-03 (+)	7.60E-03 (+)	6.92E-03 (+)	3.25E-03 (+)	2.91E-01 (+)	1.03E-02 (+)	3.02E-03 (+)	2.75E-01 (+)
F17	3.40E-02 (+)	1.58E-02 (+)	2.02E-02 (+)	4.13E-02 (+)	2.61E-02 (+)	2.21E-02 (+)	1.81E-02 (+)	2.17E-02 (+)	1.80E-02 (+)	6.65E-02 (+)	2.73E-02 (+)	1.82E-02 (+)	5.95E-02 (+)
F18	2.62E-05 (∞)	4.93E-05 (∞)	3.86E-05 (∞)	2.86E-05 (∞)	7.00E-03 (+)	2.06E-05 (∞)	4.58E-05 (∞)	2.60E-05 (∞)	4.39E-05 (∞)	1.80E-04 (∞)	2.40E-05 (∞)	3.52E-05 (∞)	3.32E-02 (+)
F19	1.01E-05 (∞)	1.34E-06 (∞)	3.49E-06 (∞)	1.49E-05 (∞)	9.38E-04 (∞)	2.31E-06 (∞)	2.32E-06 (∞)	9.64E-06 (∞)	7.38E-06 (∞)	1.91E-03 (+)	6.79E-05 (∞)	1.64E-06 (∞)	1.76E-02 (+)
F20	6.69E-03 (+)	3.68E-03 (+)	2.74E-03 (+)	1.81E-02 (+)	7.04E-03 (+)	2.29E-03 (+)	3.77E-03 (+)	1.08E-02 (+)	4.59E-03 (+)	3.54E-02 (+)	6.73E-03 (+)	3.51E-03 (+)	4.19E-02 (+)
F21	5.53E-01 (+)	5.55E-01 (+)	4.89E-01 (+)	5.58E-01 (+)	1.17E+00 (≈)	6.42E-01 (+)	5.88E-01 (+)	5.94E-01 (+)	5.58E-01 (+)	6.13E-01 (+)	5.53E-01 (+)	5.12E-01 (+)	1.19E+00 (≈)
F22	8.65E-01 (+)	8.42E-01 (+)	8.20E-01 (+)	6.69E-01 (+)	8.90E-01 (+)	8.34E-02 (+)	9.95E-01 (+)	8.90E-01 (+)	7.25E-01 (+)	9.83E-01 (+)	8.90E-01 (+)	3.33E-01 (+)	1.32E+00 (≈)
F23	9.62E-01 (+)	8.98E-01 (+)	8.25E-01 (+)	9.53E-01 (+)	9.45E-01 (+)	8.75E-01 (+)	9.47E-01 (+)	9.75E-01 (+)	9.36E-01 (+)	1.00E+00 (≈)	9.68E-01 (+)	8.65E-01 (+)	9.90E-01 (+)
F24	7.61E-01 (+)	7.03E-01 (+)	6.17E-01 (+)	7.29E-01 (+)	9.35E-01 (+)	6.77E-01 (+)	7.19E-01 (+)	7.52E-01 (+)	7.29E-01 (+)	7.70E-01 (+)	7.35E-01 (+)	6.96E-01 (+)	8.87E-01 (+)
F25	9.73E-01 (+)	9.38E-01 (+)	9.66E-01 (+)	9.80E-01 (+)	9.53E-01 (+)	8.71E-01 (+)	9.34E-01 (+)	9.51E-01 (+)	9.09E-01 (+)	1.03E+00 (≈)	9.76E-01 (+)	9.40E-01 (+)	1.14E+00 (≈)
F26	8.89E-01 (+)	4.25E-01 (+)	4.26E-01 (+)	6.32E-01 (+)	7.22E-01 (+)	2.42E-01 (+)	5.93E-01 (+)	6.18E-01 (+)	4.46E-01 (+)	7.39E-01 (+)	5.67E-01 (+)	3.31E-01 (+)	1.11E+00 (≈)
F27	9.54E-01 (+)	8.89E-01 (+)	8.43E-01 (+)	9.68E-01 (+)	9.61E-01 (+)	9.49E-01 (+)	9.56E-01 (+)	9.61E-01 (+)	9.40E-01 (+)	9.73E-01 (+)	9.66E-01 (+)	8.89E-01 (+)	1.02E+00 (≈)
F28	7.13E-01 (+)	6.46E-01 (+)	5.87E-01 (+)	9.90E-01 (+)	8.00E-01 (+)	9.05E-01 (+)	6.94E-01 (+)	7.63E-01 (+)	6.57E-01 (+)	1.05E+00 (≈)	6.90E-01 (+)	6.87E-01 (+)	8.77E-01 (+)
F29	8.90E-01 (+)	5.52E-01 (+)	5.35E-01 (+)	8.01E-01 (+)	7.93E-01 (+)	6.23E-01 (+)	7.23E-01 (+)	8.65E-01 (+)	7.59E-01 (+)	9.97E-01 (+)	8.56E-01 (+)	6.97E-01 (+)	1.00E+00 (≈)
F30	6.93E-04 (∞)	7.05E-04 (∞)	1.03E-03 (+)	3.63E-03 (+)	2.60E-04 (∞)	2.64E-04 (∞)	1.48E-03 (+)	1.19E-03 (+)	1.69E-04 (∞)	3.20E-03 (+)	2.35E-02 (+)	4.02E-04 (∞)	9.79E-01 (+)

TABLE 16. Comparison of the acceleration rates of SL-GWO with the thirteen competitor algorithms for the CEC2018 benchmarking suite (30 dimensions).

SL-GWO Vs.	GWO	WOA	HHO	SMA	GTO	ChOA	EGWO	SOGWO	CGWO	IGWO	IGWO-DE	AGWO	MEGWO
F1	1.02E-13 (∞)	3.33E-11 (∞)	6.75E-12 (∞)	9.30E-09 (∞)	1.77E-08 (∞)	3.20E-15 (∞)	2.92E-13 (∞)	1.30E-13 (∞)	3.68E-14 (∞)	6.94E-10 (∞)	1.48E-13 (∞)	1.24E-14 (∞)	2.59E-08 (∞)
F3	2.92E-05 (∞)	4.00E-06 (∞)	1.28E-04 (∞)	2.69E+00 (≈)	2.05E+03 (-)	1.51E-05 (∞)	4.13E-05 (∞)	2.16E-05 (∞)	1.52E-05 (∞)	6.10E-04 (∞)	2.03E-04 (∞)	1.31E-05 (∞)	2.75E-03 (+)
F4	4.19E-01 (+)	4.28E-01 (+)	4.67E-01 (+)	6.90E-01 (+)	8.13E-01 (+)	1.70E-02 (+)	5.94E-01 (+)	4.57E-01 (+)	2.98E-01 (+)	6.64E-01 (+)	5.06E-01 (+)	1.21E-01 (+)	8.44E-01 (+)
F5	5.68E-01 (+)	1.67E-01 (+)	2.02E-01 (+)	6.08E-01 (+)	2.61E-01 (+)	1.49E-01 (+)	3.99E-01 (+)	5.77E-01 (+)	4.02E-01 (+)	7.88E-01 (+)	5.56E-01 (+)	1.85E-01 (+)	7.24E-01 (+)
F6	1.50E-03 (+)	7.95E-05 (∞)	8.99E-05 (∞)	8.71E-03 (+)	1.39E-04 (∞)	9.43E-05 (∞)	1.38E-03 (+)	1.48E-03 (+)	2.51E-04 (∞)	3.40E-02 (+)	3.16E-03 (+)	1.09E-04 (∞)	1.42E-02 (+)
F7	6.72E-01 (+)	1.50E-01 (+)	1.42E-01 (+)	6.66E-01 (+)	2.40E-01 (+)	1.68E-01 (+)	3.87E-01 (+)	5.62E-01 (+)	4.23E-01 (+)	7.97E-01 (+)	6.34E-01 (+)	2.24E-01 (+)	6.90E-01 (+)
F8	8.07E-01 (+)	2.84E-01 (+)	3.90E-01 (+)	7.43E-01 (+)	4.14E-01 (+)	2.75E-01 (+)	3.55E-01 (+)	7.41E-01 (+)	6.40E-01 (+)	1.88E+00 (≈)	9.73E-01 (+)	3.43E-01 (+)	8.99E-01 (+)
F9	9.90E-03 (+)	4.80E-04 (∞)	5.93E-04 (∞)	2.58E-03 (+)	1.20E-03 (+)	6.21E-04 (∞)	2.31E-03 (+)	7.22E-03 (+)	2.15E-03 (+)	1.18E+01 (+)	1.09E-02 (+)	8.32E-04 (∞)	1.17E+00 (≈)
F10	1.19E+00 (≈)	7.68E-01 (+)	8.13E-01 (+)	1.06E+00 (≈)	8.64E-01 (+)	5.30E-01 (+)	5.15E-01 (+)	1.23E+00 (≈)	9.83E-01 (+)	7.93E-01 (+)	5.82E-01 (+)	6.82E-01 (+)	8.93E-01 (+)
F11	1.62E-01 (+)	9.92E-02 (+)	2.53E-01 (+)	3.29E-01 (+)	3.10E-01 (+)	1.60E-02 (+)	1.01E-01 (+)	1.04E-01 (+)	2.16E-02 (+)	8.27E-01 (+)	2.37E-01 (+)	2.50E-02 (+)	7.16E-01 (+)
F12	3.27E-04 (∞)	2.97E-04 (∞)	6.56E-04 (∞)	5.24E-03 (+)	2.00E-01 (+)	2.69E-06 (∞)	2.16E-02 (+)	1.24E-04 (∞)	1.95E-04 (∞)	1.17E-02 (+)	7.13E-04 (∞)	3.55E-05 (∞)	3.58E-01 (+)
F13	8.27E-04 (∞)	6.57E-04 (∞)	3.19E-04 (∞)	4.15E-03 (+)	7.24E-03 (+)	1.51E-07 (∞)	2.39E-03 (+)	6.62E-05 (∞)	1.07E-03 (+)	9.91E-04 (∞)	1.05E-03 (+)	4.90E-06 (∞)	6.91E-02 (+)
F14	2.05E-04 (∞)	3.87E-05 (∞)	1.24E-03 (+)	1.29E-03 (+)	1.32E-01 (+)	1.63E-04 (∞)	1.87E-03 (+)	6.33E-04 (∞)	2.47E-04 (∞)	1.56E-02 (+)	1.24E-03 (+)	7.36E-05 (∞)	5.29E-01 (+)
F15	3.36E-04 (∞)	7.81E-04 (∞)	9.89E-04 (∞)	2.40E-03 (+)	2.41E-02 (+)	9.84E-06 (∞)	2.58E-03 (+)	3.75E-04 (∞)	2.51E-04 (∞)	5.97E-03 (+)	1.13E-03 (+)	7.36E-05 (∞)	3.42E-01 (+)
F16	4.00E-01 (+)	1.79E-01 (+)	2.15E-01 (+)	3.96E-01 (+)	2.88E-01 (+)	1.68E-01 (+)	2.48E-01 (+)	5.17E-01 (+)	3.10E-01 (+)	1.10E+00 (≈)	5.09E-01 (+)	2.39E-01 (+)	4.33E-01 (+)
F17	3.06E-01 (+)	1.12E-01 (+)	1.12E-01 (+)	1.61E-01 (+)	1.35E-01 (+)	1.02E-01 (+)	1.87E-01 (+)	2.85E-01 (+)	2.18E-01 (+)	8.26E-01 (+)	4.30E-01 (+)	1.43E-01 (+)	2.88E-01 (+)
F18	6.20E-04 (∞)	2.11E-04 (∞)	3.27E-04 (∞)	7.39E-04 (∞)	1.68E-02 (+)	2.32E-04 (∞)	1.05E-03 (+)	5.01E-04 (∞)	1.66E-04 (∞)	1.94E-03 (+)	3.97E-04 (∞)	1.93E-04 (∞)	9.87E-01 (+)
F19	9.08E-05 (∞)	9.49E-06 (∞)	1.02E-04 (∞)	9.98E-04 (∞)	4.86E-02 (+)	2.49E-07 (∞)	1.48E-03 (+)	6.94E-05 (∞)	3.10E-05 (∞)	4.71E-03 (+)	3.63E-05 (∞)	5.13E-06 (∞)	5.80E-01 (+)
F20	4.73E-01 (+)	2.57E-01 (+)	2.17E-01 (+)	4.37E-01 (+)	3.42E-01 (+)	1.78E-01 (+)	2.20E-01 (+)	5.87E-01 (+)	3.04E-01 (+)	1.01E+00 (≈)	4.87E-01 (+)	2.61E-01 (+)	7.11E-01 (+)
F21	8.79E-01 (+)	5.36E-01 (+)	5.60E-01 (+)	8.43E-01 (+)	7.65E-01 (+)	5.25E-01 (+)	6.71E-01 (+)	8.99E-01 (+)	8.55E-01 (+)	1.07E+00 (≈)	9.12E-01 (+)	5.98E-01 (+)	9.26E-01 (+)
F22	6.67E-02 (+)	2.27E-02 (+)	2.63E-02 (+)	2.83E-02 (+)	1.58E-01 (+)	1.43E-02 (+)	2.03E-02 (+)	3.30E-02 (+)	3.68E-02 (+)	1.76E-01 (+)	1.52E-02 (+)	2.86E-02 (+)	4.15E-02 (+)
F23	9.18E-01 (+)	5.24E-01 (+)	4.84E-01 (+)	9.03E-01 (+)	6.73E-01 (+)	5.42E-01 (+)	8.25E-01 (+)	8.97E-01 (+)	7.46E-01 (+)	9.65E-01 (+)	8.59E-01 (+)	5.95E-01 (+)	9.42E-01 (+)
F24	8.39E-01 (+)	5.78E-01 (+)	4.53E-01 (+)	8.12E-01 (+)	7.17E-01 (+)	5.53E-01 (+)	8.52E-01 (+)	8.50E-01 (+)	7.93E-01 (+)	9.73E-01 (+)	8.69E-01 (+)	5.79E-01 (+)	9.00E-01 (+)
F25	8.55E-01 (+)	8.42E-01 (+)	9.27E-01 (+)	9.90E-01 (+)	9.41E-01 (+)	2.29E-01 (+)	8.93E-01 (+)	8.34E-01 (+)	7.51E-01 (+)	9.90E-01 (+)	8.97E-01 (+)	6.24E-01 (+)	9.67E-01 (+)
F26	6.02E-01 (+)	2.06E-01 (+)	2.33E-01 (+)	5.43E-01 (+)	5.22E-01 (+)	2.87E-01 (+)	5.05E-01 (+)	6.08E-01 (+)	4.36E-01 (+)	8.95E-01 (+)	5.43E-01 (+)	2.80E-01 (+)	8.66E-01 (+)
F27	9.36E-01 (+)	7.58E-01 (+)	7.82E-01 (+)	9.73E-01 (+)	9.09E-01 (+)	6.49E-01 (+)	9.51E-01 (+)	9.51E-01 (+)	8.31E-01 (+)	1.00E+00 (≈)	9.49E-01 (+)	7.75E-01 (+)	1.00E+00 (≈)
F28	6.60E-01 (+)	7.20E-01 (+)	7.65E-01 (+)	8.05E-01 (+)	8.79E-01 (+)	2.40E-01 (+)	7.04E-01 (+)	6.55E-01 (+)	4.93E-01 (+)	8.57E-01 (+)	7.16E-01 (+)	4.29E-01 (+)	7.17E-01 (+)
F29	6.81E-01 (+)	2.56E-01 (+)	3.45E-01 (+)	6.21E-01 (+)	4.66E-01 (+)	3.18E-01 (+)	6.03E-01 (+)	6.53E-01 (+)	3.95E-01 (+)	9.42E-01 (+)	6.40E-01 (+)	3.18E-01 (+)	9.17E-01 (+)
F30	4.91E-04 (∞)	1.62E-04 (∞)	1.27E-03 (+)	1.54E-01 (+)	3.30E-01 (+)	5.82E-05 (∞)	7.58E-02 (+)	4.78E-04 (∞)	2.84E-04 (∞)	2.77E-02 (+)	4.52E-04 (∞)	2.06E-04 (∞)	7.73E-02 (+)

extremities of the landscape. SL-GWO's computational times are similar to SOGWO's as both these algorithms operate with two sets of populations and while SOGWO creates an opposite point for each and every solution and selects the best one based on fitness, SL-GWO relies on multiple symbiotic strategies with adaptive control schemes to generate newer population and include them into the population pool. Although the selective opposition strategy from SOGWO is promising at averting entrapment, it cannot always deliver the same level of exploration-exploitation balance in SL-GWO given that the opposite points may not be the fittest at all times. SL-GWO overcomes this disadvantage through the

TABLE 17. Comparison of the acceleration rates of SL-GWO with the thirteen competitor algorithms for the CEC2018 benchmarking suite (50 dimensions).

SL-GWO Vs.													
	GWO	WOA	HHO	SMA	GTO	ChOA	EGWO	SOGWO	CGWO	IGWO	IGWO-DE	AGWO	MEGWO
F1	2.79E-11 (∞)	5.37E-09 (∞)	1.74E-09 (∞)	5.37E-06 (∞)	1.51E-05 (∞)	1.67E-12 (∞)	3.79E-11 (∞)	2.78E-11 (∞)	9.38E-12 (∞)	3.73E-08 (∞)	5.72E-11 (∞)	3.44E-12 (∞)	2.40E-07 (∞)
F3	2.61E-03 (+)	3.79E-03 (+)	1.36E-02 (+)	1.99E+01(-)	4.64E+00 (≈)	1.35E-03 (+)	2.31E-03 (+)	2.66E-03 (+)	1.90E-03 (+)	2.19E-02 (+)	1.46E-02 (+)	1.36E-03 (+)	1.20E-02 (+)
F4	3.12E-01 (+)	3.37E-01 (+)	3.81E-01 (+)	6.25E-01 (+)	6.95E-01 (+)	1.09E-02 (+)	2.68E-01 (+)	2.26E-01 (+)	1.22E-01 (+)	6.29E-01 (+)	3.94E-01 (+)	2.83E-02 (+)	2.16E-01 (+)
F5	5.96E-01 (+)	2.27E-01 (+)	2.61E-01 (+)	5.12E-01 (+)	2.85E-01 (+)	1.77E-01 (+)	3.54E-01 (+)	5.60E-01 (+)	4.30E-01 (+)	8.11E-01 (+)	3.95E-01 (+)	2.04E-01 (+)	6.58E-01 (+)
F6	1.03E-02 (+)	1.21E-03 (+)	1.21E-03 (+)	3.91E-02 (+)	1.65E-03 (+)	1.22E-03 (+)	2.06E-02 (+)	1.39E-02 (+)	2.45E-03 (+)	1.74E-01 (+)	2.35E-02 (+)	1.31E-03 (+)	5.83E-02 (+)
F7	5.59E-01 (+)	1.68E-01 (+)	1.45E-01 (+)	6.91E-01 (+)	2.18E-01 (+)	1.78E-01 (+)	3.36E-01 (+)	5.75E-01 (+)	4.31E-01 (+)	6.50E-01 (+)	4.67E-01 (+)	2.21E-01 (+)	6.71E-01 (+)
F8	5.49E-01 (+)	2.04E-01 (+)	2.44E-01 (+)	5.76E-01 (+)	3.06E-01 (+)	1.90E-01 (+)	2.36E-01 (+)	5.36E-01 (+)	4.10E-01 (+)	8.09E-01 (+)	5.76E-01 (+)	1.98E-01 (+)	6.17E-01 (+)
F9	1.12E-02 (+)	1.88E-03 (+)	1.85E-03 (+)	5.12E-03 (+)	3.76E-03 (+)	1.26E-03 (+)	6.69E-03 (+)	1.25E-02 (+)	3.50E-03 (+)	5.28E-01 (+)	1.47E-02 (+)	1.41E-03 (+)	1.66E-01 (+)
F10	1.14E+00 (≈)	6.70E-01 (+)	7.43E-01 (+)	1.09E+00 (≈)	7.62E-01 (+)	4.48E-01 (+)	5.19E-01 (+)	1.06E+00 (≈)	8.18E-01 (+)	6.86E-01 (+)	5.90E-01 (+)	5.28E-01 (+)	6.12E-01 (+)
F11	5.82E-02 (+)	1.74E-01 (+)	2.74E-01 (+)	4.51E-01 (+)	4.44E-01 (+)	1.07E-02 (+)	1.53E-01 (+)	1.25E-01 (+)	2.16E-02 (+)	4.69E-01 (+)	2.60E-01 (+)	1.77E-02 (+)	5.14E-01 (+)
F12	2.11E-04 (∞)	3.86E-04 (∞)	1.54E-03 (+)	7.41E-03 (+)	6.66E-02 (+)	3.00E-06 (∞)	1.35E-04 (∞)	3.11E-04 (∞)	3.71E-05 (∞)	5.03E-03 (+)	7.26E-04 (∞)	1.72E-05 (∞)	3.23E-02 (+)
F13	6.73E-05 (∞)	6.95E-03 (+)	1.03E-03 (+)	5.67E-02 (+)	1.20E-01 (+)	1.77E-07 (∞)	4.96E-02 (+)	3.76E-05 (∞)	7.91E-06 (∞)	1.11E-02 (+)	6.21E-05 (∞)	6.81E-06 (∞)	2.90E-02 (+)
F14	5.85E-04 (∞)	1.88E-04 (∞)	3.51E-04 (∞)	8.97E-04 (∞)	1.03E-01 (+)	1.05E-04 (∞)	1.16E-03 (+)	4.30E-04 (∞)	1.69E-04 (∞)	3.68E-03 (+)	3.44E-04 (∞)	9.15E-05 (∞)	3.09E-01 (+)
F15	4.85E-05 (∞)	1.59E-03 (+)	5.54E-04 (∞)	6.86E-03 (+)	1.36E-02 (+)	1.71E-06 (∞)	1.06E-02 (+)	2.51E-05 (∞)	1.58E-05 (∞)	3.99E-03 (+)	2.25E-04 (∞)	6.20E-06 (∞)	2.81E-02 (+)
F16	9.31E-01 (+)	3.18E-01 (+)	4.37E-01 (+)	6.86E-01 (+)	6.19E-01 (+)	3.35E-01 (+)	7.69E-01 (+)	1.13E+00 (≈)	6.94E-01 (+)	1.96E+00 (≈)	9.47E-01 (+)	4.83E-01 (+)	5.43E-01 (+)
F17	8.40E-01 (+)	3.76E-01 (+)	3.87E-01 (+)	6.24E-01 (+)	4.70E-01 (+)	3.34E-01 (+)	4.94E-01 (+)	1.03E+00 (≈)	5.65E-01 (+)	1.62E+00 (≈)	6.83E-01 (+)	4.49E-01 (+)	9.04E-01 (+)
F18	2.04E-03 (+)	9.26E-04 (∞)	2.00E-03 (+)	5.18E-03 (+)	1.28E-01 (+)	6.55E-04 (∞)	4.03E-03 (+)	1.12E-03 (+)	5.82E-04 (∞)	1.85E-02 (+)	3.37E-03 (+)	9.05E-04 (∞)	4.97E-01 (+)
F19	6.66E-05 (∞)	2.02E-05 (∞)	1.68E-04 (∞)	3.44E-03 (+)	5.46E-03 (+)	4.88E-07 (∞)	1.52E-03 (+)	3.44E-05 (∞)	2.46E-05 (∞)	2.36E-03 (+)	4.79E-05 (∞)	1.28E-06 (∞)	1.04E-01 (+)
F20	6.46E-01 (+)	2.33E-01 (+)	3.00E-01 (+)	4.46E-01 (+)	4.29E-01 (+)	2.18E-01 (+)	1.92E-01 (+)	5.50E-01 (+)	4.35E-01 (+)	1.40E+00 (≈)	5.21E-01 (+)	3.84E-01 (+)	3.91E-01 (+)
F21	8.00E-01 (+)	4.03E-01 (+)	3.91E-01 (+)	7.53E-01 (+)	5.80E-01 (+)	3.73E-01 (+)	6.12E-01 (+)	8.23E-01 (+)	6.91E-01 (+)	9.70E-01 (+)	8.93E-01 (+)	4.19E-01 (+)	8.44E-01 (+)
F22	8.59E-01 (+)	5.55E-01 (+)	6.28E-01 (+)	1.00E+00 (≈)	6.89E-01 (+)	3.81E-01 (+)	3.84E-01 (+)	8.26E-01 (+)	7.77E-01 (+)	5.29E-01 (+)	4.43E-01 (+)	4.62E-01 (+)	5.87E-01 (+)
F23	8.53E-01 (+)	3.91E-01 (+)	4.00E-01 (+)	8.25E-01 (+)	5.96E-01 (+)	4.30E-01 (+)	7.70E-01 (+)	8.62E-01 (+)	7.07E-01 (+)	9.23E-01 (+)	8.73E-01 (+)	4.56E-01 (+)	1.01E+00 (≈)
F24	8.00E-01 (+)	4.82E-01 (+)	3.23E-01 (+)	7.96E-01 (+)	6.07E-01 (+)	4.24E-01 (+)	7.86E-01 (+)	8.59E-01 (+)	6.81E-01 (+)	9.33E-01 (+)	8.16E-01 (+)	4.48E-01 (+)	8.39E-01 (+)
F25	7.05E-01 (+)	8.84E-01 (+)	8.81E-01 (+)	1.07E+00 (≈)	9.80E-01 (+)	1.15E-01 (+)	8.38E-01 (+)	7.88E-01 (+)	4.40E-01 (+)	1.02E+00 (≈)	8.94E-01 (+)	2.64E-01 (+)	1.07E+00 (≈)
F26	3.24E-01 (+)	9.60E-02 (+)	1.16E-01 (+)	3.29E-01 (+)	2.99E-01 (+)	1.23E-01 (+)	2.52E-01 (+)	3.45E-01 (+)	2.26E-01 (+)	4.92E-01 (+)	3.66E-01 (+)	1.11E-01 (+)	5.33E-01 (+)
F27	6.87E-01 (+)	3.48E-01 (+)	4.20E-01 (+)	7.55E-01 (+)	5.50E-01 (+)	3.18E-01 (+)	8.02E-01 (+)	6.67E-01 (+)	4.14E-01 (+)	9.75E-01 (+)	7.43E-01 (+)	3.44E-01 (+)	9.78E-01 (+)
F28	5.08E-01 (+)	7.97E-01 (+)	8.58E-01 (+)	1.00E+00 (≈)	9.71E-01 (+)	1.51E-01 (+)	3.51E-01 (+)	5.33E-01 (+)	3.02E-01 (+)	9.96E-01 (+)	6.63E-01 (+)	1.85E-01 (+)	9.98E-01 (+)
F29	3.93E-01 (+)	1.10E-01 (+)	2.00E-01 (+)	3.62E-01 (+)	2.15E-01 (+)	1.28E-01 (+)	4.44E-01 (+)	3.96E-01 (+)	2.37E-01 (+)	7.76E-01 (+)	4.22E-01 (+)	1.59E-01 (+)	4.12E-01 (+)
F30	3.42E-04 (∞)	2.68E-04 (∞)	1.36E-03 (+)	1.36E-02 (+)	3.21E-02 (+)	6.97E-05 (∞)	1.50E-02 (+)	3.96E-04 (∞)	2.45E-04 (∞)	6.75E-03 (+)	4.88E-04 (∞)	1.06E-04 (∞)	4.11E-01 (+)

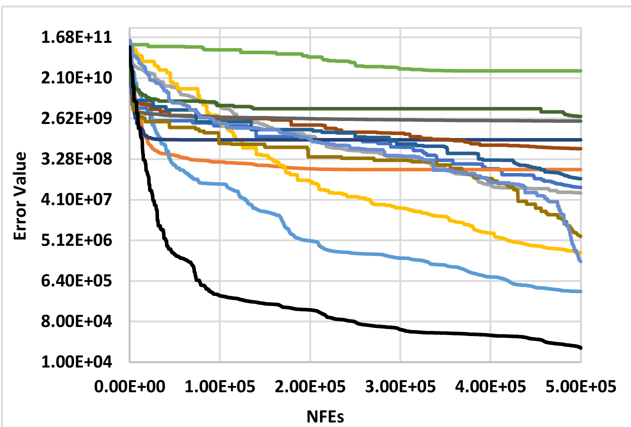

FIGURE 15. Convergence graph for function F12 with 50D.

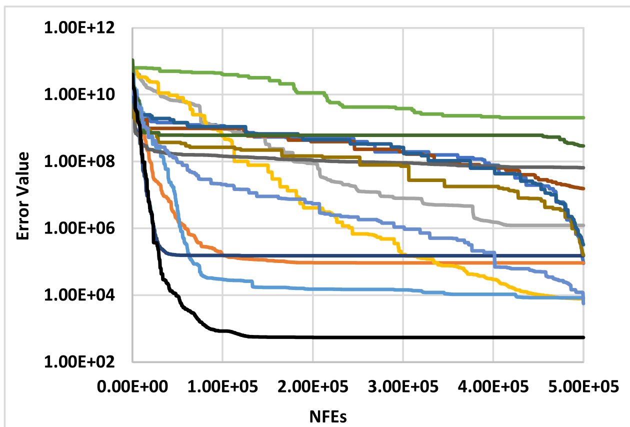

FIGURE 16. Convergence graph for function F13 with 50D.

constant update of fitter solutions from the newer solutions while replacing the older population with fitter new wolves through a greater emphasis on the three leaders at all times such that the newer solutions are not too far off or too near to the dominant wolves to prevent the functions evaluations being futile and fall victim to early entrapment. Fortunately, the higher computational times in SL-GWO can be overcome through the use of parallel computational capabilities since both the population groups operate independently while the other variants of GWO cannot be exploited through the power of parallel computational capabilities.

Table 19 compares the statistical results of SL-GWO with the recent GGWO [26] for 10, 30 and 50 dimensions of the CEC2018 test suite. It is obvious that the performances of SL-GWO and GGWO are quite competitive with each other. However, it is to be noted that GGWO utilized three times higher computational budget and GGWO has a computational complexity (CC) which is three times that of SL-GWO as it generates three new solutions for every member of grey wolf whose fitness is evaluated every iteration to such that the best one of the three solutions survive to make it to the next iteration. SL-GWO on the other hand generates only one new solution vector for every member of the wolfpack thereby reducing its computational requirements to one fitness evaluation for every member in an iteration. Despite the higher computational budget, the performances of SL-GWO are identical and at times and even outperforms the GGWO algorithm for the 50-dimensional case.

TABLE 18. Comparison of the computational times (seconds)of the fourteen algorithms for the CEC2018 benchmarking suite.

		GWO	WOA	HHO	SMA	GTO	ChOA	EGWO	SOGWO	CGWO	IGWO	IGWO-DE	AGWO	MEGWO	SL-GWO
F1	10D	0.720	0.563	0.855	2.580	1.007	5.851	0.954	1.822	2.712	5.593	0.643	2.466	0.588	6.915
	30D	3.693	2.182	3.189	10.770	3.848	44.638	5.777	11.861	5.080	9.828	3.172	15.154	2.116	12.655
	50D	15.668	9.620	13.851	41.448	15.368	194.524	26.253	65.185	18.312	30.122	13.582	71.028	10.558	50.483
F3	10D	0.679	0.493	0.800	2.670	0.965	5.740	0.956	1.839	2.471	2.931	0.588	2.281	0.540	6.134
	30D	3.494	2.153	3.164	10.075	3.851	44.290	5.585	11.933	5.265	9.952	3.044	15.300	2.107	12.865
	50D	15.489	9.359	13.279	41.029	14.966	193.061	25.021	63.180	18.370	29.785	13.497	67.496	10.451	52.742
F4	10D	0.696	0.502	0.758	2.494	0.959	5.649	0.963	1.730	2.496	2.918	0.603	2.314	0.501	6.256
	30D	3.558	2.184	3.064	10.577	3.875	44.454	5.361	11.702	5.191	9.720	3.002	14.941	2.144	13.019
	50D	16.154	9.741	13.889	40.353	14.897	191.146	25.133	62.813	18.400	30.003	13.586	67.258	10.280	56.802
F5	10D	0.747	0.565	0.802	2.533	1.001	5.691	0.962	1.810	2.580	2.957	0.637	2.295	0.590	6.473
	30D	3.802	2.460	3.484	10.663	4.286	44.948	5.981	12.307	5.557	10.481	3.435	15.617	2.637	13.940
	50D	17.425	11.276	15.719	41.476	16.246	195.675	28.560	68.292	20.765	34.028	15.985	71.876	12.781	53.790
F6	10D	0.858	0.726	1.036	2.716	1.205	5.763	1.110	1.930	2.781	3.146	0.927	2.574	0.734	6.573
	30D	5.309	3.862	5.315	11.775	5.556	46.258	7.218	13.296	7.127	11.783	4.758	16.793	3.891	15.263
	50D	25.799	19.765	26.818	54.618	25.246	220.357	37.127	75.366	29.277	41.161	22.951	81.580	20.271	44.922
F7	10D	0.746	0.598	0.842	2.495	1.096	5.848	1.064	1.937	2.561	2.995	0.663	2.383	0.596	7.173
	30D	3.866	2.529	3.631	10.401	3.971	43.400	5.816	11.983	5.594	10.383	3.504	15.052	2.527	13.726
	50D	19.618	12.270	17.600	45.494	17.669	207.139	29.697	72.144	22.815	36.282	16.339	75.542	13.637	54.914
F8	10D	0.729	0.579	0.844	2.598	1.017	5.767	1.007	1.898	2.703	2.988	0.660	2.362	0.601	6.364
	30D	3.819	2.401	3.523	10.387	3.998	43.140	5.696	11.986	5.730	10.188	3.371	15.052	2.449	13.650
	50D	19.737	12.471	17.331	48.486	18.184	215.949	29.704	71.181	22.018	33.907	16.309	75.015	13.048	41.018
F9	10D	0.841	0.618	0.921	2.586	1.078	5.678	1.023	1.898	2.557	2.961	0.667	2.410	0.632	5.265
	30D	3.746	2.533	3.658	10.460	4.099	43.722	5.745	11.782	5.506	10.377	3.380	14.686	2.489	15.516
	50D	21.893	16.346	21.805	48.863	21.369	1533.315	33.563	75.497	26.191	39.198	20.552	77.493	16.444	38.070
F10	10D	0.780	0.613	0.901	2.709	1.095	5.843	1.115	1.882	2.597	3.002	0.696	2.423	0.631	6.546
	30D	4.065	2.920	4.095	10.636	4.381	44.395	6.116	12.412	5.881	10.851	3.812	852.870	3.175	16.249
	50D	20.859	14.569	19.931	47.855	20.080	214.039	29.955	67.744	22.313	32.943	16.916	70.171	13.895	56.365
F11	10D	0.733	0.530	0.774	2.501	1.000	5.645	0.957	1.822	2.521	2.957	0.616	2.225	0.625	6.538
	30D	3.850	2.380	3.468	10.508	3.753	43.472	5.598	11.297	5.175	9.879	3.168	15.039	2.363	13.840
	50D	16.619	10.358	14.913	40.723	15.383	193.874	26.674	63.600	19.566	30.999	14.294	66.793	10.979	53.593
F12	10D	0.745	0.552	0.856	2.650	1.023	5.753	0.982	1.899	2.579	2.999	0.649	2.286	0.612	6.561
	30D	3.933	2.555	3.636	10.278	3.992	44.899	5.801	11.758	5.514	10.108	3.424	15.073	2.604	22.445
	50D	17.715	11.787	16.207	42.137	18.549	205.264	28.427	66.471	22.042	36.556	17.397	77.064	13.964	57.345
F13	10D	0.785	0.567	0.885	2.689	1.014	5.722	1.018	1.851	2.485	2.968	0.643	2.361	0.579	7.198
	30D	3.685	2.371	3.266	9.975	3.816	42.932	5.713	11.525	5.324	9.521	3.252	14.699	2.338	22.336
	50D	18.800	12.472	17.158	47.996	18.643	211.765	27.992	70.294	20.341	32.737	16.659	77.652	12.183	50.652
F14	10D	0.778	0.640	0.879	2.611	1.047	5.666	1.136	1.939	2.615	3.008	0.670	2.325	0.631	6.341
	30D	4.088	2.753	3.837	10.463	4.425	43.779	6.188	12.043	5.916	10.262	3.766	15.342	2.847	21.067
	50D	20.096	13.725	19.078	48.656	19.728	211.489	30.587	68.933	22.739	34.003	17.992	78.204	16.067	43.079
F15	10D	0.725	0.573	0.773	2.634	1.009	5.665	1.001	1.861	2.597	2.954	0.608	2.338	0.553	6.431
	30D	3.513	2.191	3.141	9.971	3.747	42.828	5.423	11.205	5.007	9.472	3.133	14.388	2.206	20.684
	50D	18.560	11.008	16.590	47.288	17.046	210.556	27.828	71.141	22.022	33.441	15.651	74.849	11.891	35.043
F16	10D	0.805	0.607	0.897	2.638	1.022	5.746	1.023	1.904	2.612	2.989	0.662	2.294	0.574	6.704
	30D	3.916	2.451	3.375	10.051	3.969	43.226	5.607	11.676	5.382	10.321	3.263	14.684	2.472	14.739
	50D	19.252	12.368	16.774	46.024	16.805	203.159	28.188	68.334	21.663	33.085	16.970	75.329	12.509	36.699
F17	10D	0.860	0.721	1.001	2.644	1.111	6.129	1.266	2.083	2.719	3.182	0.811	2.576	0.740	6.992
	30D	4.825	3.529	4.758	11.005	5.109	44.877	6.758	12.714	6.468	11.462	4.151	16.127	3.644	16.446
	50D	21.771	15.882	21.773	46.476	21.548	209.033	31.324	72.770	27.784	38.816	21.179	76.549	17.950	57.358
F18	10D	0.711	0.597	0.847	2.720	1.075	5.849	1.053	1.891	2.675	3.012	0.635	2.263	0.595	6.356
	30D	3.855	2.344	3.414	9.993	3.944	43.498	5.569	11.554	5.428	10.066	3.209	14.894	2.357	15.809
	50D	17.847	11.577	15.929	43.007	16.450	199.562	28.033	66.070	20.303	31.575	15.311	70.494	11.695	43.640
F19	10D	1.646	1.488	2.057	3.676	2.041	6.762	1.975	2.856	3.692	4.171	1.612	3.476	1.604	7.430
	30D	10.833	9.373	12.261	17.366	11.137	50.724	12.804	18.715	12.556	18.173	10.431	22.254	591.550	13.207
	50D	48.591	42.884	54.151	75.651	47.981	227.144	58.731	100.841	53.278	67.044	47.672	104.103	44.555	64.627
F20	10D	0.969	0.810	1.069	2.820	1.183	6.129	1.258	2.152	2.903	3.321	0.826	2.776	0.800	6.895
	30D	5.365	3.963	5.218	11.794	5.340	45.547	6.876	13.120	6.743	11.649	4.394	16.388	3.798	20.008
	50D	23.727	17.216	23.822	49.024	23.592	198.041	33.484	71.303	26.124	38.042	21.282	76.787	17.625	51.095
F21	10D	0.975	0.869	1.182	2.902	1.164	6.201	1.313	2.193	2.919	3.355	0.858	2.517	0.853	7.285
	30D	5.941	4.514	5.964	12.296	6.121	45.895	7.830	13.790	7.526	12.383	5.376	17.325	4.552	24.360
	50D	30.327	24.553	31.890	55.751	30.729	213.548	41.505	80.471	36.050	46.982	29.240	84.228	26.184	51.581
F22	10D	1.026	0.912	1.197	3.073	1.272	6.093	1.350	2.109	3.023	3.498	0.946	2.834	0.854	7.385
	30D	6.333	5.028	6.573	12.718	6.397	46.155	8.120	13.921	7.799	13.131	5.961	17.402	5.012	19.072
	50D	32.602	27.350	34.494	55.167	30.341	203.004	40.633	76.807	34.699	45.136	29.751	82.295	26.952	57.478
F23	10D	1.068	0.924	1.295	2.892	1.398	6.403	1.321	2.193	3.090	3.599	1.026	2.811	0.918	7.507
	30D	6.895	5.509	7.352	13.580	7.206	47.751	9.160	15.450	9.206	14.542	6.754	18.470	5.900	19.145
	50D	37.237	30.644	37.761	59.183	36.425	211.972	47.984	85.792	39.792	51.993	36.373	92.815	33.308	56.686
F24	10D	1.152	0.928	1.304	3.184	1.347	6.480	1.412	2.166	3.100	3.543	0.983	2.788	0.947	7.713
	30D	7.749	6.400	7.872	14.636	7.662	48.278	9.286	16.032	9.509	14.735	7.074	19.107	6.112	21.073
	50D	39.946	33.426	41.186	63.035	37.622	208.714	46.298	83.758	41.492	51.377	38.953	1408.386	32.211	51.355
F25	10D	1.035	0.799	1.147	3.014	1.364	6.285	1.209	2.158	2.919	3.452	0.903	2.675	0.825	7.691
	30D	7.013	5.489	7.200	13.787	7.326	48.094	9.152	15.616	9.050	14.279	6.528	18.439	5.587	18.850
	50D	40.935	34.359	42.440	66.389	40.553	221.941	48.729	84.282	44.555	1374.662	37.473	91.857	33.637	57.321
F26	10D	1.139	0.968	1.340	3.119	1.434	6.435	1.370	2.211	2.986	3.477	1.019	2.720	0.965	7.820
	30D	8.543	6.953	9.110	15.316	8.602	49.153	10.378	16.816	10.233	15.479	7.752	19.341	6.928	18.874
	50D	43.422	37.197	48.348	68.983	43.952	234.361	57.941	98.287	50.565	63.351	43.556	95.747	39.551	57.606
F27	10D	1.196	0.999	1.338	3.065	1.487	6.355	1.467	2.375	3.065	3.661	1.188	2.902	1.055	7.634
	30D	9.687	7.936	9.649	16.155	9.473	50.718	11.266	17.805	11.262	16.604	8.576	20.522	7.878	32.060
	50D	51.996	46.278	56.647	78.737	50.660	227.771	61.576	103.048	56.851	67.978	49.589	108.984	48.186	53.394
F28	10D	1.089	0.932	1.201	3.172	1.384	6.238	1.319	2.219	2.948	3.443	1.031	2.810	1.020	7.496
	30D	8.267	6.714	8.838	14.975	8.531	48.902	9.871	16.270	10.174	15.402	8.041	19.928	6.913	21.370

TABLE 18. *(Continued.)* **Comparison of the computational times (seconds)of the fourteen algorithms for the CEC2018 benchmarking suite.**

	50D	51.098	41.968	51.063	73.725	44.779	226.512	59.650	96.553	49.116	61.062	45.770	102.641	44.113	55.183
F29	10D	1.187	0.937	1.301	3.017	1.342	6.156	1.304	2.228	2.952	3.450	0.988	2.813	0.995	7.505
	30D	7.043	5.801	7.514	14.125	7.327	48.354	8.742	15.082	8.988	14.626	6.405	19.864	5.762	17.911
	50D	38.201	30.616	38.751	60.032	33.933	224.117	47.681	84.257	38.211	51.618	33.744	86.677	29.292	51.655
F30	10D	1.911	1.647	2.197	3.830	2.206	7.078	2.237	3.101	3.839	4.384	1.752	3.708	1.774	8.384
	30D	13.857	12.299	15.171	20.626	13.378	54.587	15.703	22.360	15.910	22.103	13.267	26.222	12.711	24.124
	50D	61.751	56.208	70.325	83.566	58.909	237.475	72.499	106.132	62.457	74.904	59.387	116.182	54.963	54.158

TABLE 19. **Comparison of the statistical results of SL-GWO and GGWO for the CEC2018 benchmarking suite.**

	10D					30D					50D				
	SL-GWO (1×CC)		GGWO (3×CC)			SL-GWO (1×CC)		GGWO (3×CC)			SL-GWO (1×CC)		GGWO (3×CC)		
	Mean	Std.	Mean	Std.	Result	Mean	Std.	Mean	Std.	Result	Mean	Std	Mean	Std.	Result
F1	3.93E-10	1.51E-09	0.00E+00	0.00E+00	≈	7.43E-05	2.23E-04	0.00E+00	0.00E+00	≈	7.84E-02	1.83E-01	4.24E+02	3.67E+02	+
F3	1.17E-13	1.59E-13	0.00E+00	0.00E+00	≈	6.77E-01	8.05E-01	0.00E+00	0.00E+00	-	1.70E+02	1.40E+02	0.00E+00	0.00E+00	-
F4	1.39E-10	2.80E-10	0.00E+00	0.00E+00	≈	6.12E+01	3.15E+01	1.36E+00	1.95E+00	-	9.94E+01	3.42E+01	1.09E+01	1.33E+01	-
F5	5.98E+00	2.50E+00	2.68E+00	1.29E+00	-	4.47E+01	1.25E+01	6.52E+01	1.05E+01	+	9.41E+01	1.03E+01	1.30E+02	2.57E+01	+
F6	4.62E-05	9.91E-05	0.00E+00	0.00E+00	-	5.31E-03	4.55E-03	2.24E-03	2.32E-03	≈	8.80E-02	5.82E-02	4.87E-01	3.37E-01	+
F7	1.50E+01	2.63E+00	1.20E+01	2.01E+00	≈	7.86E+01	7.86E+00	9.80E+01	1.18E+01	≈	1.64E+02	1.73E+01	2.10E+02	2.45E+01	+
F8	5.12E+00	2.35E+00	2.28E+00	7.97E-01	-	6.24E+01	1.95E+01	5.53E+01	1.22E+01	≈	9.38E+01	2.41E+01	1.63E+02	2.64E+01	+
F9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	≈	3.23E+00	2.28E+00	1.43E+00	7.15E-01	≈	3.24E+01	2.96E+01	6.79E+01	4.41E+01	+
F10	1.68E+02	1.11E+02	1.15E+02	9.50E+01	-	3.73E+03	9.43E+02	2.97E+03	3.72E+02	-	5.86E+03	4.86E+02	5.73E+03	4.98E+02	-
F11	1.80E+00	1.90E+00	4.97E-02	2.25E-01	-	3.62E+01	1.59E+01	5.61E+01	2.49E+01	+	9.19E+01	2.99E+01	1.45E+02	3.17E+01	+
F12	1.34E+02	2.94E+01	1.22E+01	5.84E+00	-	9.91E+03	8.79E+03	9.04E+02	4.53E+02	-	5.94E+04	4.99E+04	1.17E+04	4.80E+03	-
F13	2.87E+00	2.50E+00	2.17E-01	3.03E-01	≈	9.26E+01	4.29E+01	4.10E+01	1.31E+01	-	1.76E+03	2.14E+03	1.34E+03	7.93E+02	-
F14	2.09E+00	1.36E+00	5.47E-01	5.07E-01	≈	4.76E+01	9.69E+00	2.41E+01	7.89E+00	-	1.36E+02	6.40E+01	5.88E+01	1.11E+01	-
F15	8.39E-01	6.38E-01	2.21E-02	8.87E-03	≈	5.81E+01	3.36E+01	1.41E+01	3.57E+00	-	1.82E+02	4.67E+01	3.26E+02	2.16E+02	+
F16	7.68E-01	3.54E-01	2.96E-01	1.42E-01	≈	3.23E+02	2.28E+02	3.29E+02	1.01E+02	≈	1.15E+03	1.92E+02	8.60E+02	1.94E+02	-
F17	1.41E+00	6.01E-01	8.68E-01	3.88E-01	≈	8.43E+01	3.65E+01	8.97E+01	4.37E+01	-	7.86E+02	2.65E+02	6.72E+02	1.58E+02	-
F18	6.41E-01	4.96E-01	1.76E-01	7.80E-02	≈	2.97E+02	1.40E+02	2.41E+01	1.19E+00	-	4.75E+03	1.44E+03	1.16E+02	2.59E+01	-
F19	3.46E-02	3.47E-02	4.07E-02	2.41E-02	≈	2.72E+01	7.66E+00	8.63E+00	2.14E+00	-	4.78E+01	1.77E+01	8.86E+01	6.24E+00	-
F20	4.53E-01	5.88E-01	7.80E-02	1.38E-01	≈	1.58E+02	6.79E+01	7.10E+01	5.05E+01	-	4.14E+02	1.01E+02	4.07E+02	1.51E+02	≈
F21	1.19E+02	3.87E+01	1.36E+02	5.10E+01	+	2.46E+02	1.19E+01	2.50E+02	1.08E+01	+	2.92E+02	1.66E+01	3.27E+02	1.82E+01	+
F22	9.43E+01	2.61E+01	9.65E+01	1.74E+01	+	1.00E+02	1.91E-11	1.00E+02	2.02E-13	≈	5.45E+03	3.93E+03	5.67E+03	1.99E+03	+
F23	3.07E+02	1.78E+00	3.04E+02	1.74E+00	≈	3.92E+02	1.27E+01	4.25E+02	1.40E+01	+	5.16E+02	1.97E+01	5.88E+02	3.77E+01	+
F24	2.58E+02	1.16E+02	2.73E+02	1.03E+02	+	4.45E+02	1.38E+01	4.92E+02	1.39E+01	+	5.60E+02	2.08E+01	6.77E+02	2.71E+01	+
F25	4.17E+02	2.28E+01	4.13E+02	2.21E+01	≈	3.83E+02	6.57E+00	3.86E+02	1.29E+00	+	5.68E+02	3.57E+01	5.29E+02	3.66E+01	≈
F26	2.93E+02	2.58E+01	3.00E+02	0.00E+00	+	1.06E+03	5.36E+02	1.61E+03	5.14E+02	+	9.70E+02	8.66E+02	3.29E+03	4.68E+02	+
F27	3.79E+02	2.10E+00	3.90E+02	1.48E+00	≈	5.00E+02	2.42E-04	5.14E+02	7.98E+00	+	5.09E+02	2.76E+01	5.83E+02	2.78E+01	+
F28	3.99E+02	9.52E+01	3.00E+02	0.00E+00	-	3.55E+02	4.84E+01	3.10E+02	3.17E+01	≈	4.95E+02	2.05E+01	4.70E+02	1.35E+01	-
F29	2.47E+02	7.19E+00	2.27E+02	2.23E+00	-	4.90E+02	7.01E+01	5.12E+02	3.16E+01	+	5.11E+02	1.47E+02	9.09E+02	2.00E+02	+
F30	4.06E+02	2.65E+02	4.00E+02	1.62E+00	≈	1.93E+03	2.71E+03	1.99E+03	1.56E+01	+	2.35E+04	5.32E+04	6.08E+05	3.05E+04	+

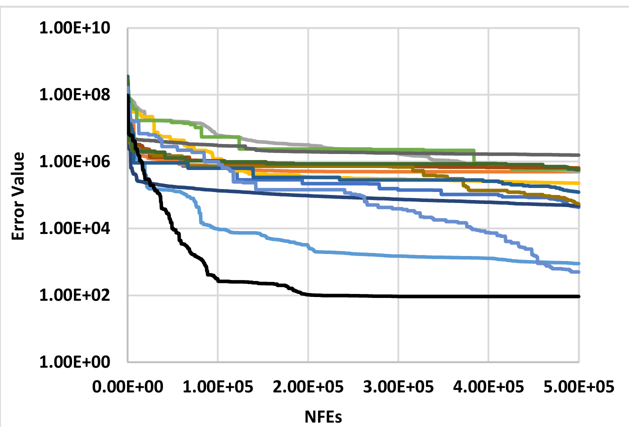

FIGURE 17. Convergence graph for function F14 with 50D.

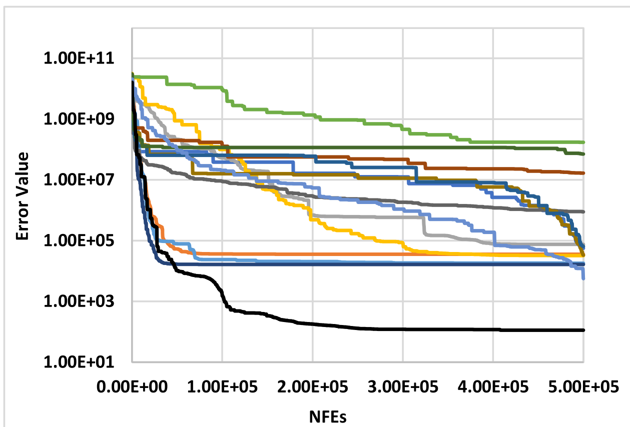

FIGURE 18. Convergence graph for function F15 with 50D.

E. PERFORMANCE ANALYSIS WITH CEC2019 BENCHMARK FUNCTIONS (TEST FOR LOCAL MINIMA AVOIDANCE)

The 2019 Special Session and Competition on Single Objective Numerical Optimization introduced the 100-Digit Challenge, which required the minimization of ten special functions (having the global optimum fitness value of "1") with restricted control parameter "tuning" for each function [14]. The test functions were methodically constructed with several local optima and a single global optimum solution to

TABLE 20. Description of the 10 CEC2019 benchmark functions (composition functions) used to determine the algorithms' ability to avoid local entrapment.

Function No.	Function	$F_i^*=F_i(X^*)$	Dimensions	Search Range	Properties
C1	Storn's Chebyshev Polynomial Fitting Problem	1	9	[-8192, 8192]	• Multimodal with one global minimum • Very highly conditioned • Non-separable; fully parameter-dependent
C2	Inverse Hilbert Matrix Problem	1	16	[-16384, 16384]	• Multi-modal with one global minimum • Highly conditioned • Non-separable; fully parameter-dependent
C3	Lennard-Jones Minimum Energy Cluster Problem	1	18	[-4,4]	• Multi-modal with one global minimum • Non-separable; fully parameter-dependent
C4	Shifted and Rotated Rastrigin's Function	1	10	[-100,100]	• Multi-modal • Non-separable • Local optima's number is huge and the penultimate optimum is far from the global optimum.
C5	Shifted and Rotated Griewank's Function	1	10	[-100,100]	• Multi-modal • Non-separable
C6	Shifted and Rotated Weierstrass Function	1	10	[-100,100]	• Multi-modal • Non-separable • Local optima's number is huge
C7	Shifted and Rotated Schwefel's Function	1	10	[-100,100]	• Multi-modal • Non-separable • Local optima's number is huge
C8	Shifted and Rotated Expanded Schaffer's F6 Function	1	10	[-100,100]	• Multi-modal • Non-separable • Local optima's number is huge
C9	Shifted and Rotated Happy Cat Function	1	10	[-100,100]	• Multi-modal • Non-Separable
C10	Shifted and Rotated Ackley Function	1	10	[-100,100]	• Multi-modal • Non-Separable

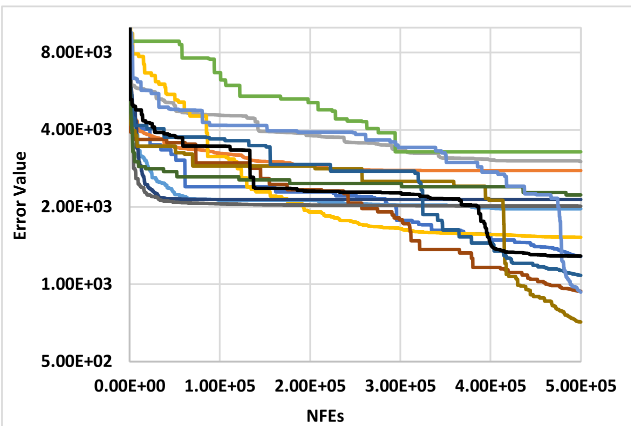

FIGURE 19. Convergence graph for function F16 with 50D.

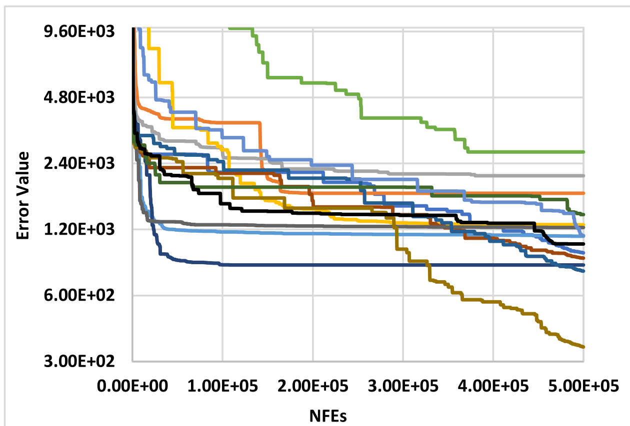

FIGURE 20. Convergence graph for function F17 with 50D.

guarantee that the exploratory ability and avoidance of local minima are put to the test. As with the previous CEC session's composition functions, the CEC 2019 benchmark suite includes difficult exploration circumstances with their landscapes shifted and rotated to further confound an algorithm's search process. It is worth noting that these functions make it extremely difficult for any global optimization algorithm to determine the global optimal solution because their formulation is intended to trap algorithms in local best positions, which is especially true for algorithms with a tendency to converge to the search landscape's central point. Additionally, these issues have a huge number of dimensions, making the search process even more complicated, and only algorithms with a strong exploratory inclination of the whole search space can discover the global optimal solution or solutions that are near to it.

The description of the CEC2019 benchmarking suite is shown in Table 20. The benchmarking results (best, worst, mean and standard deviation) are shown in Table 21, the results of Wilcoxon's rank-sum test are shown in Table 22, followed by the results of Friedman's non-parametrical test are shown in Table 23, and the mean absolute errors (MAE) for all the fifteen algorithms are given in Table 24. Furthermore, the average computational times are shown in

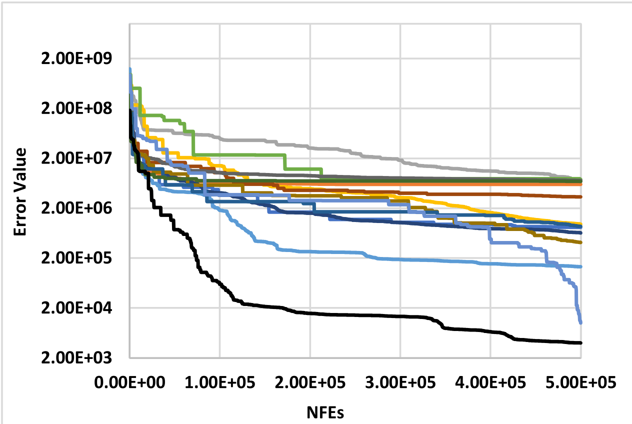

FIGURE 21. Convergence graph for function F18 with 50D.

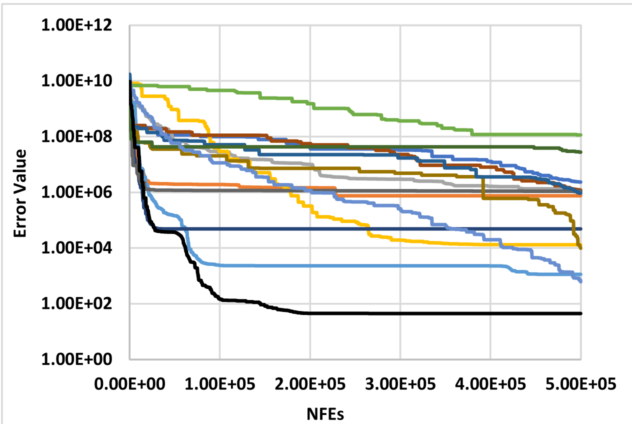

FIGURE 22. Convergence graph for function F19 with 50D.

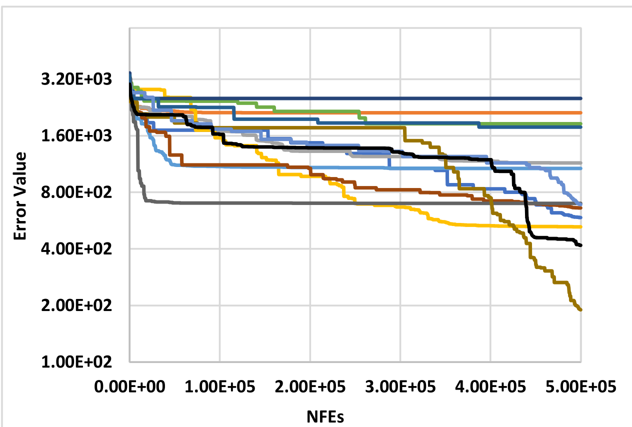

FIGURE 23. Convergence graph for function F20 with 50D.

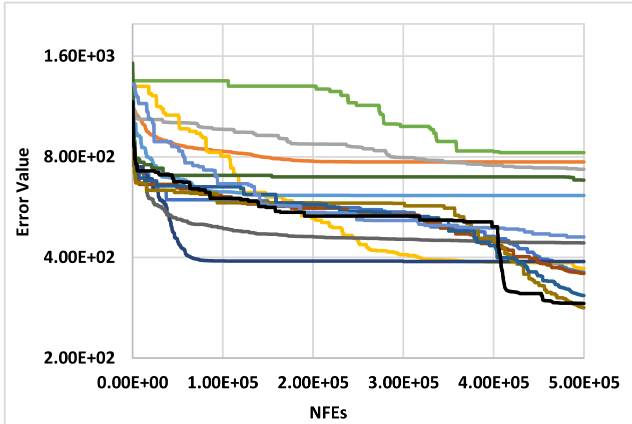

FIGURE 24. Convergence graph for function F21 with 50D.

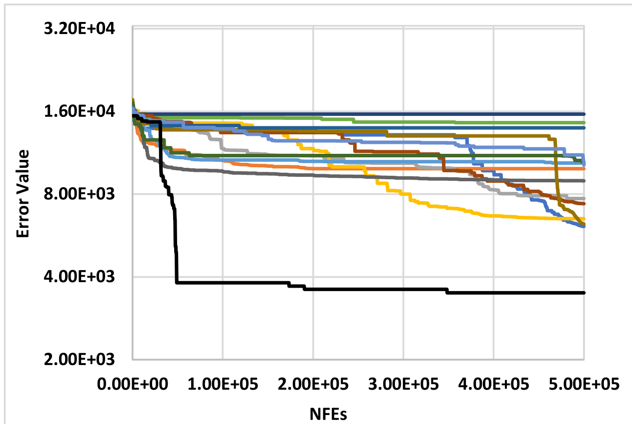

FIGURE 25. Convergence graph for function F22 with 50D.

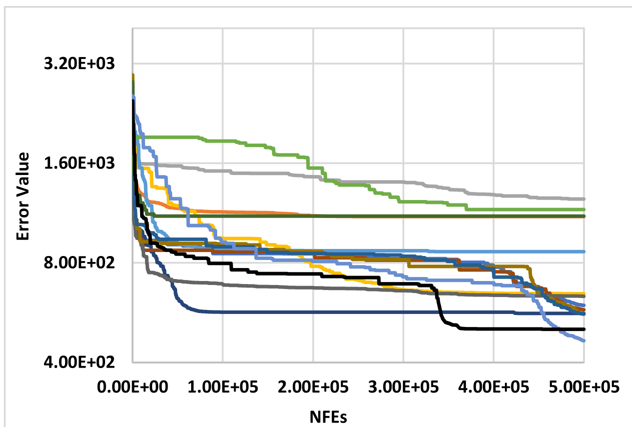

FIGURE 26. Convergence graph for function F23 with 50D.

Table 25 and the acceleration rates comparing SL-GWO with the competitor algorithms in Table 26. The NFEs are set to 1,000,000 (1E+06), the population size is set to 500 and the NFEs are chosen as the termination criteria for all the algorithms in the test bench.

- It is evident from Tables 21, 22, 23 and 24, that SL-GWO had the best optimal fitness for F1, F4, F5, F7, F8, F9 and F10. It is indicative of the algorithm's exploratory and local minima avoidance capabilities. The proposed method generated solutions closer to the global optimal solutions for F4, F5, F6, F7, F8 and F10. For the

TABLE 21. The values of best, worst, mean and the standard deviation of the fourteen algorithms for the CEC2019 benchmark functions.

		GWO	WOA	HHO	SMA	GTO	ChOA	EGWO	SOGWO	CGWO	IGWO	IGWO-DE	AGWO	MEGWO	SL-GWO
F1	Best	1*	5.182617	**1***	**1***	**1***	1*	1*	1*	1*	1*	1*	**1***	**1***	**1***
	Worst	127.4078	404.1862	**1***	**1***	**1***	155.1066	389.1412	191.3974	277.0857	54.10603	178.1388	**1***	**1***	**1***
	Mean	22.52426	128.8238	**1***	**1***	**1***	24.64043	86.88555	33.12739	45.85922	8.499513	18.22008	**1***	**1***	**1***
	Std.	40.60275	122.5669	**0**	**0**	**0**	45.03129	122.3749	53.55249	83.35811	16.35614	47.01046	**0**	**9.90E-08**	**0**
F2	Best	11.97762	2865.218	4.46541	4.215963	4.062343	70.93859	221.8733	80.13705	204.7159	124.8515	79.44772	4.880345	33.02547	**1.981103**
	Worst	408.2154	15775.06	5	4.322536	4.350098	1586.731	640.45	431.869	1188.965	325.6898	480.1753	602.2483	171.1883	**3.14342**
	Mean	206.8311	7173.363	4.767045	4.264915	4.252689	718.2843	426.0879	183.5043	697.6898	185.1859	228.2978	175.8705	121.142	**2.451158**
	Std.	119.4006	3508.898	0.234985	0.031388	0.064289	501.1633	122.8011	93.8092	291.672	56.0391	112.1118	188.7635	35.60881	**0.439519**
F3	Best	1.000022	1.409135	**1***	1.40915	1.409135	2.959369	1.409135	1.000012	1.409263	1.000037	1.00005	1.022774	1.000014	**1***
	Worst	5.130621	1.409173	**3.080261**	4.608218	1.409135	5.247427	10.56709	4.608053	9.704288	1.410602	6.31403	2.472208	3.140395	**1.409135**
	Mean	1.528463	1.409143	**1.646975**	2.475325	1.409135	3.819341	5.482546	1.857715	3.631941	1.10932	2.189343	1.720913	1.562622	**1.327308**
	Std.	1.016893	1.27E-05	**0.519969**	1.560618	2.48E-15	0.62499	3.970843	1.078453	2.560347	0.187517	1.685857	0.40069	0.641357	**0.169398**
F4	Best	**1.914893**	14.92947	10.02817	2.989946	7.964713	25.29679	5.045748	3.015361	5.043363	2.044942	2.014357	5.219578	**1.996759**	**1.984877**
	Worst	**9.954955**	64.67699	59.74784	12.93958	35.49558	49.09714	38.16105	14.66971	16.92434	14.32964	17.9305	37.26582	**7.930515**	**6.969754**
	Mean	**6.203545**	41.57473	33.99209	7.794235	18.82155	35.35297	16.24607	7.553902	11.00004	5.851114	8.667957	20.74565	**5.105148**	**5.375467**
	Std.	**2.068601**	10.6304	14.57824	3.494949	7.603824	6.194415	10.53652	3.587578	3.722766	4.094981	4.214381	8.346737	**2.206822**	**1.586334**
F5	Best	1.099373	1.125771	1.277282	1.052248	1.081209	2.307962	1.073786	1.106539	1.16319	1.095784	1.073726	1.170442	**1.033847**	**1.031972**
	Worst	1.658745	2.523525	2.243491	1.322431	1.351684	30.41824	1.884674	1.747378	1.883391	1.484234	1.681806	3.44604	**1.086952**	**1.14274**
	Mean	1.285944	1.486617	1.841541	1.178286	1.18975	9.130439	1.407972	1.342985	1.473901	1.366588	1.36296	1.728803	**1.062952**	**1.067111**
	Std.	0.199344	0.375308	0.262796	0.0721	0.069092	8.507033	0.303178	0.205421	0.19166	0.096875	0.171443	0.53538	**0.016681**	**0.03374**
F6	Best	1.048533	2.603241	2.877999	1.517307	1.131663	5.249032	1.002057	1.069189	1.718328	1.081448	1.092244	1.450882	1.095667	**1***
	Worst	3.152365	9.716595	8.843802	5.197858	5.673642	7.845568	4.08939	1.775424	5.191097	3.579961	3.111812	6.090753	3.153516	**5.00781**
	Mean	1.489345	6.430797	6.44891	3.451665	2.683345	6.282463	2.156465	1.26924	2.711985	1.36396	1.542668	2.516059	2.351599	**2.347113**
	Std.	0.621879	2.232808	1.535766	1.256909	1.173143	0.722238	1.057496	0.164639	1.019018	0.642608	0.505781	1.152387	1.310052	**1.06983**
F7	Best	19.85909	510.2021	277.0856	112.3819	276.6657	886.0968	83.02305	4.740047	4.629285	**1.53627**	119.8185	386.337	139.4641	**1.727233**
	Worst	722.5952	1705.861	1168.896	649.0355	1007.909	1358.547	1944.025	679.3928	1662.16	**644.9884**	1052.256	1422.255	655.9564	**340.3059**
	Mean	319.2437	1062.852	791.1472	342.3093	670.8519	1174.873	943.3592	393.5033	638.9174	**173.2459**	356.0407	841.2187	398.0469	**192.2208**
	Std.	186.286	334.1775	266.1401	186.3834	244.6345	164.9668	707.0831	192.5807	402.1895	**233.3579**	278.5072	262.953	166.7969	**106.7745**
F8	Best	1.262403	3.759314	3.591678	1.523856	2.721658	4.105131	1.317224	1.678525	2.196284	**1.154578**	1.225363	2.764922	1.78815	1.289127
	Worst	3.988731	4.588582	4.612167	3.546137	4.152813	4.989725	4.490767	3.544838	4.532701	**3.612046**	3.570196	4.47043	3.617594	3.656607
	Mean	2.757833	4.214821	4.141289	2.580959	3.535584	4.464796	2.870556	2.874393	3.439589	**1.791706**	2.398738	3.558824	2.575856	2.25857
	Std.	0.668974	0.291747	0.289994	0.49394	0.344145	0.275208	0.821073	0.566501	0.757603	**0.677034**	0.724678	0.481475	0.524463	0.602363
F9	Best	1.043286	1.271269	1.17035	1.058974	1.063212	1.206626	1.041786	1.033019	1.032972	1.034873	1.037921	1.093225	1.046624	**1.024161**
	Worst	1.126709	1.582216	1.501497	1.179999	1.176593	1.438677	1.155371	1.109695	1.299707	1.105061	1.16128	1.263727	1.105879	**1.095906**
	Mean	1.072743	1.3871	1.358672	1.102944	1.114161	1.323553	1.085021	1.057132	1.134912	1.070087	1.075488	1.149773	1.072324	**1.053311**
	Std.	0.020898	0.097244	0.102611	0.032829	0.03286	0.076296	0.029732	0.019741	0.069346	0.020197	0.033731	0.051609	0.019384	**0.025311**
F10	Best	2.186017	20.99715	20.91834	21.00149	**1***	21.18278	21.34208	1.562957	21.13324	1.035228	21.18056	21.12274	21.10494	**1***
	Worst	21.2991	21.17644	21.03848	21.03069	**21.13345**	21.34976	21.59065	21.29114	21.32242	21.39192	21.3819	21.34798	21.33211	**21.08455**
	Mean	19.96245	21.03511	20.99408	21.01054	**12.02608**	21.27331	21.46481	19.90384	21.24371	14.52755	21.28201	21.24977	21.20804	**10.77972**
	Std.	4.918128	0.060643	0.025206	0.009764	**10.02955**	0.045383	0.077896	5.074175	0.05319	9.865409	0.061395	0.067144	0.059673	**9.975035**

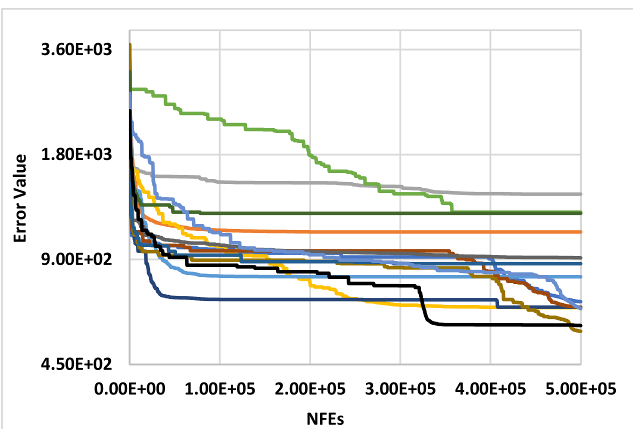

FIGURE 27. Convergence graph for function F24 with 50D.

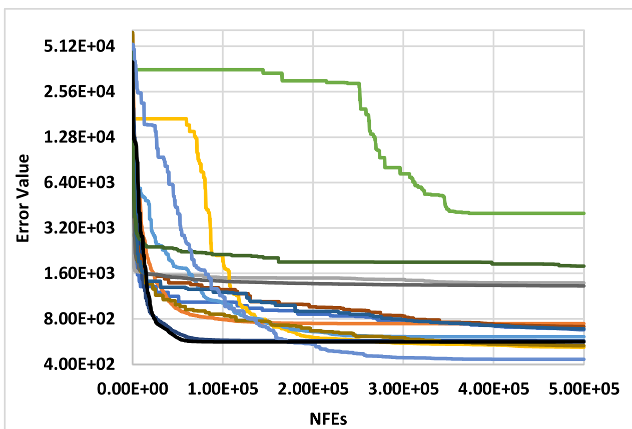

FIGURE 28. Convergence graph for function F25 with 50D.

functions F2, F4 and F5, the prosed method outperformed the other variants of GWO and the modern meta-heuristics.

- Function F2 proved to be the most challenging for all the meta-heuristics in comparison except SL-GWO. All the algorithms were victims of local entrapment at some point or the other during the course of exploration. F2 also has a higher dimensionality at 16 dimensions and its extreme multi-modality resulted in several optimizers including IGWO, MEGWO etc. being entrapped despite

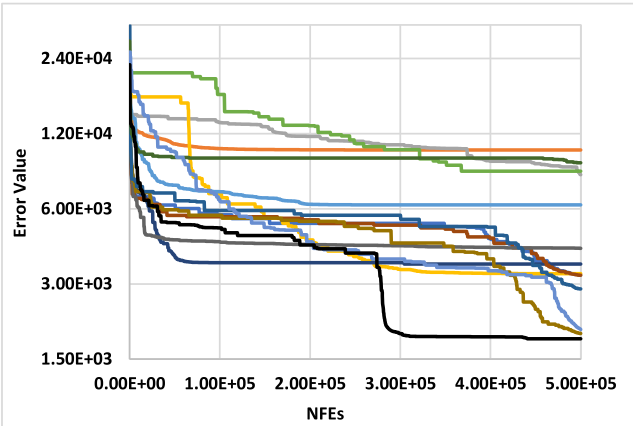

FIGURE 29. Convergence graph for function F26 with 50D.

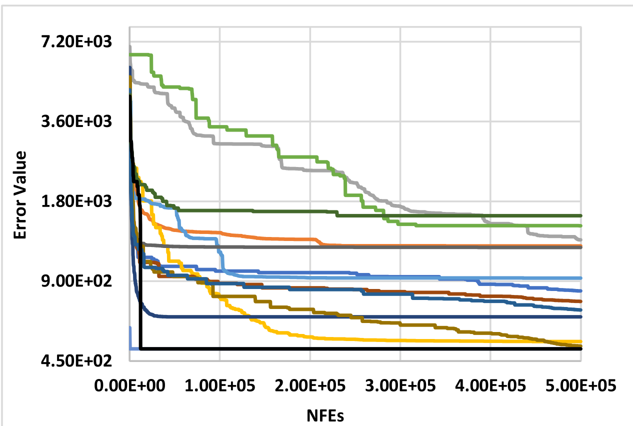

FIGURE 30. Convergence graph for function F27 with 50D.

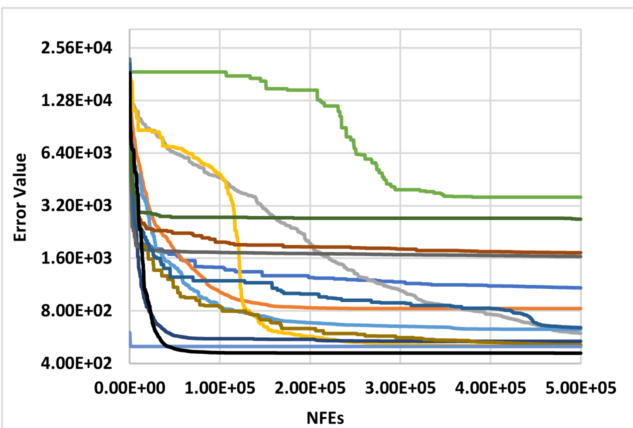

FIGURE 31. Convergence graph for function F28 with 50D.

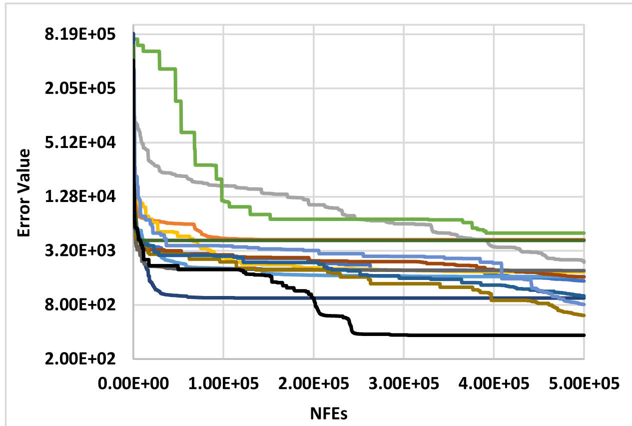

FIGURE 32. Convergence graph for function F29 with 50D.

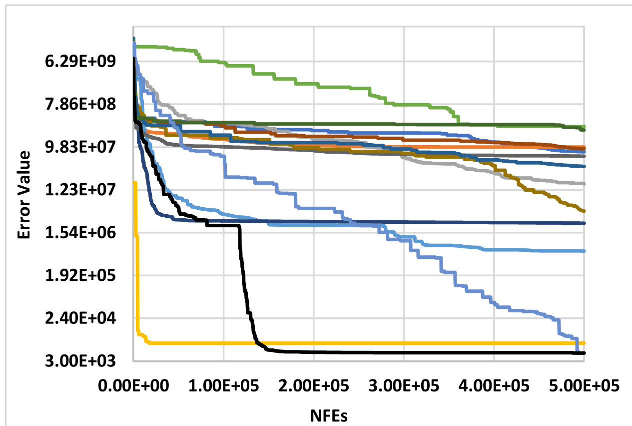

FIGURE 33. Convergence graph for function F30 with 50D.

FIGURE 34. Legend depicting the various algorithms.

their competitive performance in the previous test suite. SL-GWO's entrapment evasion capabilities are demonstrated as it generated optimal fitness scores closer to the global best fitness. The linear control strategy in SL-GWO enables a smoother transition from exploration to exploitation while the adaptive strategy controls the exploration and adapts in a dynamic manner to prevent stagnation. The linear control strategy also forces exploration at the initial stages of the search thereby preventing premature convergence and the adaptive control strategy guides the wolves to the most promising and diverse areas within the search landscape.

TABLE 22. Results of the Wilcoxon's rank sum test comparing SL-GWO with the ten modern meta-heuristics for the CEC2019 benchmarking suite.

SL-GWO vs.	R+	R-	*p* value	Result
GWO	54	1	2.12E-02	+
WOA	55	0	7.57E-03	+
HHO	54	0	3.64E-02	+
SMA	54	0	4.06E-02	+
GTO	54	0	4.96E-02	≈
ChOA	55	0	2.57E-02	+
EGWO	54	1	8.90E-03	+
SOGWO	54	1	2.73E-02	+
CGWO	55	0	6.40E-03	+
IGWO	45	10	4.27E-02	≈
IGWO-DE	54	1	2.12E-02	+
AGWO	54	0	3.64E-02	+
MEGWO	52	3	5.71E-02	≈

TABLE 23. Ranking the algorithms based on Friedman's rank for the CEC2020 benchmarking suite.

Rank	Algorithm	Friedman's rank
1.	SL-GWO	3.151267
2.	IGWO	4.659744
3.	MEGWO	5.223812
4.	SOGWO	6.018998
5.	GWO	6.046313
6.	SMA	6.276025
7.	GTO	6.779703
8.	IGWO-DE	7.640386
9.	AGWO	9.885268
10.	HHO	9.954687
11.	EGWO	10.26261
12.	CGWO	10.59836
13.	WOA	12.50936
14.	ChOA	13.35510

TABLE 24. Ranking the algorithms based on Mean Absolute Errors for the CEC2019 benchmarking suite.

Rank	Algorithm	Mean Absolute Error
1.	SL-GWO	21.98805
2.	SMA	38.71681
3.	IGWO	39.40117
4.	MEGWO	55.51275
5.	GWO	58.28993
6.	IGWO-DE	64.10778
7.	SOGWO	64.59942
8.	GTO	71.68842
9.	HHO	86.73379
10.	AGWO	107.0759
11.	CGWO	142.7102
12.	EGWO	150.7046
13.	ChOA	199.9445
14.	WOA	844.2578

- Besides F2, the function F7 was also one the most challenging and all the other algorithms generated local optimal solutions. SL-GWO had the best performance for F1, F3, F6 and F10 with the perfect 10-digit accuracy while none of the other algorithms had the perfect 10 digits for multiple functions.
- Competitive performances were observed from the functions F4, F5, F8 and F9 with SL-GWO and I-GWO being the most successful candidates. While the CEC2019 test suite does not pose any restrictions on the NFEs, it imposes restrictions on the tuning settings and it can help analyze the sensitivity of the tuning parameters to extreme search landscapes. The results demonstrate that the dynamic tuning from SL-GWO is quite efficient for complex landscapes and can provide results with good accuracy.
- The effect of additional NFEs or increasing population count produced no major improvements in the performance of most algorithms and the modification in the tuning parameters has been proven to be ineffectual for such complicated search landscapes of composition functions. The same NFEs count has been useful in identifying how quickly does an algorithm adapt to escape local entrapment and it was fairly obvious that a lack of diversifying measures leads to entrapment at a very quick point in the timeframe of exploration. Although several articles have demonstrated that the global optimal solutions are attainable through multiple tuning settings each unique to different functions, the current work does not modify or suggest multiple tuning modifications to suit the functions' landscape. Instead, all the algorithms have the same tuning settings specified earlier and no modifications have been enforced to ensure that a fair comparison has been made.
- Although the NFEs were higher for the CEC2019 test suite compared to the CEC2018 suite, most modern meta-heuristics fail to exploit the advantage with higher population size and quickly get entrapped and this proves that simple strategies with limited adaptive tuning can be detrimental despite higher computational budgets.
- The MAE for SL-GWO has been the least as it managed to provide decent performances across all the test functions. From the rankings, it can be inferred that SL-GWO is effective at handling complex search landscapes with a good tendency for exploration and solution intensification given that no additional tuning is required compared to the other algorithms.

The analysis of variance (ANOVA) through box-plots for all the CEC2019 benchmark functions are shown in Figures 35 to 44 for all the 10 functions.

F. PERFORMANCE ANALYSIS WITH STANDARD CONSTRAINED ENGINEERING PROBLEMS THE ALGORITHMS

The analysis of the performance of the proposed method for constrained engineering problems is carried out in this sub-section to determine its ability to generate feasible solutions with the stipulated computational budget. The performance of the fourteen competitor algorithms has been considered in comparative analysis and five different

TABLE 25. Comparison of the computational times (seconds) of the fourteen algorithms for the CEC2019 benchmarking suite.

	GWO	WOA	HHO	SMA	GTO	ChOA	EGWO	SOGWO	CGWO	IGWO	IGWO-DE	AGWO	MEGWO	SL-GWO
F1	14.41322	13.22378	20.82525	204.36615	41.65616	772.21947	19.09377	196.48026	28.51894	83.53441	15.42615	64.93453	13.57319	70.44380
F2	9.98585	7.06617	15.36321	198.92277	37.25928	1171.3287	15.01546	200.72437	22.48061	77.90645	9.09906	84.77340	7.14694	90.92580
F3	10.46235	6.65447	15.05662	191.09181	36.66658	1273.6538	15.70913	218.02557	22.44417	73.58164	8.85561	90.48031	6.75713	133.33570
F4	8.65598	6.82317	13.53496	178.02529	33.81653	667.60317	11.37519	162.37803	19.75126	69.33148	7.90850	54.57839	6.73738	90.54891
F5	8.76581	6.65706	13.26636	175.60345	32.55657	658.09461	11.50291	168.94326	19.96565	65.93235	8.35175	53.46695	6.68349	125.72560
F6	57.16296	55.09202	72.26711	217.36696	80.40800	675.10235	59.28614	208.05518	67.05704	113.80656	56.94085	101.65332	56.59949	122.11510
F7	10.43264	8.43043	16.32549	202.11850	38.26853	768.62253	14.20587	192.90859	24.32614	84.57048	10.26599	63.45835	8.66233	95.93770
F8	10.94480	8.06971	16.21372	204.15592	38.17012	742.59542	13.37612	193.68929	23.68946	78.94820	9.39206	63.07061	7.70815	83.38470
F9	9.77255	8.08081	15.35344	202.77891	36.94762	761.74998	13.15814	197.55741	23.16544	77.46679	9.23900	63.78240	7.46841	79.07470
F10	10.40679	8.18307	16.88364	206.48628	37.44194	744.45306	13.20015	189.29072	23.16847	80.65169	9.58965	62.97713	7.94259	95.47150

TABLE 26. Comparison of the acceleration rates of SL-GWO with the thirteen competitor algorithms for the CEC2019 benchmarking suite.

SL-GWO Vs.													
	GWO	WOA	HHO	SMA	GTO	ChOA	EGWO	SOGWO	CGWO	IGWO	IGWO-DE	AGWO	MEGWO
F1	0.0444 (+)	0.0078 (∞)	1.0000 (≈)	1.0000 (≈)	1.0000 (≈)	0.0406 (+)	0.0115 (+)	0.0302 (+)	0.0218 (+)	0.1177 (+)	0.0549 (+)	1.0000 (≈)	1.0000 (≈)
F2	0.0119 (+)	0.0003 (∞)	0.5142 (+)	0.5747 (+)	0.5764 (+)	0.0034 (∞)	0.0058 (∞)	0.0134 (+)	0.0035 (∞)	0.0132 (+)	0.0107 (+)	0.0139 (+)	0.0202 (+)
F3	0.8684 (+)	0.9419 (≈)	0.8059 (+)	0.5362 (+)	0.9419 (≈)	0.3475 (+)	0.2421 (+)	0.7145 (+)	0.3655 (+)	1.1965 (-)	0.6063 (+)	0.7713 (+)	0.8494 (+)
F4	0.8665 (+)	0.1293 (+)	0.1581 (+)	0.6897 (+)	0.2856 (+)	0.1521 (+)	0.3309 (+)	0.7116 (+)	0.4887 (+)	0.9187 (≈)	0.6202 (+)	0.2591 (+)	1.0530 (-)
F5	0.8298 (+)	0.7178 (+)	0.5795 (+)	0.9056 (≈)	0.8969 (+)	0.1169 (+)	0.7579 (+)	0.7946 (+)	0.7240 (+)	0.7809 (+)	0.7829 (+)	0.6173 (+)	1.0039 (≈)
F6	1.5759 (-)	0.3650 (+)	0.3640 (+)	0.6800 (+)	0.8747 (+)	0.3736 (+)	1.0884 (≈)	1.8492 (-)	0.8655 (+)	1.7208 (-)	1.5215 (-)	0.9329 (≈)	0.9981 (≈)
F7	0.6021 (+)	0.1809 (+)	0.2430 (+)	0.5615 (+)	0.2865 (+)	0.1636 (+)	0.2038 (+)	0.4885 (+)	0.3009 (+)	1.1095 (-)	0.5399 (+)	0.2285 (+)	0.4829 (+)
F8	0.8190 (+)	0.5359 (+)	0.5454 (+)	0.8751 (+)	0.6388 (+)	0.5059 (+)	0.7868 (+)	0.7858 (+)	0.6566 (+)	1.2606 (-)	0.9416 (≈)	0.6346 (+)	0.8768 (+)
F9	0.9819 (≈)	0.7594 (+)	0.7753 (+)	0.9550 (≈)	0.9454 (≈)	0.7958 (+)	0.9708 (≈)	0.9964 (≈)	0.9281 (≈)	0.9843 (≈)	0.9794 (≈)	0.9161 (≈)	0.9823 (≈)
F10	0.5400 (+)	0.5125 (+)	0.5135 (+)	0.5131 (+)	0.8964 (+)	0.5067 (+)	0.5022 (+)	0.5416 (+)	0.5074 (+)	0.7420 (+)	0.5065 (+)	0.5073 (+)	0.5083 (+)

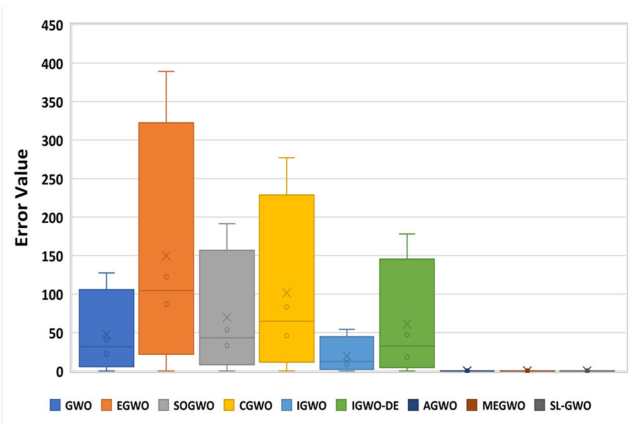

FIGURE 35. Box-plot for function F1 from CEC2019 test suite.

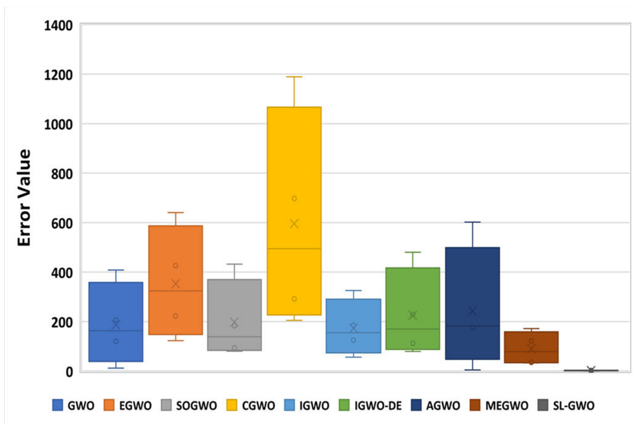

FIGURE 36. Box-plot for function F2 from CEC2019 test suite.

optimization problems with a varying number of dimensions and constraints are chosen. A computational budget of 8,000 function evaluations (NFEs) has been allotted to all the fifteen algorithms for a fair comparison.

The constraint handling for all the five optimization problems is implemented through the standard static penalty approach method [81].

1) PRESSURE VESSEL DESIGN

The Pressure vessel design problem comprises four decision variables (x_1: length of the cylindrical section, x_2: thickness of the head, x_3: inner radius and x_4: thickness of the shell) and four inequality constraints with respect to the first, third and fourth decision variables and requires the total cost minimization with respect to its design [82]. Table 27 gives the optimum costs obtained and the optimal values of decision variables by the fifteen algorithms and a comparison of the performance of the algorithms in terms of best cost, worst cost, average costs, deviation and computational times for the 30 independent runs are tabulated in Table 28.

2) WELDED BEAM DESIGN

The welded beam design problem comprises of four decision variables (x_1: weld thickness, x_2: clamping bar length, x_3: bar height and x_4: bar thickness) and four inequality constraints including bending stress, shear stress, buckling load, and beam end deflection are levied. The problem requires the total cost minimization with respect to its manufacturing cost [83]. Table 29 gives the optimum costs obtained and the optimal values of decision variables by the fifteen algorithms

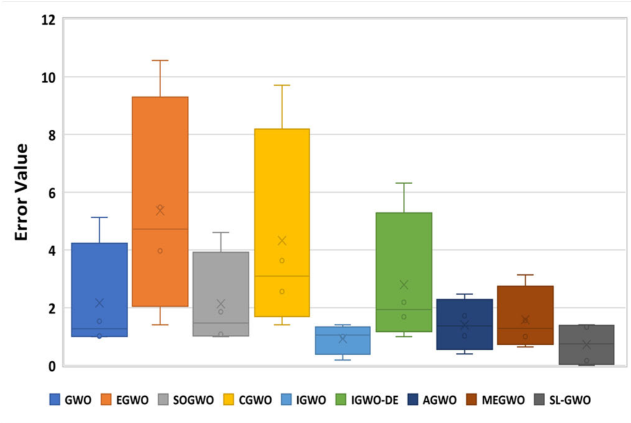

FIGURE 37. Box-plot for function F3 from CEC2019 test suite.

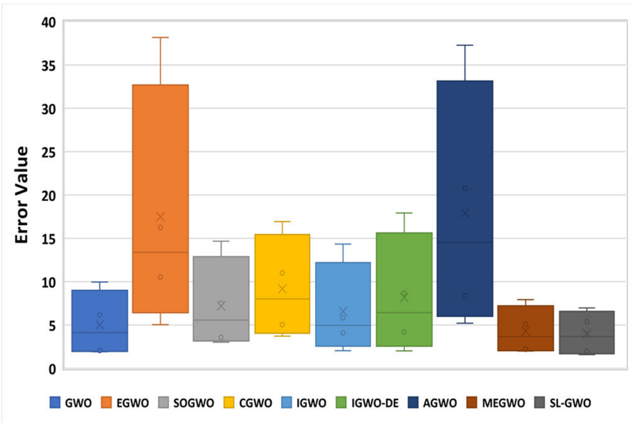

FIGURE 38. Box-plot for function F4 from CEC2019 test suite.

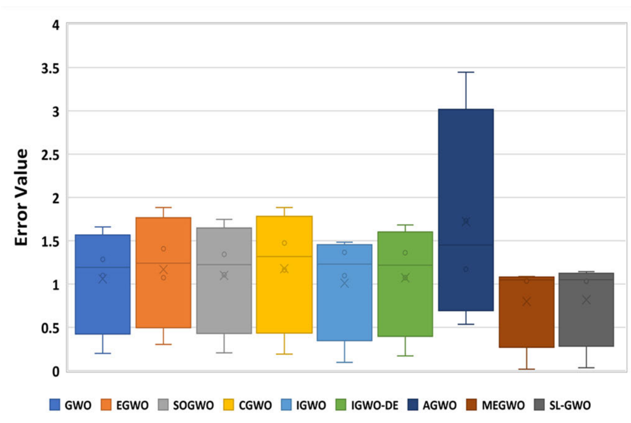

FIGURE 39. Box-plot for function F5 from CEC2019 test suite.

and a comparison of the performance of the algorithms in terms of best cost, worst cost, average costs, deviation and computational times for the 30 independent runs are tabulated in Table 30.

3) TENSION/COMPRESSION SPRING DESIGN

The tension/compression spring design problem comprises of three decision variables (x_1: wire diameter, x_2: mean coil diameter and x_3: number of active coils) and four inequality

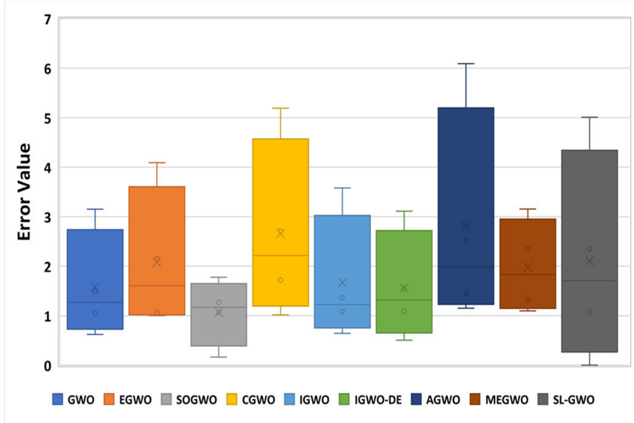

FIGURE 40. Box-plot for function F6 from CEC2019 test suite.

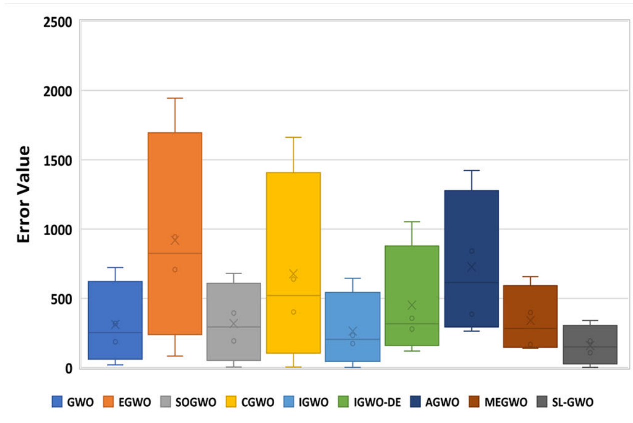

FIGURE 41. Box-plot for function F7 from CEC2019 test suite.

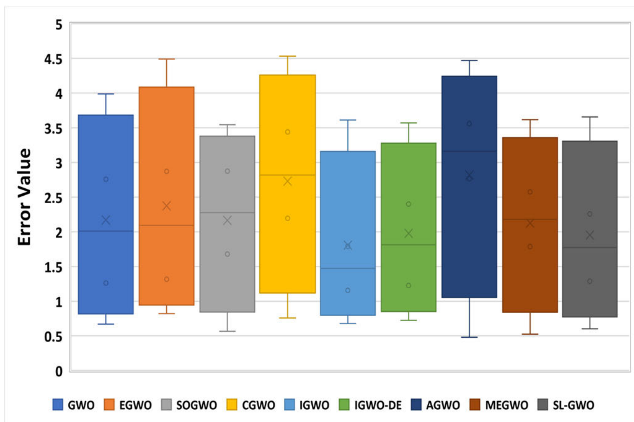

FIGURE 42. Box-plot for function F8 from CEC2019 test suite.

constraints including frequency, deflection and shear stress are levied. The problem requires the total cost minimization with respect to the weight of the spring [84]. Table 31 gives the optimum costs obtained and the optimal values of decision variables by the fifteen algorithms and a comparison of the performance of the algorithms in terms of best cost, worst

TABLE 27. The optimal costs and the optimal values of the four decision variables for the pressure vessel design obtained by the fifteen algorithms.

Algorithms	Optimal values of the decision variables obtained				Optimal Cost obtained
	x_1	x_2	x_3	x_4	
I-GWO	0.7794343556	0.3855339068	40.3837637858	199.1388121997	5889.08580256
SL-GWO	0.7796784264	0.3857205333	40.3916095060	199.0276061998	5890.28713632
EGWO	0.7872079895	0.3891173460	40.7879972605	193.5809927910	5900.96330259
GWO	0.7836986020	0.3925073584	40.6006507233	196.5582874592	5920.04068976
IGWO-DE	0.7812461382	0.3919807095	40.4279962545	198.9524402414	5924.18338773
MFO	0.8065712831	0.3986886197	41.7912762059	180.4804566083	5935.67501139
MEGWO	0.8153116347	0.4034208473	42.2432703963	174.8671869964	5953.71588307
P-ObGWO	0.7943069326	0.4003718170	41.0425810963	190.5338044521	5959.57309163
CGWO	0.8073888740	0.4051434048	41.8216392323	180.2336848268	5960.68479758
SOGWO	0.7869031318	0.4145618906	40.7107538245	194.7100797070	5985.86353196
SSA	0.7937152257	0.4045491578	41.0599155331	191.8617019128	6000.32876781
SOA	0.8039533691	0.3948298631	40.9317708625	192.4383730144	6036.32268070
ACGWO	1.4058612676	0.6416852375	64.7741722612	17.9694433599	6189.91040663
WOA	1.1782194920	0.6725792451	56.2480400639	55.3914396988	7861.10197366
ChOA	1.2884339257	0.6478661675	65.5179334709	14.3892554056	7934.46745769

TABLE 28. Comparison of the best, worst, average (mean), standard deviation and the average computational times (seconds) of the fifteen algorithms for the pressure vessel design.

Algorithms	Best	Worst	Mean	Standard Deviation	Average Computational Time
I-GWO	5889.085803	5908.671138	5898.807425	7.940496	0.3214
SL-GWO	5890.287136	5986.251721	5932.492615	56.800672	0.1356
EGWO	5900.963303	7287.901360	6591.208223	419.565473	0.1637
GWO	5920.040690	7127.880476	6159.582457	444.143389	0.1150
IGWO-DE	5924.183388	7109.254927	6167.917009	401.433442	0.1708
MFO	5935.675011	7318.998921	6568.863438	658.352589	0.1256
MEGWO	5953.715883	6491.270154	6151.271274	210.986323	0.1537
P-ObGWO	5959.573092	6917.539912	6238.299434	385.365233	0.5398
CGWO	5960.684798	7254.803346	6540.933252	463.880684	0.3384
SOA	5985.149958	7484.625566	6518.407250	620.925022	0.1120
SOGWO	5985.863532	7045.591476	6392.753534	349.731809	0.2360
SSA	6000.328768	7291.722254	6321.327363	440.524924	0.1481
ACGWO	6189.910407	8458.090517	6933.067546	823.442848	0.3156
WOA	7861.101974	14745.795100	10684.763509	2901.804623	0.1058
ChOA	7934.467458	9090.311204	8406.731199	403.404711	0.3538

TABLE 29. The optimal costs and the optimal values of the four decision variables for the welded beam design obtained by the fifteen algorithms.

Algorithms	Optimal values of the decision variables obtained				Optimal Cost obtained
	x_1	x_2	x_3	x_4	
SL-GWO	0.2056432497	3.4726052692	9.0368246210	0.2057351822	1.72508107
I-GWO	0.2057023021	3.4720946580	9.0363597481	0.2057475673	1.72511839
MFO	0.2044884399	3.4969868447	9.0376970345	0.2057242863	1.72663944
GWO	0.2054069248	3.4753168692	9.0448915700	0.2059010107	1.72773369
SOGWO	0.2049197265	3.4967107990	9.0403014545	0.2057425798	1.72787430
IGWO-DE	0.2018894317	3.5571278106	9.0363992220	0.2057909581	1.73093020
P-ObGWO	0.2046444613	3.5178443515	9.0077541220	0.2071279732	1.73518138
CGWO	0.1984442426	3.6343894227	9.0380719416	0.2057271681	1.73558675
SSA	0.2055664450	3.4621918964	9.1244251339	0.2058236907	1.73936163
SOA	0.2073495857	3.4745945267	8.9986670734	0.2087246858	1.74407327
ACGWO	0.1946076495	3.9186768165	9.0348484884	0.2086551898	1.78909070
ChOA	0.1948298001	3.4928525506	9.7618162546	0.2074044564	1.85036920
EGWO	0.1763306080	5.3676636412	7.3116706788	0.3142505432	2.32531318
WOA	0.2047350792	8.9562592073	8.9530217609	0.2095897863	2.48713692
MEGWO	N/A	N/A	N/A	N/A	Infeasible

cost, average costs, deviation and computational times for the 30 independent runs are tabulated in Table 32.

4) 10-BAR TRUSS

Truss bar optimization is optimizing the structural weight of the truss bars while taking into account design restrictions such as stress, deflection, and displacement [85]y. The decision variables are proportional to the size of the truss bars, which can be 10, 15, 25, 50, 72, or 200 in number. The truss bar optimization is applicable to continuous and discrete choice variables, and the current testing considers the 10-bar truss optimization for continuous variables. Article

TABLE 30. Comparison of the best, worst, average (mean), standard deviation and the average computational times (seconds) of the fifteen algorithms for the welded beam design.

Algorithms	Best	Worst	Mean	Standard Deviation	Average Computational Time
SL-GWO	1.725081	1.737450	1.729285	0.004690	0.1552
I-GWO	1.725118	1.727434	1.725981	0.000769	0.2705
MFO	1.726639	3.263819	2.119567	0.499748	0.1323
GWO	1.727734	1.734021	1.730376	0.002520	0.1113
SOGWO	1.727874	1.734678	1.731496	0.002510	0.2027
IGWO-DE	1.730930	1.742771	1.735814	0.004396	0.1427
P-ObGWO	1.735181	2.026696	1.877224	0.160578	0.4749
CGWO	1.735587	1.815979	1.769556	0.025748	0.2886
SSA	1.739362	1.802079	1.764990	0.021604	0.1242
SOA	1.744073	1.801040	1.767429	0.021578	0.1330
ACGWO	1.789091	1.945524	1.885947	0.050240	0.3808
ChOA	1.850369	1.946256	1.906391	0.043593	0.3026
EGWO	2.325313	5.854962	4.582309	1.625710	0.1293
WOA	2.487137	5.341149	3.701471	1.086077	0.1102
MEGWO	Infeasible	Infeasible	Infeasible	Infeasible	N/A

TABLE 31. The optimal costs and the optimal values of the three decision variables for the tension/compression spring design obtained by the fifteen algorithms.

Algorithms	Optimal values of the decision variables obtained			Optimal Cost obtained
	x_1	x_2	x_3	
WOA	0.0520741640	0.3660519323	10.7620732395	0.0126680144
SL-GWO	0.0525597443	0.3779963498	10.1456099590	0.0126827498
MFO	0.0528035224	0.3841257267	9.8461193481	0.0126874775
SOGWO	0.0509214352	0.3382509899	12.4760401265	0.0126966786
I-GWO	0.0500947135	0.3195244821	13.8600901163	0.0127172610
MEGWO	0.0499206975	0.3156398588	14.1769061147	0.0127247306
GWO	0.0502927918	0.3236531004	13.5498036531	0.0127296414
IGWO-DE	0.0500000000	0.3174040700	14.0498951078	0.0127357551
CGWO	0.0535826079	0.4038369182	8.9968687069	0.0127503690
ChOA	0.0500000000	0.3173369041	14.0832375310	0.0127595120
OpsGWO	0.0533853365	0.3988424710	9.2283314212	0.0127632298
SSA	0.0500000000	0.3167022684	14.1332600391	0.0127736001
EGWO	0.0548138774	0.4366555551	7.7835699669	0.0128356360
SOA	0.0500000000	0.3171113024	14.3097303158	0.0129299996
ACGWO	0.0537314143	0.4054688163	9.0840740613	0.0129751809

TABLE 32. Comparison of the best, worst, average (mean), standard deviation and the average computational times (seconds) of the fifteen algorithms for the tension/compression spring design.

Algorithms	Best	Worst	Mean	Standard Deviation	Average Computational Time
WOA	0.0126680	0.0138406	0.0131641	0.0003659	0.1109
I-GWO	0.0126719	0.0127274	0.0126977	0.0000230	0.2677
SL-GWO	0.0126828	0.0127716	0.0127320	0.0000331	0.1327
MFO	0.0126875	0.0136714	0.0130648	0.0003375	0.1197
SOGWO	0.0126967	0.0141891	0.0130565	0.0005627	0.1868
CGWO	0.0127211	0.0140222	0.0130918	0.0004439	0.2988
MEGWO	0.0127247	0.0150353	0.0135954	0.0009970	0.1529
GWO	0.0127262	0.0131141	0.0128002	0.0001406	0.1110
IGWO-DE	0.0127358	0.0257891	0.0143432	0.0043023	0.1111
ChOA	0.0127595	0.0156537	0.0135194	0.0009389	0.2676
P-ObGWO	0.0127632	0.0134857	0.0129738	0.0002695	0.4590
SSA	0.0127736	0.0138741	0.0130740	0.0003407	0.1387
EGWO	0.0128356	0.0177732	0.0164134	0.0023245	0.1313
SOA	0.0129300	0.0139908	0.0133880	0.0004102	0.1216
ACGWO	0.0129660	0.0150285	0.0134804	0.0008331	0.2938

at [86] has a detailed description of the mathematical formulation, the objective function. The best fitness values and their accompanying optimal decision variables for all fifteen algorithms are listed in Table 33, ordered ascending by fitness score. Table 34 contains a comparison table of the best, worst, average, standard deviation, and average computing times for the thirty separate runs of all fifteen algorithms.

TABLE 33. The optimal costs and the optimal values of the ten decision variables for the 10-bar truss optimization obtained by the fifteen algorithms.

Algorithms	Optimal values of the decision variables obtained										Optimized Weight
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	
SL-GWO	30.8010	0.1007	22.9557	15.3690	0.1021	0.5260	21.1518	7.5023	0.1040	21.2925	5062.93
I-GWO	30.7767	0.1162	22.8634	14.8660	0.1010	0.5377	21.2816	7.5477	0.1044	21.5804	5065.16
EGWO	30.0796	0.1000	24.3331	14.9236	0.1000	0.5124	21.7797	7.4546	0.1000	20.6456	5066.32
SSA	29.6158	0.1000	22.9399	15.4835	0.1000	0.6389	22.3967	7.6090	0.1000	20.7467	5068.59
P-ObGWO	31.2112	0.1292	23.2572	14.9846	0.1000	0.3838	20.8804	7.6260	0.1047	21.4410	5070.61
MFO	32.9172	0.1000	21.5842	14.6433	0.1000	0.4550	21.5026	7.7601	0.1000	20.9538	5074.49
IGWO-DE	30.3215	0.3105	23.4750	15.3971	0.1009	0.4809	21.2234	7.5074	0.1068	21.3306	5078.37
GWO	30.0255	0.1480	23.3904	15.1608	0.1013	0.7081	21.5921	7.4906	0.1556	21.3460	5078.56
SOGWO	31.1307	0.1128	23.8243	14.5217	0.1187	0.1840	20.9343	8.4957	0.2420	20.9112	5091.39
ACGWO	33.0932	0.1000	22.7542	17.3692	0.1000	0.2758	20.1395	8.3610	0.1000	19.5326	5111.17
MEGWO	30.7651	0.1566	22.7137	16.1071	0.1699	1.0380	21.1972	7.3328	0.3148	21.5711	5121.54
ChOA	33.5000	0.1000	23.3141	17.5998	0.1000	0.1292	18.8277	8.5611	0.2259	20.2002	5125.09
SOA	33.5000	0.2440	22.4329	14.3942	0.2383	0.1083	21.9335	8.4099	1.2323	20.0085	5179.27
CGWO	32.7826	1.8190	23.6231	14.8852	0.4636	1.1148	21.3122	7.7782	0.5021	21.0003	5264.54
WOA	23.5409	0.1008	24.7526	12.8709	0.1062	0.1059	24.9987	23.5442	0.1008	24.5252	5938.33

TABLE 34. Comparison of the best, worst, average (mean), standard deviation and the average computational times (seconds) of the fifteen algorithms for the 10-bar truss optimization.

Algorithms	Best	Worst	Mean	Standard Deviation	Average Computational Time
SL-GWO	5062.9262	5095.1561	5074.4336	13.9461	12.5210
I-GWO	5065.1569	5076.7244	5069.0871	3.6844	12.2926
EGWO	5066.3158	5202.7367	5114.0509	57.0897	12.0497
SSA	5068.5913	5121.1941	5084.2802	18.9596	12.5080
P-ObGWO	5070.6071	5088.9853	5082.1104	6.5015	19.2839
MFO	5074.4891	6294.4940	5285.6959	494.3209	12.4738
IGWO-DE	5078.3685	5191.9685	5141.9670	43.1927	13.2509
GWO	5078.5564	5120.8130	5099.4686	18.1237	12.0013
SOGWO	5091.3924	5104.5379	5100.4590	5.2241	13.2016
ACGWO	5111.1743	5205.3316	5172.4853	35.2193	14.0689
MEGWO	5121.5362	5139.4710	5126.1106	7.5815	13.1356
ChOA	5125.0930	5719.5558	5281.4562	223.5450	16.3919
SOA	5129.6190	7127.3982	5513.6512	791.6838	14.5863
CGWO	5264.5442	5409.6174	5331.6040	57.7597	11.7950
WOA	5938.3303	8398.0347	7025.0215	920.1037	12.4250

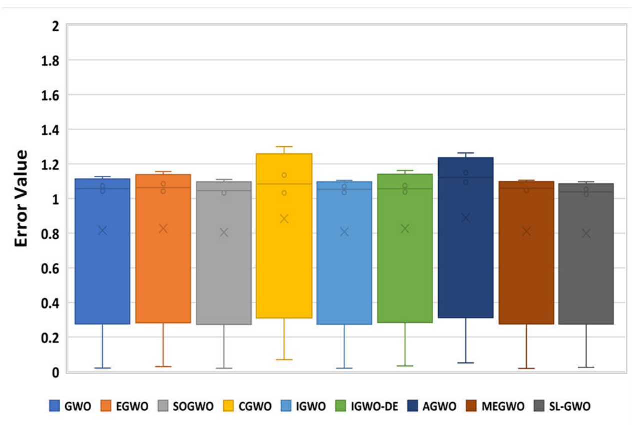

FIGURE 43. Box-plot for function F9 from CEC2019 test suite.

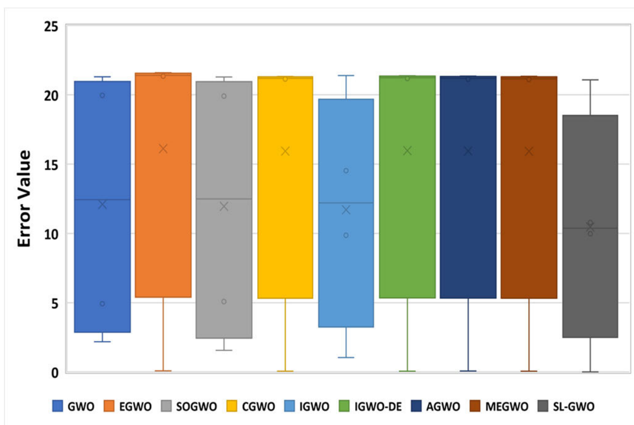

FIGURE 44. Box-plot for function F10 from CEC2019 test suite.

a: ANALYSIS OF RESULTS

- I-GWO and SL-GWO produced similar optimal costs with I-GWO having the least standard deviation. However, SL-GWO had a lower computational time (almost half that of I-GWO). The neighbourhood construction strategy based on Euclidean distance calculation in I-GWO enables the algorithm to choose the best wolves within the constructed neighbourhood favouring better exploitation and hence the least standard deviation for problems with low dimensions. Similarly, SL-GWO works to exploit the best areas in the search space through the symbiotic learning strategy while ensuring that randomized omega wolves are always chosen in this process. Although the inclusion of randomized omega wolves could impede the search process locally by a small margin resulting in slightly increased rates of standard deviation in the case of SL-GWO, it is this randomized selection that prevents local entrapment as seen in previous benchmarking tests.

- The standard GWO algorithm and its other variants although produced the best optimal solutions with higher cost function values, had larger standard deviations with their mean fitness being far away from the best fitness. ACGWO ranked last in terms of optimal fitness value and had the highest standard deviation amongst the variants of GWO considered.
- WOA and ChOA, on the other hand, couldn't deliver their best performance in both the cases of optimality and lower deviation rates for a lower setting of NFEs considered in this work. WOA, notably had the highest standard deviation for the current problem indicating that the algorithm converged to a local solution in most of the runs during the benchmarking process.
- The NFEs required by the proposed method is lower than the most cases with other modern meta-heuristics indicating that the proposed method is able to achieve a good trade-off between the exploration and exploitation quickly for constrained optimization problems.
- MEGWO produced infeasible solutions with constraint violations in some cases and hence its optimal cost and its corresponding optimal decision variable have not been included in the comparison.

V. POWER FLOW OPTIMIZATION PROBLEMS

The power flow optimization problems in power systems are complex and non-linear multi-constrained optimization tasks and present a challenging environment to measure the effectiveness of optimization algorithms. In this work13 cases of optimal power flow (OPF) problem for the IEEE 30 and 57 bus systems and 8 cases of optimal reactive power dispatch (ORPD) problem for the IEEE 30 and 57 bus systems have been investigated through a combination of linear incremental penalty and constraint correction methods using the 15 optimization algorithms.

A. OPTIMAL POWER FLOW (13 CASES)

The first task is to determine the optimal power flow (OPF) for the IEEE 30 and IEEE 57 bus systems using multiple OPF objectives from [87]. These objectives include cost, emission, power loss, and voltage stability. OPF is a highly nonlinear, difficult optimization problem in which the steady-state characteristics of an electrical network must be found in order

TABLE 35. Summary of the case studies of the OPF for IEEE 30 bus system.

Case Studies	Objectives of various case studies					
	Basic Fuel Cost	Voltage Stability	Emission	Power Loss	Voltage Deviation	Valve-point Effect
Case 1	✓					
Case 2		✓				
Case 3			✓			
Case 4				✓		
Case 5	✓					✓
Case 6	✓			✓		
Case 7	✓				✓	
Case 8	✓	✓				
Case 9	✓		✓	✓	✓	

TABLE 36. Summary of the case studies of the OPF for IEEE 57 bus system.

Case Studies	Objectives of various case studies		
	Basic Fuel Cost	Voltage Stability	Voltage Deviation
Case 10	✓		
Case 11	✓		✓
Case 12	✓	✓	
Case 13			✓

for the network to operate economically and efficiently. The complexity of the problem increases as a result of the problem's multiple equality and inequality constraints. Solving OPF continues to be a prominent yet difficult issue for power system researchers. Numerous evolutionary algorithms (EAs) and swarm intelligence-based optimization algorithms have been researched in the last couple of decades to identify optimum solutions to various OPF objectives.

The OPF for IEEE 30 bus system has 24 control/decision variables and the IEEE 54 bus system has 33 control variables to be optimized. The different cases for the formulation of the objective function are provided in Table 35. The other test cases have been considered as described in Table 36.

The multi-objective optimization cases have been dealt with as single-objective optimization problems through the weighing factors techniques whose weights have been set based on the modelling at [87].

The equality constraints are defined for the power balance of active and reactive power followed by three inequality constraints for the generator limits, two security constraints, transformer constraint and shunt compensator constraint. A comprehensive description of the mathematical formulation of the OPF, control (independent) variables, state (dependent) variables and the various constraints is available at [87].

Constraint handling in the current work is performed through a combination of the linear penalty incremental

TABLE 37. Tabulation of the best solutions of OPF for the IEEE 30-bus system.

Decision Variables	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
Algorithm with the best Fitness	**SL-GWO**	**IGWO**	**SL-GWO**	**SL-GWO**	**SL-GWO**	**MEGWO**	**IGWO**	**SL-GWO**	**MEGWO**
$PG2$ (MW)	49.16096	34.28687	75.75164	78.06647	44.05076	58.49807	47.67656	47.13485	46.44822
$PG5$ (MW)	21.04587	44.98785	48.97812	49.14295	16.23561	39.58482	23.06215	20.59888	38.06697
$PG8$ (MW)	22.32641	14.80434	34.87579	32.65859	12.60191	34.44590	17.85054	25.11835	28.82674
$PG11$ (MW)	12.02927	24.33649	29.80028	28.29173	11.07212	29.98112	13.86330	11.27504	25.96054
$PG13$ (MW)	12.36555	27.63811	25.94268	39.08216	12.76670	32.69126	14.30494	12.83797	13.51348
$V1$ (p.u.)	1.04741	1.03642	1.04166	1.06906	1.07342	1.00158	1.04412	1.06386	1.07326
$V2$ (p.u.)	1.02944	1.02076	1.02416	1.05608	1.05143	0.98761	1.03406	1.03765	1.05890
$V5$ (p.u.)	1.00160	1.02820	0.99154	1.04517	1.01026	0.95908	1.01399	1.00629	1.03364
$V8$ (p.u.)	1.00970	0.99189	0.99200	1.03700	1.03026	0.95919	1.00369	1.01086	1.03963
$V11$ (p.u.)	1.08551	1.06896	0.99510	1.00976	1.03853	1.07126	1.05849	1.07331	1.10000
$V13$ (p.u.)	1.06231	1.09407	1.03432	1.04269	1.05015	1.04362	1.01505	1.07411	1.03313
$Qc10$ (MVAr)	3.10097	2.36410	3.77824	1.16758	0.76108	4.97450	0.84672	4.82947	4.67320
$Qc12$ (MVAr)	0.54088	0.55153	1.31844	2.89736	0.52017	4.91902	0.37899	1.45761	0.73166
$Qc15$ (MVAr)	1.96823	3.41402	4.75606	0.14133	0.79650	4.61021	0.39187	3.35010	4.51538
$Qc17$ (MVAr)	0.38381	4.22586	3.21943	4.57108	4.23639	2.51985	1.11284	0.61330	3.56062
$Qc20$ (MVAr)	1.68748	3.20267	2.30104	0.71893	3.94110	4.56658	3.01947	0.56199	0.01959
$Qc21$ (MVAr)	4.84155	4.56808	0.23365	1.06300	4.42275	4.86642	2.73198	4.19029	0.53308
$Qc23$ (MVAr)	0.47317	4.68481	4.17776	4.25754	4.12906	3.86036	3.58635	4.11419	0.18679
$Qc24$ (MVAr)	2.82962	0.55909	3.70016	4.75702	2.91539	0.29185	4.97138	0.77434	2.32117
$Qc29$ (MVAr)	0.22118	3.97317	3.94070	2.72275	2.07902	3.26564	1.58695	0.89350	0.56201
$T11$ (p.u.)	0.94664	0.94130	1.05676	1.08619	0.97993	0.95474	1.04724	0.97924	1.01839
$T12$ (p.u.)	1.02302	1.01247	0.92455	0.91537	1.02071	0.92300	0.91109	1.02871	1.07753
$T15$ (p.u.)	1.01870	1.02441	1.02214	1.02117	0.97773	0.96500	0.95722	1.01715	1.01179
$T36$ (p.u.)	0.95517	0.92935	0.96980	1.02921	1.03613	0.92943	0.95936	0.94109	0.97801
Fuel cost ($/h)	802.72742	849.34969	926.56451	922.66064	806.68686	870.89124	803.82826	802.60411	845.78164
Emission (t/h)	0.36956	0.30074	0.21420	0.20943	0.43309	0.22333	0.37636	0.35105	0.59170
Ploss (MW)	9.72516	7.64548	4.23361	4.04928	11.06319	5.05861	10.07738	9.05968	7.19882
VD (p.u.)	0.26959	0.48907	0.47230	0.47138	0.47963	0.49242	0.15384	0.52667	0.17793
L-index (max)	0.14426	0.13998	0.15004	0.14294	0.15489	0.14620	0.14760	0.13972	0.14208
Fitness	802.59275	0.13889	0.21403	3.58417	835.54198	1071.72092	818.34152	816.55087	971.48195
Computational Time (Sec)	92.11	84.94	94.04	92.65	92.35	90.98	84.28	90.21	89.02

method (LPIM) and constraint correction approaches. The LPIM is the primary constraint handling mechanism that communicates with the algorithm conveying the fitness value which is the sum of the objective function value and the total incremental penalty levied on each population member. In LPIM, the total penalty added to the fitness is dependent on the individual constraint violations amplified through a linearly rising penalty factor resulting in unique penalty values. The advantage of using an incremental penalty factor is that the initial lower penalty allows initial infeasible solutions to enter the population pool such that exploitation around the infeasible zones is enhanced. As the penalty increases with the progression of the iterations, a very large penalty for minimal constraint violations forces the algorithms to avoid the infeasible zones and exploit the feasible zones only and as a result, the total constraint violation decreases. The total penalty added to the fitness function is the total sum of the products of individual constraint violations and the linear incremental penalty levied on each of them. This is represented by Eq. (5.1).

$$TP = \sum_{i=1}^{C} V_i \times L_p \tag{5.1}$$

where

$i = 1, 2, 3 \ldots C$

$$L_p = K_{min} + (K_{max} - K_{min}) \times \left(\frac{NFE_t}{TotalNFEs}\right)$$

where,

TP denotes the total penalty added to the fitness value, i = 1,2,3 … C, are the individual constraints, C denotes the total number of equality and inequality constraints, V_i denotes the individual constraint violation, L_p is the linear incremental penalty, K_{min} and K_{max} are the lower and upper penalty values set to 1E+03 and 1E+09 through empirical analysis, NFE_t is the current number of function evaluations, *TotalNFEs* denotes the total number of function evaluations.

Despite the LPIM effectiveness to prevent infeasible solutions from making it into the final population pool, it is necessary that the search process be guided to feasible zones for individual constraints. This is implemented through an archive-based constraint correction (ABCC) system acting as the secondary constraint handling mechanism which occurs after the LPIM. The two-constraint handing (CH) mechanisms are balanced based on the number of function

TABLE 38. Tabulation of the best solutions of OPF for the IEEE 57-bus system.

Decision Variables	Case 10	Case 11	Case 12	Case 13
Algorithm with the best Fitness	**IGWO**	**SL-GWO**	**SL-GWO**	**SL-GWO**
$PG2$ (MW)	88.14738	87.10601	86.70550	41.44772
$PG3$ (MW)	45.50677	46.38526	44.78901	65.83948
$PG6$ (MW)	72.54891	70.77966	71.71972	42.01418
$PG8$ (MW)	459.32328	458.94229	461.85992	290.74098
$PG9$ (MW)	95.50988	98.97297	96.81363	99.53646
$PG12$ (MW)	362.49941	360.53067	361.24335	244.69062
$V1$ (p.u.)	1.04706	1.02935	1.03748	1.01816
$V2$ (p.u.)	1.04613	1.02800	1.03632	1.01113
$V3$ (p.u.)	1.04360	1.02864	1.03353	1.00822
$V6$ (p.u.)	1.05686	1.04972	1.04922	1.00312
$V8$ (p.u.)	1.07331	1.07030	1.07301	1.02039
$V9$ (p.u.)	1.04157	1.03162	1.03917	1.00246
$V12$ (p.u.)	1.03547	1.01611	1.03130	1.01916
$Qc18$ (MVAr)	12.47841	18.83357	8.25771	9.80188
$Qc25$ (MVAr)	14.79948	14.73812	8.75901	17.99168
$Qc53$ (MVAr)	12.78896	11.17668	13.94863	19.88165
$T19$ (p.u.)	1.06623	1.09472	0.97609	0.93935
$T20$ (p.u.)	0.95907	0.98341	0.96774	1.05358
$T31$ (p.u.)	1.03129	0.99227	1.00744	0.97065
$T35$ (p.u.)	1.00610	1.09961	0.94160	0.99349
$T36$ (p.u.)	1.05139	0.90891	1.02070	1.08505
$T37$ (p.u.)	1.02283	1.01575	1.03088	1.00032
$T41$ (p.u.)	0.99702	1.01163	0.98959	0.99403
$T46$ (p.u.)	0.94752	0.93888	0.94443	0.92582
$T54$ (p.u.)	0.90375	0.90541	0.90929	0.90228
$T58$ (p.u.)	0.96742	0.96408	0.95799	0.92989
$T59$ (p.u.)	0.95423	0.96330	0.94343	0.96274
$T65$ (p.u.)	0.96752	0.97929	0.96329	0.99260
$T66$ (p.u.)	0.92070	0.93338	0.92167	0.90188
$T71$ (p.u.)	0.96358	0.97121	0.95791	0.94312
$T73$ (p.u.)	0.98830	0.99706	0.98147	1.01457
$T76$ (p.u.)	0.98202	0.93396	0.97022	0.90307
$T80$ (p.u.)	0.98837	1.00689	0.98998	0.98414
Fuel cost ($/h)	41672.64502	41699.40795	41680.34852	53436.56815
Emission (t/h)	1.35631	1.34510	1.36079	1.86441
Ploss (MW)	14.91662	15.58998	15.19510	30.12907
VD (p.u.)	1.60137	0.83056	1.50347	0.63065
L-index (max)	0.27961	0.29203	0.27807	0.29830
Fitness	41672.74878	41781.56532	41707.94424	0.60894
Computational Time (Sec)	89.82	82.60	92.84	97.58

evaluations to prevent the additional burden of a secondary mechanism for all of the available budget. LPIM is implemented for the entire budget of the function evaluations while ABCC is subject to various conditions and probabilities and limited to only the last 50% of the function evaluations. This is given in Eq. (5.2).

$$CH = \begin{cases} LPIM \\ \qquad if\ NFE_t < 0.5 \times TotalNFEs \\ \begin{cases} ABCC & if\ rand < 0.5 \\ LPIM & otherwise \end{cases} \\ \qquad otherwise \end{cases} \tag{5.2}$$

In ABCC, an archive constantly stores the best solutions with the least possible constraint violations across each constraint. For a newer solution to enter the archive, its individual constraint violations must be lower than that of the existing members (to prevent a very higher number of such comparisons the archive size is set to 10 in this work). This is given in Eq. (5.3).

$$Ar(p) = \begin{cases} X_{new} & if\ V^{new}_{i=1,2..C} < V^{p=1,2..P}_{i=1,2..C} \\ Retain\ X_p & Otherwise \end{cases} \tag{5.3}$$

were,

Ar denotes the archive of the best solutions with the least individual constraint violations, $p = 1, 2 \ldots P$ denotes the members of the archive, X_{new} is the new solution vector obtained from the optimization algorithm, X_p is the current solution vector from the archive.

The advantage of the proposed elite archival system is that the solutions with the least possible violations across individual solutions are constantly updated despite the variation in the penalty. For the remaining 50% of the budget evaluations, ABCC is implemented with a random probability such that the solutions from LPIM with higher constraint violations are corrected to accept the decision variables from a random member of the archive with lesser violations across individual constraints. This method has the added advantage of improving the feasibility irrespective of its objective function value and enables the algorithm to effectively exploit the new solution with improved diversity across its decision variables. The only disadvantage is the sacrifice of a fitness evaluation to evaluate its newer fitness and incremental penalty (if the violation persists). Hence, to prevent the excessive modifications from the ABCC resulting in higher functions evaluations utilized, a probability factor is assigned to ensure that its involvement is balanced with the LPIM. This is given in Eq. (5.4).

$$X_{j=1,2..D}^{Corrected} = \begin{cases} X_j^p & if\ V_{i=1,2..C}^{p=1,2..P} < V_{i=1,2..C}^{new} \\ X_j & Otherwise \end{cases} \tag{5.4}$$

where,

$X_j^{Correccted}$ is the corrected solution with the corrected decision variables across the j dimensions, D denotes the total number of dimensions, X_j^p denotes the decision variable for the j^{th} dimension from the member of archive, X_j is the decision variable for the j^{th} dimension retained from the new solution.

Once the corrected solutions are evaluated, its possibility of entering the archive is checked and the new fitness with the corrected decision variables are passed on the algorithm for further improvement.

The NFEs are set to 25,000 for both the OPFs of IEEE 30 bus system and IEEE 57 bus systems to ensure fair comparison amongst all the algorithms.

The best solutions and their decision variables obtained for the various cases of OPF for the IEEE 30 and IEEE 57-bus system are shown in Table 37 and Table 38 respectively.

From Table 37 and Table 38:

- The performance of SL-GWO has been good for five out of the nine cases for the OPF of the IEEE 30-bus systems and three out of the four cases for the OPF of the IEEE 57-bus system.
- For all the 15 algorithms chosen in the current comparative analysis, no constraint violations have been reported.
- Next to SL-GWO, I-GWO and MEGWO performed well for the other cases in both the bus systems.

A comparison of the best results, and average computational times recorded by the other algorithms chosen for the comparative analysis are provided in Table 39 and Table 40 for the OPF of various cases for the IEEE 30 and IEEE 57-bus system respectively.

From Table 39 and Table 40:

- The performance of I-GWO, MEGWO and GWO was competitive in most of the test cases. It can also be noticed that for the same fitness score, the other parameters have been different for the various algorithms considered. This is on account of the complexity and high non-linearity associated with the OPF problem.
- The performance of SL-GWO stands out in terms of optimality, lower standard deviation and mean values closer to the best values. The exploitation system through the different symbiotic learning strategies has been the stronghold for SL-GWO enabling the algorithm to intensify and further refine the quality of solutions. The neighbourhood operator-based improvement from I-GWO and the multi-strategy ensemble techniques from MEGWO also proved successful at generating optimal solutions while handling multiple constraints but performed next to SL-GWO.
- The computational times of ACGWO, P-ObGWO and SOGWO were the highest followed by ChOA for the modern meta-heuristics. MEGWO and I-GWO had the lowest computational times followed by MFO and SL-GWO.

A comparison of the best, worst, mean and standard deviation of the optimal costs recorded by the other algorithms chosen for the comparative analysis is provided in Table 41 and Table 42 for the OPF of various cases for the IEEE 30 and IEEE 57-bus system respectively.

B. REACTIVE POWER DISPATCH (8 CASES)

The second problem is that of the optimal reactive power dispatch (ORPD) on base configurations of IEEE 30-bus and 57-bus systems from [88]. Optimizing reactive power flow in an electrical network is an important aspect of system study as the reactive power supports network voltage which needs to be maintained within desirable limits for system reliability. A network consisting of only conventional thermal generators has been extensively studied for optimal active and reactive power dispatch.

The ORPD for IEEE 30 bus system has 19 control/decision variables and the IEEE 54 bus system has 27 control variables to be optimized. The objectives for case 1, case 1a, case 11 and case 11a are the minimization of the real power loss (P_{loss}) in the network and for case 2, case 2a, case 12 and case 12a is the minimization of the aggregate voltage deviation (VD) in the network. The equality constraints are defined for the power balance of active and reactive power followed by three inequality constraints for the generator limits, two security constraints, transformer constraint and shunt compensator constraint. A comprehensive description of the mathematical formulation of the ORPD, control (independent) variables, state (dependent) variables and the various constraints is available at [88].

In the current work, the NFEs are set to 20,000 for all the algorithms to have a fair comparison. The best solutions and their decision variables obtained for the various cases of

TABLE 39. Tabulation of the best solutions and computational times of OPF for the IEEE 30-bus system for all the algorithms in comparative analysis.

Case 1	IGWO	GWO	EGWO	IGWO-DE	WOA	SSA	ChOA	MFO	SOA	MEGWO	P-ObGWO	SOGWO	CGWO	ACGWO
Fuel cost ($/h)	803.9833	805.9453	805.0757	804.5364	805.4003	804.0997	809.4282	804.1577	805.3499	808.0218	804.4814	809.6840	817.4454	814.3000
Emission (t/h)	0.3609	0.3543	0.3638	0.3790	0.3734	0.3548	0.3545	0.3421	0.3307	0.3505	0.3608	0.3377	0.3114	0.3044
*P*loss (MW)	9.7000	9.1776	9.6811	10.0958	10.5249	9.3825	9.7170	9.0627	8.4164	10.2864	9.9312	9.5188	8.0470	7.9792
VD (p.u.)	0.2521	0.5840	0.5440	0.2605	0.3626	0.2204	0.3349	0.6699	0.7601	0.3564	0.3668	0.2415	0.3853	0.3407
L-index (max)	0.1494	0.1473	0.1460	0.1533	0.1439	0.1478	0.1410	0.1450	0.1397	0.1468	0.1477	0.1479	0.1497	0.1410
Fitness	803.9406	805.9077	804.4170	803.9480	805.2804	803.9101	809.1316	803.3171	805.0028	806.2742	804.4132	809.5346	812.7059	813.7577
Time (Sec)	81.76	94.87	171.37	118.79	90.46	97.81	169.78	87.97	96.46	82.10	235.62	262.21	103.45	352.07
Case 2	**SL-GWO**	**GWO**	**EGWO**	**IGWO-DE**	**WOA**	**SSA**	**ChOA**	**MFO**	**SOA**	**MEGWO**	**P-ObGWO**	**SOGWO**	**CGWO**	**ACGWO**
Fuel cost ($/h)	831.8637	847.0487	881.0895	886.7250	849.9982	870.4216	837.5085	814.5011	820.8834	881.7237	817.3429	842.8133	854.5879	819.9193
Emission (t/h)	0.2672	0.2542	0.2682	0.2397	0.2635	0.2687	0.2455	0.3368	0.2847	0.2267	0.3487	0.2786	0.2963	0.3117
*P*loss (MW)	6.5952	7.4744	6.6790	6.0487	6.6999	8.4014	5.9897	10.2848	7.7412	5.3297	9.0466	7.8245	7.2936	8.7037
VD (p.u.)	0.6497	0.3460	0.4987	0.2969	0.3147	0.4645	0.4362	0.3956	0.3813	0.3839	0.3765	0.5229	0.4350	0.4107
L-index (max)	0.1392	0.1403	0.1401	0.1399	0.1401	0.1405	0.1424	0.1426	0.1395	0.1399	0.1397	0.1401	0.1398	0.1404
Fitness	0.1392	0.1408	0.1400	0.1398	0.1400	0.1393	0.1411	0.1410	0.1393	0.1397	0.1394	0.1393	0.1398	0.1402
Time (Sec)	94.66	100.70	174.58	121.69	91.74	92.85	169.89	96.68	94.67	85.32	239.60	269.34	102.49	357.32
Case 3	**IGWO**	**GWO**	**EGWO**	**IGWO-DE**	**WOA**	**SSA**	**ChOA**	**MFO**	**SOA**	**MEGWO**	**P-ObGWO**	**SOGWO**	**CGWO**	**ACGWO**
Fuel cost ($/h)	887.8448	893.2787	897.2737	872.6953	896.2554	903.9941	885.7356	900.7051	903.8474	905.6590	905.2058	843.3638	906.2716	861.4414
Emission (t/h)	0.2205	0.2231	0.2318	0.2315	0.2245	0.2226	0.2447	0.2290	0.2208	0.2194	0.2268	0.2522	0.2268	0.2351
*P*loss (MW)	5.0441	5.0176	5.4245	5.8294	5.5050	6.5400	6.0951	6.5870	4.9731	4.8316	5.2286	6.2737	5.9553	6.3325
VD (p.u.)	0.4551	0.4921	0.3309	0.3718	0.5920	0.4125	0.5153	0.8433	0.2933	0.2259	0.5263	0.3684	0.3432	0.3735
L-index (max)	0.1385	0.1535	0.1537	0.1416	0.1437	0.1518	0.1496	0.1676	0.1466	0.1476	0.1508	0.1479	0.1425	0.1411
Fitness	0.2213	0.2152	0.2231	0.2269	0.2193	0.2162	0.2377	0.2252	0.2181	0.2157	0.2217	0.2499	0.2201	0.2236
Time (Sec)	86.18	101.27	177.30	123.45	93.64	96.57	169.34	96.26	97.58	87.08	242.44	268.17	105.30	358.10
Case 4	**IGWO**	**GWO**	**EGWO**	**IGWO-DE**	**WOA**	**SSA**	**ChOA**	**MFO**	**SOA**	**MEGWO**	**P-ObGWO**	**SOGWO**	**CGWO**	**ACGWO**
Fuel cost ($/h)	937.3789	913.9079	903.8567	914.7613	913.6664	908.7511	913.6206	930.7940	898.3719	925.3792	924.4493	945.8031	918.0299	872.9075
Emission (t/h)	0.2116	0.2246	0.2257	0.2138	0.2182	0.2167	0.2185	0.2137	0.2186	0.2114	0.2087	0.2126	0.2147	0.2292
*P*loss (MW)	3.8584	4.9659	4.4094	4.2393	4.2362	4.1240	4.4245	3.8817	4.8505	4.0307	3.9260	3.8472	4.7935	5.1023
VD (p.u.)	0.2865	0.3627	0.4795	0.4544	0.3549	0.3171	0.2886	0.4658	0.4529	0.4355	0.5026	0.3631	0.3097	0.3048
L-index (max)	0.1482	0.1408	0.1443	0.1532	0.1514	0.1457	0.1500	0.1537	0.1474	0.1521	0.1478	0.1456	0.1475	0.1418
Fitness	3.6110	4.0615	4.2219	4.1463	4.0747	3.9711	4.2822	3.8256	4.2873	3.7543	3.8230	3.7701	4.0776	4.7933
Time (Sec)	86.93	101.71	178.62	121.06	94.82	95.50	169.76	94.79	98.29	86.61	241.62	267.39	105.03	356.96
Case 5	**IGWO**	**GWO**	**EGWO**	**IGWO-DE**	**WOA**	**SSA**	**ChOA**	**MFO**	**SOA**	**MEGWO**	**P-ObGWO**	**SOGWO**	**CGWO**	**ACGWO**
Fuel cost ($/h)	807.5715	806.6181	811.3217	809.6988	807.4496	804.4475	818.8777	808.0607	809.0008	805.0199	816.6917	809.7646	809.0209	823.3392
Emission (t/h)	0.3969	0.4089	0.4013	0.4157	0.4184	0.3983	0.2834	0.4217	0.4161	0.4082	0.4083	0.4163	0.3591	0.3949
*P*loss (MW)	10.8470	10.9693	9.9635	10.9218	10.3078	10.4921	7.9229	10.4541	11.9185	10.3391	11.1495	10.6552	10.0520	9.7062
VD (p.u.)	0.5164	0.6003	0.5540	0.3857	0.3942	0.2897	0.3676	0.5822	0.6191	0.2977	0.4431	0.2577	0.4545	0.4163
L-index (max)	0.1571	0.1548	0.1419	0.1447	0.1454	0.1449	0.1486	0.1532	0.1548	0.1440	0.1498	0.1432	0.1383	0.1559
Fitness	844.7861	841.0672	842.5062	843.1217	839.6750	839.7907	855.0277	838.8985	841.8591	837.4052	848.5185	842.2027	846.9876	858.8394
Time (Sec)	84.25	102.87	177.93	121.02	92.72	94.63	167.27	95.59	97.37	87.02	239.14	265.75	103.62	359.08
Case 6	**SL-GWO**	**IGWO**	**GWO**	**EGWO**	**IGWO-DE**	**WOA**	**SSA**	**ChOA**	**MFO**	**SOA**	**P-ObGWO**	**SOGWO**	**CGWO**	**ACGWO**
Fuel cost ($/h)	859.4417	855.9680	863.2529	879.3434	860.3550	886.1724	851.4467	887.7211	843.5025	830.0982	878.3033	870.1330	824.4190	877.0902
Emission (t/h)	0.2429	0.2446	0.2345	0.2356	0.2439	0.2191	0.2350	0.2319	0.2468	0.2632	0.2374	0.2246	0.2847	0.2411
*P*loss (MW)	5.5886	5.5077	5.6225	5.2856	5.5045	5.1982	5.6197	5.3985	5.8191	6.5358	6.1530	5.6857	6.9327	5.5030
VD (p.u.)	0.5144	0.5832	0.4129	0.3597	0.3997	0.5158	0.2198	0.3043	0.5379	0.3994	0.3791	0.4201	0.3248	0.5450
L-index (max)	0.1552	0.1394	0.1580	0.1529	0.1491	0.1553	0.1470	0.1462	0.1468	0.1473	0.1406	0.1446	0.1490	0.1539
Fitness	1072.7779	1077.8689	1081.8408	1079.9584	1074.4104	1073.1575	1073.5909	1087.1835	1075.2336	1087.6286	1075.3076	1078.7900	1090.9538	1092.8159
Time (Sec)	97.06	89.81	104.51	180.80	126.32	97.00	99.66	173.28	100.14	99.80	245.90	272.82	108.22	361.67
Case 7	**SL-GWO**	**GWO**	**EGWO**	**IGWO-DE**	**WOA**	**SSA**	**ChOA**	**MFO**	**SOA**	**MEGWO**	**P-ObGWO**	**SOGWO**	**CGWO**	**ACGWO**
Fuel cost ($/h)	806.7395	805.7157	807.9841	815.8959	812.1537	822.4331	836.2400	805.3794	805.9845	819.7316	825.4741	822.9353	807.4576	855.3140
Emission (t/h)	0.3513	0.3738	0.3442	0.2910	0.3214	0.3115	0.3678	0.3452	0.3661	0.3212	0.2714	0.3189	0.3638	0.2463
*P*loss (MW)	9.3656	10.2967	9.9213	7.6421	9.1431	8.2285	9.7747	9.2723	10.1426	8.3808	6.9948	8.6497	9.2520	6.5445
VD (p.u.)	0.1962	0.2532	0.2153	0.2933	0.2318	0.2691	0.2306	0.3123	0.2156	0.1625	0.3268	0.3249	0.4031	0.4146
L-index (max)	0.1465	0.1459	0.1496	0.1448	0.1429	0.1504	0.1466	0.1465	0.1435	0.1488	0.1424	0.1471	0.1523	0.1397
Fitness	825.0310	827.4148	828.9426	836.3961	834.6756	838.8519	843.1904	829.5853	827.3612	835.7102	832.4634	832.1626	839.8758	853.9033
Time (Sec)	94.99	99.67	177.84	120.35	91.42	95.81	171.53	98.14	97.30	84.28	240.15	265.31	105.33	358.45
Case 8	**IGWO**	**GWO**	**EGWO**	**IGWO-DE**	**WOA**	**SSA**	**ChOA**	**MFO**	**SOA**	**MEGWO**	**P-ObGWO**	**SOGWO**	**CGWO**	**ACGWO**
Fuel cost ($/h)	809.5927	807.3191	812.0688	817.5376	822.7287	823.9069	832.2609	810.9403	813.8780	818.0459	818.7982	813.7185	873.6576	833.4160
Emission (t/h)	0.3453	0.3410	0.3715	0.3168	0.3457	0.3262	0.2851	0.3743	0.3676	0.3262	0.3210	0.3100	0.2600	0.2846
*P*loss (MW)	8.6634	9.7941	10.4612	9.0513	9.5213	7.8254	7.1201	10.5613	9.8725	8.8941	8.8839	9.1878	5.8524	8.1641
VD (p.u.)	0.3921	0.5003	0.1963	0.3382	0.5161	0.4697	0.2820	0.4138	0.5544	0.3966	0.4866	0.3529	0.4485	0.5523
L-index (max)	0.1492	0.1488	0.1474	0.1499	0.1522	0.1442	0.1517	0.1476	0.1592	0.1576	0.1567	0.1417	0.1580	0.1486
Fitness	824.5139	822.1990	821.1655	832.3187	829.9858	838.0249	847.0018	828.0694	822.3863	827.8792	833.3612	828.9275	841.2996	839.3221
Time (Sec)	86.24	102.53	179.77	122.64	91.68	95.47	170.26	95.19	97.35	87.34	241.00	266.18	104.74	355.71
Case 9	**SL-GWO**	**IGWO**	**GWO**	**EGWO**	**IGWO-DE**	**SSA**	**ChOA**	**MFO**	**SOA**	**WOA**	**P-ObGWO**	**SOGWO**	**CGWO**	**ACGWO**
Fuel cost ($/h)	827.1319	827.1228	819.0939	832.1327	818.4140	842.6760	830.0410	848.3156	823.8110	833.4370	831.8413	818.2210	820.2493	831.9914
Emission (t/h)	0.2666	0.2583	0.2845	0.2581	0.2848	0.2484	0.2604	0.2371	0.2813	0.2548	0.2600	0.2878	0.2742	0.2775
*P*loss (MW)	6.4232	6.2403	7.3428	6.4148	7.3640	6.0128	6.5951	5.6415	6.7497	6.2768	6.3611	7.3623	7.2237	7.5020
VD (p.u.)	0.1800	0.2887	0.2225	0.2980	0.4689	0.2145	0.3511	0.4016	0.3090	0.2296	0.3155	0.4235	0.2885	0.2536
L-index (max)	0.1492	0.1456	0.1416	0.1507	0.1419	0.1469	0.1451	0.1466	0.1441	0.1436	0.1499	0.1482	0.1488	0.1481
Fitness	977.0839	974.5799	987.5201	983.4123	995.6815	983.8153	987.3692	985.2893	983.7487	980.7635	981.9286	984.7328	990.1335	1006.2443
Time (Sec)	95.44	88.06	103.31	179.68	124.73	97.81	171.35	98.32	98.65	95.45	243.01	270.32	106.56	359.39

TABLE 40. Tabulation of the best solutions and computational times of OPF for the IEEE 50-bus system for all the algorithms in comparative analysis.

Case 10	SL-GWO	GWO	EGWO	IGWO-DE	WOA	SSA	ChOA	MFO	SOA	MEGWO	P-ObGWO	SOGWO	CGWO	ACGWO
Fuel cost ($/h)	41678.55	41688.16	41691.41	41722.15	41682.51	41684.04	41707.71	41680.58	41689.35	41680.17	41686.75	41691.58	41714.55	41709.38
Emission (t/h)	1.3502	1.3427	1.3525	1.3649	1.3653	1.3547	1.3549	1.3775	1.3603	1.3528	1.3478	1.3523	1.3440	1.3812
*P*loss (MW)	15.1761	15.2426	15.4031	16.1834	15.1978	15.2908	15.7762	15.2756	15.4424	15.1786	15.2626	15.2670	15.6853	15.1874
VD (p.u.)	1.5080	1.2960	1.2919	1.2598	1.3413	1.4226	1.1650	1.5883	1.4192	1.4724	1.3024	1.4122	1.1635	1.0706
L-index (max)	0.2815	0.2826	0.2827	0.2893	0.2824	0.2804	0.2861	0.2783	0.2824	0.2808	0.2841	0.2831	0.2905	0.2855
Fitness	41678.52	41687.75	41690.80	41721.34	41681.75	41683.99	41707.57	41680.47	41689.07	41679.96	41687.21	41691.66	41713.02	41705.80
Time (Sec)	92.25	105.39	183.28	127.23	97.36	98.97	175.80	99.49	90.22	90.41	280.76	275.74	108.70	369.49
Case 11	IGWO	GWO	EGWO	IGWO-DE	WOA	SSA	ChOA	MFO	SOA	MEGWO	P-ObGWO	SOGWO	CGWO	ACGWO
Fuel cost ($/h)	41688.20	41711.74	41721.12	41710.70	41696.86	41703.83	41732.19	41700.37	41718.86	41716.29	41743.11	41723.71	41746.20	41711.24
Emission (t/h)	1.3578	1.3661	1.3649	1.3384	1.3571	1.3594	1.4470	1.3650	1.3433	1.3767	1.3619	1.3406	1.3615	1.3474
*P*loss (MW)	15.3677	16.0258	16.4107	15.7839	15.4737	15.6163	16.3453	15.7299	16.0929	15.6735	16.4641	16.2790	16.1483	15.6897
VD (p.u.)	0.9664	0.8144	0.7726	0.8915	0.9372	0.9564	0.9354	0.9366	0.8400	0.7885	0.7264	1.0130	0.8107	1.0417
L-index (max)	0.2905	0.2922	0.2927	0.2906	0.2869	0.2918	0.3015	0.2914	0.2923	0.2923	0.2920	0.2905	0.2957	0.2888
Fitness	41785.40	41792.92	41795.31	41797.36	41787.06	41789.67	41814.51	41788.30	41802.10	41791.90	41814.92	41820.87	41821.64	41811.60
Time (Sec)	82.66	99.25	179.15	127.34	97.10	98.74	168.21	99.47	83.13	84.75	278.80	267.49	102.89	363.17
Case 12	IGWO	GWO	EGWO	IGWO-DE	WOA	SSA	ChOA	MFO	SOA	MEGWO	P-ObGWO	SOGWO	CGWO	ACGWO
Fuel cost ($/h)	41691.49	41693.96	41734.22	41728.26	41701.92	41698.78	41762.64	41712.18	41709.79	41702.16	41761.22	41705.63	41751.43	41739.75
Emission (t/h)	1.3603	1.3655	1.3499	1.3608	1.3726	1.3551	1.3972	1.3898	1.3117	1.3732	1.3653	1.3593	1.4403	1.3537
*P*loss (MW)	15.4193	15.5328	16.4044	15.6271	15.7344	15.2350	16.2283	15.6389	15.4787	15.3908	16.6511	15.6763	15.9618	16.6000
VD (p.u.)	1.3517	1.3674	1.2491	1.0354	1.1049	1.1538	1.0016	1.0251	0.9777	1.2737	1.0636	1.3034	1.1256	0.8633
L-index (max)	0.2825	0.2797	0.2961	0.2939	0.2861	0.2865	0.2994	0.2884	0.2866	0.2818	0.2863	0.2805	0.2947	0.2930
Fitness	41716.18	41721.36	41757.95	41750.20	41730.15	41726.02	41771.66	41739.36	41737.76	41726.28	41790.00	41728.81	41777.30	41765.26
Time (Sec)	97.07	112.99	183.57	133.74	100.90	102.91	184.28	101.92	98.68	91.34	290.00	276.59	109.74	374.12
Case 13	IGWO	GWO	EGWO	IGWO-DE	WOA	SSA	ChOA	MFO	SOA	MEGWO	P-ObGWO	SOGWO	CGWO	ACGWO
Fuel cost ($/h)	44912.66	47115.79	52097.03	45443.07	46601.41	49688.96	51610.18	52947.70	48108.57	45628.21	43191.16	56911.17	47074.98	59459.67
Emission (t/h)	1.3269	1.6399	1.8398	1.2335	1.5913	2.0213	1.7021	1.8023	1.6660	1.3955	1.3227	2.2764	1.5643	2.0792
*P*loss (MW)	17.7333	23.2346	30.0671	20.0605	21.8053	26.7577	28.7924	31.4474	21.8837	21.1965	15.7432	41.2448	28.6063	46.1367
VD (p.u.)	0.6201	0.7037	0.7087	0.7327	0.6675	0.6885	0.7284	0.6722	0.6503	0.6741	0.9497	0.7275	0.7203	0.8185
L-index (max)	0.3015	0.2930	0.2971	0.2992	0.2993	0.2945	0.2975	0.2989	0.2984	0.3010	0.3089	0.2999	0.2946	0.2913
Fitness	0.6108	0.6602	0.6862	0.7086	0.6629	0.6853	0.7451	0.6667	0.6477	0.6477	0.7424	0.7045	0.7045	0.7004
Time (Sec)	92.41	109.94	186.00	130.00	106.66	103.65	178.74	102.46	90.92	90.73	285.37	277.36	112.33	376.23

ORPD for the IEEE 30 and IEEE 57-bus system are shown in Table 43 and Table 44 respectively.

From Table 43 and Table 44:

- Similar to the first problem, the performance of SL-GWO has been good for two out of the four cases for the ORPD of the IEEE 30-bus systems and three out of the four cases for the ORPD of the 57-bus system.
- SL-GWO is good at handling problems with a higher number of decision variables as seen from the first and second problems.
- The adaptive symbiotic learning system is quite effective at achieving a good trade-off between exploration and exploitation and the static penalty approach has been efficient at generating feasible solutions without any violations of any of the constraints.
- SL-GWO exhibited better local search capabilities as it was able to generate solutions with better accuracy and the least deviation from the best values obtained.
- For all the 15 algorithms chosen in the current comparative analysis, no constraint violations have been reported.
- Next to SL-GWO, I-GWO and MEGWO performed well for the other cases in both the bus systems.

A comparison of the best results, and average computational times recorded by the other algorithms chosen for the comparative analysis are provided in Table 45 and Table 46 for the ORPD of various cases for the IEEE 30 and IEEE 57-bus system respectively.

A comparison of the best, worst, mean and standard deviation of the optimal costs recorded by the other algorithms chosen for the comparative analysis are provided in Table 47 and Table 48 for the ORPD of various cases for the IEEE 30 and IEEE 57-bus system respectively

From Table 45, Table 46, Table 47 and Table 48:

- The performances of SL-GWO, MEGWO and I-GWO have been excellent for the ORPD for both IEEE 30-bus and IEEE 57-bus systems. These algorithms reported the best fitness scores and had the least standard deviations for most of the cases. The computational times of these algorithms are lower as well. This is indicative of the algorithms' capability at handling multiple constraints with a good exploitation system.
- Unlike the chaotic variants (CGWO and ACGWO), which ended up with the highest fitness scores, the performance of ChOA was notably better by a small margin. Although the computational times of ACGWO were twice that of ChOA, it could not effectively explore and exploit the search landscape. Setting aside the average performance, the chaotic variants (CGWO and ACGWO) had a good population diversity with a greater difference in the fitness score for every iteration.

TABLE 41. Comparison of the best, worst, mean and standard deviation of the fifteen considered algorithms for OPF (IEEE 30-bus system.).

	IGWO	GWO	EGWO	IGWO-DE	WOA	SSA	ChOA	MFO	SOA	MEGWO	P-ObGWO	SOGWO	CGWO	ACGWO	SL-GWO
Case 1															
Best	803.9406	805.9077	804.4170	803.9480	805.2804	803.9101	809.1316	803.3171	805.0028	806.2742	804.4132	809.5346	812.7059	813.7577	802.5928
Worst	813.6517	817.1566	815.6659	815.1970	845.0436	815.1590	885.8444	814.5660	844.6710	809.2627	843.7818	820.7836	889.5378	890.5156	803.9053
Mean	810.8104	808.4002	807.0084	805.9723	815.7648	807.0350	815.4141	805.1068	808.8195	806.8867	810.9386	813.2621	825.0232	834.0240	802.9250
Std	2.3426	4.8986	4.8464	3.4862	11.1845	4.0765	20.3962	4.0210	10.5241	0.8339	14.0147	5.1318	27.5208	32.4212	0.4132
Case 2															
Best	0.1389	0.1408	0.1400	0.1398	0.1400	0.1393	0.1411	0.1410	0.1393	0.1397	0.1394	0.1393	0.1398	0.1401	0.1392
Worst	0.1445	0.1894	0.1830	0.1895	0.1671	0.1809	0.1986	0.1752	0.1719	0.1579	0.2013	0.1680	0.1682	0.1861	0.1590
Mean	0.1388	0.1501	0.1515	0.1499	0.1466	0.1451	0.1614	0.1495	0.1488	0.1468	0.1484	0.1479	0.1450	0.1490	0.1440
Std	0.0080	0.0215	0.0400	0.0183	0.0329	0.0404	0.0342	0.0139	0.0490	0.0055	0.0649	0.0133	0.0446	0.0547	0.0076
Case 3															
Best	0.2213	0.2152	0.2231	0.2269	0.2193	0.2162	0.2377	0.2252	0.2181	0.2157	0.2217	0.2499	0.2200	0.2236	0.1389
Worst	0.2400	0.2533	0.2538	0.2424	0.2208	0.2591	0.2681	0.2447	0.2701	0.2431	0.2515	0.3043	0.2690	0.2683	0.1445
Mean	0.2268	0.2284	0.2314	0.2316	0.2136	0.2345	0.2466	0.2200	0.2317	0.2236	0.2342	0.2681	0.2328	0.2356	0.1388
Std	0.0064	0.0152	0.0411	0.0205	0.0511	0.0375	0.0604	0.0305	0.0799	0.0071	0.0565	0.0234	0.0656	0.0436	0.0080
Case 4															
Best	3.6110	4.0615	4.2219	4.1463	4.0747	3.9711	4.2822	3.8256	4.2873	3.7543	3.8230	3.7701	4.0776	4.7932	3.5842
Worst	3.7809	4.5615	4.7219	4.6463	4.4525	4.4711	4.7822	4.3256	4.8712	3.9659	4.0175	4.3746	4.5776	5.2972	3.8565
Mean	3.6748	4.1566	4.3989	4.2940	4.1932	4.0960	4.4751	3.9831	4.2801	3.8256	3.8579	3.9780	4.1433	4.9456	3.7236
Std	0.0610	0.1276	0.1876	0.4492	0.3420	0.1948	0.2181	0.3508	0.2613	0.0817	0.0724	0.2746	0.2734	0.1866	0.1175
Case 5															
Best	844.7861	841.0672	842.5062	843.1217	839.6750	839.7907	855.0277	838.8985	841.8591	837.4052	848.5185	842.2027	846.9876	858.8394	835.5420
Worst	856.0350	842.6252	848.9582	849.1432	881.1782	851.0396	897.2662	846.5588	853.1080	845.3572	854.0337	847.9910	888.3985	901.0981	838.5395
Mean	847.9338	841.6050	843.6318	844.7185	844.9550	841.5203	860.5118	839.9989	845.0308	839.0070	849.5276	844.1370	854.8537	867.8759	836.1784
Std	3.4783	0.5675	1.8144	1.9329	12.4796	3.7394	10.4013	2.8881	4.2003	2.5089	1.3440	6.0511	13.5490	16.6990	1.0226
Case 6															
Best	1077.8689	1077.8689	1081.8408	1079.9584	1074.4104	1073.1575	1073.5909	1087.1835	1075.2336	1071.7209	1075.3076	1078.7901	1090.9538	1092.8159	1072.7779
Worst	1085.3104	1085.3104	1093.0897	1085.5127	1126.8644	1084.4065	1126.2656	1140.7210	1086.4825	1074.7109	1081.6851	1090.0391	1144.2064	1146.4360	1074.9560
Mean	1079.8712	1079.8712	1083.7137	1081.0646	1082.2249	1076.5301	1079.4866	1093.1640	1077.6962	1072.9441	1076.6653	1081.6118	1100.2208	1097.0826	1073.3504
Std	2.8790	2.8790	3.3238	1.7576	14.5623	4.6448	13.0473	12.9605	3.4631	1.3191	1.9993	5.0186	17.3917	13.4465	0.6869
Case 7															
Best	818.3415	827.4148	828.9426	836.3961	834.6756	838.8519	843.1904	829.5853	827.3612	835.7102	832.4634	832.1626	839.8757	853.9033	825.0310
Worst	819.5409	838.6638	840.1916	847.6451	875.4624	850.1009	884.7856	840.8342	838.6102	841.7016	873.3801	843.4116	918.9836	895.6063	827.7732
Mean	819.0971	829.9406	832.6161	839.6729	841.9845	842.0026	850.4364	832.4891	829.6601	836.7544	838.5862	836.3537	846.0214	866.1580	825.5994
Std	0.5834	3.9534	4.8513	6.1705	13.5635	7.3753	13.7168	4.0935	3.1898	1.5294	11.7364	5.4461	19.6456	17.7753	0.9360
Case 8															
Best	824.5139	822.1990	821.1655	832.3187	829.9858	838.0249	847.0018	828.0694	822.3863	827.8792	833.3612	828.9275	841.2996	839.3221	816.5509
Worst	827.5677	833.4480	822.4200	843.5676	870.6514	879.3912	888.8043	839.3183	833.6353	829.1849	840.9481	869.4338	882.5944	880.4725	822.4194
Mean	825.2599	823.5510	821.8117	836.2486	837.1039	842.5269	853.6219	830.7595	825.8022	828.3216	834.8995	832.7225	847.7066	843.5907	817.1022
Std	1.0310	3.8780	0.4690	4.0676	11.4374	10.4973	14.2536	6.1949	4.2647	0.5236	2.4728	8.0926	13.7644	11.1917	1.4940
Case 9															
Best	974.5799	987.5201	983.4123	995.6815	980.7635	983.8153	987.3692	985.2893	983.7487	971.4820	981.9286	984.7328	990.1335	1006.2443	977.0839
Worst	980.5464	998.7690	994.6613	1044.7627	1028.7502	995.0643	1035.6454	991.5441	994.9976	982.7309	993.1775	995.9817	1038.6244	1055.3880	982.7108
Mean	975.7155	989.7524	986.7223	999.7782	984.2894	985.6712	997.4921	986.0318	986.0488	972.9386	983.9211	988.6103	994.9589	1018.1556	978.2672
Std	2.9258	3.5030	4.7745	12.1504	10.8610	5.6832	15.5402	1.5191	5.0864	2.7474	3.6672	4.1808	12.0525	18.7173	1.7965

TABLE 42. Comparison of the best, worst, mean and standard deviation of the fifteen considered algorithms for OPF (IEEE 57-bus system.).

	IGWO	GWO	EGWO	IGWO-DE	WOA	SSA	ChOA	MFO	SOA	MEGWO	P-ObGWO	SOGWO	CGWO	ACGWO	SL-GWO
Case 10															
Best	**41672.75**	41687.75	41690.80	41721.34	41681.75	41683.99	41707.57	41680.47	41689.07	41679.96	41687.21	41691.66	41713.02	41705.80	41678.52
Worst	**41675.22**	41698.99	41696.58	41732.59	41707.20	41704.30	41732.89	41705.51	41699.78	41690.46	41702.25	41712.21	41738.96	41730.82	41679.64
Mean	**41673.52**	41689.26	41692.27	41723.43	41687.78	41689.57	41717.02	41686.44	41691.71	41681.98	41693.70	41696.74	41719.74	41714.60	41679.00
Std	**0.8646**	4.1792	1.7364	5.5258	8.0171	7.4637	10.6698	9.0190	3.8117	1.9888	5.0655	8.6336	12.5512	11.6712	0.3488
Case 11															
Best	41785.40	41792.92	41795.31	41797.36	41787.06	41789.67	41814.51	41788.30	41802.10	41791.90	41814.92	41820.87	41821.64	41811.60	**41781.57**
Worst	41790.11	41804.17	41811.18	41810.33	41808.87	41805.40	41830.29	41810.60	41824.17	41803.15	41827.60	41843.32	41846.74	41837.55	**41787.42**
Mean	41787.02	41793.93	41801.38	41800.13	41792.37	41795.93	41820.38	41792.43	41810.29	41793.57	41819.31	41825.04	41832.50	41821.66	**41783.09**
Std	1.7054	3.5871	5.8718	4.0516	8.2947	6.4557	7.4507	6.5354	8.6469	2.8783	4.8049	7.5427	10.8794	12.4112	**1.8091**
Case 12															
Best	**41691.49**	41693.96	41734.22	41728.26	41701.92	41698.78	41762.64	41712.18	41709.79	41702.16	41761.22	41705.63	41751.43	41739.75	41707.94
Worst	**41697.24**	41704.83	41745.47	41739.50	41724.35	41720.91	41788.14	41734.02	41732.76	41708.03	41772.06	41718.01	41777.11	41765.31	41714.03
Mean	**41692.81**	41695.50	41736.52	41732.02	41708.51	41705.30	41769.82	41716.59	41714.10	41703.91	41764.04	41708.53	41758.70	41752.13	41709.76
Std	**1.3360**	2.6046	5.6536	3.7146	9.5109	7.7075	10.9446	6.9160	8.8851	1.8728	4.0426	4.5819	11.1877	14.4117	2.2591
Case 13															
Best	0.61084	0.66017	0.68618	0.70857	0.66295	0.68527	0.74505	0.66667	0.64774	0.64767	0.74237	0.70445	0.70451	0.70038	**0.60894**
Worst	0.67022	0.69687	0.73189	0.74713	0.67673	0.72297	0.78658	0.71166	0.70652	0.65989	0.80372	0.72909	0.72797	0.75792	**0.63568**
Mean	0.63695	0.66906	0.69856	0.72349	0.66031	0.69450	0.75315	0.67657	0.67959	0.65129	0.74628	0.70564	0.70681	0.71325	**0.61303**
Std	0.01614	0.01502	0.05457	0.02196	0.03562	0.03394	0.03928	0.01418	0.02895	0.00517	0.05565	0.02337	0.06357	0.06649	**0.00625**

- The modern meta-heuristics could not deliver on par with the variants of GWO with WOA and SSA delivering the best performance amongst them.
- Similar to the previous case, the computational times of ACGWO, P-ObGWO and SOGWO were the highest followed by ChOA for the modern meta-heuristics.

TABLE 43. Tabulation of the best solutions of ORPD for the IEEE 30-bus system.

Decision Variables	Case 1	Case 1a	Case 2	Case 2a
Algorithm with the best Fitness	SL-GWO	MEGWO	IGWO	SL-GWO
V1 (p.u.)	1.066982	1.070304	1.016391	1.017222
V2 (p.u.)	1.058566	1.06318	1.017527	1.01637
V5 (p.u.)	1.035146	1.041312	1.020023	1.020442
V8 (p.u.)	1.041041	1.040253	1.006107	1.005665
V11 (p.u.)	1.090586	1.081645	1.026519	1.019371
V13 (p.u.)	1.056507	1.060993	0.987012	1.005967
T11 (p.u.)	1.052998	1.03101	1.035965	1.031045
T12 (p.u.)	0.911824	0.944873	0.909582	0.901893
T15 (p.u.)	1.009935	0.994442	0.931305	0.975098
T36 (p.u.)	0.986103	0.994831	0.966773	0.966217
QC10 (MVAr)	0.318036	3.177253	3.561997	4.361713
QC12 (MVAr)	4.820141	2.141947	1.239576	2.937927
QC15 (MVAr)	3.500367	4.675227	4.956077	4.912182
QC17 (MVAr)	4.765342	4.820366	1.670654	0.012509
QC20 (MVAr)	3.693621	0.697719	4.950221	4.995945
QC21 (MVAr)	4.957431	4.556395	4.976898	4.997213
QC23 (MVAr)	3.243832	1.695415	4.969468	4.996983
QC24 (MVAr)	4.90515	3.716244	4.949874	4.997173
QC29 (MVAr)	2.694546	3.976038	1.945717	2.584383
Ploss (MW)	4.423948	4.902463	5.239073	5.648152
VD (p.u.)	0.842723	0.824006	0.092961	0.091278
Fitness	4.423948	4.902463	0.092961	0.091278
Computational Time (Sec)	103.36	103.07	114.30	100.91

TABLE 44. Tabulation of the best solutions of ORPD for the IEEE 57-bus system.

Decision Variables	Case 11	Case 11a	Case 12	Case 12a
Algorithm with the best Fitness	SL-GWO	SL-GWO	IGWO	SL-GWO
V1 (p.u.)	1.080156	1.082052	1.015548	1.006178
V2 (p.u.)	1.072367	1.070723	1.00549	0.997087
V3 (p.u.)	1.056883	1.062784	1.004898	1.004601
V6 (p.u.)	1.056491	1.057305	1.000671	1.004212
V8 (p.u.)	1.069494	1.068796	1.016925	1.034857
V9 (p.u.)	1.042096	1.038669	0.993802	1.005608
V12 (p.u.)	1.042777	1.046749	1.008357	1.026848
T19 (p.u.)	1.008795	0.974603	0.960536	0.968411
T20 (p.u.)	1.075684	1.076653	1.001066	1.031683
T31 (p.u.)	1.045331	1.041548	0.983396	0.972239
T35 (p.u.)	1.022584	1.025371	0.997549	1.05359
T36 (p.u.)	1.066536	0.962227	1.041091	0.920526
T37 (p.u.)	1.069486	1.006292	0.990024	1.014981
T41 (p.u.)	1.014294	0.992809	0.971565	0.994178
T46 (p.u.)	0.975538	0.94956	0.925734	0.948408
T54 (p.u.)	0.963517	1.055088	0.944535	0.900031
T58 (p.u.)	0.986275	0.98541	0.931295	0.954169
T59 (p.u.)	0.980051	0.981969	0.913039	0.946821
T65 (p.u.)	0.988505	0.991298	0.988173	1.006273
T66 (p.u.)	0.94997	0.946293	0.900554	0.900146
T71 (p.u.)	1.014177	0.956622	0.929609	0.930552
T73 (p.u.)	1.023043	1.006242	0.966261	1.064027
T76 (p.u.)	0.963781	0.983483	0.927647	0.900524
T80 (p.u.)	1.015433	0.979868	0.976583	0.988818
QC18 (MVAr)	14.44706	8.907383	6.169421	9.912319
QC25 (MVAr)	13.61141	12.97264	13.57551	14.87556
QC53 (MVAr)	13.71829	9.894456	19.34501	19.902
Ploss (MW)	18.75129	23.70714	21.30825	28.15686
VD (p.u.)	1.267441	1.415128	0.687064	0.641429
Fitness	18.75129	23.70714	0.687064	0.641429
Computational Time (Sec)	116.47	121.67	117.20	121.04

MEGWO and I-GWO had the lowest computational times followed by SSA and SL-GWO.

VI. MERITS AND DEMERITS

Meta-heuristic improvement and enhancement strategies have benefited researchers in extending the potential of optimization to new heights. While the improvement tactics are frequently highly successful and efficient, they are not without their share of criticism and challenges. Thus, in order to provide a balanced and unprejudiced assessment, the pros and demerits of SL-GWO are outlined below.

A. MERITS

- SL- GWO's performance was unaffected by the increasing number of problem dimensions. The standard GWO algorithm or any of its variants could match SL- GWO's

TABLE 45. Tabulation of the best solutions and computational times of ORPD for the IEEE 30-bus system for all the algorithms in comparative analysis.

Case 1	IGWO	GWO	EGWO	IGWO-DE	WOA	SSA	ChOA	MFO	SOA	MEGWO	P-ObGWO	SOGWO	CGWO	ACGWO
Ploss (MW) (Fitness)	4.4327	4.4580	4.4714	4.5307	4.4490	4.5979	4.5393	4.7437	4.6245	4.4435	4.6191	4.4649	4.5193	4.5274
VD (p.u.)	0.8742	0.8003	0.8018	0.5668	0.8531	0.5661	0.7417	0.2905	0.7664	0.7815	0.4079	0.6529	0.5364	0.6666
Time (Sec)	117.56	120.13	200.56	148.37	107.42	108.82	201.63	96.11	112.29	100.93	276.22	303.67	123.48	406.01
Case 1a	**SL-GWO**	**IGWO**	**GWO**	**EGWO**	**IGWO-DE**	**WOA**	**SSA**	**ChOA**	**MFO**	**SOA**	**P-ObGWO**	**SOGWO**	**CGWO**	**ACGWO**
Ploss (MW) (Fitness)	4.9083	4.9114	4.9204	4.9429	4.9542	4.9401	4.9367	5.0251	4.9302	5.0115	4.9990	4.9430	5.0007	5.1979
VD (p.u.)	0.8548	0.7098	0.8302	0.8067	0.8340	0.6213	0.7127	0.5299	0.7855	0.3970	0.5220	0.7917	0.4515	0.2618
Time (Sec)	105.93	109.12	116.39	201.47	141.01	106.40	112.77	201.09	95.75	112.59	274.53	310.68	122.15	412.41
Case 2	**SL-GWO**	**GWO**	**EGWO**	**IGWO-DE**	**WOA**	**SSA**	**ChOA**	**MFO**	**SOA**	**MEGWO**	**P-ObGWO**	**SOGWO**	**CGWO**	**ACGWO**
Ploss (MW)	5.4039	5.4057	5.2022	5.3963	5.2301	5.0850	5.4092	5.3604	5.2330	5.3995	5.2015	5.3673	5.2505	5.1754
VD (p.u.) (Fitness)	0.0993	0.1104	0.0993	0.1126	0.1248	0.1100	0.1018	0.1151	0.1145	0.1090	0.1215	0.1267	0.1194	0.1394
Time (Sec)	103.48	117.12	208.39	148.58	112.45	109.86	203.24	103.01	116.56	97.41	281.36	311.59	125.34	409.48
Case 2a	**IGWO**	**GWO**	**EGWO**	**IGWO-DE**	**WOA**	**SSA**	**ChOA**	**MFO**	**SOA**	**MEGWO**	**P-ObGWO**	**SOGWO**	**CGWO**	**ACGWO**
Ploss (MW)	5.6659	5.7298	5.8891	5.5543	5.4973	5.5343	5.7541	5.5721	5.9119	5.6546	5.6475	5.8895	5.5144	6.3509
VD (p.u.) (Fitness)	0.0960	0.1144	0.1393	0.1120	0.1293	0.1092	0.1340	0.1168	0.1111	0.0969	0.1355	0.1315	0.1533	0.1177
Time (Sec)	108.24	115.27	200.47	139.16	106.49	104.50	197.18	95.07	107.75	97.01	273.41	301.60	118.89	403.63

TABLE 46. Tabulation of the best solutions and computational times of ORPD for the IEEE 57-bus system for all the algorithms in comparative analysis.

Case 11	IGWO	GWO	EGWO	IGWO-DE	WOA	SSA	ChOA	MFO	SOA	MEGWO	P-ObGWO	SOGWO	CGWO	ACGWO
Ploss (MW) (Fitness)	18.7882	19.0803	18.9256	19.8057	19.2048	19.5198	19.2490	18.8104	19.4486	18.7671	19.8057	19.7094	19.0954	20.0756
VD (p.u.)	1.6566	1.3672	1.4592	1.1152	1.1463	1.2410	1.2235	1.4723	1.0566	1.3626	1.1152	1.1219	1.2307	1.3912
Time (Sec)	115.80	138.04	236.75	166.32	130.28	110.27	287.24	112.26	101.34	98.74	319.03	343.02	144.61	468.84
Case 11a	**IGWO**	**GWO**	**EGWO**	**IGWO-DE**	**WOA**	**SSA**	**ChOA**	**MFO**	**SOA**	**MEGWO**	**P-ObGWO**	**SOGWO**	**CGWO**	**ACGWO**
Ploss (MW) (Fitness)	23.8770	24.3287	24.0072	23.7918	23.7930	23.8962	24.2096	24.4338	24.0986	23.7986	23.9774	25.0798	24.9565	25.1836
VD (p.u.)	1.3054	1.2486	1.5618	1.5056	1.4744	1.3793	1.3037	1.1851	1.3874	1.3675	1.3272	1.0709	1.2044	1.1439
Time (Sec)	117.49	135.07	236.42	165.60	126.38	107.57	282.31	108.94	109.42	105.94	309.24	350.67	141.63	464.52
Case 12	**SL-GWO**	**GWO**	**EGWO**	**IGWO-DE**	**WOA**	**SSA**	**ChOA**	**MFO**	**SOA**	**MEGWO**	**P-ObGWO**	**SOGWO**	**CGWO**	**ACGWO**
Ploss (MW)	21.4760	23.5849	21.0464	21.8505	23.3271	21.2212	22.3823	22.3117	21.7182	21.2761	21.6278	22.9019	21.7697	22.5905
VD (p.u.) (Fitness)	0.7177	0.7447	0.7946	0.7729	0.7512	0.7408	0.8089	0.7643	0.7794	0.7614	0.7252	0.7356	0.8181	0.8760
Time (Sec)	124.57	135.45	228.71	166.11	129.28	106.40	283.23	112.27	109.70	104.18	315.25	346.13	139.86	467.21
Case 12a	**IGWO**	**GWO**	**EGWO**	**IGWO-DE**	**WOA**	**SSA**	**ChOA**	**MFO**	**SOA**	**MEGWO**	**P-ObGWO**	**SOGWO**	**CGWO**	**ACGWO**
Ploss (MW)	28.5747	26.4387	26.6930	26.7102	28.6407	26.6038	27.5835	26.3982	27.2496	27.1455	26.6107	27.0192	27.8448	30.1219
VD (p.u.) (Fitness)	0.6777	0.6891	0.6583	0.6865	0.6810	0.6824	0.7438	0.6785	0.7432	0.7209	0.7163	0.7777	0.8026	0.8064
Time (Sec)	117.68	137.76	236.88	166.38	128.37	108.84	287.12	106.48	109.30	106.51	309.62	350.13	142.59	466.34

TABLE 47. Comparison of the best, worst, mean and standard deviation of the fifteen considered algorithms for ORPD (IEEE 30-bus system.).

	IGWO	GWO	EGWO	IGWO-DE	WOA	SSA	ChOA	MFO	SOA	MEGWO	P-ObGWO	SOGWO	CGWO	ACGWO	SL-GWO
Case 10															
Best	4.4327	4.4580	4.4714	4.5307	4.4490	4.5979	4.5393	4.7437	4.6245	4.4435	4.6191	4.4649	4.5193	4.5274	**4.4239**
Worst	4.7910	4.5637	4.9714	4.7506	4.6881	4.8735	5.1920	4.9485	5.0453	4.8012	5.1191	5.1272	5.1054	5.0274	**4.5682**
Mean	4.5110	4.4883	4.6856	4.5707	4.4906	4.6805	4.7179	4.8325	4.7294	4.5196	4.7730	4.8299	4.6169	4.7668	**4.5251**
Std	0.0980	0.0339	0.2230	0.3231	0.3889	0.4009	0.6059	0.7916	0.6581	0.1764	0.5765	0.6634	0.7513	0.4237	**0.0493**
Case 11															
Best	4.9114	4.9204	4.9429	4.9542	4.9401	4.9367	5.0251	4.9302	5.0115	**4.9025**	4.9990	4.9430	5.0007	5.1979	4.9083
Worst	5.1966	5.4140	5.1618	5.4542	5.5825	5.1540	5.5251	5.4302	5.4729	**5.1642**	5.5764	5.4489	5.6838	5.8123	5.2954
Mean	5.0365	5.0753	4.8731	5.0871	5.0347	5.0410	5.1273	5.0081	5.1150	**4.9960**	5.0687	5.1210	5.1495	5.3246	4.9883
Std	0.1229	0.4082	0.2072	0.5723	0.8860	0.6799	0.7024	0.3443	0.2223	**0.0971**	0.4454	0.8731	0.8515	0.7294	0.0989
Case 12															
Best	**0.0930**	0.1104	0.0993	0.1126	0.1248	0.1100	0.1018	0.1151	0.1145	0.1090	0.1215	0.1267	0.1194	0.1394	0.0993
Worst	**0.0980**	0.1153	0.1057	0.1216	0.1333	0.1133	0.1066	0.1253	0.1210	0.1148	0.1272	0.1341	0.1235	0.1455	0.1043
Mean	**0.0949**	0.1120	0.1024	0.1149	0.1268	0.1109	0.1034	0.1191	0.1144	0.1110	0.1234	0.1284	0.1200	0.1412	0.1003
Std	**0.0020**	0.0019	0.0041	0.0053	0.0033	0.0043	0.0020	0.0031	0.0020	0.0033	0.0027	0.0077	0.0039	0.0037	0.0028
Case 13															
Best	0.0960	0.1144	0.1393	0.1120	0.1293	0.1092	0.1340	0.1168	0.1111	0.0969	0.1355	0.1315	0.1533	0.1177	**0.0913**
Worst	0.1073	0.1237	0.1483	0.1214	0.1397	0.1190	0.1421	0.1264	0.1198	0.1048	0.1472	0.1409	0.1643	0.1293	**0.1015**
Mean	0.0996	0.1161	0.1423	0.1161	0.1306	0.1131	0.1353	0.1191	0.1151	0.0990	0.1400	0.1363	0.1597	0.1197	**0.0946**
Std	0.0041	0.0052	0.0038	0.0062	0.0046	0.0038	0.0052	0.0036	0.0051	0.0035	0.0051	0.0068	0.0046	0.0045	**0.0042**

performance and the five modern meta heuristics used in the comparison show that it is impervious to *the curse of dimensionality*. There are two reasons for this: the symbiotic phase of learning and the greedy selection technique, which promotes elitism and avoids local trapping while essentially relocating the wolf's location from the standard GWO process. The other reason for its superior performance in every

TABLE 48. Comparison of the best, worst, mean and standard deviation of the fifteen considered algorithms for ORPD (IEEE 57-bus system.).

	IGWO	GWO	EGWO	IGWO-DE	WOA	SSA	ChOA	MFO	SOA	MEGWO	P-ObGWO	SOGWO	CGWO	ACGWO	SL-GWO
Case 11															
Best	18.7882	19.0803	18.9256	19.8057	19.2048	19.5198	19.2490	18.8104	19.4486	18.7671	19.8057	19.7094	19.0954	20.0756	**18.7513**
Worst	21.4260	23.1446	22.0141	24.0005	22.4778	23.0198	23.0827	22.6071	22.9486	22.5803	24.0383	24.2838	23.2624	24.0823	**21.2937**
Mean	19.9168	20.1075	20.2787	21.1107	19.8548	21.3671	20.5657	20.2760	20.8791	19.7262	21.7736	21.0566	20.9317	21.1616	**19.6598**
Std	0.8671	3.6457	1.1107	2.4998	2.9745	4.6274	1.9819	5.3237	2.4113	1.3549	3.5670	1.5501	4.6560	3.2449	**1.0815**
Case 11a															
Best	23.8770	24.3287	24.0072	23.7918	23.7930	23.8962	24.2096	24.4338	24.0986	23.7986	23.9774	25.0798	24.9565	25.1836	**23.7071**
Worst	28.0033	28.6624	27.5072	28.2823	27.2930	27.9548	28.0946	28.6949	26.2850	25.7669	28.3000	29.4554	29.4719	29.1627	**27.7019**
Mean	25.4889	26.3776	25.2426	25.3496	24.8367	25.9033	26.3934	25.3878	24.4311	24.5513	25.8604	27.5283	27.0111	27.4993	**25.4395**
Std	1.5247	3.6385	1.2830	1.9566	4.1514	2.6173	3.9545	4.0768	2.8474	0.7683	4.9451	1.7988	4.8294	3.7281	**1.3068**
Case 12															
Best	**0.6871**	0.7447	0.7946	0.7729	0.7512	0.7408	0.8089	0.7643	0.7794	0.7614	0.7252	0.7356	0.8181	0.8760	0.7177
Worst	**0.7083**	0.7911	0.8262	0.7808	0.7902	0.7858	0.8368	0.8019	0.8429	0.7829	0.7659	0.7841	0.8569	0.9253	0.7568
Mean	**0.6957**	0.7588	0.8036	0.7667	0.7491	0.7547	0.8190	0.7745	0.7919	0.7680	0.7390	0.7463	0.8258	0.8859	0.7232
Std	**0.0072**	0.0182	0.0108	0.0348	0.0423	0.0389	0.0625	0.0520	0.0767	0.0073	0.0333	0.0459	0.0616	0.0596	0.0099
Case 12a															
Best	0.6777	0.6891	0.6583	0.6865	0.6810	0.6824	0.7438	0.6785	0.7432	0.7209	0.7163	0.7777	0.8026	0.8064	**0.6414**
Worst	0.7295	0.7476	0.6897	0.7422	0.7316	0.7268	0.7803	0.7089	0.8136	0.7405	0.7614	0.8205	0.8636	0.8468	**0.6824**
Mean	0.6846	0.7046	0.6677	0.7076	0.7004	0.6871	0.7482	0.6896	0.7611	0.7308	0.7280	0.7782	0.8158	0.8225	**0.6584**
Std	0.0238	0.0305	0.0084	0.0194	0.0419	0.0567	0.0095	0.0668	0.0527	0.0077	0.0415	0.0132	0.0623	0.0512	**0.0189**

benchmarking scenario is the equilibrium of the exploration and exploitation system by the three symbiotic learning strategies.

- A good exploration and exploitation balance has been possible through the dynamic and adaptive control of the S_r and D_1. The proposed method delivered solutions at the global optimum or closer to it in most complex benchmarking cases avoiding local entrapment and also had good convergence characteristics with the empirically tuned parametric settings.
- The system of population sub-grouping of the wolfpack into attacking hunters and experienced hunters has been a positive reinforcement, allowing the hunting strategies and adaptive control mechanisms to work in synergy and assist in repositioning omega wolves with increased population diversity, thereby increasing coverage of the search landscape.
- The first and second symbiotic learning strategies, combined with the Greedy selection strategy, promoted elitism by selecting the best wolves, and the best solution is assigned as the alpha wolf and passed to the next iteration to refine and explore the solutions further, as well as to assist the algorithm in gaining a better understanding of the search landscape.
- SL-GWO reduces GWO's reliance on alpha, beta, and delta wolves to relocate each omega wolf. The addition of randomized omega wolves facilitates information transmission between the various wolves and minimizes local trapping that occurs in the standard GWO's population system with less diversity.
- The greedy selection technique encourages elitism and accelerates convergence times by directing the algorithm's search to the possibly best places within the search landscape.

B. DEMERITS

- The inclusion of sorting the wolves based on the fitness in every iteration can append an additional layer of computational complexity and can result in slightly higher computational times.
- SL-GWO's reliance on greedy selection during the symbiotic learning phase may have an effect on its local search capabilities. The greedy selection technique tries to promote elitism by selecting only superior solutions while discarding inferior alternatives. This can sometimes result in slower convergence for a certain hybrid landscape (hybrid test functions) as seen in the CEC2018 test suite.
- Due to SL- GWO's reliance on random omegas (at least seven distinct omega wolves), the population size must always be more than seven. With a population size of less than seven, the algorithm may fail to operate.

VII. CONCLUSION AND FUTURE SCOPE

This article realizes an improved meta-heuristic optimization technique known as SL-GWO to combat *the curse of dimensionality* and improve population diversity through different symbiotic hunting and learning strategies. SL-GWO restructures the standard hierarchical hunting system in GWO through population sub-grouping such that each group acts individually with its own uniquely crafted hunting and control mechanisms. Dynamic tuning through linear and adaptive tuning mechanisms for the two sub-groups of wolves aid the hunt of individual wolves to evolve stronger and fitter over time with diverse hunting instances for the solution dimensions. Despite the computational complexity of SL-GWO being slightly higher than the standard GWO due to the addition of a quick sort mechanism, the revised algorithmic structure achieves excellent local optima avoidance and limits the stagnation occurring within the wolfpack at an individual level. The uniquely crafted four symbiotic hunting schemes incorporating random omega wolves extend the scope of exportation of every grey wolf preventing their collapse into the vicinity of the three dominant wolves. The first and second symbiotic strategies contribute significantly to driving the wolfpack to all the corners of the search landscape to explore and exploit adaptively while the third and fourth hunting strategies encourage convergence capabilities while ensuring that the search mechanism is rotationally invariant. The population selection and progression preserve the personal best contributions of individual grey wolves and encourage the fittest wolves to emerge as the dominant wolves to guide the next generation of wolves to hunt effectively while preserving elitism and diversity.

The proposed method has a better performance in handling constrained and unconstrained problems and has been effective at avoiding local entrapment in complex search landscapes. Extensive benchmarking analysis with the 29 test functions from the CEC2018 benchmarking suite with 10, 30 and 50 dimesons reinforce the algorithms' immunity towards *the curse of dimensionality* while operating on a limited computational budget. The validation with the CEC2019 test suite provides a comprehensive outlook of the algorithm's ability to adapt to complex landscapes proving an enhanced balance at exploration and exploitation while being less prone to local entrapment. A better balance of exploration and exploitation with accelerated convergence has been witnessed across the various test cases with statistically significant performance. Following comprehensive and extensive testing and validation through multiple benchmarking standards and complex real-world optimization problems, the efficiency and efficacy of SL-GWO is validated and its performance is compared against the standard GWO, eight of its latest advanced variants and five recent meta-heuristics. SL-GWO stands out as the best performing meta-heuristic with improved optimality and higher consistency for the various test cases and ranked in the top spots for most of the testing reinforcing its supremacy as an improved and advanced variant of GWO.

SL-GWO may be used to solve a wide range of issues in AI, power systems, machine learning, and related fields. The suggested approach may be customized by practitioners to meet their specific needs, hence SL-GWO was designed with an eye toward extensibility. Other optimization areas in power systems, such as EV optimization, power electronics, smart grid integration, distribution systems, power dispatch issues and control systems can also benefit from the proposed method. The proposed approach may be used to train neural networks (NN) in computer science (feed-forward NNs and convolution NNs). Images, data, and patterns may all be classified more effectively using SL-GWO. Currently, a support vector classifier-based technique for detecting COVID-19 infection from X-ray images is being considered for deployment. This method's binary SL-GWO formulation might be used to make feature selection more efficient. The implementation of a multi-objective variant is a possibility for dealing with issues that need a Pareto-optimal front. A project has been designed to build a multi-objective option for optimizing energy management in electric vehicles. Symbiotic learning methodologies can be used for different meta-heuristics in order to conduct experiments that will enhance the system.

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AALA KALANANDA VAMSI KRISHNA REDDY received the B.Tech. degree in electrical and electronics engineering from Jawaharlal Nehru Technological University, Ananthapur, India, in 2017, and the M.Tech. degree in electrical engineering from Lovely Professional University, Phagwara, India, in 2019. He is currently pursuing the Ph.D. degree with the School of Electrical Engineering, Vellore Institute of Technology, Vellore, India.

His research interests include swarm and evolutionary computation, electric vehicles, and power systems.

KOMANAPALLI VENKATA LAKSHMI NARAYANA (Senior Member, IEEE) received the B.Tech. degree in instrumentation engineering from Nagarjuna University, Guntur, India, in 2001, the M.Tech. degree in electrical engineering from the Motilal Nehru National Institute of Technology, Allahabad, India, in 2006, and the Ph.D. degree in instrumentation engineering from Andhra University, Visakhapatnam, in 2013.

He has 20 years of experience in both teaching and research. He is currently a Professor with the School of Electrical Engineering, Vellore Institute of Technology, Vellore, India. As a Published Researcher, he has authored or coauthored over 50 research papers (five articles in IEEE Sensors Journal and ten more in other SCI journals) and published in various peer-reviewed journals, book chapters, and conferences of international repute. His research interests include sensors and signal conditioning, measurements, wireless sensor networks, optimization, control design, and electric vehicles. He has reviewed a good number of research papers of *IET Science, Measurement and Technology*, IEEE Access, *Artificial Intelligence Review* (MDPI), *Journal of Engineering Science and Technology* (JESTEC), and *Recent Advances in Electrical and Electronic Engineering*. He was a recipient of the Gold Medal Award for his Outstanding Academic Performance in M.Tech. Program at MNNIT, in 2006.