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RESEARCH ARTICLE

Inventory Positioning in Supply Chain Network: A Service-Oriented Approach

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ABSTRACT This study develops a mixed-integer linear programming model based on a guaranteed service approach for an inventory positioning problem in a supply chain under the base stock inventory policy. Our proposed model aims to determine appropriate inventory positions and amounts and the optimal service level for the supply chain to minimize the total cost of safety inventory holding and shortage. Two demand scenarios, based on normal and empirical distributions, are investigated. An extensive numerical experiment is conducted to illustrate the applicability and effectiveness of our model, especially under empirical distribution. The experiment features a practical network structure and demand data from an industrial user. Moreover, to further validate the experimental results from the mathematical model, they are compared with the result from a simulation model, which is constructed to imitate the operations of the supply chain. The comparison result indicates that the model solution under the empirical demand distribution is close to the simulation regarding the difference in the total cost (less than 1%). This solution significantly outperforms the model solution under the normal demand, which results in a significant difference in total cost (more than 25%) compared to the simulation.

INDEX TERMS Base stock policy, empirical demand, guaranteed service approach, inventory positioning, mixed-integer linear programming, multi-echelon inventory system, multiple cycle service levels, safety stock placement.

I. INTRODUCTION

A supply chain is a network of production and storage facilities of raw material and part suppliers, component and semi-finished item producers, the final product manufacturer, distribution centers, wholesalers, and retailers. These facilities are connected by material, information, and financial flows [1]. A supply chain's performance is influenced by internal and external factors such as network structure, customer demand, replenishment lead time, and inventory policy. Among these factors, inventory, in conjunction with a service level, is an essential driver of supply chain efficiency. A recent report released by the Office of the National Economic and Social Development Council (NESDC) shows that the total inventory value in 2020 is around 46.5%, constituting the most significant portion of the total logistics cost structure [2]. Hence, optimizing inventories across the supply chain is motivated by economic reasons. To substantially reduce inventory cost in a supply chain, inventories of all supply chain members are jointly, rather than separately, considered as a target to improve. A supply chain usually experiences demand uncertainty propagated downstream to upstream. This uncertainty and operational constraints may result in inventory shortages at many locations. These shortages subsequently affect the supply chain's performance. Therefore, keeping safety stocks at suitable locations is a countermeasure that mitigates the impact of demand variability and maintains a desired customer service level [3], [4]. Identifying the suitable locations and levels of safety stocks of different materials (parts, components, semi-finished, and finished goods) throughout the supply chain is essential and a challenge for both supply chain practitioners and researchers.

This paper involves the problem of choosing the locations and amount of safety stock at each location in a supply

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chain, usually referred to as the safety stock placement or inventory positioning problem in the research literature. The solution to this problem is explored by numerous studies using either a stochastic service approach (SSA) or a guaranteed service approach (GSA). These two methods are introduced in the pioneering works of Clark and Scarf [4] and Simpson [3], respectively. The main difference between the two approaches involves how the demand uncertainty is handled. For SSA, it is assumed that safety stock is the only means to cope with demand uncertainty [4]. Therefore, if a shortage occurs, the shortage is backordered and fulfilled when inventory is available. Since no other action is taken during the shortage, the timing of demand fulfillment becomes random, resulting in a stochastic service level. GSA, on the other hand, divides demand uncertainty into two parts, bounded and unbounded. The former is covered by safety stock, while the latter is handled by external measures, such as expediting shipment and outsourcing. These measures result in a consistent fulfillment time and service level. Since the model in this study is developed using GSA, the literature related to GSA is the primary subject of the literature review. An in-depth discussion of SSA and its integration with GSA can be found in the studies of Wang [5], Simchi-Levi and Zhao [6], Eruguz *et al.* [7], and de Kok *et al.* [8].

Even though the GSA model was introduced decades ago, most research on this subject has recently been published [7]. Based on the GSA, originally presented by Kimball in 1955 and later published in Kimball [9], for a single-stage inventory system, Simpson [3] developed a GSA model to determine the inventory policy for a serial supply chain. The solution to this model is obtained using a dynamic programming algorithm developed by Graves [10]. After that, Simpson's model [3] is extended to accommodate various practical supply chain settings through the studies of Graves and Willems [11], [12], [13], Humair and Willems [14], Funaki [15], Moncayo-Martınez and Zhang [16], Jiang *et al.* [17], Aouam and Kumar [18], Ghadimi *et al.* [19], Aouam *et al.* [20].

When a safety stock placement problem is modeled using GSA, two common inventory policies are usually specified for each stocking location in a supply chain. They are the (R, Q) policy, where an order of Q is placed when the inventory position falls on or below a reorder point R, and the base stock policy (or order-up-to policy), in which an order is placed every review period to bring the inventory position to a pre-specified base stock level. Shenas *et al.* adopted the R, Q policy for a continuous review two-stage serial supply chain [21]. The authors propose a model to compute the reorder point and determine the upstream stage's inventory. Similarly, Li and Chen [22] consider a variant of (R, Q) policy, i.e., echelon (R, nQ) policy for a general serial supply chain. A dynamic programming algorithm is developed to optimize inventory in the supply chain. The solution approach of Li and Chen is adapted by Li *et al.* [23] for an assembly system with a(nR, Q) policy. The problems of Li and Chen [22] and Li *et al.* [23] are also explored by Chen and Li [24] under various operating flexibilities. Despite its popularity in research, the (R, Q) policy is not widely applied in practice. The most common policy to handle inventory systems like warehouses and distribution centers is the base stock policy [25]. This policy is often implemented for a periodic review system, where ordering costs can be minimized when orders are arranged and consolidated [26]. Therefore, a majority of researchers consider the base stock policy in their safety stock placement problems [11], [15], [16], [18], [19], [20], [27], [28].

In addition to inventory policy, another important assumption in the safety stock placement problem is the underlying assumption of the distribution for customer demand. To simplify the problem characteristic, most studies assume that the demand follows either a theoretical distribution, such as Normal [11], [14], [16], [18], [19], [20], [25], [27], [28], Poisson distribution [22], [23], [24], or a stochastic process, in which the demand still follows a normal distribution with a dynamic variance [12], [15]. Although these demand assumptions are widely applied in inventory management literature, they are well-known for the poor approximation of several real demand patterns, which are uncertain, intermittent, and unpredictable [29]. Given a demand distribution, the total demand during a replenishment cycle of a supply chain is split into two unequal parts. The larger part, referred to as bounded demand, is fulfilled by available inventory, while the smaller one, known as unbounded demand, is handled by operating flexibilities, such as accelerated production [19], and subcontracting [18], [20]. Under this demand-splitting scheme, the timing of fulfillment (or lead time) is always guaranteed. In addition, since the bounded demand represents the fraction of total demand in a replenishment cycle that is satisfied by the inventory system, it implicitly determines the cycle service level of the supply chain. Therefore, many GSA studies determine the size of bounded demand by specifying a cycle service level, such as 90% [11], 95% [15], [16], [27], 97.5% [19], [20], or 97.7% [28]. It is conventional wisdom that service level is prescribed by either customers or managers. Therefore, most GSA research studies treat the service level as a given input and focus on minimizing the total inventory cost without considering the impact of handling an additional amount of unbounded demand. However, most managers and customers usually indicate a service level based on experience and preference rather than a comprehensive analysis of trade-offs among factors such as operating flexibility and inventory carrying costs [24]. Indeed, carefully evaluating these factors would provide a better service level that minimizes the total inventory cost [25]. This approach is demonstrated in the study of Aouam and Kumar [18]. The authors show that optimizing service level in addition to the safety stock placement decision by considering extra measures, including subcontracting and overtime, results in a lower total inventory cost than taking a service level as an input parameter.

TABLE 1. Summary of studies on the safety stock placement problem.

Papers	Inventory policy	Demand distribution	Cycle service level	Shortage
Graves and Willems (2000) [11]	Base stock	Normal	Fixed (90%)	
Graves and Willems (2005) [10]	Base stock	Normal distribution with a dynamic variance	Fixed (95%)	
Simchi-Levi and Zhao (2005) [4]	Base stock	Poisson	N/A	✓
Kaminsky and Kaya (2008) [25]	Base stock	Normal	Fixed (95%)	
Jung *et al.* (2008) [23]	Base stock	Normal	Flexible	✓
Funaki (2012) [13]	Base stock	Normal distribution with a dynamic variance	Fixed (95%)	
Li *et al.* (2013) [21]	(R, Q)	Poisson	Fixed	
Moncayo-Martínez and Zhang (2013) [14]	Base stock	Normal	Fixed (95%)	
Chen and Li (2015) [22]	(R, Q)	Poisson	Fixed	
Graves and Schoenmeyr (2016) [26]	Base stock	Normal	Fixed (97.7%)	✓
Aouam and Kumar (2019) [16]	Base stock	Normal	Flexible	✓
Ghadimi *et al.* (2020) [17]	Base stock	Normal	Fixed (97.5%)	
Aouam *et al.* (2021) [18]	Base stock	Normal	Fixed (95%, 97.5%)	
This study	Base stock	Normal and empirical	Flexible	✓

Table 1 presents how the current research expands upon the previous studies by concurrently considering various realistic aspects of the safety stock placement problem.

To the best of our knowledge, no studies have explored the inventory placement problem, where the safety stock placement and service level are optimized together while considering the behavior of the stochastic demand that is discrete and intermittent. Therefore, we develop a model to address this research question in our study. The contributions of our paper are summarized as follows.

- A mathematical programming model, which integrates safety stock placement and multiple service levels, is developed under the assumption of normal demand.
- The assumption of normal demand is then relaxed to accommodate the modeling of the demand using the empirical distribution.
- To evaluate the performance of our model under both demand assumptions, a simulation model of the inventory system is developed.
- The experiment results indicate that the system measures of performance produced by the solution of the optimization model with the empirical distribution match those of the simulation model. This result is not the case for the solution of the optimization model with the normal distribution.

The remainder of this paper is organized as follows. The description of the safety stock placement problem is presented in Section 2. The mathematical model formulations that consider multiple service levels are provided in Section 3. Then, a simulation model that evaluates the performance of the proposed models is presented in Section 4. A comparison of results between the mathematical model and the simulation is given in Section 5. Finally, the conclusions are made in Section 6.

II. PROBLEM STATEMENT

This paper considers a multi-echelon inventory optimization problem for a production company that coordinates a supply chain network. The network consists of many stages at different locations to manufacture a single product. The final assembly of this product is commenced at the most downstream stage, referred to as Stage 1, in the network. In other words, end-customer orders are received and fulfilled by Stage 1. Upon receiving an order, the end customer is quoted a lead time for delivery. The delivery lead time is also interchangeably referred to as committed service time in the remaining parts of this paper. In the meantime, orders for components and subassemblies required to produce the finished product are sent to upstream stages. These stages also quote different lead times for the received orders. Stage 1 must wait for all components and subassemblies to be delivered to start its production. Most stages in the network are centrally managed by the company, while some are operated by suppliers and subcontractors. These stages are defined as internal and external stages, respectively. Since the company is uncertain about customer demand, inventories, especially safety inventories, may be required at several stages to protect the supply chain against any demand fluctuation and maintain a consistent customer service level. These inventories can be located at and carried by internal stages. The company can minimize its total supply chain inventory cost by carefully determining the stages to keep inventories and their respective amounts. The modeling of the problem uses the following parameters and notations:

A. SETS

- N: set of nodes i, j, that represent supply chain stages, $i, j \in N = \{1, 2, \ldots\}$;
- E: set of external stages in the network, $E \subset N$;
- I: set of internal stages in the network, $I \subset N$;
- A: set of arcs connecting a predecessor stage j and a successor stage i in the network, $(j, i) \in A$;
- L: set of options for the length of net replenishment time, $L = \{1, 2, \ldots, l_{max}\}$;
- K: set of discrete cycle service levels (CSLs), $K = \{90\%, 91\%, \ldots, 99\%, 99.1\%, \ldots, 99.9\%\}$

B. PARAMETERS

- μ_D: average customer demand D per period;
- σ_D : standard deviation of demand D per period;
- $\Phi^{-1}(\cdot)$: the inverse of standard normal cumulative distribution function;
- $\mathcal{L}(k)$: the standard normal loss function corresponding to a CSL of k;
- $f(d)$: probability mass function of demand d in one period;
- $f(x|l)$: the probability mass function of demand X during a given replenishment time of l period(s);
- $F(x|l)$: the cumulative distribution function of demand X, given the replenishment time of l period(s);
- $G(k|l)$: the inverse of $F(x \mid l)$ associated with a CSL of k.
- $E[\mathrm{OH}]$: the expected inventory on-hand.
- v_i: product value at stage i;
- B: the base stock level;
- $T_{j,i}$: transportation time from stage j to stage i, if they are at different locations;
- O_j: minimum quoted service time of external stage j;
- P_i: processing time of internal stage i;
- R: service time that Stage 1 commits to end customers;
- $Q_{l,k}$: the known amount of safety stock to cover demand during a given replenishment time of l at a CSL of k;
- $\mathrm{ES}_{l,k}$: the expected amount of shortage within a replenishment cycle when holding l days of inventory to maintain a CSL of k.
- $H_i = h \times v_i$: inventory holding cost per unit per year of the output item of stage i, where h is a fraction of v_i.
- $\mathrm{CS}_i = p \times v_i$: shortage cost per unit at stage i, where p is a fraction of v_i.

C. DECISION VARIABLES

- $S_{i,k}^{in}$: incoming service time of stage i at CSL of k;
- $S_{i,k}^{out}$: outgoing service time of stage i at CSL of k;
- $x_{i,k}$: net replenishment time of an output item at stage i at CSL k;
- $y_{j,i,k}$: net replenishment time of an output item of stage j, which is held as an input item in front of stage i, at CSL k;
- $u_{i,l,k}$: binary variable, which takes a value of 1 if the net replenishment time of finished part at stage i is l periods at a CSL of k, or 0 otherwise. It should be noted that when $u_{i,l,k} = 1$, the safety stock of the output item will be kept at stage i.
- $w_{j,i,l,k}$: binary variable, which takes a value of 1 if the net replenishment time of the finished part that comes from stage j, which is held as input material at stage i, is l periods, at a CSL of k or 0 otherwise;
- z_k: binary variable, which takes a value of 1 if a CSL of k is chosen for the whole supply network, or 0 otherwise;

III. MODEL DEVELOPMENT

A. MODEL ASSUMPTIONS

- The supply chain under study is an assembly network.
- Demand is assumed to be either normally or empirically distributed.
- All internal stages are under the same ownership such that all stages in the network share and observe the same demand information.
- Delivering and processing times are independent of order quantity.
- Multiple service levels are considered.
- Both production and storage capacities are not considered.
- Inventory positioning decisions are for the safety inventory, while cycle inventory is managed in a lot-for-lot manner.

B. SUPPLY CHAIN NETWORK

The supply chain network consists of N stages. Stage 1, which is closest to the end customers, performs the final assembly operation. The other $(N - 1)$ stages are associated with each of the $(N - 1)$ items, i.e., raw materials, components, or subassemblies, that the final product requires. Each item is either produced by an internal manufacturing stage or procured from an external supplier stage. The network is modeled as a directed graph $G(N, A)$, where N is the set

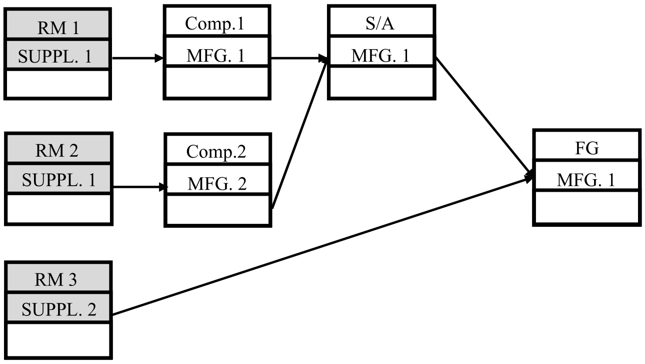

FIGURE 1. A typical supply chain network of an assembled product.

of nodes and A is the set of arcs representing the stages and material flows from one stage to another. The node set N is partitioned into two subsets, I and E, such that $N = I \cup E$, where I consists of internal stages, E contains external ones. An illustration of an example network is presented in Figure 1.

Figure 1 represents a supply chain network consisting of four internal manufacturing stages, producing component 1 (Comp. 1), component 2 (Comp. 2), subassembly (S/A), and the finished goods (FG), and three external supplier stages, where raw materials 1, 2, and 3 (RM 1 to RM 3) are purchased. The four internal stages are at two locations, i.e., MFG. 1 and MFG. 2. Similarly, the three external supplier stages belong to two different suppliers, i.e., SUPPL. 1 and SUPPL. 2. Transportation is required between two stages at different locations.

C. SUPPLY CHAIN OPERATIONS

In the network diagram, each internal stage has potential stocking locations: input items from an upstream stage may be kept before the internal stages, and output items from production may be kept after the stages before being sent to the downstream stage. When an internal stage i receives an order from its downstream stage (i.e., internal customer), the customer is given a quoted lead time, after which the order will be fulfilled. This quoted time is defined as the outgoing service time S_i^{out} of stage i. Every stage i can properly decide a quoted lead time to its customer, except for Stage 1, whose maximum quoted time to the end customers is subject to a given R periods, i.e., $S_i^{out} \geq 0\,(\forall i \in I)$ and $S_1^{out} \leq R$. In addition, upon receiving an order, stage i immediately places orders for items required for production to its upstream stages $j \in N$, where $(j, i) \in A$, to maintain the base stock level. Each upstream stage j then gives a quoted delivery time to stage i, similarly defined as the outgoing service time S_j^{out} of stage j. If stage j is an external stage, its S_j^{out} is subject to a given minimum quoted service time O_j from a supplier j, i.e., $S_j^{out} \geq O_j\,(\forall j \in E)$.

Since delivery times quoted by upstream stages may differ, stage i must wait until all items in which stage i does not have safety stocks are delivered before its production is commenced. This waiting time is defined as incoming service time S_i^{in} of stage i and is equal to the largest total, among all stages j, of outgoing service times, quoted to stage i, including the required transportation time $T_{j,i}$, i.e., $S_i^{in} = \left(S_j^{out} + T_{j,i}\right)$. The production time at stage i is assumed to take $P_i > 0$ time periods and is assumed to be independent of the production quantity [3].

D. CUSTOMER DEMAND

In the inventory positioning problem, the information about the end customer demand is instantaneously passed to the upstream stages of the supply chain through a series of orders. In order words, all internal stages are under single ownership and are assumed to share information so that every stage observes the same demand pattern as Stage 1. If the customer demand for the finished product of Stage 1 in a period follows a probability distribution with a mean of μ_D and a standard deviation of σ_D, the demand for the output item of an upstream stage j follows the same distribution.

Modelling the demand in a period by using the normal distribution is a common practice in some inventory management studies [11], [14], [16], [18], [19], [20], [25], [27], [28]. This assumption allows the demand over multiple periods, i.e., replenishment lead time, to be approximated using the normal distribution. However, this may provide a poor estimation of the system behavior when the underlying shape of the demand distribution is non-normal. This is especially the case for many medium- and slow-moving items that experience intermittent demand and, for some items, a few outliers. Under this demand pattern, the normal distribution is effective only when the lead time is extremely long such that it can overcome the intermittency and presence of outliers, which is rarely the case in practice. To properly model the demand with such characteristics, empirical distribution is used to obtain a better estimation. Description of the base stock policy under the two demand modeling approaches and a comparison of system performance measures between them are provided in the subsequent section and a numerical experiment.

E. BASE STOCK POLICY UNDER NORMAL DEMAND

For an internal stage $i \in I$ that chooses to keep the safety stock of its output item, a proper base stock level B is determined. To maintain this level, a stage i always generates orders for its input items and sends them to upstream stages immediately after it receives a customer order. Typically, it takes S_i^{in} periods for the orders from the upstream stages to arrive and P_i periods to manufacture item i for the customer. Since stage i commits to fulfilling its customer demand after S_i^{out} periods, stage i requires a net outgoing replenishment time, $l \geq 0$, to fulfill the customer order. In addition to keeping the safety stock of the output item, each internal stage $i \in I$ may choose to keep some inventory of the input item from an upstream stage j in storage as a buffer to shorten the supply lead time, i.e., $S_j^{out} + T_{j,i}$, $\forall\,(j, i) \in A$. The net replenishment time of these two scenarios is expressed by the following Equation.

$$l = \begin{cases} S_i^{in} + P_i - S_i^{out} & if\ i \in I \\ S_j^{out} + T_{j,i} - S_i^{in} & if\ (j, i) \in A \end{cases} \quad (1)$$

As stage i faces the demand with a mean of μ_D and SD of σ_D in every period, the total demand over l periods has the mean of $\mu_D l$ and SD of $\sigma_D\sqrt{l}$ [11].

In the case that the demand of stage i is assumed to follow the normal distribution, the base stock level of an item, either output from stage i or input item from stage j, is usually specified as,

$$B = \mu_D l + \Phi^{-1}(k)\,\sigma_D\sqrt{l} \quad (2)$$

where $\Phi^{-1}(k)$ is the inverse of the standard normal cumulative distribution function associated with the cycle service level (CSL) of k in a replenishment cycle. Based on the specified base stock level, on average, a fraction $(1-k)$ of the demand cannot be fulfilled by the inventory on-hand. This amount is assumed to be backlogged, and its expected value is estimated as,

$$\mathrm{ES}_{l,k} = \mathcal{L}(k)\,\sigma_D\sqrt{l} \quad (3)$$

Also, using the base stock level as specified above, the expected inventory of an item, which is kept in storage at stage i, is determined in (4).

$$E\,[\mathrm{OH}] = \mu_D P + \Phi^{-1}(k)\sigma_D\sqrt{l} \quad (4)$$

where the first term represents work-in-process or pipeline inventory, and the second term the safety stock. Since the pipeline inventory is constant under our problem's setting, the system performance measures only depend on l, which is a function of S_j^{out}, S_i^{in}, and S_i^{out}. In other words, optimizing the supply chain inventory in our study is equivalent to determining the quoted service times, which leads to the safety stock location and safety stock amount carried by each internal stage in the supply chain.

F. BASE STOCK POLICY UNDER EMPIRICAL DEMAND

When Stage 1 observes a demand pattern modeled by an empirical distribution, the mean of the demand is measured using (5).

$$\mu_D = \sum df(d) \quad (5)$$

where $f(d)$ is the probability mass function fitted from using the empirical distribution.

The functions $f(x|l)$ and $F(x|l)$ are derived from $f(d)$, given l [30], [31]. With these estimations, the base stock levels and expected inventories for each internal stage are expressed as,

$$B = G(k|l) \quad (6)$$

$$E\,[\mathrm{OH}] = \mu_D P + (B - \mu_D l) \quad (7)$$

The safety stock to minimize can be determined by subtracting (6) with the average demand during l periods, i.e., $G(k|l) - \mu_D l$. Similar to the case of the normal demand, the expected amount of unsatisfied demand is estimated as,

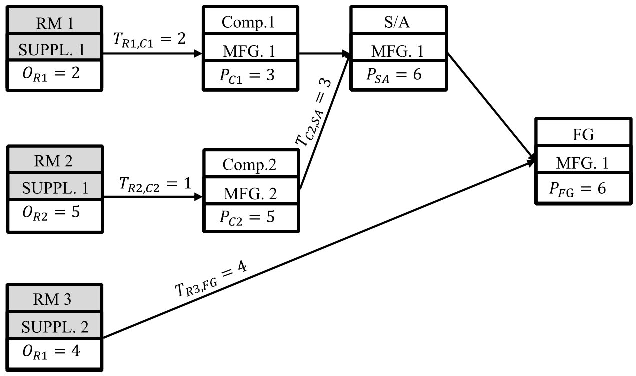

FIGURE 2. An example of supply chain network.

$$\mathrm{ES}_{l,k} = \sum\nolimits_{x>B} (x - B) f(x|l) \quad (8)$$

G. TOTAL SAFETY STOCK COST

Generally, an inventory positioning problem aims to minimize the total safety stock cost for input and output items for a given service level. Under the normal and the empirical demands, the total safety stock cost can be respectively expressed as,

$$Z = \sum_{i\in I} H_i \Phi^{-1}(k)\,\sigma_D\sqrt{l} + \sum_{j\in N|(j,i)\in A} H_j \Phi^{-1}(k)\,\sigma_D\sqrt{l} \quad (9)$$

$$Z = \sum_{i\in I} H_i\,(G(k|l) - \mu_D l) + \sum_{j\in N|(j,i)\in A} H_j (G(k|l) - \mu_D l) \quad (10)$$

While Equation (9) contains non-linear terms of decision variables, i.e., l, and Equation (10) contains $G(k|l)$, which is dependent on l. In this research, Equations (9) and (10) are linearized by using binary variables to select values of l among all possible values that result in the minimum total cost. To accommodate this approach, all possible values of l are enumerated in an interval between zero and a predetermined maximum value. The maximum value is the total time on the longest path in the network, from one of the most upstream stages to Stage 1. This procedure is illustrated in the following example.

The example network in Figure 2 has three possible paths to Stage 1 (FG), starting from the most upstream external stages, SUPPL. 1, SUPPL. 2, and SUPPL. 3. The total time of a path can be determined based on O_j, $T_{j,i}$, and P_i of all stages on that path. The maximum possible value for l of the network is calculated as, $l_{max} = \max\{O_{R1} + T_{R1,C1} + P_{C1} + P_{SA} + P_{FG}, O_{R2} + T_{R2,C2} + P_{C2} + T_{C2,SA} + P_{SA} + P_{FG}, O_{R1} + T_{R3,FG} + P_{FG}\}$

$$= \max\{2{+}2{+}3{+}6{+}6,\ 5{+}1{+}5{+}3{+}6{+}6,\ 4{+}4{+}6\}$$

$$= \max\{19, 26, 14\} = 26$$

That is, for this network, l can be at most 26 periods for any event.

Let $L = \{0, 1, \ldots, l_{max}\}$ be the set of possible values of l in a supply chain network. The total safety stock cost for normal and empirical demand are respectively re-formulated as follows.

$$TAC = \sum_{i \in I} H_i \sum_{l \in L} u_{i,l} Q_{l,k} + \sum_{j \in N|(j,i) \in A} H_j \sum_{l \in L} w_{j,i,l} Q_{l,k} \tag{11}$$

where

$$Q_{l,k} = \begin{cases} \Phi^{-1}(k)\sigma_D \sqrt{l} & \text{if demand is normally distributed} \\ G(k|l) - \mu_D l & \text{if demand is empirically distributed} \end{cases} \tag{12}$$

H. TOTAL SHORTAGE COST

Our study considers CSL as a decision variable rather than the input parameter. In other words, the optimal inventory positions should provide the service level that minimizes the total cost, consisting not only of the safety stock cost but also the shortage cost. To accommodate the decision of CSL, a set K containing discretized levels of service is included as part of the two binary variables u and w. We add the total shortage cost to (11) to give the total annual cost (TAC) of the supply chain network.

$$\begin{aligned} \text{TAC} = & \sum_{i \in I} H_i \sum_{l \in L} \sum_{k \in K} u_{i,l,k} Q_{l,k} \\ & + \sum_{j \in N|(j,i) \in A} H_j \sum_{l \in L} \sum_{k \in K} w_{j,i,l,k} Q_{l,k} \\ & + \sum_{i \in I} \text{CS}_i \sum_{l \in L} \sum_{k \in K} \frac{365}{l} \times \text{ES}_{l,k} u_{i,l,k} \\ & + \sum_{j \in N|(j,i) \in A} \text{CS}_j \sum_{l \in L} \sum_{k \in K} \frac{365}{l} \times \text{ES}_{l,k} w_{j,i,l,k} \end{aligned} \tag{13}$$

Since the holding cost is charged on an annual basis, the shortage cost is computed in the same manner. As the shortage is counted within a replenishment cycle, and the shortage cost is applied for each unit short, the total number of shortages within a year is determined by multiplying the number of units short in a cycle (i.e., $\text{ES}_{l,k} u_{i,l,k}$ or $\text{ES}_{l,k} w_{j,i,l,k}$) by the number of cycles within a year, i.e., $365/l$.

I. MIXED-INTEGER LINEAR PROGRAMMING MODEL

The mathematical model for the inventory placement problem under study is formatted as follows.

Minimize TAC

Subject to
$$\sum_{k \in K} S_{j,k}^{out} \geq O_j \quad \forall j \in E \tag{14}$$

$$\sum_{k \in K} \left(S_{j,k}^{out} - S_{i,k}^{in} \right) + T_{j,i} = \sum_{k \in K} y_{j,i,k} \quad \forall (j, i) \in A \tag{15}$$

$$\sum_{k \in K} \left(S_{i,k}^{in} - S_{i,k}^{out} \right) + P_i = \sum_{k \in K} x_{i,k} \quad \forall i \in I \tag{16}$$

$$\sum_{k \in K} S_{1,k}^{out} \leq R \tag{17}$$

$$x_{i,k} = \sum_{l \in L} (l \times u_{i,l,k}) \quad \forall i \in I;\ k \in K \tag{18}$$

$$y_{j,i,k} = \sum_{l \in L} (l \times w_{j,i,l,k}) \quad \forall (j, i) \in A;\ k \in K \tag{19}$$

$$\sum_{l \in L} u_{i,l,k} \leq z_k \quad \forall i \in I;\ k \in K \tag{20}$$

$$\sum_{l \in L} w_{j,i,l,k} \leq z_k \quad \forall (j, i) \in A;\ k \in K \tag{21}$$

$$S_{i,k}^{in} \leq l_{max} z_k \quad \forall i \in N;\ k \in K \tag{22}$$

$$S_{i,k}^{out} \leq l_{max} z_k \quad \forall i \in N;\ k \in K \tag{23}$$

$$\sum_{k \in K} z_k = 1 \tag{24}$$

$$\begin{aligned} & S_{i,k}^{in}, S_{i,k}^{out}, x_{i,k} \geq 0 \quad \forall i \in N\ k \in K \\ & y_{j,i,k} \geq 0 \quad \forall (j, i) \in A;\ k \in K \\ & u_{i,l,k}, w_{j,i,l,k} \in \{0, 1\}, \quad \forall (j, i) \in A;\ l \in L;\ k \in K \\ & z_k \in \{0, 1\}, \quad \forall k \in K \end{aligned} \tag{25}$$

The objective function is to minimize the total annual cost of the supply chain for both input and output items across all stages. Constraints (14) force the outgoing service time of each external stage in the supply network to be no shorter than its minimum quoted service time. Constraints (15) and (16) determine the net replenishment time for each inventory of input and output items, respectively. Both constraints are derived from Equation (1), which represents the net replenishment time. In addition, each constraint chooses a specific cycle service level k among all possible levels and a value of the net replenishment time. Constraint (17) ensures that the outgoing service time at Stage 1 would not exceed the minimum service time committed to the end customers. Constraints (18) and (19) represent the correspondence between the net replenishment time of either output or input items and the selected net replenishment time. Constraints (20) and (21) imply that not more than one net replenishment time is selected for each inventory position. Constraints (22) and (23) represent the selected incoming and outgoing service times bounded by l_{max}. Constraint (24) indicates that only one customer service level is chosen for the entire network. Constraints (25) represent the non-negativity and integer nature of the lead time of different supply chain stages.

IV. MODEL EVALUATION USING SIMULATION

To evaluate the solution of the proposed MILP model, a simulation model is developed to imitate the operation of every inventory position. The optimal solution from the optimization model, including safety stock amount $Q_{l,k}$, the net replenishment time $x_{i,k}$ or $y_{j,i,k}$ of each inventory position, is used as inputs to the simulation model. Based on the safety stock amount, the base stock level B of stage i in the simulation model can be computed as follows:

$$B = \begin{cases} \mu_D x_{i,k} + Q_{l,k} & \text{if stage i keeps its output item} \\ \mu_D y_{j,i,k} + Q_{l,k} & \text{if stage j keeps its input item.} \end{cases} \tag{26}$$

The inventory system follows the base stock policy, where the inventory position is reviewed, and an order is placed every period to bring the inventory position up to the base stock level of output (or input) items after $x_{i,k}$ (or $y_{j,i,k}$). In addition, unmet demand at the end of a period can be fulfilled by the arrived order of that period (if any). If the unmet demand cannot be satisfied in the current period, it will be accumulated and fulfilled when there is available inventory in the following periods.

Simulation Notation:

- $\mathrm{bOH}_{i,t}$: Beginning inventory on-hand of stage i in period t.
- $\mathrm{OQ}_{i,t}$: Order quantity of stage i in period t.
- $\mathrm{OA}_{i,t}$: Order arrival of stage i in period t.
- $d_{i,t}$: Demand of stage i in period t.
- $\mathrm{dm}_{i,t}$: Amount of satisfied demand at stage i in period t.
- $\mathrm{Sh}_{i,t}$: Amount of shortage at stage i in period t.
- $\mathrm{CSh}_{i,t}$: Cumulative shortage at stage i in period t.
- $\mathrm{eOH}_{i,t}$: Ending inventory on-hand of stage i in period t.

To simulate the behaviors of safety inventory of output items at an internal stage i, the simulation input parameters include the net replenishment time $x_{i,k}$, the base stock level B, unit holding cost H_i, and unit shortage cost CS_i. At the beginning of the simulation when $t = 0$, $\mathrm{bOH}_{i,t}$, $\mathrm{eOH}_{i,t}$, $\mathrm{Sh}_{i,t}$, $\mathrm{CSh}_{i,t}$ are initialized such that $\mathrm{bOH}_{i,t} = \mathrm{eOH}_{i,t} = B$, and $\mathrm{Sh}_{i,t} = \mathrm{CSh}_{i,t} = \mathrm{OA}_{i,t} = 0$. In every upcoming period $t = t + 1$, the beginning inventory $\mathrm{bOH}_{i,t}$ is updated using the ending on-hand of the previous period $\mathrm{eOH}_{i,t-1}$. After that, the customer demand $d_{i,t}$, randomly generated using the underlying demand distribution, arrives and is fulfilled by available inventory at that period, i.e., $\mathrm{bOH}_{i,t}$. The amount of demand that can be satisfied $\mathrm{dm}_{i,t}$, amount of shortage $\mathrm{Sh}_{i,t}$, and cumulative shortage $\mathrm{CSh}_{i,t}$, are then determined, respectively. In addition, an order $\mathrm{OQ}_{i,t}$ is created and sent upstream. This order is delivered to stage i after $x_{i,k}$ periods. In the first $x_{i,k}$ periods of the simulation, although there is a stream of orders sent upstream from stage i, no order arrives at stage i, i.e., the first order is delivered in period $t = x_{i,k} + 1$. From then, there is a stream of arriving orders at stage i. In other words, $\mathrm{OA}_{i,t} = 0$ during the first $x_{i,k}$ periods, and $\mathrm{OA}_{i,t} = \mathrm{OQ}_{i,t-x_{i,k}}$ starting from period $x_{i,k} + 1$. Since a shortage may occur before an order arrival within a period, the shortage amount is either partially or fully satisfied by the incoming order $\mathrm{OA}_{i,t}$. After the clearance

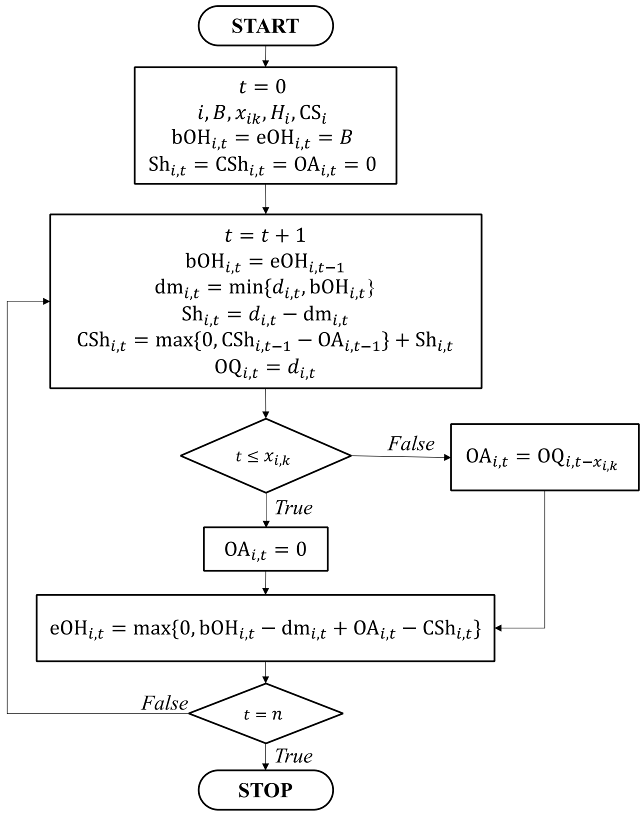

FIGURE 3. The logic of the simulation model.

of shortage, the ending inventory $\mathrm{eOH}_{i,t}$ is updated. When the simulation reaches period n, it is terminated, and the statistics for this inventory system of stage i are collected. The total inventory cost of a stage TAC_i is computed as follows:

$$\mathrm{TAC}_i = \mathrm{EOH}_i \times H_i + \frac{365}{l} \times \mathrm{ES}_i \times \mathrm{CS}_i \tag{27}$$

where $l = x_{ik}$ (or $l = y_{ijk}$), EOH_i and ES_i are the expected inventory on-hand and the expected shortage of stage i over a cycle during n days of simulation, respectively.

$$\mathrm{EOH}_i = \frac{\sum_{t=1}^{n} \mathrm{bOH}_{i,t}}{n}; \quad \mathrm{ES}_i = \frac{\sum_{t=1}^{n} \mathrm{Sh}_{i,t}}{n} \tag{28}$$

For an internal stage i keeping input items from stage j in its storage, the simulation of its inventory system is conducted with the same logic. Since all stages in the supply chain network receive the same demand information as it is sent upstream from Stage 1 and the service times between stages are guaranteed, each can be simulated as an independent inventory system based on the same demand dataset and simulation logic.

V. NUMERICAL STUDY

A. PROBLEM INSTANCE

A numerical study features a problem instance adapted from the case study in Simchi-levi *et al.* (2008). The supply

FIGURE 4. A supply chain network of an assembly product.

chain network is illustrated in this problem as a diagram, including vertices and arcs. A vertex (or node) represents a manufacturing stage, while an arc represents the flow of materials from one vertex to another (see Figure 4).

TABLE 2. Summary of input data and decision variable values of the network at 98% service level under normal demand.

Input data					Decision variables				System measures of performance (per cycle)		Annual costs			
j	i	$T_{j,i}$	P_j	O_j	$S_{j,k}^{in}$	$S_{j,k}^{out}$	$x_{j,k}$	$y_{j,i,k}$	$Q_{l,k}$	$ES_{l,k}$	CS_j	H_j	Holding cost	Shortage cost
FG		0	4		26	30	-	-	-	-	160.5	53.5	0	0
C1.4	FG	0	1		25	26	-	-	-	-	16.5	5.5	0	0
C12	FG	0	50		0	26	24	-	119.79	0.43	1.8	0.6	71.87	11.73
S3.3	FG	0	3		23	26	-	-	-	-	85.5	28.5	0	0
C13	FG	0	4		22	26	-	-	-	-	0.9	0.3	0	0
C14	FG	0	14		12	26	-	-	-	-	5.4	1.8	0	0
C15	FG	0	3		23	26	-	-	-	-	2.7	0.9	0	0
C9.4	FG	0	30		26	56	-	30	133.93	0.48	19.5	6.5	870.55	113.61
C10.2	FG	0	1		25	26	-	-	-	-	9	3	0	0
C11.3	FG	0	12		14	26	-	-	-	-	10.5	3.5	0	0
C1.3	C1.4	0	3		22	25	-	-	-	-	12	4	0	0
S3.2	S3.3	0	1		22	23	-	-	-	-	78	26	0	0
R8	C14	8	0	24	0	24	-	20	109.35	0.39	3.3	1.1	120.29	23.55
R9	C15	8	0	10	0	15	-	-	-	-	1.2	0.4	0	0
C9.3	C9.4	0	5		21	26	-	-	-	-	18	6	0	0
C10.1	C10.2	0	5		20	25	-	-	-	-	6.3	2.1	0	0
C11.2	C11.3	0	3		11	14	-	-	-	-	9	3	0	0
C1.2	C1.3	0	1		21	22	-	-	-	-	10.5	3.5	0	0
S3.1	S3.2	0	6		16	22	-	-	-	-	63	21	0	0
C9.2	C9.3	3	6		12	18	-	-	-	-	12.6	4.2	0	0
R6	C10.1	35	0	41	0	41	-	56	182.98	0.65	5.4	1.8	329.37	23.03
C11.1	C11.2	0	16		7	11	12	-	84.71	0.30	8.4	2.8	237.18	77.38
C1.1	C1.2	0	4		17	21	-	-	-	-	9	3	0	0
C6.3	S3.1	2	6		8	14	-	-	-	-	9	3	0	0
S2	S3.1	3	3		10	13	-	-	-	-	22.5	7.5	0	0
S1	S3.1	0	3		13	16	-	-	-	-	24	8	0	0
C5	S3.1	2	1		13	14	-	-	-	-	1.2	0.4	0	0
C2.2	S3.1	0	7		9	16	-	-	-	-	3.6	1.2	0	0
C9.1	C9.2	0	12		0	12	-	-	-	-	12	4	0	0
R7	C11.1	3	0	4	0	4	-	-	-	-	1.8	0.6	0	0
R1	C1.1	6	0	40	0	40	-	29	131.68	0.47	6	2	263.36	35.56
C6.2	C6.3	0	2		6	8	-	-	-	-	6.6	2.2	0	0
C7	S2	0	9		1	10	-	-	-	-	9	3	0	0
C8	S2	0	2		8	10	-	-	-	-	7.5	2.5	0	0
C4	S1	2	3		8	11	-	-	-	-	10.5	3.5	0	0
C3	S1	2	13		7	20	-	9	73.36	0.26	12	4	293.43	127.65
C2.1	C2.2	0	4		5	9	-	-	-	-	2.4	0.8	0	0
C6.1	C6.2	0	8		0	6	2	-	34.58	0.12	2.4	0.8	27.66	54.16
R4	C7	28	0	5	0	5	-	32	138.32	0.49	4.5	1.5	207.49	25.39
R5	C8	35	0	12	0	12	-	39	152.70	0.55	6.6	2.2	335.95	33.73
R3	C3	1	0	6	0	6	-	-	-	-	9.6	3.2	0	0
R2	C2.1	3	0	3	0	6	-	4	48.90	0.17	0.6	0.2	9.78	9.57
												Total	**2,766.93**	**535.34**

TABLE 3. Summary of input data and decision variable values of the network at 98% service level under empirical demand.

Input data					Decision variables				System measures of performance (per cycle)		Annual costs			
j	i	$T_{j,i}$	P_j	O_j	$S_{j,k}^{in}$	$S_{j,k}^{out}$	$x_{j,k}$	$y_{j,i,k}$	$Q_{l,k}$	$ES_{l,k}$	CS_j	H_j	Holding cost	Shortage cost
FG		0	4		25	29	-	-	-	-	160.5	53.5	0	0
C1.4	FG	0	1		24	25	-	-	-	-	16.5	5.5	0	0
C12	FG	0	50		0	25	25	-	137.97	0.63	1.8	0.6	82.78	16.40
S3.3	FG	0	3		22	25	-	-	-	-	85.5	28.5	0	0
C13	FG	0	4		21	25	-	-	-	-	0.9	0.3	0	0
C14	FG	0	14		11	25	-	-	-	-	5.4	1.8	0	0
C15	FG	0	3		22	25	-	-	-	-	2.7	0.9	0	0
C9.4	FG	0	30		26	25	31	-	152.41	0.66	19.5	6.5	990.64	152.11
C10.2	FG	0	1		24	25	-	-	-	-	9	3	0	0
C11.3	FG	0	12		13	25	-	-	-	-	10.5	3.5	0	0
C1.3	C1.4	0	3		21	24	-	-	-	-	12	4	0	0
S3.2	S3.3	0	1		21	22	-	-	-	-	78	26	0	0
R8	C14	8	0	24	0	24	-	21	128.02	0.58	3.3	1.1	140.82	33.41
R9	C15	8	0	10	0	14	-	-	-	-	1.2	0.4	0	0
C9.3	C9.4	0	5		21	26	-	-	-	-	18	6	0	0
C10.1	C10.2	0	5		19	24	-	-	-	-	6.3	2.1	0	0
C11.2	C11.3	0	3		10	13	-	-	-	-	9	3	0	0
C1.2	C1.3	0	1		20	21	-	-	-	-	10.5	3.5	0	0
S3.1	S3.2	0	6		15	21	-	-	-	-	63	21	0	0
C9.2	C9.3	3	6		12	18	-	-	-	-	12.6	4.2	0	0
R6	C10.1	35	0	41	0	41	-	57	200.62	0.84	5.4	1.8	361.11	29.04
C11.1	C11.2	0	16		7	10	13	-	104.11	0.50	8.4	2.8	291.50	117.27
C1.1	C1.2	0	4		16	20	-	-	-	-	9	3	0	0
C6.3	S3.1	2	6		7	13	-	-	-	-	9	3	0	0
S2	S3.1	3	3		9	12	-	-	-	-	22.5	7.5	0	0
S1	S3.1	0	3		12	15	-	-	-	-	24	8	0	0
C5	S3.1	2	1		12	13	-	-	-	-	1.2	0.4	0	0
C2.2	S3.1	0	7		8	15	-	-	-	-	3.6	1.2	0	0
C9.1	C9.2	0	12		0	12	-	-	-	-	12	4	0	0
R7	C11.1	3	0	4	0	4	-	-	-	-	1.8	0.6	0	0
R1	C1.1	6	0	40	0	40	-	30	150.17	0.66	6	2	300.33	47.82
C6.2	C6.3	0	2		5	7	-	-	-	-	6.6	2.2	0	0
C7	S2	0	9		0	9	-	-	-	-	9	3	0	0
C8	S2	0	2		7	9	-	-	-	-	7.5	2.5	0	0
C4	S1	2	3		7	10	-	-	-	-	10.5	3.5	0	0
C3	S1	2	13		7	10	10	-	93.39	0.46	12	4	373.56	199.73
C2.1	C2.2	0	4		4	8	-	-	-	-	2.4	0.8	0	0
C6.1	C6.2	0	8		0	8	-	3	56.72	0.39	2.4	0.8	45.37	113.05
R4	C7	28	0	5	0	5	-	33	155.88	0.69	4.5	1.5	233.83	34.58
R5	C8	35	0	12	0	12	-	40	170.56	0.74	6.6	2.2	375.22	44.29
R3	C3	1	0	6	0	6	-	-	-	-	9.6	3.2	0	0
R2	C2.1	3	0	3	0	11	-	10	93.39	0.46	0.6	0.2	18.68	9.99
												Total	**3,213.84**	**797.68**

From the figure, the network contains nine external stages (Ext.) that provide raw materials (RMs) to the manufacturing plants (Mfg.), and 33 internal stages, each of which performs a manufacturing operation or assembly operation. Each vertex presents key information about the manufacturing operation at each stage. These include component code (Comp. for component and S/A for sub-assembly) and its index number, facility location code, process cycle time for internal stages or quoted service time for the external stage, and a unit value of the component after operation completion.

The index number of a component (or sub-assembly) indicates whether it is processed in multiple sequential steps. For example, starting at the top left corner, raw material 1 (RM 1), supplied by external supplier 1 (Ext. 1), is processed into Component 1 in four sequential steps. Thus, the components are denoted Comp. (1.1) to Comp. (1.4). When two or more components (or sub-assemblies) are assembled, they become a sub-assembly (S/A), e.g., Comp. (3) and Comp. (4) become S/A 1.

In each vertex, the 2$^{\text{nd}}$ row contains information about the facility location, including manufacturing plants 1 to 3, denoted Mfg. 1 to Mfg. 3; external suppliers 1 to 5, denoted Ext. 1 to Ext. 5. For an internal stage, its process cycle time P_j and value of the component (or sub-assembly) V_j are given. For example, it takes a process cycle time, $P_{C1.1}$, of four days (waiting time and processing time) to turn RM 1 into Comp. (1.1), and the unit value, $v_{C1.1}$, after completing the stage is 30 THB. Fractions of h and p used for calculating the unit holding and shortage costs are assumed to be 10% and 30%, respectively. For an external stage, the quoted service time O_j to its successor stage is given instead of the process cycle time. It represents the time (in days) it takes the supplier to deliver the RM to the requested stage. For the final assembly stage FG, a 30-day response time is quoted to the customers. The required transportation time to deliver materials from one stage to another stage at different facility locations is T_{ji} provided on the arcs. For instance, the transportation time between Ext. 2 and Mfg. 1 is three days.

The internal stages are eligible candidates for the placement of safety stock. The final stage is an assembly operation producing finished goods (FG). The finished goods consist of eight components and one sub-assembly, all of which are manufactured at the same facility (Mfg. 1). Bins representing safety stock positions in the network are also placed on the arcs. A bin placed at the beginning of an arc represents the safety stock of the finished part (output from the stage) kept at an upstream stage before being transported to the downstream stage upon request. A bin at the end of an arc indicates safety stocks of finished parts from its upstream stage that is kept in front of a downstream stage.

B. OPTIMAL SOLUTIONS

The proposed MILP model in Section 3-I is validated with the problem instance of an assembly product comprising components produced in the adapted network. Using CPLEX 12.9.0 Solver, the optimal solution contains the quoted service times of all pairs of upstream and downstream stages. The solution can be obtained in approximately 20 minutes of computational time on a personal computer with a 4.70 GHz Intel Core i7-10710U processor and RAM of 32.0 GB 64-bit. Optimal positions of safety stocks in the network are then derived from the optimal quoted service times from the model. If an outgoing service time exceeds an incoming service time of a stage, then that stage requires a safety stock placement.

Stochastic demand for finished goods is based on one-year historical data of an actual product. In addition, it should be noted that the demand data are not normally distributed. Instead, it is intermittent by nature. As previously mentioned, demand and shortage during net replenishment time are usually approximated by a normal distribution in some studies. However, there are situations where the normal distribution provides a poor approximation in practice, specifically for medium- and slow-moving items, when the demand is intermittent, in which case the empirical distribution provides a better estimation. Therefore, the two distributions are experimented with in our numerical study to evaluate their performance in estimating demand and shortage during the net replenishment time under such demand characteristics.

The average daily demand, μ_D, and standard deviation, σ_D, are computed from the actual demand data for the normal demand scenario. The amount of safety stock and the expected shortage are determined by the second term of Eq. (2) and Eq. (3), respectively. For the empirical distribution, the amount of safety stock and the expected shortage are computed by the second term of Eq. (7) and Eq. (8). In addition, the maximum possible quoted service time between any two stages is determined based on the critical path, which is the longest path through the network. This network's critical path is the path from stages producing the following components, R6→C10.1→C10.2→FG. In other words, lead times can vary between 0 and 86 days in this problem instance. Other model parameters are given in the input data column in Table 2.

In the table, stages indexed with the letter R are external stages (R1 to R9), while the others are internal stages. Stages at the beginning of the network have no predecessor, while the other stages may have one or more upstream stages. Each stage has only one downstream stage, except for the final assembly stage, which has no successor. The proposed model aims to determine the safety stock locations and their quantities that minimize the total inventory cost, including inventory holding cost and shortage cost. The holding and shortage costs are computed as 10% and 30% of the unit product values, respectively. The service level is defined in 19 scenarios from 90%, 91%, …, 99%, 99.1%, 99.2%, …99.9%. Tables 2 and 3 show the optimal solutions at the service level of 98%, where demand is under normal and empirical distributions, respectively.

In this problem instance, the safety factor associated with the service level of 98% is $\Phi^{-1}(0.98) = 2.0537$. From the table, we can identify supply chain stages that should use a make-to-stock production strategy and keep the safety stock. On the contrary, the remaining stages without safety stock would operate with a make-to-order strategy. The results indicate that four stages keep safety stocks of their output component after completion and seven other stages keep safety stocks of their input raw materials. The indicator for keeping stock is from the optimal net replenishment lead time from the MILP model. Specifically, a safety stock placement is needed for each stage that does not have enough time to fulfill its downstream stage request since its optimal outgoing service time is shorter than its optimal incoming service time. For example, from Table 2, stage C11.1 is promised to receive an item from its upstream stage R7 in $S^{in}_{C11.1,98\%}$ =7 days, while it commits a service time of 11 days to its downstream stage FG. However, C11.1 takes $P_{C11.1} = 16$ days to process an order. Therefore, 12-day of FG safety stock ($S^{in}_{C11.1,98\%} + P_{C11.1} - S^{out}_{C11.1,98\%} = 7 + 16 - 11 = 12$ days) must be kept to satisfy the customer demand.

Similarly, a safety stock placement is needed for an external stage that cannot satisfy the demand from a downstream stage because the actual time they can serve their customer exceeds their committed service time. The amount of safety stock is computed from the outgoing service time of a stage and the transportation time from that stage to the downstream stage. For example, stage R8 commits a service time of $S^{in}_{C14,98\%} = 24$ days to stage C14 with an additional 8 days to deliver the order. However, stage C14 expects their orders to arrive 12 days after placing the order. Therefore, it is essential to keep a safety stock of 20 days between external stage R8 and internal stage C14 to satisfy the demand on time.

It should be noted that our proposed MILP model does not consider the manufacturing cost or other costs related to producing the products. Hence, with the same quantity of safety stock, the holding cost of a completed component (before shipment) at an upstream stage and the holding cost of the incoming shipment of the component at the downstream stage have the same value. For example, with the same amount of safety stock of 24 days, approximately 120 units, the holding cost of this component as an output item at stage C12 is the same as the cost of holding the component as an input item at stage FG. Tables 2 and 3 show that the optimal total safety stock cost at 98% of the service level is 3,302.27 THB/year and 4,011.52 THB/year for normal and empirical distributions, respectively.

Table 4 shows the total safety stock cost at different service levels, ranging from 90% to 99.9%, for the case of using the normal distribution to model the demand data. Note that these results are obtained by fixing the CSL in our MILP model. To accommodate this, Equation (24) is revised to $z_k = 1$ for the selected CSL k and set this variable for other service levels equal to zero. Then, the MILP model is solved for one CSL at a time. The experiment aims to demonstrate that the optimal CSL from our proposed MILP matches the best CSL from solving the model with different CSLs separately.

Generally, service level represents the trade-off between inventory holding and shortage costs. A low service level would increase stock-out, which leads to an increase in the shortage cost. On the other hand, a high service level indicates more safety stock to be kept, which reduces shortage but leads to a higher inventory holding cost. The result shows that the minimum total cost is achieved at the service level of 98% from both approaches, i.e., solving k CSLs at once and one at a time. This level of customer service suggests keeping more safety stock rather than experiencing shortage.

TABLE 4. The total inventory cost of different service levels – normal distribution (in THB).

Service level	Holding cost of output item	Holding cost of input item	Shortage cost of output item	Shortage cost of input item	Total cost
0.9	976.52	1,268.89	1,251.25	1,220.67	4,717.33
0.91	1,315.95	885.84	1,547.93	766.68	4,516.39
0.92	1,035.81	1,150.46	1,181.20	939.50	4,306.98
0.93	948.92	1,273.78	941.82	935.56	4,100.08
0.94	751.12	1,514.50	589.24	1,045.04	3,899.90
0.95	58.75	2,269.46	32.69	1,349.06	3,709.96
0.96	1,090.60	1,286.41	660.94	494.03	3,531.98
0.97	577.08	1,959.73	428.63	414.67	3,380.11
0.98	336.71	2,430.22	143.26	392.07	3,302.27
0.99	1,098.85	1,980.83	82.83	195.67	3,358.18
0.991	1,117.40	2,014.26	73.72	174.15	3,379.53
0.992	1,053.55	2,133.06	60.50	159.17	3,406.28
0.993	1,694.95	1,555.62	132.94	56.69	3,440.20
0.994	1,732.80	1,590.36	112.23	47.86	3,483.26
0.995	1,776.73	1,630.68	91.91	39.20	3,538.52
0.996	1,829.32	1,678.94	72.04	30.72	3,611.02
0.997	1,895.34	1,739.54	52.68	22.46	3,710.02
0.998	1,985.27	1,822.08	33.95	14.48	3,855.78
0.999	2,131.55	1,956.33	16.10	6.87	4,110.85

From Table 5, when the demand is modeled using empirical distribution, the total inventory costs are higher than the normal demand for all the CSLs. This result is because empirical distribution can accurately capture the uncertainty of actual demand, while normal distribution fails to capture this skewness. Therefore, if we compute the safety stock using the normal distribution, the estimation of inventory cost may not be accurate. The next section will show the accuracy in estimating the total inventory cost by comparing the results from the MILP model under each of these two demand distributions with results from a simulation model.

C. SIMULATION RESULTS

The preliminary results during the simulation model validation process give an estimate of the standard error of the key system measure of performance. Based on the standard error, the required number of replications for the simulation

TABLE 5. The total inventory cost of different service levels – empirical distribution (in THB).

Service level	Holding cost of output item	Holding cost of input item	Shortage cost of output item	Shortage cost of input item	Total cost
0.9	1,634.92	1,029.15	1,939.55	2,802.44	5,466.51
0.91	1,143.14	1,468.03	1,458.58	2,669.84	5,281.00
0.92	759.30	1,862.80	1,018.30	2,446.37	5,068.47
0.93	393.79	2,158.42	689.42	2,322.40	4,874.61
0.94	1,540.15	1,062.01	1,439.59	2,062.40	4,664.57
0.95	857.70	1,839.58	744.50	1,765.39	4,462.66
0.96	1,571.71	1,191.03	1,084.59	1,501.98	4,264.72
0.97	1,269.76	1,677.56	609.92	1,160.27	4,107.60
0.98	1,738.48	1,475.37	485.51	797.68	4,011.52
0.99	1,510.50	2,143.79	215.24	429.28	4,083.57
0.991	2,065.84	1,661.00	291.64	383.04	4,109.88
0.992	1,773.41	2,044.43	190.99	317.89	4,135.73
0.993	456.47	3,452.07	46.42	273.38	4,181.92
0.994	531.05	3,481.94	66.19	233.32	4,246.31
0.995	2,197.05	1,903.76	167.83	226.40	4,327.21
0.996	2,279.55	1,968.96	133.21	180.03	4,428.54
0.997	2,383.25	2,057.21	99.21	130.21	4,570.68
0.998	2,533.15	2,173.91	65.06	85.38	4,792.45
0.999	2,770.35	2,368.21	32.72	42.63	5,181.20

model for a given optimal solution from the MILP model is estimated to be 15 replications. The comparison between the results from the MILP model and 15-replication simulation is shown in Tables 6 and 7 for normal and empirical distributions, respectively.

Computing the amounts of safety inventory using the normal distribution when the actual demand data do not follow the normal distribution illustrates that the MILP model's result underestimates the total cost by 28.46% compared to the simulation results. Of this difference, 3.17% is attributed to holding cost and 25.29% to shortage cost. On the other hand, when the safety stock amounts are estimated from the empirical distribution, the MILP model can give accurate estimates of the system measure of performance with a 0.10% difference compared to the simulation results. This 0.10% results from the 0.84% underestimating of holding cost and 0.93% overestimating of the shortage cost. The comparison results demonstrate the effectiveness of determining the safety stocks based on the empirical distribution that matches the characteristic of the demand during replenishment time, which is superior to simply assuming the normal distribution.

In addition, a statistical test is performed to ensure that the difference between the solution from the MILP under the empirical distribution is not statistically different from the simulation results. In addition, a similar test is

TABLE 6. Comparison of cost components between MILP model and simulation in the case of normal distribution.

Stage	MILP			Simulation		
	Holding Cost	Shortage Cost	Total	Holding Cost	Shortage Cost	Total
C12	71.87	11.72	83.60	74.62	17.55	92.17
C9.4	870.55	113.62	984.17	915.03	144.15	1059.18
R8	120.29	23.55	143.84	124.58	40.78	165.35
R6	329.37	23.03	352.40	366.55	16.06	382.61
C11.1	237.18	77.39	314.57	242.55	202.37	444.93
R1	263.36	35.55	298.91	276.35	45.88	322.23
C3	293.43	127.65	421.08	298.15	444.81	742.96
C6.1	27.66	54.14	81.80	28.19	664.07	692.26
R4	207.49	25.39	232.87	218.99	30.07	249.07
R5	335.95	33.73	369.68	358.09	32.97	391.06
R2	9.78	9.58	19.36	9.98	64.15	74.13
Total	2,766.93	535.35	3,302.28	2,913.08	1,702.86	4,615.95
				-3.17%	-25.29%	28.46%

TABLE 7. Comparison of cost components between MILP model and simulation in the case of the empirical distribution.

Stage	MILP			Simulation		
	Holding Cost	Shortage Cost	Total	Holding Cost	Shortage Cost	Total
C12	82.78	16.40	99.18	85.45	11.20	96.64
C9.4	990.64	152.11	1142.75	853.37	101.63	955.01
R8	140.82	33.41	174.23	174.78	23.31	198.09
R6	361.11	29.04	390.15	445.98	6.31	452.29
C11.1	291.50	117.27	408.77	295.56	110.99	406.55
R1	300.33	47.82	348.16	312.56	30.84	343.39
C3	373.56	199.73	573.29	377.55	214.07	591.61
C6.1	45.37	113.05	158.42	45.70	201.75	247.45
R4	233.83	34.58	268.41	244.80	20.02	264.82
R5	375.22	44.29	419.51	398.19	23.78	421.97
R2	18.68	9.99	28.66	18.88	10.70	29.58
Total	3213.84	797.69	4011.53	3252.82	754.60	4007.40
				-0.84%	0.93%	0.10%

TABLE 8. Statistical test results.

Demand distribution vs. simulation	T-statistic	P-value
Empirical	0.450	0.662
Normal	11.140	0.000

applied for the case between the MILP solution under the normal distribution and the simulation results. The test results are presented in Table 8.

From the p-values, the solution from the MILP under the empirical distribution and the simulation are not statistically different, whereas the MILP solution under the normal

TABLE 9. Sensitivity analysis results in the case of normal distribution.

Service level	Shortage Cost							
	10%	15%	20%	25%	30%	35%	40%	45%
0.9	2,857.64	3,363.46	3,839.75	4,293.24	4,717.33	5,114.33	5,488.89	5,844.87
0.91	2,814.17	3,272.98	3,705.86	4,119.67	4,516.39	4,890.67	5,244.67	5,580.94
0.92	2,772.25	3,187.85	3,576.56	3,948.34	4,306.98	4,653.83	4,984.49	5,299.85
0.93	2,739.66	3,109.43	3,453.19	3,782.13	4,100.08	4,408.61	4,709.09	5,000.06
0.94	**2,722.14**	3,034.55	3,337.74	3,624.14	3,899.90	4,168.23	4,429.91	4,685.68
0.95	2,723.73	2,976.83	3,229.35	3,476.20	3,709.96	3,937.27	4,158.94	4,375.84
0.96	2,750.97	**2,947.06**	3,142.64	3,337.87	3,531.98	3,720.60	3,901.76	4,079.24
0.97	2,808.13	2,957.39	**3,098.57**	3,239.44	3,380.11	3,520.61	3,660.99	3,800.25
0.98	2,919.82	3,020.56	3,121.05	**3,212.97**	**3,302.27**	**3,391.47**	3,480.52	3,569.50
0.99	3,171.08	3,217.93	3,264.78	3,311.63	3,358.18	3,404.60	**3,451.02**	**3,497.44**
0.991	3,212.72	3,254.42	3,296.11	3,337.81	3,379.53	3,420.85	3,462.16	3,503.47
0.992	3,259.83	3,296.45	3,333.06	3,369.67	3,406.28	3,442.90	3,479.19	3,515.46
0.993	3,313.78	3,345.38	3,376.99	3,408.59	3,440.20	3,471.81	3,503.41	3,534.82
0.994	3,376.53	3,403.21	3,429.90	3,456.58	3,483.26	3,509.94	3,536.63	3,563.31
0.995	3,451.11	3,472.97	3,494.82	3,516.67	3,538.52	3,560.37	3,582.23	3,604.08
0.996	3,542.52	3,559.64	3,576.77	3,593.90	3,611.02	3,628.15	3,645.27	3,662.40
0.997	3,659.92	3,672.45	3,684.97	3,697.49	3,710.02	3,722.54	3,735.06	3,747.59
0.998	3,823.49	3,831.56	3,839.64	3,847.71	3,855.78	3,863.85	3,871.92	3,880.00
0.999	4,095.54	4,099.37	4,103.19	4,107.02	4,110.85	4,114.68	4,118.50	4,122.33

TABLE 10. Sensitivity analysis results in the case of the empirical distribution.

Service level	Shortage Cost							
	10%	15%	20%	25%	30%	35%	40%	45%
0.9	3,271.04	3,897.67	4,479.48	4,987.11	5,466.51	5,918.87	6,342.67	6,749.59
0.91	3,232.43	3,803.01	4,337.43	4,828.78	5,281.00	5,708.56	6,112.68	6,497.94
0.92	3,194.64	3,709.36	4,191.57	4,645.86	5,068.47	5,468.03	5,845.11	6,202.80
0.93	3,178.03	3,635.55	4,072.38	4,481.69	4,874.61	5,248.68	5,600.57	5,933.40
0.94	**3,163.87**	3,567.58	3,950.85	4,312.73	4,664.57	4,999.59	5,322.35	5,628.06
0.95	3,174.43	3,518.59	3,845.74	4,157.80	4,462.66	4,755.02	5,035.99	5,312.07
0.96	3,201.70	**3,486.72**	3,755.64	4,014.39	4,264.72	4,512.37	4,746.31	4,972.31
0.97	3,271.29	3,495.35	**3,707.66**	3,912.07	4,107.60	4,300.98	4,488.51	4,667.70
0.98	3,434.56	3,582.13	3,729.25	**3,876.17**	**4,011.52**	**4,144.47**	4,274.72	4,403.40
0.99	3,783.44	3,867.08	3,940.45	4,012.03	4,083.57	4,155.02	**4,226.23**	4,296.56
0.991	3,837.47	3,912.02	3,982.20	4,046.04	4,109.88	4,173.48	4,236.85	4,300.16
0.992	3,903.92	3,966.23	4,028.11	4,082.75	4,135.73	4,188.71	4,241.43	**4,294.07**
0.993	3,976.66	4,030.38	4,084.11	4,136.24	4,181.92	4,227.49	4,273.05	4,318.55
0.994	4,065.02	4,110.67	4,156.32	4,201.97	4,246.31	4,285.20	4,324.08	4,362.97
0.995	4,176.28	4,214.01	4,251.75	4,289.48	4,327.21	4,364.95	4,397.12	4,429.20
0.996	4,308.52	4,338.53	4,368.53	4,398.54	4,428.54	4,458.55	4,488.56	4,518.34
0.997	4,483.87	4,505.57	4,527.27	4,548.97	4,570.68	4,592.38	4,614.08	4,635.78
0.998	4,735.53	4,749.76	4,763.99	4,778.22	4,792.45	4,806.68	4,820.91	4,835.14
0.999	5,152.78	5,159.88	5,166.99	5,174.10	5,181.20	5,188.31	5,195.41	5,202.52

distribution is significantly different from the simulation results. The results indicate the effectiveness of the empirical distribution over the normal distribution in modelling the demand.

D. SENSITIVITY ANALYSIS

Sensitivity analysis is performed to examine the effects of the model parameters on the objective function value and the point at which the optimum is reached [32]. Notably, a sensitivity analysis of the shortage cost CS is performed to evaluate how the solutions react to changes in this cost component. Note that varying this cost component alone is sufficient since it already represents the trade-off between the two cost components. The numerical experiment is re-run with the ratio of CS to H varying from 1 to 4.5 with the step size of 0.5, i.e., $\mathrm{CS}/H = \{1, 2, \ldots, 4.5\}$. The result of the sensitivity analysis, including inventory holding costs and shortage costs of both finished goods and raw materials, are shown in Tables 9-10.

Tables 9 and 10 indicate the relationship between shortage cost and service level. While holding the unit inventory holding cost fixed, the optimal service level becomes higher as the unit shortage cost increases. The increase in the service level prevents the rise in the shortage cost. This trend is also observed in the case of the empirical distribution.

VI. CONCLUSION

This study introduces a MILP model for positioning safety stock in an assembly supply chain network. All the stages operate under a base stock inventory policy and face the same demand information. Due to the demand uncertainty and operational constraints, some stages are required to keep safety stock to maintain an acceptable CSL. In addition to determining the safety stock locations, the MILP model can select the CSL that minimizes the total safety stock holding and shortage costs for the whole supply chain. The model is tested under two demand distributions, a commonly assumed normal and empirical.

To validate the results from the MILP model, a simulation model is developed to imitate the behavior of the base stock policy for stages that keep safety stocks in the supply chain. The net replenishment time of each safety stock position obtained from the optimization model is used as input to the simulation model. The MILP model results under both demand distributions are compared with the simulation results in terms of cost components and percentages of their contribution to the total cost. The results confirm the accuracy of the MILP model under the empirical demand, with slight differences compared to the simulation. However, the model under the normal demand underestimates the amount of holding and shortage at most inventory positions. This behavior demonstrates the effectiveness of empirical over normal distributions in capturing the demand uncertainty.

Our proposed model can be extended in different directions. One of them should accommodate the presence of multiple products sharing the same facility. In addition, the production capacity of each facility in the supply chain network should be considered. Moreover, uncertainty in production and transportation time can be included in the model.

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