IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 27 July 2022, accepted 23 August 2022, date of publication 29 August 2022, date of current version 2 September 2022.
Digital Object Identifier 10.1109/ACCESS.2022.3202630

APPLIED RESEARCH

Mathematical Simulation of Behavior of Female Breast Consisting Malignant Tumor During Hormonal Changes

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ABSTRACT The thermal models of human organs for normal, as well as malignant tissues, can be used for generating thermal information for detecting and analyzing tumors. Thermography suffers from the issues of accuracy and specificity. This is due to the non-availability of complete thermal information of human body organs due to benign and malignant disorders. The complete study of thermal patterns in human body organs due to benign and malignant disorders can be useful for generating thermal information for improving the specificity and sensitivity in existing thermographic techniques for the detection of cancer. We have proposed a thermal model of a woman's breast involving two benign disorders, namely cysts and hormonal changes due to the menstrual cycle. In the proposed model, various critical factors like thermal conductivity and metabolic-heat generation are correlated with blood mass flow while the boundary conditions for acceptance are framed. The model is developed in terms of a boundary-value problem involving the partial equations for a 2-D steady-state case. Thermal patterns within a woman's breast due to cysts during the presence and absence of various phases of the mensuration cycle are obtained and analyzed. Also, the thermal patterns due to the cyst becoming malignant neoplasm in the Female breast are obtained and analyzed. The substantial differences within thermal patterns in the breast due to malignant tumors and benign disorders like cysts and various phases of the cycle are observed. The thermal information generated from such models is useful to differentiate between benign and malignant disorders by thermography.

INDEX TERMS Female breast, menstrual cycle, breast cyst, numerical simulation.

I. INTRODUCTION

Due to the non-invasive radiation methods, thermography is always a preferred choice over mammography or ultrasound. It is a pain-free imaging method, which uses infrared to sense heat and increased vascularity that may be related to angiogenesis. It is also capable of detecting physiological changes that can work as early detection of breast cancer. Many other issues, like premenstrual breast swelling and tenderness, or cyclical mastalgia, also play a vital role in detecting fibrocystic breast disease. As the hormones rise and fall during a normal menstrual cycle, it is important to observe the behavior of the female breast during these hormonal changes. The various benign changes and disorders take place in human body organs during the life of human subjects. These changes may take place due to aging, and hormonal changes in the size, shape, and structure of human organs which lead to thermal stress affecting the performance of human subjects. One of the notable examples is a woman's breast. The various physical and physiological changes can take place in a woman's breast because of aging, hormonal changes, and menstrual cycle and they lead to various benign

The associate editor coordinating the review of this manuscript and approving it for publication was Shuai Liu.

and malignant disorders [1], [2], [3]. The cyst is a common benign disorder among 57% women. The menstrual cycle is another cause of benign disorder in the Female breast. Monthly hormone changes often cause cysts to urge bigger and become painful and more noticeable just before the menstrual period. During the cycle, the ladies feel some pain and tenderness, lumpiness in their breasts. There's also a risk of cyst delivery to malignant tumors. Thus, the study of thermal behavior in a woman's breast because of cyst in the presence and absence of varied phases of the cycle is often useful to get the thermal information, which successively is often useful in the development of clinical thermography. Patterson [4], [5], [6], [7], [8] performed the experimental investigations to review the behavior of temperature in a peripheral area of the human body. Many recent studies on temperature distribution in peripheral tissues of flat regions of the physical body like trunk [9], [10], [11], [12], [13], [14], [15], [16], the cylindrical-shaped human organs like limbs [13], [17], [18], [19], [20], [21] and spherical-shaped organs like head [22], [23], [24], [25], [26], [27] and woman's breast [28], [29], [30], [31], [32], [33] are well-analyzed under all the essential environmental conditions. Also, theoretical studies on the thermal effect of malignant tumors in various regions of the physical body like flat, cylindrical, and spherical shaped organs [4], [34], [35], [36], [37], [38], [39], [40], [41], [42] are reported by various researchers. References [30], [43], [44], [45], [45], [46], [47], [48] have analyzed the thermal behavior in both genders under different environmental subjects for 1-D and 2-D cases. Acharya *et al.* have also presented the study of the nature of temperature during hormonal changes. It is, however, evident that little or no attention is given to analyzing the effect of benign disorders on temperature distribution in physical body organs while very few investigations were focused on to study of thermal patterns in Female breast during various phases of the cycle [49], [50], [51], [52], [53], [54]. The various sorts of benign changes can occur in physical body organs especially in a woman's breast simultaneously. Further, no model is reported in the past for the study of cyst due to thermal changes or a malignant neoplasm in a woman's breast in various phases of the cycle. In this paper, we have proposed a model to review the thermal changes in the Female breast due to two different simultaneously occurring benign disorders like a cyst and different phases of the cycle. Further, the model is modified to review the thermal effect of a cyst turning into a malignant neoplasm in various phases of the cycle.

II. MATHEMATICAL MODEL

A breast with a hemispherical shape can be assumed to have five layers namely, 1. epidermis, 2. dermis, 3. subcutaneous tissue, 4. glandular layer, and 5. muscle with the thoracic wall. The domain of the breast is assumed to be 72mm from the areola to the body core with the individual thickness of various layers as follows: 1. epidermis (≅1.5 mm), 2. dermis (≅2 mm), 3. subcutaneous tissue (≅1.5mm), 4. glandular layer (≅45mm) and 5. muscle with a thoracic wall (≅22mm). In the presented study, a tumor or cyst is assumed in a glandular layer at the central line of the breast. This is because most breast tumor/cyst develops in lobules and milk ducts of the glandular layer and medically glandular is known as the breast.

The equation to model the thermal behavior of female breast for 2D - case when it is not depended on time in spherical coordinates is as follows [10], [16]:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(Kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(K\sin\theta\frac{\partial T}{\partial\theta}\right) + M\left(T_A-T_v\right)+S=0 \quad (1)$$

where K = thermal conductivity of tissue, M = blood mass flow rate, TA = arterial blood temperature, Tv = Vein blood temperature, T = tissue temperature at position r measured from the center towards the skin surface, S = rate of metabolic heat generation in tissue.

The external surface of hemispherical district is presented to the climate and warmth misfortune at this surface happens primarily because of conduction, dissipation, radiation, and convection. In this way, the condition forced at the external surface is as per the following [15]:

$$-K\frac{\partial T}{\partial r}=h\left(T-T_a\right)+LE \text{ at } r=r_n,\quad \theta\in(0,\pi) \quad (2)$$

where h is the heat transfer coefficient, Ta is the atmospheric temperature, and L and E are latent heat and rate of sweat evaporation, respectively. rn is the radius of the outer surface of the breast.

For female body organs, h and E are as follows [44]:

$$E = 8.47\times10^{-5}\left(0.1\times T_0+0.7\times T_b\right)-36.6^\circ\text{C}$$
$$h = 1.32\left(T_w-T_a/2\cdot r_5\right)^{1/4}\quad w/\text{m}^2\text{C}$$
$$L = 2.4\times10^6\text{cal/g}$$

The woman's breast is assumed to be hemispherical consisting of various tissue layers. At higher and medium atmospheric temperatures, the inner surface is maintained at a uniform body core temperature T_b. Therefore, the adiabatic condition at the inner boundary is as follows,

$$T(r,\theta)=T_b\text{atr}=r_0 \quad (3)$$

The normal tissues of breast have self-controlled metabolic activity which is expressed as [21]:

$S=\bar{S}\left[1+Q_d\left(T_b-T\right)\right]$, where $Q_d=\frac{2}{T_a+T_b}$ and $\bar{S}$ is the normal rate of metabolic heat generation. In order to make the model non-dimensional the following scaling of parameters is used:

$$U=\frac{T_b-T}{T_b}, TT_b(1-U),$$
$$U_A=\frac{T_b-T_A}{T_b}, T_A=T_b\left(1-U_A\right),$$
$$U_V=\frac{T_b-T_V}{T_b}, T_V=T_b\left(1-U_V\right)$$

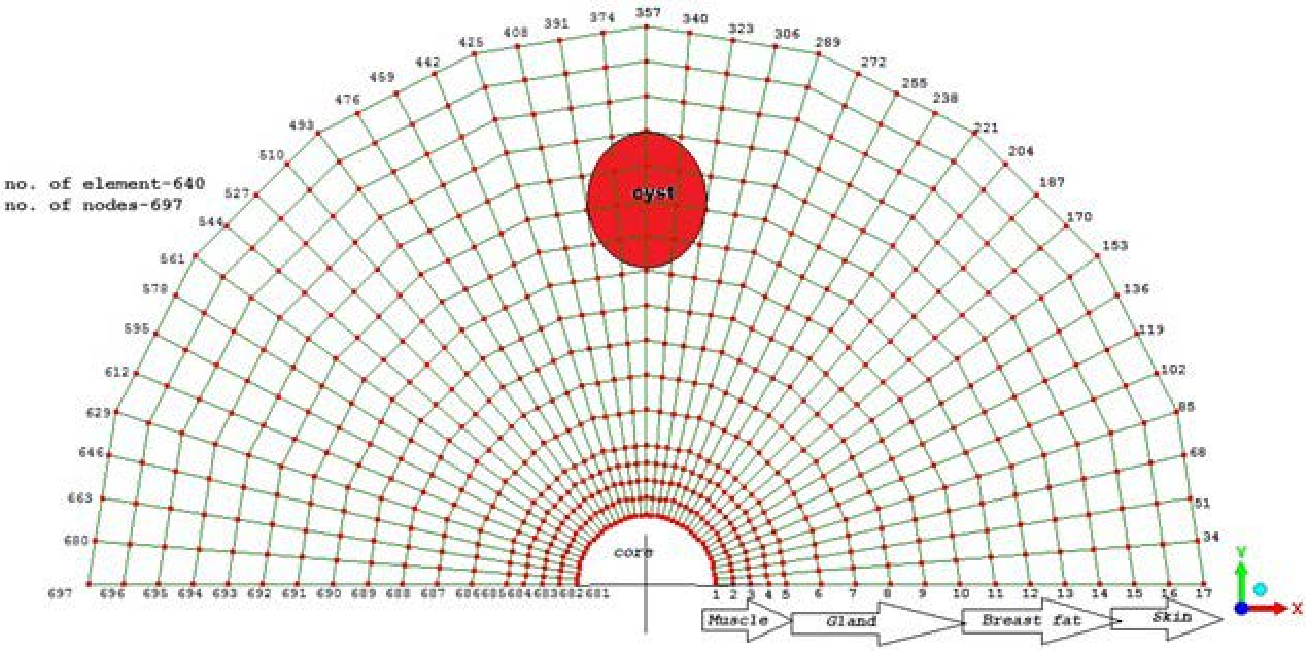

FIGURE 1. Element wise discretization of female breast involving cyst or a malignant tumor.

$$T_a = T_b (1 - U_a)$$
$$r = r' r_n \Rightarrow r' = \frac{r}{r_n}, \theta = \theta' \pi \Rightarrow \theta' = \frac{\theta}{\pi},$$

where U is dimensionless temperature.

Using these parameters in equation (1), we get a non-dimensional form of the equation given by

$$\frac{\pi^2}{r'^2} \frac{\partial}{\partial r'} \left(K r'^2 \frac{\partial U}{\partial r'} \right) + \frac{1}{r'^2 \sin(\theta' \pi)} \frac{\partial}{\partial \theta'} \left(K \sin(\theta' \pi) \frac{\partial U}{\partial \theta'} \right) + M (U_A - U_V) + S^* = 0 \quad (4)$$

where $M = \rho_b \omega_b c_b r_n^2 \pi^2$
$S^* = \bar{S} [1 + Q_d U T_b] r_n^2 \pi^2$

$S'' = \eta S$ and $Q_d = 0$ in malignant tissues, η is a parameter that represents the ratio of metabolic activity in tumor and normal tissues, and the metabolic activity in tumors is found to vary between 1 to 7 times of that in normal tissues [36], [37]. The non-dimensional form of boundary conditions (2) and (3) is given by:

$$-K \frac{\partial U}{\partial r'} = h^* (U - U_a) + L^* E^*, \quad r = r_n \& \theta \in (0, \pi) \quad (5)$$

$$h r_n = h^* \quad \text{and } E^* = E r_n T_b$$
$$\text{and} \quad U = \frac{T_b - T}{T_b} = 0 \quad \text{at} = \quad r' = r_0 / r_n \quad (6)$$

III. PROPOSED METHOD

The female breast has 16 layers surrounding the inner core (Figure 1). The inner core typically consists of blood vessels, lobes, and glandular tissues. The first four layers above the core are muscles. Above the muscles, there are five layers of glands. Above the glands, the fat region is divided into four layers. The breast fat is covered by three layers of skin and sub-dermal tissues. A spherical-shaped lump is assumed to be present in the fat layers. Initially, this lump is assumed to be a cyst and alternatively, this lump is assumed a malignant tumor. The whole region is divided into 640 coaxial circular sector elements and 697 nodes.

$$I^{(e)} = \frac{1}{2} \iint \left[K^{(e)} \pi^2 r'^2 \left(\frac{\partial U^{(e)}}{\partial r'} \right)^2 + K^{(e)} \left(\frac{\partial U^{(e)}}{\partial (\theta' \pi)} \right)^2 + \left\{ M^* \left(U_V^e - U_A^{(e)} \right) + S^{4(e)} \right\} U^{(e)} r'^2 dr' d(\theta' \pi) \right] + \frac{\lambda^{(\epsilon)}}{2} \int_{\theta_i}^{\theta_i} \left\{ h^* \left(U^{(\varepsilon)} - U_a \right)^2 + 2 L^* E^* U^{(\epsilon)} \right\} r'^2 d(\theta' \pi)$$
$$\text{for} \quad e = 1(1)640 \quad (7)$$

Here we assume non-dimensionalized arterial blood temperature equal to the average of non-dimensionalized nodal temperatures of the previous element along the radial direction as the arterial blood will enter the eth element from its previous element below it and the blood will have an almost average temperature of its previous element because blood gives heat to the element from which it passes and cools down to an almost same temperature of that element. In the same way, the venous blood temperature is also taken as the average of nodal temperatures of the element from which it is coming

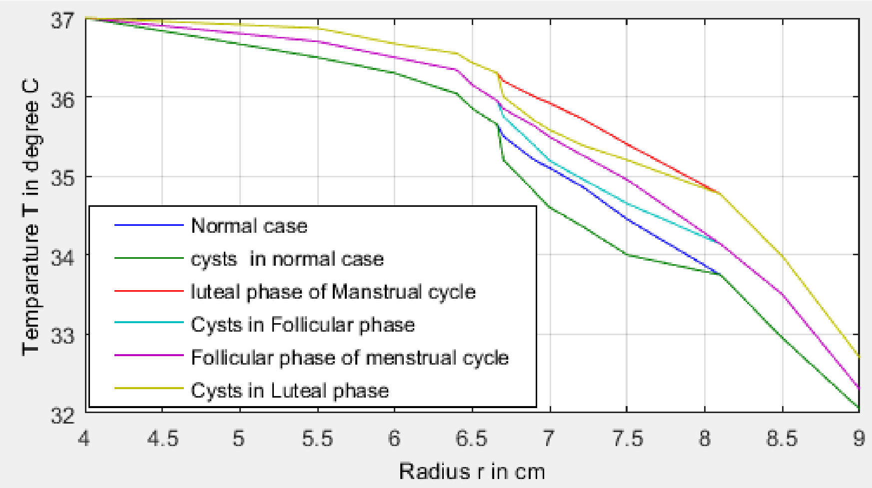

FIGURE 2. Radial thermographic behavior in Female breast due to cyst in absence and presence of menstrual cycle. For $T_a = 23°C$, $E = 0.24 \times 10^{-3}$ gm/cm^2 min. Normal case means in absence of menstrual cycle.

into the next element.

$$U_A^{(\epsilon)} = \left(\frac{U_i + U_j + U_k + U_l}{4}\right)^{(\epsilon-1)} \text{ and}$$

$$U_V^{(\epsilon)} = \left(\frac{U_i + U_j + U_k + U_i}{4}\right)^{(\epsilon+1)}$$

Here:

The shape function for the behavior of temperature at the each node is proposed as:

$$U^{(\epsilon)} = c_1^{(\epsilon)} + c_2^{(\epsilon)} r' + c_3^{(\epsilon)} \theta' + c_4^{(\epsilon)} r'\theta' \tag{8}$$

$$U^{(\epsilon)} = U_p, P = i, j, k, l \tag{9}$$

$$I^{(\epsilon)} = \sum_{\epsilon=1}^{N} I^{(\epsilon)} \tag{10}$$

$$\frac{dI}{d\bar{U}} = 0 \tag{11}$$

where

$\overline{U} = U_1 U_2 U_3 \ldots U_{697}$

From (11) we get a system of linear algebraic equations as follows;

$$[X]_{697\times697}\left[\overline{U_{697\times1}}\right] = [Y]_{697\times1} \tag{12}$$

Here, we applied the Gauss elimination method which is used to obtain the solution of equation 12 and MATLAB is used to develop a computerized numerical solution for the entire problem.

IV. RESULT ANALYSIS

The reproduction is first accomplished for $N = 640$ components and afterward for $N = 1280, 2560$ and 5120 components. We got a temperature of 32.5666 at hub number 352 in pimple during the luteal period of the feminine cycle for the model with N = 640 components and temperature 32.5676 at hub number 692 in sore for the model with N = 1120 components. The error is $[(32.5676 - 32.5666)/32.5676] * 100$ which works out to be $27.2 \times 10 - 6\%$ as it were. In case of four decimal places $(0.5666/0.5676) * 100 = 99.82\%$ is the certainty level.

The experimental results are plotted for temperature dispersion in Female bosom including blister and threatening tumor in presence and nonappearance of period. Figure 2 and 3 discusses the temperature dispersion along spiral course in Female bosom including blister in nonattendance and presence of various periods of monthly cycle for Ta = 230C and various estimations of E. In Figure 4, 5 and 6 temperature profiles are appeared along with spiral bearing in Female bosom including growth and threatening tumor for typical case (without period) and follicular and luteal periods of feminine cycle for Ta $= 23^0$C, gm/cm^2 min. In Figure 7, 8 and 9 the temperature profiles are appeared along with precise heading in Female bosom including growth and threatening tumor in nonappearance and presence for feminine cycle Ta $= 23^0$C, gm/cm^2 min.

In Figure 2 and 3 it is observed that the temperature tumbles down continuously from the inward center $r = 4.0$ cm of the bosom towards the external surface of the bosom. This is because of the warmth from the external surface of the bosom to the climate. The temperature is discovered to be higher in all through the tissues of the bosom in the luteal stage when contrasted with the follicular stage and the temperature in the follicular stage is higher than that in the typical case for example without a period. Further, we observed the adjustment in the slant of the bend at the intersections of each layer which is because of the various properties of each layer. Likewise, we observed the sharp change in slant of the bend and fall in temperature at the intersections of ordinary tissues and blister, and temperature is most reduced at the focal piece of the sore. The adjustment in the incline of the bend at the intersections of ordinary tissues and pimple demonstrates the limit and area of the blister.

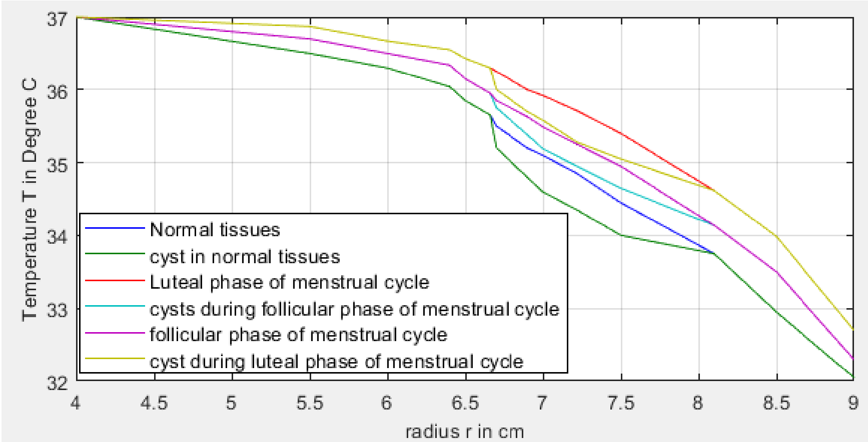

FIGURE 3. **Radial thermographic behavior in Female breast due to cyst in absence and presence of menstrual cycle. For $T_a = 23°C$, $E = 0.48 \times 10^{-3}$ gm/cm^2 min. Normal case means in absence of menstrual cycle.**

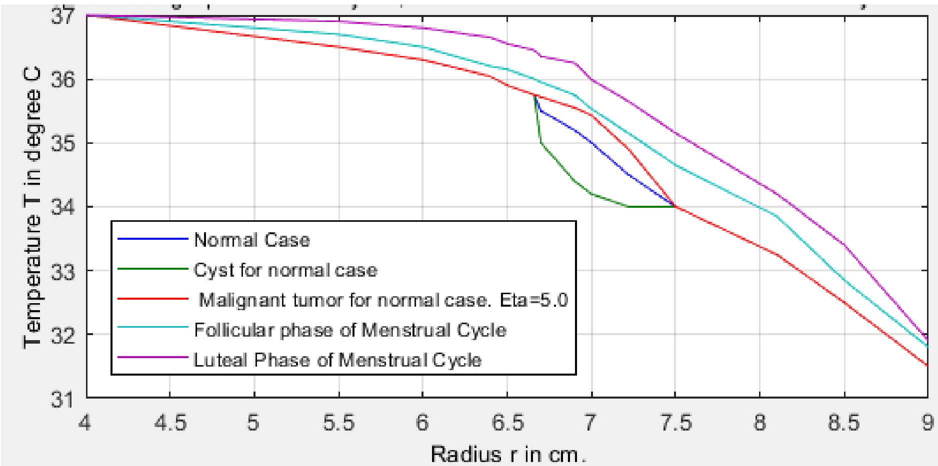

FIGURE 4. **Figure:4 Radial thermographic behavior in Female breast due to lump in absence of menstrual cycle for $T_a = 23°C$, $E = 0.24 \times 10^{-3}$ gm/cm^2 min. Lump is cyst or malignant tumor. Normal case means absence of menstrual cycle.**

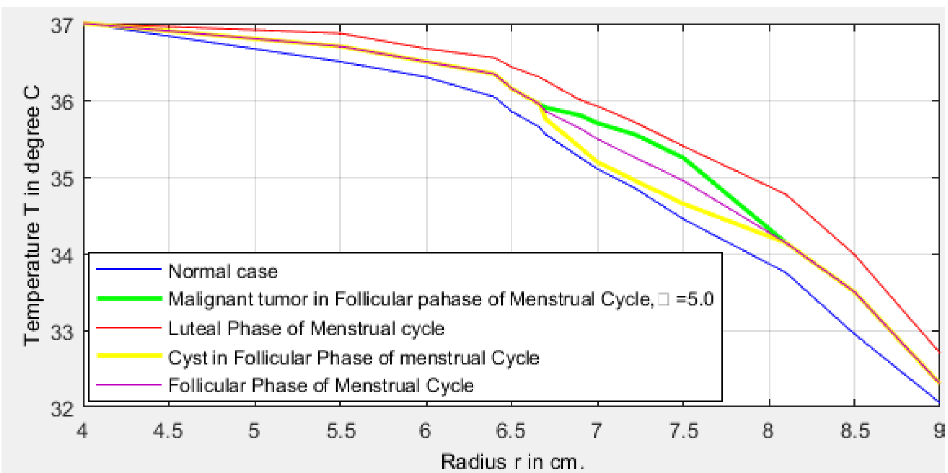

FIGURE 5. **Radial thermographic behavior in Female breast due to lump during follicular phase of menstrual cycle for $T_a = 23°C$, $E = 0.24 \times 10^{-3}$ gm/cm^2 min. Lump is cyst or malignant tumor.**

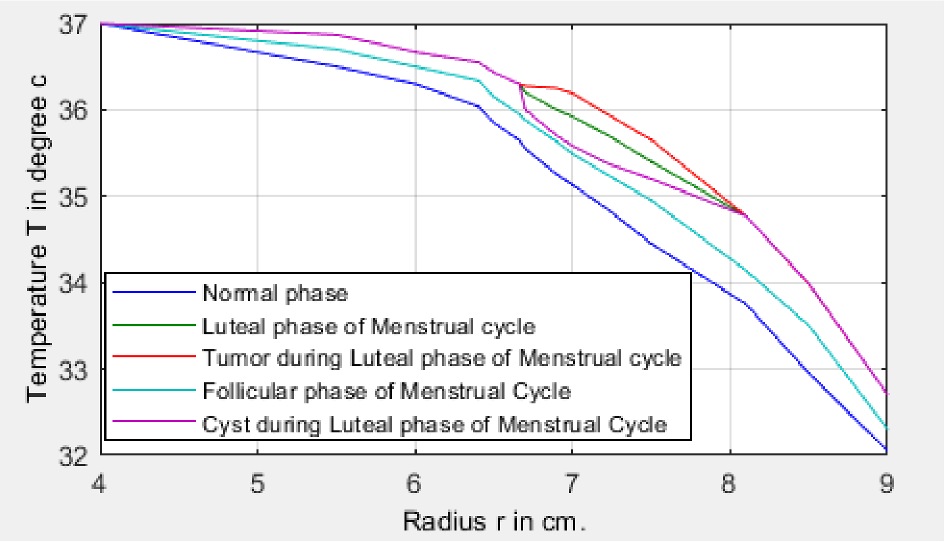

FIGURE 6. **Radial thermographic behavior in Female breast due to lump during luteal phase of menstrual cycle for $T_a = 23^\circ C$, $E = 0.24 \times 10^{-3}$ gm/cm^2 min. Lump is cyst or malignant tumor.**

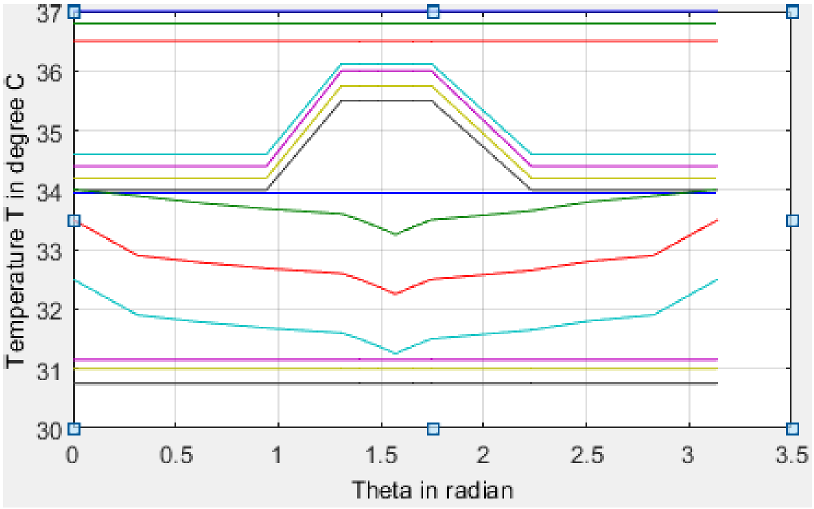

FIGURE 7. **Angular thermographic behavior in Female breast due to lump in absence of menstrual cycle for $T_a = 23^\circ C$, $E = 0.24 \times 10^{-3}$ gm/cm^2 min. Lump is cyst or malignant tumor. Normal case means absence of menstrual cycle.**

In Figures 4, 5 and 6 we observed the sharp changes in the incline of the bend at the intersections of the protuberance and ordinary tissues. At the point when this bump is sore, we observed the deviation of temperature profiles downwards from the intersection of ordinary tissues and the sore and this fall in temperature is maximum at the focal piece of the blister and afterward, rise in temperature started from the focal point of growth up to the intersection of pimple and typical tissues. At a point when this protuberance is a threatening tumor, we observed the rise in temperature profiles at the intersection of ordinary tissues and tumor and this rise of temperature is consistent up to a focal piece of the tumor where it is extreme. From the focal piece of the tumor, a fall in temperature profile is seen up to the intersection of the tumor and typical tissues. The Figures 4, 5 and 6 depicted that the temperature in the tissues without a monthly cycle (typical case), is lower than that for the follicular stage which is again lower than that for the luteal stage. Similarly, the temperature in the blister and tumor in a typical case (nonappearance of the feminine cycle) is lower than that in pimple and tumor individually in a follicular stage which is further lower than that in the luteal stage. In Figures 7, 8 and 9 we observed the adjustment in the slant of the bend along a precise course at the intersections of bump and typical tissues. We noticed that when the irregularity is a harmful tumor, there is a rise of temperature from the intersection of typical and tumor tissues till the focal piece of the tumor and afterward fall in temperature is noticed from the focal portion of tumor up to

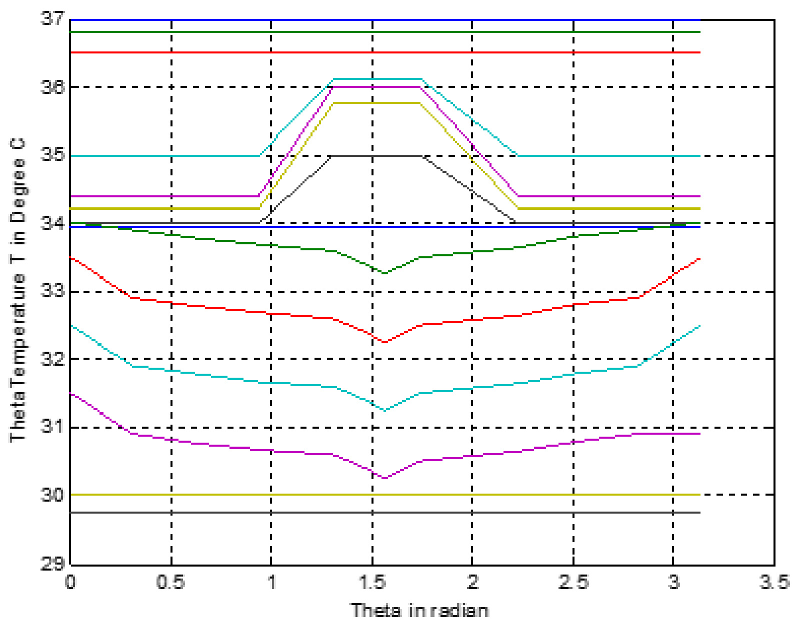

FIGURE 8. Angular thermographic behavior in Female breast due to lump during follicular phase of hormonal changes for $T_a = 23°C$, $E = 0.24 \times 10^{-3}$ gm/cm^2 min. Lump is cyst or malignant tumor.

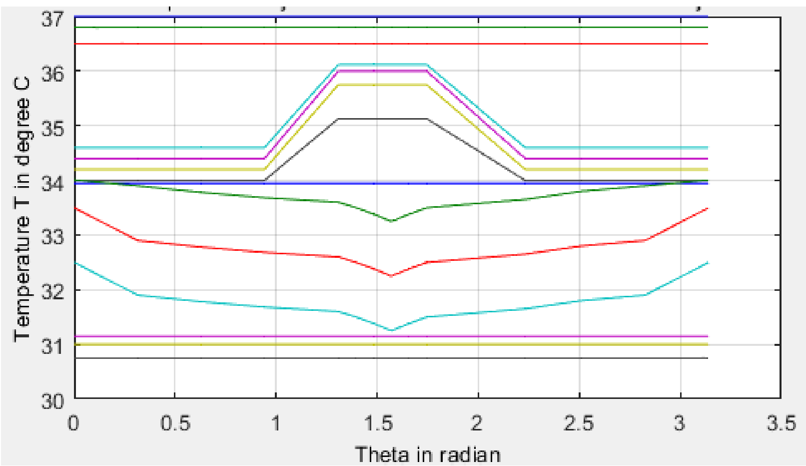

FIGURE 9. Angular thermographic behavior in female breast due to lump during luteal phase of hormonal changes for $T_a = 23°C$, $E = 0.24 \times 10^{-3}$ gm/cm^2 min. Lump is cyst or malignant tumor.

ordinary tissues. We noted that the fall in temperature profiles along a precise course from the intersection of typical tissues and pimples at various outspread positions and this fall was more at the focal piece of growth, and afterward, this fall in temperature profiles diminishes as we move away from a focal portion of sore up to the ordinary tissues. This fall in temperature is most extreme in the tissues in spiral situations, close to the blister and this fall in temperature in tissues is lower at outspread positions from the pimple.

Table -1 shows the change of temperature because of growth and tumor for the ordinary case and during follicular and luteal period of the monthly cycle. In the event of growth, there is a fall in temperature and consequently, the values are prefixed by the (-) sign which demonstrates that this is the most extreme fall in temperature from the comparing reference profiles (typical, follicular, or luteal stage). Here reference profiles imply that the temperature profiles during the absence of period or follicular stage or luteal period of the

TABLE 1. Maximum temperature differences in female breast with & with lump for various cases.

S. No.	Normal breast	During Follicular	During Luteal phase
Cyst	$(-)0.4051°C$	$(-)0.449°C$	$(-)0.48°C$
Tumor	$0.44°C$	$0.472°C$	$0.56°C$

monthly cycle in the typical tissues of Female bosom without tumor and blister. The positive estimations of temperature are mentioned in Table-1 for harmful tumors demonstrate that the major change of the temperature of the tumor from the reference profile is because of the rise of temperature brought about by the tumor. For a typical case, in follicular stage and luteal stage in Table-1, it is observed that the most extreme temperature distinction is 0.48°C because of sore in a luteal stage which is higher when contrasted with that in the follicular stage and ordinary case. It is observed in Table -1 that temperature has changed because of dangerous tumor in the luteal stage is 0.56°C which is noteworthy when compared during the follicular stage and ordinary case. No test or hypothetical outcomes are accessible for correlation of temperature profiles in Female bosom including sore or dangerous tumor during the period. Nonetheless, the outcomes of this experiment for the ordinary case are in concurrence with our early researches [36], [55]. Further, the experimental results for temperature appropriation in Female bosom without tumor or sore during various periods of the feminine cycle are in concurrence with the results of Acharya *et.al.* [44].

Thus, from the Figures 4, 5, 6, 7, 8 and 9 and Table-1 we can conclude that the malignant tumor acts as a source of heat and cyst acts as a sink. These thermal patterns indicating properties like source and sink can be useful in distinguishing between tumor and cyst. Further, these properties of tumor and cyst are more visible during the luteal phase as compared to the follicular phase and normal case. The sudden noticeable changes in temperature profiles indicate the junctions of the lumps and normal tissues, which in turn indicate the boundary and location of the lump in the breast.

V. CONCLUSION

A 2-D (Finite component Model) is proposed to review the thermal patterns generated due to the cyst and malignant tumor in the Female breast during hormonal changes. The analysis and results show that the malignant tumor acts as a local heat source in the breast and the cyst acts as a local sink in the breast. These warm properties of tumor and growth give us the clear picture for recognizing the pimple and dangerous tumor by thermography. Further, the sharp changes in the slant of bends at the intersections of irregularities and ordinary tissues give a way of area and size of the protuberance in the bosom. The FEM technique is assisted with a variety of the properties of different tissues in the female bosom. More models can be developed to explore the identification and categorization of sudden change in temperature due to harmful issues in the Female bosom by thermography. The presented study reveals the major issues and disorders in female breasts specifically related to harmful tumors. The immediate extension of the presented work could be the analysis of the proposed model in the case of cervical cancer and other diseases related to hormonal changes. The presented model contributes to the early detection of harmful tumors and may assist the medical prognosis at the early stage of cancer. This model can further be extended for time series analysis to understand the underlying causes of the growth of tumors over time. The generic version of the presented model is available on request to the authors and is programmed using the modular programming method in MATLAB. The presented model can further be refined by Black-box simulation, and in turn, it can be used for deep learning using Convolutional Neural Networks or Deep Belief Networks, to find out the hidden pattern analysis.

ACKNOWLEDGMENT

The research work follows Mathematics Subject Classification (MSC) Classification Codes: 92C35 (Physiological flow) and 65L60 (Finite elements, Rayleigh-Ritz, Galerkin, and Collocation Methods).

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