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RESEARCH ARTICLE

Flocking With Informed Agents Based on Incomplete Information

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ABSTRACT In this study, the problem of multi-agent flocking with partially informed agents is investigated, by considering the incomplete information factor in a flocking process. Incomplete information includes two aspects: receiver and sender. One is resisted or distorted information by the agents when they receive information from the virtual leader or others, and the other is passive loss of information sent by the virtual leader or others to the agents. In a flocking process with a fraction of informed agents, to make informed agents drive more uninformed agents to track the virtual leader, we first discuss the derivative of the potential function in the flocking algorithm: the force function. The relationship between repulsion and attraction among agents is directly shown. Subsequently, an improved flocking algorithm is proposed based on Morse potential function. The stability of the algorithm is proved by using the Lyapunov stability theorem and LaSalle's invariance principle. Consider the initial distribution of agents with low connectivity and density, based on the above modified algorithm, a novel method of selecting informed agents as propagandists is presented. Propagandists are created in the vicinity of virtual leaders. Before flocking, propagandists move regularly within an arbitrarily distributed group, disseminating information to other uninformed agents. This approach can reduce the unfavorable effects caused by incomplete information. Eventually, the simulation results show that even though only one informed agent is selected as the propagandist, most agents can track the common objective.

INDEX TERMS Flocking, informed agents, potential function, incomplete information, propagandist.

I. INTRODUCTION

As we all know, flocking is a phenomenon in nature, such as flocking of bacteria, schooling of fishes, grouping of ants, crowding of people and so on [1], [2]. A great number of agents can organize a coordinated movement finally by using simple rules and local environmental information. The flocking movement has the characteristics of adaptability, robustness, dispersion and self-organization. Thus, the flocking of multi-agents has attracted widespread attention from researchers in multiple fields [3], [4], [5], [6], [7], [8], [9], [10]. Flocking is used in many control areas including massive distributed sensor networks [11], unmanned aircraft systems [12], [13], [14], and swarm robots [15], [16].

In1986, Reynolds [3] proposed three heuristic rules to simulate the flocking behavior of animals on a computer. The three rules are shown below, in descending order of priority. 1) Collision avoidance: no collision between adjacent companions; 2) Speed matching: match the speed of the surrounding neighbors; 3) Cohesion: try to get as close to your peers as possible. In 1995, Vicsek *et al.* [17] began to study collective behavior from a theoretical perspective and then proposed a simple mode of multi-agent of flocking. In 2003, Jadbabaie *et al.* [18] revisited the Vicsek model without considering noise. By using graph theory and other knowledge, it was proved that the Vicsek model is a stable linear system, and the velocity can converge under certain conditions. In 2006, Olfati-Saber [19] was a pioneer in cluster research, providing the theoretical framework for three flocking algorithms. The first algorithm is a concentrated embodiment of Reynolds' three rules. The results usually lead to the

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fragmentation of the agents and cannot achieve flocking, while the second and third algorithms can implement flocking. The second algorithm is a group of multi-agents that move freely in space. The group of multi-agents tracks the virtual leader by providing speed and position navigational feedback mechanism to each agent. Based on the second algorithm, the third algorithm sets up obstacles during flocking. This flocking algorithm has an assumption, which guarantees all agents would have access to information about the virtual leader. Such assumption requires each agent to have the ability of navigation and positioning, which may be difficult to achieve. However, the assumption does not exist in nature. To overcome this constraint, in 2009, Su *et al.* [20] further explored the flocking algorithm of Olfati-Saber. Although a small number of agents are notified of location and speed information of virtual leader during flocking process, but flocking can also be achieved. The premise is that the uninformed multi-agent is affected by the informed agent in the process of flocking, and follows the informed multi-agent to move at the expected speed, eventually forming a larger group. However, during the flocking process, there are still some agents unable to follow the virtual leader. In addition, [21] and [22] discussed the multi-agent system with switching topology.

In nature, it is normal that only a few individuals know related information. For example, a few fish know migratory route information. Recently, how to solve these problems, some studies have received substantial attention. Yu *et al.* [23], based on observer navigation feedback, proposed a time-varying distributed multi-agent dynamic system flocking algorithm. In this algorithm, each informed agent only knows location information about the virtual leader, but the velocity can still converge. Su *et al*, to maintain initial network connectivity, [24] presented a second-order consistency algorithm. In the case, adaptive strategy is introduced to speed navigation feedback weight and speed coupling strength. The virtual leader can keep synchronization with the multi-agents. Zhou *et al.* [25] combined the idea of virtual force and pseudo-leader mechanism. A method to select informed agents in weighted networks is proposed. Haeri *et al.* [26] used the essential attribute of artificial potential field flocking algorithm. Based on the agent's initial location distribution, the attribute is used to determine the optimal number of agents to be informed, to reduce communication consumption. Atrianfar *et al.* [27] described three methods for selecting the number and initial location of informed agents. Select the least number of informed agents, to maximize the number of uninformed agents gathering towards the virtual leader. Lou *et al.* [28] designed a new control algorithm based on Olfati-Saber flocking algorithm. An adaptive controller and a virtual leader feedback controller are added to the algorithm. Even with only a few individuals in the flocking process, all agents form large networks, maintain connectivity and track virtual leaders. Wu *et al.* [29] introduced a cohesion item to force the multi-agent to approach the center of mass in the flocking algorithm. In terms of the initial distribution of community structures, Ganganath *et al.* [30] adopted Newman fast algorithm to select informed agents.The comparison shows that this method is superior to random and cluster-based selections of informed agents, and can better realize large-scale agents tracking virtual leaders.

From the review of the above works with some informed agents flocking, there are two main ways to maximize the number of uninformed agents that track virtual leaders. 1) The informed agent is selected according to its initial position [27], [30]; 2) Change the original flocking algorithm [24], [28]. As far as the author knows, when people choose informed agents according to the initial distribution of agents, they usually consider the initial distribution of agents with strong connectivity and high density, for example, references [27], [28], [30]. So, it is meaningful to study the flocking of partially informed agents with low connectivity and low density in the initial distribution of agents, which is the first motivation of this article.

It is worth noting that most of the above studies did not take into account individual differences, as well as broad characteristics between members of any group. Taking into account the characteristics of each individual, [31] divided multi-agent system into homogeneous and heterogeneous. Wei [32] researched the different equilibrium distance during the flocking process. To reduce communication losses, only some agents know the information. Considering the cooperative interaction and antagonistic interaction between agents, [33], [34], [35] studied the bipartite consistency problem of linear multi-agent systems. For different situations, different compensators are designed to estimate the state of leaders. However, in most articles, people pay more attention to the cooperative relationship between agents. In effect, agents can choose cooperation or non cooperation in the flocking process. As a society, individuals will have different ideas on one thing due to different life experiences and educational levels. For example, two groups with common interests will choose to cooperate, otherwise they may not cooperate. Therefore, it is of great significance to show the cooperative or non cooperative relationship in the agent flocking process, which is the second motivation of this article.

Except for the heterogeneity of agents, incomplete information in multi-agent systems is seldom concerned in previous articles. Even few articles focused on incomplete information, their research backgrounds are different from this paper. In addition, incomplete information also includes fault detection approaches [36], [37], [38] and various uncertainties in the real systems [39], [40], etc. In 2004, Li *et al.* [41] converted incomplete information into complete information to obtain the optimal solution of the model. Basu *et al.* [42] reconstructed the system equations, and allowed incomplete information to exist. Therefore, it is necessary to consider incomplete information in the process of multi-agent flocking, which is the third motivation of this article.

Inspired by the above work, this paper studies the flocking of partially informed agents under incomplete information. Firstly, an improved flocking algorithm is proposed to achieve better flocking. Then, under incomplete information, the flocking method of partially informed agent is proposed. The main contributions of this paper are as follows:

1) Based on the analysis of the derivative force function of potential function, this paper improves the classical flocking algorithm. On the one hand, a better and simple potential function can reduce the computational complexity to some extent. On the other hand, it will reduce the flocking time and produce better flocking effect. Finally, in the flocking process of some informed agents, there will be more uninformed agents tracking virtual leaders.
2) This paper considers the non cooperative relationship between agents, and the relationship between agents and the virtual leader. Thus, influenced by opinion dynamics [43], each agent is given different degrees of resistance in the flocking process. These resistances are generated randomly, and in order to be more realistic, the degree of resistance of each agent will change over time. At the same time, there will be passive information loss between the unmanned aerial vehicles and the unmanned aerial vehicles leader during the mission. We also discuss the passive loss of information between agents and virtual leaders during the flocking. For the sake of brevity, the above two aspects are called incomplete information.
3) This paper presents a method of flocking with only one informed agent. This informed agent is called a propagandist. The main function is to disseminate information, and make uninformed agents get information. This method solves the flocking problem of partially informed agents with incomplete information and low initial distribution density of agents.

The rest of the paper is organized as follows. Section II introduces the relevant knowledge. Section III presents a novel flocking algorithm with a fraction of informed agents, and especially, the propagandist is selected as an only informed agent. In Section IV, the stability of the improved algorithm is analyzed by using LaSalle's invariance principle and Lyapunov stability theorem. Section V includes some simulation results of flocking. Finally, some conclusions are presented in Section VI.

II. PRELIMINARIES

A. GRAPH THEORY

A system consists of N mobile agents. $G = (V, E, A)$ represents the network topology. Where $V = \{1, 2, \ldots, N\}$is a node (or vertex) set, and N is the total number. $E = \{e_{ij} = (i, j);\ i, j \in V\}$ represents all edges, which is also called edge set. if $(i, j) \in E$ is equivalent to $(j, i) \in E$, G is called an undirected graph. Otherwise, G is called an directed graph. In this paper, the network topology is considered as an undirected graph. The adjacency matrix $A = (a_{ij})_{N\times N}$ represents the interconnection between nodes. If $(i, j) \in E$, $a_{ij} = 1$, the two agents are connected. Otherwise $a_{ij} = 0$. The Laplacian matrix $L = D - A$, a $N \times N$ positive semi-definite matrix of network topology G, has eigenvectors of $I_n = (1, \ldots, 1)^T$, and the eigenvalues are 0.

B. FORMULATION

In a $2D$ Euclidean space with double integrator dynamics, consider a set of N agents moving, which is described as:

$$\begin{cases} \dot{q}_i = p_i \\ \dot{p}_i = u_i \end{cases}, \quad i = 1, 2, \ldots, N \tag{1}$$

where $q_i, p_i \in \mathbb{R}^2$ are the position vector, velocity vector, respectively, $u_i \in \mathbb{R}^2$ is the control protocol.

In a multi-agent system of this paper, without special emphasis, all agents are equal under other conditions, such as mass and size. In practical application, due to the limited technology, the sensing range of the agents is limited. Therefore, agents only interacts with other agents in the sensing range. Then, the neighboring of agent i can be set to:

$$N_i = \{j \in V : ||q_j - q_i|| < r\} \tag{2}$$

where N_i represents the set of all the neighbors of agent i within the sensing range. $||q_j - q_i||$ is the Euclidean norm. The r (usually $r > 0$) is a limited sensing distance.

Furthermore, according to the flocking algorithm proposed in [19], when the system is stable, there is a certain distance between agents, which should satisfy the following constraints:

$$||q_i - q_i|| = d_e, \quad i = 1, 2, \ldots, N \tag{3}$$

where d_e is balance distance, and usually $0 < d_e < r$.

III. FLOCKING CONTROL ALGORITHM

In 2006, Olfati-Saber proposed the classical flocking algorithm [19]. The algorithm has laid the foundation for flocking research. In this section, we have made some improvements to the algorithm. The selection method of informed agent is also introduced.

A. FLOCKING ALGORITHM

In [19], each agent has an equal control input and consists of three components:

$$u_i = f_i^g + f_i^d + f_i^\gamma \tag{4}$$

where f_i^d, f_i^g are the gradient-based term and velocity consensus term, respectively. f_i^γ is the navigational feedback term, which can be defined as:

$$f_i^\gamma = c_1(q_i - q_\gamma) + c_2(p_i - p_\gamma), c_1, c_2 > 0. \tag{5}$$

In addition, by analyzing the classical potential functions [19], [44], [45], an improved flocking algorithm is proposed. Compared with other flocking algorithms, we simplify the interaction rules, and ensure that the improved algorithm can achieve flocking. The main difference is to change the first term of the control protocol (4). The interaction rules between agents and their neighbors are: close-range repulsion and long-distance attraction. Going further, the control protocol u_i can be rewritten as:

$$u_i = \sum_{j\in N_i} f_\alpha(||q_j - q_i||)c_{ij} + \sum_{j\in N_i} a_{ij}(p_j - p_i) - c_1(q_i - q_\gamma) - c_2(p_i - p_\gamma) \quad (6)$$

where $c_{ij} = (q_j - q_i)\Big/\sqrt{1+\varepsilon||q_j - q_i||^2}$, $||q_j - q_i||$ is the Euclidean norm, $\varepsilon \in(0,1)$ is a weighted unit vector, and a_{ij} is the adjacency matrix.

$$a_{ij}(q) = \begin{cases} \rho_h\left(\dfrac{||q_j - q_i||_\sigma}{||r||_\sigma}\right), & if\ j \neq i \\ 0, & otherwise \end{cases} \quad (7)$$

where, the σ-*norm* is defined as $||z||_\sigma = \frac{1}{\varepsilon}[\sqrt{1+\varepsilon||z||^2} - 1]$, $\varepsilon > 0$. Then, the bump function $\rho_h(z)$, $h \in (0, 1)$, which is defined as:

$$\rho_h(z) = \begin{cases} 1, & z \in [0, h) \\ \dfrac{1}{2}\left[1 + \cos(\pi\dfrac{z-h}{1-h})\right], & z \in (h, 1] \\ 0, & otherwise \end{cases} \quad (8)$$

In addition, the force function only considers the repulsion and attraction between agents. Applying this force function to the control protocol can effectively reduce the calculation difficulties during the flocking process. The computational complexity of this paper and paper [19] is O(n) and O(n^2), respectively. Finally, the force function $f_\alpha(x)$ is defined as:

$$f_\alpha(x) = \begin{cases} k_1(1 - e^{-0.2(x-d_e)})e^{-0.2(x-d_e)} & x \le d_e \\ k_2\dfrac{(x-d_e)(x-r)}{d+r+5-x} & d_e < x \le r \\ 0 & x > r \end{cases} \quad (9)$$

where k_1 and k_2 are the coefficients of interaction between agents. x is the Euclidean distance between two agents. The potential function $F_\alpha(x)$ is defined as:

$$F_\alpha(x) = \int_{d_e}^{x} f_\alpha(s)ds \quad (10)$$

Note that the potential function is a nonnegative function.

Assume that the balance distance d_e is 5 and the sensing distance r is 6. This force function is depicted in Figure 1. The two agents will repel each other if they are within the balance distance (0-5). It shows attraction in the range of balance distance and sensing distance (5-6).

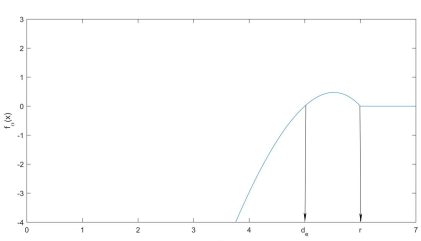


FIGURE 1. **The force function fα(x) with a finite cut-off.**

In [20], Su *et al.* modified flocking algorithm of [19]. In the process of flocking, assume that only a small number of agents know the objectives of the group. Some agents that know information about the virtual leader are called informed agents. This information includes the location and velocity of the group objective (or the virtual leader). Similarly, the flocking algorithm (6) proposed in this paper is also modified. Eventually the control protocol (6) is restructured as:

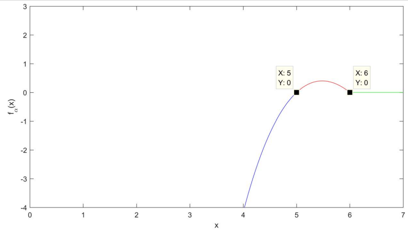
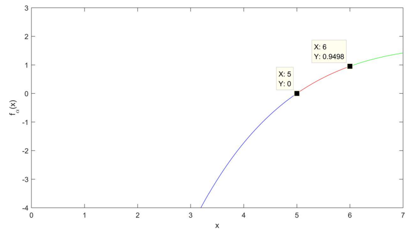
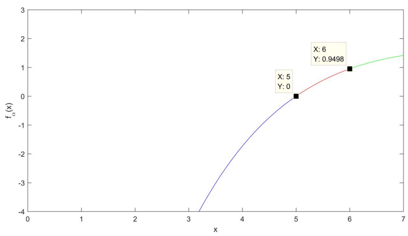
$$u_i = \sum_{j\in N_i} f_\alpha(||q_j - q_i||)c_{ij} + \sum_{j\in N_i} a_{ij}(p_j - p_i) - h_i[c_1(q_i - q_\gamma) + c_2(p_i - p_\gamma)]. \quad (11)$$

Here, if the agents are informed $h_i = 1$, otherwise $h_i = 0$.

B. THE FORCE FUNTION

In this section, the potential functions are divided into three major categories by analyzing the force functions. Through analysis and comparison, the effectiveness of the algorithm is illustrated in this paper. In the force function graph, repulsion is negative value (blue line) and attraction is positive value (red line). There is no force beyond the sensing distance (green line). The first and second categories are shown in Table 1(1) and Table 1(2), respectively. The value of the force function at the balance position is 0 ($d_e = 5$). At the position of maximum sensing distance ($r = 6$), the value of the force function is equal to or greater than 0, respectively. The third category is shown in Table 1(3). The value of the force function is less than 0 at the balance position and greater than 0 at the sensing distance position. Generally speaking, when the distance between two agents is equal to the balance distance, repulsion and attraction are equal. When the distance is greater than the sensing distance, the force will disappear. It is worth noting that in Table 1(3), when $d_e = 5$, the value is less than 0, and it is repulsion. But at the balance distance, the correct value is zero. Due to the above reasons, it has an adverse impact on the multi-agent cluster. To illustrate the validity of the function proposed in this paper, the definite integral of the force function is calculated piecewise. The repulsive potential can be obtained by $F_\alpha(x) = \int_0^{d_e} f_\alpha(x)dx$. On the condition that the repulsive of the two types of potential functions are equal, the force function in this paper has a maximum value, and Morse force function has a maxima value, and make the two value equal. As shown in Figure 2, this paper has improved the attraction section of Morse force function. In most cases, the force function image in this article is above the Morse force function, accounting for 74.3 percent of the total. Meanwhile, the result shows that the attraction between agents is greater between the balance point and the intersection point. This conclusion will be fully proved in the next simulation. Therefore, this paper chooses to construct the first type of potential function, which is helpful for flocking of partially informed agents.

TABLE 1. The force function and force function image.

Sequence number	The force function	force function image	Source	name
(1)	$f_\alpha(x)=\begin{cases}k_1(1-e^{-0.2(x-d_e)})e^{-0.2(x-d_e)} & x\le d_e\\ k_2\dfrac{(x-d_e)(x-r)}{d+r+5-x} & d_e<x\le r\\ 0 & x>r\end{cases}$		This paper	This paper
(2)	$f_\alpha(x)=2ak_1(1-e^{-a(x-d_e)})e^{-a(x-d_e)}$		Reference [44]	Morse
(3)	$f_\alpha(x)=24\varepsilon[\dfrac{d_e^{\ 6}}{x^7}-2\dfrac{d_e^{\ 12}}{x^{13}}]$		Reference [45]	L-J

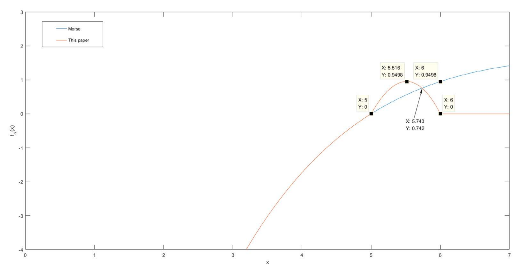


FIGURE 2. Force function graph comparison.

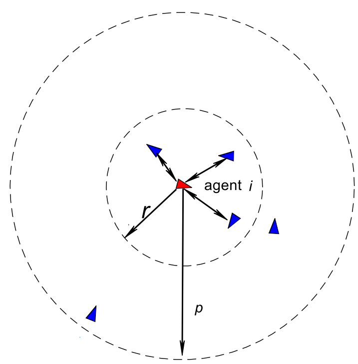


FIGURE 3. Interaction diagram between propagandist and neighbor.

C. A FLOCKING OF PARTIALLY INFORMED AGENTS WITH PROPAGANDISTS

Random creation of informed agents does not inform all other uninformed agents. The result is that some agents are not affected by the virtual leader in the network. In order to overcome this difficulty, many scholars have carried out relevant studies, such as community-based informed agents selection [30] and cluster-based informed agents selection [46]. In this paper, the proposed method is different from other studies. This paper mainly aims at the problems of low density and wide distribution of agents. So a flocking approach based on partially informed agents is proposed. Informed agents are called propagandists. The propagandist is the only informed agent in the whole flocking process.

The method indication picture is shown in Figure 4. A red cross mark virtual leader and a red dot represent a propagandist. The green square is the movement track of the propagandist, and the arrows represent the direction. The blue dotted line shows the propaganda radius. The black dotted line indicates the initial distribution of uninformed agents.

We summarize the method as follows:

1) The virtual leader stays still and delivers the information to the propagandist.

2) The propagandist moves along a certain route (green track) and transmits information about the virtual leader to uninformed agents. Eventually the propagandist returned to the starting point. During this process, the uninformed agent moves at a lower speed before being notified.

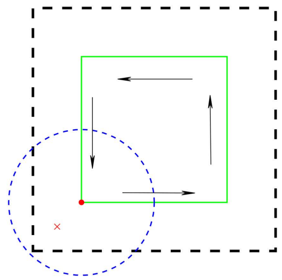


FIGURE 4. The movement of the propagandist.

3) When the propagandist comes back, the virtual leader starts to move. Then, agents track virtual leaders under the influence of the propagandist.

It is worth noting that the propagandist not only has the characteristics of ordinary agents (the balance distance d_e and the sensing distance r), but also has a unique propaganda radius p. In Figure 3 below, the propaganda radius will not disappear during the whole flocking process. The propaganda is still the only informed agent. To reduce the adverse effects caused by incomplete information, we give the propagandists autonomy. The control protocol is described as:

$$u_i = \sum_{j\in N_i} f_\alpha(||q_j - q_i||)c_{ij} + \sum_{j\in N_i} a_{ij}(p_j - p_i) - c_1(q_i - q_\gamma) - c_2(p_i - p_\gamma) - l\left[(q_i - q_\gamma) + (p_i - p_\gamma)\right] \quad (12)$$

IV. STABILITY ANALYSIS

In the previous section, we proposed an improved flocking algorithm, which can realize the flocking of partially informed agents. In this section, by giving the related theorems of the flocking, Lyapunov theorem and LaSalle's invariance principle are used to prove the stability of the algorithm.

Theorem1: Consider N mobile agents in two dimensions. Satisfy double-integrator dynamics equations (1) and control protocols (11). Assume that in the initial state, the velocity is random, and the positions of the agents are uniformly distributed. The following statements apply:

(a) Agents will not collide with each other during flocking.

(b) Eventually all agents match the speed.

(c) All agents gather together and form a group to follow the virtual leader.

Proof: Let $\hat{q}_i = q_i - q_\gamma$ and $\hat{p}_i = p_i - p_\gamma$ are position and velocity error vector, respectively. Then, according to (1), the double-integrator error dynamics equations of the agent i is defined as:

$$\begin{cases} \dot{\hat{q}}_i = \hat{p}_i \\ \dot{\hat{p}}_i = u_i \end{cases} \quad (13)$$

where $i = 1, 2, \ldots, N$. Let $q_{ij} = q_i - q_j$ and $\hat{q}_{ij} = \hat{q}_i - \hat{q}_j$. Clearly, $\hat{q}_{ij} = q_{ij}$. Hence, the control protocol (11) of agent i can be rewritten as:

$$u_i = \sum_{j\in N_i} f_\alpha(||\hat{q}_{ij}||)c_{ij} + \sum_{j\in N_i} a_{ij}(\hat{p}_j - \hat{p}_i) - h_i(c_1\hat{q}_i + c_2\hat{p}_i) \quad (14)$$

According to [19], [20], the collective potential function is modified as:

$$\begin{aligned} \hat{V}_i(\hat{q}_{ij}) &= \sum_{j\in v\setminus\{i\}} F_\alpha(||\hat{q}_{ij}||) \\ &= \sum_{j\notin N_i, j\neq i} F_\alpha(r) + \sum_{j\in N_i} F_\alpha(||\hat{q}_{ij}||) \end{aligned} \quad (15)$$

An energy-like Lyapunov function is selected as follows:

$$Q(\hat{q}, \hat{p}) = \frac{1}{2}\sum_{i=1}^{N}(U_i(\hat{q}) + \hat{p}_i^T\hat{p}_i) \quad (16)$$

where

$$\begin{aligned} U_i(\hat{q}) &= \sum_{j=1, j\neq i}^{N} F_\alpha(||\hat{q}_{ij}||) + h_i c_1 \hat{q}_i^T \hat{q}_i \\ &= \hat{V}_i(\hat{q}_{ij}) + h_i c_1 \hat{q}_i^T \hat{q}_i \end{aligned} \quad (17)$$

and

$$\hat{q} = \begin{bmatrix} \hat{q}_1 \\ \hat{q}_2 \\ \vdots \\ \hat{q}_N \end{bmatrix}, \hat{p} = \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \vdots \\ \hat{p}_N \end{bmatrix}.$$

Obviously, $Q(\hat{q}, \hat{p})$ is a positive semi-definite function of $(\hat{q}, \hat{p})$. Due to the symmetry of the potential function $F_a(x)$ and the adjacent matrix $A(t)$, one has

$$\frac{\partial F_\alpha(||\hat{q}_{ij}||)}{\partial \hat{q}_{ij}} = \frac{\partial F_\alpha(||\hat{q}_{ij}||)}{\partial \hat{q}_i} = -\frac{\partial F_\alpha(||\hat{q}_{ij}||)}{\partial \hat{q}_j} \quad (18)$$

Then,

$$\frac{1}{2}\sum_{i=1}^{N}\dot{U}_i = \sum_{i=1}^{N}(\hat{p}_i^T \nabla_{\hat{q}_i}\hat{V}_i(\hat{q}_{ij}) + h_i c_1 \hat{p}_i^T \hat{q}_i) \quad (19)$$

where $\dot{U}_i = dU_i/dt$. Therefore, the derivative of Q is given below:

$$\begin{aligned} \dot{Q} &= \frac{1}{2}\sum_{i=1}^{N}\dot{U}_i + \sum_{i=1}^{N}\hat{p}_i^T\dot{\hat{p}}_i \\ &= -\hat{p}^T[(L(t) + c_2 H(t)) \otimes I_n]\hat{p} \end{aligned} \quad (20)$$

where

$$H(t) = \begin{bmatrix} h_1 & 0 & \cdots & 0 \\ 0 & h_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_N \end{bmatrix}$$

Due to $L(t)$ and $H(t)$ are positive semi-definite matrices. Likewise, $L(t) += c2H(t)$ is positive semi-definite matrices. So, $\dot{Q} \leq 0$ implies this is a non-increasing function over time t. Thus $Q(t) \leq Q_0$ for all $t \geq 0$. From Equations (16) and (17), we obtain $c_1\hat{q}_i^T\hat{q}_i \leq 2Q_0$ for agent i, which guarantees the flocking. The part (c) is proven.

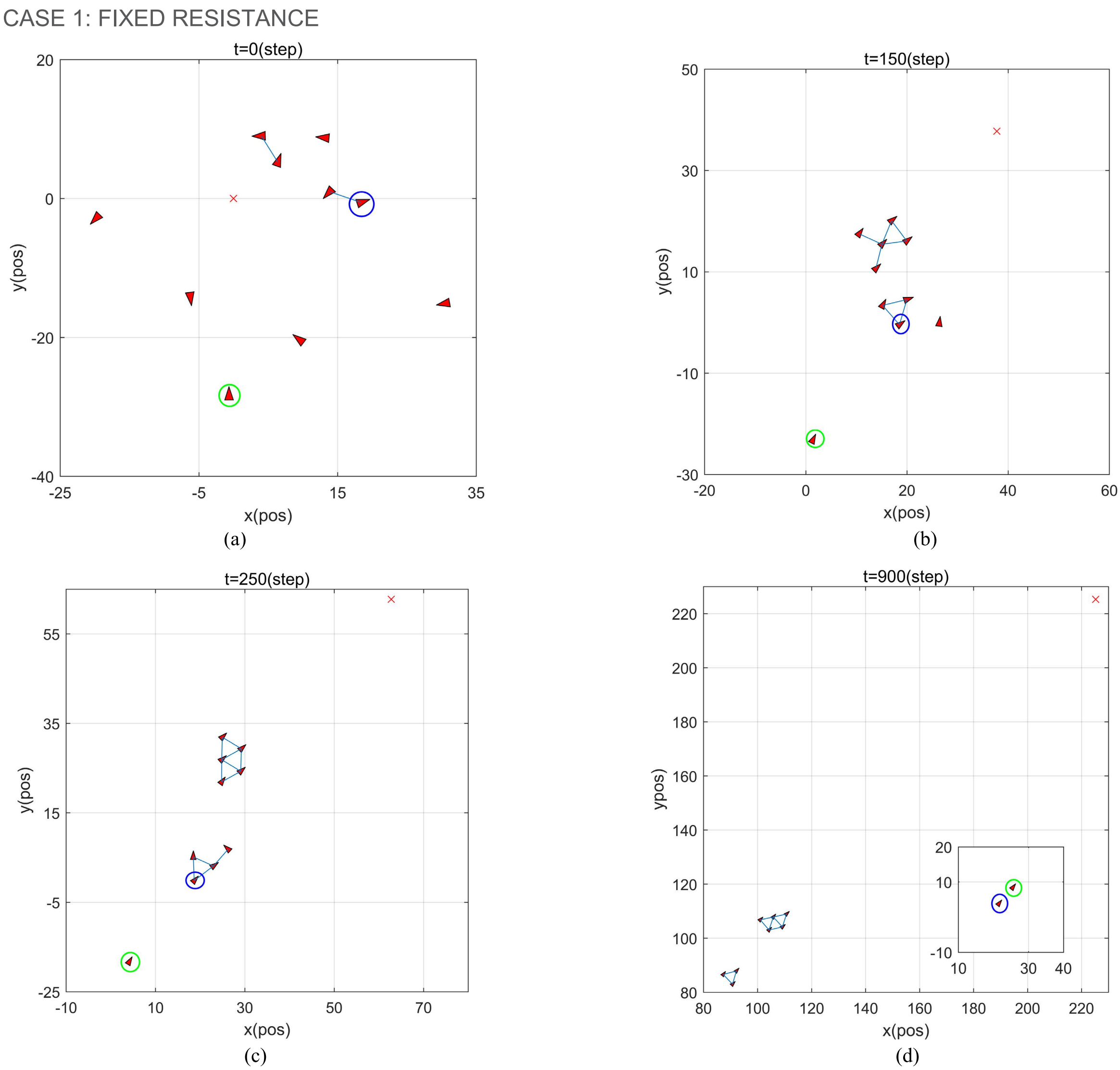


FIGURE 5. Each agent has a fixed degree of resistance during the flocking. (a) t=0 step. (b) t=150 step. (c) t=250 step. (d) t=900 step.

Take into account $Q(t) \leq Q_0$, we define an invariant set (21)

$$\Omega = \{(\hat{q}^T, \hat{p}^T)^T : Q(t) \leq Q_0\} \tag{21}$$

According to LaSalle's invariance principle, the state of all agents will converge from Ω to the largest set $\Omega_{\max} = \{(\hat{q}^T, \hat{p}^T)^T : \dot{Q}(t) = 0\}$. From the equation (16), we obtain:

$$\begin{aligned} \dot{Q} &= \frac{1}{2}\sum_{i=1}^{N} \dot{U}_i + \sum_{i=1}^{N} \hat{p}_i^T \dot{\hat{p}}_i \\ &= -\hat{p}^T[(L(t) + c_2 H(t)) \otimes I_n]\hat{p} \\ &= -\hat{p}^T (L(t) \otimes I_n)\hat{p} - \hat{p}^T (c_2 H(t) \otimes I_n)\hat{p} \end{aligned} \tag{22}$$

The matrices $L(t)$ and $H(t)$ are positive semi-definite matrices. Thus, the non-negative energy function is the smallest. $\dot{Q} = 0$ is equivalent to $\hat{p} = [\hat{p}_1, \hat{p}_2, \cdots, \hat{p}_N]^T \equiv [0, 0, \cdots, 0]^T$. The condition is satisfied only when $P_1 \equiv P_2 \ldots \equiv P_N$. The part (b) is proven.

In view of the (16), Q is a non-increasing function over time t. Thus $Q(t) \leq Q_0$ for all $t \geq 0$. Assume that at least two agents collide during flocking. Then,

$$\begin{aligned} Q(\hat{q}, \hat{p}) &= \frac{1}{2}\sum_{i=1}^{N} (U_i(\hat{q}) + \hat{p}_i^T \hat{p}_i) \\ &= \frac{1}{2}\sum_{i=1}^{N} (\hat{V}_i(\hat{q}_{ij}) + h_i c_1 \hat{p}_i^T \hat{p}_i) + \frac{1}{2}\sum_{i=1}^{N} \hat{p}_i^T \hat{p}_i \\ &\geq \frac{1}{2}\sum_{i=1}^{N} \hat{V}_i(\hat{q}_{ij}) \geq Q_0 \end{aligned} \tag{23}$$

Which contradicts the condition $Q(t) \leq Q_0$. Therefore, the above assumption is wrong. The part (a) is proven.

V. SIMULATION RESULT

In this section, lots of simulation results are presented. The effectiveness of the algorithm proposed in the third section is

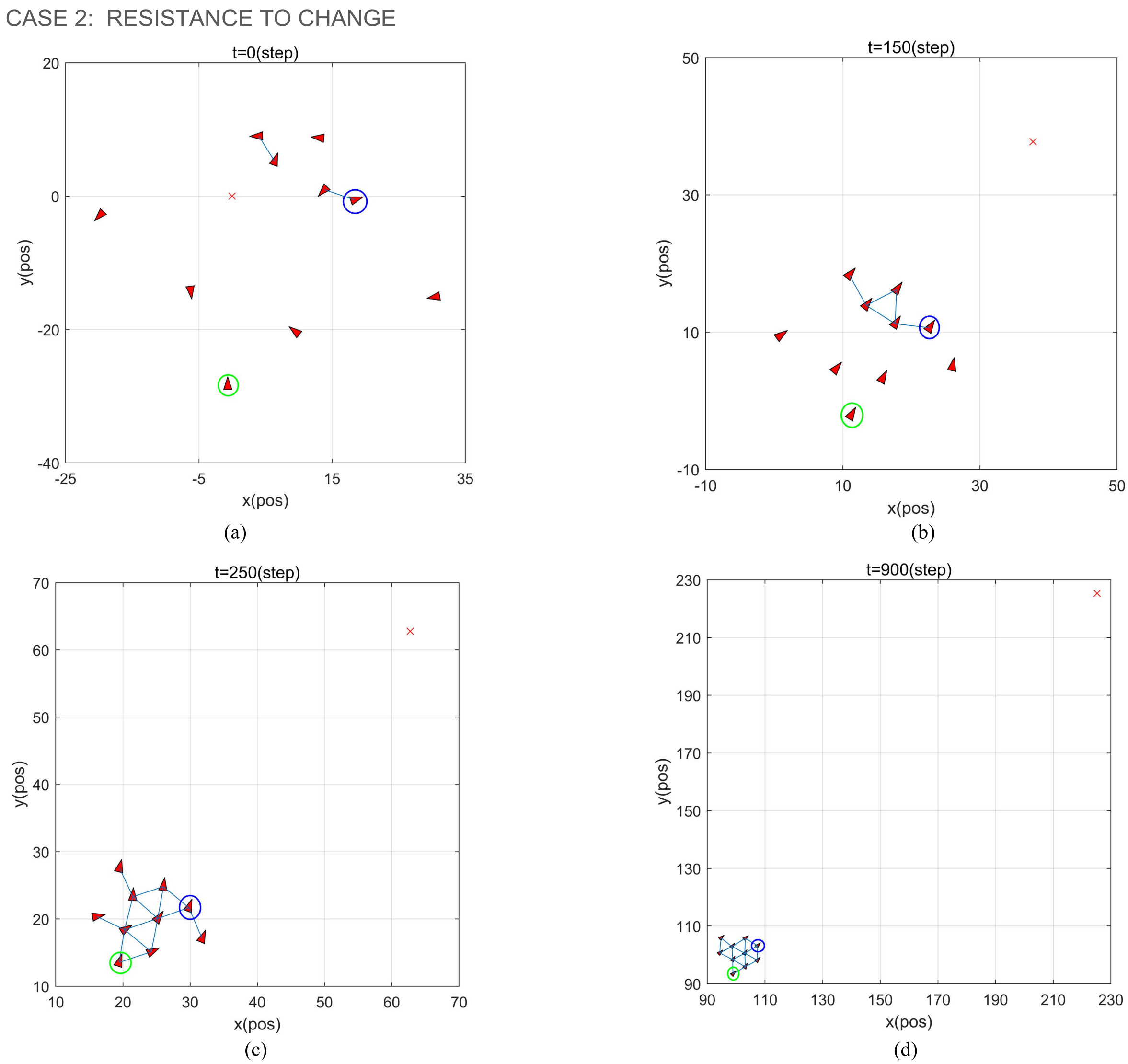


FIGURE 6. **Each agent has a varying degree of resistance during the flocking. (a) t=0 step. (b) t=150 step. (c) t=250 step. (d) t=900 step.**

proved. At the same time, the flocking method proposed in this paper is verified.

A. FLOCKING WITH INCOMPLETE INFORMATION

Before the formal simulation, the flocking simulation under incomplete information is briefly described. This will lay the foundation for the following simulations. In this section, two cases are discussed, in both cases, the virtual leader information is randomly lost. First, each agent has a fixed resistance (randomly generated) that does not change over time. Secondly, each agent has different resistance and changes over time.

To better reflect the impact of incomplete information on flocking, large-scale agents are not selected, and only 10 agents are selected in two-dimensional space for simulation. At the initial moment, the positions of the agents are randomly distributed in the box $[-40, 40]\times[-40, 40]$, and satisfy the Gaussian distribution, as shown in Figure 5(a). The initial velocities are randomly chosen from the box $[-1, 1]\times[-1, 1]$ (corresponding to the velocity in the x-axis and y-axis direction). The virtual leader is marked with a red "$\times$". The initial position q_γ and velocity p_γ are set as $q_\gamma(0) = [0, 0]^T$ and $p_\gamma(0) = [0.5, 0.5]^T$, respectively. Other parameters are given in Table 2.

The initial conditions remain the same in both cases as shown in Figure 5(a) and Figure 6(a). When agents track the virtual leader, they do not collide with each other and keep a certain distance. Eventually, in both cases, because each agent has varying degrees of resistance and the lack of virtual leader information, agents failed to track to the virtual leader. Note the two agents marked with blue and green circles in Figure 5 and Figure 6. Compared with other agents, the two agents have relatively large resistance in Figure 5. Hence, the two agents barely moved, and the distance between the two agents and the virtual leader is the largest. Due to the small degree of resistance of green agents, so the green agents

TABLE 2. Parameters of the proposed algorithm.

Parameters	Notation and value
The balance distance	$d_e = 5$
The sensing distance	$r = 6$
The bump function $\rho_h(.)$	$h = 0.7$
σ - norm	$\varepsilon = 0.1$
Coefficients of control protocol (11)	c_1=0.01, c_2=0.05
Repulsion coefficient (9)	$k_1 = 6.4$
Attraction coefficient (9)	$k_2 = -39.8$
Simulation steps	$t = 0 - 900$
Compensation coefficient	l =0.7

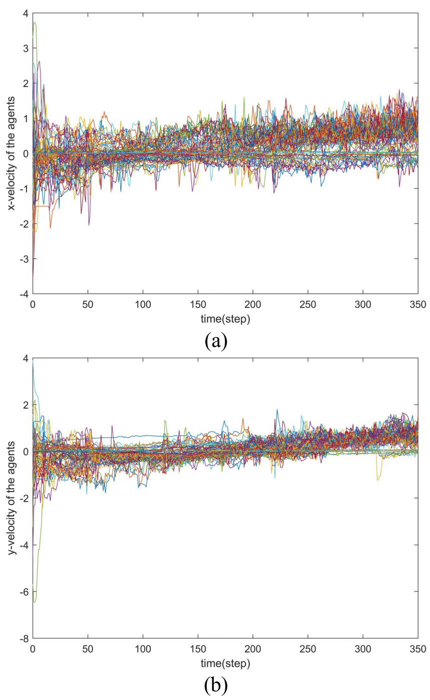


FIGURE 7. Velocity curve of all agents. (a) x-axis direction. (b) y-axis direction.

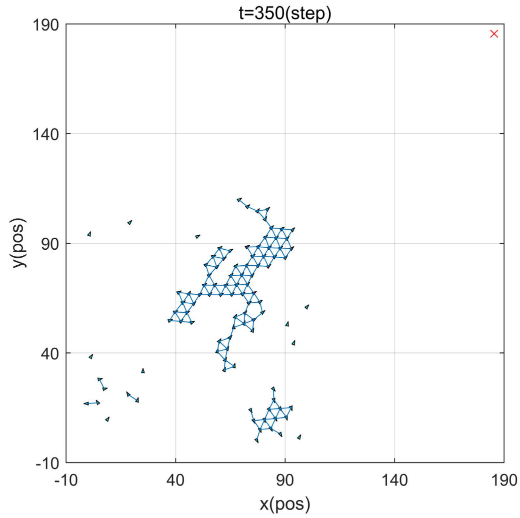


FIGURE 8. Final flocking picture using Morse potential functions.

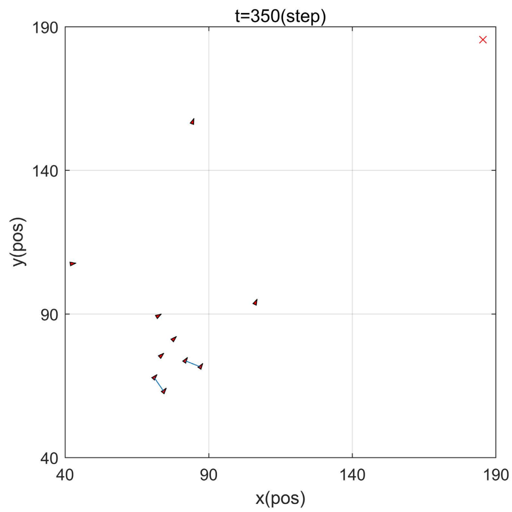


FIGURE 9. Final flocking picture using L-J potential functions.

eventually exceeded the blue agents. In the end, the distance between the remaining agents and the virtual leader is directly proportional to the degree of resistance, shown in Figure 5(d). In Figure 6, two agents (marked with blue and green circles) have different degrees of resistance with other agents. As time goes on, all agents gather together and form a small group. Taking into account the real life, individuals will constantly adjust and change themselves as the environment changes, because information received by individuals from the outside world is not always complete. On this account, the following simulation is based on case 2.

B. RANDOM SELECTION OF INFORMED AGENTS AND FLOCKING WITH INCOMPLETE INFORMATION

Figure 10 shows a flocking simulation with incomplete information. Based on the control protocol (11), 100 agents moving in two-dimensional space are simulated, including 10 informed agents and 90 uninformed agents. The initial position and velocity of both informed and uninformed agents are distributed in the boxes [0, 100]×[0, 100] and [0, 0.01]×[0, 0.01], respectively, as shown in Figure 10(a). The initial position q_γ and velocity p_γ of the virtual leader are set as $q_\gamma(0) = [10,\ 10]^T$ and $p_\gamma(0) = [1,\ 1]^T$. Some other parameters of the algorithm are given in Table 2. In this section, the incomplete information is added in the simulation. Related simulations with incomplete information are described in section A.

In Figure 10(a), red triangles represent informed agents, and blue triangles represent uninformed agents. Solid lines indicate interconnections. The red "×" indicates the virtual leader. The virtual leader moves diagonally in the coordinate system. Figure 10(a) shows the initial state of the high disconnect. With the passage of time, the scale of the cluster continues to increase. From Figure 10(b) to Figure 10(f), more and more agents gather together and form a group with

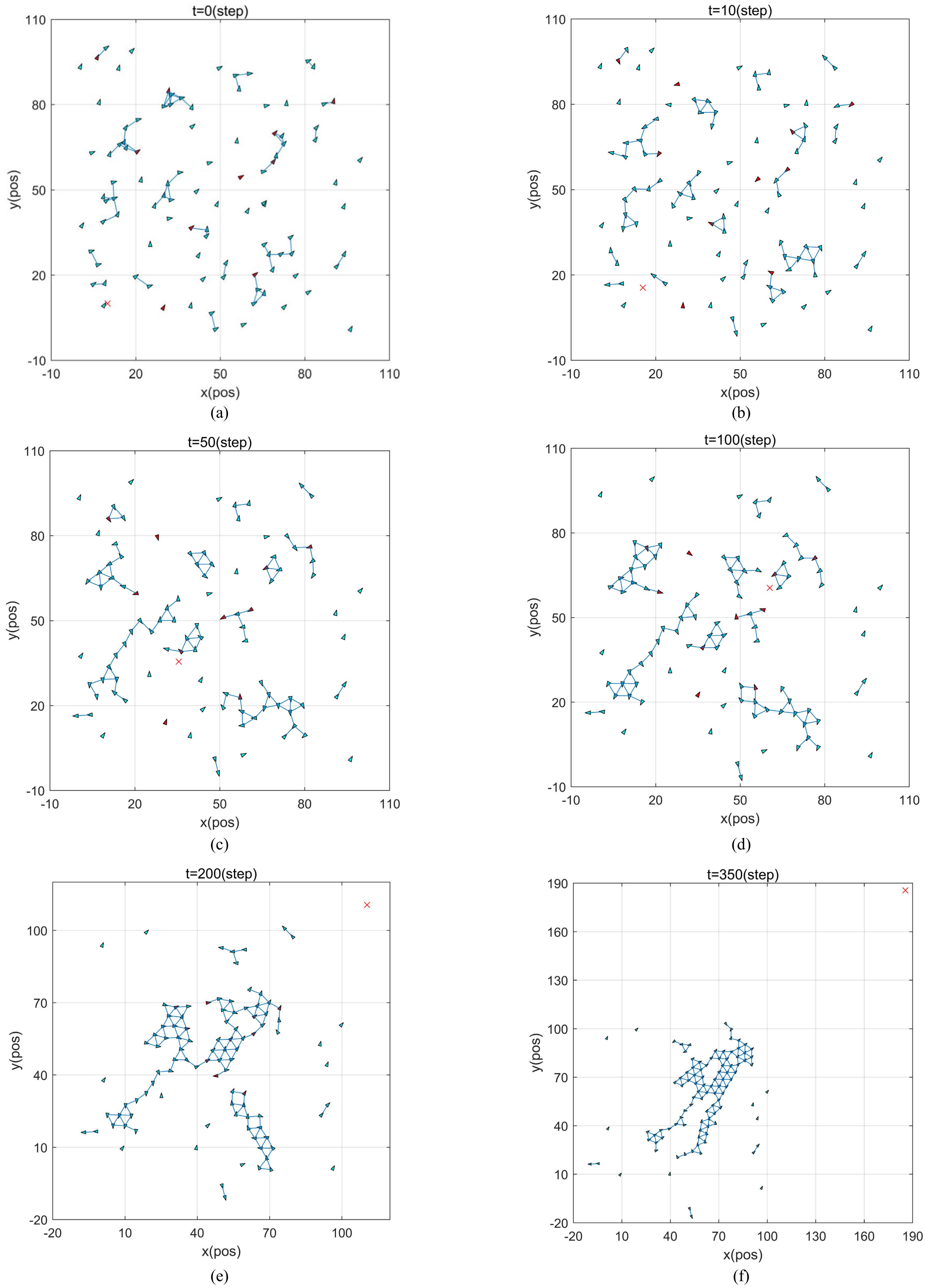


FIGURE 10. Flocking under incomplete information, including 10 informed agents and 90 uninformed agents. The number of informed agents are selected randomly. The red triangle represents the informed agent, and the blue triangle represents the uninformed agent. A red "×" denotes a virtual leader. Solid lines indicate the interconnection between agents. (a) t=0 step. (b) t=10 step. (c) t=50 step. (d) t=100 step. (e) t=200 step. (f) t=350 step.

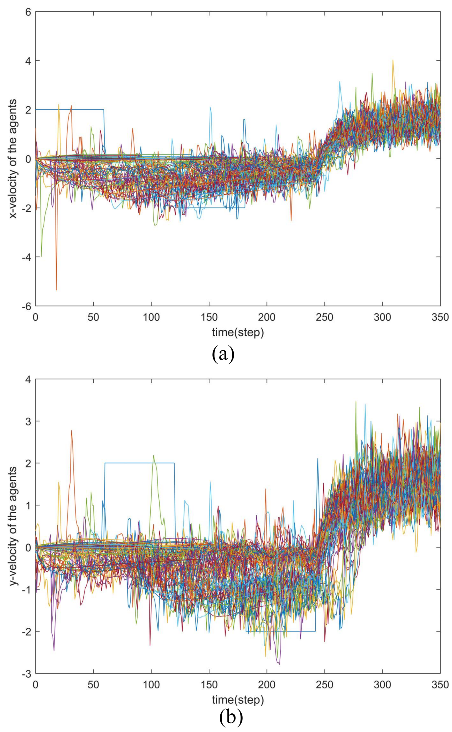


FIGURE 11. Velocity curve of all agents. (a) x-axis direction. (b) y-axis direction.

the same velocity. Finally, there are 81 agents in the group (see Figure 10(f)). Figure 7 shows the velocity curve of the agents during the flocking process. It is clear that the uninformed agent, which is not affected by the informed agent, keeps moving at its original velocity. The velocity information of the virtual leader is dynamically missing, leading to the constant adjustment of the speed of the agents, but stable within a certain range.

In order to better compare the effectiveness of the improved potential function in this paper, the initial state remains the same, as shown in Figure 10(a). The flocking process is much the same as above. The results under different potential functions are shown in Figure 8 and Figure 9. It is clear that, using Morse and L-J potential functions, the final number of clusters reached is 70 and 10, respectively. L-J potential functions produces unfavorable results, which is not conducive to flocking.

C. MULTI-AGENTS FLOCKING WITH PROPAGANDIETS AND INCOMPLETE INFORMATION

In section *B*, flocking simulation is carried out based on three different potential functions under incomplete information. This proves the superiority of the proposed potential function. To further improve the adverse consequences caused by incomplete information, in this section, multi-agents flocking simulation based on propagandist is implemented, as shown in Figure 14. The parameter settings involved in this section are shown in Table 2. The difference is that 100 agents are generated, including 99 uninformed agents and 1 propagandist (the informed agent). The initial distribution of

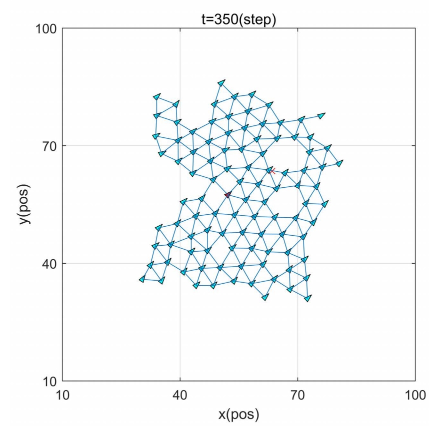


FIGURE 12. Final flocking picture using Morse potential functions, propagandist is the only informed agent.

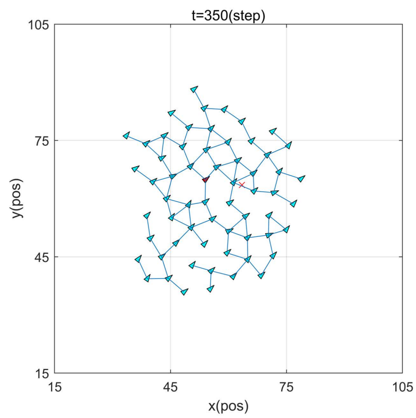


FIGURE 13. Final flocking picture using L-J potential functions, Propagandist is the only informed agent.

uninformed agents satisfies the uniform distribution. The initial position and velocity of the propagandist (the red agent) are [20, 20] and [2, 0], respectively. The position of the virtual leader (the red cross) is [10, 10], as shown in Figure 14(a).

The propagandist radius of propagandist is 30. To begin with, the propagandist receives orders from the virtual leader. Then inform the uninformed agents. The movement track of the propagandist is square, as described in section III. Here there is no need to overstate it. The simulation pictures are from Figure 14(a) to Figure 14(e). During this process, the virtual leader stays still. In Figure 14(e), all agents have already received the propagandist's message. Most agents gather around the virtual leader. At this point, the propagandist has returned to the initial position [20, 20]. Most agents are already connected and form a large network. The virtual leader starts moving at [1, 1] speed. After that, it is still the propagandist who knows the information about the virtual leader (see Figure 14(f)). In the end, it is shocking that the number of agents in this small group reached 100. Similarly, under these conditions, Figure 11 indicates the velocity curve during the flocking process. As the control protocol of propagandist is improved, this group can successfully track the virtual leader. The method proposed in this

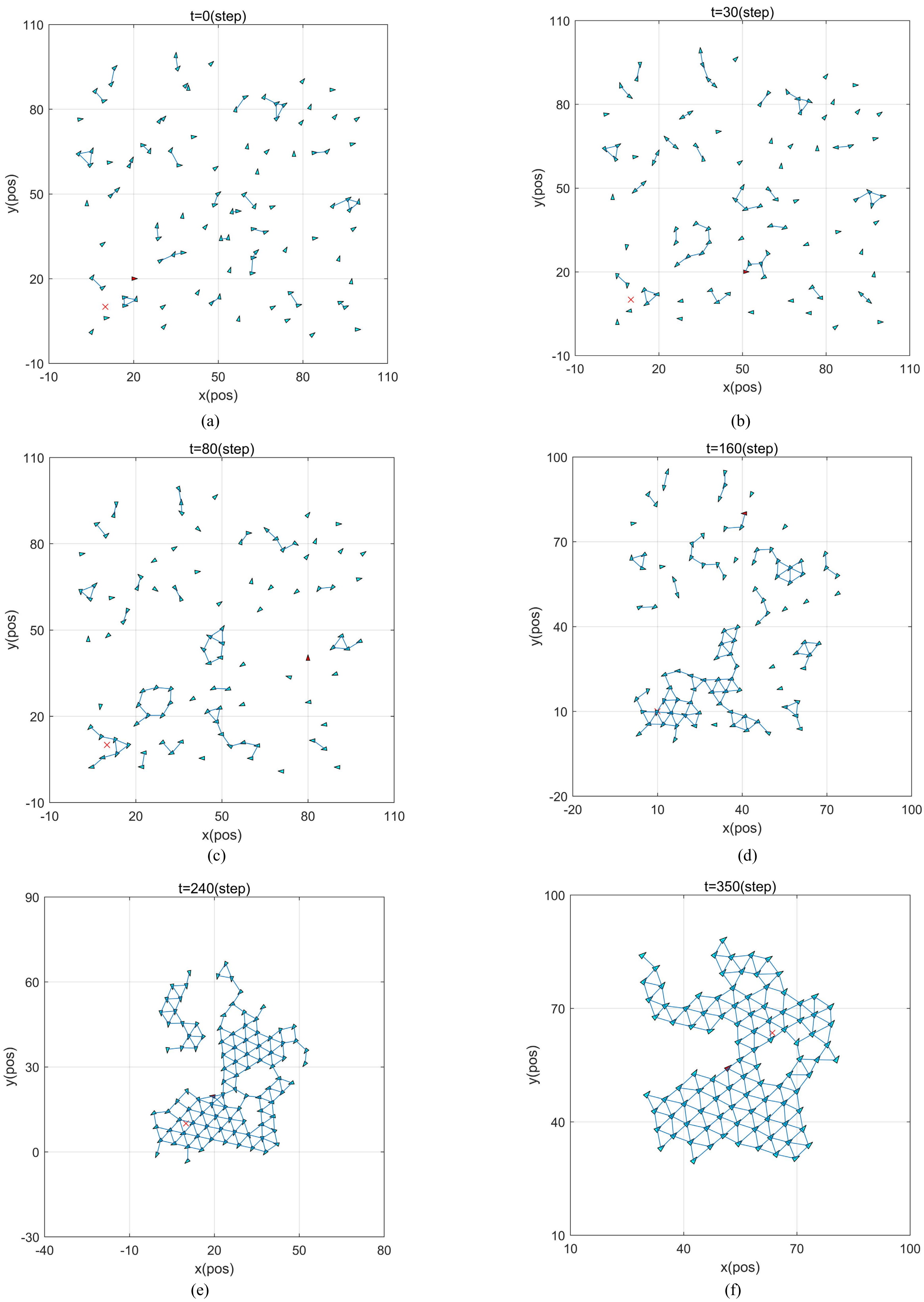


FIGURE 14. Flocking under incomplete information, including 1propagandist (informed agent) and 99 uninformed agents. The red triangle represents the informed agent, and the blue triangle represents the uninformed agent. A red "×" denotes a virtual leader. Solid lines indicate the interconnection between agents. (a) t=0 step. (b) t=30 step. (c) t=80 step. (d) t=160 step. (e) t=240 step. (f) t=350 step.

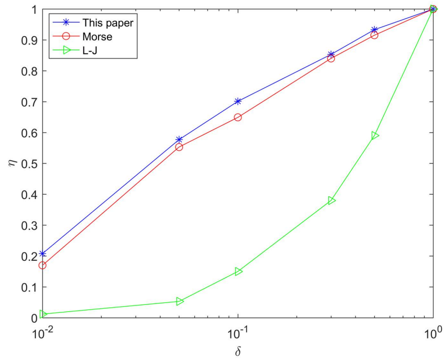


FIGURE 15. Fraction of agents with desired velocity as a function of the fraction of informed agents, under different potential functions with incomplete information. Informed agents are randomly selected. The estimated results are taken as the average of 50 simulation.

paper is that only one informed agent can realize interconnection of the whole network. The simulation results also benefit from the influence of incomplete information. However, it is worth noting that before the virtual leader movement, the propagandist needs to take time to inform the uninformed agents.

Similarly, the potential function in this paper is replaced by Morse and L-J potential functions, respectively. The initial states remain the same, as shown in Figure 14(a). Run the simulation again. The results are shown in Figure 12 and Figure 13 below. The number of eventually formed groups is 100 and 62. It can be seen from Figure 12 and Figure 13, even with Morse and L-J potential functions, the results are good. So this method plays a decisive role in solving the problem of incomplete information. Compared with other methods [25], [28], it shows the advantages of this method.

In order to further prove the feasibility of the improved flocking algorithm and the effectiveness of the flocking method proposed in this paper, more simulations were performed. Based on the assumptions of the above simulation, the initial positions are randomly selected form the box [0, 100] × [0, 100]. The initial velocities are chosen form [0.01,0.01]. The q_γ and p_γ of the virtual leader are set as $q_\gamma(0) = [10, 10]^T$ and $p_\gamma(0) = [1, 1]^T$, respectively. Other parameter settings are the same as in Table 2.

We randomly select some informed agents from 100 agents. Figure15 presents the relationship between the proportion η of agents that track the virtual leader and the proportion δ of the informed agents. The simulation results are taken as the average value of 50 times. This result is a cluster of partially informed agents under incomplete information. The three lines represent different potential functions, which come from Table 1. Obviously, the potential function in this paper is better than other potential functions. At the same time, the validity of the potential function proposed in this paper is illustrated again.

Since the propagandist is the only informed agent, there is no need to set the proportion of informed agents. In the same way, based on different potential functions under incomplete information, 50 simulations were performed to calculate the average value. The resulting values are 99.8, 99.5, and 44.9, corresponding to the potential function in this paper, Morse, L-J, respectively. Compared with random selection of informed agent [20], the results show that the method presented in this paper is better.

VI. CONCLUSION

In this paper, firstly based on the consideration of reality, flocking of informed agents is simulated under incomplete information. In order to drive more agents to track virtual leaders at the same speed, based on the analysis of the potential function, we improve the original flocking algorithm and prove the stability of the algorithm. The conclusion of simulation is that the potential function in this paper is beneficial to the flocking of multi-agent. Secondly, this paper regards a informed agent as a propagandist to spread information to other agents. This method is superior to the method of randomly selecting agents, and has a better flocking effect under incomplete information. This method reduces the adverse effects caused by incomplete information. Moreover, simulation results also verify the effectiveness of the above approaches.

The method proposed in this paper has a certain lag. And before the propagandist notifies the uninformed agents, the uninformed agents move at a low speed. For future work, we will consider how to generate flocking more quickly and reduce the lag time. Meanwhile, it is better to reduce the adverse consequences caused by incomplete information.

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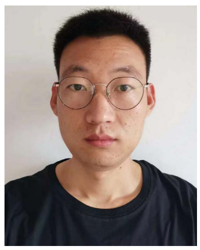

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