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RESEARCH ARTICLE

Synthesis of a Transfer Function With Real Poles From Tabulated Frequency Response Data for Transmission-Line Impedance Modeling

TAKU NODA, (Senior Member, IEEE)
Grid Innovation Research Laboratory, Central Research Institute of Electric Power Industry (CRIEPI), Yokosuka, Kanagawa 240-0196, Japan
e-mail: takunoda@ieee.org

ABSTRACT Synthesis of a transfer function from tabulated frequency response data is an important topic in engineering. Especially in transmission line (TL) modeling, the impedance of a TL must be synthesized in the form of a transfer function of complex frequency, for instance, to be used in time-domain simulations. The TL impedance whose frequency response is in most cases calculated by analytical formulas and prepared in the form of tabulated data shows variation with respect to frequency due to skin effects in conductors and the ground, and synthesis of the frequency variation theoretically requires real poles only. This paper proposes an algorithm to synthesize a transfer function only with real poles from tabulated frequency response data for TL impedance modeling. Since the skin effects are a phenomenon which is continuous with respect to frequency and theoretically does not have poles at specific positions, the problem is essentially to approximate such a frequency response by a transfer function with a finite number of poles. Considering this point, a pole allocation method is investigated using practical TL impedance-matrix data. Once the poles have been allocated, their residue matrices are identified by a least-squares method using the singular value decomposition algorithm with column scaling. For validation, the method is applied to the synthesis of impedance matrices of an overhead power TL and a submarine-cable power TL.

INDEX TERMS Frequency response, impedance, poles and zeros, skin effect, system identification, transfer functions, transmission lines.

I. INTRODUCTION

Synthesis of a transfer function from tabulated frequency response data is an important topic in engineering, and it has a relatively long history of research. First, the history is briefly reviewed here.

A fundamental study for obtaining a transfer function from tabulated frequency response data was presented by Levy [1]. In the study, a rational function of s is used as the transfer function and identified by a least-squares method in which the denominator is multiplied to both sides in the formulation. Note that the complex frequency is denoted by s throughout the paper. Later, Sanathanan and Koerner proposed to apply weighting values in the least-squares formulation to cancel out the effects of the denominator [2]. This stream of studies are well described by Whitefield [3]. Later, it was recognized that the s^n terms in the least-squares formulation take a wide range of values whose ratios exceed machine arithmetic accuracy [4], especially when a wide range of frequency is considered. This fact leads to frequency-range partitioning methods [5], [6], [7], [8] to avoid ill-conditioning in the least-squares formulation. Another successful method called Vector Fitting uses a partial-fraction form of s or a pole-residue form of s as the transfer function [9], [10]. The identification is carried out by relocating poles with introducing a scaling factor in an iterative procedure.

In some engineering applications, it may be a priori known that the target system to be modeled has no complex poles. In transmission-line (TL) modeling, ranging from a TL interconnecting electronic circuits to a power TL, the impedance

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of a TL shows variation with respect to frequency due to skin effects in conductors and the ground, and it is theoretically known that the impedance has no complex poles. If this is the case, we may want to synthesize a transfer function with real poles only, for instance, to be used in time-domain simulations. All those methods mentioned in the previous paragraph, however, cannot guarantee to give real poles only and may give complex poles due to numerical errors. Notable early work to identify an *RL* circuit thus a transfer function only with real poles includes the following. Yen *et al.* proposed a method to synthesize an *RL* circuit from the conductor cross-section data of a TL [11]. Semlyen *et al.* proposed a Gauss-Seidel-type iteration scheme to identify a multiphase *RL* circuit from given frequency response data for three-phase overhead TL modeling [12]. The former is not applicable to a multiphase line, and the latter is restricted to the three-phase case. Note that the frequency response of a TL is in most cases calculated by analytical formulas [13] and prepared in the form of tabulated data.

This paper proposes an algorithm to synthesize a transfer function only with real poles from tabulated frequency response data for TL impedance modeling. The method is applicable to the general multiphase case. Since the skin effects are a phenomenon which is continuous with respect to frequency and theoretically does not have poles at specific positions, the problem is essentially to approximate such a frequency response by a transfer function with a finite number of poles. In other words, the continuous response is reproduced by the sum of a finite number of discrete frequency responses. Considering this point, a pole allocation method is investigated. Different pole allocations are generated by a geometric sequence and tested using practical TL impedance-matrix data from [14]. As a result, it is found that equidistant poles allocated on the logarithmic axis give reasonably accurate results. Once the poles have been allocated, their residue matrices are identified as a least-squares problem. It is solved by the singular value decomposition (SVD) algorithm with a column-scaling technique. A primitive version of the proposed algorithm is partially shown in [14], and this paper presents the improved and complete version with sufficient investigations. For validation, the method is applied to the synthesis of impedance matrices of an overhead power TL and a submarine-cable power TL, the same data mentioned above, and accurate results are obtained.

II. PREPARATION

A. PROBLEM DESCRIPTION

In this paper, we denote the complex frequency by $s = \sigma + j\omega$. Consider a linear system whose frequency response is $H(s)$. We assume that the system has real poles only. The input-output relationship is written as

$$y(s) = H(s)x(s), \tag{1}$$

where $x(s)$ and $y(s)$ are respectively the input to and the ouput from the linear system. In electrical engineering applications, $H(s)$ is usually an impedance or an admittance. In a single-phase case, $x(s)$, $y(s)$ and $H(s)$ are all scalar. In a multiphase case, $x(s)$ and $y(s)$ are column vectors of order N, and $H(s)$ is an N-by-N matrix, where N is the number of phases. $H(s)$ is often obtained by numerical computation using analytical formulas, and we assume that the result is given in the form of numerical data at K discrete frequency points:

$$H_1, H_2, \cdots, H_K \tag{2}$$

They are frequency response data of the system at angular frequency points ω_k for $k = 1, 2, \cdots, K$. Table 1 shows an example of tabulated frequency response data in the case of $N = 2$. Our problem is to synthesize a transfer function of the partial-fraction form

$$H(s) = \sum_{m=1}^{M} \frac{1}{s - p_m} R_m \tag{3}$$

whose response closely approximates the given frequency response data (2) at the discrete angular frequency points. In the equation above, p_m and R_m are real poles and their residues for $k = 1, 2, \cdots, K$. In a multiphase case, the dimension of R_m is the same as that of $H(s)$.

B. TL IMPEDANCE EQUIVALENT CIRCUIT AND ITS EQUATION

A TL basically consists of more than or equal to two conductors and insulating space and/or material(s) around them. The currents propagating along the conductors show skin effects depending on the frequency. The equivalent circuit of the series impedance of a TL, when one of the conductors is focused on with its surrounding space, can be expressed by Fig. 1 (a) according to [15], and this Cauer ladder network can be converted to the Foster form shown in Fig.1 (b) [16]. To describe the Foster-form equivalent circuit by immittance, the admittance form

$$Y(s) = \sum_{m=1}^{M} \frac{1}{r_m + s l_m} = \sum_{m=1}^{M} \frac{1/l_m}{s + r_m/l_m} \tag{4}$$

is suitable, where r_m and l_m are the resistance and the inductance of the mth layer of the Foster-form equivalent circuit. Since (3) and (4) are in the same form, we can conclude the following. To model the series impedance matrix Z of a TL,

TABLE 1. Example of tabulated frequency response data.

f	$\|h_{11}\|$	$\angle h_{11}$	$\|h_{12}\|$	$\angle h_{12}$	$\|h_{21}\|$	$\angle h_{21}$	$\|h_{22}\|$	$\angle h_{22}$
1.0E-1	3.19E+3	-0.2	8.96	-96.8	8.96	-96.8	3.19E+3	-0.2
1.0	3.18E+3	-1.4	7.45E+1	-100.4	7.45E+1	-100.4	3.18E+3	-1.4
1.0E+1	2.97E+3	-9.4	5.36E+2	-119.6	5.36E+2	-119.6	2.97E+3	-9.4
1.0E+2	1.90E+3	-16.1	1.39E+3	-166.8	1.39E+3	-166.8	1.90E+3	-16.1
1.0E+3	1.31E+3	-31.9	1.20E+3	152.6	1.20E+3	152.6	1.31E+3	-31.9
1.0E+4	4.05E+2	-43.6	3.82E+2	137.8	3.82E+2	137.8	4.05E+2	-43.6
1.0E+5	1.29E+2	-44.9	1.24E+2	135.7	1.24E+2	135.7	1.29E+2	-44.9
1.0E+6	4.07E+1	-45.2	3.94E+1	135.0	3.94E+1	135.0	4.07E+1	-45.2
1.0E+7	1.28E+1	-45.1	1.25E+1	135.0	1.25E+1	135.0	1.28E+1	-45.1

f: frequency [Hz], $|h_{ij}|$, $\angle h_{ij}$: magnitude and phase angle [deg] of the ij element of H_k

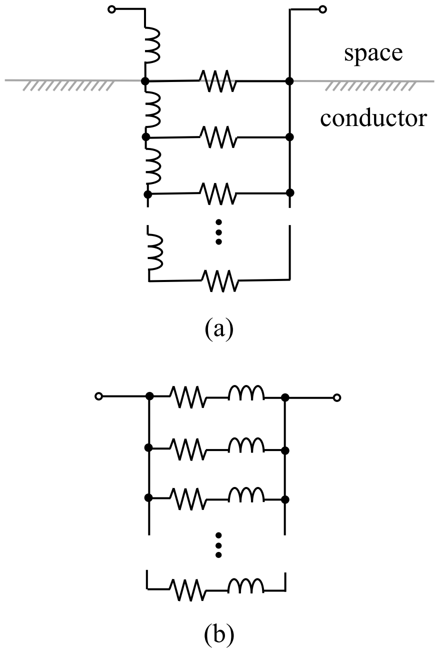

FIGURE 1. **Equivalent circuit of the series impedance of a TL, when one of the conductors is focused on with its surrounding space, can be expressed by the Cauer ladder network shown in (a), and it can be converted to the Foster form shown in (b).**

Z can be calculated at discrete frequency points for instance by skin-effect formulas, and they are inverted to obtain their admittance matrices in the form of frequency response data as shown in (2). Actually, Table 1 shows the inverse of the impedance matrix of two OC 60 mm^2 wires (hard drawn copper wires) horizontally placed at 10 m in height. The distance between the two wires is 50 cm, and the ground resistivity is 100 Ωm. The admittance matrix data obtained are the input for the synthesis. Then, a transfer function is synthesized in the partial-fraction form (3). Note that the resultant transfer function matrix reproduces the dynamics of the admittance matrix that is the inverse of the series impedance matrix of the given TL.

III. SYNTHESIS ALGORITHM

In the proposed algorithm, a transfer function of the form (3) is synthesized from the frequency response data, which are conceptually or mathematically given in the form (2) and actually given in a tabulated form such as the one shown in Table 1 in a certain computer file format. The algorithm first determines the distribution of poles, and then, their residue matrices are identified to obtain the transfer function.

A. POLE ALLOCATION

The poles of the transfer function are allocated along the negative real axis, or the negative σ axis, on the complex s plane. As understood from skin effect formulas [13], the skin effects are a phenomenon which is continuous with respect to frequency and theoretically does not have poles at specific positions. Therefore, our problem is essentially to approximate such a continuous frequency response by a transfer function with a finite number of poles. In other words, the continuous frequency response is reproduced by the sum of a finite number of discrete frequency responses as shown in (3). The TL impedance is generally monotonically-increasing due to the $j\omega$ term multiplied to the inductance and also due to the $\sqrt{j\omega}$ terms appearing in skin effect expressions. This fact suggests that an optimal pole allocation can be achieved roughly by one of the three distributions, a distribution with more poles at lower frequencies, that with more poles at higher frequencies, or a equidistant distribution. Since it is well-known that the frequency response of TL impedances can be well captured using a logarithmic frequency axis, we denote the base 10 logarithm of frequency f by φ. Consider the geometric sequence

$$\Delta\varphi_m = \Delta\varphi_1 \alpha^{m-1} \qquad (m = 1, 2, \cdots, M) \tag{5}$$

If $\Delta\varphi_m$ is considered the distance between two neighboring poles in logarithmic frequency, then we can generate the three pole distributions mentioned above by changing the value of the common ratio α. Calculating the sum of the sequence up to the $(m-1)$th term as

$$\varphi_m = \varphi_1 + \sum_{\mu=1}^{m-1} \Delta\varphi_1 \alpha^{\mu-1} \tag{6}$$

gives the position of the mth pole in logarithmic frequency. Thus, the mth pole is given by

$$p_m = -2\pi\, 10^{\varphi_m} \tag{7}$$

for $m = 1, 2, \cdots, M$. When α, the first frequency point f_1 and the last frequency point f_K are given, we obtain

$$\Delta\varphi_1 = (\varphi_K - \varphi_1)\frac{1-\alpha}{1-\alpha^{M-1}} \tag{8}$$

$$\varphi_1 = \log_{10} f_1, \quad \varphi_K = \log_{10} f_K \tag{9}$$

If α is smaller than one, (7) gives more poles at higher frequencies. If α is larger than one, (7) gives more poles at lower frequencies. And, if $\alpha = 1$, then we get equidistant pole allocation by (7) on the logarithmic scale. We will discuss an optimal value of α using numerical results obtained by practical TL impedance-matrix data in Section IV.

B. IDENTIFICATION OF RESIDUE MATRICES

In the previous section, poles have been allocated. Therefore, the next task is to identify their residue matrices. The residue matrices are identified so that the response of the transfer function (3) reproduces the given frequency response (2) as closely as possible. This leads to

$$\sum_{m=1}^{M} \frac{1}{j\omega_k - p_m} R_m \simeq H_k \tag{10}$$

at the discrete frequency points $k = 1, 2, \cdots, K$.

For various kinds of simulations, a perfect reproduction of dc response is desirable. Let us assume that the dc resistance matrix R_{dc} of a TL is given. The dc response of the left-hand side of (10) must agree with G_0 that is the inverse of R_{dc}.

$$\sum_{m=1}^{M}\left(-\frac{1}{p_m}R_m\right)=G_0 \tag{11}$$

Both R_{dc} and G_0 are diagonal matrices. The diagonal entries of R_{dc} are dc resistance values of the conductors, and their reciprocals are the diagonal entries of G_0. Therefore, the entries of G_0 are considered known values, and the equation above can be rearranged as

$$R_M=\sum_{m=1}^{M-1}\left(-\frac{p_M}{p_m}R_m\right)-p_M G_0 \tag{12}$$

Substituting this equation into (10) gives

$$\sum_{m=1}^{M-1}\left(\frac{1}{j\omega_k-p_m}-\frac{p_M}{p_m}\frac{1}{j\omega_k-p_M}\right)R_m \simeq H_k+\frac{p_M}{j\omega_k-p_M}G_0 \tag{13}$$

If the residue matrices are determined using the equation above, the dc response is perfectly reproduced. If R_{dc} is not available, we can use (10) instead.

Equations (10) and (13) are both matrix equations. Any element of both matrix equations results in the same form

$$\sum_{m=1}^{M'} a_{k,m}\, r_{ij,m}\simeq b_{ij,k} \tag{14}$$

where $M'=M$ in the case of (10) and $M'=M-1$ in the case of (13). The (i,j) element of R_m is denoted by $r_{ij,m}$. In the case of (10), the coefficients $a_{k,m}$ and $b_{ij,k}$ are given by

$$a_{k,m}=\frac{1}{j\omega_k-p_m},\qquad b_{ij,k}=h_{ij,k} \tag{15}$$

In the case of (13),

$$a_{k,m}=\frac{1}{j\omega_k-p_m}-\frac{p_M}{p_m}\frac{1}{j\omega_k-p_M} \tag{16}$$

$$b_{ij,k}=h_{ij,k}+\frac{p_M}{j\omega_k-p_M}g_{0,ij} \tag{17}$$

In the equations above, $h_{ij,k}$ is the (i,j) element of H_k, and $g_{0,ij}$ is the (i,j) element of G_0. For each of the matrix elements, tabulating all frequency points for $k=1,2,\cdots,K$ gives the overdetermined linear equations of the form

$$Ax\simeq b \tag{18}$$

where x is a column vector of order M' containing $r_{ij,m}$ for $m=1,2,\cdots,M'$ as unknowns. Note that x is a real vector, since R_m is real. A is an K-by-M' matrix containing the coefficients $a_{m,k}$, and b is a column vector of order K containing the coefficients $b_{ij,k}$. Since A and b are complex-valued, they can be decomposed into their real and imaginary components. This leads to the following real-valued overdetermined linear equations.

$$\begin{bmatrix}\mathrm{Re}\{A\}\\ \mathrm{Im}\{A\}\end{bmatrix}x\simeq\begin{bmatrix}\mathrm{Re}\{b\}\\ \mathrm{Im}\{b\}\end{bmatrix} \tag{19}$$

Solving the equation above for each matrix element gives the residue matrices R_m for $m=1,2,\cdots,M'$. In the case where R_{dc} is available and (13) is used, R_M is obtained by (12).

For accurate and robust solution, the following numerical techniques are utilized. To solve (19) accurately, its condition number should be improved [17]. For this purpose, column scaling [18], [19] which balances the Euclidean norms of the matrix columns to one is applied. Finally, we want to avoid the situation where the solution algorithm such as the QR algorithm fails due to a very small pivot value. To cope with this situation, the SVD procedure proposed in [20] is used and briefly reviewed here. If the left-hand-side matrix of (19) is denoted by A', its SVD is expressed by $A'=USV^T$, where U and V are $2K$-by-M' and M'-by-M' orthogonal matrices respectively. S is a diagonal matrix whose diagonal entries $s_1,s_2,\cdots,s_{M'}$ are real and positive and called the singular values of A'. The singular values can be sorted such that $s_1\geq s_2\geq\cdots\geq s_{M'}$ by permuting the elements of U and V. If the right-hand-side vector of (19) is denoted by b' and if we define $\xi=V^Tx$ and $\beta=U^Tb'$, (19) can be brought to

$$S\xi\simeq\beta \tag{20}$$

Since the singular values can be viewed as the contributions of the equations to the solution, the elements of very small ones compared to the largest one s_1 can be replaced with zeros. As a result, the first M'' elements are retained.

$$\hat{\xi}=\begin{bmatrix}\frac{\beta_1}{s_1} & \frac{\beta_2}{s_2} & \cdots & \frac{\beta_{M''}}{s_{M''}} & 0\cdots 0\end{bmatrix} \tag{21}$$

In this way, we can avoid divisions by extremely small numbers and ensure the computational stability of the solution process. The final solution is given by

$$x=V\hat{\xi} \tag{22}$$

The small singular values whose elements are replaced by zeros can be determined by comparing their values to a user-defined tolerance considering the machine epsilon of the computer used. In the comparison with the tolerance, the singular values should be normalized so that $s_1=1$.

C. MODEL ORDER DETERMINATION

The number of poles M is the order of the transfer function to be synthesized in the partial fraction form (3). An optimal order of the transfer function can be searched for by repeating the synthesis procedures mentioned above with changing M, and the optimality may be assessed by the following error index.

$$\delta=\max(\delta_R,\delta_L) \tag{23}$$

$$\delta_R=\max_{i,j,k}\left|\log\mathrm{Re}\left\{z_{ij}(j\omega_k)\right\}-\log\mathrm{Re}\left\{z_{ij,k}\right\}\right| \tag{24}$$

$$\delta_L=\max_{i,j,k}\left|\log\frac{\mathrm{Im}\left\{z_{ij}(j\omega_k)\right\}}{\omega_k}-\log\frac{\mathrm{Im}\left\{z_{ij,k}\right\}}{\omega_k}\right| \tag{25}$$

where $z_{ij}(j\omega_k)$ and $z_{ij,k}$ are the ij element of the inverse of $H(j\omega_k)$ and that of H_k respectively. Since $H(j\omega_k)$ and H_k are admittance matrices, $z_{ij}(j\omega_k)$ and $z_{ij,k}$ are impedance matrix elements. For the calculation of the index above, the inverse of the synthesized transfer function matrix $H(j\omega_k)$ and that of the given frequency response data H_k are calculated at frequency points $k = 1, 2, \cdots, K$ to obtain the values of $z_{ij}(j\omega_k)$ and $z_{ij,k}$. Then, (23)–(25) are evaluated. For TL modeling, the resistance and the inductance are important for reproducing the loss and the propagation velocity respectively. Thus, the reproduction of the resistance is evaluated by (24), and that of the inductance is evaluated by (25). Finally, the larger one is selected as the overall error index as in (23).

Starting from a designated value of M, the synthesis algorithm described in Sections III-A and III-B is performed, and the error index δ is evaluated by (23)–(25). If δ is larger than the error tolerance ε specified by the user, M is increased by one and the synthesis algorithm is repeated. This process is iterated until δ becomes smaller than or equal to ε. When this condition is satisfied, the synthesized transfer function is considered the final model.

IV. NUMERICAL EXAMPLES

A. DOUBLE-CIRCUIT OVERHEAD POWER TRANSMISSION LINE

Fig. 2 shows the conductor arrangement of a 500-kV double-circuit overhead power TL. This is actually an existing TL in Japan. Considering the skin effects in the conductors and the ground [13], the impedance matrix of the TL has been calculated at 161 frequency points from 0.1 Hz to 10 MHz. The frequency points are sampled equidistantly on the logarithmic scale including the boundaries. Since there are 6 power wire bundles (three phases of two circuits) and 2 ground wires, the size of the original impedance matrix is 8 by 8. The voltages and currents of the ground wires are not of interest in most power system simulations, and the rows and columns corresponding to the ground wires are thus eliminated assuming zero voltages taking their existence into account. The resultant matrix size is 6 by 6. The dc resistance of each power wire bundle is calculated to be 17.55 mΩ/km, and it is taken into account in the residue matrix identification. The space inductance is subtracted from the impedance matrix.

To investigate an optimal value of the common ratio α for pole allocation, both the values of α and the model order M are varied, and the pole allocation procedure described in Section III-A and the residue matrix identification procedure described in Section III-B are carried out for each combination of value sets. The value of α was varied from 0.95 to 1.05 at an interval of 0.001, while that of M was varied from 10 to 50. It had been tested in advance that the values of α outside the range above did not give accurate results. Fig. 3 (a) shows the error index δ given in (23) for all combinations of the value sets. When one of the arguments of the logarithmic functions in (24) and (25) is negative at any frequency point in any matrix element, δ is forced to one. The case where the synthesized transfer function achieves the smallest value of δ or the smallest error is at $\alpha = 0.978$ and

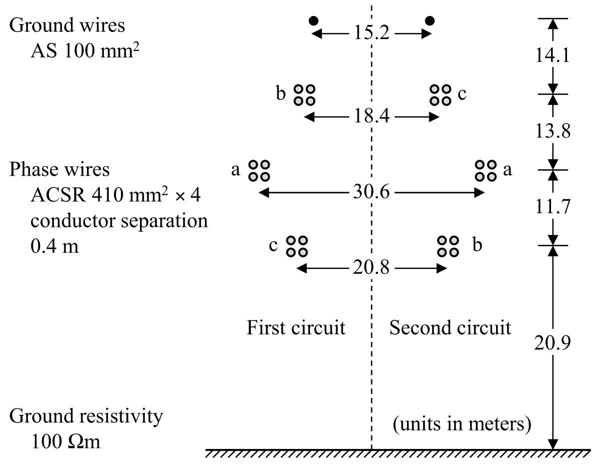

FIGURE 2. Conductor arrangement of a 500-kV double-circuit overhead power transmission line.

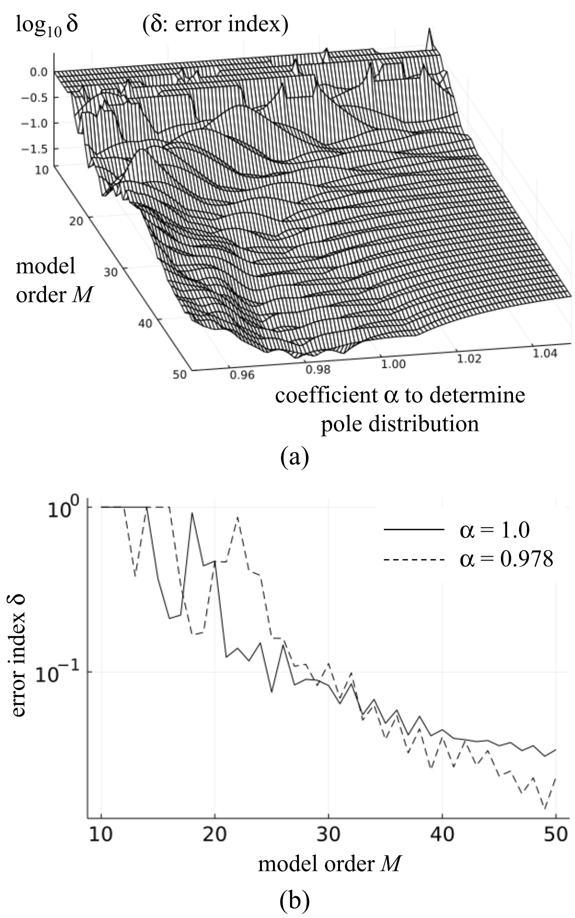

FIGURE 3. Variation of the error index δ given in (23) with respect to the common ratio α used for pole allocation and the model order M for the double-circuit overhead power TL example: (a) shows the entire view. (b) shows the equidistant pole allocation case ($\alpha = 1$) and the case where the synthesized transfer function achieves the smallest value of δ ($\alpha = 0.978$). When one of the arguments of the logarithmic functions in (24) and (25) is negative at any frequency point in any matrix element, δ is forced to one.

$M = 49$ as shown in Fig. 3 (b). It should be noted however that the equidistant pole allocation case or $\alpha = 1$ gives a slightly less accurate but accurate enough especially in the range $\delta < 10^{-1}$ which is practicable for simulations.

With the equidistant pole allocation ($\alpha = 1$), the proposed synthesis algorithm including the order determination procedure described in Section III-C has been performed. The error tolerance ε is set to 0.05, and an optimal order is searched for within the range from 10 to 50. Equations whose singular values are smaller than 1,000 times of the machine epsilon of the computer used are eliminated in the SVD solution. Fig.4 shows the synthesis result, where the frequency responses of the synthesized model and the given data are compared in resistance and inductance. Since the synthesized transfer function is quite accurate, dashed lines cannot be distinguished. The error index δ becomes smaller than the error tolerance ε at $M = 35$, the model order has been determined to be 35. Since the synthesized model with $\alpha = 1$ is quite accurate, the equidistant pole allocation may be recommended.

B. SUBMARINE-CABLE POWER TRANSMISSION LINE

The cross section of a 250-kV dc submarine power cable is shown in Fig. 5. It is a 600-mm^2 oil-filled (OF) cable used for an hvdc submarine power TL. Since hvdc submarine cables are installed with a large distance between cables, mutual couplings among different cables are negligible. The cable is laid on a seabed whose resistivity is 100 Ωm, and the resistivity of the sea water is 0.21 Ωm. Taking the skin effects in the cable conductors, the sea water and the seabed into account [13], the impedance matrix of the TL has been calculated at 161 frequency points from 0.1 Hz to 10 MHz. The frequency points are sampled equidistantly on the logarithmic scale including the boundaries. The calculation of the ground return impedance is due to Sunde's formula [21]. There are 3 conductor layers in the cable, and the size of the original impedance matrix is thus 3 by 3. Since the steel armor is soaked in the sea water, its voltage can be considered zero. With this condition, the impedance matrix is reduced to 2 by 2. The dc resistances of the core conductor and the lead sheath are 28.67 mΩ/km and 29.37 mΩ/km respectively, and they are taken into account in the residue matrix identification. The space inductance is subtracted from the impedance matrix.

For the investigation of an optimal value of the common ratio α for pole allocation, the values of α and M are varied, and the synthesis algorithm described in Sections III-A and III-B are carried out for each combination of value sets. The values of α was varied from 0.95 to 1.05 at an interval of 0.001, while M was varied from 20 to 60. It had been also tested in advance that the values of α outside the range above did not give accurate results. Fig. 6 (a) shows the error index δ for all combinations of the value sets. In the same way as the previous example, δ is forced to one, when one of the arguments of the logarithmic functions in (24) and (25) is negative at any frequency point in any matrix

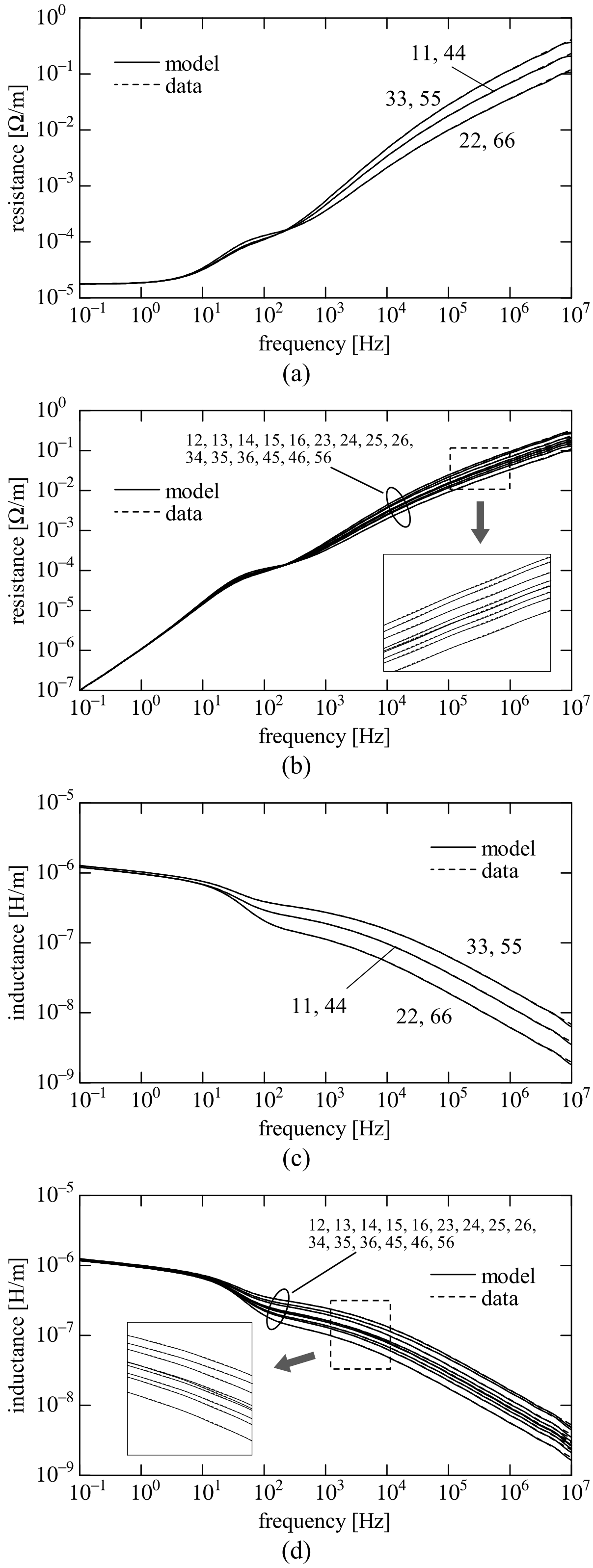

FIGURE 4. Synthesis result of the double-circuit overhead power TL example: (a) resistance of the diagonal elements, (b) resistance of the off-diagonal elements, (c) inductance of the diagonal elements, and (d) inductance of the off-diagonal elements. The two digit numbers shown in the figures designate matrix indices.

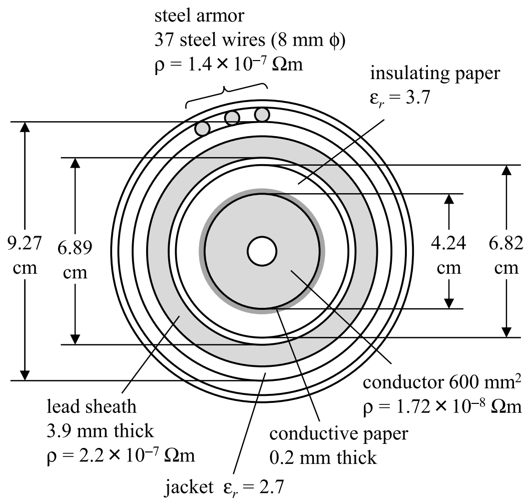

FIGURE 5. Cross section of a 250-kV dc submarine power cable.

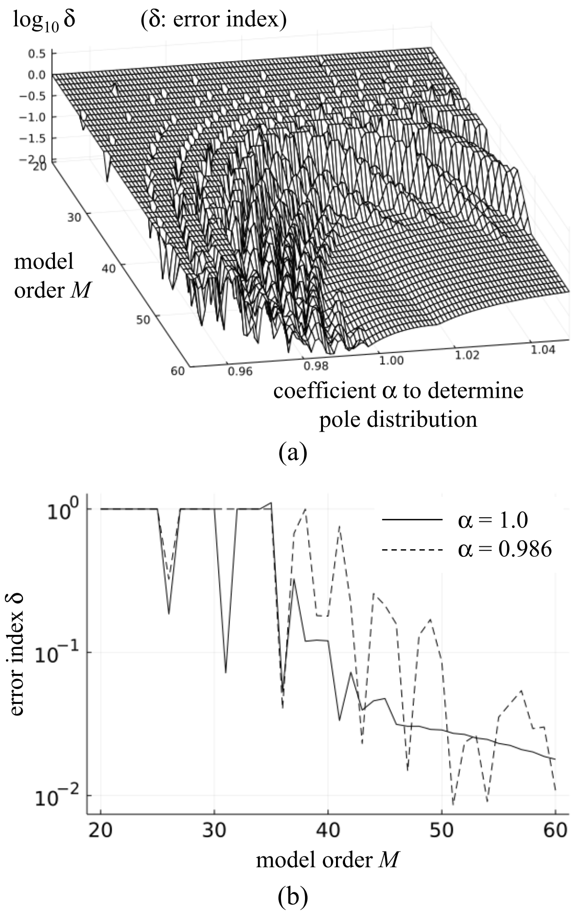

FIGURE 6. Variation of the error index δ given in (23) with respect to the common ratio α used for pole allocation and the model order M for the submarine-cable power TL example: (a) shows the entire view. (b) shows the equidistant pole allocation case ($\alpha = 1$) and the case where the synthesized transfer function achieves the smallest value of δ ($\alpha = 0.986$). When one of the arguments of the logarithmic functions in (24) and (25) is negative at any frequency point in any matrix element, δ is forced to one.

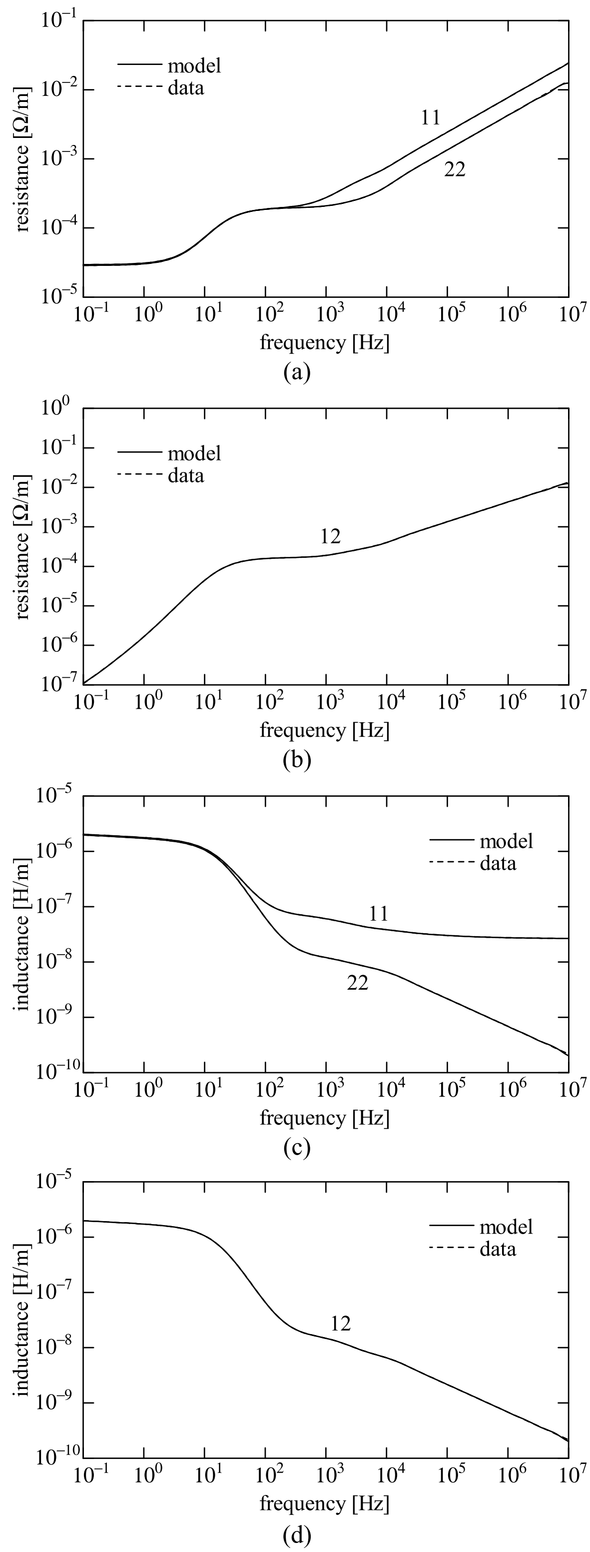

FIGURE 7. Synthesis result of the submarine-cable power TL example: (a) resistance of the diagonal elements, (b) resistance of the off-diagonal elements, (c) inductance of the diagonal elements, and (d) inductance of the off-diagonal elements. The two digit numbers shown in the figures designate matrix indices.

element. The case where the synthesized transfer function achieves the smallest value of δ thus the smallest error is at $\alpha = 0.986$ and $M = 51$. Fig. 6 (b) compares the result of this case with that of the equidistant pole allocation case ($\alpha = 1$). Both the present and previous examples indicate that a value which is slightly smaller than 1 or slightly more poles at high frequencies may give the smallest value of the error index δ. This point may worthwhile to investigate deeper as a further advanced research topic. At the same time, however, the equidistant pole allocation case or $\alpha = 1$ stably achieves the practicable range $\delta < 10^{-1}$ where $M > 40$ as shown in Fig. 6 (b).

With the equidistant pole allocation ($\alpha = 1$), the proposed synthesis algorithm including the order determination procedure has been performed. The settings of the error tolerance ε and the SVD solution are the same as the previous example. The search range for an optimal order is set to 20 to 60. The synthesis result is shown in Fig. 7, where the frequency responses of the synthesized model and the given data are compared in resistance and inductance. Again, dashed lines cannot be distinguished, since the synthesized transfer function is quite accurate. The model order has been determined to be 41. As observed, the synthesized model is quite accurate, and we may conclude from the two examples that the equidistant pole allocation on the logarithmic scale ($\alpha = 1$) is recommended for practical simulations.

V. CONCLUSION

This paper has proposed an algorithm to synthesize a transfer function only with real poles from tabulated frequency response data for transmission-line (TL) impedance modeling. The pole allocation method has been investigated using the impedance-matrix data of a 500-kV double-circuit overhead power TL and a 250-kV dc submarine power cable. The former is a typical ac transmission line, and the latter is used for an hvdc submarine transmission. As a result, it has been found from practical examples that equidistant poles allocated on the logarithmic axis give accurate results. A least-squares method for identifying the residue matrices of the poles has been proposed. The singular value decomposition (SVD) algorithm with a column-scaling technique is used for obtaining an accurate result with enhanced numerical stability. An order determination method has also been proposed to find an optimal model order. For validation, the synthesis algorithm that combines the aforementioned methods is applied for synthesizing the impedance matrices of the double-circuit overhead power TL and the dc submarine power cable, and accurate results are obtained. As the next task, we plan to use the proposed method for time-domain simulations with the Bergeron Cell line model [14]. This work may realize variable-length modeling of TLs, which can be considered a new trend different from recent developments [22], [23].

APPENDIX A
EFFECT OF ERRORS

As mentioned in the main text, the tabulated frequency response data of a TL impedance are in most cases obtained by numerical computation using analytical formulas. In some cases, however, such data are obtained by measurement, and the data may involve measurement errors. To investigate such cases, the impedance matrix of the 500-kV double-circuit overhead power TL used in Section IV-A is calculated at 41 frequency points from 0.1 Hz to 10 MHz, in other words, 5 points per decade, and errors are superimposed on the response to simulate measurement. The errors are generated by normally-distributed random numbers and superimposed independently on both the resistance and the inductance part. The standard deviation used in the random number generation is set to the half of the given error constant ε_m, and the error is bounded by ε_m. In order to well observe the effects of the errors, an optimal order is searched for within the range from 10 to 100, and the error tolerance ε is relaxed to 0.1. Transfer function synthesis is performed by the proposed algorithm with varying the error constant ε_m from 0 % to 5 % by 1 %. The result is shown in Table 2. Note that the result changes each time the program is executed due to the random number generation, and a typical result is shown in Table 2 by repeating the synthesis several times. As the superimposed errors become larger, the determined model order M becomes larger at the beginning but saturated to a certain value, 83, above $\varepsilon_m = 2$ %. It seems that the characteristics of the superimposed errors are reproduced by the model in that region. From this result, we can draw an interesting conclusion. When the proposed synthesis algorithm is applied to measured data, they should be smoothed out for instance by applying a local polynomial approximation so that the identified transfer function will not reproduce the error characteristics.

TABLE 2. Effect of superimposed errors on model order.

error constant ε_m (%)	model order M
0	25
1	35
2	83
3	83
4	83
5	83

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TAKU NODA (Senior Member, IEEE) was born in Osaka, Japan, in 1969. He received the B.Eng., M.Eng., and Ph.D. degrees from Doshisha University, Kyoto, Japan, in 1992, 1994, and 1997, respectively. In 1997, he joined the Central Research Institute of Electric Power Industry (CRIEPI), where he is currently the DX Strategist and a Senior Research Scientist with the Grid Innovation Research Laboratory and also works as the Chairperson of the IEEE Power & Energy Society Japan Joint Chapter. He held the following positions in his carrier, such as a Visiting Scientist of the University of Toronto, ON, Canada, from 2001 to 2002; an Adjunct Professor of Doshisha University, from 2005 to 2008; an Editor of IEEE Transactions on Power Delivery, from 2008 to 2014; and a Lecturer of the Shibaura Institute of Technology, Tokyo, Japan, from 2012 to 2015. He received the Best Paper Award twice, in 2008 and 2022, and the Progress Award twice, in 2009 and 2016, from the Institute of Electrical Engineers of Japan (IEEJ) and also received the Electrical Science and Engineering Promotion Award from its foundation, in 2016.