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RESEARCH ARTICLE

Transient and Steady Current in a Series RL Circuit

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ABSTRACT Part of electrical engineering is circuit theory, which includes methods for calculating steady and transient currents in a series RL circuit. Power systems are typically inductive and can be modeled with RL circuits. An RL circuit is formed by a real voltage source, inductors and resistors. The inductance and resistance of the circuit elements do not depend on time. The ideal voltage source, which is part of the real source, has voltage depending on time. Current in a series RL circuit is the solution to an ordinary differential equation, which is a mathematical expression of Kirchhoff's voltage law. The goal is the maximum possible reduction of mathematical manipulations in solving the equation for current calculation. Methods for solving the equation in a series RL circuit was proposed. In contrast to the literature, neither the phasor technique, the Fourier circuit analysis, the Laplace transform nor the convolution integral technique is used in the calculation of current. A mathematically correct solution to the equation is supplemented with solutions of illustrative examples. This solution is confronted with solution methods in electrical engineering and physics textbooks. The importance of the initial condition of current whose first derivative occurs in the equation is emphasized. An original definition of steady current and transient current is proposed.

INDEX TERMS RL circuit, numerical analysis, ordinary differential equations, transient and steady current.

I. INTRODUCTION

Part of electrical engineering is circuit theory, which includes methods for calculating steady and transient currents in a series *RL* circuit. In the article, the series *RL* circuit is denoted by the symbol $\mathscr{C}$. The circuit $\mathscr{C}$ is formed by a non-ideal voltage source, resistors and inductors. The ideal voltage source, which is part of the non-ideal voltage source, has the voltage $U(t)$, where t is time. In $\mathscr{C}$, there can also be sources that implement the initial conditions, and also switches which connect and short-circuit the voltage sources. A real voltage source can substitute for the non-ideal current source or a part of the circuit, according to Thevenin's theorem. The symbols L and R denote not only the parameters that characterize a circuit element but also the respective elements (e.g. resistor R). The parameters L and R are assumed to be constant, i.e. independent of t. The current in $\mathscr{C}$ is analysed when connecting and short-circuiting the voltage source. The disconnection of the source can be modelled in two ways. One of them consists in interrupting the circuit with the source disconnected at time t_{off}. No current can flow through the interrupted circuit, $I(t) = 0$ for $t > t_{\text{off}}$. With the other way of disconnecting, which, however, is beyond the scope of the article, the source disconnection is implemented using a circuit breaker, in which an electric arc is produced between the breaker contacts. In this case it is necessary to replace the breaker by a physical and a mathematical model of the breaker.

For given circuit parameters and a voltage $U(t)$, the current is the solution to the ordinary differential equation

$$L\dot{I}(t) + RI(t) = U(t), \tag{1}$$

with the dot above the quantity symbol denoting its derivative with respect to t. (1) is Kirchhoff's voltage law for $\mathscr{C}$. In textbooks on physics and electrical engineering, in the

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part dealing with the circuit theory, the calculation of current in $\mathscr{C}$ and the so-called transient phenomena are discussed. In the literature, attention is mainly devoted to the sinusoidal voltage $U(t)$. The topic calculation of current in $\mathscr{C}$ has in the literature been dealt with so many times that it is not within an individual's power to master the relevant literature. It can only be hoped that the choice [1], [2], [3], [4], [5], with which the results given in the article are compared, is sufficiently representative. The word literature further means [1], [2], [3], [4], [5].

In view of the fact that the voltage $U(t)$ can in reality take on only real values and R, L are real constants, the solution to (1) is real too. This is in agreement with reality since the solution to (1) is current in $\mathscr{C}$ and current can only take on real values. The solution to (1) is expressed by the formula derived in mathematics

$$I(t) = \exp(-t/\tau)\left[\frac{1}{L}\int U(t)\exp(t/\tau)\,\mathrm{d}t + C\right], \qquad (2)$$

where $\tau = L/R$ and C is a constant. The calculation of current is thus a calculation of the right side of (1) for R, L and $U(t)$. In the literature, the solution to (1) is often derived via solving the homogeneous differential equation relevant to (1), and the particular integral of equation (1). The derivation is sometimes accompanied by physical interpretation of the mathematical procedure.

The original contribution of the article is the presentation of mathematically correct pieces of knowledge about the calculation of current in $\mathscr{C}$, and about the interpretation of the calculation results. The solving of illustrative examples supplements the theoretical interpretation. Definitions are given of steady, conditioned steady and transient currents. The essence of the calculation of current in the circuit $\mathscr{C}$ for sinusoidal voltage is the same as for non-sinusoidal voltages. In contrast to the literature, neither the phasor technique, the Fourier circuit analysis, the Laplace transform nor the convolution integral technique is used in the calculation of current.

II. CURRENT IN A CIRCUIT

The series RL circuit is a series connection of a resistor (with resistance R), inductor (with inductance $L > 0$), and non-ideal voltage source. The non-ideal voltage source can have an internal resistance R_U and internal inductance L_U, which are included in R and L. The ideal voltage source in a non-ideal voltage source has a given voltage $U(t)$. The schematic of $\mathscr{C}$ is given in Fig. 1. The branch between the nodal points 2 and 3 in Fig. 1 is interrupted by the switch S_3, and the two-position switch without interruption of the circuit S_2 is in position 5. L and R are constant, they do not depend on t. The source U_{ini} in Fig. 1 implements the initial condition. Equation (1) is an ordinary differential equation, linear and inhomogeneous, (1) is a mathematical model for the calculation of current in $\mathscr{C}$.

If $U(t)$ is continuous over the interval $\mathcal{J}$ (which can even be infinite), then for an arbitrary $t_0 \in \mathcal{J}$ and an arbitrary

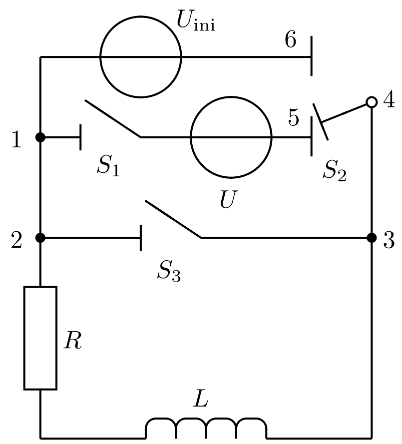

FIGURE 1. Circuit $\mathscr{C}$ is between nodal points 1 and 4, with S_2 in position 5 and $\mathscr{C}$ is closed by switch S_1. Switch S_3 serves to short-circuit voltage source U. Two-position switch S_2 without interruption of circuit connects voltage source U_{ini} and voltage source U in positions 6 and 5 respectively.

I_0 there is just one solution to equation (1) that satisfies the initial condition $I(t_0) = I_0$. The solution is defined over the whole interval $\mathcal{J}$, see [6], article 17.11. According to [6], [7], [8] The solution to (1) is expressed by (2), where the constant C in (2) depends on the initial condition and is the solution to the equation $I(t_0) = I_0$. The solution $I(t)$, satisfying the initial condition $I(t_0) = I_0$, exists and is unambiguous in the interval $[t_0, m]$, $t_0 < m$ also for $U(t)$, which is piecewise continuous in the interval $[t_0, m]$. In this case, $I(t)$ is obtained in the following way. $U(t)$ is piecewise continuous, which means that there exist points $t_0, t_1, \dots, t_\ell; t_\ell = m$, that divide the interval $[t_0, m]$ into partial intervals

$$\mathcal{J}_i = (t_{i-1}, t_i], \quad i = 1, 2, \dots, \ell, \qquad (3)$$

such that in each of these intervals the function $U(t)$ is continuous. Let us denote

$$\bar{t}_i = t_i - t_{i-1}, \quad i = 1, 2, \dots, t_\ell.$$

According to the theorem on the existence and unambiguity of the solution to (1) there exists an unambiguous solution $I_k(t)$ to (1) in each interval $[0, \bar{t}_k]$ that satisfies the initial condition $I_k(0) = I_{k-1}(\bar{t}_{k-1})$, with $I_1(0) = I_0$. The functions $I_k(t), k = 1, 2, \dots, t_\ell$, are determined by (2) and the initial condition $t_0 = 0, I_0 = I_{k-1}(\bar{t}_{k-1})$, while in (2) there is $U(t + t_{k-1})$ instead of $U(t)$. The sought solution $I(t)$ for a piecewise continuous voltage is determined by the relation

$$I(t) = \check{I}_1(t) + \check{I}_2(t) + \dots,$$

where

$$\check{I}_k(t) = \begin{cases} I_k(t - t_{k-1}) & \text{if } t \in \mathcal{J}_k \\ 0 & \text{otherwise} \end{cases}$$

The current $I(t)$ is continuous, but at the points $t_1, t_2, \dots, t_{\ell-1}$ it need not be smooth. There are not only mathematical but also physical reasons for the current continuity. A current discontinuity (jump) would cause a finite change in the energy in the inductor magnetic field at a single instant, which is not possible because it would be related either to an infinitely large power supplied into the circuit or to an infinitely large power drawn from the circuit.

A concrete solution to (1) will be given below for several sources of t-dependent voltage, where $t \in \mathcal{J} = [t_0, \infty)$.

A. CALCULATION OF THE INTEGRAL IN (2)

In the calculation of the current, difficulties can only appear when calculating the integral

$$\int U(t)\exp(t/\tau)\,\mathrm{d}t \tag{4}$$

in (2). According to the assumption, $U(t)$ is piecewise continuous and thus the integrand in (4) is also piecewise continuous and therefore (4) always exists. Some integrals of the product of the exponential function $\exp(t/\tau)$ and some other function, which is $U(t)$, can be found in published tables on indefinite integrals, for example in [6], [9], and [10]. The integral (4) can be expressed as the sum of a finite number of elementary functions if $U(t)$ is, among other things, a linear combination of integer powers of time (polynomial) or a linear combination of integer powers of the functions $\sin(\omega t)$ and $\cos(\omega t)$, where $\omega \neq 0$. If (4) cannot be expressed as the sum of a finite number of elementary functions, then there remain three possibilities. The first and second possibilities come into consideration also for $U(t)$ given by the table.

The first possibility is the numerical quadrature [6], [11], because the solution to (1), which satisfies the initial condition $I(t_0) = I_0$, can according to [8] be expressed by the formula

$$\begin{aligned} I(t) &= \exp\left(-\frac{1}{\tau}\int_{t_0}^{t}\mathrm{d}t\right) \\ &\quad\times\left[I_0 + \frac{1}{L}\int_{t_0}^{t} U(t)\exp\left(\frac{1}{\tau}\int_{t_0}^{t}\mathrm{d}t\right)\mathrm{d}t\right] \end{aligned}$$

and after rewriting

$$\begin{aligned} I(t) &= \exp[-(t-t_0)/\tau] \\ &\quad\times\left[I_0 + \frac{1}{L}\int_{t_0}^{t} U(t)\exp[(t-t_0)/\tau]\mathrm{d}t\right]. \end{aligned} \tag{5}$$

In this formula, the integral from t_0 till t can for an arbitrary t be calculated numerically.

The second possibility consists in approximating $U(t)$ by $U_{\mathrm{app}}(t)$ such that

$$\int U_{\mathrm{app}}(t)\exp(t/\tau)\,\mathrm{d}t \tag{6}$$

can be expressed in the finite form. The integral (6) is an approximation of (4). $U(t)$ is assumed to be piecewise continuous and each continuous part can be approximated by a polynomial with arbitrary precision [6], [11]. When approximating the voltage by a polynomial and calculating (6) by parts, the value of (6) is obtained in the finite form [6], [9], [10]. If $U_{\mathrm{app}}(t) = P_U(t, n)$, where $P_U(t, n)$ is a polynomial of the variable t of degree n, then according to [10] the integral (6) is equal to $\exp(t/\tau)P_I(t, n)$, where

$$\begin{aligned} P_I(t, n) &= \tau P_U(t, n) - \tau^2 \dot{P}_U(t, n) + \ldots \\ &\quad + (-1)^n \tau^{n+1} P_U^{(n)}(t, n). \end{aligned}$$

After substituting in (2) and after rewriting, the approximation of the current in $\mathscr{C}$ is

$$I_{\mathrm{app}}(t) = P_I(t, n)/L + C\exp(-t/\tau). \tag{7}$$

It is to be noted that the approximation of $U(t)$ by a polynomial does not mean that all the possibilities of an appropriate approximation have been exhausted. If $U_{\mathrm{app}}(t)$ is equal to the first N terms of the Fourier series of the function $U(t)$, then the integral in (2) can be expressed in the finite form, provided the Fourier series only contains the sine and cosine functions [6], section 13.5, [9], A-38.

Via a numerical quadrature the values of the integral under calculation are obtained for specific discrete values of t. If $U_{\mathrm{app}}(t)$ is used, the value of the integral under calculation is expressed analytically by the function of t, where t lies in an interval.

The third possibility is to not calculate the integral in (2) and perform the calculation of $I(t)$ by the numerical method. Equation (1) adjusted to $\dot{I}(t) = U(t)/L - I(t)/\tau$ is in a form that is appropriate when the numerical method is used for solving ordinary differential equations [6], [7], [8], [11]. Its application does not require calculating any integrals.

B. LINEARLY TIME-DEPENDENT VOLTAGE

A voltage that is linearly dependent on t is determined by the formula

$$U_{\mathrm{lin}}(t) = U_0 + s(t - t_0). \tag{8}$$

When connecting $U_{\mathrm{lin}}(t)$, the current is at the instant t_0 equal to I_0, then according to (2)

$$\begin{aligned} I_{\mathrm{lin}}(t) &= \frac{U_{\mathrm{lin}}(t) - s\tau}{R} \\ &\quad + \exp[(t_0 - t)/\tau]\left[I_0 - \frac{U_0 - s\tau}{R}\right]. \end{aligned} \tag{9}$$

If $U_{\mathrm{lin}}(t)$ does not depend on t, i.e. $s = 0$, $U_{\mathrm{lin}}(t) = U_0$, then according to (9) the current is

$$I_{\mathrm{con}}(t) = \frac{U_0}{R} + \exp[(t_0 - t)/\tau]\left[I_0 - \frac{U_0}{R}\right]. \tag{10}$$

When $I_0 = U_0/R$, then $I_{\mathrm{con}}(t) = U_0/R$. For $I_0 \neq U_0/R$, the graph asymptote of the function $I_{\mathrm{con}}(t)$ is the straight line $I = U_0/R$. If $I_0 < U_0/R$, then $I_0 - U_0/R < 0$ and the graph of $I_{\mathrm{con}}(t)$ lies below the asymptote and (10) can be adjusted and its logarithms found such that it holds

$$\ln\left[\frac{U_0}{R} - I_{\mathrm{con}}(t)\right] = \frac{t_0 - t}{\tau} + \ln\left[\frac{U_0}{R} - I_0\right].$$

The term on the right-hand side is a linear function of t and its graph in semilogarithmic coordinates is a straight line. Equation (10) can be adjusted in a similar way when $I_0 > U_0/R$.

When short-circuiting an arbitrary voltage source, there is a current I_0 in the circuit at the instant $t_{sho} \geq 0$, and for $t \geq t_{sho}$ it holds $U(t) = 0$. According to (2) the current is

$$I_{sho}(t) = I_0 \exp[(t_{sho} - t)/\tau], \quad \text{for } t \geq t_{sho}. \tag{11}$$

$I_{sho}(t)$ depends on t, I_0 and τ and does not depend on what voltage the source produced prior to short-circuiting, i.e. for $t < t_{sho}$. For a given I_0 and τ, $I_{sho}(t)$ is thus the same for all voltage sources. Similar to the connection of the source the logarithm $\ln I_{sho}(t)$ is a linear function of time.

Fig. 1 was partially described at the beginning of Section II. To complete the description it is necessary to describe the function of the branch between nodal points 1 and 4 if S_2 is in position 6. To determine $I(t)$ unambiguously, it is necessary to set the values t_0 and I_0 for which it holds $I(t_0) = I_0$. Probably the most usual is to assume that S_2 is in position 5, S_3 is in the same position as in Fig. 1, there is no current flowing through $\mathscr{C}$, and at the instant t_0 the source of voltage U is connected by S_1. In this case the initial condition is $I(t_0) = 0$. Another option is that in $\mathscr{C}$ the source $U = U_1(t)$ is connected and $I_1(t)$ is flowing through the circuit. At the instant t_0, when the magnitude of the current is $I_1(t_0)$, $U_1(t)$ changes to $U_2(t)$, and it is necessary to determine $I_2(t)$ that satisfies the initial condition $I_2(t_0) = I_1(t_0)$. In this case, the change in the voltage is taken to mean a change in the kind of voltage, for example the constant voltage changing to the sinusoidal voltage. This method is limited by the properties of the voltage source U. A universal method for implementing the initial condition consists in using the source $U_{ini} = U_0$, where U_0 is a constant voltage of appropriate magnitude, and S_2 is in position 6. If the initial current is to have the value I_0, it suffices to choose U_0 such that it holds $U_0 = RI_0$. After a sufficiently long time, the current will, by (10), be practically equal to I_0, and, after connecting the source U at time t_0 using S_2, the current in $\mathscr{C}$ will satisfy the initial condition $I(t_0) = I_0$.

C. SINUSOIDAL VOLTAGE

Sinusoidal voltage with a period $T_{sin} = 1/f_{sin}$ is determined by the formula

$$U_{sin}(t) = \hat{U} \sin(\omega t + \alpha), \quad \omega = 2\pi f_{sin}. \tag{12}$$

With the source (12) connected at the instant t_0, when the current equals I_0, the current in the circuit is according to (2)

$$\begin{aligned} I_{sin}(t) = {} & \hat{I} \sin(\omega t + \beta) \\ & + \exp[(t_0 - t)/\tau][I_0 - \hat{I} \sin(\omega t_0 + \beta)], \end{aligned} \tag{13}$$

where

$$\hat{I} = \frac{\hat{U}}{\sqrt{R^2 + \omega^2 L^2}}, \quad \beta = \alpha - \arctan\frac{\omega L}{R}. \tag{14}$$

The fraction $\omega L/R$ is always of non-negative value, consequently $\arctan(\omega L/R) \in [0, \pi/2]$.

D. VOLTAGE OF RECTANGULAR PULSES

The voltage of rectangular pulses $U_{rec}(t)$ is periodical with the frequency f_{rec}, period T_{rec} and amplitude $\hat{U}_{rec}$. In the first period it is determined by the formula

$$U_{rec}(t) = \begin{cases} \hat{U}_{rec}, & t \in (t_0, t_0 + \theta], \ 0 < \theta < T_{rec}, \\ 0, & t \in (t_0 + \theta, t_0 + T_{rec}]. \end{cases} \tag{15}$$

In this case, $U_{rec}(t)$ is not continuous in $\mathcal{J}$. The interval $\mathcal{J}$ is divided by the points

$$t_0 + (i-1)T_{rec}, t_0 + (i-1)T_{rec} + \theta, t_0 + iT_{rec}, \quad i = 1, 2, \ldots,$$

into partial intervals such that in each of these intervals $U_{rec}(t)$ is constant and thus also continuous. The method for calculating $I_{rec}(t)$ was described in Subsection II-B.

E. VOLTAGE OF TRIANGULAR OSCILLATION

The periodical voltage of triangular oscillation with the frequency f_{tri}, period T_{tri} and amplitude $\hat{U}_{tri}$ is determined in the first period by the formula

$$U_{tri}(t) = \begin{cases} pt, & t \in [0, T_{tri}/4), \\ p(T_{tri}/4 - t) + \hat{U}_{tri}, & t \in [T_{tri}/4, 3T_{tri}/4), \\ p(t - 3T_{tri}/4) - \hat{U}_{tri}, & t \in [3T_{tri}/4, T_{tri}), \end{cases} \tag{16}$$

where $p = 4\hat{U}_{tri}/T_{tri}$. The calculation of $I_{tri}(t)$ can be performed directly, using (2) or using the same procedure as in the case of piecewise continuous voltage on the assumption that $\mathcal{J}$ is divided by the points

$$t_0, iT_{tri}/4, \ldots, \quad i = 1, 3, 5, \ldots,$$

into partial intervals. In each partial interval, $U_{tri}(t)$ is an increasing or decreasing linear function of t.

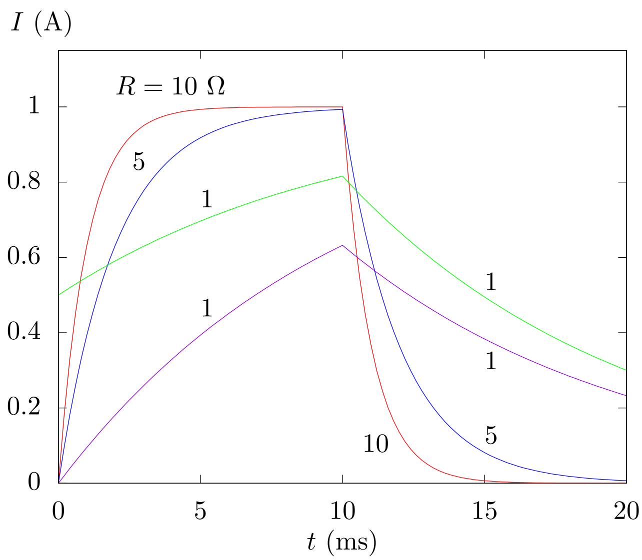

FIGURE 2. Currents $I_{con}(t)$ and $I_{sho}(t)$ in $\mathscr{C}$ when connecting ($t_0 = 0$) and short-circuiting ($t_{sho} = 10$ ms) the voltage sources respectively, and their dependence on R. Constant voltage U_0 is chosen such that it holds $U_0/R = 1$ A. $R = 10\ \Omega, I_0 = 0$: red line; $R = 5\ \Omega, I_0 = 0$: blue line; $R = 1\ \Omega, I_0 = 0$: violet line; $R = 1\ \Omega, I_0 = 0.5$ A: green line.

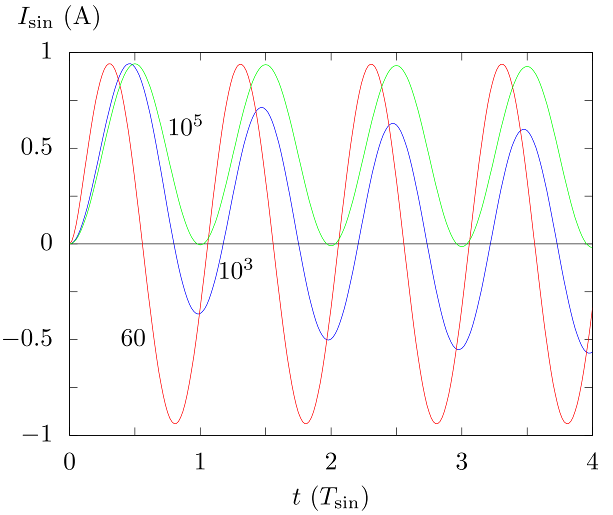

FIGURE 3. Current $I_{\rm sin}(t)$ in circuit $\mathscr{C}$, for three frequencies $f_{\rm sin}$, after connecting source of sinusoidal voltage (12), $\alpha = 0$, at instant $t_0 = 0, I_0 = 0$. $f_{\rm sin} = 60$ Hz: red line; $f_{\rm sin} = 10^3$ Hz: blue line; $f_{\rm sin} = 10^5$ Hz: green line.

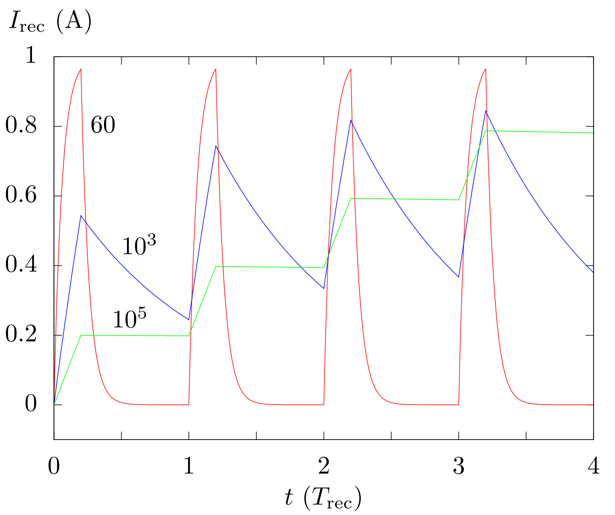

FIGURE 4. Current $I_{\rm rec}(t)$ in $\mathscr{C}$, for three frequencies $f_{\rm rec}$, after connecting source of rectangular pulses (15), $\theta = 0.2\,T_{\rm rec}$, at instant $t_0 = 0, I_0 = 0$. $f_{\rm rec} = 60$ Hz: red line; $f_{\rm rec} = 10^3$ Hz: blue line; $f_{\rm rec} = 10^5$ Hz: green line.

F. EXAMPLE

Specific currents in $\mathscr{C}$, with the parameters $R = 10\ \Omega, L = 10$ mH, for the voltages considered will be shown in the Figures. The values R and L were chosen the same as in [1], p. 92, Problem 13.

In Fig. 2, the currents $I_{\rm con}(t)$ and $I_{\rm sho}(t)$ in $\mathscr{C}$ are shown for several values of R; $t_0 = 0$. U_0 is chosen such that after connecting the voltage source, the curves $I_{\rm con}(t)$ have the same asymptote, $I = U_0/R = 1$ A, for all the R values considered.

In Fig. 3, the dependence of $I_{\rm sin}$ on t in $\mathscr{C}$ is illustrated for the three frequencies $f_{\rm sin} = 60, 10^3, 10^5$ Hz after the connection of the source (12), $\alpha = 0$, at the instant $t_0 = 0, I_0 = 0$. The voltage amplitude $\hat{U}$ was chosen such that the values of $I_{\rm sin}$ in the first local maximum are the same for the frequencies considered.

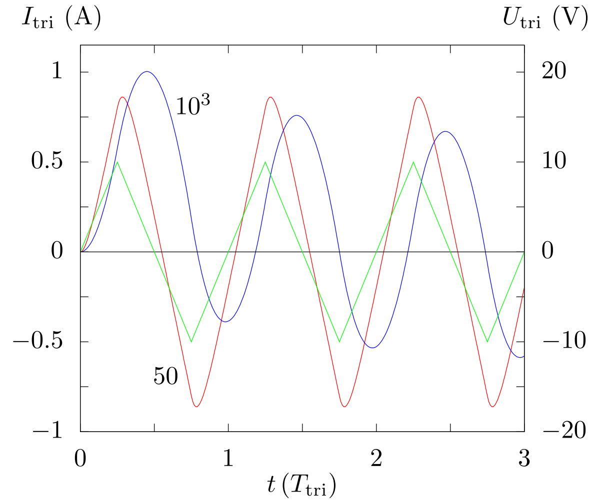

FIGURE 5. Current $I_{\rm tri}(t)$ in $\mathscr{C}$, for two frequencies $f_{\rm tri}$, after connecting source of triangular oscillations $U_{\rm tri}(t)$ (16), at instant $t_0 = 0, I_0 = 0$. $U_{\rm tri}(t)$ for $f_{\rm tri} = 50$ Hz is shown by green line, for $f_{\rm tri} = 10^3$ Hz the amplitude $\hat{U}_{\rm tri}$ is equal to 50 V. $I_{\rm tri}(t)$ for $f_{\rm tri} = 50$ Hz and 10^3 Hz is shown by red line and blu line respectively.

Shown in Fig. 4 is the dependence of $I_{\rm rec}$ on t in $\mathscr{C}$ for the three frequencies $f_{\rm rec} = 60, 10^3, 10^5$ Hz after the connection of the source of rectangular pulses (15), $\theta = 0.2\,T_{\rm rec}$, at the instant $t_0 = 0, I_0 = 0$. The voltage $\hat{U}_{\rm rec}$ chosen for $f_{\rm rec} = 60, 10^3, 10^5$ Hz was 10, 30 and 10^3 V respectively.

Fig. 5 shows the dependence of $I_{\rm tri}$ on t in $\mathscr{C}$ after the connection of the source of periodic triangular oscillations (16) at the instant $t_0 = 0, I_0 = 0$.

G. CIRCUIT WITHOUT RESISTOR

The circuit $\mathscr{C}_L$ is only formed by an inductor and ideal voltage source. The existence of such a circuit in reality is debatable. The subject of the present Subsection is the calculation of current in this circuit because such a case is given in the literature. The current $I(t)$ in a circuit is the solution to (1) for $R = 0$, i.e.

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{U(t)}{L}. \tag{17}$$

The same that holds for equation (17) in $\mathscr{C}$ also holds for the existence and unambiguity of the solution to (1). The solution to (17) that satisfies the initial condition $I(t_0) = I_0$ is expressed by (5) and it can be obtained for $R = 0$ by integrating (17)

$$\int_{I_0}^{I(t)} \mathrm{d}I = \frac{1}{L}\int_{t_0}^{t} U(x)\,\mathrm{d}x.$$

After integrating and rewriting, the solution to (17) is

$$I(t) = I_0 + \frac{F(t) - F(t_0)}{L}, \quad \text{where } F(x) = \int U(x)\,\mathrm{d}x. \tag{18}$$

It follows from (18) that, among other things, after short-circuiting any voltage source in $\mathscr{C}_L$ at the instant t_0, when there is I_0 in the circuit, the current remains constant and independent of time, $I(t) = I_0$ for $t \geq t_0$. From this case and from (18) it also follows that after connecting the source of rectangular pulses (15) the current will be a non-decreasing function of time. In the first part (of length θ) of every period T_{rec} of the voltage, the current will increase by $\theta \hat{U}_{\mathrm{rec}}/L$, and in the second part (of length $T_{\mathrm{rec}} - \theta$) of the voltage period, the current will be constant.

After connecting the source of sinusoidal voltage (12) at the instant t_0, when the current in $\mathscr{C}_L$ is equal to I_0, it holds for $t \geq t_0$

$$I(t) = I_0 + \hat{I}[\sin(\omega t + \beta) - \sin(\omega t_0 + \beta)], \tag{19}$$

where

$$\hat{I} = \frac{\hat{U}}{\omega L}, \ \beta = \alpha - \frac{\pi}{2}.$$

Formula (19) is consistent with (13) and (14) since it can be obtained from these formulae via the limit transition for $R \to 0+$.

The circuit $\mathscr{C}_L$ is the limit case of $\mathscr{C}$ (for $R \to 0$) in which the value R is small with respect to L. When short-circuiting the source in $\mathscr{C}_L$, the current is from the instant t_0 steady, its value is I_0, while in $\mathscr{C}$ the current is from the instant t_0 exponentially decreasing with the exponent $(t_0 - t)/\tau$. When the source of steady voltage in $\mathscr{C}_L$ is connected at time t_0, the current is steady immediately, while in $\mathscr{C}$ there is transient current, which becomes steady in a time proportional to $\exp[(t_0 - t)/\tau]$.

III. STEADY AND TRANSIENT CURRENT

In this Section, the steady current $I_{\mathrm{ste}}(t)$ and transient current $I_{\mathrm{tra}}(t)$ are defined precisely, and the relation is analysed between these currents and the current $I(t)$, where $I(t)$, $t \in [t_0, \infty)$, is the current in $\mathscr{C}$. The starting point for the formulation of the definition of steady current is the definition of steady state in [13], item 101-14-01: the steady state is the state of a physical system in which the characteristic parameters remain constant in time.

Definition 1: Steady current is expressed by the function $I_{\mathrm{ste}}(t)$, which is determined unambiguously by characteristic parameters independent of time.

The steady voltage can be defined analogously. It should be noted that steady voltage or steady current need not be oscillating and periodic. For example, the voltage $U_{\mathrm{lin}}(t)$ (see (8)) is steady.

Definition 2: The transient current is taken to mean a current that is not steady.

Currents in $\mathscr{C}$ can be steady only on the assumption that the voltage $U(t)$ is steady. The right sides of (9), (10) and (13) for the calculation of current have two parts. The first part is the steady current $I_{\mathrm{ste}}(t)$ on the assumption of the steady voltage $U(t)$ and zero second part. The second part is the product of the function $\exp[(t_0 - t)/\tau]$ and the term $E_1 = [I_0 - E_2]$, which does not depend on t. In the initial condition $I(t_0) = I_0$ it can be chosen $I_0 = E_2$, then for $t \geq t_{\mathrm{ste}}$, $t_{\mathrm{ste}} = t_0$ it holds $I(t) = I_{\mathrm{ste}}(t)$. For example, for the sinusoidal voltage (12) the equality $E_1 = 0$ can be obtained by suitably choosing the initial phase α of the voltage $U(t)$, as follows from (13) and (14). If $I_0 \leq \hat{I}$, then $E_1 = 0$ for

$$\alpha = \arctan\frac{\omega L}{R} + \arcsin\frac{I_0}{\hat{I}} - \omega t_0 \pm i2\pi, \quad i = 0, 1, \ldots.$$

The formula for the current in the circuit $\mathscr{C}_L$ does not include the function $\exp[(t_0 - t)/\tau]$. It follows from Subsection II-G and from this Section that for the steady voltage $U(t)$ also the current in $\mathscr{C}_L$ is steady for $t \geq t_0$ and cannot be transient.

Definition 3: If for an arbitrary $\varepsilon > 0$ there exists the time t_{ste}, for which it holds

$$|I(t) - I_{\mathrm{ste}}(t)| < \varepsilon, \text{ if } t \geq t_{\mathrm{ste}}, \tag{20}$$

then the current $I(t)$ is transient for $t \in [t_0, t_{\mathrm{ste}})$, and for $t \geq t_{\mathrm{ste}}$ it is called conditioned steady current.

The current $I(t)$, which is expressed by a mathematical expression such as (9), (10) and (13), has a limit for $t \to \infty$. In this case it holds

$$I_{\mathrm{ste}}(t) = \lim_{t\to\infty} I(t). \tag{21}$$

From the definition of the limit of the function and from (21) it follows that for an arbitrary $\varepsilon > 0$ there exists the time t_{ste}, for which (20) holds. The inequality in (20) is the inequality $|E_1 \exp[(t_0 - t)/\tau]| < \varepsilon$, which enables the calculation

$$t_{\mathrm{ste}} > t_0 + \tau \ln\frac{|E_1|}{\varepsilon}. \tag{22}$$

For example, for the sinusoidal voltage with the currents as given in Fig. 3, i.e. for $f_{\mathrm{sin}} = 60, 10^3, 10^5$ Hz, the time t_{ste}, after rounding to an integer multiple of the period T_{sin}, is successively equal to $1\,T_{\mathrm{sin}}, 9\,T_{\mathrm{sin}}, 846\,T_{\mathrm{sin}}$ for $\varepsilon = 10^{-4}$ A.

On the right-hand side of (7) for the calculation of $I_{\mathrm{app}}(t)$, the first term is a polynomial that is determined by coefficients that do not depend on t, so that $I_{\mathrm{app}}(t)$ is steady, if the constant C in the second member on the right side of (7), which is determined by the initial condition, is equal to zero. If $C \neq 0$, then $I_{\mathrm{app}}(t)$ is a conditioned steady current.

Using the condition that expresses the measure of stability of the characteristic parameters, t_{ste} can be determined for the periodic $U(t)$. If the voltage $U(t)$ of the source in the circuit $\mathscr{C}$ is periodic, with the period T, then it can be assumed that the steady current $I_{\mathrm{ste}}(t)$ will also be periodic, with the period T. During the ith period of the voltage $U(t)$ the current reaches the local maximum $I_{\max,i}$ and local minimum $I_{\min,i}$. For example, for (12), $t_{\mathrm{ste}} = i\,T_{\mathrm{sin}}$ if it holds

$$\left(\frac{|\hat{I} - I_{\max,i}|}{\hat{I}} < \delta\right) \wedge \left(\frac{|\hat{I} + I_{\min,i}|}{\hat{I}} < \delta\right),$$

where $\delta > 0$ is the chosen small number. For the source of triangular oscillations the following condition can be chosen

$$\left(\frac{|I_{\max,i} - I_{\max,i-1}|}{|I_{\max,i}|} < \delta\right) \wedge \left(\frac{|I_{\min,i} - I_{\min,i-1}|}{|I_{\min,i}|} < \delta\right). \tag{23}$$

For the currents in Fig. 5, i.e. for $f_{\mathrm{tri}} = 50$ Hz and $f_{\mathrm{tri}} = 10^3$ Hz, it holds $t_{\mathrm{ste}} = 1\,T_{\mathrm{tri}}$ and $t_{\mathrm{ste}} = 8\,T_{\mathrm{tri}}$ respectively for $\delta = 10^{-3}$. The condition (23) can also be chosen in the case of rectangular pulses with currents as in Fig. 4. For $f_{\mathrm{rec}} = 60, 10^3, 10^5$ Hz the time t_{ste} is successively equal to $2\,T_{\mathrm{rec}}, 10\,T_{\mathrm{rec}}, 463\,T_{\mathrm{rec}}$ for $\delta = 10^{-3}$. For a voltage with the frequency $f_{\mathrm{rec}} = 60$ Hz (23) is not very suitable, in particular for low values of δ, because $\lim I_{\mathrm{min},i} = 0$ for $i \to \infty$. In such a case it is better to replace the relative deviation in the second part of (23) with the absolute deviation.

IV. COMPARISON OF OBTAINED RESULTS WITH PUBLISHED RESULTS

When comparing the author's own results with the results in the literature, the symbols used are those used in the present article and they may differ from the symbols used in the literature.

In [2] it is stated: "Note that steady-state behaviour sets in after all 'transients' have died down." This definition covers the conditioned steady current as defined in Section III. Part of [2] deals with attempts to solve circuits with the source of sinusoidal voltage at some angular frequency ω and it is assumed that the currents, too, are sinusoidal at the same frequency ω. Thus the steady sinusoidal current is assumed and the circuits are solved exclusively by the symbolic method (using the phasors). By Subsection II-C, (13), the current in $\mathscr{C}$ is sinusoidal only in the case $I_0 = \hat{I}\sin(\omega t_0 + \beta)$. In the other cases, the sinusoidal current is conditioned steady.

A comparison of the method of calculating current as proposed in the present article with the method using the Fourier series can be demonstrated by calculating current in $\mathscr{C}$, with $R = 10\ \Omega$, $L = 10$ mH, with the source of sawtooth voltage

$$U_{\mathrm{saw}}(t) = U_0 + s(t - t_0), \quad s = -\frac{2U_0}{T}, U_0 = 1\ \mathrm{V}\cdot\mathrm{m}^{-1}, \tag{24}$$

when this source is connected. The source is assumed to be connected at time $t_0 = 0$, when the initial current is $I_0 = 0.01$ A. The voltage $U_{\mathrm{saw}}(t)$ is periodic with a period $T = 10^{-3}$ s. In each interval $t \in ((i-1)T, iT]$, $i = 1, 2, \ldots$ the voltage is continuous. According to the subsection II-B the current $I_i(t)$ is in the ith interval determined by the formula (9), where $t_0 = (i-1)T, I_0 = I_{i-1}((i-1)T)$ for $i = 2, 3, \ldots$, while for $i = 1$ I_0 is the given initial current 0.01 A. The dependence of current and voltage on t is shown in Fig. 6. The current in Fig. 6 is transient, becoming steady with growing t. The difference between the maximum current value in the ninth and tenth periods is less than 5 μA and therefore it can be regarded as conditioned steady for $t \geq 10\,T$. Steady voltage which is not sinusoidal is in the literature replaced with the Fourier series. For the steady voltage (24) it holds

$$U_{\mathrm{saw}}(t) = \frac{2U_0}{\pi}\sum_{k=1}^{\infty}\frac{\sin(k\omega t)}{k}, \quad \omega = \frac{2\pi}{T}. \tag{25}$$

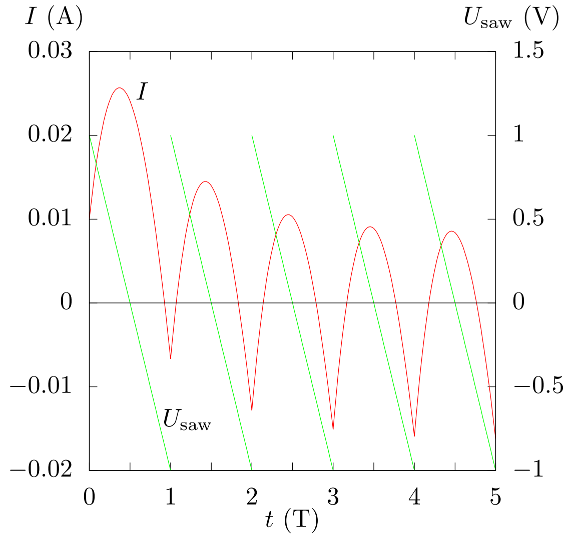

FIGURE 6. Voltage $U_{\mathrm{saw}}(t)$ (green straight lines) and current $I(t)$ (red line) in the course of five periods.

The current in $\mathscr{C}$ satisfies equation (1), in which $U(t)$ is replaced with the right side of the relation (25)

$$\dot{I}(t) + \frac{I(t)}{\tau} = \frac{2U_0}{L\pi}\sum_{k=1}^{\infty}\frac{\sin(k\omega t)}{k}. \tag{26}$$

It is assumed that in steady state the current $I(t)$ in $\mathscr{C}$ is the sum $I_1(t) + I_2(t) + \ldots$, where $I_k(t)$, $k = 1, 2, \ldots$, is the solution to equation (26) for the kth harmonic. Since the steady state is concerned here, this equation can be solved using the phasors $\underline{U}_k = \hat{U}_k\angle 0, \underline{I}_k = \hat{I}_k\angle\beta_k$, where $\hat{U}_k = 2U_0/(k\pi)$. $\underline{I}_k$ is the solution to the algebraic equation

$$\mathrm{j}k\omega\underline{I}_k + \frac{\underline{I}_k}{\tau} = \frac{\underline{U}_k}{L}.$$

It holds

$$I_k(t) = |\underline{I}_k|\sin[k\omega t + \arg(\underline{I}_k)],$$

where $\arg(\underline{I}_k) = \beta_k$. The calculation of $I(t)$ is performed with the first N terms of the Fourier series and therefore $I(t)$ is conditioned steady; with increasing N $I(t)$ converges to steady current. Fig. 7 gives the calculation results for $N = 5$ during one period. It can be seen in Fig. 7 that the black line differs little from the red line. For $N > 5$ the two lines are practically the same. However, it is necessary to observe that the calculation using the formula (2) is simpler in the case when there are no problems with calculating the integral in (2). In the contrary case it is more efficient to calculate the transient current via approximating the voltage $U(t)$ for example by the first N terms of the Fourier series in the integral in (2), as stated in II-A.

For the calculation of transient current in $\mathscr{C}$ the Laplace transform technique is in [2] proposed to be used. Using the Laplace transform, differential equation (1) is converted to an algebraic equation and the result of its solution is with the help of the inverse transform converted to a current in a circuit. In [2], the Laplace transform is used to solve Example 7.1: $R = 4\ \Omega$, $L = 5$ H, $t_0 = 0$, $U(t) = 6$ V

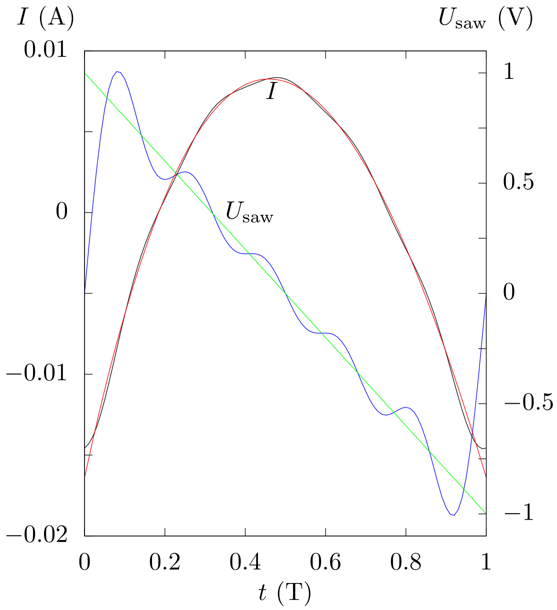

FIGURE 7. **Function $I(t)$ (black line) calculated using the Fourier series for $N = 5$ differs only a little from current calculated using formula (2) (red line). Green curve is the graph of $U_{\mathrm{saw}}(t)$ and blue line is its approximation by five terms of Fourier series.**

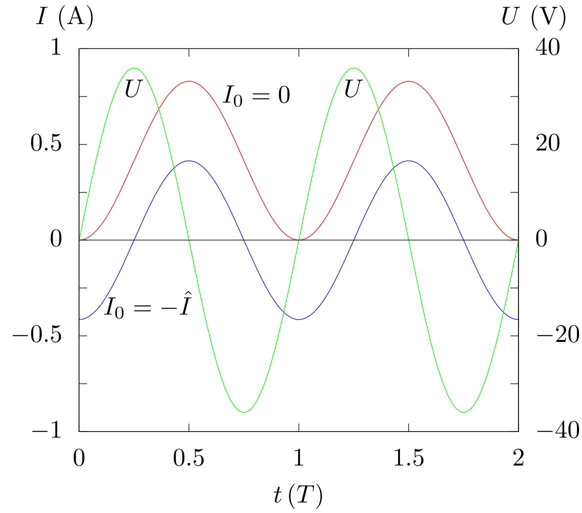

FIGURE 8. **Sample Problem 31.05 in [5]. Voltage $U(t)$ (green line) and current $I(t)$ during two voltage periods in circuit $\mathscr{C}_L$ for initial values $I_0 = 0$ and $I_0 = -\hat{I}$.**

for $t \geq t_0$, $I_0 = 4$ A. The solution derived in [2] is $I(t) = 1.5 + 2.5\exp(-0.8t)$, which is the same as (10) after the substitution of assigned values. Using the Laplace transform can be of advantage in the calculation of transient currents which are the solution to a system of integro-differential equations. In view of the fact that current in the series *RL* circuit is determined by the formula (2), the application of the Laplace transform is superfluous.

In [5], Section 30-6 *RL* CIRCUITS, the formula is given for the current $I(t)$ in $\mathscr{C}$ after connecting the source of constant voltage U_0 for $t_0 = 0$, $I_0 = 0$, $I(t) = U_0[1 - \exp(-t/\tau)]/R$. This formula corresponds to (10). In [5], Section Sample Problem 31.05, the calculation is performed of the current in the circuit $\mathscr{C}_L$ with the parameters: $L = 230$ mH, $U(t) = \hat{U}\sin(120\pi t)$, where $\hat{U} = 36$ V. The result given in [5] is

$$I(t) = \hat{I}\sin(120\pi t - \pi/2), \quad \text{where } \hat{I} = 0.415 \text{ A}. \tag{27}$$

The voltage source and current (27) in Fig. 8 are illustrated by curves denoted by U and $I_0 = -\hat{I}$ respectively. Fig. 8 corresponds to Figure 31-13 (a) in [5]. According to (19), the current (27) corresponds to the current in $\mathscr{C}_L$ for $t_0 = 0$ and $I_0 = -\hat{I}$. In [5], no mention is made of the initial condition. With no initial condition given, there is no unambiguous solution to (17). If it were assumed, for example, that at the instant $t = 0$ the current in $\mathscr{C}_L$ is zero, then the graph of the current $I(t)$ would be the curve denoted by $I_0 = 0$ in Fig. 8.

The introduction of the chapter Transient Analysis in [1] includes the definition: "Transients are non-steady-state time variations of voltages and currents which occur as a result of switching circuit elements and (or) changing interconnections of electric circuits". In [1] steady-state is taken to mean the circuit state in which all voltages and currents are sinusoidal. Unlike in [5], attention in [1] is devoted to the method of implementing the initial conditions. For the calculation of current in the circuit $\mathscr{C}$ the initial conditions are of fundamental significance because without their knowledge the current cannot be determined unambiguously; however, indispensable for the calculation itself are only the values t_0 and I_0 while the way they were obtained is irrelevant. There is no doubt that at the instant t_0 the current in the circuit $\mathscr{C}$ has some value and this value may not be ignored or trivialized.

In [1] the cases of connecting and short-circuiting the source of constant voltage and connecting the source of sinusoidal voltage are solved. In the solution it is assumed that the solution to equation (1) is the sum of the solution to the homogeneous equation ($U(t) = 0$) pertaining to (1) and the particular solution to (1). Such a solution must, by the theorem on the existence and unambiguity of the solution to (1), be the same as the solution (2). The particular solution can be calculated by the variation of constants method [6], [8]. The particular solution in [1] is determined using impedance and Ohm's law in the complex form. However, using the phasor technique is only possible for sinusoidal voltage (12) and steady sinusoidal current.

The first to be solved in [1] are circuits which contain only dc or ac sources. At the beginning of Subsection 7.5 in [1] it is said: "But, how can we calculate a response of an electric circuit to an *arbitrary source?* This can be accomplished by using a special technique called the *convolution integral technique*." The convolution integral is in essence the integral (4) and the convolution integral technique is a method of calculating this integral. In contrast to Subsection II-A, this technique is always used when $U(t)$ is not the voltage of a dc or ac source, and it is based on approximating $U(t)$ using the unit step function and unit impulse function [1]. In [1], EXAMPLE 7.17, the convolution integral technique is used to calculate the current $I(t)$ in the circuit $\mathscr{C}$: $R = 1\ \Omega$, $L = 2$ H, $U(t) = 2t$ V for $t \in [0, 1]$ s, $U(t) = 2$ V for $t > 1$ s; the initial condition $I(0) = I_0$ is not specified. The solution is given by (9) and (10). This solution, unlike the solution given in [1], contains constants that are determined by the initial condition. It follows from a comparison of the two solutions that the solution given in [1] satisfies the initial condition $I(0) = 0$.

In Section III, the steady current was defined generally, without any assumption regarding the voltage $U(t)$, although from a physical point of view it follows that the current can get steady only after the stabilization of the voltage $U(t)$. In the literature, steady current is usually discussed only in the case of the sinusoidal voltage $U_{\sin}(t)$, (12). According to [1], there is steady state in $\mathscr{C}$ if for the source voltage $U_{\sin}(t)$ there is in the circuit the sinusoidal current

$$I_{ss}(t) = \hat{I}_{ss} \sin(\omega t + \beta_{ss}).$$

$I_{ss}(t)$ is the current in $\mathscr{C}$ if and only if it satisfies (1). After substituting $U_{\sin}(t)$ and $I_{ss}(t)$ for $U(t)$ and $I(t)$ respectively in (1), and after rewriting, two equations are obtained for the calculation of $\hat{I}_{ss}$ and β_{ss}

$$R \cos \beta_{ss} - \omega L \sin \beta_{ss} = \hat{U} \cos \alpha / \hat{I}, \tag{28}$$

$$\omega L \cos \beta_{ss} + R \sin \beta_{ss} = \hat{U} \sin \alpha / \hat{I}. \tag{29}$$

Solving these equations yields $\hat{I}_{ss} = \hat{I}$ and $\beta_{ss} = \beta$, where $\hat{I}$ and β are determined by (14) and therefore it holds

$$I_{ss}(t) = \hat{I} \sin(\omega t + \beta). \tag{30}$$

This solution is determined by the circuit parameters R and L and the source voltage $U_{\sin}(t)$, and it does not contain constants that might be determined by the initial condition. (30) is a steady particular solution to (1) that passes through the point (t_0, I_0), where t_0 is an arbitrary chosen instant, and $I_0 = \hat{I} \sin(\omega t_0 + \beta)$. For $I_0 \neq \hat{I} \sin(\omega t_0 + \beta)$, $I_{ss}(t)$ is conditioned steady current, as follows from a comparison of (30) with (13).

V. CONCLUSION

The article discusses the calculation of current in a circuit formed by one real source of voltage, inductor and resistor. The inductance and resistance of circuit elements do not depend on time. An original definition of steady and transient currents is proposed. The series circuit is analysed. The current in a circuit is the solution to an ordinary differential equation which is a mathematical expression of Kirchhoff's voltage laws. The current in a series *RL* circuit is determined by (2) for an arbitrary piecewise continuous source voltage $U(t)$. Solving this equation the way it usually is done in mathematics is confronted with methods published in the literature.

In the calculation of current, neither the phasor technique, the Fourier circuit analysis, the Laplace transform nor the convolution integral technique is used. An unambiguous calculation of current in the series *RL* circuit is not possible without the knowledge of the initial value of current whose first derivatives with respect to time appear in the equation for the calculation of current. The knowledge of initial condition prior to current calculation is often trivialized or ignored in the literature. The theoretical explanation is supplemented with solutions of illustrative examples.

Currents in a parallel *RL* circuit are the solution to differential-algebraic equations (DAE), which are a mathematical expression of Kirchhoff's voltage and current laws. Unlike a series RL circuit, a parallel circuit can have many different configurations. However, when calculating currents in a parallel RL circuit by solving the appropriate DAE, mathematical manipulations can be significantly reduced compared to published works [14], [15], [16], [17].

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