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RESEARCH ARTICLE

Containment Tracking Control of Multiple Airships With Noise

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ABSTRACT This paper studies the containment control of multiple airships with multiple leaders disturbed by second-order moment processes. Firstly, the stochastic airship system is introduced by modeling in the mixed coordinate system. Then by combining backstepping method and graph theory, a new controller with adjustable parameters is designed to ensure that all states of the closed-loop system are bounded in probability, the mean-square of tracking errors between followers' output and leaders' output can be made arbitrarily small. A simulation example is finally given, which illustrates the feasibility of the designed control scheme.

INDEX TERMS Containment tracking control, multiple airships, second-order moment processes.

I. INTRODUCTION

With the wide application of airship in many fields, the research on its control problem has attracted extensive attention. Reference [1] considers the modeling problem by introducing the physical principle of airship flight, and proposes six degrees of freedom (6-DOF) airship dynamic equation. Reference [2] designs the controller by using backstepping method, analyzes by Lyapunov stability theorem, and gives the nonlinear 6-DOF simulation results of airship model. Nevertheless, [1], [2] only investigate the control problem for airship without noise, and do not consider the control problem for airship under stochastic disturbance.

Stochastic disturbance exists widely in engineering practice, so we further study the control problem for airship with random disturbance, which is usually regarded as white noise in [3] and [4]. Reference [5] points out that second-order moment processes can more reasonably describe stochastic disturbance for physical systems, which is more practical in systems. Reference [6] designs a controller for the airship model with second-order moment processes, then solves trajectory tracking problem for airship.

For stochastic systems, [7], [8], [9], [10], [11], [12], [13], [14], [15], [16] study the stochastic designs. They develop two approaches of controller design based on Lyapunov functions to investigate stochastic control: quartic Lyapunov functions are introduced in [9], [10], [11], and [12] introduces the weighted quadratic Lyapunov functions. In addition, [13], [14], [15] investigate the control problem of multi-agent systems (MASs), due to their widespread applications. Reference [13] considers the output-feedback tracking problem for MASs and gives a new homogeneous domination design scheme. Reference [16] solves the problem of cooperative control for multiple nonlinear systems with second-order moment processes. A common point of the above works is that study the MASs have only one leader.

The consensus problem for MASs with multiple leaders is called containment control [17]. Recently, [18] studies the distributed containment control problem for MASs with multiple dynamic leaders, then analyzes its stability. In [19], the containment problem of upper-triangular MASs is investigated.

By the backstepping method, the containment problem for multiple airships with multiple leaders disturbed by second-order moment processes is studied. The contributions related to this paper are as follows:

(1) We consider more general system models. It is worth noting that most of the results on airships are only for single agent systems, such as [6], [20], [21], and [22]. In this paper, we consider multiple airships, which is more general than the results in [6], [20], [21], and [22].

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(2) We propose new design and analysis methods. The designs in [3] and [4] can only deal with white noise and the method in [6] is only invalid for single-agent system. In this paper, we propose a new design by constructing new distributed coordinate transformations and developing a new step by step distributed design. This design method can effectively deal with white noise and the information interactions between agents simultaneously.

The remainder of this paper can be organized as follows. Section II gives some preliminaries. The problem is formulated in Section III. In Section IV, the new controller design and performance analysis are proposed to deal with the containment problem. A simulation example is given in Section V. The conclusion is introduced in Section VI.

II. PRELIMINARIES

Assumption 1: A stochastic process $\xi(t)$ is $\mathcal{F}_t-$adapted, it is also piecewise continuous, such that

$$\sup_{t\geq t_0} E|\xi(t)|^2 < K,$$

where $K > 0$.

Definition 1 [23]: If $|z(t)|$ satisfies the following formula

$$\lim_{c\to\infty} \sup_{t>t_0} P\{|z(t)| > c\} = 0, \tag{1}$$

then $z(t)$ is bounded in probability.

The relationship among N followers is depicted by a digraph $\mathcal{G} = (\mathcal{V}_f, \mathcal{E}, A)$ in this paper. $\mathcal{V}_f = \{1, 2, \cdots, N\}$ is the set of nodes, the set of arcs is described by $\mathcal{E} \subset \mathcal{V}_f \times \mathcal{V}_f$, and $A = (a_{ij})_{n\times n}$ with nonnegative elements is a weighted adjacency matrix. The set of neighbors of vertex i can be described by $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E},\ i \neq j\}$. In addition, $a_{ij} > 0$ shows that node j can directly send information to node i and $a_{ij} = 0$, otherwise. Define $D = diag(\sum_{j\in\mathcal{N}_1} a_{1j}, \sum_{j\in\mathcal{N}_2} a_{2j}, \cdots, \sum_{j\in\mathcal{N}_N} a_{Nj})$ is the degree matrix. And the Laplacian of $\mathcal{G}$ can be set as $L = D - A$.

We study the MASs with $N(N > 1)$ followers and $K(K > 1)$ leaders, and we use a graph $\overline{\mathcal{G}} = (\overline{\mathcal{V}}, \overline{\mathcal{E}})$ to depict the relationship, where $\overline{\mathcal{V}} = \mathcal{V}_f \cup \mathcal{V}_l$, $\mathcal{V}_f = \{1, 2, \cdots, N\}$, $\mathcal{V}_l = \{1, 2, \cdots, K\}$, and the set of arcs is denoted by $\overline{\mathcal{E}} \subset (\mathcal{V}_f \times \mathcal{V}_f \cup \mathcal{V}_f \times \mathcal{V}_l)$. If for every node i in $\mathcal{V}_f$, one can find a node j in $\mathcal{V}_l$, so there exists a path in $\overline{\mathcal{G}}$ from node j to i, and it shows that the set $\mathcal{V}_l$ is globally reachable in $\overline{\mathcal{G}}$. Define matrix $B = diag(\sum_{r=1}^{K} b_{1r}, \cdots, \sum_{r=1}^{K} b_{Nr})$ is the leader adjacency matrix related to $\overline{\mathcal{G}}$, where $b_{ir} > 0$ if node $r \in \mathcal{V}_l$ can directly send information to node $i \in \mathcal{V}_f$ and $b_{ir} = 0$, otherwise.

Lemma 1 [18]: Denote $H = L + B$, and all eigenvalues of H have positive real parts if and only if the set $\mathcal{V}_l$ is globally reachable in $\overline{\mathcal{G}}$.

III. PROBLEM FORMULATION

Now, we describe the airship model.

As shown in [6], we establish the appropriate coordinate system including the ERF and BRF to describe the airship's position, attitude and speed in Figure 1. We use $q_1 = (x, y, z)^T$ and $q_2 = (\psi, \theta, \varphi)^T$ to represent the position and

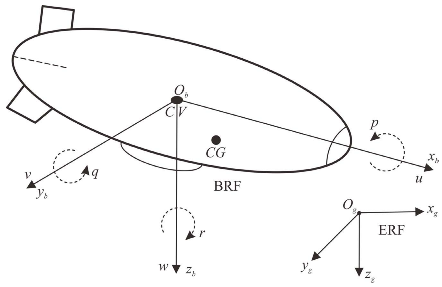

FIGURE 1. Depiction of airship.

attitude of airship, respectively. ψ is yaw angle, pitching angle is described by θ, and φ is called roll angle. The linear velocity and angular velocity of airship are depicted by $\nu_1 = (u, v, \omega)^T$, $\nu_2 = (p, q, r)^T$. The more information about airship is introduced by [6].

Damping Kν and airflow ξ are considered in this paper, so the equation of airship with 6-DOF is described as

$$\begin{aligned} \dot{q} &= T(q)\nu, \\ M\dot{\nu} + C(\nu)\nu + K\nu + G(q) &= u + \xi, \end{aligned} \tag{2}$$

where $q = (q_1^T, q_2^T)^T$, $\nu = (\nu_1^T, \nu_2^T)^T$, $T(q)$=diag$\{T_1, T_2(q)\}$, and

$$T_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$T_2(q) = \begin{pmatrix} 1 & \sin\varphi\tan\theta & \cos\varphi\tan\theta \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi\sec\theta & \cos\varphi\sec\theta \end{pmatrix},$$

$$M = \begin{pmatrix} mE + M_a & mS^T(r_c) \\ mS(r_c) & J + J_a \end{pmatrix} = M^T,$$

$$C(\nu) = \begin{pmatrix} 0 & -(mE + M_a)(S(r_c)S(\nu_2))^T \\ 0 & S(\nu_2)(J + J_a) \end{pmatrix},$$

$$J = \begin{pmatrix} J_x & -J_{xy} & -J_{xz} \\ -J_{yx} & J_y & -J_{yz} \\ -J_{zx} & -J_{zy} & J_z \end{pmatrix},$$

$$G(q) = \begin{pmatrix} (B_f - mg)\sin\theta \\ -(B_f - mg)\cos\theta\sin\varphi \\ -(B_f - mg)\cos\theta\cos\varphi \\ -z_g mg\cos\theta\sin\varphi \\ -z_g mg\sin\theta - x_g mg\cos\theta\cos\varphi \\ x_g mg\cos\theta\sin\varphi \end{pmatrix}.$$

$T(q)$ is transformation matrix between ERF and BRF. m is the mass of airship, the additional mass is depicted by M_a, J is moment of inertia matrix, and J_a is additional inertia, the airship's center of gravity is $r_c = (x_g, y_g, z_g)^T$, let $C(\nu)$ be Coriolis matrix in this paper. In addition, $u = (F, N)^T$, the external forces and external moments are described by F and N, respectively. ξ is second-order moment process.

In this paper, we consider systems with K leaders and N followers. The ith follower's dynamic equation can be depicted as

$$\dot{q}_i = T_i(q_i)\nu_i,$$

$$M\dot{\nu}_i + C_i(\nu_i)\nu_i + K\nu_i + G_i(q_i) = u_i + \xi_i,$$
$$y_i = q_i, \quad i = 1, \cdots, N, \quad (3)$$

where $q_i = (q_{i1}^T, q_{i2}^T)^T \in R^6$, $\nu_i = (\nu_{i1}^T, \nu_{i2}^T)^T \in R^6$, $T_i(q_i)$=diag$\{T_{i1}, T_{i2}(q_i)\}$, $\xi_1, \cdots, \xi_i$ are second-order moment processes, and $u_i \in R^6$, $y_i \in R^6$ are the input, output of the ith follower, respectively. Let $q_i, \nu_i, T_i(q_i), C_i(\nu_i), G_i(q_i)$ and $q, \nu, T(q), C(\nu), G(q)$ have the similar definitions, so we replace $q, \nu, T(q), C(\nu), G(q)$ with $q_i, \nu_i, T_i(q_i), C_i(\nu_i), G_i(q_i)$, for $i = 1, \cdots, N$.

As shown in [6], we need the following assumption on airship.

Assumption 2: The attitude q_{i2} of airship satisfies $|\psi_i| < \pi$, $|\theta_i| < \frac{\pi}{2}$, $|\varphi_i| < \frac{\pi}{2}$, it means that matrix $T_i(q_i)$ is always invertible.

IV. CONTROLLER DESIGN AND PERFORMANCE ANALYSIS

For system (3), the following assumptions are imposed.

Assumption 3: The leaders set $\mathcal{V}_l$ is globally reachable in the directed topology $\overline{\mathcal{G}}$.

Assumption 4: The leaders' output $r_s(t)$ and $\dot{r}_s(t)$, $s = 1, \cdots, K$, are bounded, they are only available for the ith follower satisfying $s \in \mathcal{N}_i$, $i = 1, \cdots, N$.

Remark 1: It is necessary to use Assumption 3 to study containment problem of the MASs (3). If Assumption 3 not holds, it means that all leaders are separated from some followers, and they do not have information interactions, which makes it impossible for those followers to track leaders.

In this section, the new controller is first developed for the MASs (3), then the stability analysis is introduced.

Step 1: Define

$$\zeta_{i1} = \sum_{s=1}^{N} a_{is}(y_i - y_s) + \sum_{j=1}^{K} b_{ij}(y_i - r_j), \quad (4)$$

combining (3) and (4), we get

$$\begin{aligned}\dot{\zeta}_{i1} &= \sum_{s=1}^{N} a_{is}(T_i(q_i)\nu_i - T_s(q_s)\nu_s) \\ &\quad + \sum_{j=1}^{K} b_{ij}(T_i(q_i)\nu_i - \dot{r}_j) \\ &= d_i T_i(q_i)\nu_i - \sum_{s=1}^{N} a_{is}T_s(q_s)\nu_s - \sum_{j=1}^{K} b_{ij}\dot{r}_j, \quad (5)\end{aligned}$$

where $d_i = \sum_{s=1}^{N} a_{is} + \sum_{j=1}^{K} b_{ij}$.

Choosing $V_{i1} = (1/2)\zeta_{i1}^T\zeta_{i1}$, from (5) we can have

$$\begin{aligned}\dot{V}_{i1} &= \zeta_{i1}^T\Big(d_i T_i(q_i)\nu_i - \sum_{s=1}^{\mathbb{N}} a_{is}T_s(q_s)\nu_s - \sum_{j=1}^{K} b_{ij}\dot{r}_j\Big) \\ &= d_i\zeta_{i1}^T T_i(q_i)(\nu_i - \alpha_i) + d_i\zeta_{i1}^T T_i(q_i)\alpha_i \\ &\quad - \zeta_{i1}^T \sum_{s=1}^{N} a_{is}T_s(q_s)(\nu_s - \alpha_s) \\ &\quad - \zeta_{i1}^T \sum_{s=1}^{N} a_{is}T_s(q_s)\alpha_s - \sum_{j=1}^{K} b_{ij}\zeta_{i1}^T\dot{r}_j. \quad (6)\end{aligned}$$

From Assumption 3, Lemma 1, it can be concluded that H is invertible, we choose

$$\begin{bmatrix} T_1(q_1)\alpha_1 \\ \vdots \\ T_N(q_N)\alpha_N \end{bmatrix} = -(H^{-1} \otimes I_6)\begin{bmatrix} c_{11}\zeta_{11} \\ \vdots \\ c_{N1}\zeta_{N1} \end{bmatrix} + (H^{-1} \otimes I_6)\left(\begin{bmatrix} \sum_{j=1}^{K} b_{1j} \\ \vdots \\ \sum_{j=1}^{K} b_{Nj} \end{bmatrix} \otimes \dot{r}_j\right), \quad (7)$$

which is equivalent to

$$d_i T_i(q_i)\alpha_i - \sum_{s=1}^{N} a_{is}T_s(q_s)\alpha_s - \sum_{j=1}^{K} b_{ij}\dot{r}_j = -c_{i1}\zeta_{i1}. \quad (8)$$

By Assumption 3, we get $d_i > 0$, for $i = 1, \cdots, N$.

Then from Assumption 2 and (8), we have

$$\begin{aligned}\alpha_i &= -T_i^{-1}(q_i)\Big(\frac{c_{i1}}{d_i}\zeta_{i1} - \frac{\sum_{j=1}^{K} b_{ij}}{d_i}\dot{r}_j\Big) \\ &\quad + \frac{T_i^{-1}(q_i)}{d_i}\sum_{s=1}^{N} a_{is}T_s(q_s)\alpha_s, \ i = 1, \cdots, N, \quad (9)\end{aligned}$$

substituting (9) into (6) yields

$$\begin{aligned}\dot{V}_{i1} &= -c_{i1}\zeta_{i1}^T\zeta_{i1} + d_i\zeta_{i1}^T T_i(q_i)(\nu_i - \alpha_i) \\ &\quad - \zeta_{i1}^T\sum_{s=1}^{N} a_{is}T_s(q_s)(\nu_s - \alpha_s), \quad (10)\end{aligned}$$

where $c_{i1} > 0$ are parameters to be designed, for $i = 1, \cdots, N$.

Step 2: Introducing the coordinate transformation $\zeta_{i2} = \nu_i - \alpha_i$.

Let $V_i = V_{i1} + (1/2)\zeta_{i2}^T M\zeta_{i2}$, by (10) we have

$$\begin{aligned}\dot{V}_i &= -c_{i1}\zeta_{i1}^T\zeta_{i1} + d_i\zeta_{i1}^T T_i(q_i)(\nu_i - \alpha_i) \\ &\quad - \zeta_{i1}^T\sum_{s=1}^{N} a_{is}T_s(q_s)(\nu_s - \alpha_s) + \zeta_{i2}^T\xi_i \\ &\quad + \zeta_{i2}^T(u_i - C_i(\nu_i)\nu_i - K\nu_i - G_i(q_i) - M\dot{\alpha}_i). \quad (11)\end{aligned}$$

According to Young's inequality, we get

$$\begin{aligned}& d_i\zeta_{i1}^T T_i(q_i)(\nu_i - \alpha_i) \\ &\quad \le \frac{1}{2}d_i|\zeta_{i1}|^2 + \frac{1}{2}d_i\|T_i(q_i)\|^2|\zeta_{i2}|^2, \\ &\quad \zeta_{i1}^T\sum_{s=1}^{N} a_{is}T_s(q_s)(\nu_s - \alpha_s) \\ &\quad \le \frac{1}{2}\sum_{s=1}^{N} a_{is}|\zeta_{i1}|^2 + \frac{1}{2}\sum_{s=1}^{N} a_{is}\|T_s(q_s)\|^2|\zeta_{s2}|^2, \\ & \zeta_{i2}^T\xi_i \le \epsilon|\zeta_{i2}|^2 + \frac{1}{4\epsilon}|\xi_i|^2, \quad (12)\end{aligned}$$

where $\epsilon > 0$ is constant to be designed.

From (11) and (12), we have

$$\begin{aligned}\dot{V}_i \le & -\Big(c_{i1} - \frac{1}{2}d_i + \frac{1}{2}\sum_{s=1}^{N} a_{is}\Big)|\zeta_{i1}|^2 \\ & + \zeta_{i2}^T\Big(u_i - C_i(\nu_i)\nu_i - K\nu_i - G_i(q_i) - M\dot{\alpha}_i \\ & + \epsilon\zeta_{i2} + \frac{1}{2}d_i\|T_i(q_i)\|^2\zeta_{i2}\Big) \\ & - \frac{1}{2}\sum_{s=1}^{N} a_{is}\|T_s(q_s)\|^2|\zeta_{s2}|^2 + \frac{1}{4\epsilon}|\xi_i|^2. \end{aligned} \tag{13}$$

Denote $V = \sum_{i=1}^{N} V_i$, by (13) we get

$$\begin{aligned}\dot{V} \le & -\sum_{i=1}^{\mathbb{N}}\Big(c_{i1} - \frac{1}{2}d_i + \frac{1}{2}\sum_{s=1}^{\mathbb{N}} a_{is}\Big)|\zeta_{i1}|^2 \\ & + \sum_{i=1}^{\mathbb{N}}\zeta_{i2}^T\Big(u_i - C_i(\nu_i)\nu_i - K\nu_i - G_i(q_i) \\ & - M\dot{\alpha}_i + \epsilon\zeta_{i2} + \frac{1}{2}d_i\|T_i(q_i)\|^2\zeta_{i2}\Big) \\ & - \frac{1}{2}\sum_{i=1}^{\mathbb{N}}\sum_{s=1}^{\mathbb{N}} a_{is}\|T_s(q_s)\|^2|\zeta_{s2}|^2 + \frac{N}{4\epsilon}|\xi_i|^2 \\ \le & -\sum_{i=1}^{N}\Big(c_{i1} - \frac{1}{2}d_i + \frac{1}{2}\sum_{s=1}^{N} a_{is}\Big)|\zeta_{i1}|^2 \\ & + \sum_{i=1}^{N}\zeta_{i2}^T\Big(u_i - C_i(\nu_i)\nu_i - K\nu_i - G_i(q_i) \\ & - M\dot{\alpha}_i + \epsilon\zeta_{i2} + \frac{1}{2}d_i\|T_i(q_i)\|^2\zeta_{i2} \\ & - \frac{1}{2}\sum_{s=1}^{N} a_{si}\|T_i(q_i)\|^2\zeta_{i2}\Big) + \frac{N}{4\epsilon}|\xi_i|^2. \end{aligned} \tag{14}$$

Choosing the actual control law

$$\begin{aligned} u_i = & C_i(\nu_i)\nu_i + K\nu_i + G_i(q_i) + M\dot{\alpha}_i - \epsilon\zeta_{i2} \\ & - \frac{1}{2}d_i\|T_i(q_i)\|^2\zeta_{i2} + \frac{1}{2}\sum_{s=1}^{N} a_{si}\|T_i(q_i)\|^2\zeta_{i2} - c_{i2}\zeta_{i2}, \end{aligned} \tag{15}$$

such that

$$\begin{aligned}\dot{V} \le & -\sum_{i=1}^{N}\Big(c_{i1} - \frac{1}{2}d_i + \frac{1}{2}\sum_{s=1}^{N} a_{is}\Big)|\zeta_{i1}|^2 \\ & - \sum_{i=1}^{N} c_{i2}|\zeta_{i2}|^2 + \frac{N}{4\epsilon}|\xi_i|^2, \end{aligned} \tag{16}$$

where $c_{i2} > 0$ are parameters to be designed, for $i = 1, \cdots, N$.

By (16), we have

$$\dot{V} \le -c_0 V + \frac{N}{4\epsilon}|\xi_i|^2, \tag{17}$$

where $c_0 = \min_{1\le i\le N}\{2c_{i1} - d_i + \sum_{s=1}^{N} a_{is}, 2c_{i2}\}$.

Remark 2: This paper investigates the control problem of multiple airships. Unlike the design scheme of single airship under second-order moment processes in [6], our new design method handle the information interactions among agents effectively, which is more practical in real applications.

Remark 3: Compared with the MASs with only one leader in [16], this paper studies multiple airships systems with multiple leaders, it makes more difficult to consider the directed graph, and the tracking signal is also more complex than [16].

Define

$$B_l = \begin{bmatrix} b_{11} & \cdots & b_{1K} \\ \vdots & \ddots & \vdots \\ b_{N1} & \cdots & b_{NK} \end{bmatrix}.$$

Lemma 2 [18]: Under Assumption 3, the ith element for $r_c = H^{-1}B_l r$ is depicted by $r_{ci} = \sum_{j=1}^{K} k_{ij} r_j$ with nonnegative constant k_{ij}, and it satisfies $\sum_{j=1}^{K} k_{ij} = 1,\ i = 1, \cdots, N$, where $r = (r_1, \cdots, r_K)^T$.

By Lemma 2, the main results can be obtained in this paper.

Theorem 1: Under Assumptions 1-4, for the MASs (3), using the controller (15), with

$$c_{i1} > \frac{1}{2}d_i - \frac{1}{2}\sum_{s=1}^{N} a_{is}, \quad c_{i2} > 0, \tag{18}$$

we can obtain:

1) The closed-loop system has a globally unique solution on $[t_0, \infty)$.

2) All the states of the closed-loop system are bounded in probability.

3) For arbitrary positive constant ε, initial value $q(t_0) = (q_1^T(t_0), \cdots, q_N^T(t_0))^T$, $\nu(t_0) = (\nu_1^T(t_0), \cdots, \nu_N^T(t_0))^T$, and we can find nonnegative constants $k_{ij} = 1 (j = 1, \cdots, K)$ satisfying $\sum_{j=1}^{K} k_{ij} = 1$, there exists a finite-time $T(q(t_0), \nu(t_0), \varepsilon)$, leads to

$$E|y_i(t) - \sum_{j=1}^{K} k_{ij} r_j(t)|^2 < \varepsilon, \quad \forall t > T(q(t_0), \nu(t_0), \varepsilon), \quad i = 1, \cdots, N.$$

In other words, the mean-square of tracking errors between each follower's output and the convex combination of all leaders' output can be made arbitrarily small.

Proof: Define $\zeta = (\zeta_{11}^T, \zeta_{12}^T, \cdots, \zeta_{N1}^T, \zeta_{N2}^T)^T$, and let

$$\sigma_l = \inf\{t \ge t_0 : |\zeta(t)| \ge l\}, \quad \forall l > 0.$$

By (17), implies that

$$\begin{aligned} & EV(\zeta(t\wedge\sigma_l)) - V(\zeta(t_0)) \\ & \le -c_0 E\int_{t_0}^{t\wedge\sigma_k} V(\zeta(s))ds + \frac{N}{4\epsilon}E\int_{t_0}^{t}|\xi_i(s)|^2\, ds. \end{aligned} \tag{19}$$

From V, Assumption 1 and (19), such that

$$EV(\zeta(t\wedge\sigma_l)) \le V(\zeta(t_0)) + \frac{NK}{4\epsilon}(t - t_0). \tag{20}$$

Then according to [24, Lemma 3] and (20), it can be concluded that the existence of system's solution on $[t_0, \infty)$ holds ture. So the proof of conclusion 1) is given.

With (20) and Fatou Lemma in [16], such that

$$\begin{aligned} EV(\zeta(t)) &= E\Big(\liminf_k V(\zeta(t\wedge\sigma_l))\Big) \\ &\le \liminf_k E(V(\zeta(t\wedge\sigma_l))) \\ &\le V(\zeta(t_0)) + \frac{NK}{4\epsilon}(t-t_0) \\ &< \infty, \end{aligned}$$

it shows that

$$EV(\zeta(t)) < \infty. \tag{21}$$

By (17), and (21), we obtain

$$V(\zeta(t)) < \infty, \quad \dot V(\zeta(t)) < \infty, \ \ a.s.. \tag{22}$$

According to Fubini's theorem, and with (21), (22), we get

$$\begin{aligned} \int_{t_1}^t E\dot V(\zeta(s))ds &= E\int_{t_1}^t \dot V(\zeta(s))ds \\ &= EV(\zeta(t)) - EV(\zeta(t_1)), \end{aligned}$$

Therefore, we can get

$$\frac{dEV(\zeta(t))}{dt} = E\frac{dV(\zeta(t))}{dt}. \tag{23}$$

Let $\gamma(t) = EV(\zeta(t))$, by Assumption 1, (17), and(23), we obtain

$$\begin{aligned} \dot\gamma(t) &\le -c_0\gamma + \frac{N}{4\epsilon}E|\xi_i|^2 \\ &\le -c_0\gamma + \frac{N}{4\epsilon}K. \end{aligned} \tag{24}$$

By [16, Lemma 5], (24) is expressed by

$$\gamma(t) \le e^{-c_0(t-t_0)}\gamma(t_0) + \frac{NK}{4c_0\epsilon}\Big(1-e^{-c_0(t-t_0)}\Big),$$

which yields

$$EV(\zeta(t)) \le e^{-c_0(t-t_0)}EV(\zeta(t_0)) + \frac{NK}{4c_0\epsilon}\Big(1-e^{-c_0(t-t_0)}\Big). \tag{25}$$

Now, we show conclusion 3).

Let $\zeta_1 = (\zeta_{11}^T, \cdots, \zeta_{N1}^T)^T$. By (25), we have

$$\begin{aligned} E|\zeta_1|^2 &= E\Big(|\zeta_{11}|^2 + \cdots + |\zeta_{N1}|^2\Big) \\ &\le 2\Big(e^{-c_0(t-t_0)}EV(\zeta(t_0)) \\ &\quad + \frac{NK}{4c_0\epsilon}\Big(1-e^{-c_0(t-t_0)}\Big)\Big). \end{aligned} \tag{26}$$

From the definitions of $\zeta_{11}, \cdots, \zeta_{N1}$, it can be seen that

$$\begin{aligned} \zeta_1 &= \Big(\sum_{s=1}^N a_{1s}(y_1^T - y_s^T) + \sum_{j=1}^K b_{1j}(y_1^T - r_j^T), \cdots, \\ &\quad \sum_{s=1}^N a_{Ns}(y_N^T - y_s^T) + \sum_{j=1}^K b_{Nj}(y_N^T - r_j^T)\Big)^T \\ &= \Big(d_1y_1^T - \sum_{s=1}^N a_{1s}y_s^T - \sum_{j=1}^K b_{1j}r_j^T, \cdots, \\ &\quad d_Ny_N^T - \sum_{s=1}^N a_{Ns}y_s^T - \sum_{j=1}^K b_{Nj}r_j^T\Big)^T \\ &= (H\otimes I_6)y - (B_l\otimes I_6)r, \end{aligned} \tag{27}$$

where $y = (y_1^T, \cdots, y_N^T)^T$, $r = (r_1^T, \cdots, r_K^T)^T$.

From Assumption 3, Lemma 1, (26) and (27), we get

$$\begin{aligned} &E|y - (H^{-1}\otimes I_6)(B_l\otimes I_6)r|^2 \\ &\quad \le 2|H^{-1}\otimes I_6|^2\Big(e^{-c_0(t-t_0)}EV(\zeta(t_0)) \\ &\quad\quad + \frac{NK}{4c_0\epsilon}\Big(1-e^{-c_0(t-t_0)}\Big)\Big). \end{aligned} \tag{28}$$

From definitions of c_0, ϵ, for arbitrary $\varepsilon > 0$ and $q_i(t_0), \nu_i(t_0)$, we select c_{i1}, c_{i2} and ϵ appropriately, for $i = 1, \cdots, N$, and we can get a finite-time $T(q(t_0), \nu(t_0), \varepsilon)$, it follows from (28) that

$$\begin{aligned} &E|y - (H^{-1}\otimes I_6)(B_l\otimes I_6)r|^2 < \varepsilon, \\ &\qquad\qquad \forall t > T(q(t_0), \nu(t_0), \varepsilon). \end{aligned} \tag{29}$$

Therefore, with (29), for $i = 1, \cdots, N$, we get

$$\begin{aligned} &E|y_i - r_{ci}|^2 \le E|y - (H^{-1}\otimes I_6)(B_l\otimes I_6)r|^2 < \varepsilon, \\ &\qquad\qquad \forall t > T(q(t_0), \nu(t_0), \varepsilon). \end{aligned}$$

According to Lemma 2, the conclusion 3) is given.

Next, conclusion 2) will be proved.

It follows from (25) that

$$EV(\zeta(t)) \le EV(\zeta(t_0)) + \frac{NK}{4c_0\epsilon}. \tag{30}$$

Noticing

$$\begin{aligned} EV(\zeta) &\ge \int_{|\zeta|>c} V(\zeta)P(dw) \\ &\ge \inf_{|\zeta|>c} V(\xi_i)P(|\zeta|>c). \end{aligned} \tag{31}$$

Combining (30) and (31), such that

$$P(|\zeta|>c) \le \frac{EV(\zeta(t_0)) + \frac{NK}{4c_0\epsilon}}{\inf_{|\zeta|>c} V(\zeta)}. \tag{32}$$

With $V(\zeta)$ and (32), implies that

$$\begin{aligned} \lim_{c\to\infty}\sup_{t>t_0} P(|\zeta|>c) &\le \lim_{c\to\infty}\sup_{t>t_0} \frac{EV(\zeta(t_0)) + \frac{NK}{4c_0\epsilon}}{\inf_{|\zeta|>c} V(\zeta)} \\ &= 0. \end{aligned} \tag{33}$$

From (33) and Definition 1, we can get that ζ is bounded in probability. With (27) and Assumption 4, we obtain the conclusion that $y_i = q_i$, is bounded in probability, $i = 1, \cdots, N$.

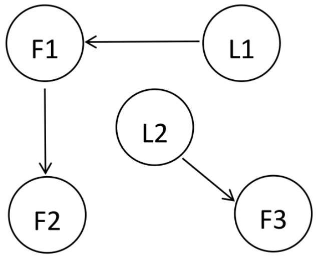

FIGURE 2. Communication topology $\bar{\mathcal{G}}$.

Noting that ζ_{i1}, ζ_{i2}, and q_i are bounded in probability, by $\zeta_{i2} = \nu_i - \alpha_i$, we get the conclusion that $\nu_i, i = 1, \cdots, N$ is bounded in probability. So we can get the conclusion 2).

Therefore, we have proved the Theorem 1.

V. SIMULINK EXAMPLE

We consider the MASs with three followers and two leaders in this part, which is depicted in Figure 2.

The leaders' outputs are given as

$$r_1(t) = (2\sin t,\ \sin t,\ \frac{1}{2}\cos t,\ \sin t,\ e^{-t},\ \frac{4}{5}e^{-t})^T,$$
$$r_2(t) = (\frac{3}{2}\cos t,\ \frac{1}{2}\sin t,\ \frac{1}{1+t},\ \cos t,\ 2e^{-t},\ \frac{1}{3}e^{-t})^T.$$

From Figure 2, we have

$$L = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

From the definition of H, we have

$$H = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad H^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{34}$$

By Lemma 2, we get

$$r_c = (H^{-1} \otimes I_6)(B_l \otimes I_6)r = \begin{bmatrix} r_1 \\ r_1 \\ r_2 \end{bmatrix}. \tag{35}$$

Define the output tracking errors as

$$e_{1j} = y_1 - r_{c1},\ e_{2j} = y_2 - r_{c2},\ e_{3j} = y_3 - r_{c3}. \tag{36}$$

In the simulation, choosing the parameters $c_{11} = 1$, $c_{12} = 0.6$, $c_{21} = 0.5$, $c_{22} = 0.1$, $c_{31} = 1$, $c_{32} = 0.1$, and $\epsilon = 0.4$. By setting the initial values $q_1(0) = (0.8,\ 1,\ 0.15,\ -0.5,\ 0.5,\ 0.4)^T$, $v_1(0) = (1.2,\ 0.1,\ 0.3,\ 0.7,\ -0.1,\ 0.1)^T$, $q_2(0) = (1.2,\ -0.2,\ 0.35,\ 0.5,\ 0.4,\ 0.7)^T$, $v_2(0) = (1,\ 0.3,\ 0.2,\ 0.5,\ 0.1,\ 0.05)^T$, $q_3(0) = (0.7,\ 0.3,\ -0.1,\ 0.4,\ 0.3,\ 0.9)^T$, $v_3(0) = (0.8,\ 0.2,\ 0.2,\ 0.6,\ 0.2,\ -0.3)^T$. We can see that Figures 3 and 4 show the responses of the tracking errors and controllers, respectively. Figure 5 shows the trajectories of three followers and two leaders. We can get Figure 3, which shows that $|e_{ij}| < \varepsilon = 0.5, \forall t > T(\varepsilon) = 5s, i = 1, 2, 3; j = 1, 2, 3, 4, 5, 6$. Thus, the efficiency of controllers is proved.

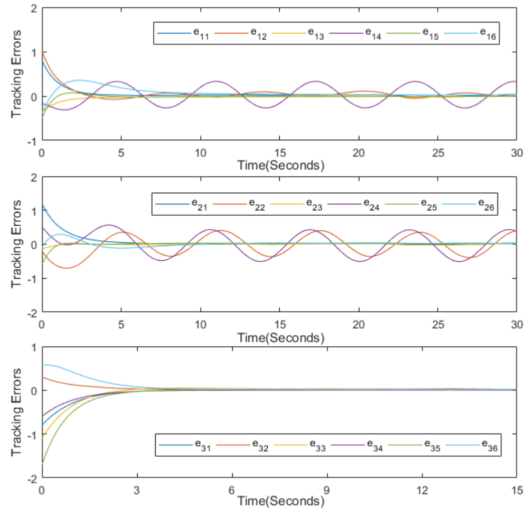

FIGURE 3. Responses of tracking errors.

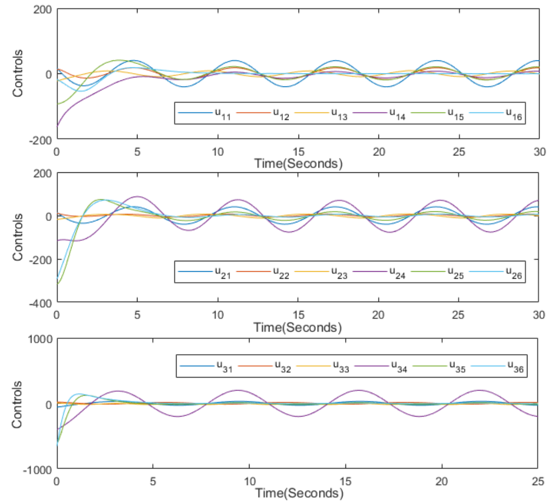

FIGURE 4. Responses of controllers.

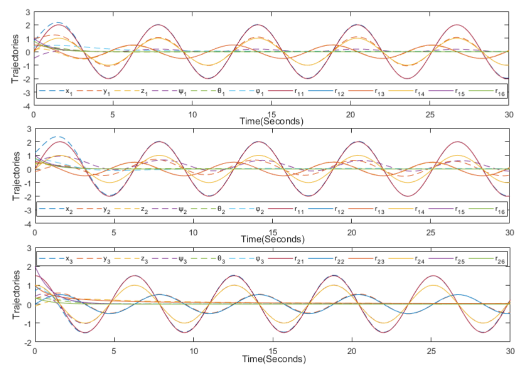

FIGURE 5. Trajectories of three followers and two leaders.

VI. CONCLUSION

This paper solves the containment tracking control problem for multiple airships with multiple leaders disturbed by second-order moment processes. By the backstepping method, we propose the new controllers to prove that all states of the closed-loop system are bounded in probability and the mean-square of tracking errors achieve arbitrarily small.

For future work, the containment problem for multiple airships with second-order moment processes under switching topologies can be considered. Another interesting work is extending the results to fuzzy models in [25] and [26] or more general stochastic systems in [27], [28], [29], and [30].

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