RESEARCH ARTICLE

A Novel Two-Mode Integral Approach for Thermal Error Modeling in CNC Milling-Turning Machining Center

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ABSTRACT Thermal errors have the largest contribution, as much as about 70%, to the machining inaccuracy of computer-numerical-controlled (CNC) machining centers. The error compensation method so far plays the most popular and effective way to minimize the thermal error. How to accurately and quickly build an applicable thermal error model (TEM) is the kernel work of thermal error compensation. On the basis of some comprehensive machine-learning schemes, past proposed TEMs had impressive merits for dealing with the thermal error modeling of single-function (milling or turning cutting) machine tools with only considering one set of thermal key points. These proposed modelling methodologies become worse when applied to CNC compound milling-turning machining centers in actual cutting applications. This paper proposes a two-mode integral TEM based on the Lasso and the random forest regression schemes to quickly and accurately predict the thermal deformations of such a machine. The first mode is the thermal error modeling for milling cutting conditions, and the second mode is that for turning cutting conditions. For data reduction, two different sets of temperature key points, one for milling and the other for turning, are obtained. Then, on the basis of the random forest regression scheme, we separately establish two TEMs but concurrently use them to predict the tool-center-point deformations of both milling as well as turning spindles. Further, we compare our proposed TEM with several frequently-used machine-learning-based TEMs and the results show that our proposed TEM are the best among all, no matter in the modelling experiment or in the test experiment. The proposed TEM has a maximum prediction error of 6.08 μm for milling cutting and that of 1.455 μm for turning cutting in the modeling experiment. By our proposed two-mode integral TEM, the thermal error of a multi-function milling-turning machining center can be accurately predicted and quickly compensated.

INDEX TERMS Thermal error model, thermal error compensation, CNC milling-turning machine tools, Lasso regression, machining learning.

I. INTRODUCTION AND LITERATURE REVIEW

The accuracy of machine tools plays a crucial role in modern precision manufacturing. An investigation revealed that up to over 70% of the error on a computer-numerical-controlled (CNC) machine is caused by the effect of thermal deformation [1]. How to successfully suppress the thermal deformation is an important work. Against this issue, various methods including the avoidance of thermal displacement,

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control of heat transfer, and compensation of thermal error have been developed [2], [3], [4]. In general, the aforementioned first two methods have already been fully considered in the design stage for modern machine tools development. The leftover method of thermal error compensation (TEC) becomes the main stream for overcoming the thermal deformation generated in machining. The TEC method is a kind of indirect or software compensation scheme which is based on the principle of a series of corrections computed by a specific mathematical thermal-error model [5]. From the detected continuous temperature signals of a machine tool,

this model may correctly predict the thermal deformation at the tool center point (TCP). Accordingly, correct mechanical positions of TCP can be compensated by the machine's control system during machining. Due to the cost-effectiveness and ease of implementation, TEC method is the most widely used scheme to reduce the thermal error of a CNC machine during machining.

The manipulation procedure of indirect TEC includes the following seven key steps:

(1) Select a target CNC machining center.
(2) Choose proper experimental machining conditions.
(3) Determine characteristic measurement points of temperature.
(4) Perform experimental apparatus setup and measure.
(5) Collect and reduce data.
(6) Build and verify thermal error model (TEM) building and verification.
(7) Test the proposed TEM with real-cutting conditions.

Overall, the above seven items significantly affect the success or not for a TEC method and its actual applications. Through decades of efforts on TEC investigations, lots of excellent mathematic TEMs were proposed. In the early stage, a linear and a modified linear regression model were proposed to analyze the thermal error for simple CNC vertical machining centers or lathes [6], [7], [8], [9]. However, to overcome the limitations of low accuracy and bad robustness for the regression model, a grey system model based on a first-order differential equation, such as GM(1, n), was proposed [10]. Thereafter, for further enhancing the prediction accuracy, a lot of mathematic schemes based on machine learning, such as the adaptive network-based fuzzy inference system [11], the ridge regression [12], the fuzzy art map of artificial neural network [13], and some statistics methods [14], were proposed to deal with TEM building and characteristic temperature point selection problems. During this period, a quite successful mapping scheme of support vector regression (SVR) combined with parameter-optimized methods (e.g. genetic algorithm) were proposed [15]. Meanwhile, some data mining schemes were used to capture the characteristic points of temperature and reduce the measured data set in the thermal error modelling. For example, Wang *et al.* [16] used the rough set theorem to reduce the measured temperature data and found the knowledge kernel for help establishing a precise TEM. Recently, accompanied by prosperous developing of deep learning neural network schemes, scholars begun to use the convolution neural network scheme as the mapping function of TEM [17], [3], [18], whose prediction ability is regarded as better than the other aforementioned machine learning schemes.

Despite the progress in building a precise TEM using various mathematic models, there are other deterministic factors needed to be considered first, which may affect the accuracy and robustness of a TEM. As shown previously, there are seven items involved in building a TEM for a CNC machine tool. Among the related parameters contained in these steps, the machine type, experimental

cutting conditions, and measured data play the most crucial roles. Different types of machine tools (three-axes or five-axes, vertical or horizontal, and C-type or Column-type) will lead to different selections of TEMs, because the structure complexities markedly affect the thermal behavior or thermal deformation of a machine tool. So far, studies on the thermal deformation as well as TEM of five-axis or milling-turning machine tools draw much attention due to the challenge coming from structure complexity. Focusing on the principle of thermal deformation for a complex CNC five-axis machining center, Hong and Ibaraki [19] investigated the thermal influence on error motions of rotary axes by static R test. And, Martin *et al.* [5] used the transfer function based on the concept of partial linearization to establish TCM for a head-to-head type five-axis machine tool. Mares [20] proposed a simple TEM to deal with the problem of thermal error minimization of a turning-milling center. It is noticeable that, for complicated machine tools, the built TEM should be as accurate in the modelling but simple in engineering applications as possible. These kind of issues are seldom discussed. On the other hand, it is known that different experimental cutting conditions significantly influence the adoption of thereafter data handling schemes and TEM selections [18]. Although we have some ISO standards of thermal test conditions for checking the thermal deformation of simple CNC machine tools, such as CNC lathes or milling machines, it still lacks the specs. or standards of how to establish an accurate TEM, especially for complicated five-axis or milling-turning machine tools.

Concluding, there are three major problems unsolved by existed thermal error modeling methods. First, for a compound milling-turning machine, there coexists two different types of thermal behaviors, one appears in the milling operation and the other in the turning operation. This matter severely blocks efforts to build an accurate TEM with using only one general set of temperature key points. Second, arbitrarily assigning experimental cutting conditions and accordingly retrieving the measured data to build a TEM has a great risk to cause fatal prediction error since the adopted cutting conditions as well as the induced corresponding thermal behaviors in a machine are usually different from those happened in actual machining applications. Third, there lacks a transparent, quick, and accurate TEM for future error compensation on machine tools. In view of the shortcomings of the existed TEMs, we propose a novel two-mode integral thermal error modelling approach based on a kernel machine learning scheme of the least absolute shrinkage and selection operator (Lasso) regression to solve the above three problems to promote the machining accuracy of a milling-turning machining center. Beside, simulation comparisons and validation experiments are carried out to verify the proposed thermal error modelling approach.

II. EXPERIMENTS

We choose a vertical-type CNC milling-turning machining center (shown in Fig. 1) as the target of TEC. This study is focused on minimizing of thermally induced errors at

the TCPs of milling and turning cuttings for this complex machine tool. All experiments were performed on this machine tool.

A. EXPERIMENTAL SETUP

The measurement units were primarily composed of temperature probes and displacement sensors. According to the potential rules of selecting temperature key points on a machine tool suggested by Ruijun *et al.* [21], We stamp a total of twelve PT-100 resistance thermometers (T1-T12) on the thermal key points of the machine tool, which are close to the main heat sources or capable of reflecting the temperature change of machine, as listed in Table 1 and shown in Fig.2. On the other hand, two non-contact eddy-current displacement sensors (D1 and D2, resolution: 0.1 μm, Model KD-2300, produced by KAMAN Co. Ltd.) clutched in the fixture are used to detect the spindle's displacement at TCP (represented by a test mandrel clamped at the leading end of the milling spindle) in the Z1-direction (up and down). And, another two eddy-current displacement sensors (D3 and D4, same model as previous) and assembles are used to detect the spindle's displacement at TCP (represented by a test mandrel clamped at the leading end of the turning spindle) in the Z2-direction (up and down). The positions of the above displacement sensors are listed in Table 1 and shown in Fig. 2.

B. EXPERIMENTAL CONDITIONS

A successful TEM is significantly affected by the adopted experimental cutting conditions which cannot be set in an arbitrary manner. They should be carefully set in the way of reflecting cutting situations as real as possible. Although there exist some ISO standards for testing a machine's final precision, the regulations about thermal precision issues including TEMs or experimental cutting
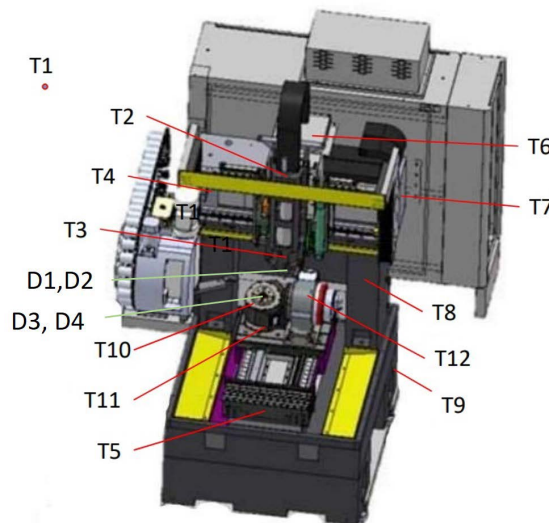
FIGURE 2. Temperature key points on target machine.

TABLE 1. Locations of temperature and displacement sensors.

Sensors	Locations	Sensors	Locations
T1	Environment	T7	Cross beam
T2	MS back end	T8	Vertical column
T3	MS front end	T9	Machine bed
T4	X-axis servo-motor	T10	TS front end
T5	Yaxis servo-motor	T11	TS back end
T6	Z-axis servo-motor	T12	RT motor
D1, D2	Leading end of milling spindle	D3, D4	Leading end of turning spindle

MS: MILLING SPINDLE, TS:TURNING SPINDLE, RT:ROTARY TABLE

conditions for TEM building still lack. Many scholars used simple cutting paths (such as a zig-zag planar motion or just turning without moving) or fixed rotational speed of spindle as the experimental conditions. These measures might not only induce improper temperature distributions and TCP displacements, but also lead to wrong TEMs. Eventually, a final bad compensation result appears in real-cutting tests.

In this study, we plan some off-line experimental cutting conditions that may simulate the on-line real cutting situations as close as possible. Two experimental cutting conditions, one for TEM building and the other for TEM verification, are planned as listed in Table 2 and 3, respectively. These experimental cutting conditions are set in the way of simulating real cutting conditions for making the metal shell of a cell phone (shown in Fig. 3).

C. MEASUREMENT RESULTS

The temperature and displacement sensors are integrated into one system so as to retrieve the temperature and deformation reading synchronically. The measurement results of temperature variations at different thermal key points are shown in Fig. 4. It is found from Fig. 4(b) that the temperature of machine may vary from the room temperature of 24°C to a maximum of 81°C at thermal key point T2. Overall, the temperatures at different thermal key points change markedly

FIGURE 1. Configuration of target CNC milling-turning machining center, Model S5, made by Zheng Feng CNC Technology Co. LTD.

TABLE 2. Experimental CUTTING conditions for tem building.

Steps	1.Coarse cutting	2.Tool changing	3.Coarse cutting	4.Tool changing	5.Fine cutting	6.Tool changing	7.Fine cutting	8.Tool changing	9.Grinding	10.Tool changing	11.Grinding
Running periods (min)	1st period 0-50	Stop 50-60	2nd period 60-140	Stop 140-150	3rd period 150-250	Stop 250-260	4th period 260-420	Stop 420-430	5th period 430-490	Stop 490-500	6th period 500-600
MS(rpm)	2000	0	2000	0	4000	0	4000	0	0	0	0
TS(rpm)	0	0	0	0	0	0	0	0	6000	6000	6000
X-FS(m/s)	15	0	15	0	15	0	15	0	15	0	15
Y-FS(m/s)	15	0	15	0	15	0	15	0	15	0	15
Z-FS(m/s)	30	0	30	0	30	0	30	0	30	0	30

MS: MILLING SPINDLE, TS: TURNING SPINDLE, FS: FEEDING SPEED

TABLE 3. Experimental CUTTING conditions for tem testing.

Steps	1.Coarse cutting	2.Tool changing	3.Coarse cutting	4.Tool changing	5.Fine cutting	6.Tool changing	7.Fine cutting	8.Tool changing	9.Grinding	10.Tool changing	11.Grinding
Running periods (min)	1st period 0-30	Stop 30-40	2nd period 40-80	Stop 80-90	3rd period 90-130	Stop 130-140	4th period 140-200	Stop 200-210	5th period 210-270	Stop 270-280	6th period 280-350
MS(rpm)	3000	0	3000	0	5000	0	5000	0	0	0	0
TS(rpm)	0	0	0	0	0	0	0	0	8000	8000	8000
X-FS(m/s)	30	0	20	0	10	0	10	0	15	0	15
Y-FS(m/s)	30	0	20	0	10	0	10	0	15	0	15
Z-FS(m/s)	35	0	25	0	15	0	15	0	20	0	20

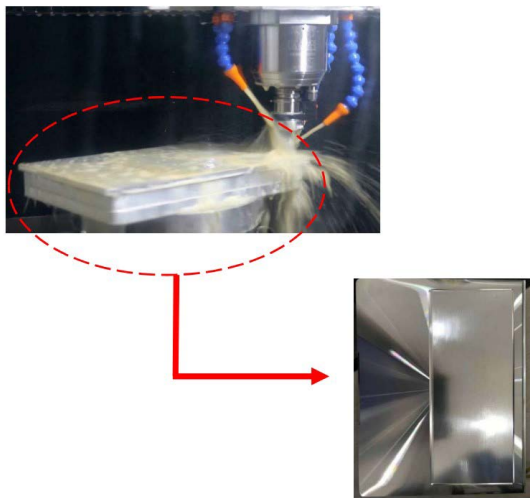MS: MILLING SPINDLE, TS:TURNING SPINDLE, FS: FEEDING SPEED

FIGURE 3. Real cutting conditions for making the metal shell of a cell phone.

with time. Meanwhile, the variation of the deformation with time at the tip of the milling as well as the turning spindles in the Z-axis direction are shown in Fig. 5. The thermal drift at the tip of the milling spindle reaches a maximum of 78.2 μm at about t=415 min. Similarly, the thermal drift at the tip of the turning spindle reaches a maximum of 61.5 μm at about t= 600 min.

III. TWO-MODE THERMAL ERROR MODELLING
We propose a two-goal and two-mode integral approach for modelling the thermal error. Two goals of TEM are related to the promotion of compensation performance in practical applications. First, it is desired that the temperature key points
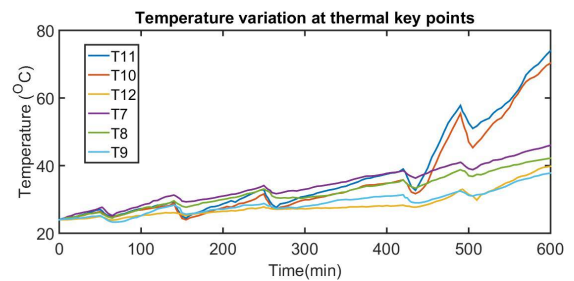
(a) Temperature variations at T7, T8, T9, T10, T11, and T12.
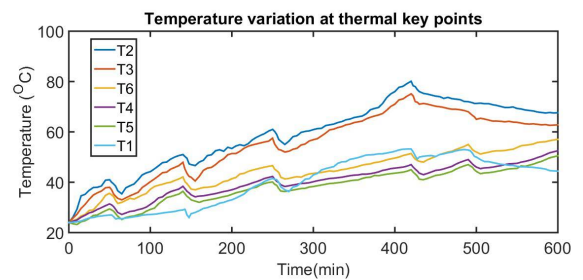
(b) Temperature variations at T1, T2, T3, T4, T5, and T6.

FIGURE 4. Temperature variations at thermal key points.

should be as less as possible for simple and easy manipulation of TEC. Therefore, we use the Lasso regression method to trim the redundant thermal key points. Second, in engineering application, it is preferred to use an explicit regression model rather than an implicit black-box model to deal with the non-linear relationship between thermal displacements at TCPs and temperatures at thermal key points. Therefore, we employ some outstanding machine learning schemes, such as the support vector regression and the random forest scheme, to solve this kind of problem. On the other hand,
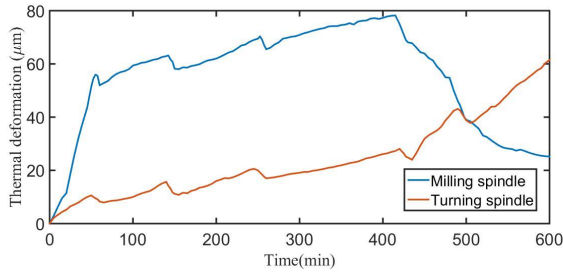
FIGURE 5. Thermal drifts of milling and turning spindles in the Z-axis direction.

two modes of TEM are proposed for accurately predicting the thermal error of a compound CNC machine tool that operates under the milling-cutting or the turning-cutting condition, or both conditions.

A. DATA REDUCTION

To optimally select and meanwhile reduce the thermal key points, we adopt the Lasso regression to optimally determine the weight coefficient for each thermal key point. Meanwhile, we also calculate the weight coefficient using the frequently-used ordinary least square (OLS) regression [22] for comparison. A general linear regression model can be expressed as

$$y_i = \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_j x_{ij} + \beta_p x_{ip} + \beta_0 + \varepsilon_i, \quad (1)$$

where y_i is the thermal displacement of the milling or turning spindle, x_{ij} is the temperature at the thermal key point, β_j is the weight coefficient of x_{ij}, and ε_i is the error.

Conventionally, we use OLS regression to estimate β_j. However, OLS regression has several drawbacks, such as the over-fitting problem and the limitation on dealing with multicollinearity, which may undermine the generalizability of the model. The multicollinearity of a linear regression function can be measured by the condition number, defined as:

$$\lim_{\varepsilon \to 0} \sup_{\|\Delta x\| \leq \varepsilon} \frac{\|\Delta y\|}{\|\Delta x\|}, \quad (2)$$

in which Δy means the change of the output value for the linear regression function, and Δx is a small change in the input parameter.

Therefore, we use the Lasso regression instead of OLS regression, in which it can effectively and properly deal with the multicollinearity problem. Lasso regression adopts the loss function as

$$\sum_{i=1}^{n} (y_i - \sum_{j}^{x_{ij}} \beta_j)^2 + \lambda \sum_{j=1}^{p} |\beta_j|, \quad (3)$$

where λ is the amount of shrinkage. In Eq. (3), the first term means the residual sum of squares and the second term stands for the penalty function. Lasso regression shrinks the input size of prediction model by minimizing the penalty function using least squares method. By this manipulation, the coefficient of unimportant features becomes zero and

the features with the corresponding non-zero coefficients are considered as the selected features [23], [24].

Based on the measured data of temperature variations at thermal key points and displacements of milling and turning spindles at TCPs, we may obtain the influence of each temperature at thermal key point on the thermal drifts of spindles at TCPs via OLS and Lasso regression methods.

1) MILLING CUTTING CASE

For the deformation of the milling spindle, firstly the obtained OLS regression result, expressed in terms of the weight coefficient for each temperature at thermal key point, with a mean-root-squared error of 0.942 and a condition number of 6390, is shown in Fig. 6a. The condition number of a linear regression function measures how much the output value of the function can change for a small change in the input parameter. Here, it is defined as the absolute error of the output function divided by the absolute error of Since the weight coefficient of T4 is only -0.22, very small compared to the other eleven weight coefficients, we deem that the influence of the temperature at T4 (X-axis motor) is negligible. Besides, owing to the high value of condition number in this case, which means there exists strong multi-collinearity among temperature variables, this method is apparently not suitable for estimating the weight coefficient of temperature at thermal key point. Alternatively, we adopt the Lasso regression method to overcome the multi-collinearity problem. Now, we analyze the same problem as previously stated. The obtained result (shown in Fig. 6b) reveals that, scare influence occurs for temperatures at T4 (weight value: 0) and T7 (weight value: 0) where are the thermal key points of cross beam and X-axis servo motor, respectively. The temperatures at other key points: T2, T3, T5, T6, T8, T9, T10, T11, and T12 are considered to be the influential input variables for further TEM works.

2) TURNING CUTTING CASE

On the other hand, for the turning spindle, by using OLS regression, we may obtain none of the temperature at the thermal key point can be neglected (shown in Fig. 7a), but the condition number is still as high as 6390, which means there exists strong multi-collinearity among temperature variables and we need other suitable scheme such as Lasso regression. Through the calculation by Lasso regression scheme, we may obtain a new set of weight coefficients for temperatures at thermal key points, as shown in Fig 7b. It is seen that only the temperatures at T2 and T11 with weight coefficients of 0.184 and 0.89, respectively need to be considered as the input variables for further TEM.

B. THERMAL ERROR MODEL BUILDING AND COMPARISON

In engineering applications, the less temperature key point, the better practical TEC can be achieved. Since we have already obtained nine influential temperature key points with respect to the milling-spindle deformation and two with respect to the turning-spindle deformation, as indicated
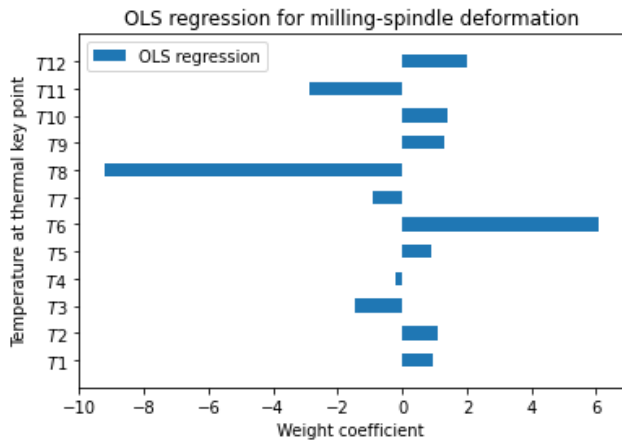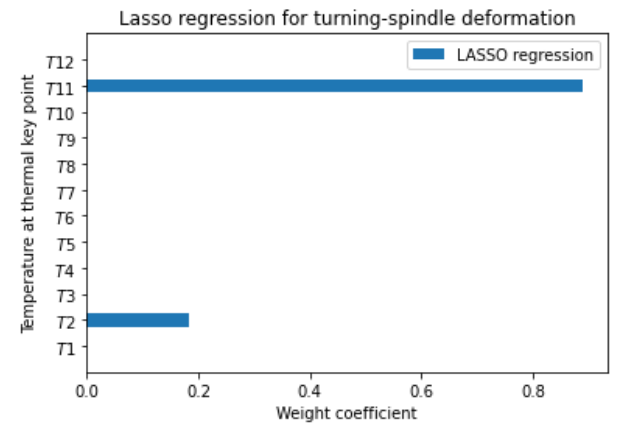
(a) Weight coefficients obtained by OLS.
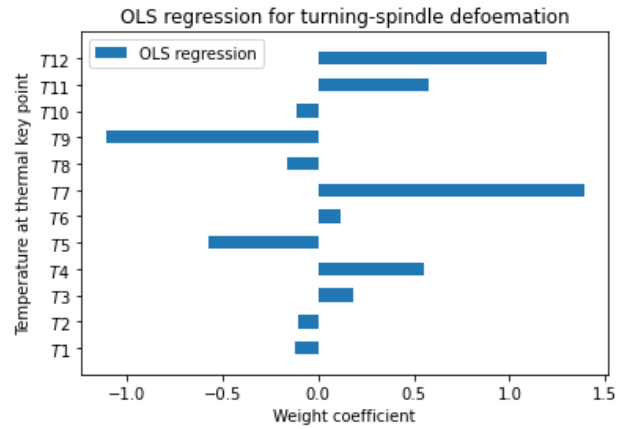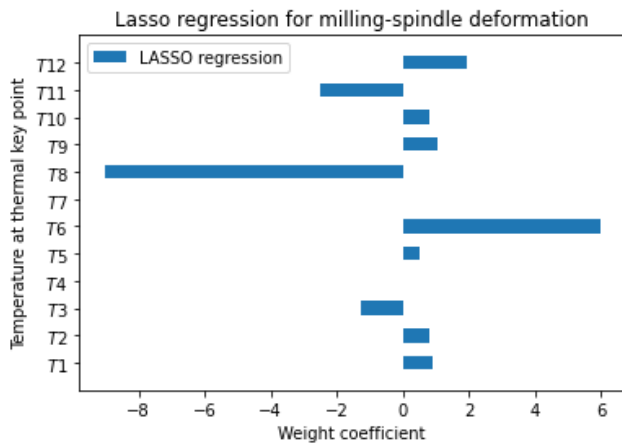
(b) Weight coefficients obtained by Lasso.

FIGURE 6. Influence of temperature at thermal key points on milling-spindle deformations using OLS regression.

(a) Weight coefficients obtained by OLS.

(b) Weight coefficients obtained by Lasso.

FIGURE 7. Influennce of temperatures at thermal key points on turning-spindle deformations using Lasso regression.

previously. After that we need to build an accurate TEM with suitable machine learning schemes to predict the deformations of the milling as well as the turning spindle. Recently, SVR and deep-learning neural network (DNN) schemes, known as good methods for dealing with highly non-linear mapping problems, were frequently used in the thermal error modeling problems of CNC machine tools [3], [18]. Therefore, to build a proper TEM of our complex milling-turning machine tool, we adopt SVR, DNN, and an excellent random forest (RF) methods to predict thermal deformations. Followings are the manipulation details of the thermal error modeling works using these schemes.

1) MODELLING VIA DNN

At the first stage, we carry out TEM via DNN method. Details of DNN can be found in [25].

In the case of predicting milling-spindle deformations, we initially select the known nine influential temperatures at thermal key points as the input variables and the deformation of the milling spindle as the output variable. Then, we construct a back-propagation DNN which has five layers with sequentially 10, 8, 6, 4, and 1 neuron in respective layer (from the input (1st) layer to the output (5th) layer), and using the rectified linear unit (ReLu) as the activation function. Through calculations, we obtain a convergence error of 0.004 after 10000 epochs of training. The calculation results of measured and predicted thermal deformations, and the error between them are shown in Fig. 8. A maximum prediction error of 8.34 μm is obtained using DNN scheme.

In the case of predicting turning-spindle deformations, we select the known two influential temperatures at thermal key points as the input variables and the deformation of the turning spindle as the output variable. Then, we construct a back-propagation DNN which has five layers with sequentially 2, 2, 2, 2, and 1 neuron in respective layer (from the input (1st) layer to the output (5th) layer), and using the rectified linear unit (ReLu) as the activation function. And, through calculations, we obtain a convergence error of 0.0049 after 10000 epochs of training. The calculation results of measured and predicted thermal deformations, and the error between them are shown in Fig. 9. A maximum prediction error of 2.72 μm is obtained using DNN scheme.
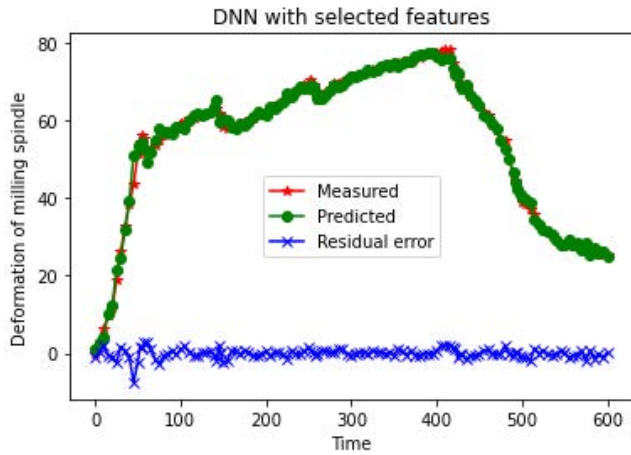
FIGURE 8. Thermal error prediction results by DNN with selected features for the milling-spindle deformation.

FIGURE 9. Thermal error prediction results by DNN with selected features for the turning-spindle deformation.

2) MODELLING VIA SVR

SVR is a superior machine learning scheme developed by Vapnik [26], which was successfully applied in different engineering applications [27], [28], [25], including the thermal error prediction problems for CNC machine tools [29]. The SVR attempts to minimize the error based on the principle of structural risk minimization (SRM). Generally speaking, a linear-regression problem involves the determination of a linear function

$$g(\mathbf{x}) = \mathbf{w} \cdot \mathbf{x} + b \tag{4}$$

that best interpolates the training data $\{(x_1, y_1), \ldots, (x_m, y_m)\}$. The goal of learning is to find a proper function that predicts the actual y data as close as possible with a deviation of ε. The task of SRM in SVR is to find a proper function $g(\mathbf{x})$ with ε as flat as possible, which can be determined by minimizing the Euclidean norm $|w|^2$. The minimization problem is constructed as:

$$\text{minimize } \frac{1}{2} |w|^2 + P \sum_{i=1}^{m} (\delta_i + \delta_i^*) \tag{5}$$

$$\text{subject to } \begin{cases} y_i - w \cdot x_i - b \leq \varepsilon + \delta_i \\ w \cdot x_i + b - y_i \leq \varepsilon + \delta_i^* \end{cases} \tag{6}$$

where P ($P > 0$) is the penalty factor that balances the empirical risk and model flatness, δ_i and δ_i^* means loose variables. The SVR achieves these by mapping the training patterns from the input space to a high-dimensional feature space where the original data can be separated by a linear function. The mapping function is expressed as

$$g(x) = \sum_{i=1}^{m} w_i K_i(x_i, x) + \theta \tag{7}$$

where w is the weight, K is the kernel mapping function, and θ is the offset. Using different forms of the kernel function will result in different regression deviations, depending on the data characteristics. Here, we adopt the radial basis function
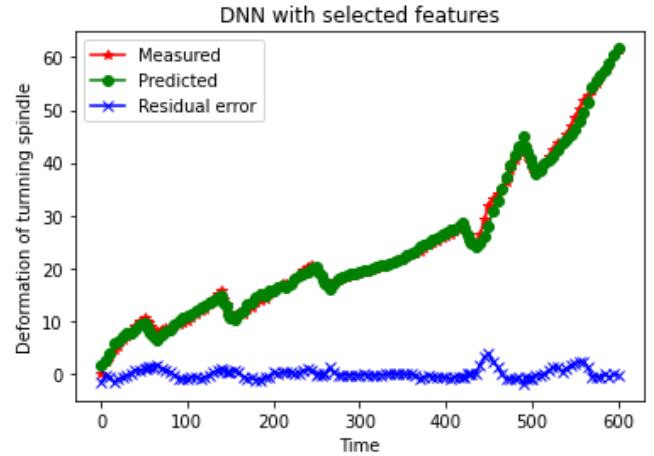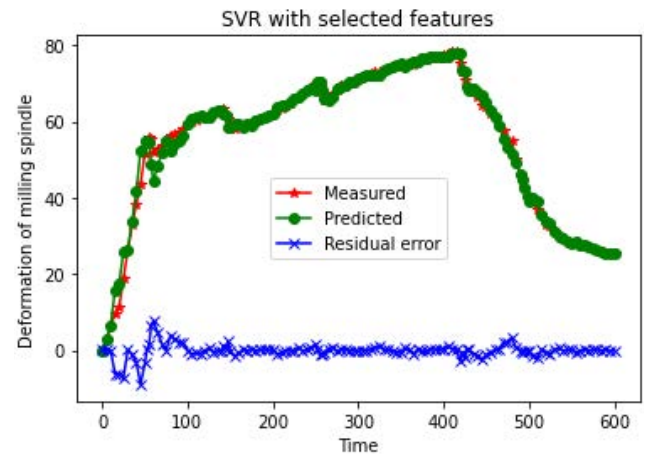

FIGURE 10. Thermal error prediction results by SVR with selected feature points for the milling-spindle deformation.

as the kernel

$$K(x, x') = \exp(-\frac{|x - x_i|^2}{2\gamma^2}) \tag{8}$$

where γ is the width of the radial basis function. The prediction accuracy of SVR model (Eq. (4) – (8)) is mainly determined by two hyper-parameters: P and γ. In the following thermal error modelling works using SVR, a frequently-used grid search method for these two hyper-parameters is adopted. The research range of P is set to $(1, 10, 20, \ldots, 1000)$ with spacing 10, respectively.

In the case of predicting milling-spindle deformations, we use the measured data to train SVR model, in which the 5-fold cross validation is adopted. The modelling result is shown in Fig. 10. We get an average accuracy of 97.3%. The try-and-error obtained optimal γ, θ, and P values are 0.1, -2.02 and 550, respectively. The maximum compensation error between the predicted and the measured spindle deformations is 8.98 μm which is a little bit larger than that obtained via DNN scheme.

In the case of predicting turning-spindle deformations, with the same SVR structure parameters and training settings

as those in the previous case, we may get the modelling results, as shown in Fig. 11. The obtained average accuracy is 99.4%. The try-and-error obtained optimal γ, θ, and P values are 0.2, -1.89 and 300, respectively. The maximum prediction error between the predicted and the measured spindle deformations is 4.79 μm which is larger than that obtained via DNN scheme.
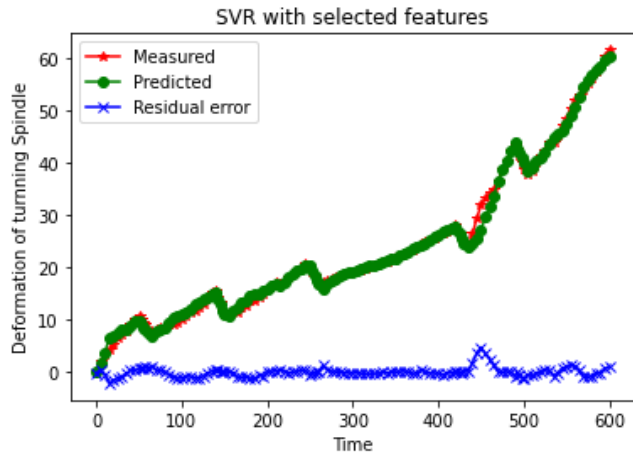

FIGURE 11. Thermal error prediction results by SVR with selected feature points for the turning-spindle deformation.

3) MODELLING VIA RF REGRESSION

The RF regression is a popular machine-learning scheme proposed by Breiman [27]. The RF scheme accumulates tree predictors associated with various random vectors. In the training process of RF regression, the overall output is obtained by averaging the output values of all individual trees. The learner bagging algorithm is used in RM regression for training any single tree [28]. The bootstrap samples of the training sets are repeatedly selected, and Gini impurity fits t_b trees in these samples.

The predicted values for unseen complexes are calculated via following equation:

$$y = \frac{1}{B} \sum_{b=1}^{B} t_b(x), \tag{9}$$

where y is the output and B is the tree number.

In the case of predicting milling-spindle deformations, we set the related parameters in RF regression scheme as: (1) the maximum tree depth: 15, and (2) the number of features: 6. Based on the measured data, we may train RF and get the result (shown in Fig. 12) as: (1) the average accuracy using 5-fold cross validation: 94.1%, (2) the maximum prediction error: 6.08 μm which is the best among all three regression schemes (8.98 μm for SVR and 8.34 μm for DNN) for predicting thermal deformations of the milling spindle. In other words, for the deformation prediction of a milling spindle in a CNC milling-turning machining center, RF regression has the best prediction ability among all three schemes.
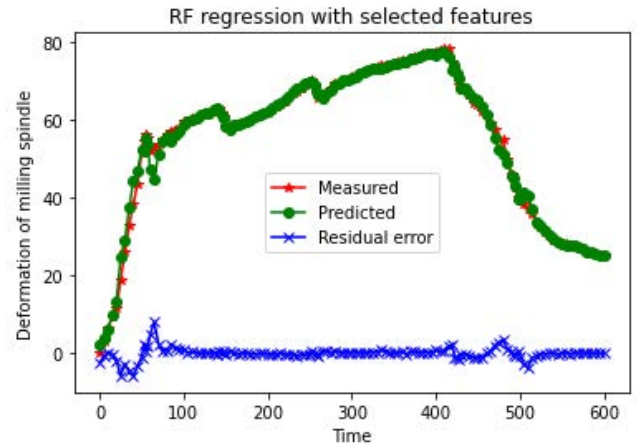

FIGURE 12. Thermal error prediction results by RF with selected feature points for the milling-spindle deformation.
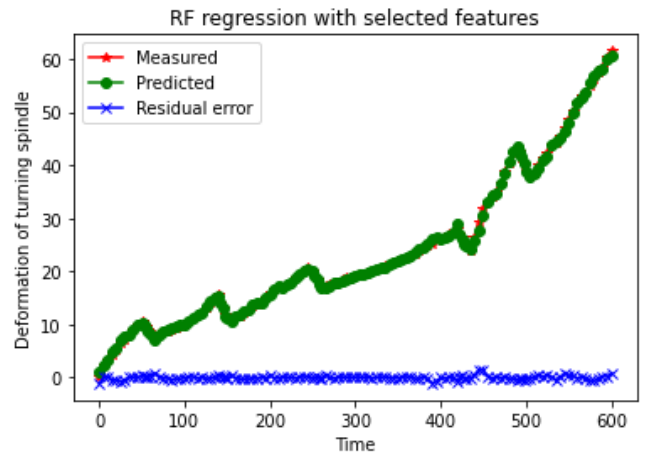

FIGURE 13. Thermal error prediction results by RF regression with selected feature points for the turning-spindle deformation.

On the other hand, in the case of predicting turning-spindle deformations, we may similarly set the related parameters of RF as: (1) the maximum tree depth: 10, and (2) the number of features: 2. Based on the measured data, we train RF and get the result (shown in Fig. 13) as: (1) the average accuracy using 5-fold cross validation: 99.4%, (2) the maximum compensation error: 1.455 μm which is the best among all three regression schemes (4.79 μm for SVR and 2.72 μm for DNN) for predicting thermal deformations of the turning spindle. In other words, for the deformation prediction of a turning spindle in a CNC milling-turning machining center, RF regression has the best prediction ability among all three schemes.

IV. COMPARISON DISCUSSION

For the sake of data reduction and well error compensation, seeking less temperature key points and accordingly building accurate prediction TEM have drawn much attention for the recent decade. However, different machine structures and internal mechanisms lead to different thermal behaviors at TCPs. For our target of a multi-function milling-turning

machine, it is obtained that different sets of thermal key points exist in the milling-cutting deformation (T2, T3, T5, T6, T8, T9, T10, T11, and T12) and in the turning-cutting deformation (T2 and T11). This phenomenon is noteworthy since past studies only report one set of thermal key points with respect to only milling or turning deformation for the target of a single-function machine. One set of thermal key points is not sufficient nor adequate to describe the thermal behaviors for a milling-turning machine tool.

A summary of comparisons between measured and predicted results via various proposed schemes is listed in Table 4. The rank order of average prediction accuracy for different TEMs are: RF (94.9%), DNN (92.5%), and SVR (90.4%). Apparently, the frequently-used SVR or DNN scheme predicts not so well compared to RF regression scheme in this study. On the basis of measured deformations of the milling spindle, the average maximal residual error via RF model (6.08 μm) has improvements of 32.3% better than that by SVR model (8.98 μm) and 27% than that by DNN (8.34 μm). On the other hand, for the turning spindle, the predicted errors by RF model (1.46 μm) has improvements of 69.6% better than that by SVR model (4.79 μm) and 27% than that by DNN (2.72 μm). These facts indicate that the prediction result via the TEM of Lasso-based RF regression scheme are satisfactory.

TABLE 4. comparison between measured and predicted results via various proposed schemes with selected feature points.

TEM	Meas.	DNN prediction	SVR prediction	RF prediction
Maximal thermal deformation at TCP	MS: 78.2 μm TS: 61.5 μm	MS:69.86 μm TS:64.22 μm	MS:69.22 μm TS:66.29 μm	MS:72.12 μm TS:62.96 μm
Maximal residual error	MS:- TS:-	MS:8.34 μm TS:2.72 μm	MS:8.98 μm TS:4.79 μm	MS:6.08 μm TS:1.46 μm
Prediction accuracy	MS:- TS:- Avg.:-	MS:89.3% TS:95.6% Avg.:92.5%	MS:88.5% TS:92.2% Avg.:90.4%	MS:92.2% TS:97.6% Avg.:94.9%

MS: MILLING SPINDLE, TS:TURNING SPINDLE

To further verify our proposed new TEM, we now perform another experiment. The new test condition is arranged as listed in Table 3 to simulate actual cutting conditions for a similar metal work piece of the cell phone shown in Fig. 1. We measure the temperature variations at two thermal key points of T2 and T3 as well as the thermal deformation of the turning spindle at TCP. The obtained maximum measured thermal deformation is 53.6 μm. On the basis of the measured temperatures, the maximum predictive deformations via trained DNN, SVR, and RF schemes are obtained as 54.978 μm, 58.489 μm, and 56.242 μm, respectively. In other words, after compensation with our proposed model, the maximum thermal error can be improved by 95.07%, 90.87%, and 97.43% via DNN, SVR, and RF models, respectively. The test results not only show that our proposed Lasso-based RF model performs the best among all three models, but also exhibits satisfactory prediction of thermal deformations for the CNC milling-turning machine tool.

V. CONCLUSION

In this study, we propose a two-mode integral TEM for well predicting the thermal deformations at TCPs of the milling as well as the turning spindles for a CNC compound milling-turning machining center. Due to structural complexities of the target machine, the traditional way of using only one set of thermal key points to establish one TEM for this machine that has simultaneously milling-cutting and turning-cutting functions is no longer suitable. At the stage of data reduction, we use Lasso regression to acquire two sets of thermal key points according to different thermal behaviors induced by milling-cutting and turning-cutting conditions. One set of T2, T3, T5, T6, T8, T9, T10, T11, and T12 is obtained for the milling-cutting condition, and the other set of T2 and T11 is obtained for the turning-cutting condition. At the stage of TEM building, we compare and obtain that RF regression model is the optimal TEM that has the best prediction ability among frequently-used OLS regression, SVR, and DNN models in the model building as well as the testing experiments. Furthermore, the proposed integral Lasso-based RF-TEM predicts well for the milling-spindle and the turning-spindle TCP deformations with maximum prediction errors of 1.455 μm and 6.08 μm, respectively. Recently, in the marketplace, CNC compound milling-turning machine tools are becoming hot in tool machinery. This paper provides an explicit (not a black box) and accurate thermal error modeling methodology to markedly reduce the thermal error induced in machining for this type of complicated machine tool. Moreover, this methodology may also be applied to solve the thermal error modelling problems for other CNC multi-function (multiple milling-cutting, multiple turning-cutting, grinding, etc.) machine tools.

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