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RESEARCH ARTICLE

Analysis of S-Box Based on Image Encryption Application Using Complex Fuzzy Credibility Frank Aggregation Operators

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ABSTRACT This article is about a criterion based on credibility complex fuzzy set (CCFS) to study the prevailing substitution boxes (S-box) and learn their properties to find out their suitability in image encryption applications. Also these criterion has its own properties which is discussed in detailed and on the basis of these properties we have to find the best optimal results and decide the suitability of an S-box to image encryption applications. S-box is the only components which produces the confusion in the every block cipher in the formation of image encryption. So, for this first we have to convert the matrix having color image using the nonlinear components and also using the proposed algebraic structure of credibility complex fuzzy set to find the best S-box for image encryption based on its criterion. The analyses show that the readings of GRAY S-box is very good for image data.

INDEX TERMS Fuzzy set, complex fuzzy set, fuzzy credibility numbers, Frank T-norm, S-Box.

I. INTRODUCTION

Multiple criteria group decision-making (MCGDM) issues is that of decision making problems, In which we have to find the best solution using various aggregation operators having alternatives and some criteria. There are many problems which is face in the real stage but we have try to solve these decision making problems using various aggregation operators and methods. As normally the decision making problems has unclear and uncertain data in the form of crisp set and there is some issues which is not discussed and solved by crisp set.

A. A BRIEF REVIEW ON THE DEVELOPMENT OF FUZZY SET

So to solve this type of issues Zadeh [1] define the generalize form of classical set (crisp set) which is called fuzzy set (FS) to deal with such kind of issues having uncertainties and uncertain information. After the detailed study of the MCGDM problems we conclude that there are three aspects (parts) which is in the case of MCGDM problems. 1) The first part is that we have to represent the information and the types of information. 2) The second part is that we have to collect the data having alternatives and criteria. 3) The third part is that we have to show the best alternatives of the MCGDM problems using some define score or accuracy function. So by using of basic properties and operational laws of FS the Song *et al.* [2] introduced its work and their application to real life problems. Merigó and Gil-Lafuente [3] explained the generalized aggregation operators under FS information.

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B. A BRIEF REVIEW ON THE DEVELOPMENT OF INTUITIONISTIC FUZZY SET

After defining the concept of fuzzy set, which has one degree called membership degree (MD) and that degree belong to the close interval zero and one. So there are some problems which is not deal by the fuzzy set and it has a lack of information. So to overcome this type of problems Atanassov [4] define the concept of intuitionistic fuzzy set (IFS) having two degrees called MD and non-membership degree (NMD) and that degrees belong to the close interval zero and one also having condition that their sum must belong to the close interval zero and one. Thao *et al.* [5] explained the similarity measure under IFS information and their application to real life decision making problems.

C. A BRIEF REVIEW ON THE DEVELOPMENT OF FUZZY CREDIBILITY SET

There are some problems using the fuzzy set and its generalize form which is discussed as above but there is some problems in which the degree of accuracy or credibility is not discussed by the previous ideas. So the Ye *et al.* [6] explained these types of problems in detail and that idea is name as fuzzy credibility set (FCS). As the fuzzy set is define as the grading system but that grades is only the one grade. After that the Atanassov explained the generalized form of FS called intuitionistic fuzzy set (IFS) having two degrees one is called membership degree and the other is called non-membership degree (NMD) having the condition that the sum of MD and NMD belong to the close interval zero and one. Beside these all of the information is correct but lack of some terms is not discussed which is the accuracy degree or credibility degree. The generalize form of IFS is called FCS but in the FCS we have also two degree one is called MD and the other is called degree of accuracy or credibility. Ye *et al.* [7] explained the series of aggregation operators under cubic FCS and their application to DMs problems.

D. A BRIEF REVIEW ON THE DEVELOPMENT OF COMPLEX FUZZY SET

The fuzzy set and its generalization only deal with uncertainty in data, but didnot explain at a given phase of time. But there are some problems in which there is a need of phase term and amplitude, So to discuss the phase of time and amplitude and membership degree range is exceed from real subset to a unit disc of the complex plane Ramote *et al.* [8] define a new set called complex fuzzy set (CFS). Furthermore the detailed about the CFS and its properties are discussed by Ramote *et al.* [9]. The correlation of CFS and Pythagorean Fuzzy set are explained by Dick *et al.* [10]. A concise outline of the CFS and logic are presented by Yazdanbakhsh and Dick [11]. Mehmood *et al.* [12] explained the similarity measure under bipolar complex fuzzy infromation. Garge *et al.* [13] explained a series aggregation operators under complex q-Rung orthopair fuzzy information and their application to DMs problems.

E. A BRIEF REVIEW ON THE DEVELOPMENT OF COMPLEX INTUITIONISTIC FUZZY SET

After the description about CFS there are some problems which is not solved by CFS because it contained more than one degree and having two degrees called MD and NMD which give us the idea of complex intuitionistic fuzzy set (CIFS) and explained by Alkouri and Salleh [14]. The distance measure, relations and composition of any two CIFS are discussed by Alkouri and Salleh [15]. The distance and entropy measure for complex intuitionistic fuzzy soft set are explained by Kumar and Bajaj [16]. In the solving of decision making problems there is a need of aggregation operators (Aops), So the Power AOps as well as the distance measure for the pair of CIFS are discussed by Rani and Garg [17], [18]. Under CIFS information the correlation measure was discussed by Garg and Rani [19]. Also Ali and Smarandache [21] extended the theory of CFS to complex neutrosophic sets.

F. A BRIEF REVIEW ON THE DEVELOPMENT OF FRANK t-norm AND t-conorm

The Frank t-norm and t-conorm [22] are a wide and adjustable family of continuously triangular norms that are important generalizations of the probabilistic and Lukasiewicz t-norm and t-conorm. Because the Frank t-norm and t-conorm both have a parameter, they are more adjustable in the information fusion process and are more suited to modeling DMs issues. Using Frank norms a series of power aggregation operators is discussed by Mahnaz *et al.* [23] and their application to DMs problems under CFS information. Yahya *et al.* [24] explained a series of aggregation operators on the basis of Frank norms and their application to DMs problems.

G. A BRIEF REVIEW ON THE DEVELOPMENT OF CoCoSo METHOD

The aggregation operators is used in the decision making problems which is basically a MCGDM problems and also will helpful to collect all the data in the form of aggregated data and give the final results. But the approach (methods) is another way to find the best result but can used for multi criteria decision making problems. Yazdani *et al.* [25] defined the Combined compromise solution (CoCoSo) method and their application to DMs problems.Using the CoCoSo method Qiyas *et al.* [26] discussed all the steps and their application to DMs problems. Karasan *et al.* [27] explained all the steps of CoCoSo method and their application to DMs problems. Wen *et al.* [28] explored the CoCoSo method in hesitant fuzzy linguistic environment. Wang *et al.* [29] discussed the CoCoSo method and theri application to real life problems.

As in light of above literature reveiw, in this paper we have discussed a series of aggregation operators on the basis of Frank norms for MCGDM problems and also we have discussed all the step for the CoCoSo method under complex fuzzy credibility (CFCS) information. Also there are steps in the MCGDM problems which are to represent the expert informations, after that we have to aggregate all the expert informations and lastly we have to apply the define score function to the aggregated values to display the final results. Also there are some novelty of this paper which is as follows.

1) The basic set operational laws of CFCS are consider and are extended to the operational laws of Frank norms for CFCS.

2) Also the aggregation operators on the basis of extended operational laws of Frank norms, we define the fuzzy credibility Frank average and geometric aggregation operators.

3) The CoCoSo method is discussed under CFCS information.

4) The S-box image encryption under various criteria is discussed in this paper and also to find the best image encryption. we have applied all the develop work in this case of real life decision making example.

H. MOTIVATION AND OBJECTIVE OF PROPOSED WORK

The concept of FS, IFS, FCS and CFCS has its own limitation and their application to many real life decision problems. The motivation of proposed model is given step by step in the whole manuscript. Now, we discuss some important objectives of this paper.

1) Some cases of the FS, IFS are failed in the real life decision making problems, because all of these has discussed only the MD and NMD. But here the main goal of this paper to discussed the MD and degree of accuracy (credibility). which mean that it provide more detailed about the membership degree for any decision making (DMs) problems.

2) The second goal of our proposed model is that we have discussed in detail about the MD and degree of credibility (accuracy). Which give us more detailed about the correctness (accuracy) of the MD. For example, if we assign (0.4, 0.7) and (0.5, 0.9) to any DMs problems, where the membership degrees is 0.4 and 0.5 and degree of credibility is 0.7 and 0.9 respectively, which shows that (0.5, 0.9) is better than (0.4, 0.7) because the degree of accuracy (credibility) of the second MD is greater than the first MD.

3) Our third objective is to construct a strong relationship between proposed models and MCGDM problems. We develop two novel algorithms to deal with the uncertainties in the data with multiple-attributes group decision making problems. We use different score function and accuracy function for the ranking of alternatives in MCGDM. It is interesting to note that both algorithms yield the same result.

4) We may take the Frank t-norm and t-conorm, which we will describe as the core operational laws, and create a series of aggregation operators based on their Frank operational laws, which will provide a high and broad range for decision making issues. Another feature is that the Frank t-norm and t-conorm are used to organize data in a certain format while also assisting in the removal of ambiguity and vagueness in operational laws.

5) The CoCoSo method a good way to find final results for any MADM problems and also the aggregation operators also help in the collection of experts information and after applied a well defined score function we have to find the best optimal results. So here we have discussed the aggregation operators as well as the technique for the finding of best optimal solutions.

6) The most significant purpose is to establish a solid connection between the proposed model and MCGDM problems. Under the influence of the suggested model's operator estimate approaches, we offer novel operators for determining MCGDM concerns. To illustrate the usefulness and effectiveness of the desired solution, a useful example applicable to select and study the S-box image encryption.

I. SETUP OF THE PAPER

The setup of our paper is that in the first section the introduction is presented and in the second and third section the operational laws and the proposed weighted average aggregation operator and its properties is displayed. The proposed ordered weighted average and hybrid weighted average aggregation operators is discussed in the section four and five respectively. In the section six and seven the algorithms and the real life example is discussed in detailed. Lastly in the section eight and nine we have talked about the comparison with other methods and conclusion of our work is presented.

J. EXISTING DEFINITIONS

This section contained some definitions which is helpful in the formation of new work in this paper. These existing definitions contained fuzzy set, intuitionistic fuzzy set, fuzzy credibility set, complex fuzzy set, complex intuitionistic fuzzy set and their operational laws.

K. FUZZY SET [1]

Let $X \neq \phi$. Then, the FS A in X is define and mathematically we can write as. $A = \{\langle x, a_{iA}(x)\rangle \,|x \in X\}$, where $a_{iA}(x)$ show the degree of membership and also $a_{iA}(x)$ contain to the close interval zero and one.

L. INTUITIONISTIC FUZZY SET [4]

Let $X \neq \phi$. Then, the IFS A in X is define and mathematically we can write as. $A = \{\langle x, a_{iA}(x), b_{iA}(x)\rangle \,|x \in X\}$, where $a_{iA}(x)$, $b_{iA}(x)$ show the degree of membership and degree of non-membership and also $a_{iA}(x)$, $b_{iA}(x)$ contain to the close interval zero and one and a condition that their sum must belong to the close interval zero and one.

M. COMPLEX FUZZY SET [8]

Let $X \neq \phi$. Then, the CFS A in X is define and mathematically we can write as. $A = \{\langle x, a_{iA}(x)\rangle \,|x \in X\}$, where $a_{iA}(x)$ show the degree of membership that is basically a complex membership degree and also $a_{iA}(x)$ contain in the unit circle of a complex plane and is of the form $z = a_{iA}(x)e^{i2\pi u_A(x)}$, where $i = \sqrt{-1}$ and $a_{iA}(x)$ belong to the close interval zero and one, also the value of the $e^{i2\pi}$ is real valued function.

N. COMPLEX INTUITIONISTIC FUZZY SET [14]

Let $X \neq \phi$. Then, the CIFS A in X is define and mathematically we can write as. $A = \{\langle x, a_{iA}(x), b_i(x)\rangle \,|x \in X\}$, where $a_{iA}(x), b_i(x)$ show the degree of membership and non-membership respectively that is basically a complex membership and non-membership degrees and also $a_{iA}(x)$, $b_i(x)$ contain in the unit circle of a complex plane and is of the form $z = a_{iA}(x)e^{i2\pi a_{iA}(x)}$, where $i = \sqrt{-1}$ and $a_{iA}(x)$ belong to the close interval zero and one, also the value of the $e^{i2\pi}$ is real valued function.

O. FUZZY CREDIBILITY SET [6]

Let $X \neq \phi$. Then, the FCS $\tilde{I}$ is defined and mathematically we can write as. $\tilde{I} = \{\langle x, a_{iA}(x), b_{iA}(x)\rangle | x \in X\}$, where $a_{iA}(x) \to [0,1]$, $b_{iA}(x) \to [0,1]$ are the membership function of x in $\tilde{I}$ and the degree of credibility related to $a_{iA}(t)$ respectively, Then the pair $(x, a_{iA}(x), b_{iA}(x))$ are called fuzzy credibility numbers (FCNs).

P. COMPLEX FUZZY CREDIBILITY SET

Let $X \neq \phi$. Then, the CFCS A in X is define and mathematically we can write as. $A = \{\langle x, a_{iA}(x)e^{i2\pi a_{iA}(x)}, b_{iA}(x)e^{i2\pi b_{iA}(x)}\rangle | x \in X\}$, where $a_{iA}(x)e^{i2\pi a_{iA}(x)}$, $b_{iA}(x)e^{i2\pi b_{iA}(x)}$ show the complex membership degree and degree of credibility and also $a_{iA}(x)e^{i2\pi a_{iA}(x)}$, $b_{iA}(x)e^{i2\pi b_{iA}(x)}$ contain in the unit circle of a complex plane and is of the form $z = a_{iA}(x)e^{i2\pi a_{iA}(x)}$, $b_{iA}(x)e^{i2\pi b_{iA}(x)}$ where $i = \sqrt{-1}$ and $a_{iA}(x)e^{i2\pi a_{iA}(x)}$, $b_{iA}(x)e^{i2\pi b_{iA}(x)}$ belong to the close interval zero and one.

Q. OPERATIONAL LAWS OF COMPLEX FUZZY CREDIBILITY SET

Here the proposed properties of related CFCS, Let we have two CFCNs $s_i = (a_{iA}e^{i2\pi a_{iA}(x)}, b_{iA}e^{i2\pi b_{iA}(x)})$ then we write as,

1) $\alpha_1 \oplus^* \alpha_2 = (a_1e^{i2\pi a_1} + a_2e^{i2\pi a_2} - a_1e^{i2\pi a_1}a_2e^{i2\pi a_2}, b_1e^{i2\pi b_1} + b_2e^{i2\pi b_2} - b_1e^{i2\pi b_1}b_2e^{i2\pi b_2})$
2) $\alpha_1 \otimes^* \alpha_2 = (a_1e^{i2\pi a_1}a_2e^{i2\pi a_2}, b_1e^{i2\pi b_1}b_2e^{i2\pi b_2})$
3) $\lambda^{\alpha^*} = (1-(1-a_ie^{i2\pi a_i})^\lambda, b_i^\lambda e^{i2\pi b_i})$, for $\lambda > 0$.
4) $\alpha^{*\lambda} = ((a_ie^{i2\pi a_i})^\lambda, 1-(1-b_ie^{i2\pi b_i})^\lambda)$, for $\lambda > 0$.

R. SCORE FUNCTION FOR CFCNs

Let we have CFCNs which is denoted by $s_i = (a_{iA}e^{i2\pi a_{iA}(x)}, b_{iA}e^{i2\pi b_{iA}(x)})$, Then the score function can be defined as, $S(s_i) = [a_ie^{i2\pi a_{iA}(x)}b_ie^{i2\pi b_{iA}(x)} + (a_ie^{i2\pi a_{iA}(x)} + b_ie^{i2\pi b_{iA}(x)})/2]/2$.

S. NEW SCORE FUNCTION FOR CoCoSo METHOD

Let we have CFCNs which is denoted by $s_i = (a_{iA}e^{i2\pi a_{iA}(x)}, b_{iA}e^{i2\pi b_{iA}(x)})$, Then the new score function can be defined as, $S(s_i) = [a_ie^{i2\pi a_{iA}(x)} - b_ie^{i2\pi b_{iA}(x)} + 1 + \frac{e^{a_ie^{i2\pi a_{iA}(x)} - b_ie^{i2\pi b_{iA}(x)}} - 1}{1+\pi}]$. Where the degree of hesistancy is define as follows, $\pi = \sqrt{1 - a_i - b_i}$. $S(s_i) \in [e^{-1}, 2+e]$

T. OPERATIONAL LAWS OF COMPLEX FUZZY CREDIBILITY NUMBERS

Here, we proposed some properties of related CFCNs. Let we have two CFCNs then we write as,

1) $\alpha_1 \oplus^* \alpha_2 = \left(\begin{matrix} u_1e^{i2\pi u_1} + u_2e^{i2\pi u_2} - u_1e^{i2\pi u_1}u_2e^{i2\pi u_2}, \\ c_1e^{i2\pi \upsilon_1} + c_2e^{i2\pi \upsilon_2} - c_1e^{i2\pi \upsilon_1}c_2e^{i2\pi \upsilon_2} \end{matrix}\right)$.
2) $\alpha_1 \otimes^* \alpha_2 = (u_1e^{i2\pi u_1}u_2e^{i2\pi u_2}, c_1e^{i2\pi \upsilon_1}c_2e^{i2\pi \upsilon_2})$.
3) $\lambda^{\alpha^*} = (1-(1-u_ie^{i2\pi u_i})^\lambda, c_i^\lambda e^{i2\pi \upsilon_i})$, for $\lambda > 0$.
4) $\alpha^{*\lambda} = ((u_ie^{i2\pi u_i})^\lambda, 1-(1-c_ie^{i2\pi \upsilon_i})^\lambda)$, for $\lambda > 0$.

II. FRANK AGGREGATION OPERATORS FOR COMPLEX FUZZY CREDIBILITY NUMBERS

Using the basic operational laws of Frank t-norm and t-conorm, we have define a series of aggregation operators like weighted average aggregation operators, in which we have to add the corresponding alternatives and criteria having the corresponding weights. These aggregation operators are namely complex credibility fuzzy weighted aggregation operators which will helpful in the collections of more than one decision matrix.

A. FRANK OPERATIONAL LAWS FOR COMPLEX FUZZY CREDIBILITY SET

The properties and operational laws of Frank t-norm and t-conorm are discussed in detailed which are addition, multiplication, scalar multiplication and some power scalar multiplication. These properties are helpful to define a new aggregation operators, where $\lambda > 1$.

1) $$\alpha_1 \oplus^* \alpha_2 = \left\{\begin{matrix} 1-\log_\lambda\left(1+\frac{(\lambda^{1-a_i}-1)^{w_i}}{\lambda-1}\right) \\ e^{i2\pi\, 1-\log_\lambda\left(1+\frac{(\lambda^{1-a_i}-1)^{w_i}}{\lambda-1}\right)}, \\ \log_\lambda\left(1+\frac{(\lambda^{b_i}-1)^{w_i}}{\lambda-1}\right) \\ e^{i2\pi\, \log_\lambda\left(1+\frac{(\lambda^{b_i}-1)^{w_i}}{\lambda-1}\right)} \end{matrix}\right\},$$

2) $$\alpha_1 \otimes^* \alpha_2 = \left\{\begin{matrix} \log_\lambda\left(1+\frac{(\lambda^{a_i}-1)^{w_i}}{\lambda-1}\right) \\ e^{i2\pi\, \log_\lambda\left(1+\frac{(\lambda^{a_i}-1)^{w_i}}{\lambda-1}\right)}, \\ 1-\log_\lambda\left(1+\frac{(\lambda^{1-b_i}-1)^{w_i}}{\lambda-1}\right) \\ e^{i2\pi\, 1-\log_\lambda\left(1+\frac{(\lambda^{1-b_i}-1)^{w_i}}{\lambda-1}\right)} \end{matrix}\right\},$$

3) $$\lambda.\alpha_1^* = \left\{\begin{matrix} 1-\log_\lambda\left(1+\frac{(\lambda^{1-a_i}-1)^{w_i}}{(\lambda-1)^{w_i-1}}\right) \\ e^{i2\pi\, 1-\log_\lambda\left(1+\frac{(\lambda^{1-a_i}-1)^{w_i}}{(\lambda-1)^{w_i-1}}\right)}, \\ \log_\lambda\left(1+\frac{(\lambda^{b_i}-1)^{w_i}}{(\lambda-1)^{w_i-1}}\right) \\ e^{i2\pi\, \log_\lambda\left(1+\frac{(\lambda^{b_i}-1)^{w_i}}{(\lambda-1)^{w_i-1}}\right)} \end{matrix}\right\},$$

4) $$\alpha_1^{*\lambda} = \left\{\begin{matrix} \log_\lambda\left(1+\frac{(\lambda^{a_i}-1)^{w_i}}{(\lambda-1)^{w_i-1}}\right) \\ e^{i2\pi\, \log_\lambda\left(1+\frac{(\lambda^{a_i}-1)^{w_i}}{(\lambda-1)^{w_i-1}}\right)}, \\ 1-\log_\lambda\left(1+\frac{(\lambda^{1-b_i}-1)^{w_i}}{(\lambda-1)^{w_i-1}}\right) \\ e^{i2\pi\, 1-\log_\lambda\left(1+\frac{(\lambda^{1-b_i}-1)^{w_i}}{(\lambda-1)^{w_i-1}}\right)} \end{matrix}\right\}.$$

III. COMPLEX FUZZY CREDIBILITY FRANK AVERAGING AGGREGATION OPERATORS

Consider we have a mapping which is defined as $\alpha^n \rightarrow \alpha$, then this mapping is said to be CFCFWA operator, and these weighted aggregation operators is defined on the basis of Frank norm having weight vector $w = (w_1, w_2, \ldots, w_n)^T$, with $\Sigma_{i=1}^n w_i = 1$ and $w_i \in [0, 1]$.

$$CFCFWA(s_1, s_2, \ldots, s_n) = \oplus_{i=1}^n s_i e^{i2\pi} w_i$$

$$\begin{aligned} &CFCFWA(s_1, s_2, \ldots, s_n) = \left\{\oplus_{i=1}^n s_i e^{i2\pi} w_i\right\} \\ &= \left\{ \begin{array}{c} 1-\log_\lambda 1+\Pi_{i=1}^n(\lambda^{1-a_i}-1)^{\omega_i} \\ e^{i2\pi\left(1-\log_\lambda 1+\Pi_{i=1}^n(\lambda^{1-a_i}-1)^{\omega_i}\right)}, \\ \log_\lambda 1+\Pi_{i=1}^n(\lambda^{b_i}-1)^{\omega_i} e^{i2\pi\left(\log_\lambda 1+\Pi_{i=1}^n(\lambda^{b_i}-1)^{\omega_i}\right)} \end{array} \right\}. \end{aligned} \tag{1}$$

In the next theorem we prove that the Equ. (1) is also a CFCNs and the aggregated value of CFCNs $\vartheta = \{s_i, i = 1, 2, \ldots, n\}$ is also a CFCNs.

Theorem 1: Let we have a family of CFCNs that is denoted as $\vartheta = \{s_i, i = 1, 2, \ldots, n\}$, having weight vector like $w = (w_1, w_2, \ldots, w_n)^T$. So, the aggregated values of the proposed aggregation operators is still a CFCNs.

$$\begin{aligned} &CFCFWA(s_1, s_2, \ldots, s_n) \\ &= \left\{\oplus_{i=1}^n s_i e^{i2\pi} w_i\right\} \\ &= \left\{ \begin{array}{c} 1-\log_\lambda 1+\Pi_{i=1}^n(\lambda^{1-a_i}-1)^{\omega_i} \\ e^{i2\pi\left(1-\log_\lambda 1+\Pi_{i=1}^n(\lambda^{1-a_i}-1)^{\omega_i}\right)}, \\ \log_\lambda 1+\Pi_{i=1}^n(\lambda^{b_i}-1)^{\omega_i} \\ e^{i2\pi\left(\log_\lambda 1+\Pi_{i=1}^n(\lambda^{b_i}-1)^{\omega_i}\right)} \end{array} \right\} \end{aligned}$$

Proof: Using mathematical induction we have to prove the above theorem, using $n = 2$,

$$\begin{aligned} &s_1 + s_2 \\ &= \left\{ \begin{array}{c} 1-\log_\lambda(\lambda^{1-a_1}-1)^{\omega_1}(\lambda^{1-a_2}-1)^{\omega_2} \\ e^{i2\pi\left(1-\log_\lambda(\lambda^{1-a_1}-1)^{\omega_1}(\lambda^{1-a_2}-1)^{\omega_2}\right)}, \\ \log_\lambda 1+(\lambda^{b_1}-1)^{\omega_1}(\lambda^{b_2}-1)^{\omega_2} \\ e^{i2\pi\left(\log_\lambda(\lambda^{b_1}-1)^{\omega_1}(\lambda^{b_2}-1)^{\omega_2}\right)}. \end{array} \right\} \\ &= \left\{ \begin{array}{c} 1-\log_\lambda 1+\Pi_{i=1}^n(\lambda^{1-a_i}-1)^{\omega_i} \\ e^{i2\pi\left(1-\log_\lambda 1+\Pi_{i=1}^n(\lambda^{1-a_i}-1)^{\omega_i}\right)}, \\ \log_\lambda 1+\Pi_{i=1}^n(\lambda^{b_i}-1)^{\omega_i} \\ e^{i2\pi\left(\log_\lambda 1+\Pi_{i=1}^n(\lambda^{b_i}-1)^{\omega_i}\right)}. \end{array} \right\} \end{aligned}$$

Let $n = k$, then

$$\begin{aligned} &CFCFWA(s_1, s_2) \\ &= \left\{\oplus_{i=1}^2 s_i e^{i2\pi} w_i\right\} \\ &= \left\{ \begin{array}{c} 1-\log_\lambda 1+\Pi_{i=1}^k(\lambda^{1-a_k}-1)^{\omega_i} \\ e^{i2\pi\left(1-\log_\lambda 1+\Pi_{i=1}^k(\lambda^{1-a_k}-1)^{\omega_i}\right)}, \\ \log_\lambda 1+\Pi_{i=1}^k(\lambda^{b_k}-1)^{\omega_i} \\ e^{i2\pi\left(\log_\lambda 1+\Pi_{i=1}^k(\lambda^{b_k}-1)^{\omega_i}\right)}. \end{array} \right\} \end{aligned}$$

Further, we check for $n = k + 1$, we have,

$$\begin{aligned} &CFCFWA(s_1, s_2, \ldots, s_{k+1}) \\ &= \left\{ \begin{array}{c} 1-\log_\lambda 1+\Pi_{i=1}^k(\lambda^{1-a_k}-1)^{\omega_i} \\ e^{i2\pi\left(1-\log_\lambda 1+\Pi_{i=1}^k(\lambda^{1-a_k}-1)^{\omega_i}\right)}, \\ 1-\log_\lambda 1+\Pi_{i=1}^{k+1}(\lambda^{1-a_{k+1}}-1)^{\omega_i} \\ e^{i2\pi\left(1-\log_\lambda 1+\Pi_{i=1}^{k+1}(\lambda^{1-a_{k+1}}-1)^{\omega_i}\right)}, \\ \log_\lambda 1+\Pi_{i=1}^k(\lambda^{b_k}-1)^{\omega_i} \\ e^{i2\pi\left(1-\log_\lambda 1+\Pi_{i=1}^k(\lambda^{b_k}-1)^{\omega_i}\right)}, \\ \log_\lambda 1+\Pi_{i=1}^{k+1}(\lambda^{b_{k+1}}-1)^{\omega_i} \\ e^{i2\pi\left(\log_\lambda 1+\Pi_{i=1}^{k+1}(\lambda^{b_{k+1}}-1)^{\omega_i}\right)}. \end{array} \right\} \end{aligned}$$

The result is true for $n = k + 1$ and hence true for $n \geq 1$.

Now, there are some basic properties which are discussed in detailed for the proposed aggregation operators that is idempotency, monotonicity and boundedness.

A. IDEMPOTENCY PROPERTY

Let we have a family of CFCNs that is denoted as $\vartheta = \{s_i e^{i2\pi}, i = 1, 2, \ldots, n\}$, having weight vector like $w = (w_1, w_2, \ldots, w_n)^T$. Then, we can write the idempotency property as and can exists, but here we have a family of CFCNs that is $s_i e^{i2\pi} = s e^{i2\pi} = (u e^{i2\pi}, c e^{i2\pi})\ \forall i = 1, \ldots, n$.

$$CFCFWA(s_1, s_2, \ldots, s_n)e^{i2\pi} = s e^{i2\pi}$$

B. BOUNDEDNESS PROPERTY

Let we have a family of CFCNs that is denoted as $\vartheta = \{s_i e^{i2\pi}, i = 1, 2, \ldots, n\}$, having weight vector like $w = (w_1, w_2, \ldots, w_n)^T$. Then, we can write the idempotency property as and can exists, but here we have $s_i e^{i2\pi}\ (i = 1, \ldots, n)^-$ is a family of CFCNs then a set $s_{\min} e^{i2\pi} = \left(\min_i u_i e^{i2\pi}, \max_i c_i e^{i2\pi}\right)$ and $s_{\max} = \left(\max_i u_i e^{i2\pi}, \min_i c_i e^{i2\pi}\right)$ are the maximum and minimum CFCNs.

$$s_{\min} e^{i2\pi} \leq CFCFWA(s_1, s_2, \ldots, s_n)e^{i2\pi} \leq s_{\max} e^{i2\pi}$$

C. MONOTONICITY PROPERTY

Let we have a family of CFCNs that is denoted as $\vartheta = \{s_i e^{i2\pi}, i = 1, 2, \ldots, n\}$, having weight vector like $w = (w_1, w_2, \ldots, w_n)^T$. Then, we can write the idempotency property as and can exists, but here we have $s_i e^{i2\pi}\ (i = 1, \ldots, n)$, $s_i^* e^{i2\pi}\ (i = 1, \ldots, n)$ be the family of CFCNs such that $s_i e^{i2\pi} \leq s_i^* e^{i2\pi}$,

$$\begin{aligned} &CFCFWA(s_1, s_2, \ldots, s_n)e^{i2\pi} \\ &\qquad \leq CFCFWA(s_1^*, s_2^*, \ldots, s_n^*)e^{i2\pi} \end{aligned}$$

Proof: We have a family of $s_i e^{i2\pi} = se^{i2\pi} = (ue^{i2\pi}, ce^{i2\pi})$ $(i = 1, \ldots, n)$. Then,

$$CFCFWA(s_1, s_2, \ldots, s_n)e^{i2\pi} = \left\langle \oplus_{i=1}^{n} s_i e^{i2\pi} w_i \right\rangle = \left\{ \begin{array}{c} 1 - \log_\lambda 1 + \Pi_{i=1}^{n} (\lambda^{1-a_i} - 1)^{\omega_i} \\ e^{i2\pi\left(1-\log_\lambda 1+\Pi_{i=1}^{n}(\lambda^{1-a_i}-1)^{\omega_i}\right)}, \\ \log_\lambda 1 + \Pi_{i=1}^{n} (\lambda^{b_i} - 1)^{\omega_i} \\ e^{i2\pi\left(\log_\lambda 1+\Pi_{i=1}^{n}(\lambda^{b_i}-1)^{\omega_i}\right)} \end{array} \right\}$$

For all i. Therefore,

$$= \left\{ \begin{array}{c} 1 - \log_\lambda 1 + \Pi_{i=1}^{n} (\lambda^{1-a_i} - 1)^{\omega_i} \\ e^{i2\pi\left(1-\log_\lambda 1+\Pi_{i=1}^{n}(\lambda^{1-a_i}-1)^{\omega_i}\right)}, \\ \log_\lambda 1 + \Pi_{i=1}^{n} (\lambda^{b_i} - 1)^{\omega_i} \\ e^{i2\pi\left(\log_\lambda 1+\Pi_{i=1}^{n}(\lambda^{b_i}-1)^{\omega_i}\right)} \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left(1 - (1-u)^{\sum_{k=1}^{n} p_k}\right) \\ \left(1 - (1-e^{i2\pi(u)})^{\sum_{k=1}^{n} p_k}\right), \\ \left((1 - (1-c)^{\sum_{k=1}^{n} p_k}\right) \\ \left((1 - (1-e^{i2\pi(v)})^{\sum_{k=1}^{n} p_k}\right) \end{array} \right\}$$

$$= \left(1 - (1 - ue^{i2\pi}), 1 - (1 - ce^{i2\pi})\right)$$

$$= (ue^{i2\pi}, ce^{i2\pi}) = se^{i2\pi}$$

Thus we have,

$$CFCFWA(s_1, s_2, \ldots, s_n)e^{i2\pi} = se^{i2\pi}$$

We have $s_{\min}e^{i2\pi}, s_{\max}e^{i2\pi}$ are the maximum and minimum CFCNs, as $s_{\min}e^{i2\pi} \le s_i e^{i2\pi} \le s_{\max}e^{i2\pi}$. Then,

$$\oplus_{i=1}^{n} s_{\min}e^{i2\pi} w_i \le \oplus_{i=1}^{n} s_i e^{i2\pi} w_i \le \oplus_{i=1}^{n} w_i s_{\max} e^{i2\pi}$$

is exist and now we have,

$$s_{\min}e^{i2\pi} \le \oplus_{i=1}^{n} s_i e^{i2\pi} w_i \le s_{\max}e^{i2\pi}$$

which implies that,

$$s_{\min}e^{i2\pi} \le CFCFWA(s_1, s_2, \ldots, s_n)e^{i2\pi} \le s_{\max}e^{i2\pi}$$

If we have a family of $s_i \le s_i^*, \oplus_{i=1}^{n} s_i w_i \le \oplus_{i=1}^{n} s_i^* w_i^*$. Then,

$$CFCFWA(s_1, s_2, \ldots, s_n)e^{i2\pi} \le CFCFWA(s_1^*, s_2^*, \ldots, s_n^*)e^{i2\pi}$$

Which is the required prove of the property.

IV. COMPLEX FUZZY CREDIBILITY FRANK ORDERED AVERAGING AGGREGATION OPERATORS

Consider we have a mapping which is defined as $\alpha^n \to \alpha$, then this mapping is said to be CFCFOWA operator, and these weighted aggregation operators is defined on the basis of Frank norms having following format having weight vector that is represented by $w = (w_1, w_2, \ldots, w_n)^T$, with $\Sigma_{i=1}^{n} w_i = 1$ and $w_i \in [0, 1]$.

$$CFCFOWA(s_{\sigma_1}, s_{\sigma_2}, \ldots, s_{\sigma_n}) = \oplus_{i=1}^{n} s_{\sigma_i} e^{i2\pi} w_i.$$

$$CFCFOWA(s_1, s_2, \ldots, s_n) = \left\{ \oplus_{i=1}^{n} s_{\sigma_i} e^{i2\pi} w_i \right\} = \left\{ \begin{array}{c} 1 - \log_\lambda 1 + \Pi_{i=1}^{n} (\lambda^{1-a_{\sigma_i}} - 1)^{\omega_i} \\ e^{i2\pi\left(1-\log_\lambda 1+\Pi_{i=1}^{n}(\lambda^{1-a_{\sigma_i}}-1)^{\omega_i}\right)}, \\ \log_\lambda 1 + \Pi_{i=1}^{n} (\lambda^{b_{\sigma_i}} - 1)^{\omega_i} \\ e^{i2\pi\left(\log_\lambda 1+\Pi_{i=1}^{n}(\lambda^{b_{\sigma_i}}-1)^{\omega_i}\right)} \end{array} \right\}, \quad (2)$$

In the next theorem we prove that the equation (2) is also a CFCNs and the aggregated value of CFCNs $\vartheta = \{s_{\sigma_i}, i = 1, 2, \ldots, n\}$ is also a CFCNs.

Theorem 2: Let we have a family of CFCNs that is denoted as $\vartheta = \{s_{\sigma_i}, i = 1, 2, \ldots, n\}$, having weight vector like $w = (w_1, w_2, \ldots, w_n)^T$. So, the aggregated values of the proposed aggregation operators is still a CFCNs.

$$CFCFOWA(s_1, s_2, \ldots, s_n) = \left\{ \oplus_{i=1}^{n} s_{\sigma_i} e^{i2\pi} w_i \right\} = \left\{ \begin{array}{c} 1 - \log_\lambda 1 + \Pi_{i=1}^{n} (\lambda^{1-a_{\sigma_i}} - 1)^{\omega_i} \\ e^{i2\pi\left(1-\log_\lambda 1+\Pi_{i=1}^{n}(\lambda^{1-a_{\sigma_i}}-1)^{\omega_i}\right)}, \\ \log_\lambda 1 + \Pi_{i=1}^{n} (\lambda^{b_{\sigma_i}} - 1)^{\omega_i} \\ e^{i2\pi\left(\log_\lambda 1+\Pi_{i=1}^{n}(\lambda^{b_{\sigma_i}}-1)^{\omega_i}\right)} \end{array} \right\}.$$

Proof: The proof is obvious.

Now there are some basic properties which are discussed in detailed for the proposed aggregation operators that is idempotency, monotonicity and boundedness.

A. IDEMPOTENCY PROPERTY

Let we have a family of CFCNs that is denoted as $\vartheta = \{s_{\sigma_i}e^{i2\pi}, i = 1, 2, \ldots, n\}$, having weight vector like $w = (w_1, w_2, \ldots, w_n)^T$. Then, we can write the idempotency property as and can exists, but here we have a family of CFCNs that is $s_{\sigma_i}e^{i2\pi} = s_\sigma e^{i2\pi} = (ue^{i2\pi}, ce^{i2\pi})$ $\forall i = 1, \ldots, n$.

$$CFCFOWA(s_{\sigma_1}, s_{\sigma_2}, \ldots, s_{\sigma_n})e^{i2\pi} = s_\sigma e^{i2\pi}$$

B. BOUNDEDNESS PROPERTY

Let we have a family of CFCNs that is denoted as $\vartheta = \{s_{\sigma i}e^{i2\pi}, i = 1, 2, \ldots, n\}$, having weight vector like $w = (w_1, w_2, \ldots, w_n)^T$. Then, we can write the idempotency property as and can exists, but here we have $s_{\sigma_i}e^{i2\pi}$ $(i = 1, \ldots, n)^-$ is a family of CFCNs then a set $s_{\sigma\ \min} e^{i2\pi} = \left(\min_i u_i e^{i2\pi}, \max_i c_i e^{i2\pi}\right)$ and $s_{\sigma\ \max} = \left(\max_i u_i e^{i2\pi}, \min_i c_i e^{i2\pi}\right)$ are the maximum and minimum CFCNs.

$$s_{\sigma\ \min} e^{i2\pi} \leq CFCFOWA(s_{\sigma_1}, s_{\sigma_2}, \ldots, s_{\sigma_n}) e^{i2\pi} \leq s_{\sigma\ \max} e^{i2\pi}$$

C. MONOTONICITY PROPERTY

Let we have a family of CFCNs that is denoted as $\vartheta = \{s_{\sigma_i} e^{i2\pi}, i = 1, 2, \ldots, n\}$, having weight vector like $w = (w_1, w_2, \ldots, w_n)^T$. Then, we can write the idempotency property as and can exists, but here we have $s_{\sigma_i} e^{i2\pi}\ (i = 1, \ldots, n)$, $s^*_{\sigma_i} e^{i2\pi}\ (i = 1, \ldots, n)$ be the family of CFCNs such that $s_{\sigma_i} e^{i2\pi} \leq s^*_{\sigma_i} e^{i2\pi}$

$$CFCFOWA(s_{\sigma_1}, s_{\sigma_2}, \ldots, s_{\sigma_n}) e^{i2\pi} \leq CFCFOWA(s^*_{\sigma_1}, s^*_{\sigma_2}, \ldots, s^*_{\sigma_n}) e^{i2\pi}$$

Proof: The proof is obvious.

V. COMPLEX FUZZY CREDIBILITY FRANK HYBRID AVERAGING AGGREGATION OPERATORS

Consider we have a mapping which is defined as $\alpha^n \to \alpha$, then this mapping is said to be CFCFHWA operator, and these weighted aggregation operators is defined on the basis of Frank norms having following format having weight vector that is represented by $w = (w_1, w_2, \ldots, w_n)^T$, with $\Sigma_{i=1}^n w_i = 1$ and $w_i \in [0, 1]$.

$$\begin{aligned} &CFCFHWA(s_{\sigma_1}, s_{\sigma_2}, \ldots, s_{\sigma_n})' \\ &= \oplus_{i=1}^n s'_{\sigma_i} e^{i2\pi} w_i. \\ &CFCFHWA(s_1, s_2, \ldots, s_n)' \\ &= \left\{\oplus_{i=1}^n s'_{\sigma_i} e^{i2\pi} w_i\right\} \\ &= \left\{\begin{matrix} 1 - \log_\lambda 1 + \Pi_{i=1}^n (\lambda^{1-a'_{\sigma_i}} - 1)^{\omega_i} \\ e^{i2\pi\left(1 - \log_\lambda 1 + \Pi_{i=1}^n (\lambda^{1-a'_{\sigma_i}} - 1)^{\omega_i}\right)}, \\ \log_\lambda 1 + \Pi_{i=1}^n (\lambda^{b'_{\sigma_i}} - 1)^{\omega_i} \\ e^{i2\pi\left(\log_\lambda 1 + \Pi_{i=1}^n (\lambda^{b'_{\sigma_i}} - 1)^{\omega_i}\right)} \end{matrix}\right\}. \end{aligned} \tag{3}$$

In the next theorem we prove that the equation (3) is also a CFCNs and the aggregated value of CFCNs $\vartheta = \{s'_{\sigma_i}, i = 1, 2, \ldots, n\}$ is also a CFCNs.

Theorem 3: Let we have a family of CFCNs that is denoted as $\vartheta = \{s'_{\sigma_i}, i = 1, 2, \ldots, n\}$, having weight vector like $w = (w_1, w_2, \ldots, w_n)^T$. So, the aggregated values of the proposed aggregation operators is still a CFCNs.

$$\begin{aligned} &CFCFHWA(s_1, s_2, \ldots, s_n)' \\ &= \left\{\oplus_{i=1}^n s'_{\sigma_i} e^{i2\pi} w_i\right\} \\ &= \left\{\begin{matrix} 1 - \log_\lambda 1 + \Pi_{i=1}^n (\lambda^{1-a'_{\sigma_i}} - 1)^{\omega_i} \\ e^{i2\pi\left(1 - \log_\lambda 1 + \Pi_{i=1}^n (\lambda^{1-a'_{\sigma_i}} - 1)^{\omega_i}\right)}, \\ \log_\lambda 1 + \Pi_{i=1}^n (\lambda^{b'_{\sigma_i}} - 1)^{\omega_i} \\ e^{i2\pi\left(\log_\lambda 1 + \Pi_{i=1}^n (\lambda^{b'_{\sigma_i}} - 1)^{\omega_i}\right)} \end{matrix}\right\}. \end{aligned}$$

Proof: The proof is obvious.

Now there are some basic properties which are discussed in detailed for the proposed aggregation operators that is idempotency, monotonicity and boundedness.

A. IDEMPOTENCY PROPERTY

Let we have a family of CFCNs that is denoted as $\vartheta = \{s'_{\sigma_i} e^{i2\pi}, i = 1, 2, \ldots, n\}$, having weight vector like $w = (w_1, w_2, \ldots, w_n)^T$. Then, we can write the idempotency property as and can exists, but here we have a family of CFCNs that is $s'_{\sigma_i} e^{i2\pi} = s'_\sigma e^{i2\pi} = (ue^{i2\pi}, ce^{i2\pi})\ \forall i = 1, \ldots, n$.

$$CFCFHWA(s'_{\sigma_1}, s'_{\sigma_2}, \ldots, s'_{\sigma_n}) e^{i2\pi} = s'_\sigma e^{i2\pi}.$$

B. BOUNDEDNESS PROPERTY

Let we have a family of CFCNs that is denoted as $\vartheta = \{s'_{\sigma_i} e^{i2\pi}, i = 1, 2, \ldots, n\}$, having weight vector like $w = (w_1, w_2, \ldots, w_n)^T$. Then, we can write the idempotency property as and can exists, but here we have $s'_{\sigma_i} e^{i2\pi}\ (i = 1, \ldots, n)^-$ is a family of CFCNs then a set $s'_{\sigma\ \min} e^{i2\pi} = \left(\min_i u_i e^{i2\pi}, \max_i c_i e^{i2\pi}\right)$ and $s'_{\sigma\ \max} = \left(\max_i u_i e^{i2\pi}, \min_i c_i e^{i2\pi}\right)$ are the maximum and minimum CFCNs.

$$s'_{\sigma\ \min} e^{i2\pi} \leq CFCFHWA(s'_{\sigma_1}, s'_{\sigma_2}, \ldots, s'_{\sigma_n}) e^{i2\pi} \leq s'_{\sigma\ \max} e^{i2\pi} \tag{4}$$

C. MONOTONICITY PROPERTY

Let we have a family of CFCNs that is denoted as $\vartheta = \{s'_{\sigma_i} e^{i2\pi}, i = 1, 2, \ldots, n\}$, having weight vector like $w = (w_1, w_2, \ldots, w_n)^T$. Then, we can write the idempotency property as and can exists, but here we have $s'_{\sigma_i} e^{i2\pi}\ (i = 1, \ldots, n)$, $s'^*_{\sigma_i} e^{i2\pi}\ (i = 1, \ldots, n)$ be the family of CFCNs such that $s'_{\sigma_i} e^{i2\pi} \leq s'^*_{\sigma_i} e^{i2\pi}$

$$CFCFHWA(s'_{\sigma_1}, s'_{\sigma_2}, \ldots, s'_{\sigma_n}) e^{i2\pi} \leq CFCFHWA(s'^*_{\sigma_1}, s'^*_{\sigma_2}, \ldots, s'^*_{\sigma_n}) e^{i2\pi} \tag{5}$$

Proof: The proof is obvious.

VI. DETAIL DESCRIPTION OF MCGDM PROBLEMS UNDER COMPLEX FUZZY CREDIBILITY INFORMATION

In this section we have to discuss the detailed of our proposed aggregation operators that is apply to the real life group decision making problems and also will helpful to aggregate the data in the form one collective information which will be easy for finding the best optimal solution. So, these types of MCGDM problems has alternatives, criteria and weight vectors which is denoted by $p_i(i = 1, 2, \ldots, m)$, $c_j(j = 1, 2, \ldots, n)$ and $w_i = (w_1, w_2, \ldots, w_n)$. The the alternatives is count for the things or objective of the given decision making problems, the criteria is count for the properties or objective of thing of the given decision making problems and the weight vectors are used be the proposed aggregation operators. And the group decision matrix is denoted by D_1, D_2, D_3 respectively.

Furthermore, the detailed of proposed algorithm is define as follow which has some steps under the complex fuzzy credibility information.

A. ALGORITHM-I

The algorithm-I is based on the proposed aggregation operator by apply on the MCGDM problems to find out the best result. There are some step which will helpful in the finding of best result.

Step-1. Represent the data in the form of decision matrix having alternatives and criteria.

$$M = [w_i(h_{ij}^k)]_{m\times n}$$

$$D = \begin{matrix} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{matrix} \begin{bmatrix} c_1 & c_2 & c_3 & \cdots & c_m \\ h_{11}^k & h_{12}^k & h_{13}^k & \cdots & h_{1m}^k \\ h_{21}^k & h_{22}^k & h_{23}^k & \cdots & h_{2m}^k \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_{n1}^k & h_{n2}^k & h_{n3}^k & \cdots & h_{mn}^k \end{bmatrix}$$

Step-2. Evaluate the aggregated decision matrix using the proposed aggregation operators like as; CFCFWA, CFCFOWA and CFCFHWA.

Step-3. Evaluate the score or accuracy function of the aggregated decision matrix.

Step-4. Represent the final ranking result based on the score or accuracy functions.

Step-5. The end.

B. ALGORITHM-II

The aggregation operators will helpful in the collection of data which is collected by a group for a specific problems that is normally taken as group data called MCGDM problems. After that we have have apply our proposed method steps to find the final results for the given problems.

But the algorithm-II is normally an approach or technique that is used for that type of data which is collected by a single information for a specific problems which is consider as MCDM problems. There are some steps which will helpful in the finding of best result.

Step-1. Represent the data in the form of decision matrix having alternatives and criteria.

$$M = [\tau(k_{ij}^l)]_{m\times n}$$

$$D = \begin{matrix} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{matrix} \begin{bmatrix} c_1 & c_2 & c_3 & \cdots & c_m \\ h_{11}^k & h_{12}^k & h_{13}^k & \cdots & h_{1m}^k \\ h_{21}^k & h_{22}^k & h_{23}^k & \cdots & h_{2m}^k \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_{n1}^k & h_{n2}^k & h_{n3}^k & \cdots & h_{nm}^k \end{bmatrix}$$

Step-2. Evaluate the aggregated decision matrix using the proposed aggregation operators like as; CFCFWA, CFCFOWA and CFCFHWA.

Step-3. Represent the normalized decision matrix.

$$[w_i(h_{ij}^k)]'_{m\times n} = \begin{Bmatrix} (a_i, b_i) = (a_i, b_i)^c \\ (b_i, a_i) \end{Bmatrix}. \tag{6}$$

Step-4. Compute the score or accuracy function.

$$S(A_i) = a_{ij} - b_{ij} + 1 + \frac{e^{a_{ij}-b_{ij}}}{1+\pi_{ij}}.$$

Step-5. Compute the Renyi entropy measure R_j.

$$R_j = \frac{1}{1-\alpha}\log\left(\sum_{i=1}^{m} h_{ij}^k\right)$$

Step-6. Compute the objective weight w_j.

$$w_j = \frac{1-R_j}{\sum_{i=1}^{n}(1-R_j)}$$

Step-7. Determine the combined weight vector β_j.

$$\beta_j = \frac{w_{jR_j}}{\sum_{j=1}^{n} w_{jR_j}}$$

Step-8. Determine the total of the weighted comparability sequence for every alternatives on X_i.

$$X_i = \sum_{j=1}^{n} w_j ab_{ij}$$

Step-9. Determine the whole of the power weight of comparability sequences of each for each alternatives as Y_i.

$$Y_i = \sum_{j=1}^{n} (ab_{ij})^{w_j}$$

Step-10. Determine the whole aggregations which is as Y_i.

$$L_{ia} = \frac{Y_i + X_i}{\sum_{j=1}^{n}(Y_i + X_i)}.$$

$$L_{ib} = \frac{Y_i}{\min_i Y_i} + \frac{X_i}{\min_i X_i}.$$

$$L_{ic} = \frac{\lambda Y_i + (1-\lambda)X_i}{\lambda \max_i Y_i + (1-\lambda)\max_i Y_i}.$$

Step-11. Compute the assessment values.

$$p_i = (L_{ia}L_{ib}L_{ic})^{\frac{1}{3}} + \frac{L_{ia}+L_{ib}+L_{ic}}{3}$$

Step-12. The end.

C. PROBLEM DESCRIPTION ABOUT S-BOX

There are some inputs and outputs which is used in the S-box that is represented by c, d and consider as bits and $c \neq d$ respectively. If we assign some values to the c, d S-box will give us various types of S-box, which can take the values as in 2^{cd} words in the form of c, d bits, when $c = 8$, $d = 8$ will give us i.e $c \times d = 8 \times 8$ S-box. There are some special block ciphers (AES, APA, Gray, Lui, Residue Prime and S_8) which is basically studied in this paper. Tran *et al.* [30] explained the accuracy and application of a S-box to image encryption application. If we assign various values to the inputs of any type of S-box to check the encryption process and also to check which types of S-box are best. So, to solve this type of problem we have to develop a method to solve these types of problems under complex fuzzy credibility information.

To study the pixel detailed and encryption steps of any image, we may use the correlation information. The various methods is used to find the best result in the image encryption process, but here we develop some methods to find a best and suitable ways of image encryption using various types of S-box. So there are some methods like entropy analysis, contrast analysis, homogeneity analysis, energy analysis and mean of absolute deviation analysis are used to know about the strength of image encryption. These analyses will provide us a good results in the formation of image using various S-box. Furthermore our aim is that we have to used various kind of S-box to improve the image encryption under the various analysis. we have to repattern the plain image and transform it into the best way using S-box.

There are various stages in which a plain image is encrypted and Finally, we didn't say for these S-box through these parameters to which one is better and suitable for image encryption everyone has own quality and disquietly at different stages. So there are some vagueness and uncertainties in this analyses. Now, we use the CoCoSo method under complex credibility fuzzy set to find the suitable S-box for an image encryption.

D. ALGORITHMS UNDER COMPLEX FUZZY CREDIBILITY SET

Decision support systems are a precise class of computer-based information systems that support your decision-making activities. A decision support system analyzes data and provide interactive information support to professionals during the decision-making process. Decision making implies selection of the best decision from a set of possible options. In some cases, this selection is based on past experience. Past experience is used to analyze the situations and the choice made in these situations.

Now, here we apply our proposed work to real life decision making problems which is basically an image encryption. The detailed of the alternatives and criteria is discussed as under. Consider we have set of alternatives which are four alternatives as $p_i = \{p_1, p_2, p_3, p_4\}$, where p_1 is for plain image, p_2 is for AES image, p_3 is for Gray and p_4 is for S_8. And the decision maker take the decision on the basis of four criteria which is as follow. c_1 is for energy of image, c_2 is for contrast of image, c_3 is for average correlation of image and c_4 is for homogeneity of an image.

Now, to make a decision under credibility complex fuzzy information, we have used the above aspects of criteria. Also we have to find the suitable S-box. The four possible alternatives are to be evaluated using the complex fuzzy credibility information from the decision maker under the above four criteria.

E. COMPUTATIONAL RESULT OF ALGORITHM-I

The computational results of our proposed algorithm-I is that we have to find all the steps which is needful and also to find the best results of the given problems using these steps.

Step-1 Represent the data in the form of decision matrix having alternatives and criteria which is denoted by Table-1,Table-2 and Table-3.

TABLE 1. CFCNs information by D_1.

	c_1	c_2
p_1	$(0.1e^{i2\pi(0.1)}, 0.8e^{i2\pi(0.9)})$	$(0.5e^{i2\pi(0.3)}, 0.4e^{i2\pi(0.2)})$
p_2	$(0.8e^{i2\pi(0.9)}, 0.2e^{i2\pi(0.1)})$	$(0.2e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.1)})$
p_3	$(0.4e^{i2\pi(0.8)}, 0.1e^{i2\pi(0.2)})$	$(0.4e^{i2\pi(0.3)}, 0.1e^{i2\pi(0.5)})$
p_4	$(0.6e^{i2\pi(0.7)}, 0.4e^{i2\pi(0.3)})$	$(0.6e^{i2\pi(0.7)}, 0.4e^{i2\pi(0.2)})$
	c_3	c_4
p_1	$(0.8e^{i2\pi(0.2)}, 0.9e^{i2\pi(0.5)})$	$(0.8e^{i2\pi(0.3)}, 0.7e^{i2\pi(0.2)})$
p_2	$(0.1e^{i2\pi(0.5)}, 0.7e^{i2\pi(0.6)})$	$(0.6e^{i2\pi(0.2)}, 0.2e^{i2\pi(0.5)})$
p_3	$(0.8e^{i2\pi(0.3)}, 0.5e^{i2\pi(0.4)})$	$(0.9e^{i2\pi(0.5)}, 0.8e^{i2\pi(0.7)})$
p_4	$(0.3e^{i2\pi(0.6)}, 0.8e^{i2\pi(0.7)})$	$(0.5e^{i2\pi(0.2)}, 0.9e^{i2\pi(0.4)})$

TABLE 2. CFCNs information by D_2.

	c_1	c_2
p_1	$(0.6e^{i2\pi(0.1)}, 0.8e^{i2\pi(0.9)})$	$(0.5e^{i2\pi(0.4)}, 0.6e^{i2\pi(0.3)})$
p_2	$(0.5e^{i2\pi(0.7)}, 0.6e^{i2\pi(0.4)})$	$(0.4e^{i2\pi(0.2)}, 0.6e^{i2\pi(0.8)})$
p_3	$(0.9e^{i2\pi(0.8)}, 0.7e^{i2\pi(0.6)})$	$(0.3e^{i2\pi(0.7)}, 0.2e^{i2\pi(0.2)})$
p_4	$(0.7e^{i2\pi(0.7)}, 0.8e^{i2\pi(0.5)})$	$(0.7e^{i2\pi(0.2)}, 0.1e^{i2\pi(0.5)})$
	c_3	c_4
p_1	$(0.8e^{i2\pi(0.2)}, 0.7e^{i2\pi(0.5)})$	$(0.4e^{i2\pi(0.3)}, 0.7e^{i2\pi(0.2)})$
p_2	$(0.1e^{i2\pi(0.5)}, 0.7e^{i2\pi(0.6)})$	$(0.2e^{i2\pi(0.2)}, 0.6e^{i2\pi(0.5)})$
p_3	$(0.7e^{i2\pi(0.3)}, 0.5e^{i2\pi(0.4)})$	$(0.3e^{i2\pi(0.5)}, 0.2e^{i2\pi(0.7)})$
p_4	$(0.5e^{i2\pi(0.6)}, 0.8e^{i2\pi(0.3)})$	$(0.5e^{i2\pi(0.2)}, 0.5e^{i2\pi(0.4)})$

Step -2 Compute the aggregated decision matrix which is represented by Table-4, Table-5 and Table-6.

TABLE 3. CFCNs information by D_3.

	c_1	c_2
p_1	$(0.3e^{i2\pi(0.8)}, 0.8e^{i2\pi(0.5)})$	$(0.5e^{i2\pi(0.1)}, 0.3e^{i2\pi(0.3)})$
p_2	$(0.5e^{i2\pi(0.4)}, 0.9e^{i2\pi(0.7)})$	$(0.2e^{i2\pi(0.9)}, 0.4e^{i2\pi(0.2)})$
p_3	$(0.8e^{i2\pi(0.8)}, 0.7e^{i2\pi(0.6)})$	$(0.3e^{i2\pi(0.8)}, 0.3e^{i2\pi(0.1)})$
p_4	$(0.5e^{i2\pi(0.5)}, 0.9e^{i2\pi(0.4)})$	$(0.5e^{i2\pi(0.7)}, 0.1e^{i2\pi(0.6)})$
	c_3	c_4
p_1	$(0.5e^{i2\pi(0.3)}, 0.3e^{i2\pi(0.1)})$	$(0.2e^{i2\pi(0.4)}, 0.4e^{i2\pi(0.6)})$
p_2	$(0.3e^{i2\pi(0.4)}, 0.6e^{i2\pi(0.6)})$	$(0.4e^{i2\pi(0.4)}, 0.2e^{i2\pi(0.1)})$
p_3	$(0.2e^{i2\pi(0.2)}, 0.6e^{i2\pi(0.8)})$	$(0.3e^{i2\pi(0.6)}, 0.4e^{i2\pi(0.4)})$
p_4	$(0.6e^{i2\pi(0.7)}, 0.5e^{i2\pi(0.2)})$	$(0.5e^{i2\pi(0.3)}, 0.5e^{i2\pi(0.7)})$

TABLE 4. The aggregated matrix using proposed CFCFWA.

Alternatives	
p_1	$(0.455e^{i2\pi(0.392)}, 0.554e^{i2\pi(0.404)})$
p_2	$(0.460e^{i2\pi(0.670)}, 0.504e^{i2\pi(0.375)})$
p_3	$(0.590e^{i2\pi(0.672)}, 0.384e^{i2\pi(0.368)})$
p_4	$(0.581e^{i2\pi(0.594)}, 0.415e^{i2\pi(0.380)})$

TABLE 5. The aggregated matrix using proposed CFCFOWA.

Alternatives	
p_1	$(0.474e^{i2\pi(0.366)}, 0.546e^{i2\pi(0.384)})$
p_2	$(0.444e^{i2\pi(0.538)}, 0.500e^{i2\pi(0.410)})$
p_3	$(0.586e^{i2\pi(0.568)}, 0.456e^{i2\pi(0.473)})$
p_4	$(0.573e^{i2\pi(0.599)}, 0.552e^{i2\pi(0.334)})$

TABLE 6. The aggregated matrix using proposed CFCFHWA.

Alternatives	
p_1	$(0.797e^{i2\pi(0.750)}, 0.283e^{i2\pi(0.189)})$
p_2	$(0.795e^{i2\pi(0.866)}, 0.266e^{i2\pi(0.192)})$
p_3	$(0.854e^{i2\pi(0.890)}, 0.188e^{i2\pi(0.166)})$
p_4	$(0.859e^{i2\pi(0.860)}, 0.209e^{i2\pi(0.174)})$

Step-3 Compute the score or accuracy function which are denoted by Table-7. Also the final ranking result is also shown in Table-7.

F. COMPUTATIONAL RESULT OF ALGORITHM-II

The computational results of our proposed algorithm-II is that we have to find all the numerical values of the steps which is needful and also to find the best results of the given problems using these proposed steps.

Step-1 Represent the data in the form of decision matrix which is as discussed in the algorithm-I and denoted as Table-1, Table-2 and Table-3.

Step -2 Compute the aggregated matrix by using proposed aggregation operators which is represented in the Table-4, Table-5 and Table-6 in the algorithm-I.

TABLE 7. The ranking results of the suggested technique using Algorithm -I.

Proposed Methods	score values p_1	p_2	p_3	p_4	Ranking
CFCFWA	0.121	0.145	0.162	0.153	$p_3 > p_4 > p_2 > p_1$
CFCFOWA	0.114	0.135	0.175	0.163	$p_3 > p_4 > p_2 > p_1$
CFCFHWA	0.178	0.202	0.209	0.207	$p_3 > p_4 > p_2 > p_1$

Step-3 Normalize the decision matrix, but in this step we does not normalized the data because the data is in the form of benefit criteria and also the data is uniform.

Step-4 Compute the overall score function of the aggregated decision matrix. which is shown as in Table-8.

TABLE 8. The result of score function of the aggregated decision matrix.

Alternatives	c_1	c_2	c_3	c_4
p_1	1.637	1.904	0.991	2.010
p_2	2.007	2.134	1.115	1.823
p_3	1.455	2.208	1.854	1.930
p_4	1.632	1.792	1.834	1.532

Step-5 Compute the combined weight vector which is as follows.

$$\beta_1 = 0.407973, \quad \beta_2 = 0.323473, \ \beta_3 = 0.188152, \ \beta_4 = 0.80399$$

Step-5 Compute the whole weighted comparability sequences.

$$X_1 = 0.449, \quad X_2 = 0.451, \ X_3 = 0.540, \ X_4 = 0.581$$

Step-7 Compute the power of weight comparability sequences which is as follows.

$$Y_1 = 3.28, \quad Y_2 = 3.26, \ Y_3 = 3.38, \ Y_4 = 3.49$$

Step-8 Compute the three aggregated values of all the functions.

$$L_{1a} = 2.40, \quad L_{2a} = 2.41, \ L_{3a} = 2.53, \ L_{4a} = 2.63$$
$$L_{1a} = 2, \quad L_{2a} = 2.00, \ L_{3a} = 2.23, \ L_{4a} = 2.35$$
$$L_{1a} = 13.34, \quad L_{2a} = 13.29, \ L_{3a} = 14.01, \ L_{4a} = 14.55$$

Step-9 Compute the assessment values which is denoted by Table-9.

TABLE 9. The ranking results of the suggested technique using Algorithm -II.

Proposed Methods	Score values p_1	p_2	p_3	p_4
CoCoSo Approach	7.055	7.035	7.492	7.276

Ranking
$p_3 > p_4 > p_1 > p_2$

G. SENSITIVITY ANALYSIS

To check the stability of our proposed method we have to do sensitivity analysis, that is in the Frank norms and its coefficient (parameter) of decision method is critical to the ranking results. So we assign various values to the parameter in the proposed aggregation operators from 1 to 15 to find the various results and also to see the ranking results.

TABLE 10. The Ranking results for different values of λ.

λ	p_1	p_2	p_3	p_4	Ranking
1	0.130	0.127	0.345	0.260	$p_3 > p_4 > p_1 > p_2$
3	0.128	0.110	0.254	0.137	$p_3 > p_4 > p_1 > p_2$
5	0.124	0.118	0.347	0.286	$p_3 > p_4 > p_1 > p_2$
6	0.135	0.122	0.390	0.298	$p_3 > p_4 > p_1 > p_2$
9	0.123	0.110	0.390	0.267	$p_3 > p_4 > p_1 > p_2$
11	0.124	0.104	0.202	0.186	$p_3 > p_4 > p_1 > p_2$
13	0.160	0.125	0.278	0.195	$p_3 > p_4 > p_1 > p_2$
15	0.126	0.113	0.207	0.164	$p_3 > p_4 > p_1 > p_2$

As we can see from above Table-10, the ranking results is same by assign different values to the parameter in the proposed aggregation operators. If we put $\lambda = 1$ in the proposed aggregation operator then the score values of the alternatives are $p_1 = 0.130$, $p_2 = 0.127$, $p_3 = 0.345, p_4 = 0.260$. So from here the best score values of p_3 is best. If we put $\lambda = 3$ in the proposed aggregation operator then the score values of the alternatives are $p_1 = 0.128$, $p_2 = 0.110$, $p_3 = 0.254$, $p_4 = 0.137$. So from here the best score values of p_3 is best. If we put $\lambda = 5$ in the proposed aggregation operator then the score values of the alternatives are $p_1 = 0.124$, $p_2 = 0.118$, $p_3 = 0.347, p_4 = 0.286$. So from here the best score values of p_3 is best. If we put $\lambda = 6, 9, 11, 13$ and 15 in the proposed aggregation operator then the score values of the alternatives are $p_1 = 0.135, 0.123, 0.124, 0.160, 0.126$ $p_2 = 0.122, 0.110, 0.104, 0.125, 0.113$, $p_3 = 0.390, 0.390, 0.202, 0.278, 0.207, p_4 = 0.298, 0.276, 0.186, 0.195, 0.164$ respectively. So from here the best score values of p_3 is best among all the alternatives.

VII. COMPARISON AND DISCUSSION ANALYSIS

In this section we have to compare our proposed method with other existing methods to check the validity of our proposed work. So there are two ways of comparison which is to compare with an aggregation operators wise and the other is in the form of technique (approach) wise comparison. But here we can compare our result as of two types namely aggregation operators wise and method wise comparison.

A. EXISTING AGGREGATION OPERATORS

There are some work which is done in the field of aggregation operators using various form of fuzzy data. But here we have to compare with an intuitionistic weighted average aggregation operators, intuitionistic ordered weighted average aggregation operators and intuitionistic hybrid weighted average aggregation operators. Also with complex intuitionistic weighted average aggregation operators, complex intuitionistic ordered weighted average aggregation operators and complex intuitionistic hybrid weighted average aggregation operators. And also we have compare our method with TOPSIS method under IFS information.

B. PROPOSED AGGREGATION OPERATORS

While the proposed aggregation operators is develop by using the operational laws of Frank norms under complex fuzzy information that is the complex fuzzy credibility Frank weighted average aggregation operators, complex fuzzy credibility Frank ordered weighted average aggregation operators and complex fuzzy credibility Frank hybrid weighted average aggregation operators.

C. COMPARISON WITH IFWA

In this section the comparison of our proposed method with existing is discussed in detailed. This comparison analysis is a types of aggregation wise comparison in which we have to check the accuracy of our proposed aggregation operators.

D. EXISTING AGGREGATION OPERATORS

In the existing aggregation operators the data is in the form of intuitionistic fuzzy numbers (IFNs) and series of aggregation operators is defined on the intuitionistic fuzzy operational laws. The series of aggregation operators is a type of IFWA [32], IFOWA and IFHWA. So by comparing our results with this result is same.

E. RESULT OF COMPARISON

The result of comparison is that the ranking result will be different because the develop work discussed the phase term as well as the amplitude term for any information. While as in case of intuitionistic fuzzy information the data is in the real valued fuzzy data which can only explain the phase term for the membership degree. So that is why the new proposed work is more generalized and more accurate than the existing methods.

F. COMPARISON WITH CIFWA

In this section the comparison of our proposed method with existing is discussed in detailed. This comparison analysis is a types of aggregation wise comparison in which we have to check the accuracy of our proposed aggregation operators.

TABLE 11. The result of comparison with IFWA.

Approaches	Score values			
IFWA [32]	0.119	0.120	0.189	0.132
CFCFWA	0.121	0.145	0.162	0.153

Ranking
$p_3 > p_4 > p_2 > p_1$
$p_3 > p_4 > p_2 > p_1$

G. EXISTING AGGREGATION OPERATORS

In the existing aggregation operators the data is in the form of complex intuitionistic fuzzy numbers (CIFNs) and series of aggregation operators is defined on the complex intuitionistic fuzzy operational laws. The series of aggregation operators is a type of CIFWA [31], CIFOWA and CIFHWA. So by comparing our results with this result is same as our develop results.

H. RESULT OF COMPARISON

The result of comparison is that the ranking result will be same as our develop work. because the proposed work and existing work discussed the phase term as well as amplitude term for any decision making problems. The result of comparison is discussed as follows,

TABLE 12. The result of comparison with CIFWA.

Approaches	Score values			
CIFWA [31]	0.123	0.110	0.160	0.154
CFCFWA	0.121	0.145	0.162	0.153

Ranking
$p_3 > p_4 > p_1 > p_2$
$p_3 > p_4 > p_2 > p_1$

I. COMPARISON WITH TOPSIS METHOD UNDER IFS INFORMATION

In this section we have compare our result which obtained through CoCoSo method with the existing TOPSIS method. Furthermore we see that the result obtained is same and accurate to the develop work.

J. EXISTING TOPSIS METHOD UNDER IFS INFORMATION

In the existing method, the data is in the form of IFS information and also there are some steps which is defined for the TOPSIS method [33]. So we have compare our develop method to check the accuracy.

Now in the existing method we have taken the data is in the form of CFCS information and we have discussed all the steps for the CoCoSo method. The method is also used for the ranking of any decision making problems. So by comparing our result with the existing results, we get the same results. The comparison results are discussed as follows,

K. RESULT OF COMPARISON

The result of comparison is that the ranking result will be same as our develop work. The result of comparison is discussed as follows,

TABLE 13. The result of comparison with TOPSIS method.

Approaches	Score values			
TOPSIS method [33]	0.123	0.109	0.180	0.159
CoCoSo method	0.121	0.145	0.162	0.153

Ranking
$p_3 > p_4 > p_1 > p_2$
$p_3 > p_4 > p_2 > p_1$

TABLE 14. The overall result of comparison analysis with other existing methods.

Suggested Techniques	Score values p_1	p_2	p_3	p_4
IFWA [32]	0.119	0.120	0.189	0.132
TOPSIS method [33]	0.123	0.109	0.180	0.159
CIFWA [31]	0.123	0.110	0.160	0.154
CoCoSo Approach	7.055	7.035	7.492	7.276
CFCFWA	0.121	0.145	0.162	0.153
CFCFOWA	0.114	0.135	0.175	0.163
CFCFHWA	0.178	0.202	0.209	0.207

Ranking
$p_3 > p_4 > p_2 > p_1$
$p_3 > p_4 > p_1 > p_2$
$p_3 > p_4 > p_1 > p_2$
$p_3 > p_4 > p_1 > p_2$
$p_3 > p_4 > p_2 > p_1$
$p_3 > p_4 > p_2 > p_1$
$p_3 > p_4 > p_2 > p_1$

The Table-14 show the overall comparison results, in which we have compare our develop work with the existing work. In this Table-14, we can see that there are also two way of comparison in which the first way is the existing aggregation operators and the other way is the existing method. In the two way of comparison the result is same.

L. RESULT AND DISCUSSION

In this paper we have discussed the Frank norm operational laws, After that we have develop as series of aggregation operators like CFCFWA, CFCFOWA and CFCFHWA. Then, we explained and prove all the basic properties for the develop aggregation operators. In the next section we have develop two algorithms which is named as algorithm-I and algorithm-II. In the algorithm-I we have discussed all the steps and which is based on the develop aggregation operators and lastly we have to applied our proposed score function to display the final best optimal results. Also in the algorithm-II we have explained all the steps of CoCoSo method which is also helpful in the best optimal solution for any decision making problems. In lastly we have applied these two algorithms to a real life decision making problems which is a S-box image encryption, in which we get the best results.

M. LIMITATIONS AND DESCRIPTION OF OUR PROPOSED WORK

As this paper we have discussed a series aggregation operators like CFCFWA, CFCFOWA and CFCFHWA aggregation operators and its basic properties were explained in the form of theorem. The aggregation operator helps in the collection of expert information and there a Frank norms which can explain the flexibility on the basis of various parameters, the various cases of paremeter values which is change in the proposed aggregation operators and that provide the same best results.

This paper does not discussed the multi criteria decision making, nor does consider different experts weight. So we will extend this process for multi criteria decision making in future work and make it practical.

VIII. CONCLUSION AND FUTURE WORK

The practical application of complex fuzzy credibility numbers is studied in the various field of computer science and many other field. Decision making process is a small class of computer based knowledge that support your decision making activities. A decision support systems analyzes data and give interactive knowledge support to professionals during the decision-making process. With comparison to other analysis the result of complex fuzzy credibility numbers is accurate. So in this paper we have used the develop aggregation operators to collect the decision expert information under CFCS information and also to find a suitable S-box transformation in the image encryption process. Basically an S-box is very important component in a block cipher cryptosystem which have the responsibility to induce confusion in the data. So there is a need to find the confusion capability of different S-box for image encryption application and also which box is best among others. In this paper we have analyzed prevailing S-box and come to know that GRAY S-box have very good readings. Our study will start a new direction in the field of decision support systems and cryptosystems. In future we will focus some other types of S-box transformation based on other criterion.

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