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RESEARCH ARTICLE

Sparse Signal Representation, Sampling, and Recovery in Compressive Sensing Frameworks

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ABSTRACT Compressive sensing allows the reconstruction of original signals from a much smaller number of samples as compared to the Nyquist sampling rate. The effectiveness of compressive sensing motivated the researchers for its deployment in a variety of application areas. The use of an efficient sampling matrix for high-performance recovery algorithms improves the performance of the compressive sensing framework significantly. This paper presents the underlying concepts of compressive sensing as well as previous work done in targeted domains in accordance with the various application areas. To develop prospects within the available functional blocks of compressive sensing frameworks, a diverse range of application areas are investigated. The three fundamental elements of a compressive sensing framework (signal sparsity, subsampling, and reconstruction) are thoroughly reviewed in this work by becoming acquainted with the key research gaps previously identified by the research community. Similarly, the basic mathematical formulation is used to outline some primary performance evaluation metrics for 1D and 2D compressive sensing.

INDEX TERMS Compressed sensing, compressive sampling, reconstruction algorithms, sensing matrix.

I. INTRODUCTION

We have seen an exponential increase in digital data production and utilisation since the inception of modern digital evolution. The available datasets are massive and complex, necessitating a significant amount of power and communication bandwidth. Furthermore, current storage capacities have abandoned the big data problem. As a result, data compression can be beneficial in addressing the issues and has been used in a variety of applications. Figure 1 depicts the overall steps involved in various stages of signal compression.

In conventional data compression techniques, it is clear that all samples must first be accumulated and then processed to remove redundant information. As a result, the sampling end has a higher computational load.

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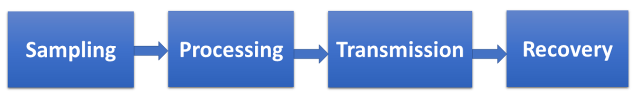

FIGURE 1. Signal compression stages.

Another method, which takes advantage of sparsity in natural signals, has been proposed in recent years. Due to its simultaneous sampling and compression capabilities, this is known as Compressive Sensing (CS) [1]. It tends to estimate the original signal from an under-determined set of measurements, resulting in lower sampling costs at the transmitter end.

There are three key components of a CS framework:

- Measurement Matrix (Φ).
- Sparse Transformation Basis (Ψ).

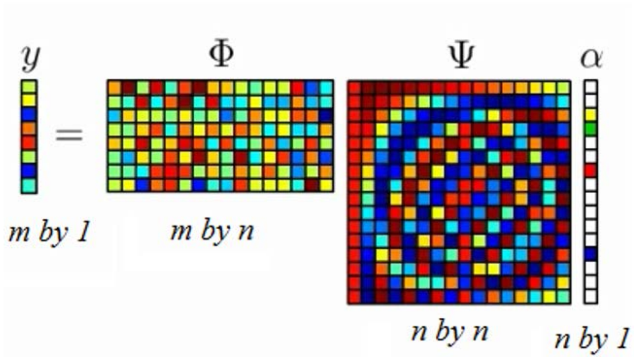

FIGURE 2. **Compressive sensing components.**

- Reconstruction Technique.

The first two primary components of a CS framework are shown in Figure 2.

Similarly, the sparse transformation bases can be either built-in or learned. Some widely used built-in transforms are:

- Discrete Fourier Transform (DFT) [2]
- Discrete Cosine Transform (DCT) [3]
- Discrete Sine Transform (DST) [4]
- Hadamard Transform (HT) [5]
- Lapped Transform (LT) [6]
- Discrete Wavelet Transform (DWT) [7]

Apart from the predefined dictionaries, there are some other techniques, such as ksvd, that can be used to provide learned transformation basis.

Using optimization methodologies, recovery algorithms are employed in compressed sensing to faithfully recover the original signal from a random and incomplete set of samples. There are some widely used classes of sparse signal recovery techniques, which are mentioned below:

- Convex Relaxation Algorithms:
  These are the optimization techniques which possess better reconstruction accuracy, with lower speed, such as Basis Pursuit (ℓ_1-minimization).
- Greedy Iterative Algorithms:
  These algorithms have higher reconstruction speed with approximate solution, it includes Matching Pursuit and Orthogonal Matching Pursuit (OMP).
- Iterative Algorithms:
  Some widely used algorithms in this class are Iterative Hard Thresholding (IHT), and Iterative Soft Thresholding (IST).
- Bayesian Method:
  These techniques are used to reconstruct original signal by taking into consideration the correlation between the signal elements.
- Deep Learning Techniques:
  Deep learning methods for signal reconstruction are based on the capabilities of training based deep neural networks.

Figure 3 illustrates the complete outline of CS frameworks along with the categories of deployed techniques. The sections that follow describe each category of the illustrated figure by going through the current advances listed in the literature. In this work, the fundamental CS components are linked to the challenges and future prospects to target the specific application areas by going through current trends in CS literature. The fundamental challenges faced by the conventional CS frameworks are summarized below.

- The availability of sparse transformation basis is of utmost importance for the CS frameworks. Therefore, the availability of a feasible basis for the available datasets is one of the key challenges.
- The redundant and incomplete information projected on to the transformed domain needs efficient recovery algorithms for signal recovery to restore the information lost during inefficient sampling process. So computational models has to be developed by comparing its efficiency with the existing signal recovery algorithms.
- Efficient signal recovery from the incomplete set of information is also fundamentally needed. It is also necessary to demonstrate working knowledge of existing signal recovery techniques as well as to deploy advanced machine learning models for efficient signal recovery.
- Even when using computationally expensive signal recovery methods, using random measurement matrices in the existing compressed sensing domain degrades performance. Therefore, the development of optimized and efficient measurement matrix is required, to gather the samples which carries higher information content.

This paper is organized as follow:

Section II demonstrate all the recent developments and deployments of sparse signal transformation. Section III illustrates the conventionally used random, deterministic, and data driven sampling matrices and latest trends in developing training based methods for sampling matrices development. Section IV is related to CS based reconstruction algorithms, while section V describes major application areas of CS, where it is successfully deployed in recent years. The performance evaluation metrics used in CS are explained in section VI. The last section VII is regarding the major highlights of this paper, and concludes it with major takeaways of this paper.

II. SPARSE SIGNAL REPRESENTATION

Compressed sensing has gained popularity in recent years due to its data recovery ability from a small subset of samples. It was proposed by Emmanuel J. Candes in [1] by stating the possibility of original information retrieval from a relatively small number of sparse data samples. The ability of CS to recover information from an incomplete set of data has increased its applicability in diverse range of application areas. Prior to CS, it was thought that the Nyquist theorem was the fundamental condition for sampling the available information. The Nyquist sampling theorem states that a signal can be accurately recovered from its samples if the sampling rate is twice or more than the rate of the signal's highest frequency component. Before the necessary processing can take place, a signal that is band-unlimited must be

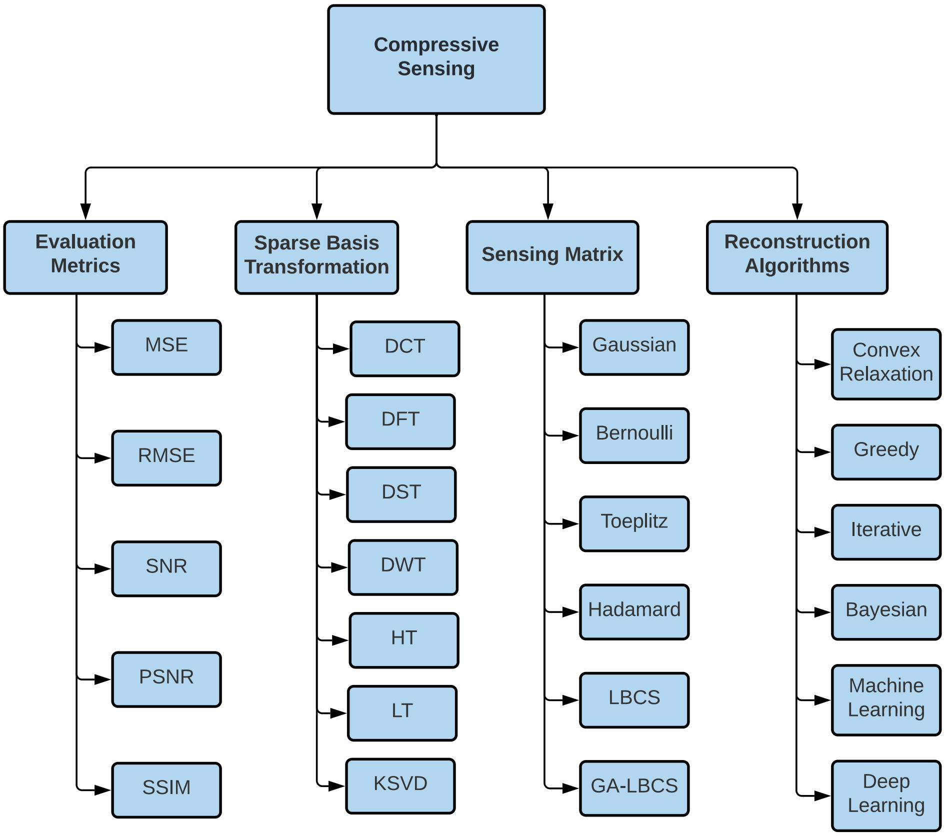

FIGURE 3. Developed components of a compressive sensing framework.

transformed into a signal that is band-limited. This method previously required more computing power, transmission bandwidth, and storage memory, in addition to being prone to a high number of reconstruction errors. The majority of available signals possess sparsity and can be reconstructed with a limited number of data samples if presented in the appropriate transform basis Ψ [1]. This study showed that data samples with two key characteristics can be precisely reconstructed from very limited content:

- Sparsity
- Incoherence

Similarly, at the sampling end, the Restricted Isometric Property (RIP) [1] ensures that the needed quantity of information content is present in random signal samples by fine-tuning the essential performance parameters at the sampling end, resulting in more information embedding in a fewer number of samples. As a result, recovery algorithms can achieve better performance in reconstructing original information.

A. SPARSITY

Sparsity is one of the necessary conditions for accurate signal reconstruction in CS frameworks. Sparse signal representation aims to preserve original information by representing 1D signals, images, and videos with a small number of non-zero coefficients. The primary goal of sparse representation is to convert the original signal into a set of basis coefficients with a small number of non-zero coefficients. Because of the scarcity of information in available sample, processing time tends to increase while computation resources and transmission bandwidth are reduced. The sparse coefficients of a signal 'x' can be expressed as follows in terms of the basis 'ψ':

$$x = \sum_{i=1}^{N} \alpha_i \psi_i = \alpha \Psi, \tag{1}$$

where α_i denotes a collection of 'N' transformed coefficients in the basis Ψ.

B. SPARSE TRANSFORMATION DICTIONARIES

CS is a subclass of sparse signal representation that acquires random data samples with the highest signal information content, combining sampling and compression in a single step. Sparsity implies that most naturally occurring signals can be represented with the fewest data elements. As a result, dictionaries were developed to aid in the representation of sparse signals.

Signals are not always completely sparse in their original form. As a result, predefined overcomplete dictionaries are employed for converting signals into a sparse version. Discrete Cosine Transform (DCT) [8], Discrete Sine Transform

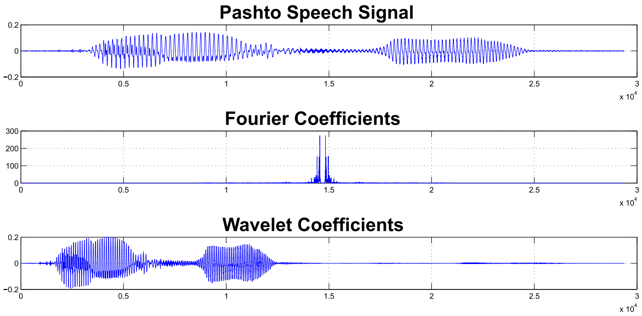

FIGURE 4. Speech signal representation in fourier & wavelet domain.

(DST) [9], and Discrete Fourier Transform (DFT) [10] are some well-known transformation bases that represents 1-D signals in terms of their sparse coefficients, while Wavelet Transform (WT) is used for sparse representation of images and videos [11]. Transform coding is the representation of natural signals using readily available bases. Modern computation methods have also aided the development of learned dictionaries that can provide sparse coefficients for a known dataset. In the column space of these learned dictionaries, similar signals are thus represented. [12].

There are also some transforms which are based on orthogonal polynomials similar to DCT and DFT. These transforms include Discrete Hahn Polynomials (DHPs) which has been used for efficient feature extraction in image processing. The work done in [13] addressed some shortcomings and proposed efficient Hahn orthogonal basis to be deployed for higher orders. Similarly, other classes of orthogonal polynomial have also shown promising results in representing higher order signals in orthogonal basis for efficient feature extraction. These polynomials are Krawtchouk Polynomials (KPs) [14], Charlier polynomials [15], Meixner orthogonal polynomials [16], and Tchebichef polynomials [17], [18].

The sparse transformation bases for speech signals are explored and studied in [19]. Figure 4 demonstrates the effectiveness of DCT and DWT bases in speech signal sparse transformation.

In addition to the predefined dictionaries, a subclass of dictionaries are employed to represent signals in sparse basis [20]. These dictionaries have made significant contributions to the field of sparse signal processing due to their effectiveness in sparse signal representation [21] at the cost of more training data and computational resources.

III. SIGNAL SAMPLING

The measurement matrix, also represented by Φ in the literature, is known as sampling modality and used in the CS framework for decomposing high-dimensional input data samples into the low-rank components. The transformation carried out by Φ can be written as:

$$\Phi : \mathbb{R}^n \rightarrow \mathbb{R}^m \quad \forall\, n \gg m \tag{2}$$

By capturing informative samples, a well-chosen Φ can aid in the faithful signal reconstruction process. Therefore, efficient measurement matrix design is critical to the overall performance improvement of the sampling and recovery process. Zaeemzadeh *et al.* [22] addressed the adaptivity of measurement matrix by proposing the adaptive method for designing measurement matrices based on a generative approach. The developed method proposed an adaptive sensing matrix that samples the signal's most important features which indicates whether a specific set of data samples is helpful in signal reconstruction process or not.

A. RESTRICTED ISOMETRIC PROPERTY

The efficiently designed sensing matrix assists in the reconstruction process by preserving the significant data elements, if it satisfies Restricted Isometric Property (RIP) [1], [23]. In CS framework, RIP ensures the information preservation property of a sensing matrix. RIP states that a matrix 'A' collects significant data samples of a signal, if it satisfies 3

$$(1-\sigma_S)||b||_2 \leq ||Ab||_2 \leq (1+\sigma_S)||b||_2, \tag{3}$$

where σ_S is a restricted isometric constant and its value ranges between 0 and 1.

RIP ensures that a sensing matrix maps two input signals, a_1 and a_2, with two distinct output signals, b_2 and b_2, as:

$$a_1 - a_2 \approx ||\Phi a_1 - \Phi a_2||_2^2 = ||b_1 - b_2||_2^2. \tag{4}$$

The smallest possible number of samples required for exact signal recovery to satisfy equation 4, is:

$$m \geq S \log\left(\frac{n}{S}\right). \tag{5}$$

B. RANDOM AND DETERMINISTIC SAMPLING

The retrieval of higher information content from an incomplete set of information samples has been a major focus of recent work in the fields of signal estimation and statistical signal processing. Sensing matrix may have pronounced effects in achieving higher quality reconstructed signal by having least amount of signal loss at the recovery end. The importance of efficient signal sampling is illustrated in the form of Figure 5. It is shown that selecting an inefficient and limited identity matrix for the sampling matrix yields in inaccurate signal reconstruction.

The majority of earlier work in CS framework focused on the development of computationally unrealistic and complex signal recovery algorithms. Till now, researchers have worked and devised techniques for sampling modalities enabling higher information capturing of the signals [24], [25]. As a result, the devised mechanism improves recovery performance, as well as, reconstruction speed.

Random sampling techniques have been used in a variety of CS applications owing to its historical usefulness. In [26], authors demonstrate the efficiency of random sampling techniques and their usage in wireless sensor networks. In addition, [27] presented a novel gradient projection-based approach for compressively sensed signal reconstruction with a different method of initialising measurement matrix. When compared to other widely used signal sampling strategies, this

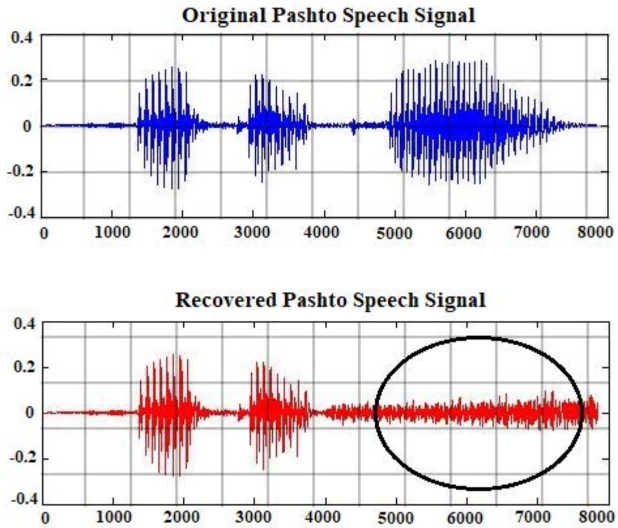

FIGURE 5. Signal loss as a result of inefficient sensing.

strategy outperforms them. However, the trade-offs associated with learning-based approaches including computational power and memory storage resources are not addressed.

Another paper [28] showed how CS can be used to analyse speech signals to preserve their quality. Figure 6 illustrates the reconstruction performance of various speech signals.

This study concentrated on the sparsity of speech signals in various transform domains. Random sampling, on the other hand, is implemented using Gaussian sensing matrix. The authors investigate the randomly selected signal for reconstruction using a greedy reconstruction algorithm of Orthogonal Matching Pursuit (OMP). DCT is deemed as a better choice for use as a sparse basis for speech and audio signals reconstruction, whereas Gaussian random matrix aided in the efficient and accurate reconstruction resulting in better performance with OMP. One such similar strategy proposed in [19] which presented the use of Bernoulli sensing matrix for an enhanced speech signal sampling than other techniques. Although the paper primarily deals with the signal recovery algorithms, however it also includes a proposal for a Bernoulli matrix-based training matrix that performs better for speech signals.

In this quest, work done in [29] explored a diverse range of random and deterministic sensing matrices to propose an efficient matrix. In this work, the experimented matrices were compared on the basis of reconstruction accuracy and reconstruction time, while DCT is applied as a sparse transformation basis owing to its efficiency in sparsity of speech signals [30]. The random matrices have shown better reconstruction accuracy while on the contrary, deterministic matrices reconstruction accuracy is of moderate accuracy. On the other hand, the speed of the ℓ_1-norm reconstruction method for the deterministic samples is higher as compared to random matrices. The results which we have taken also discussed that the deterministic matrices can be applied for real time application because of the enhanced speed of the reconstruction method at the cost of lower reconstruction accuracy. In this work, speech signals from TIMIT [31], Pashto dataset [32] and Urdu dataset [33] corpora were used for the experimentation.

Despite all the advantages associated with random sensing, it falls short in terms of fast and computationally feasible sampling for accurate signal recovery. This issue is also discussed in [34], demonstrating the advantages of using deterministic sensing. Several deterministic matrices are explored and studied in [34] showing better performance.

C. LEARNING BASED APPROACHES FOR SENSING MATRIX DESIGN

Preliminary literature in the CS recorded that Φ has a non-adaptive nature to ensure better reconstruction performance [35]. Therefore, training-based sensing matrix design methods were presented to include the inherent structure of a training dataset for CS-based signal sampling [19]. The offline training approaches for sampling matrix design are shown to perform efficiently because they preserve the inherent structure of the training dataset. Due to the performance enhancement across all 1D and 2D signals demonstrates the adaptive nature of the offline training mechanism. In [36], a similar learning-based approach was used to account for the inter-column correlation matrix. As a parent matrix, a Gaussain random matrix is selected and trained for the dataset. The results obtained were favourable and a positive sign for researchers working on learning-based techniques. Similarly, the limitations of random subsampling can be mitigated by using structured subsampling, as demonstrated in [37] and [38]. In order to improve image recovery in the CS domain, an image dataset is used to train a sampling matrix and for real time neural signals reconstruction [38]. Using deep Convolutional Neural Networks (CNN) and the linear reconstruction method, this work adopted the training mechanism proposed by [37] for real-time applications. In [39], a similar approach is used to for optimal sensing matrix for gathering the maximum information. Similarly, a GA based approach for sensing matrix design is recently proposed in [40]. In this work, evolutionary mechanism is deployed to generate the optimized sensing matrix after many iterations called generations.

An important and a basic question now becomes that which structured samples should one select and aim for? As a result, for a given application, the non-zero coefficients are chosen on the basis of the undersampling ratio. Mathematically,

$$\hat{\Phi} = \underset{\Phi:|\Phi|=n}{\arg\max} \frac{1}{m} \sum_{j=1}^{m} \sum_{i\in\Phi} |\langle \Psi_i, x_j \rangle|^2, \tag{6}$$

where Ψ is the orthonormal sparse basis operator and Φ is the sampling matrix from which $\hat{\Phi}$ is learned.

Effective signal sampling is also critical in remote sensing applications to maintain the quality of image. The remote station power consumption is crucial and must be considered

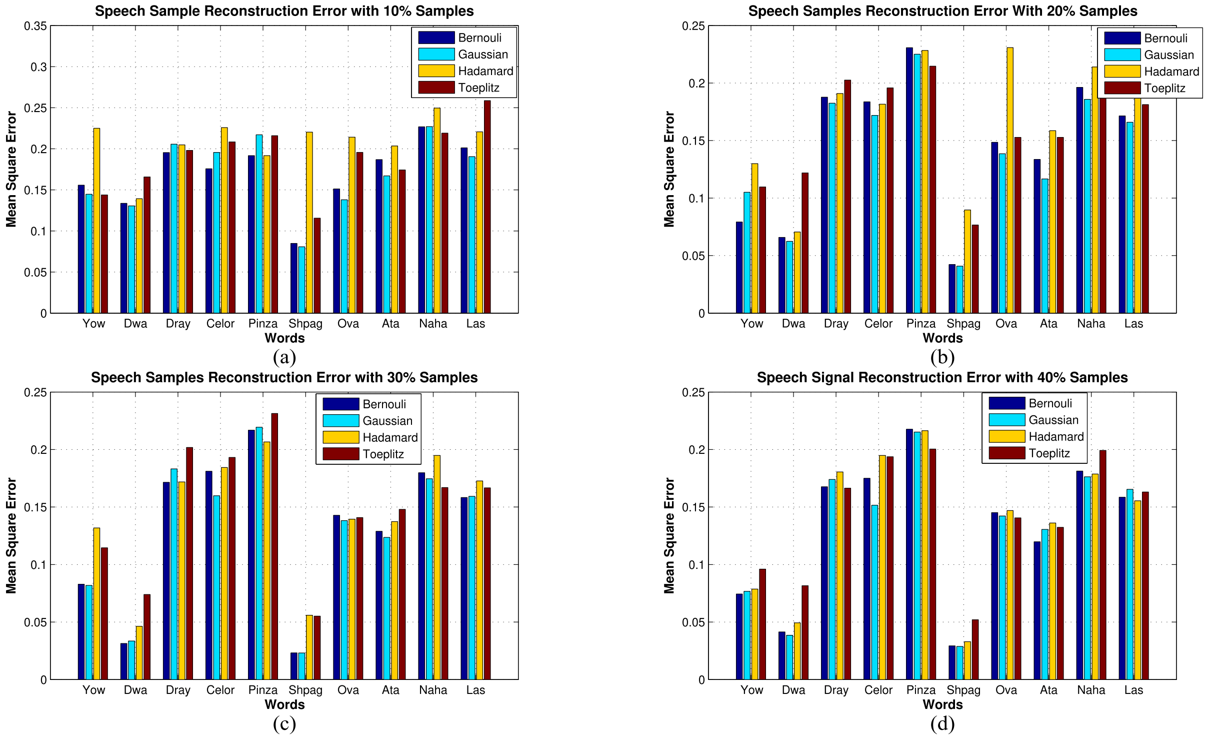

FIGURE 6. Speech samples recovery using Bernouli random matrix, Gaussian random matrix, Hadamard matrix, and Toeplitz matrix from (a) 10%, (b) 20%, and (c) 30%, (d) 40% of the original signal samples.

for in these applications, and traditional transform coding methods are not feasible. For signal sampling and reconstruction, the proposed framework in [41] used CNN. The devised setup is made up of various layers that are in charge of signal sampling and reconstruction. The obtained results demonstrate the efficacy and usefulness of the proposed framework in the CS domain for effective real-time applications.

IV. SPARSE SIGNAL RECOVERY

After sparse domain sampling, recovering an enhanced quality original signal from an incomplete set of data becomes a relatively difficult task at the receiver. The computational process for recovery is aided by limited statistical parameters of signal obtained from a small number of samples.

A. COMMONLY USED RECONSTRUCTION TECHNIQUES

Compressed sensing (CS) is commonly used to reconstruct the original signal using the least possible number of samples. Some of the optimization-based reconstruction techniques, including Basis Pursuit (BP) [42], Orthogonal Matching Pursuit (OMP) [43], Total Variation (TV) [44], Compressed Sensing Matching Pursuit (COSAMP) [45], and Iterative Hard Thresholding (IHT) [46], have started to gain attention owing to its successful and an enhanced signal reconstruction at the receiver.

BP is a type of convex relaxation algorithm based on the the following equation for sparse coefficients recovery.

$$\hat{\alpha} = \underset{x \in \mathbb{R}^n}{\text{minimize}} \; ||x||_1 \quad \text{s.t} \; ||y - Ax||_2^2 \leq \epsilon, \tag{7}$$

Here ϵ is the ℓ_2 error controlling term while $\hat{\alpha}$ is a set of sparse domain signal coefficients. With the help of (8), $alp\hat{h}a$ is utilised to locate the original signal 'x' by minimizing function.

$$x = \Psi^* \hat{\alpha}, \tag{8}$$

where $\Psi^* = \Psi^{-1}$ denotes the unitary transformation.

For statistical inference, a modified version of ℓ_1 minimization is used for sparse signal recovery. This technique is called Least Absolute Shrinkage and Selection Operator (LASSO) [47]. The mathematical formulation of LASSO can be described as:

$$\hat{\alpha} = \underset{x \in \mathbb{R}^n}{\text{minimize}} ||y - \Phi\Psi x||_2^2 + \lambda ||x||_1, \tag{9}$$

Here λ represents regularisation parameter.

The Linear reconstruction method is considered as the simplest for solving full ranked equations, and it is based on (10).

$$\hat{x} = A^{-1} y. \tag{10}$$

However, the use of (10) is restricted only to fully ranked systems. As a result, the system's least square solution is as

$$\hat{x} = \underset{x}{\text{argmin}} \, ||y - Ax||^2. \tag{11}$$

It is notable that the (11) has a solution in vectoral format as below:

$$\hat{x} = (A^T A)^{-1} A^T y, \tag{12}$$

where A^T is the transpose of matrix A and the expression $(A^T A)^{-1} A^T$ is known as pseudo inverse.

B. MODERN TECHNIQUES FOR CS BASED SIGNAL RECONSTRUCTION

1) BLOCK COMPRESSED SENSING

A performance comparison of sparse signal reconstruction techniques was done previously on this area. [24], [48], sampling modality for 1-D signals [49], sampling and reconstruction of biological and thermal images [50], [51]. The study is carried out in [52] which presented a CS configuration oqing to the distinctive time-frequency nature of audio signals. It narrated the representation of speech signals and compared the performances of widely utilised speech signal recovery algorithms. As seen earlier, linear signal reconstruction techniques are also enabled using the data fitting technique to the original data samples and are based on the principle of minimization of the norm between the original dataset and predicted data samples. The speed performance and efficieny of the system may degrade when employing the overall data in the optimized model. For this reason, Block Compressed Sensing (BCS) [53] is utilized to mitigate this issue a technique by breaking down a large dataset into mini blocks or batches. A. Shen *et al.* suggested an approach for gathering information in the form of blocks and ultimately recovering the original signal at the receiver using Lapped Transform (LT) to take the temporal correlation between various blocks [53]. The results of the recovered signal was very much interesting and enhanced from other techniques not deploying LT. It is further demonstrated that $\ell 1$-norm minimization, often known as Basis Pursuit (BP), may be used to successfully recover signals [42]. A close analysis of the techniques, namely BP, Matching Pursuit (MP), Orthogonal MP (OMP), and frames method, showed that the BP significantly outperformed the rest of methods [42].

2) BAYESIAN TECHNIQUES

Probabilistic techniques have been frequently employed to reconstruct original signals in signal recovery and estimation jobs. Bayes' theorem is used as the foundation of proposed configurations in a wide range of data recovery applications. Due to its effectiveness in information synthesis, the posterior probability distribution has become one of the most preferred algorithms for signal recovery from a small quantity of data. A wide range of CS based optimization algorithms for ECG signal reconstruction are investigated and compared with the Bayes' theorem [20]. It demonstrates that Bayesian recovery outperforms a range of traditional recovery techniques.

According to the Baye's theorem, the posterior probability of b is based on some assumed priors and can be written as:

$$P(b|a; \gamma_j X_j) = N(\mu_b, \Sigma_b) \tag{13}$$

13 finds the parameter on the basis of type II maximum likelihood estimation. As the posterior probability approaches the mean of b, Maximum-A-Posteriori (MAP) estimation can be easily determined. Such framework is called Sparse Bayesian Learning (SBL).

In the CS literature, efforts are made to build sparse structured sensing matrices using modern data computing approaches. The work done in [19] proposed Block Sparse Bayesian Learning (BSBL) after developing a sparse binary matrix for speech signal sampling and reconstruction. The temporal correlation among signal elements is exploited in BSBL based signal recovery. It demonstrated the use of Bayesian approach for signal reconstruction in conjunction with the developed matrix. The performance of proposed and baseline approaches is evaluated on the basis of Structural Similarity Index (SSIM). When comparing BSBL with basis pursuit, BSBL has shown promising results outperforms in the speech signal recovery task.

ECG, EMG, and EEG reconstruction have all been successfully accomplished using CS applications [54], [55]. BSBL has also outperformed in the reconstruction of biomedical signals [54], [56], [57]. The benefits of BSBL in respect of speed, precision, and durability have also been applied to EEG imaging [58]. Moreover, the applications of BSBL for real time montiroing of machine conditions are also demonstrated in [59]. Chun-Shien Lu *et al.* [60] also used one of these strategies to recover image information by taking advantage of the relationship between image samples. It also investigated the ability of transformed coefficients to represent signals on a sparse basis. Compressive Image Sensing (CIS) is the name given to this method, which is investigated for a variety of 2D and 1D signals and observed to provide accurate signal reconstruction with improved speed.

In the literature, numerous approaches are studied for original information retrieval from subsampled lower-dimensional data samples. The Bayes theorem is also used in the CS framework's signal reconstruction tasks. According to Shannon's sampling theorem, the work in [61] demonstrates time invariant sampling of information. Partial information sampling has been shown to save energy while simultaneously raising the cost of sampling equipment. The proposed methodology, on the other hand, employs Bayesian Compressed Sensing (BCS), a technique that allows for the collection of very small yet highly informative data samples. To analyse the present data and forecast future measurements, a BCS-based adaptive technique is proposed.

The following steps are executed in this work:

1) At first, a random subset of samples is taken.
2) For these measurements, the posterior mean and variance are determined.
3) More data is gathered to ensure a large posterior mean and variance.
4) The measurements with the highest posterior mean and variation are chosen.
5) To limit the possibility of local optima, the motion planning algorithm forecasts the next data sample from a smaller number of measurements. Accurate measurements are predicted using P_i measurements.

As previously indicated, spatial domain is used for signal sampling, in which image 'x' of 32×32 pixels is used to retrieve a 5×5 block. Following that, signal 'y' is computed as $\mathrm{y} = \phi \mathrm{x}$ and $\mathrm{y} = \phi \psi \mathrm{S}$, where 'S' is a 2D wavelet transform of a grey scale picture. The rows of ϕ has 5×5 non-zero

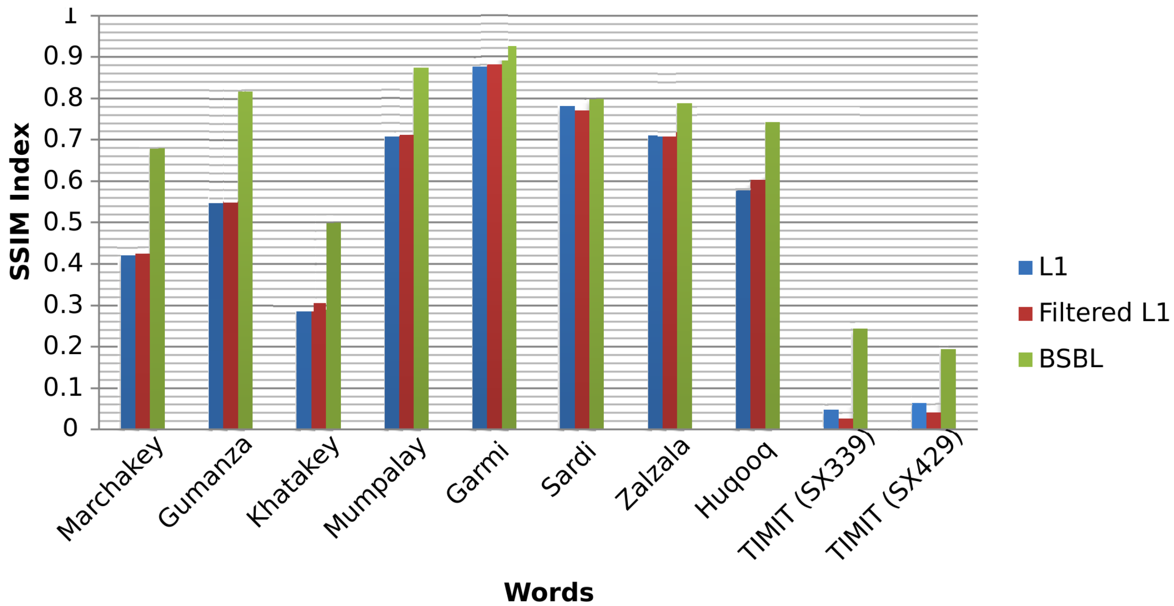

FIGURE 7. Comparison of BSBL with ℓ_1 and filtered ℓ_1 recovery.

elements. The reconstruction error is calculated using the following mathematical equation as in [61], which is then used to evaluate performance:

$$Error(y_{rec}; y) = \frac{||y_{rec} - y||_2}{||y|||_2}, \tag{14}$$

where y and y_{rec} denoted the original and reconstructed signals, respectively. When adaptive and random measuring methods are compared, adaptive sampling outperforms in terms of lower reconstruction error.

3) EVOLUTIONARY METHODS

Evolutionary methods are algorithms that are inspired by nature and are used to iteratively find the optimum answer. To generate the best offspring, a set number of iterations, known as generations, is used. Sparse recovery algorithms based on such tactics have lately been applied in the CS sector. A Genetic Algorithm (GA) is a form of evolutionary mechanism that resembles Darwin's theory of evolution. M. Heredia et al work 's in the CS [62], [63] demonstrated the use of GA rather than ℓ_0 or ℓ_1 recovery approaches. The *chromosomes* created in this study demonstrated a variety of sparse setups, and the solutions discovered for each time span are linked to them.

For constructing optimum data-driven models, evolutionary techniques such as the Genetic Algorithm (GA) have been applied. GA offers a wide range of applications that have showed promise for data recovery. In [63], which uses the Curvelet Transform (CT) [64] for sparse signal encoding, a GA-based data recovery is discussed. The sensing matrix used in this study was a Gaussian matrix, and GA was employed as a reconstruction technique. The PSNR obtained by DCT, DWT, and OMP is compared to that obtained by the proposed technique, and it is demonstrated that the new framework outperforms the other methods.

4) MACHINE LEARNING AND DEEP LEARNING

Machine Learning (ML) approaches have demonstrated their efficacy in classification, regression analysis, data mining, natural language processing, computer vision, and other areas in recent years. ML techniques are used in data estimation and recovery jobs because of their data-driven nature. To execute the needed task, ML models must be trained with a sufficient amount of data. The usage of Artificial Neural Networks (ANN) for data recovery was described in [65]. Training and testing tasks are carried by using data from a Wireless Sensor Network (WSN). ANN estimates the original signal using the data samples provided at the input side and altered weights of different layers. Increasing the number of layers in an ANN reduces training error, but at the cost of increased processing power. The performance of the ANN-based recovery approach was compared to that of IHT and OMP in another study [65]. While neural networks have a faster reconstruction speed than IHT and OMP, their reconstruction accuracy is inferior to that of high-performance convex relaxation techniques, such as $\ell 1$-norm recovery, which outperforms other reconstruction techniques [24]. Working in the same domain yielded a more complex ANN with a greater number of hidden layers. The network is therefore known as a Deep Neural Network (DNN), and many efforts are being made to retrieve high resolution data from a small amount of input [66]. Since the advent of AI applications, DL has been used in a variety of applications including signal reconstruction. In [65], an ANN is presented for reconstructing the original signal in the CS framework. In comparison to OMP and IHT, the proposed ANN-based method has better recovery performance and speed. The work done in [68] and [69] proposed novel techniques for CS based symbol detection and image reconstruction tasks. In this work, Convolutional Autoencoders are deployed for both CS based sensing and reconstruction. It pioneered the block-wise image sensing and reconstruction task, which improved a system's performance by up to 0.8 dB PSNR. Similarly, Sparse Autoencoder based CS framework is also deployed in [70] and has yielded promising results.

In CS framework, the prudently designed sampling matrix is able to assist in effective signal recovery task [29]. The proposed work in [71], have used Autoencoder wherein encoder and decoder are employed as sampling and reconstruction network, respectively. The proposed network, known as the Stacked Sparse Denoising Autoencoder Compressed Sensing (SSDAE CS) model, yielded better recovery performance and speed. A similar setup has been proposed in [72] utilising a flexible strategy reconstruction of an MR image. Adaptive learning from training images is the foundation of the proposed technique. When compared to modern reconstruction algorithms, the experimental results demonstrated the effectiveness of the obtained results.

The recently developed BSBL approach has shown promising results as compared with the widely used ℓ_1-norm minimization technique. The work done in [19] illustrated the use of BSBL for the reconstruction of speech signals. The deployment of Wavelet filtering is also checked to enhance the quality of recovered signals, but BSBL has shown improved reconstruction performance as shown in Figure 7.

V. OTHER APPLICATION AREAS OF COMPRESSIVE SENSING

From communication system design to biological data gathering systems, CS can be used in a variety of situations. Because of power and bandwidth efficiency, data gathering with a small number of samples is usually emphasised. The following are some examples of CS applications in various fields:

A. ANALOG TO DIGITAL CONVERSION

With the emergence of digital signal processing techniques and technologies, analogue filtering and processing have been put back by digital hardware. Analogue to digital conversion is one of the most significant accomplishment due to its interfacing ability between generated data and digital hardware. As a result, for a digital system to process data, it must be in a suitable format. In this context, Analog to Digital Conversion (ADC) plays an important role in the modern digital age. ADC refers to a sampling and quantization-based method. In a traditional ADC system, the Nyquist sampling theorem is employed for signal sampling, however this is superseded by CS [73]. The basic phases of demodulation, filtering, and sampling are replaced by sparse signal transformation via a conversion matrix (Ψ) and signal information to discrete sample conversion matrix (Φ). The operations of ADC is depicted in Figure 8.

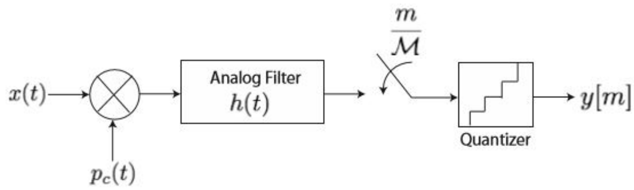

FIGURE 8. Analog to digital conversion.

The following is the system equation based on the diagram [73]:

$$X[m] = \sum_{i=1}^{N} \alpha_i \int_{-\infty}^{\infty} \psi_i(\tau) r_q(\tau) h(mM - \tau) d\tau. \tag{15}$$

The system is known as Analog Information Conversion (AIC), and it replaces traditional ADC with CS-based transformation and sampling to reap the advantages of CS.

B. INTERNET OF THINGS AND WIRELESS SENSOR NETWORKS

The capability of sparse and low rank approximation stimulates a wide range of CS frameworks. Another use of CS is in sensor networks, where the number of sensors can be reduced in order to obtain higher-dimensional data from these sensors. Information redundancy in linear measurements wastes a lot of sensor resources and time in a lot of sensor networks. As a result, adaptive sensing is utilised in mobile sensor networks to save money by eliminating static sensing, which is capable of performing reconstruction based on previously calculated distributions. The [61] looked for novel techniques to sense useful information content in the available data. This paper looked at how a small number of mobile sensors could be used to reconstruct original data samples in sensor networks.

IoT architectures benefit substantially from compressed sensing. It helps low-power, low-bandwidth sensors that don't have local processing units. As a result, it uses fewer sensors. Figure 11 depicts the outline of a traditional CS-based IoT. WSNs used for data collection typically have limited computational and power resources. Higher data rate sampling and transmission are not only more expensive, but also more susceptible to bit error. In the same way, real-time CS applications in WSN are limited due to higher resources utilisation. The algorithm of [75] facilitate to send large amount of data in smaller chunks. Each block is treated as a distinct entity at the receiver end, and CS is applied to each one. The original signal reconstruction methods use the blocked samples.

The B^2 samples converted from a B×B pixel image block are transmitted in this work using the mathematical equation below:

$$y_i = \phi B x_i, \tag{16}$$

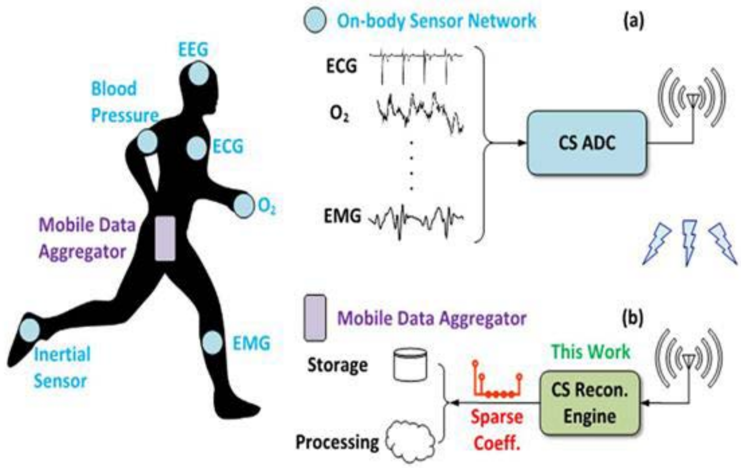

FIGURE 9. Compressed sensing in health care [67].

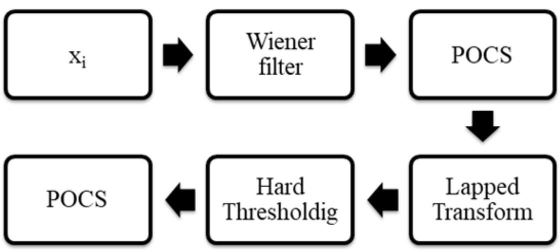

FIGURE 10. First stage of non-linear samples reconstruction.

where ϕB is an Independent and Identically Distributed (I.I.D) Gaussian matrix. Total ϕ is a diagonal matrix, where ϕB are its diagonal entries.

On the receiver side, an initial solution was obtained using MMSE-based linear estimation with autocorrelation. Non-linear signal reconstruction is used in two stages to improve reconstruction efficiency.

Stage 1: For further improvements, the first stage employs a linearly reconstructed signal x_i. The following procedure is repeated five times as shown in Figure 10.

where POCS is a term that refers to a projection on a convex set.

The Lapped transform is used in the block processing mechanism because it is more computationally efficient than the Wavelet transform. Gaussian noise is also eliminated using hard thresholding.

Stage 2: In order to store a block at its position in a second stage, a decimated Wavelet transform is used to capture information of an overall image. The frame is then processed, block-wise by using the lapped transform, which preserves local information in the frame.

The above steps are applied to a variety of imaging applications, and in all scenarios, block-based CS is found to outperform [75].

C. IMAGE AND VIDEO TRANSMISSION

The CS concepts are also used in the design of error correction modules in image and video transmission systems. Deng *et al.* proposed a codec based on CS that improves image error resilience [21]. The codec uses a multilevel 2D Discrete Wavelet Transform (DWT) to make a non-sparse image signal sparse. Because the coefficients of the low frequency sub-band contain the most information, it allocated more measurements to the coarser level. This scheme improves error robustness at high packet loss rates without relying on an explicit error resilience method, resulting in better performance with less complexity than current JSCC schemes. Single Pixel Multi Time (SPMT) and Multi Pixel Single Time (MPST) imaging cameras are two types of traditional CMOS or CCD cameras. The majority of cameras in use today are MPST, which necessitates additional power and processing resources to do the operation. With the introduction of compressive sensing, it has become increasingly desired to rebuild the original signal from a small number of meaningful samples in order to lower a process's power consumption. As a result, Rice University researchers created an SPMT Single Pixel Camera (SPC) to enable it with the inherent capabilities of CS [76].

SPC can also be used as a night vision camera when operated in the infrared range. Besides sensing flexibility, the practical advantages of SPC design stem from the fact that photodiode quantum efficiency is higher than that of pixel sensors in a typical CCD or CMOS array. As a result, the fill factor of a DMD can reach up to 90%, whereas the fill factor of a CCD/CMOS array is around 50%. Another advantage to emphasise is that each CS measurement receives approximately N/2 times more photons than the average pixel sensor, significantly reducing image distortion from dark noise and read-out noise [76]. The overall procedure is depicted in Figure 13.

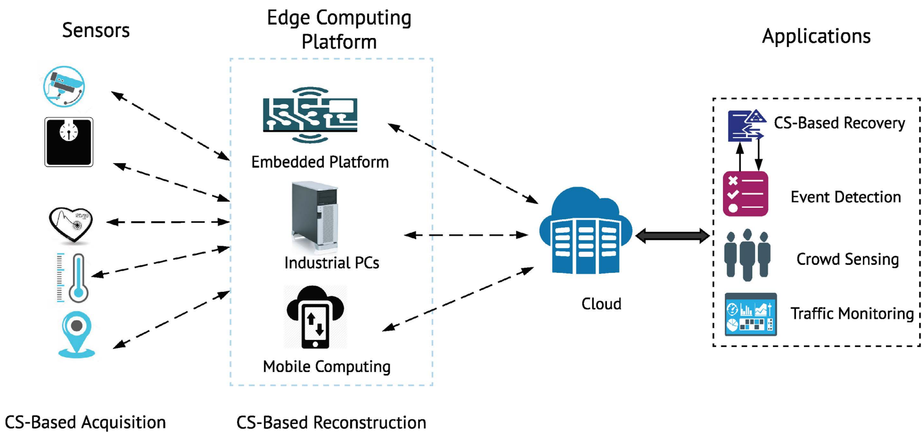

FIGURE 11. Internet of things [74].

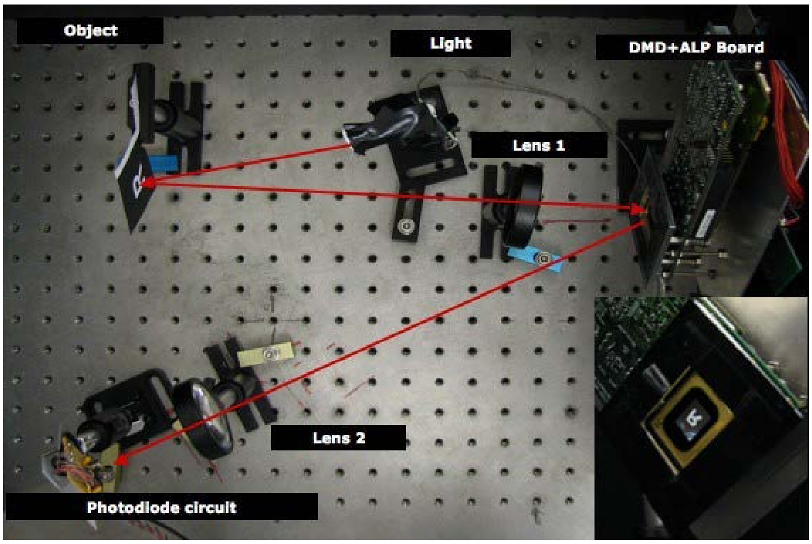

FIGURE 12. Experimental setup for single pixel camera [76].

D. CS IN DATA ACQUISITION SYSTEMS

In resource-constrained applications, CS has a number of features that make low-power signal transmission possible. The research presented in [77] provided an overview of the sensing, transmission, and recovery of the widely available CS-based techniques for EEG signals. The shortcomings of traditional EEG signal sampling and transmission mechanisms are explained and compared to the CS-based techniques.

As shown in Figure 9, signal acquisition and reconstruction have a wide range of applications in healthcare, e-commerce, smart homes, surveillance systems, and a variety of other areas. By utilising an optimised set of statistical features, a GA-based model based on the human behaviour and pattern recognition is presented in [78]. Similarly, the use of artificial intelligence (AI) in the analysis and prediction of infrastructure damage has been discussed in [79]. This paper gave an overview of ANN applications mainly for investigating the concrete structural stiffness. Recent developments on signal acquisition and recovery applications have been beneficially incorporated in several domains, including identification [80], surveillance [81], information clustering [82], automobile industry [83], and mechanical design analysis [84].

The applications of pertaining to data collection and acquisition have expanded in response to rising trends in intelligent system design and analysis with he emergence of AI. The data collected by smart sensors is processed to train data-driven intelligent models, with the goal of enhancing data sampling for a reliable signal recovery process. A similar concept has been applied in telemonitoring and telemedicines to predict and recognise activities with the aid of smart wearable sensor specifically trained for the purpose [78]. As illustrated in Figure 9, this smart wearable sensor has found many applications for remote health monitoring

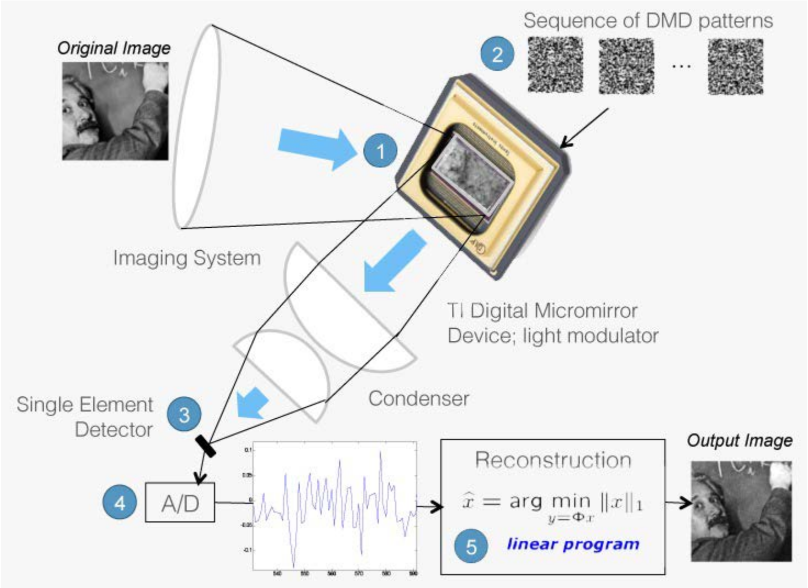

FIGURE 13. Single pixel camera.

systems in smart homes. It consists of power-efficient, portable, and bandwidth-efficient sensors that require the least amount of computation resources.

In [85], one of these designs is proposed, which uses depth cameras and Hidden Markov Models (HMM) to monitor, record, and recognise activities in smart home applications. Depth cameras were also used as smart sensors for monitoring and data acquisition tasks in the work described in [86]. In this paper, a life logging system is created by combining several steps, including data collection, feature extraction, HMM training, and deployment of trained models for healthcare applications.

VI. PERFORMANCE EVALUATION METRICS

To evaluate the performance of the system under consideration, the signal recovery procedure employs a quantitative measure of similarity between the original and recovered samples. This section provides an overview of commonly used performance evaluation measures in order to gain a basic understanding of performance metrics.

In many signal processing and estimation applications, the Mean Squared Error (MSE) is commonly employed as a performance metric to quantify the degree of similarity between signals. The mean squared sum of the differences between real and measured data samples is known as the MSE [87]. MSE can be written as

$$MSE(a, b) = \frac{1}{M} \sum_{i=1}^{M} (a(i) - b(i))^2. \tag{17}$$

There is also another type of MSE which is called the Normalized Mean Squared Error (NMSE) [88]. It is also used in many signal estimation and recovery task for performance evaluation of the reconstruction algorithms due to its error normalization capabilities [89], [90], [91]. Mathematically it can be written as:

$$NMSE(a, b) = \frac{\sum_{i=1}^{M} (a(i) - b(i))^2}{\sum_{i=1}^{M} a(i)^2}. \tag{18}$$

In many scenarios, the reconstruction performances of the proposed models is evaluated on the basis of MSE and Signal-to-Noise Ratio (SNR) obtained by comparing original and reconstructed signals. The mathematical formulation of SNR is shown as

$$SNR(a, b) = 20 \times \log_{10} \frac{||a_i||_2}{||a_i - b_i||_2}. \tag{19}$$

In CS-based applications, the SNR is also utilised as a performance parameter. Aside from that, the RMSE value of the original and reconstructed signals is also utilised as a statistic. RMSE between the original signal 'a' and reconstructed value 'b' is

$$RMSE(a, b) = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (a_i - b_i)^2}. \tag{20}$$

The squared terms in MSE cause many engineering applications to perceive it as the signal's energy deviation metric. When a similarity measure of perceptually important data (such as speech and images) is to be assessed, MSE has failed to act as an assessment tool. As a result, the SSIM index is another qualitative measuring metric utilised in CS [87]. Because of its capacity to gather perceived changes in structural information, SSIM is utilised instead of MSE and PSNR. This qualifies it as a perception-based model.

Mathematically SSIM can be written as:

$$SSIM(a, b) = \frac{(2\mu_a \mu_b + z_1)(2\sigma_{ab} + z_2)}{(\mu_a{}^2 + \mu_b{}^2 + z_1)(\sigma_a{}^2 + \sigma_b{}^2 + z_2)} \tag{21}$$

The mean values of signal a and b are μ_a and μ_b, respectively, and the variances of a and b are $\sigma_a{}^2$ and $\sigma_b{}^2$ in (21). The correlation between a and b is called σ_{ab}. The two variables z_1 and z_2 are used to stabilise the division with a weak denominator in a similar way.

The analytical comparison of different performance evaluation metrics for a tabular, audio, visual, and other types of datasets has been made and it is observed that the calculation of SSIM involves complexity as compared to MSE, RMSE, SNR, and PSNR, but it better suits the applications where CS is applied for the sampling and reconstruction of unstructured data, where perceptual information retrieval is more important. Similarly, MSE, RMSE, SNR, and PSNR etc are simple to implement and pose the numerical difference between the original and reconstructed data samples. Therefore, these performance measures can be used in applications where sampling and reconstruction of structured data is involved.

VII. MAJOR TAKEAWAYS

In this work, we presented the innovations, challenges, and future prospects of CS based signal sparsity, sampling, and its reconstruction from the limited number of sampled data. Based on the recent literature demonstrated in this paper, the prime conclusions and takeaways of this work are listed below.

- In signal sparsity, different transform bases are studied for a variety of signals including 1-D audio signals, visual images, and tabular data as well. It is shown that DWT can be efficiently deployed for images sparse transformation, while for 1-D signals, such as biomedical signals and audio signals, DCT has been shown to perform effectively. Moreover, training based transformation bases, e.g., K-SVD, demonstrates better performance at the cost of higher training time. Therefore, the deployment of readily available sparse transformation bases can be used for fast operations for a specified kind of datasets. Similarly, ksvd can be used for all datasets to provide the refined and fixed number of sparse coefficients, and perform better as compared to the readily available transforms at the cost of more training data and computation resources.
- Variety of approaches for signal subsampling are discussed in this paper, which includes random sampling (Gaussian and Bernoulli matrices), deterministic sampling (Hadamard and Toeplitz matrices), and data-driven training based sampling (LBCS, GA-LBCS, and GA trained matrices). The comparative analysis of the previously deployed sampling techniques has demonstrated significant improvements in sampling speed and reconstruction accuracy with the deployment of learning based techniques i.e., GA-LBCS, LBCS, and GA. However, the improvement is achieved at the cost of more training data, training time, and high performance hardware requirement. The graphical illustrations of Figure 15 and 14 shows the analytical comparison of customary sampling techniques and training based mechanisms i.e., LBCS and GA.

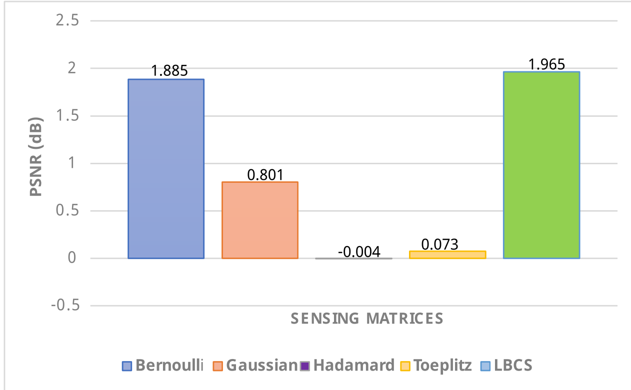

FIGURE 14. Sampling performance improvement by using learning based compressive subsampling.

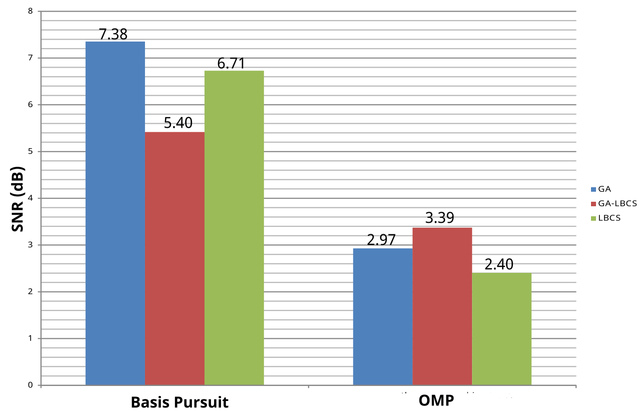

FIGURE 15. Performance attainment by GA based learning technique.

In Figure 14, the main concern was to compare the reconstruction performance of ℓ_1-norm minimization technique when samples were provided by the widely used random and deterministic sensing matrices as compared to the matrix developed by training-based mechanisms. The higher SNR obtained from the samples provided by learning-based techniques developed matrix demonstrates the performance improvement of these techniques. Although, some random sensing matrices such as Bernoulli random matrix, has also shown comparative results when compared with the developed technique, yet the computational requirements made offline learning-based stand apart, at the cost of more training data and time.

In continuation of the above mentioned work, Figure 15, the SNR of the reconstructed signals are compared, when samples were provided by the sensing matrices developed by the data-driven offline learning-based techniques i.e., GA Based Technique, Learning Based Compressive Subsampling (LBCS), and the GA based Learning Compressing Subsampling

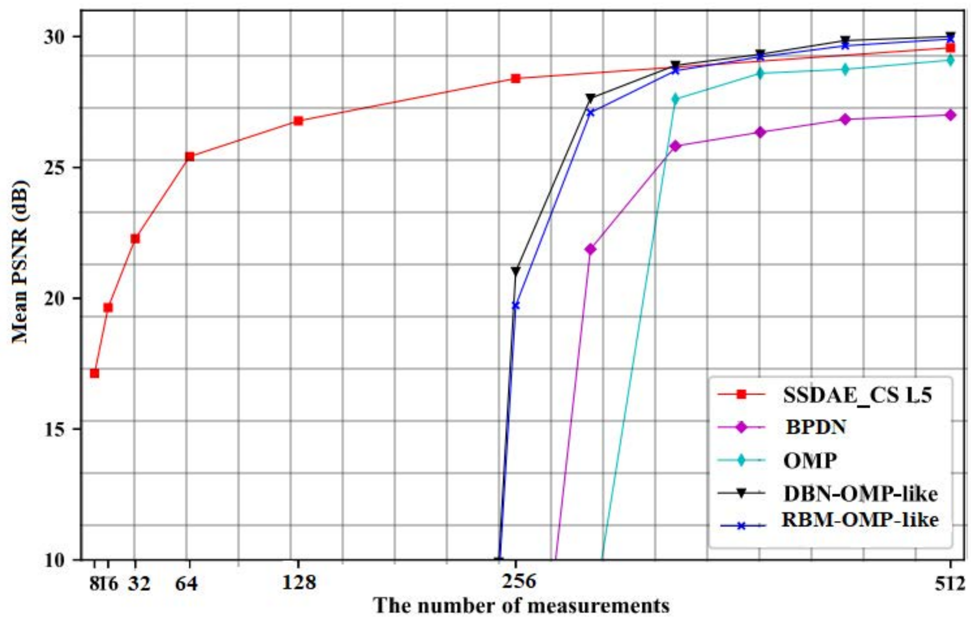

FIGURE 16. Improved CS setup by using deep Autoencoders [92].

(GA-LBCS). In this figure, the first group of bars demonstrate the SNR of the reconstructed signals with respect to the original signal, when L1-norm minimization technique is deployed for the reconstruction purposes. The *ell*$_1$-norm minimization belongs to the class of convex relaxation algorithms, which exhibits better reconstruction performance at the cost of reconstruction speed [40]. Similarly, the second group of bars is for greedy reconstruction algorithm which is the Orthogonal Matching Pursuit (OMP). The use of OMP is preferred where reconstruction speed is crucial as compared to the reconstruction accuracy. In this figure, different learning-based sensing matrix design methods are compared due to their enhanced performance in facilitating signal reconstruction algorithms for reconstructing original signal.

- The availability of efficient signal reconstruction technique can enhance the performance of CS frameworks. Therefore, we discussed and analyzed the performances of conventionally used reconstruction methods available in CS based literature. We studied the accuracy and speed improvements achieved by deploying ℓ_1-norm minimization, OMP, Bayesian method, linear reconstruction, Machine Learning methods, and Deep Autoencoders for the variety of signals. It is concluded that the utilization of Deep Autoencoders tends to enhance the quality of a signal when reconstructed from the limited number of samples. The simulation results shown in the form of Figure 16 demonstrates the performance enhancement by deploying Deep Autoencoders.
- In this paper, we also reviewed some applications of CS in variety of fields which include data acquisition systems, multimedia communication, health sector, telecommunication, speech processing, and IoTs etc. These fields are prone to resources scarcity with the exponential rise in digital data. While the deployment of CS has resolved many issues, still it has the tendency in performance improvement with the application of sophisticated sampling and reconstruction techniques, which are suggested as a future work in each field of applications.

VIII. CONCLUSION

A thorough background understanding of CS frameworks is demonstrated by going through the literature work done in the field. This work is made up of a comprehensive review of the literature on all of the processing blocks and mathematical operations that gave useful understanding of the area for grasping advanced concepts and technicalities of compressive sensing. The work given in this paper aims to encourage the development of novel strategies for efficient signal sampling and recovery in sparse basis. The inherent benefits of sparse signal modification for sampling and reconstruction contribute to the range of potential applications of CS, which is evident in its deployment for a wide range of application areas. In order to accomplish this, a set of the most widely used sampling matrices and reconstruction algorithms are investigated and analysed in terms of reconstruction accuracy and speed, and then used as baseline approaches. The majority of the existing CS literature is devoted with the development of faithful signal reconstruction algorithms. These efforts culminate in the development of technologies for signal recovery that are more power-consuming and computationally infeasible. As a result, this study also examined the deployment of data-driven, learning-based methods for their use in addressing the challenges connected with traditional CS frameworks.

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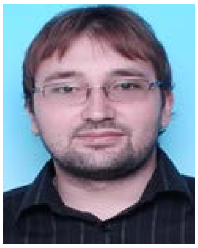

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