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RESEARCH ARTICLE

The Optimal Investment and Re-Guarantee Purchase of China's Financing Guarantee Institutions Considering Risk Preference

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ABSTRACT Financing guarantee institutions achieve capital preservation and appreciation through investment, and diversify business risks by purchasing re-guarantee. In order to study the optimal investment and re-guarantee purchase strategies of financing guarantee institutions, the geometric Brownian motion modulated by Markov chain modulation is selected to describe the price process of risk assets, the Hamilton-Jacobi-Bellman equation is constructed based on the utility maximization criterion, and the solutions of optimal investment and re-guarantee purchase strategies are discussed under the exponential utility function. Ultimately, the influence of relevant parameters on the optimal strategies is studied through computational experimental simulation method. The results showed that the risk-free interest rate, risk aversion coefficient and guarantee period have significant effects on the optimal investment and re-guarantee purchase strategies. The market mechanism only affects the trend of the optimal investment strategy, but has no effect on the optimal re-guarantee purchase strategy. However, the increase of the re-guarantee institution's safety loading and the guarantee recovery rate will significantly reduce the re-guarantee purchase ratio.

INDEX TERMS Financing guarantee institution, investment, re-guarantee purchase, utility.

I. INTRODUCTION

Because of the complexity of financing business, commercial banks inevitably face certain credit risks [1]. Fortunately, governments worldwide have established financing guarantee mechanisms for enhancing small and medium-sized enterprises' (SMEs) credit levels [2]. In China, there are currently 5,139 legal person institutions nationwide, with a guarantee balance of 3.26 trillion yuan, which has effectively promoted the development of inclusive finance business. The risk of financing guarantee compensation in developed capitalist countries is shared by multi-agent and ultimately covered by the national finance, while the original guarantee institutions, re-guarantee institutions, banks and local governments share and replace them according to an agreed proportion in China [3]. However, in the process of sharing the compensation risk, the re-guarantee institution sets a limit, and no compensation will be paid if the limit is exceeded. The local government also performs the compensation risk depending on the financial situation, so that the financing guarantee institution with lower guarantee fees has to pay most of the compensation. In order to compensate, it is necessary to make up for the compensation risk of the guarantee business with profits from investment and other businesses. Therefore, trying to maintain the balance between various businesses and risks, including the income from external investment of assets and the purchase of re-guarantee products, has become an important business issue in the operation of China's financing guarantee institutions.

With the deepening of financing guarantee practice, research results consistently show that credit guarantee has significant effects in alleviating credit constraints between

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banks and SMEs, reducing the incidence of credit rationing, and improving loan availability [4]–[6]. On this basis, scholars further explored the risk management and operating benefits of financing guarantee institutions. In terms of risk management, most scholars focused on risk assessment of guarantee products [7], [8], identification of risk factors in guarantee networks [9], [10], guarantee risk control mechanism [11]–[13], etc.; while a few scholars studied the risk diversification under the multi-agent cooperation mechanism. For instance, Wang *et al.* used evolutionary game to analyze the adjustable range of re-guarantee risk sharing operation mechanisms such as the multi-agent risk sharing ratio and government risk subsidies in the guarantee market [3]. In terms of operating benefits, relevant research results mainly focused on the formulation of guarantee pricing strategies. Based on actuarial pricing methods, He *et al.* built a credit guarantee pricing actuarial model with two parameters of guarantee fee and guarantee rate to simulate guarantee benefit under different parameter combinations [14]. Tahizadeh-Hesary *et al.* discussed the decision-making mechanism of guarantee pricing under the influence of default risk rate, economic conditions and other factors from the perspective of macroeconomics and guaranteed enterprise's financial characteristics [15].

Although the above research involves the impact of guarantee pricing or re-guarantee ''risk sharing'' on the benefits of guarantee institutions, in fact, the business pricing of guarantee institutions is controlled by the governments. Accordingly, moderate investment has become an important means of supplementing working capital [16]. Since Markowitz proposed the use of the mean-variance model to solve investment optimization problems, scholars have paid more attention to the characterization and optimization criteria of the company's surplus process [17]. Regarding the characterization of the surplus process, most of the early studies assumed that the company's surplus is a random variable, and subsequent studies extended it to the Cramer-Lundberg model with a composite Poisson process to characterize surplus [18], [19]. Since then, some studies have developed it to a more general jump-diffusion model that uses geometric Brownian motion to approximate the claim amount and overcome the discontinuity of the surplus function in the Cramer-Lundberg model [20].

As for the selection of optimization criteria, the following categories of criteria are widely used. The first is to minimize the company's bankruptcy probability, that is, to minimize the company's operational risk by formulating the optimal investment portfolio and reinsurance strategies [21]. The second is the mean-variance criterion, which converts the bi-objective planning problem of minimizing risk and maximizing expected utility into a single-objective decision model through a linear combination method, and maximizes the utility of the company by determining the ratio of investment and reinsurance [22], [23]. The third is the maximization criterion of the company's terminal wealth utility. By selecting several types of typical utility functions, the optimal investment and reinsurance strategies are solved with the help of optimal control theory, so as to maximize the final expected utility [24]. Especially, the optimal strategy problem can be solved by using the dynamic programming theory proposed by Bellman [25], which its basic idea is to consider optimal control problems with different initial times and states. These problems are solved by building Hamilton-Jacobi-Bellman (HJB) equation to establish the correlation [26]. When the initial value of the optimal function is known, HJB equation has a display solution, and the solution is unique [27]. In fact, the HJB equation has been widely used to solve the optimal investment and reinsurance problems of insurance institutions [28]–[30].

Judging from the existing research, there are many studies on reinsurance ''risk sharing'' and the optimization of corporate investment portfolios. However, there is no relevant literature to directly study the optimal investment and re-guarantee purchase strategies of guarantee institutions. As a financing intermediary, the operation of guarantee institutions taking into account the random disturbance factors of the financial market, the risk asset portfolio and the risk diversification of re-guarantee is obviously a business difficulty, and it is also a problem that needs to be studied in depth. In addition, most of the former literatures assume that the price process of risky assets satisfies geometric Brownian motion, while a large number of studies have found that financial markets generally have the property of mechanism conversion [31]. To sum up, considering that the guarantee institution invests the company's surplus in risk assets or risk-free assets, and purchases part of the re-guarantee service under the proportional re-guarantee business model, this paper studies the choice of optimal investment and re-guarantee purchase strategies.

Compared with the existing research results, the main contributions are claimed in this paper. On the one hand, the price of risk assets is characterized by the geometric Brownian motion embedded in the Markov chain. The drift and diffusion coefficients of the geometric Brownian motion are randomized through the introduction of Markov chains, thus overcoming the shortcoming of the traditional Brownian motion in the portrayal of risky asset price. The model can not only reflect the low-order random fluctuations of the underlying assets, but also capture the high-order disturbances caused by sudden industrial policies or major emergencies. On the other hand, the Hamilton-Jacoby-Bellman equation is established, the optimal investment and re-guarantee purchase strategies are discussed under the exponential utility function, then the influence of the key parameters on the optimal strategies is analyzed through the calculation experiment simulation.

II. SURPLUS MODELING OF GUARANTEE INSTITUTIONS

For the convenience of analysis, the following basic hypotheses are given. Table 1 lists main symbols included in this study.

TABLE 1. Notation.

Symbol	Description
u	The drift coefficient of asset prices
σ	The volatility coefficient of asset prices
r	The risk-free interest rate
ρ	The guarantee premium rate
ρ'	The re-guarantee premium rate
ω	The safety loading of guarantee institution
η	The safety loading of re-guarantee institution
v	The recovery rate
λ	The risk aversion coefficient
θ	The average single compensation amount
δ	The average number of compensations
α_t	The state of the market mechanism at time t
x_t	The guarantee institution's funds at time t
P_t	The price of risk-free assets at time t
S_t	The price of risky assets at time t
R_t	The surplus of guarantee institution at time t
Y_i	The amount of the i-th compensation
β_t	The risk self-retention amount of guarantee institution at time t
γ_t	The proportion of the capital invested by guarantee institution in risky assets to the investable assets at time t

Hypothesis 1: Similar to the studies of Bauerle and Leimcke [32] and Ceci *et al.* [30], consider a frictionless and arbitrage-free financial market, all random processes and random variables are defined in a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$. There is a σ flow $\{\mathcal{F}_t, t \geq 0\}$ that satisfies the usual conditions, that is, $\mathcal{F}_t$ is right continuous and $\mathcal{P}$ is complete.

Hypothesis 2: There are two types of tradable assets in the market, one is the risk-free bond P, and the other is the risky asset S [28], and the prices of both are related to the market mechanism. A continuous-time homogeneous Markov chain $\alpha = \{\alpha_s\}_{0 \leq s \leq T}$ is used to describe the state of the market mechanism, and the state space of α is $\mathbb{Z} = \{e_1, e_2, \cdots, e_N\}$ (N is a positive integer), where $e_i = (0,0,\cdots,0,1,0,\cdots,0)^{\mathrm{T}}$ represents a column vector with the i-th element being 1 and the other elements being 0. Meanwhile, we assume the transition matrix of the Markov chain $\mathcal{Q} = (q_{i,j=1,2,\cdots,\mathrm{N}})$ is a conservative $\mathcal{Q}$ matrix.

Hypothesis 3: Risk assets like stocks and stock funds allow continuous trading, regardless of relevant taxes and transaction costs, and the quantity can be divided infinitely. Drawing on the studies of Jin *et al.* [31] and Bauerle and Leimcke [32], the price process of risky assets is described by the following stochastic differential equation:

$$\begin{cases} dS_t = u(\alpha_t) S_t dt + \sigma(\alpha_t) S_t dB_t^{(1)} \\ S_0 = S > 0 \end{cases} \tag{1}$$

Among them, $B_t^{(1)}$ is the standard Brownian motion defined in space $(\Omega, \mathcal{F}, \mathcal{P})$; $u(\alpha_t)$ and $\sigma(\alpha_t)$ are the drift coefficient and volatility coefficient representing the price of the risky asset at time t respectively, which depend on the market mechanism of the risky asset. If $\alpha_t = e_i$, then $u(\alpha_t) = u_i$, $\sigma(\alpha_t) = \sigma_i$. Additionally, it is also assumed that the time-varying process of the price process P_t of the risk-free asset satisfies the following differential equation [28]:

$$\begin{cases} dP_t = rP_t dt \\ P_0 = 1 \end{cases} \tag{2}$$

Among them, r represents the risk-free interest rate, which satisfies $r > 0$.

Hypothesis 4: According to the application of Cramer-Lundberg model [33], [34], it is assumed that the surplus process of the guarantee institution satisfies:

$$R_t = R_0 + \rho t - \sum_{i=1}^{N(t)} Y_i \tag{3}$$

Among them, $R_0 \geq 0$ is the initial surplus; $\rho > 0$ is a constant, indicating the premium income of the guarantee institution per unit time; $(N_t)_{t \geq 0}$ obeys the homogeneous Poisson process with parameter δ, indicating that the total number of compensations occurred in the $[0, T]$ time period; $\{Y_1, Y_2, \cdots, Y_N\}$ (N is a positive integers) are a group of independent and identically distributed positive random variables, representing the amount of the i-th compensation, the first moment and second-order moment of Y_i is $E(Y_i) = \theta$ and $E(Y_i^2) = \kappa^2$ respectively; composite Poisson process $\sum_{i=1}^{N(t)} Y_i$, Markov process $(\alpha_t)_{t \in [0,T]}$ and Brownian motion $B_t^{(1)}$ are independent of each other.

Meanwhile, ρ can be expressed by the following formula:

$$\rho = (1 + \omega)\delta\theta \tag{4}$$

Among them, $\omega \geq 0$, representing the safety loading of guarantee institution.

Hypothesis 5: The guarantee institution chooses the method of proportional re-guarantee to diversify the risk in the operation process [3]. β_t $(0 < \beta_t < 1)$ represents the risk self-retention amount of the guarantee institution at time t, and $(1 - \beta_t)$ is the re-guarantee purchase ratio. When the compensatory loss Y_i occurs, the guarantee institution only has to bear the loss of $\beta_t Y_i$. In addition, assuming that the safety loading of re-guarantee institution is η, then the re-guarantee fee to be paid by the guarantee institution per unit time satisfies:

$$\rho' = (1 + \eta)(1 - \beta_t)\delta\theta \tag{5}$$

Introducing the re-guarantee into the Cramer-Lundberg model and considering the recovery rate ν, then the guarantee institution's surplus process satisfies:

$$dR_t = (1+\omega)\delta\theta dt - (1+\eta)(1-\beta_t)\delta\theta dt - (1-\nu)\,\beta_t d\sum_{i=1}^{N_t} Y_i \quad (6)$$

The surplus function in equation (6) does not have continuity, which is inconvenient for subsequent analysis. A similar processing method is used by Harrison [35] and Browne [36], the compensation process $\sum_{i=1}^{N(t)} Y_i$ is approximated by standard Brownian motion, and the limit form of the classical risk model is obtained. The surplus process will be characterized by the following stochastic differential equation:

$$dR_t = \left\{ \begin{array}{l} (1+\omega)\delta\theta dt - (1+\eta)(1-\beta_t)\delta\theta dt \\ -(1-\nu)(\delta\theta\beta_t dt + \zeta\beta_t dB_t^{(2)}) \end{array} \right\} \quad (7)$$

Among them, $B_t^{(2)}$ represents the random part of the compensatory expenditure, and the Markov process $(\alpha_t)_{t\in[0,T]}$, Brownian motion $B_t^{(1)}$, $B_t^{(2)}$ are independent of each other. Further simplification of equation (7) can be obtained:

$$dR_t = \delta\theta(\eta\beta_t - \eta + \omega + \nu\beta_t)dt - (1-\nu)\zeta\beta_t dB_t^{(2)} \quad (8)$$

The above hypotheses define the price process of risky assets and risk-free assets, and also specify the guarantee fee income, re-guarantee fee expenditure and compensatory loss of the guarantee institution per unit time, so as to obtain the guarantee institution's surplus represented by equation (8) process. The following will further consider the impact of investment income on terminal wealth on the basis of equation (8), and use dynamic programming theory to solve the optimal investment and re-guarantee purchase strategies under the criterion of maximum expected utility.

III. OPTIMAL INVESTMENT AND RE-GUARANTEE PURCHASE STRATEGIES UNDER RISK PREFERENCE

A. MEASURE OF RISK AVERSION PREFERNCE

In the market, investors can be divided into risk-lovers, risk-neutral and risk-averse according to their different risk preferences. To quantify investors' aversion to risk, the Arrow-Pruitt risk measure $D(x) = -\frac{U''(x)}{U'(x)}$ is introduced. Among several common utility functions, the exponential utility function has been widely used in the field of financial mathematics because it has a constant risk aversion coefficient. For instance, Delong explores the problem of maximizing the exponential utility of insurance company with wealth-related risk aversion [37], while Gan and Wang quantify the degree of risk aversion in the investment process of reinsurance company using an exponential utility function [28]. Accordingly, we will consider the optimal investment and re-guarantee purchase strategies of the guarantee institution under the exponential utility function. Suppose the exponential utility function has the following expression:

$$U(x) = c - \frac{k}{\lambda}e^{-\lambda x}$$

where $k > 0$, $\lambda > 0$, and the risk aversion coefficient is $D(x) = -\frac{U''(x)}{U'(x)} = \lambda$.

B. CONSTRUCTION OF HJB EQUATION

Within $[t, T]$, γ_t $(0 \le \gamma_t \le 1)$ represents the proportion of the capital invested by the guarantee institution in risky assets to the investable assets at time t, while $(1-\gamma_t)$ represents the proportion of the capital invested in the risk-free assets. Taking $X^{\gamma,\beta} = \left\{X_t^{\gamma,\beta}, 0 \le t \le T\right\}$ to represent the wealth process when the guarantee institution chooses the investment strategy γ and the re-guarantee self-retention strategy β, then the expression of $X_t^{\gamma,\beta}$ is as follows:

$$dX_t^{\gamma,\beta} = \gamma_t X_t^{\gamma,\beta}\frac{dS_t}{S_t} + (1-\gamma_t)X_t^{\gamma,\beta}\frac{dP_t}{P_t} + dR_t \quad (9)$$

Substitute equations (1), (2) and (8) into equation (9) to get (10), as shown at the bottom of the page.

It is assumed that all strategy combinations (γ_t, β_t) adopted by the guarantee institution are feasible, that is, (γ_t, β_t) satisfies: ① $\{\gamma_t\}_{t\ge 0}$ and $\{\beta_t\}_{t\ge 0}$ are predictable processes with respect to $\{\mathcal{F}_t\}_{t\ge 0}$; ② $0\le \gamma_t \le 1$, $0\le \beta_t \le 1$. Let the set of all feasible strategies be denoted by Π. In order to obtain the value function of the optimization problem, we assume that $U(X^{\gamma,\beta})$: $(0,\infty) \to R$ is the utility function of the guarantee institution under the wealth $X^{\gamma,\beta}$, where $U(\cdot)$ is a strictly increasing concave function on $[0, T]$, that is, $U' > 0$, $U'' < 0$. For the guarantee institution, it is necessary to choose the optimal strategy (γ_t^*, β_t^*) to maximize the expected utility brought by the terminal wealth. Therefore, the following optimization problem can be obtained:

$$V(t,x,i) = \sup_{(\gamma_t,\beta_t)\in\prod} E\left[U(X_T^{\gamma,\beta})\middle| X_T^{\gamma,\beta} = x, \alpha_t = e_i\right] \quad (11)$$

Among them, $i = 1, 2, \cdots, \mathrm{N}$, $E[\cdot\,||X_t^{\gamma,\beta} = x, \alpha_t = e_i]$ represents the expected utility when the wealth of the guarantee institution is x and the market mechanism of the risky asset is e_i. In order to solve the optimization problem and express the value function $V(t,x,i)$, the following HJB equation is constructed by the classical dynamic programming principle:

$$\sup_{(\gamma_t,\beta_t)\in\prod} \left\{ \begin{array}{l} G_t + [rx + \gamma_t x(u_i - r) + \delta\theta(\eta\beta_t - \eta + \omega \\ +\nu\beta_t)]G_x + \frac{1}{2}[\sigma_i^2\gamma_i^2 x^2 + (1-\nu)^2\zeta^2\beta_t^2]G_{xx} \\ + \sum_{j\in[1,2,\cdots,N]} q_{ij}[G(t,x,j) - G(t,x,i)] \end{array} \right\} = 0 \quad (12)$$

$$dX_t^{\gamma,\beta} = \left\{ \begin{array}{l} [rX_t^{\gamma,\beta} + \gamma X_t^{\gamma,\beta}(u(\alpha_t) - r) + \delta\theta(\eta\beta_t - \eta + \omega + \nu\beta_t)]dt \\ +\gamma_t X_t^{\gamma,\beta}\sigma(\alpha_t)dB_t^{(1)} - (1-\nu)\zeta\beta_t dB_t^{(2)} \end{array} \right\} \quad (10)$$

The above formula has the boundary condition $G(T, X_T^{\gamma,\beta}, \alpha_T) = U(X_T^{\gamma,\beta})$, where G_t is the first derivative of the value function $G(t,x,i)$ with respect to t, and G_x and G_{xx} are the first and second derivatives of the value function $G(t,x,i)$ with respect to x.

C. MODEL SOLUTION

To facilitate the expression, let

$$g^{\gamma,\beta} = \left\{ \begin{array}{l} G_t + [rx + \gamma_t x(u_i - r) + \delta\theta(\eta\beta_t - \eta + \omega \\ +\nu\beta_t)]G_x + \frac{1}{2}[\sigma_i^2\gamma_i^2x^2 + (1-\nu)^2\zeta^2\beta_t^2]G_{xx} \\ + \sum\limits_{j\in[1,2,\cdots,N]} q_{ij}\,[G(t,x,j) - G(t,x,i)] \end{array} \right\} \tag{13}$$

We can learn from the studies of Bauerle and Rieder [38] and Wang and Rong [39] that if (γ_t^*, β_t^*) maximizes the equation (13), then $G(t,x,i) = V(t,x,i) = V^{\gamma^*,\beta^*}(t,x,i)$ is established. Here, $V^{\gamma^*,\beta^*}(t,x,i) = E[U(X^{\gamma^*,\beta^*}(T))|X^{\gamma^*,\beta^*}(t) = x, \alpha_t = e_i]$. In addition, γ^* and β^* represent optimal investment and re-guarantee self-retention strategies, respectively.

Let the first derivative of $g^{\gamma,\beta}$ with respect to γ_t and β_t in equation (13) be equal to 0, we can get the optimal investment and re-guarantee self-retention strategies.

$$\gamma_t^*(t,x,i) = -\frac{(u_i - r)\,G_x}{x\sigma_i^2 G_{xx}} \tag{14}$$

$$\beta_t^*\,(t,x,i) = -\frac{\delta\theta\,(\eta+\nu)\,G_x}{\zeta^2\,(1-\nu)^2\,G_{xx}} \tag{15}$$

Substitute equations (14) and (15) into equation (12) to obtain the conditions that *G(t,x,i)* should satisfy when the guarantee institution obtains the optimal investment and re-guarantee self-retention strategies.

$$\begin{aligned} &G_t + rxG_x - \left[\frac{(u_i-r)^2}{2\sigma_i^2} + \frac{\delta^2\theta^2\,(\eta+\nu)^2}{2\,(1-\nu)^2\,\beta^2}\right] \\ &\times \frac{G_x^2}{G_{xx}} - \delta\theta(\eta-\omega)G_x \\ &+ \sum_{j=1,2,\cdots,N} q_{ij}\,[G(t,x,j) - G(t,x,i)] = 0 \end{aligned} \tag{16}$$

Equation (16) satisfies the boundary condition:

$$G(T, X_T^{\gamma,\beta}, \alpha_T) = U\left(X_T^{\gamma,\beta}\right).$$

According to the conclusion of Yang and Zhang [40], under the assumption of exponential utility function, the HJB equation satisfying equation (12) has the following solution:

$$G(t,x,i) = c - \frac{k}{\lambda}e^{-\lambda x e^{r(T-t)}}f(t,i) \tag{17}$$

Since equation (17) satisfies the boundary condition $G(T, X_T^{\gamma,\beta}, \alpha_T) = U\left(X_T^{\gamma,\beta}\right)$ and there is $f(T,i) = 1$, the first and second derivative of G_t are calculated with respect to t and x respectively, and the results are as follows:

$$G_t = -\frac{k}{\lambda}e^{-\lambda x e^{r(T-t)}}f_t(t,i) - kxre^{-\lambda x e^{r(T-t)}+r(T-t)}f(t,i) \tag{18}$$

$$G_x = ke^{-\lambda x e^{r(T-t)}+r(T-t)}f(t,i) \tag{19}$$

$$G_{xx} = -k\lambda e^{-\lambda x e^{r(T-t)}+2r(T-t)}f(t,i) \tag{20}$$

It is noteworthy that $f_t(t,i)$ is the first order derivative of the functional equation $f(t,i)$ with respect to t. Substitute equations (18), (19) and (20) into equation (13) to get:

$$\begin{aligned} f_t(t,i) - [&\frac{(u_i-r)^2}{2\sigma_i^2} + \frac{(\eta+\nu)^2\delta^2\theta^2}{2(1-\nu)^2\beta^2}]f(t,i) \\ &+ \delta\theta(\eta-\omega)e^{r(T-t)}f(t,i) \\ &+ \sum_{j\in[1,2,\cdots,N]} q_{ij}\,[f(t,j) - f(t,i)] = 0 \end{aligned} \tag{21}$$

Set $z_i = \frac{(u_i-r)^2}{2\sigma_i^2} + \frac{(\eta+\nu)^2\delta^2\theta^2}{2(1-\nu)^2\beta^2}$, the equation (21) can be simplified to:

$$\begin{aligned} f_t(t,i) - z_i f(t,i) + \delta\theta\lambda(\eta-\omega)e^{r(T-t)}f(t,i) \\ + \sum_{j\in[1,2,\cdots,N]} q_{ij}\,[f(t,j) - f(t,i)] = 0 \end{aligned} \tag{22}$$

Lemma 1: The expression $f(t,i)$ satisfying equation (22) is as follows:

$$\begin{aligned} f(t,i) = \exp[&\frac{\delta\lambda\theta(\eta-\omega)(e^{r(T-t)}-1)}{r} \\ &- \frac{(\eta+\nu)^2\delta^2\theta^2(T-t)}{2(1-\nu)^2\beta^2}] \\ &\cdot E[e^{\int_t^T \frac{-(u_{\alpha_s}-r)^2}{\sigma_{\alpha_s}^2}ds}|\alpha_t = e_i] \end{aligned} \tag{23}$$

The proof of Lemma 1 can be seen in Appendix.

From the study of Wang and Gan [41], it is clear that there is conditional expectation in the solution of $f(t,i)$ in equation (23), and the calculation of conditional expectation is discussed below. Definition:

$$J_i = \int_t^T I_{\{\alpha_s = e_i\}}ds$$

where J_i represents the cumulative stay time in the Markov chain α within $[t,T]$, and $I_{\{\cdot\}}$ is an indicative function, and the following equation holds:

$$E\left[e^{\int_t^T -\frac{(u_{\alpha_s}-r)^2}{\sigma_{\alpha_s}^2}du}|\alpha_t = e_i\right] = E\left[e^{\sum\limits_{i=1}^{N}\frac{-(u_i-r)^2J_i}{\sigma_i^2}}|\alpha_t = e_i\right] \tag{24}$$

It can be seen from equation (23) that if the conditional probability density function of J under the condition $\alpha_t = e_i$ is obtained, the conditional expectation in equation (23) can be obtained. The conditional feature function of J can be obtained through Schmidli [42], and the conditional probability density function of J can be obtained by Fourier transform. In particular, when there are only two states of the Markov chain, the condition expectation has the following solution:

$$E\left[e^{\int_t^T \frac{-(u_{\alpha_s}-r)^2}{\sigma_{\alpha_s}^2}du}|\alpha_t = e_i\right]$$

$$= E\left[e^{\frac{-(u_1-r)^2 J_1}{\sigma_1^2} - \frac{(u_2-r)^2(T-t-J_1)}{\sigma_2^2}} | \alpha_t = e_i \right]$$

From Lemma 1, we know that $G(t, x, i)$ determined by equation (17) is the solution of the HJB equation, and according to $f(t, i) > 0$, $G_x > 0$, $G_{xx} < 0$ can be obtained from equations (19) and (20), so there is an inequality:

$$|G(t, x, i)| \le \max\left\{ c, \frac{k}{\lambda} e^{-x\lambda e^{r(T-t)}} f(t, i) \right\}$$
$$\le \max\left\{ c, \frac{k}{\lambda} (1 + e^{-x\lambda e^{r(T-t)}}) f(t, i) \right\}$$

Since $V(t, x, i) = \mathrm{G}(t, x, i)$, then the optimal investment and re-guarantee strategies of guarantee institution can be obtained by substituting equations (19) and (20) into equations (14) and (15).

$$\begin{cases} \gamma_t^*(t, x, i) = \dfrac{(u_i - r)}{x\lambda\sigma_i^2} e^{-r(T-t)} \\ \beta_t^*(t, x, i) = \dfrac{\delta\theta(\eta + \nu)}{\lambda\zeta^2(1-\nu)^2} e^{-r(T-t)} \end{cases}$$

Among them, $0 \le \gamma_t^*(t, x, i) \le 1$, $0 \le \beta_t^*(t, x, i) \le 1$, and the value function $V(t, x, i)$ is satisfied $V(t, x, i) = c - \frac{k}{\lambda} e^{-\lambda x e^{r(T-t)}} f_t(t, i)$, and the expression $f(t, i)$ is shown in (23).

IV. COMPUTATIONAL EXPERIMENT SIMULATION ANALYSIS

The above functional expressions of the optimal strategies shows that the optimal investment and re-guarantee purchase strategies of guarantee institutions will be characterized by dynamic changes as the parameters such as market mechanism, risk-free interest rate, and risk aversion coefficient keep changing. In order to analyze the dynamic evolution law of these the optimal strategies, the influence results of parameters are simulated by computational experiment. ① Referring to the fluctuation range of risk assets such as stocks in China's securities market, it is assumed that there are two market mechanisms in Markov chain, and the state space is $\mathbb{Z} = \{\alpha_{\mathrm{I}}, \alpha_{\mathrm{II}}\}$, where market mechanism I: the drift coefficient of asset prices $u_1 = 0.15$, the volatility coefficient $\sigma_1 = 0.40$; market mechanism II: $u_2 = 0.10$, $\sigma_2 = 0.50$. Especially, market mechanism I has high asset returns and slightly lower volatility. ② China's re-guarantee institutions generally charge 20% of the original guarantee fee as the re-guarantee fee, and bear 30% of the compensation risk. Given the above, the safety loading of re-guarantee institution is $\eta = 0.25$, and the recovery rate is $\upsilon = 10.9\%$. ③ Other parameters such as the average single compensation amount $\theta = 0.2$, the average number of compensations $\delta = 2$, the second-order moment of the compensation amount $E(Y_i^2) = 0.5$, $\zeta^2 = E(\delta Y_i^2) = 1$; when the term T-t = 1, the guarantee institution's funds $x_t = 1$ at time t, and the risk-free interest rate is the value $r = 1.5\%$ with reference to the national debt interest rate, risk aversion coefficient $\lambda = 1$.

A. SIMULATION RESULTS

1) SIMULATION OF THE INFLUENCE OF MARKET MECHANISM ON INVESTMENT STRATEGY

According to the values of the above parameters, Figure 1 represents the simulation situation of the optimal risk asset investment strategy under the two market mechanisms with different values of wealth x and simulation period t. When other conditions are the same, the proportion of risky assets invested by the guarantee institution in market mechanism I (Figure 1(a)) is significantly higher than that in market mechanism II (Figure 1(b)). From the previous analysis, it can be seen that different market mechanisms are reflected in differentiated drift coefficient and volatility coefficient. Accordingly, we further explore the influence of volatility and drift coefficients on optimal investment strategy. Figure 2 shows that the investment ratio of risky assets decreases with the increase of volatility coefficient σ, and increases with the increase of drift coefficient u, which is also in line with the actual investment business.

Since the volatility and drift coefficients have no direct impact on the re-guarantee strategy, the simulation is not carried out.

2) SIMULATION OF THE INFLUENCE OF RISK-FREE INTEREST RATE AND RISK AVERSION COEFFICIENT ON INVESTMENT AND RE-GUARANTEE STRATEGIES

Other parameters still take the above values, and the risk-free interest rate r and the risk aversion coefficient λ take different values for simulation. The Figure 3(a) and Figure 3(b) show the influence of r and λ on the optimal risk investment strategy, respectively. The increase of risk-free interest rate and risk aversion coefficient will significantly reduce the proportion of risky investment. Additionally, through the comparison of Figure 3(a) and Figure 3(b), it is found that when other conditions are the same, the proportion of funds invested in market mechanism I is higher than that of market mechanism II. The outstanding performance is that the influence of risk-free interest rate and risk aversion coefficient on the optimal investment strategy is more significant in market mechanism I. Additionally, Figure 4 depicts the influence of risk-free interest rate and risk aversion coefficient on the optimal re-guarantee purchase strategy. With the increase of risk-free interest rate and risk aversion coefficient, the proportion of the guarantee institution choosing risk self-retention is gradually decreasing, and the proportion of purchasing re-guarantee is increasing.

3) SIMULATION OF THE INFLUENCE OF GUARANTEE PERIOD ON INVESTMENT AND RE-GUARANTEE STRATEGIES

We set two different guarantee periods, namely, T-t = 1 and T-t = 10. Figure 5(a) reflects the influence of the guarantee period on the optimal investment strategy under different market states. As can be seen, with the extension of the guarantee contract period, the proportion of the guarantee institution investing in risk assets is decreasing. Furthermore,

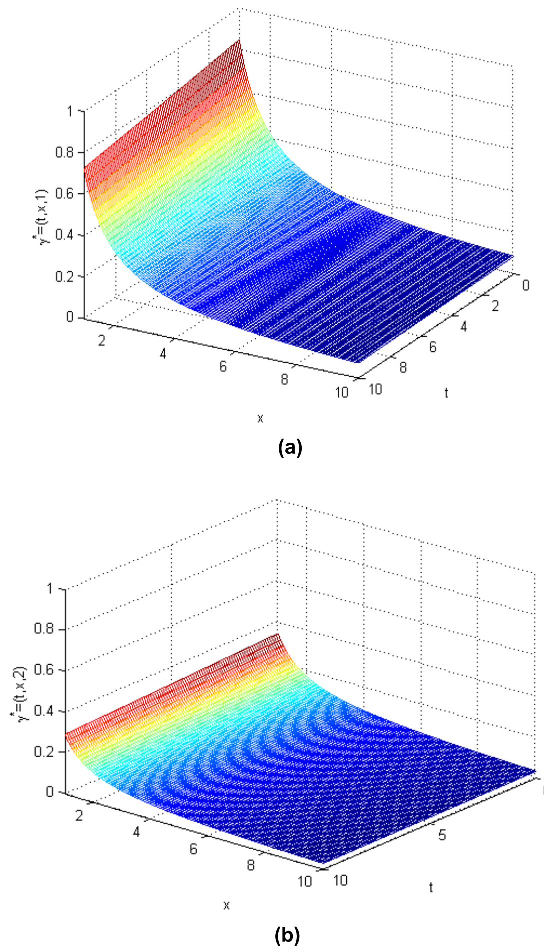

FIGURE 1. **Simulation of optimal investment under different market mechanisms. (a) market mechanism I. (b) market mechanism II.**

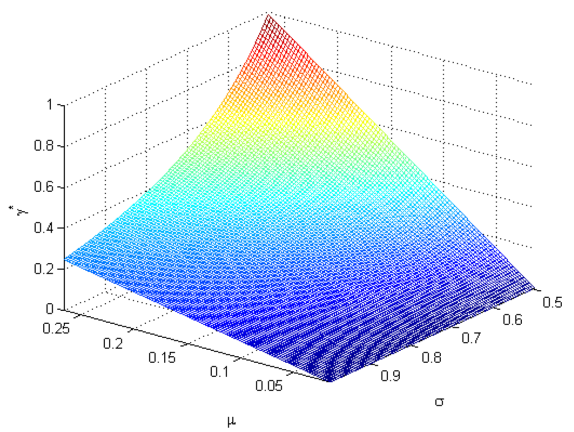

FIGURE 2. **The influence of drift and volatility coefficients on the optimal investment strategy.**

the situation of the investment market also affects the judgment of the guarantee institution on uncertainty risk to a certain extent. In other words, the proportion of investment in risky assets is significantly higher in the period of steady rise (with a larger u and a smaller σ) than during a trough

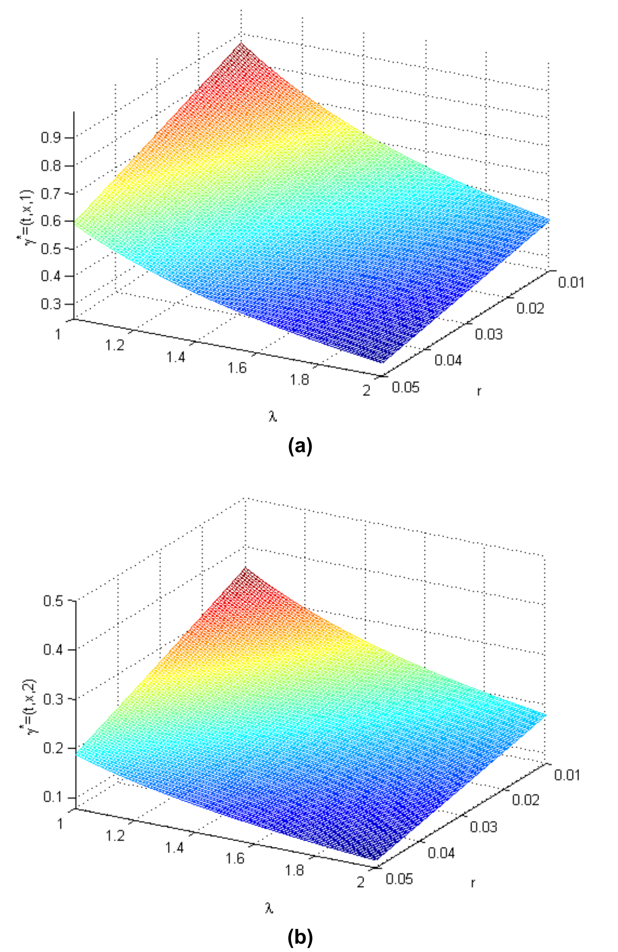

FIGURE 3. **The influence of risk aversion coefficient and risk-free interest rate on optimal risk investment strategies under different market mechanisms. (a) market mechanism I. (b) market mechanism II.**

period (with a smaller u and a larger σ) in the investment market. On the other hand, Figure 5(b) depicts the influence of guarantee period on the optimal re-guarantee strategy, showing that the extension of guarantee period has a significant negative impact on the risk self-retention ratio of the guarantee institution.

4) SIMULATION OF THE INFLUENCE OF RE-GUARANTEE INSTITUTION'S SAFETY LOADING AND RECOVERY RATE ON RE-GUARANTEE STRATEGY

Figure 6 shows the influence of re-guarantee institution's safety loading on the optimal re-guarantee self-retention strategy. The increase of this parameter has a significant negative impact on the optimal re-guarantee purchase ratio. Especially, when the guarantee institution's risk aversion coefficient is low, the change of this indicator has a more significant impact on the optimal re-guarantee self-retention strategy. Similarly, Figure 7 shows the influence of the guarantee recovery rate on the optimal re-guarantee self-retention strategy. With the improvement of the guarantee recovery rate, the proportion of the guarantee institution choosing risk self-retention is increasing.

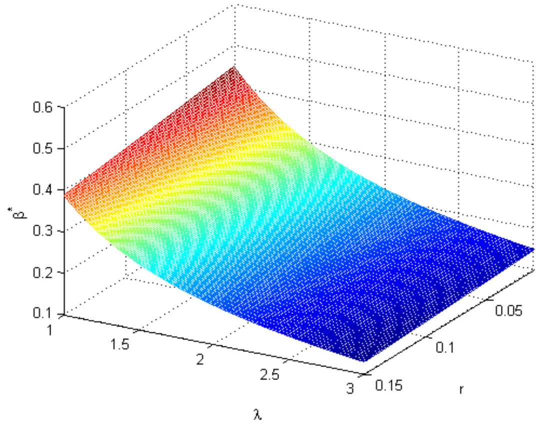

FIGURE 4. The influence of risk aversion coefficient and risk-free interest rate on the optimal re-guarantee strategy.

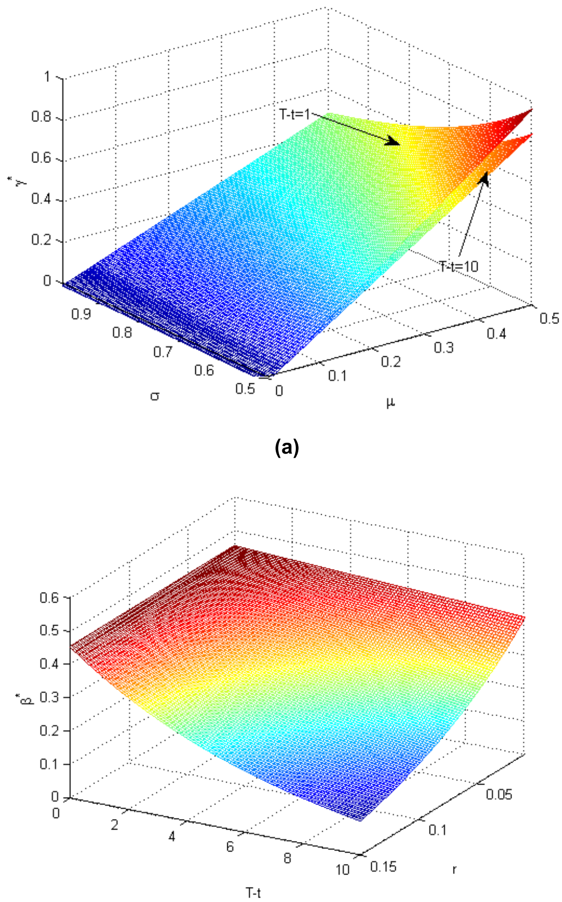

FIGURE 5. The influence of guarantee period on optimal investment and re-guarantee strategies. (a) optimal investment. (b) optimal re-guarantee.

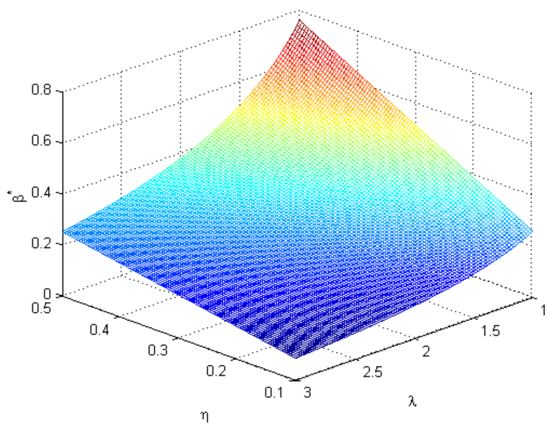

FIGURE 6. The influence of re-guarantee institution's safety loading on the optimal re-guarantee strategy.

B. RESULTS DISCUSSION

From the above simulation results, it can be seen that there are significant differences in the effects of different factors on the investment and re-guarantee purchase.

First, similar to the findings of Wang *et al.* [20] and Xiao and Qiu [29] in the field of insurance, when the overall expected return rate of the market is greater than zero and the volatility of asset value is small, the returns brought by investing in risky assets are higher than those of risk-free assets such as bonds. Conversely, when the overall expected return rate is low and the volatility is high, investing in risky assets brings lower returns and bears a higher risk of asset impairment. This suggests that investing in risk-free assets is a more reasonable choice.

Second, from the perspective of guarantee practice, when the risk-free interest rate increases, the guarantee institution tend to invest in risk-free assets, but the return from investing in risk-free assets is lower than that of risky assets, which means that the final investment income may decline. In order to diversify the compensation risk, it is a reasonable choice to appropriately increase the proportion of re-guarantee purchase. Meanwhile, the higher the risk aversion coefficient, the greater the risk aversion of the guarantee institution. Obviously, the guarantee institution will also increase the proportion of re-guarantee purchase to transfer the risk. Although few scholars have found the applicability of the above decision-making mechanism in the guarantee industry, it has been fully demonstrated in the insurance or reinsurance industry, which is consistent with the intrinsic mechanism derived from Gan and Wang [28], Deng *et al.* [43], and Zhang and Meng [44].

Third, the finding regarding the period of guarantee contract is an important addition, which was overlooked in previous studies of Wang *et al.* [3] and Gu [16]. In terms of realistic interpretations of the finding, on the one hand, the higher the degree of uncertainty, and the greater the risk borne by the guarantee institution. Therefore, the guarantee institution reduces the investment ratio of risky assets and invests more risk-free assets to effectively reduce the risk brought by the uncertainty. On the other hand, the extension of guarantee period means that the contingent risk of the guarantee business increases, so it makes sense to reduce the risk self-retention ratio and increase the re-guarantee purchase ratio.

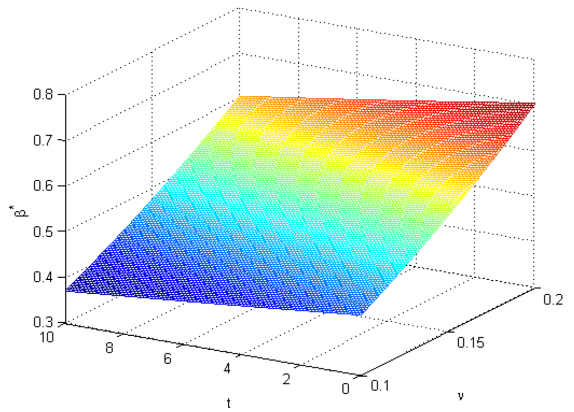

FIGURE 7. The influence of guarantee recovery rate on the optimal re-guarantee strategy.

Finally, according to the study of Yan *et al.* [45], the guarantor's behavioral decisions are more conservative in a high-risk aversion scenario. Therefore, the guarantee institution is more willing to take risks themselves under the low risk aversion coefficient, and the increase in the re-guarantee fee makes the re-guarantee purchase ratio drop significantly. On the contrary, the increase in the re-guarantee fee has no significant impact on the re-guarantee purchase ratio. Meanwhile, the improvement of the guarantee recovery rate can effectively reduce the compensation loss of the guarantee institution. Certainly, increasing the risk self-retention ratio and reducing the re-guarantee purchase can effectively improve the income of the guarantee institution. This further complements the explanation of the moderating effect of the guarantee recovery rate on the re-guarantee operating mechanism in the study of Wang *et al.* [3].

V. CONCLUSION AND MANAGEMENT IMPLICATIONS

Based on the geometric Brownian motion of Markov chain modulation, this paper describes the price process of risky assets, obtains the optimal solutions of the investment and re-guarantee purchase strategies of the guarantee institution under the exponential utility function by solving the Hamilton-Jacobi-Bellman equation, then simulates the internal influence mechanism of factors such as risk aversion and market mechanism on the optimal strategies. The main conclusions are as follows.

First, different market mechanisms have a significant impact on the investment strategy of risky assets, while the optimal re-guarantee purchase strategy has no concern with the market mechanisms. Especially, the higher the average return rate of risky assets, the proportion of the guarantee institution investing in risky assets should be appropriately increased; the greater the volatility of risky assets, the lower the proportion of investment in risky assets.

Second, the increase of risk-free interest rate, risk aversion coefficient and guarantee period will reduce the proportion of the guarantee institution investing in risky assets. On the contrary, the proportion of re-guarantee purchase of the guarantee institution will increase with the same variation trend of the above three parameters.

Finally, the re-guarantee institution's safety loading and the guarantee recovery rate have no impact on the guarantee institution's optimal risk asset investment strategy, but have a significant negative impact on the optimal re-guarantee strategy purchase strategy. Noteworthy, the greater the re-guarantee institution's safety loading, the more inclined the guarantee institution is to choose not to apply for re-guarantee, but this effect is weakened with the increase of the risk aversion coefficient of the guarantee institution.

According to the above research conclusions, we can get the following management implications. (1) Referring to the "Administrative Measures for the Asset Proportion of Financing Guarantee Companies", Chinese financing guarantee institutions take investment income and re-guarantee purchase into business considerations at the same time, focusing on dynamic balance, while regulatory authorities require regular business reporting and supervise the static balance of assets at the end of the period. Admittedly, this reality has led to a regulatory misalignment. (2) The separation phenomenon of different factors affecting the investment and re-guarantee purchase strategies is due to the fact that multiple parameters occur at different time points. If the time points happen to overlap, it may seriously affect the cash flow and even trigger a chain reaction. Therefore, it is necessary to supervise the cash flow of the guarantee institution. (3) The risk of financing guarantee business in China is shared by multi-agent cooperation, within the framework of market-oriented operation. Driven by the risk-averse preference of multiple-agent, behaviors such as risk hiding, adverse selection, and passive cooperation are common. It is necessary to gradually clarify the behavior boundaries of each agent in the system to ensure that compensation risk is shared on time and according to the contract.

In the future, the following two aspects can be considered in depth. On the one hand, considering the uncertainty of the surplus process of guarantee institutions, this study will improve the diffusion risk model into a jump-diffusion risk model and compare the differences of the findings. On the other hand, under the concept of classical solution, the HJB equation may not have a solution, or the uniqueness may not be satisfied. Therefore, applying the viscosity solution theory to verify the existence and uniqueness of the solution of the HJB equation is an improvement direction of this study.

APPENDIX

The proof of Lemma 1 is as follow.

Proof: Using Ito's lemma, we get:

$$f(T,\alpha_T) = f(t,i) + \int_t^T f_t(s,\alpha_s)ds + \sum_{t \le s \le T} [f(s,\alpha_{s_+}) - f(s,\alpha_s)]$$

Combining equation (21), we can get:

$$f_t(s,\alpha_s) = z_{\alpha_s} f(s,\alpha_s) - \delta\theta\lambda(\eta-\omega)e^{r(T-s)} f(s,\alpha_s) - \sum_{j\in[1,2,\cdots,N]} q_{\alpha_s j}\,[f(s,j) - f(s,\alpha_s)] = 0$$

Noteworthy, there is $f(T,\alpha_s) = 1$ and the following equation:

$$E[\sum_{t\le s\le T} [f(s,\alpha_{s_+}) - f(s,\alpha_s)]\,|\alpha_t = e_i] = \int_t^T \sum_{j\in[1,2,\cdots,N]} q_{\alpha_s j}\,[f(s,j) - f(s,\alpha_s)]ds\,|\alpha_t = e_i]$$

Substitute above equation into the function of $f(T,\alpha_T)$, and simultaneously calculate the conditional expectation for $\alpha_t = e_i$, we can get:

$$1 = f(t,i) + \int_t^T E[z_{\alpha_s} f(s,\alpha_s)\,|\alpha_t = e_i]ds - \int_t^T E[\delta\theta\lambda(\eta-\omega)e^{r(T-s)} f(s,\alpha_s)\,|\alpha_t = e_i]ds$$

Then,

$$f(t,i) = E[\exp\left\{\int_t^T (-z_{\alpha_s} + \delta\theta\lambda(\eta-\omega)e^{r(T-s)})ds\right\}|\alpha_t = e_i] = \exp\left\{\frac{\delta\theta\lambda(\eta-\omega)(e^{r(T-s)}-1)}{r} - \frac{(\eta+\nu)^2\delta^2\theta^2(T-t)}{2(1-\nu)^2\beta^2}\right\} \cdot E[e^{\int_t^T \frac{-(u_{\alpha_s}-r)^2}{\sigma_{\alpha_s}^2}ds}\,|\alpha_t = e_i]$$

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