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RESEARCH ARTICLE

Local Indiscernibility Relation Reduction for Information Tables

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ABSTRACT Attribute reduction comes from machine learning and is an important component of rough set theory. Research on attribute reduction has produced many important achievements. The aim of attribute reduction is to reduce the complexity of data while retaining its original characteristics to the greatest extent. The concept of attribute reduction is of great significance in machine learning research. In previous studies, a variety of attribute reduction definitions have been proposed according to different rules. Based on the binary relations among objects and local decision rules, this paper describes a local indiscernibility relation reduction for information tables. The discernibility matrix for the proposed reduction is established, and examples for single- and multi-decision classes are presented to illustrate that the proposed local indiscernibility relation reduction can be applied to decision tables. According to the reduction concept developed in this paper, and considering a heuristic algorithm for calculating the significance of attributes and a binary integer programming algorithm based on the discernibility matrix, three reduction algorithms are proposed. Experiments are conducted using four classifiers and a number of publicly available datasets. A comparison of the experimental results presented in this paper demonstrates the feasibility of the proposed algorithms.

INDEX TERMS Discernibility matrix, information table, attribute reduction, indiscernibility relation, reduction algorithm.

I. INTRODUCTION

Rough set theory [1], [2] is a data analysis tool for handling uncertainty and inconsistency. At present, a hot topic of research in rough set theory is attribute reduction. The aim of attribute reduction is to delete redundant attributes according to some specific rules while retaining an unchanged object classification in the universe. To date, many attribute reduction methods [3]–[6] have been studied, such as positive region reduction [7]–[9], variable precision reduction [10]–[12], assignment reduction [13], covering reduction [14], [15], and knowledge granularity reduction [16]–[19].

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Existing reduction algorithms include discernibility matrix-based algorithms and heuristic algorithms. Skowron and Rauszer [20]–[22] proposed reduction algorithms based on the discernibility matrix. Once the discernibility matrix has been constructed, the discernibility function can be derived and then transformed from the conjunctive normal form (CNF) to the disjunctive normal form (DNF), thus obtaining all reducts. However, the computational complexity of this approach is relatively high. Liu *et al.* [23] proposed a unified reduction algorithm based on invariant matrices for three kinds of reduction. To reduce the computational complexity of the discernibility matrix-based algorithm, a binary integer programming problem [24] can be established after the discernibility matrix has been constructed. In the process of constructing the discernibility matrix, the core attributes are obtained by finding the minimum element in the discernibility matrix and identifying the reducts [25]. Heuristic algorithms [26]–[28] typically calculate the attribute dependency to determine the reducts. Examples include test-cost-sensitive attribute reduction [29] and attribute reduction based on the conditional information entropy [30].

Obtaining all rules in a decision table is computationally intensive. Sometimes, only local rules need to be obtained, in which case it is not necessary to calculate all decision classes simultaneously. Liu *et al.* [31] considered how the local decision rules in a decision table could be best obtained, and proposed the concept of lth decision class reduction. They then proved the relationship between positive region reduction and lth decision class reduction. By again considering the local decision class, Liu *et al.* [32] proposed X-upper approximation reduction and X-lower approximation reduction for decision tables, and gave the corresponding proofs. Yao [33] showed that classification-based reduction does not work equally well for each decision class. However, class-specific attribute reduction provides a special form of classification-based reduction. Chen [34] obtained more optimized local rules by using a local reduction algorithm in a decision table. Considering the decision-making subset, the binary relation has been extended to a fuzzy relation on the discernibility matrix-based reduction algorithm, and the X-lower and X-upper approximation reductions have been derived in the fuzzy relation decision system [35]. Additionally, the concept of local reduction has been introduced to fuzzy rough sets [36], [37]. Discernibility and indiscernibility relation reductions have been proposed for information tables [38], leading to relative discernibility and relative indiscernibility relation reductions for decision tables [39]–[41].

Based on the local decision set [31]–[34], [42], [43], this paper analyzes the discernibility relation of any objects in the decision subset. This concept is applicable to communication and consultation [44] in the fields of labor arbitration, diplomatic negotiation, and so on. Considering the local set, and from the point of view of the binary relation between any objects, this study provides a new perspective for reduction research.

The remainder of this paper is structured as follows. Section II introduces the concept of rough sets and reviews some reduction definitions. In Section III, the concepts of a binary relation set and reduction are proposed, and the corresponding discernibility matrix is constructed. Section IV introduces the local indiscernibility relation reduction (LIRR) for single- and multi-decision classes, and illustrates the use of LIRR through several examples. In Section V, according to the definition of LIRR, three reduction algorithms are proposed, and the corresponding algorithms are discussed. To illustrate the effectiveness of the proposed algorithms, the results before and after reduction are compared experimentally in Section VI using selected classifiers. Finally, Section VII concludes the paper.

II. PRELIMINARIES

The tuple $S = (U, AT, \{V_a \mid a \in AT\}, \{I_a \mid a \in AT\})$ represents an information table, where U is the universe set, AT is a finite nonempty set of attributes, V_a is a nonempty set of values for $a \in AT$, and $I_a : U \to AT$ is a function in which $I_a(x)$ takes a value on a, $\forall x \in U$. If $AT = C \cup D$, $C \cap D = \emptyset$, where C is the condition attribute set and D is the decision attribute set, then the tuple S is called a decision table, written as $(U, C \cup D)$.

Given a subset of the attribute set $A \subseteq AT$, an equivalence relation is defined by,

$$\mathrm{R_A} = \{(x, y) \mid (x, y) \in U \times U, I_a(x) = I_a(y), \forall a \in A\}. \quad (1)$$

The equivalence class of an attribute set A is denoted by $[x]_A = \{y \mid (x, y) \in R_A\}$.

Definition 1: Let $(U, C \cup D)$ be a decision table with the equivalence relations R_C and R_D on U. If $[x]_C \subseteq [x]_D$ for each $x \in U$, where $R_C = \cap_{R \in C} R$, $R_D = \cap_{d \in D} d$, then $(U, C \cup D)$ is said to be consistent; otherwise, $(U, C \cup D)$ is said to be inconsistent.

Definition 2: Let (U, AT) be an information table, X be a subset of U, $B \subseteq AT$. The lower and upper approximations [1], [2] of X are characterized as

$$\underline{R}_B(X) = \{x | [x]_B \subseteq X\} \quad (2)$$
$$\overline{R}_B(X) = \{x | [x]_B \cap X \neq \emptyset\} \quad (3)$$

Definition 3: Let $(U, C \cup D)$ be a decision table, where C is the condition attribute set and D is the decision attribute set. If $B \neq \emptyset$ and $B \subseteq C$, B is called the positive region reduction [1] of C if it satisfies the following conditions:

$$\mathrm{Pos}_C\, D = \mathrm{Pos}_B\, D \quad (4)$$
$$\text{For any } B' \neq \emptyset \text{ and } B' \subset B, \mathrm{Pos}_C\, D \neq \mathrm{Pos}_{B'}\, D \quad (5)$$

where $\mathrm{Pos}_C\, D$ is the positive region in the decision table $(U, C \cup D)$.

Definition 4: Let $(U, C \cup D)$ be a decision table. If $B \neq \emptyset$ and $B \subseteq C$, B is called the variable precision reduction [10], [45] of C if B satisfies the following conditions:

$$\forall x \in U, (\mu_{CD}(x))_\beta = (\mu_{BD}(x))_\beta \quad (6)$$

$\exists x \in U$, for any $\emptyset \neq B' \subset B$,

$$(\mu_{CD}(x))_\beta \neq (\mu_{B'D}(x))_\beta \quad (7)$$

where

$$(\mu_{CD}(x))_\beta = \left(\frac{|[x]_C \cap D_1|}{|[x]_C|}, \frac{|[x]_C \cap D_2|}{|[x]_C|}, \ldots, \frac{|[x]_C \cap D_l|}{|[x]_C|}\right)_\beta,$$

with $U/R_D = \{D_1, D_2, \ldots, D_l\}$.

Definition 5: Let $(U, C \cup D)$ be a decision table with $U/R_D = \{D_1, D_2, \ldots, D_l\}$. If $\emptyset \neq B \subseteq C$, for a given D_j $(D_j \in U/R_D)$, if B satisfies the following conditions:

$$\forall x \in U, \left(P\left(D_j \mid [x]_C\right)\right)_\beta = \left(P\left(D_j \mid [x]_B\right)\right)_\beta \quad (8)$$

$\exists x \in U$, for any $\emptyset \neq B' \subset B$,

TABLE 1. An information table.

U	a_1	a_2	a_3
x_1	1	1	1
x_2	1	1	1
x_3	1	1	0
x_4	1	0	0
x_5	1	0	1

$$\left(P\left(D_j \mid [x]_C\right)\right)_\beta \neq \left(P\left(D_j \mid [x]_{B'}\right)\right)_\beta \tag{9}$$

then B is called the local attribute reduction [31] of C.

The concepts of attribute reduction introduced above are more common in decision tables. In addition, there are many types of reduction definitions.

III. LOCAL INDISCERNIBILITY RELATION REDUCTION FOR INFORMATION TABLES

Several concepts of local reduction [31]–[34] have been proposed, but they study attribute reduction from the perspective of equivalence classes. To date, there has been a lack of analysis from the perspective of the relation between any two objects. Based on this, the local indiscernibility relation is studied in this paper.

We now consider the local decision set, which is a subset of the decision set, to analyze the indiscernibility relation between any two objects. The definition of the local indiscernibility relation and its corresponding reduction concept are proposed in this section. Note that the local decision set is different from the decision equivalence class (or multi-decision equivalence classes).

Definition 6: Let (U, C) be an information table, where C is the attribute set. Given $X \subseteq U$, the local indiscernibility relation is defined as

$$\mathrm{Ind}_C X = \{(x, y) \mid [x]_C \subseteq X \Leftrightarrow [y]_C \subseteq X\} \tag{10}$$

A simple example is given to illustrate the above definition.

Example 1: Consider the information table (U, C) in Table1, where $U = \{x_1, x_2, x_3, x_4, x_5\}$, $C = \{a_1, a_2, a_3\}$. The set of all equivalence classes in U with respect to an equivalence relation R_C is denoted as $U/R_C = \{\{x_1, x_2\}, \{x_3\}, \{x_4\}, \{x_5\}\}$. Given $X_1 = \{x_1, x_2, a_4\}$, $\mathrm{Ind}_C X_1 = \underline{R_C}(X_1) \times \underline{R_C}(X_1) = \{(x_1, x_1), (x_1, x_2), (x_1, x_4), (x_2, x_1), (x_2, x_2), (x_2, x_4), (x_4, x_1), (x_4, x_2), (x_4, x_4)\}$, where the cardinality $|\mathrm{Ind}_C X_1| = 9$. Given $X_2 = \{x_1, x_5\}$, $\mathrm{Ind}\ _C X_2 = \underline{R_c}(X_2) \times \underline{R_C}(X_2) = \{(x_5, x_5)\}$, with $|\mathrm{Ind}\, d_C X_2| = 1$.

Definition 7: Let (U, C) be an information table, where C is the attribute set. Given $X \subseteq U$ and $A \subseteq C$, the significance of attribute a is defined as

$$\mathrm{Sig}(a, A, X) = \frac{|\mathrm{Ind}_A X| - \left|\mathrm{Ind}_{A-\{a\}} X\right|}{|\mathrm{Ind}_C X|} \tag{11}$$

where $|\cdot|$ is the cardinality of the set.

In Table 1, given $X_2 = \{x_1, x_2, x_3, x_4\}$, for attribute a_1, Definition 6 gives $|\mathrm{Ind}_C X_2| = 16$, $\left|\mathrm{Ind}_{C-\{a_1\}} X_2\right| = 16$, and so $\mathrm{Sig}\,(a_1, C, X_2) = 0$. For attribute a_2, $\left|\mathrm{Ind}_{C-\{a_2\}} X_2\right| = 4$, and so $\mathrm{Sig}\,(a_2, C, X_2) = 0.75$. Given an attribute subset $A = \{a_1, a_2\}$, for attribute a_1, $|\mathrm{Ind}_A X_2| = 9$, $\left|\mathrm{Ind}_{A-\{a_1\}} X_2\right| = 9$, and $\mathrm{Sig}\,(a_1, A, X_2) = 0$. For attribute a_2, $\left|\mathrm{Ind}_{A-\{a_2\}} X_2\right| = 0$ and $\mathrm{Sig}\,(a_2, A, X_2) = 0.56$.

According to Definition 6, because $B \subseteq C$, we have that $\mathrm{Ind}_B X \subseteq \mathrm{Ind}_C X$, $|\mathrm{Ind}_B X| \leq |\mathrm{Ind}_C X|$.

For each $a \in C$, if $\mathrm{Sig}(a, C, X) = 0$, then a is a dispensable attribute for the local indiscernibility relation; otherwise, a is an indispensable attribute.

Definition 8: Let (U, C) be an information table, where C is the attribute set. $B \neq \varnothing$ and $B \subseteq C$, B is called the local indiscernibility relation reduction (LIRR) of C if it satisfies the following two conditions:

$$\mathrm{Ind}_C X = \mathrm{Ind}_B X \tag{12}$$

$$\text{For any} B' \subset B, \mathrm{Ind}_C X \neq \mathrm{Ind}_{B'} X \tag{13}$$

According to Definition 8, to obtain the reduction result, the corresponding discernibility matrix $\mathrm{M} = \left(m_{ij}\right)_{n \times n}$ is defined as follows, where n is the number of elements in the universe set: $m_{ij} =$

$$\begin{cases} \left\{a \mid \left(x_i, x_j\right) \notin R_a\right\}, & x_i \in \underline{R_C}(X) \text{ and } x_j \notin \underline{R_C}(X) \\ \emptyset, & \text{otherwise} \end{cases} \tag{14}$$

This leads to the following lemma.

Lemma 1: Let (U, C) be an information table. If $x_i \in \underline{R_C}(X)$ and $x_j \notin \underline{R_C}(X)$ for $x_i, x_j \in U$, then $m_{ij} \neq \varnothing$.

Proof: Suppose that $x_i \in \underline{R_C}(X)$, $x_j \notin \underline{R_C}(X)$. We assume the contrapositive: if $m_{ij} = \emptyset$, then $\left(x_i, x_j\right) \in R_C$. This means that $x_j \in \underline{R_C}(X)$, and so $x_j \in \underline{R_C}(X)$ and $x_j \notin \underline{R_C}(X)$, which is a contradiction. ■

We can now state the following theorem.

Theorem 1: Let (U, C) be an information table and let B be the LIRR of $C, \emptyset \neq B \subseteq C$. Then, the following conditions are equivalent:

$$\mathrm{Ind}_C X = \mathrm{Ind}_B X \tag{15}$$

$$\text{If } m_{ij} \neq \varnothing, \text{ then } m_{ij} \cap B \neq \varnothing \tag{16}$$

$$\text{If } (x, y) \in R_B, \text{ then } (x, y) \in \mathrm{Ind}_C X \tag{17}$$

Proof: (15) $\Rightarrow$ (16) If $m_{ij} \neq \emptyset$, then $x_i \in \underline{R_C}(X)$ and $x_j \notin \underline{R_C}(X)$. Suppose that $m_{ij} \cap B = \emptyset$, $\left(x_i, x_j\right) \in R_B$. Then, $\left(x_i, x_j\right) \in \mathrm{Ind}_B X$. Hence, $\left(x_i, x_j\right) \in \mathrm{Ind}_C X$ by condition (15). Now, $x_i \in \underline{R_C}(X)$ and $x_j \in \underline{R_C}(X)$, which is a contradiction.

(16) $\Rightarrow$ (17) If $(x, y) \in R_B$, suppose that $(x, y) \notin \mathrm{Ind}_C X$. Then, $x_i \in \underline{R_C}(X)$, $x_j \notin \underline{R_C}(X)$, and so $m_{ij} \neq \emptyset$. Therefore, $m_{ij} \cap B \neq \emptyset$ by condition (16). This means that $\exists a \in B$ such that $(x, y) \notin R_a$, which is a contradiction.

(17) $\Rightarrow$ (15) Because $B \subseteq C$, we have that $\mathrm{Ind}_B X \subseteq \mathrm{Ind}_C X$. Now, we have to show that $Ind_C X \subseteq \mathrm{Ind}_B X$. If $(x, y) \in \mathrm{Ind}_C X$, for $(x, y) \in R_B$, $y \in \underline{R_C}(X)$ by condition (17), which means that $[y]_C \subseteq X$, and so $y \in X$. Thus, $[x]_B \subseteq X$, so $(x, y) \in \mathrm{Ind}_B X$. ■

TABLE 2. Another information table.

U	a_1	a_2	a_3	a_4
x_1	1	0	0	0
x_2	1	1	1	1
x_3	1	1	0	0
x_4	0	1	1	0
x_5	0	1	0	1
x_6	0	1	0	1

TABLE 3. A decision table.

U	a_1	a_2	a_3	a_4	d
x_1	1	0	0	0	0
x_2	1	1	1	1	1
x_3	1	1	0	0	1
x_4	0	1	1	0	2
x_5	0	1	0	1	2
x_6	0	1	0	1	1

The theorem leads to the following corollary.

Corollary 1: Let (U, C) be an information table. Then, $B(\emptyset \neq B \subseteq C)$ is an LIRR of C if and only if it is a minimal subset satisfying $m_{ij} \cap B \neq \emptyset$ for any $m_{ij} \neq \emptyset$.

According to Corollary 1, the LIRR algorithm based on the discernibility matrix for an information table (U, C) is as follows:

Algorithm The LIRR Algorithm Based on the Discernibility Matrix

Input: An information table (U, C) and $X \subseteq U$

Output: All results B for the LIRR

1: Calculate $\underline{R_C}(X)$;

2: $m_{ij} \leftarrow m_{ij} \cup a$ for $i, j \in \left\{1, 2, \ldots, \left|\underline{R_C}(X)\right|\right\}$; $//a \in C$

3: According to the matrix $\mathrm{M} = \left(m_{ij}\right)_{n\times n}$, the discernibility function $f = \prod\left(\sum_{m_{ij}\neq\varnothing} m_{ij}\right)$ in the CNF;

4: Transform the discernibility function to $f = \sum_{i=1}^{S}\left(\prod B_i\right)$ in the CNF;

5: Return B_i,which is the result of attribute reduction.

Example 2: In Table2, if $X = \{x_1, x_4, x_5\}$, $\mathrm{Ind}_C X = \{(x_1, x_1), (x_1, x_4), (x_4, x_1), (x_4, x_4)\}$. The discernibility matrix M, as shown at the bottom of the page.

According to the discernibility matrix M, the CNF of the discernibility function is $f = a_2(a_1 + a_3)(a_1 + a_4)(a_3 + a_4)$, and then the DNF of the discernibility function is $f = a_1a_2a_3 + a_1a_2a_4 + a_2a_3a_4$. Thus, $\{a_1, a_2, a_3\}, \{a_1, a_2, a_4\}$, and $\{a_2, a_3, a_4\}$ are the reduction results of C.

IV. APPLICATION OF LIRR ALGORITHM TO SINGLE- OR MULTI-DECISION CLASSES

In decision tables, the local decision set is a subset of the decision set, and single- or multi-decision classes are a special form of the local decision set. In this section, we illustrate the LIRR algorithm through examples of single- and multi-decision classes in decision tables.

Example 3: In Table3, the quotient set given by C is $U/R_C = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_5, x_6\}\}$ and the quotient set given by D is $U/R_D = \{D_1, D_2, D_3\}$, with $D_1 = \{x_1\}, D_2 = \{\mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_6\}, D_3 = \{x_4, x_5\}$. Given the single-decision class D_2, $Ind_C(D_2) = \{(x_2, x_2), (x_2, x_3), (x_3, x_2), (x_3, x_3)\}$. To obtain all LIRR results, the LIRR algorithm is used. The discernibility matrix M is constructed as follows,

$$M = \begin{bmatrix} \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \{a_2, a_3, a_4\} & \emptyset & \emptyset & \{a_1, a_4\} & \{a_1, a_3\} & \{a_1, a_3\} \\ \{a_2\} & \emptyset & \emptyset & \{a_1, a_3\} & \{a_1, a_4\} & \{a_1, a_4\} \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \end{bmatrix}$$

According to the discernibility matrix M, the discernibility function in the CNF is $f = a_2(a_1 + a_3)(a_1 + a_4)$, whereas the discernibility function in the DNF is $f = a_1a_2 + a_2a_3a_4$. Thus, $\{a_1, a_2\}, \{a_2, a_3, a_4\}$ are the two results for the LIRR of C.

In Example 3, a decision equivalence class D_2 takes the same value of $I_a(x) = 1$(*Table*3), i.e., the local decision set is a single-decision class. In Example 4, the local decision set X is a multi-decision class, which takes values of $I_a(x) = 0$ or $I_a(x) = 1$(Table 3).

Example 4: In Table3, given a set $X = D_1 \cup D_2$,

$$\mathrm{Ind}_C(X) = \{(x_1, x_1), (x_1, x_2), (x_1, x_3), (x_2, x_1), (x_2, x_2), (x_2, x_3), (x_3, x_1), (x_3, x_2), (x_3, x_3)\}$$

$$M = \begin{bmatrix} \emptyset & \{a_2, a_3, a_4\} & \{a_2\} & \emptyset & \{a_1, a_2, a_4\} & \{a_1, a_2, a_4\} \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \{a_1, a_4\} & \{a_1, a_3\} & \emptyset & \{a_3, a_4\} & \{a_3, a_4\} \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \end{bmatrix}$$

To obtain all LIRR results using the LIRR algorithm, the discernibility matrix M is constructed as follows:

$$M=\begin{bmatrix} \emptyset & \emptyset & \emptyset & \{a_1,a_2,a_3\} & \{a_1,a_2,a_4\} & \{a_1,a_2,a_4\} \\ \emptyset & \emptyset & \emptyset & \{a_1,a_4\} & \{a_1,a_3\} & \{a_1,a_3\} \\ \emptyset & \emptyset & \emptyset & \{a_1,a_3\} & \{a_1,a_4\} & \{a_1,a_4\} \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \end{bmatrix}$$

According to M, the discernibility function in the CNF is $f=(a_1+a_3)(a_1+a_4)$ and the discernibility function in the DNF is $f=a_1+a_3a_4$. Thus, $\{a_1\}$, $\{a_3,a_4\}$ are the two results for the LIRR of C.

As this is an NP-hard problem, reduction algorithms based on discernibility matrices are very time-consuming, especially when the number of attributes or objects is large. Thus, the LIRR algorithm requires a lot of time to transform from the CNF to the DNF, and we often cannot obtain the required results. To improve the efficiency of the algorithm, we can simply calculate one or a few LIRR results.

V. THREE ALGORITHMS FOR LIRR IN INFORMATION TABLES

Based on Definition 8, three algorithms are proposed in this section. The addition-deletion strategy and deletion strategy are common methods for obtaining the optimal attribute subset, and are widely used heuristic approaches for identifying the reduction results. Algorithm 1 obtains the reduction through the addition-deletion strategy, whereas Algorithm 2 obtains the reduction through the deletion strategy. Computing the core attribute set is the key operation in Algorithm 1, and the significance of the attributes is an important basis for Algorithms 1 and 2. Therefore, the core attributes and core attribute sets of LIRR are defined in this section. Algorithm 3 constructs the discernibility matrix according to the method proposed in Section 3, and then uses binary integer programming [24] to obtain the reduction results.

Definition 9: Let (U,C) be an information table. Given $X\subseteq U$, the set of core attributes of the local indiscernibility relation is defined as follows: $CORE_C(X)=\bigcap_{i=1}^{s}B_i$. where B_i is the result of attribute reduction.

In Section 4, the local decision set X given the decision table in Table 3 is $D_2=\{\mathrm{X}_2,\mathrm{X}_3,\mathrm{X}_6\}$, because $\{a_1,a_2\},\{a_2,a_3,a_4\}$ are the two reduction results for $\mathrm{Ind}_C(D_2)$ in Example 3. Thus, $\{a_2\}$ is the core set for LIRR with respect to $\mathrm{Ind}_C(D_2)$.

Theorem 2: Let (U,C) be an information table, where C is a condition set. Then, $\forall a\in C$, $a\in CORE_C(X)$ if and only if $\mathrm{Ind}_{C-\{a\}}X\neq\mathrm{Ind}_C X$.

Proof: $(\Rightarrow)$ Suppose that $\mathrm{Ind}_{C-\{a\}}X=\mathrm{Ind}_C X$. It is easy to show that attribute a is dispensable for each $a\in C$, This contradicts the statement that attribute $a\in CORE_C(X)$, so it holds. $(\Leftarrow)$ If $\mathrm{Ind}_{C-\{a\}}X\neq\mathrm{Ind}_C X$, then attribute a is indispensable for $a\in C$. Thus, a is an element in all LIRR results. ■

Using Definition 9 and Theorem 2, we state the following definition.

Definition 10: Let (U,C) be an information table. Given $X\subseteq U$, the set of core attributes of the local indiscernibility relation are defined as follows: $CORE_C(X)=\left\{a\mid a\in C,\mathrm{Ind}_{C-\{a\}}X\neq\mathrm{Ind}_C X\right\}$.

Algorithms 1 and 2 mainly calculate the reduction results according to Definitions 7 and 10. The addition-deletion strategy for the LIRR result, i.e., Algorithm 1, is as follows.

Algorithm 1 The LIRR Algorithm for the Addition-Deletion Strategy

Input: An information table (U,C) and $X\subseteq U$
Output: An LIRR result R

1: Set $B=\varnothing$; //(i.e., initialize B, a reduction result);
2: Calculate $\mathrm{Sig}(a,C,X)$ for each $a\in C$;
3: Add a to B whenever $\mathrm{Sig}(a,C,X)>0$;
4: $CORE_{\mathrm{Ind}}(C)\leftarrow B$;
5: While $\mathrm{Ind}_C X\neq\mathrm{Ind}_B X$, repeat step 6;
6: Add a to B for a satisfying $\mathrm{Sig}(a,B\cup\{a\},X)=\max_{a'\in C-B}\mathrm{Sig}\left(a',B\cup\left\{a'\right\},X\right)$
7: End while
8: Update B by deleting a from B whenever $\mathrm{Sig}(a,B,X)=0$;
9: $R=B$;
10: Return R.

Algorithm 1 calculates the core attributes and adds them to set B. Among the remaining attributes in the condition attribute set, those with the greatest significance according to Definition 7 are added to B one by one in order of significance. These two main steps are called the add strategy. We then iterate through all elements in set B to remove redundant attributes. The deletion strategy for the LIRR result, i.e., Algorithm 2, is as follows.

Algorithm 2 The LIRR Algorithm for the Deletion Strategy

Input: An information table (U,C) and $X\subseteq U$
Output: An LIRR result R

1: Let $R=C$, $C\,D=C$;
2: Calculate $\mathrm{Sig}(a,R,X)$ for each $a\in CD$;
3: Sort attributes in CD in an ascending order of significance;
4: While $CD\neq\emptyset$ do
5: $CD\leftarrow CD-\{a\}$ for $a\in CD$ satisfying $\mathrm{Sig}\left(a,C\,D,\mathrm{Ind}_C X\right)=0$;
6: If $\mathrm{Ind}_{R-\{a\}}X=\mathrm{Ind}_C X$;
7: $R\leftarrow R-\{a\}$;
8: End while
9: Return R.

According to Definitions 7 and 8, Algorithm 2 first sorts all attributes in C in an ascending order of significance. The redundant attributes continue to be deleted for any $a\in R$ until $\mathrm{Ind}_C X\neq\mathrm{Ind}_R X$.

Because of the large amount of computation involved in the transformation of the CNF to the DNF, a binary integer programming model can be used. The binary integer programming model for the LIRR result, i.e., Algorithm 3, is as follows.

Algorithm 3 The LIRR Algorithm by Applying Binary Integer Programming

Input: An information table (U, C) and $X \subseteq U$

Output: An LIRR result R

1: Calculate $\underline{R_C}(X)$;
2: $m_{ij} \leftarrow m_{ij} \cup a$ for $i, j \in \{1, 2, \ldots, |\underline{R_C}(X)|\}$; $//a \in C$;
3: Construct the corresponding discernibility matrix $\mathrm{M} = (m_{ij})_{n\times n}$, where n is the number of elements in the universe set;
4: Calculate the minimal element set $L_{\min}$ for the matrix $\mathrm{M} = (m_{ij})_{n\times n}$;
5: Minimize $\sum_{a\in m_{ij}} a > 0$ such that $m_{ij} \in T_{\min}$, where each attribute $a \in m_{ij}$ in $T_{\min}$ is labeled 1; otherwise, it is labeled 0;
6: $R = B$;
7: Return R.

To better explain the application of Algorithm 3 to the discernibility matrix, we illustrate its application through Example 5.

Example 5: Consider the decision table $(U, C \cup D)$ in Table 2 and the discernibility matrix M in Example 2, in which the minimum subsets are $\{a_2\}\{a_1, a_3\}, \{a_1, a_4\}, \{a_3, a_4\}$. Each $a_i \in C$ is considered as the binary decision variable. The binary integer programming model is,

$$\begin{aligned} &\boldsymbol{Min}\ a_1 + a_2 + a_3 + a_4,\\ &\boldsymbol{s.t.}\ a_2 > 0\\ &\quad a_1 + a_3 > 0\\ &\quad a_1 + a_4 > 0\\ &\quad a_3 + a_4 > 0 \end{aligned}$$

where $a_i = 1$ or $a_i = 0$ for $a_i \in C$.

The solutions to the model are $a_1 = a_2 = a_3 = 1$ and $a_4 = 0$; $a_1 = a_2 = a_4 = 1$ and $a_3 = 0$; or $a_2 = a_3 = a_4 = 1$ and $a_1 = 0$. Thus, the results are $\{a_1, a_2, a_3\}, \{a_1, a_2, a_4\}$, and $\{a_2, a_3, a_4\}$.

In obtaining all the results, the reduction method based on the discernibility matrix is relatively inefficient. However, in most cases, we only need to find one or a few results. Algorithm 3 obtains the reduction results by applying binary integer programming to the discernibility matrix. Compared with Algorithms 1 and 2, the calculation of Algorithm 3 is relatively complex. However, for the reduction algorithm based on the discernibility matrix, this method[24] offers improved operational efficiency.

VI. EXPERIMENTAL ANALYSIS

To demonstrate the effectiveness of the algorithms proposed in this paper, we compare Algorithms 1–3 with the existing algorithms LVP-AR[31], PAR-DM[32] and LR-DM[36]. The LVP-AR algorithm is a reduction method based on precision, whereas algorithms PAR-DM and LR-DM are reduction methods for local decision sets. Fourteen UCI datasets [46] were used in the experiments; these datasets are briefly summarized in Table 4, where $|U|$ and $|C|$ denote the number of objects and condition attributes, respectively, and $|U/D|$ denotes the number of classes. All experiments were coded in Python 3.8 and were tested on a personal computer running 64-bit Windows10 Pro with an Intel(R) Core i7 − 10750HCPU2.60GHz and 16.0 GB RAM. The glass identification, statlog (vehicle silhouettes), thoracic surgery, statlog (heart), tic-tac-toe endgame, solar flare, chess (king-rook vs. king-pawn), and breast cancer Wisconsin (diagnostic) datasets are denoted as GI, Vehicle, T.S., Heart, T.T.T., S.F., Chess, and B.C., respectively.

TABLE 4. Dataset information.

ID	Dataset	Abbreviation	$\lvert U\rvert$	$\lvert C\rvert$	$\lvert U/D\rvert$
1	Glass identification	G.I.	214	9	7
2	Statlog(vehicle silhouettes)	Vehicle	946	18	4
3	Wine	Wine	178	13	3
4	Thoracic surgery	T.S.	470	16	2
5	Zoo	Zoo	101	16	7
6	Statlog(heart)	Heart	270	13	2
7	Iris	Iris	150	4	3
8	Yeast	Yeast	1484	8	10
9	Tic-tac-toe Endgame	T.T.T.	958	9	2
10	Solar Flare	S.F.	1389	10	10
11	Abalone	Abalone	4177	8	28
12	Chess(King-Rook vs. King-Pawn	Chess	3196	36	2
13	Flags	Flags	194	28	7
14	Breast Cancer Wisconsin	B.C.	569	30	2

Four classifiers are used to evaluate the accuracy of the reduction results—kernel naive Bayes (NB), C5 decision tree (DT), fine Gaussian support vector machine (SVM), and fine K-nearest neighbors (KNN). In obtaining the results, 10-fold cross-validation was used. The experimental results are shown in Figs.1-10, in which the X-axis represents the different datasets and the Y-axis represents the runtime or the classification accuracy. Fig.1 shows the runtime of all six algorithms on the different datasets. The reduction length of the above algorithms is compared in Fig. 2. Figs.4, 6, 8, and 10 compare the classification accuracy using the kernel NB, C5 DT, fine Gaussian SVM, and fine KNN classifiers, respectively.

For convenience, the dataset before reduction is called the original dataset. Because Algorithms 3, LVP-AR, PAR-DM and LR-DM build discernibility matrices, their runtimes are relatively high in Fig.1. Fig.2 shows the reduction

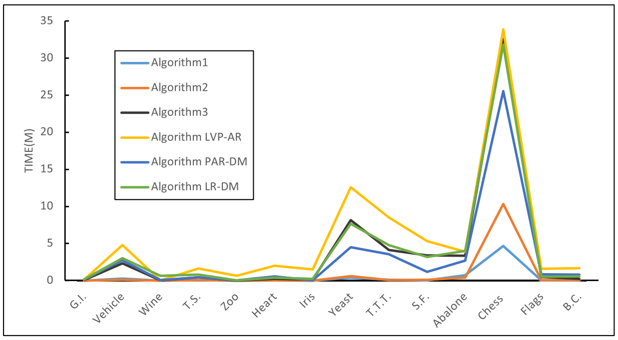

FIGURE 1. Runtime results.

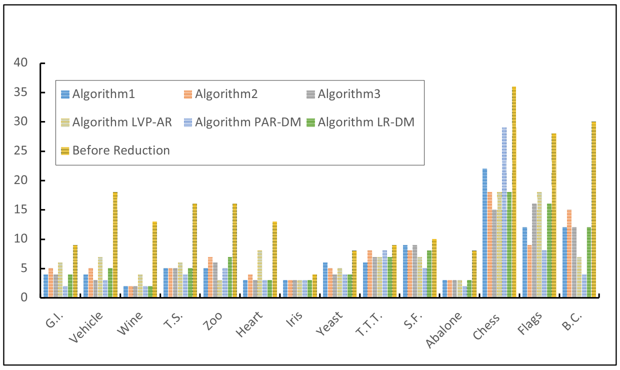

FIGURE 2. Length comparison of six algorithms.

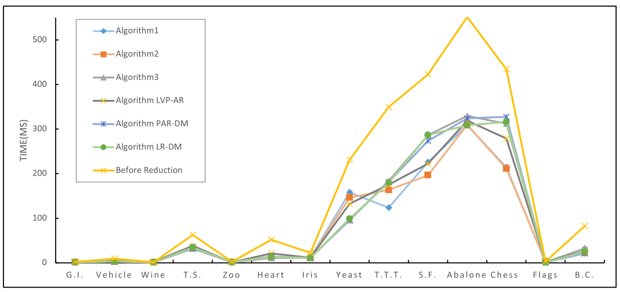

FIGURE 3. Runtime results based on kernel NB.

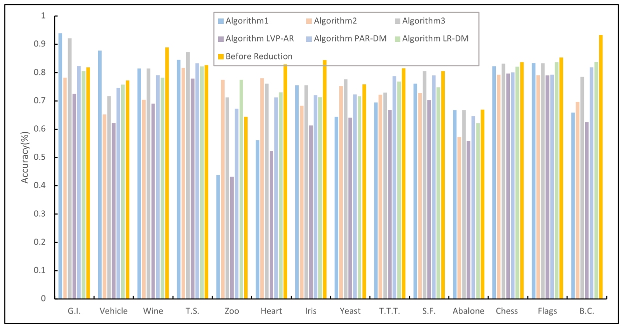

FIGURE 4. Comparison of accuracy based on kernel NB.

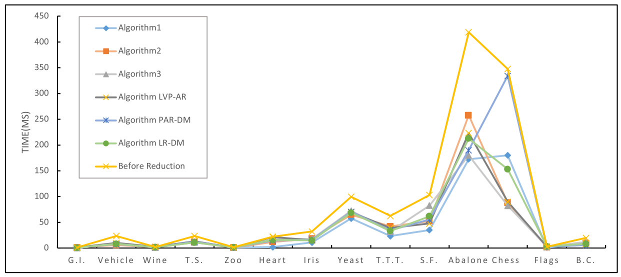

FIGURE 5. Runtime results based on C5 DT.

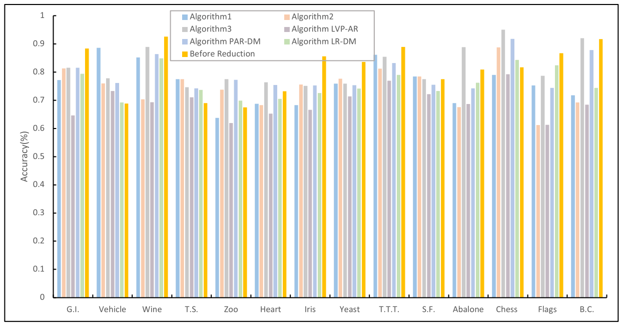

FIGURE 6. Comparison of accuracy based on C5 DT.

performance achieved on the different datasets. The algorithms involved in the experiments have no obvious differences in performance.

Figs. 3 and 4 compare the runtime and accuracy before and after reduction when using the kernel NB classifier. The runtime after applying the six algorithms is obviously better than with the original data (Fig.3). The Algorithm LVP-AR has relatively low classification accuracy compared with the other algorithms. On most datasets, the classification accuracy using the reduction obtained by Algorithm 3 is higher than that obtained by Algorithms 1 and 2. Although the accuracies are slightly lower than for the original dataset, the datasets obtained by the reduction algorithms have significantly lower runtimes, indicating that the fitting effect is ideal.

Figs. 5 and 6 compare the runtime and accuracy before and after reduction when using the C5 DT classifier. Again, the runtime of the reduced datasets is significantly better than that of the original dataset (Fig. 5). On most datasets, the classification accuracy of the reduction obtained by Algorithm 3 is slightly higher than that obtained by the other algorithms. Note that the classification accuracy of the reduction obtained by Algorithm 3 is relatively high on the Abalone and Chess datasets. For example, on the Abalone dataset, the accuracies of the reductions obtained by Algorithms 1–3, LVP-AR, PAR-DM and LR-DM are 68.98%, 67.59%, 88.82%, 68.65%, 74.22%, and 76.19%, compared with 80.89% for the original data.

Figs. 7 and 8 compare the runtime and accuracy before and after reduction when using the fine Gaussian SVM classifier. The classification accuracy of the datasets obtained by Algorithms 1-3, PAR-DM and the original datasets fluctuates slightly (Fig. 8). The LVP-AR and LR-DM algorithms have relatively low classification accuracy. The reduced datasets obtained by the proposed algorithms have very

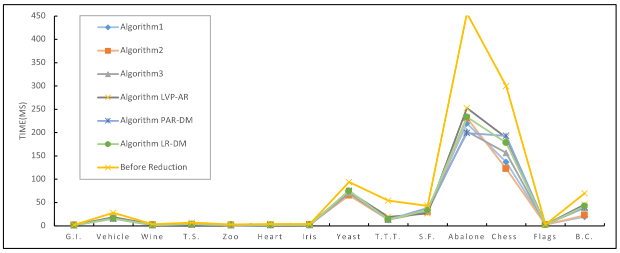

FIGURE 7. Runtime results based on fine Gaussian SVM.

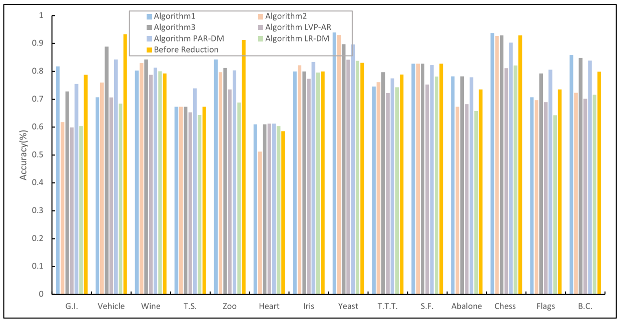

FIGURE 8. Comparison of accuracy based on fine Gaussian SVM.

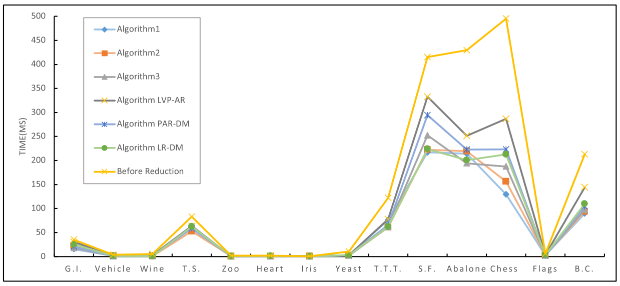

FIGURE 9. Runtime results based on fine KNN.

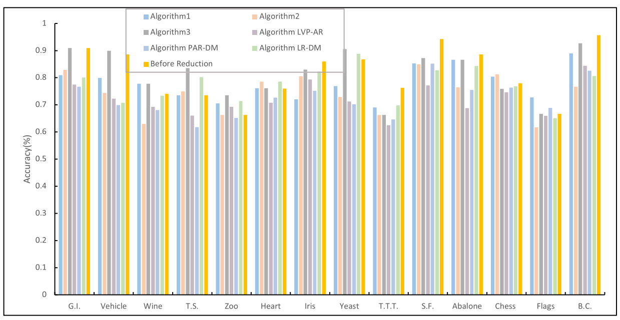

FIGURE 10. Comparison of accuracy based on fine KNN.

similar runtimes, but are generally faster than with the original data(Fig. 7).

Figs. 9 and 10 compare the runtime and accuracy before and after reduction when using the fine KNN classifier. The runtime of the reduced datasets is lower than that of the original datasets. The training accuracy is slightly different before and after reduction. For example, in the S.F. dataset, the runtimes of Algorithms 1–3, LVP-AR, PAR-DM, and LR-DM are 217.01, 222.53, 252.38, 332.73, 294.4, and 223.82 ms, respectively, compared with 415.37 ms for the original dataset. The corresponding training accuracies are 85.26%, 84.99%, 87.22%, 77.18%, 85.16%, and 82.74% for Algorithms 1–3, LVP-AR, PAR-DM, and LR-DM, compared with 94.28% for the original dataset.

With the four classifiers, there is a slight difference in training accuracy before and after reduction. However, the runtime of the reduced datasets is better than that of the original datasets, which means that the fitting effect is ideal.

VII. CONCLUSION

Considering the local decision set and the discernibility relation, this paper has proposed the concept of the local indiscernibility relation reduction. In view of the proposed LIRR, the construction of the discernibility matrix has been established and verified. To improve the operational efficiency of LIRR, three algorithms based on add-deletion, deletion, and binary integer programming strategies, respectively, were proposed. Finally, the effectiveness of the proposed algorithms was verified using 14 UCI datasets and four classifiers. The proposed LIRR not only provides a new method for reduction, but also makes it possible to study local reduction under non-equivalent relations.

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