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RESEARCH ARTICLE

Adaptive Fractional-Order Non-Singular Fast Terminal Sliding Mode Control Based on Fixed Time Disturbance Observer for Manipulators

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ABSTRACT In this study, with the help of a disturbance observer, an adaptive fractional-order sliding mode control scheme is designed to achieve trajectory tracking control of indeterminate manipulators. Firstly, the fractional-order non-singular fast terminal sliding mode (FO-NFTSM) surface is presented to increase the convergence velocity of the controller. Secondly, the adaptive reaching law is designed on the strength of the super-twisting algorithm to ensure the control performance of the approaching stage. Meanwhile, aiming at the compound disturbance existing in the manipulator system, an adaptive fixed-time sliding mode disturbance observer (AFSMDO) is introduced to settle the problem, which can estimate the lumped disturbance in real-time. Moreover, compensate for the system and raise the precision of the controller. The Lyapunov method is applied to certify the stability of the control system. Simulation experiments verify the superior property of the controller designed in this article.

INDEX TERMS Manipulator, terminal sliding mode, super-twisting algorithm, fixed-time disturbance observer.

I. INTRODUCTION

As one of the most widely used mechanical devices in the field of robotics, the manipulator is widely used in crucial fields such as aerospace, military, manufacturing, and medical [1]–[4]. The trajectory tracking control of the manipulator has always been a hot research direction. Whether the manipulator system can move in the light of the preset trajectory is the crux of realizing intricate tasks. However, the manipulator is a highly coupled, complex, time-varying non-linear system, which not only has unmodeled dynamic errors, parameter errors, etc. but also its motion state and control accuracy are disturbed by unknown factors in the external environment. All these make high-precision trajectory tracking face many challenges. Accordingly, it is essential to design a reasonable and feasible control strategy, which is of great significance for the wide application of manipulators.

In recent years, in the field of trajectory tracking control of robotic arms, scholars have proposed many control schemes, such as neural network control [5]–[7], iterative learning control [8], [9], fuzzy control [10]–[12], sliding mode control [13]–[15], and so on. Sliding mode control is a non-linear control with a discontinuous control construction. In the dynamic process, the system compensation gain is adjusted according to the current state deviation of the system so that the system state slides according to the predetermined sliding mode trajectory. At this time, the system is invariant under the condition of parameter perturbation and external interference. In short, sliding mode control has a fast response speed and strong robustness, so it is widely used in practical applications such as manipulators, cable-driven manipulators, and permanent magnet synchronous motors [16]–[19].

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Linear sliding mode control cannot ensure that the system state will reach the balance point within a certain time. In order to attain finite-time convergence of the system state, a terminal sliding mode (TSM) control is proposed in [20]. However, conventional TSM control may suffer from singularity problems. Therefore, non-singular terminal sliding mode (NTSM) control in [21] has been developed to overcome this problem. In [22], a fast NTSM control method is raised, which realizes the faster dynamic response characteristics and higher steady-state accuracy of the permanent magnet linear motor system.

The above discussions are all integer-order control schemes. It is well known that fractional-order controllers have better control accuracy than integer-order controllers due to their flexibility in fractional differentiation and integration. In [23], combining a switching reaching law, a fractional-order NTSM controller is designed, which enhances the control accuracy and robustness of the electro-optical tracking system. The availability of the control program is tested by a simulation example. In [24], the fractional-order backstepping fast TSM method is developed to ensure the robustness of the manipulator system with uncertain factors, actuator faults, and external disturbances.

However, the chattering phenomenon is a big obstacle preventing the sliding mode control method from achieving a better control effect. The super-twisting algorithm is a sort of high-order sliding mode algorithm, which can solve the chattering problem well and has outstanding control performance. The key thought of the higher-order sliding mode is to apply the sign function to the higher-order derivative of the sliding mode variable. The super-twisting algorithm does not demand derivatives of the sliding mode variables, which simplifies the structure of the controller. Wang *et al.* [25] introduced the super-twisting algorithm into fractional NTSM control so as to implement the purpose of tracking control of the cable manipulator. The super-twisting algorithm ensures excellent robustness and high control accuracy at the approaching stage. In [26], a fractional-order super-twisting control method is used to ameliorate the robustness of electromechanical systems under uncertain conditions. In [27], the super-twisting algorithm is combined with the fractional NTSM surface to achieve a better tracking control effect of the optoelectronic tracking system and ensure the strong robustness of the system.

Unfortunately, there are unknown disturbances and uncertainties that are difficult to obtain in manipulator systems, and the observer technique is a productive method to estimate the lumped disturbance in the system. For the presence of matching disturbance in the system, a fractional disturbance observer is introduced in [28] to compensate for the system, which improves the positioning performance of the double pendulum offshore crane. In [29], a reduced-order observer is raised to gauge the uncertainty in the system and realizes the tracking of the position of the manipulator in a limited time. In [30], the extended state observer is improved, and a fixed-time extended state disturbance observer is presented, which ensures the high accuracy tracking control of the two-link manipulator.

In order to precisely track and control the trajectory of the end of the manipulator with uncertain factors, a controller with superb performance is necessary. Motivated by the above works, ground on a disturbance observer, this article designs an adaptive fractional-order sliding mode controller to realize the trajectory tracking control of the manipulator with uncertain factors. The main contributions are depicted by:

(1) The fractional-order non-singular fast terminal sliding mode (FO-NFTSM) surface is presented, which realizes the finite-time convergence of the system state and enhances the precision of the controller.
(2) The adaptive reaching law is designed using the super-twisting algorithm to raise the control performance of the approaching stage.
(3) A new adaptive fixed-time sliding mode disturbance observer (AFSMDO) is raised to estimate and compensate for the lumped disturbance and increase the tracking accuracy of the controller.
(4) Theoretical stability analysis of the control system is given. Furthermore, the effectiveness and superiority of the designed controller have been verified by the simulation results.

The remainder of this work is constructed as follows. The second section gives a few basic lemmas and the dynamic model of the manipulator. The design of the controller is described in the third section. In the fourth section, the stability analysis is given. Furthermore, the effectiveness of the designed controller is demonstrated by simulation results in the fifth section. At last, the conclusion is presented in the sixth section.

II. PRELIMINARIES AND MANIPULATOR MODEL DESCRIPTION

A. PRELIMINARIES

Lemma 1 [31]: Think over a scalar dynamic system

$$\dot{y} = -c_1 \mathrm{sig}^{n_1} y - c_2 \mathrm{sig}^{n_2} y, y(0) = y_0 \tag{1}$$

where $\mathrm{sig}^{n_1} y = |y|^{n_1} \cdot \mathrm{sign(y)}$, $\mathrm{sig}^{n_2} y = |y|^{n_2} \cdot \mathrm{sign(y)}$, $c_1 > 0$, $c_2 > 0$, $n_1 > 1$, and $0 < n_2 < 1$. Afterwards, the system is fixed-time stable. In addition, the settling time T is bounded by

$$T \leq T_{\max} = \frac{1}{c_1(n_1 - 1)} + \frac{1}{c_2(1 - n_2)} \tag{2}$$

Lemma 2 [32]: Towards all positive numbers $x_i > 0, i = 1, 2, \ldots, n$, the coming inequalities hold

$$\begin{cases} \sum_{i=1}^{n} x_i^{\varepsilon} \geq \left(\sum_{i=1}^{n} x_i\right)^{\varepsilon}, & \text{if } 0 < \varepsilon < 1 \\ \sum_{i=1}^{n} x_i^{\varepsilon} \geq n^{1-\varepsilon} \left(\sum_{i=1}^{n} x_i\right)^{\varepsilon}, & \text{if } \varepsilon > 1 \end{cases} \tag{3}$$

Lemma 3 [32]: For a system $\dot{x} = f(x)$, if there are a continuous function $V(x)$ and scalars $\alpha_1 > 0$, $\alpha_2 > 0$, $\gamma_1 > 0$, $0 < \gamma_2 < 1$, and $0 < \phi < \infty$, such that

$\dot{V}(x) \leq -\alpha_1 V^{\gamma_1} - \alpha_2 V^{\gamma_2} + \phi$ holds, the system is fixed-time stable, and the region of convergence is

$$V(x) \leq \min\left\{\left\{\frac{\phi}{(1-\kappa)\alpha_1}\right\}^{\frac{1}{\gamma_1}}, \left\{\frac{\phi}{(1-\kappa)\alpha_2}\right\}^{\frac{1}{\gamma_2}}\right\} \tag{4}$$

where $0 < \kappa < 1$. The convergence time satisfies:

$$T \leq \frac{1}{\alpha_1}\frac{1}{\kappa(\gamma_1 - 1)} + \frac{1}{\alpha_2}\frac{1}{\kappa(1-\gamma_2)} \tag{5}$$

B. DYNAMICS OF MANIPULATOR

Consider the following manipulator dynamics model:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + \tau_d \tag{6}$$

where $q \in R^n$, $\dot{q} \in R^n$, and $\ddot{q} \in R^n$ are the position, angular velocity, and angular acceleration vector of the manipulator joint, respectively; $G(q) = G_0(q) + \Delta G(q) \in R^n$ denotes the gravity matrix; $C(q,\dot{q}) = C_0(q,\dot{q}) + \Delta C(q,\dot{q}) \in R^{n\times n}$ indicates the centrifugal force and Coriolis matrix; $M(q) = M_0(q) + \Delta M(q) \in R^{n\times n}$ means the inertia matrix. $M_0(q)$, $C_0(q,\dot{q})$, and $G_0(q)$ are nominal values, $\Delta M(q)$, $\Delta C(q,\dot{q})$, and $\Delta G(q)$ are uncertain values. $\tau \in R^n$ indicates the control input acting on the joint. τ_d denotes the external disturbance.

Equation (6) can be written as:

$$M_0(q)\ddot{q} + C_0(q,\dot{q})\dot{q} + G_0(q) = \tau + F_d \tag{7}$$

where $F_d = \tau_d - \Delta M(q)\ddot{q} - \Delta C(q,\dot{q})\dot{q} - \Delta G(q)$ denotes the lumped disturbance of model error and external disturbance.

The research target of this article is to devise a controller so that the actual trajectory of the manipulator can rapidly and precisely track the desired trajectory.

III. DESIGN OF CONTROLLER

In this section, in line with the sliding mode control strategy and disturbance observer technique, we design a tracking controller for the manipulator with uncertain factors.

The design process of the controller is primarily segmented into three steps. Above all, with the help of fractional-order theory, the FO-NFTSM surface is given. Subsequently, the adaptive reaching law is presented by using the super-twist algorithm. At last, a new AFSMDO is raised to deal with the lumped disturbance in the system.

A. DESIGN OF SLIDING MODE SURFACE

The position tracking error of each joint of the manipulator is

$$e = q_d - q \tag{8}$$

where q_d is taken as the desired position of the joint, q indicates the actual position of the joint.

In general, the linear sliding mode surface cannot assure that the system state reaches a certain region of zero in a limited time. The traditional linear sliding mode surface is depicted by

$$s = \dot{e} + ce \tag{9}$$

where $c > 0$.

Hence, for the purpose of gaining the superior control performance of the manipulator system, the FO-NFTSM surface is depicted by

$$s = D^{a}e + m_1 |e|^{\lambda_1}\operatorname{sign}(e) + m_2 |\dot{e}|^{\lambda_2}\operatorname{sign}(\dot{e}) \tag{10}$$

where D^{α} represents the fractional derivative, $0 < \alpha < 1$, $m_1 > 0$, $m_2 > 0$, $\lambda_1 > \lambda_2$, $1 < \lambda_2 < 2$. The time derivative of equation (10) gives

$$\dot{s} = D^{a+1}e + m_1\lambda_1 |e|^{\lambda_1 - 1}\dot{e} + m_2\lambda_2 |\dot{e}|^{\lambda_2 - 1}\ddot{e} \tag{11}$$

Remark 1: The exponent of the error e in the sliding mode surface function s is greater than 1, which can prevent the error exponent from being negative after the derivation of s, and assure the non-singularity of the sliding mode surface.

B. DESIGN OF REACHING LAW

Sliding mode control can be split into sliding stage and reaching stage. Using the above FO-NFTSM surface can effectively ensure powerful robustness, excellent control accuracy, and rapid convergence in the sliding stage. To further ensure the control property of the reaching stage, the super-twisting algorithm is adopted, and the adaptive super-twisting reaching law is designed as

$$\dot{s} = -\hat{k}_1 |s|^{\frac{1}{2}}\operatorname{sign}(s) - \int_0^t k_2 \operatorname{sign}(s)\mathrm{d}t - \hat{k}_3 s - k_4 \operatorname{sign}(s) \tag{12}$$

where k_1, k_2, k_3, and k_4 are positive matrices, $\hat{k}_1$ is the estimate of k_1, $\hat{k}_3$ is the estimate of k_3, $\tilde{k}_1 = k_1 - \hat{k}_1$, and $\tilde{k}_3 = k_3 - \hat{k}_3$.

The adaptation parameters $\hat{k}_1$ and $\hat{k}_3$ can be acquired by the ensuing adaptive law:

$$\dot{\hat{k}}_1 = \eta_1 m_2 \lambda_2 |\dot{e}|^{\lambda_2 - 1} |s|^{\frac{3}{2}} \tag{13}$$

$$\dot{\hat{k}}_3 = \eta_2 m_2 \lambda_2 |\dot{e}|^{\lambda_2 - 1} |s|^{2} \tag{14}$$

where $\eta_1 = \operatorname{diag}[\eta_{11}, \eta_{12}, \ldots, \eta_{1n}]$, $\eta_2 = \operatorname{diag}[\eta_{21}, \eta_{22}, \ldots, \eta_{2n}]$, $\eta_{1i} > 0$ and $\eta_{2i} > 0$ are adaptive gains.

According to equations (7), (11), and (12), the control law of the system is acquired:

$$\begin{aligned}\tau = {} & M_0(q)\ddot{q}_d + M_0(q)(\hat{k}_1 |s|^{\frac{1}{2}}\operatorname{sign}(s) + \int_0^t k_2 \operatorname{sign}(s)\mathrm{d}t + \hat{k}_3 s \\ & + k_4 \operatorname{sign}(s) + \frac{1}{\lambda_2 m_2}|\dot{e}|^{2-\lambda_2}\operatorname{sign}(\dot{e})(D^{\alpha}e + m_1\lambda_1 |e|^{\lambda_1 - 1})) \\ & + C_0(q,\dot{q})\dot{q} + G_0(q) - F_d\end{aligned} \tag{15}$$

C. DESIGN OF AFSMDO

In practical applications, the lumped disturbance F_d is usually difficult to obtain accurately, and its upper bound value is also difficult to obtain. For the purpose of compensating for the influence of lumped disturbance in the control system, an AFSMDO is designed to effectively estimate and compensate for the lumped disturbance. In accordance with the fixed-time control theory and the disturbance observer method, the sliding mode surface is defined by introducing auxiliary variables, and we raise a sliding mode disturbance observer with fixed-time convergence property, which brings about a valid estimation of the lumped disturbance of the system and realizes a high-precision trajectory tracking manipulator.

Assumption 1: The modeled lumped disturbance $d = M_0^{-1}F_d$ and its derivative $\dot{d}$ are unknown and bounded. $\left|\dot{d}\right| \leq b_3$, where b_3 is a constant.

Define a sliding mode surface:

$$s_a = \dot{z} \tag{16}$$

where $z = \dot{q} - \hat{m}$. $\hat{m}$ is the estimated value of $\dot{q}$. Then

$$\dot{\hat{m}} = \hat{d} + M_0(q)^{-1}(\tau - C_0(q,\dot{q})\dot{q} - G_0(q)) \tag{17}$$

where $\hat{d}$ is the estimated value of d. d is the lumped disturbance of the manipulator system.

AFSMDO is given as:

$$\dot{\hat{d}} = b_1 \left|s_a\right|^{h_1} \operatorname{sign}(s_a) + b_2 \left|s_a\right|^{h_2} \operatorname{sign}(s_a) + \hat{b}_3 \operatorname{sign}(s_a) \tag{18}$$

$$\dot{\hat{b}}_3 = v\left|s_a\right| \tag{19}$$

where $b_1, b_2 > 0$, $b_3 \geq \left|\dot{d}\right|$, $h_1 > 1$, $h_2 < 1$, $v > 0$.

Taking the derivative of the auxiliary variable z, we get

$$\begin{aligned}\dot{z} &= \ddot{q} - \dot{\hat{m}}\\ &= M_0(q)^{-1}\left(\tau + F_d - C_0(q,\dot{q})\dot{q} - G_0(q)\right)\\ &\quad - \hat{d} - M_0(q)^{-1}(\tau - C_0(q,\dot{q})\dot{q} - G_0(q))\\ &= d - \hat{d}\end{aligned} \tag{20}$$

According to equation (16), we get

$$s_a = \dot{z} = d - \hat{d} \tag{21}$$

Proof: To demonstrate the stability of the AFSMDO error and its fixed-time convergence characteristics, the following Lyapunov function is chosen:

$$V_1 = \frac{1}{2}s_a^2 + \frac{1}{2v}\tilde{b}_3^2 \tag{22}$$

Taking the derivation of the above equation, combining equation (18), equation (19), and equation (21), we get

$$\begin{aligned}\dot{V}_1 &= s_a\dot{s}_a - \frac{1}{v}\tilde{b}_3\dot{\hat{b}}_3\\ &= s_a(\dot{d} - \dot{\hat{d}}) - \frac{1}{v}\tilde{b}_3\dot{\hat{b}}_3\\ &= s_a(\dot{d} - b_1\left|s_a\right|^{h_1}\operatorname{sign}(s_a) - b_2\left|s_a\right|^{h_2}\operatorname{sign}(s_a) - \hat{b}_3\operatorname{sign}(s_a))\\ &\quad - \frac{1}{v}\tilde{b}_3\dot{\hat{b}}_3\\ &= s_a\dot{d} - b_1\left|s_a\right|^{h_1+1} - b_2\left|s_a\right|^{h_2+1} - \hat{b}_3\left|s_a\right| - \tilde{b}_3\left|s_a\right|\\ &= s_a\dot{d} - b_1\left|s_a\right|^{h_1+1} - b_2\left|s_a\right|^{h_2+1} - \hat{b}_3\left|s_a\right| - \left(b_3 - \hat{b}_3\right)\left|s_a\right|\\ &\leq -b_1\left|s_a\right|^{h_1+1} - b_2\left|s_a\right|^{h_2+1} - (b_3 - \dot{d})\left|s_a\right|\\ &\leq -b_1\left|s_a\right|^{h_1+1} - b_2\left|s_a\right|^{h_2+1}\end{aligned} \tag{23}$$

Due to $\dot{V}_1 \leq -b_1\left|s_a\right|^{h_1+1} - b_2\left|s_a\right|^{h_2+1} \leq 0$, the observer is stable in line with the Lyapunov stability theory.

Next, to demonstrate the fixed-time convergence property of the observer, equation (23) can be rewritten as:

$$\begin{aligned}\dot{V}_1 \leq{}& \left(\frac{1}{2v}\tilde{b}_3^2\right)^{\frac{h_1+1}{2}} + \left(\frac{1}{2v}\tilde{b}_3^2\right)^{\frac{h_2+1}{2}} - \left(\frac{1}{2v}\tilde{b}_3^2\right)^{\frac{h_1+1}{2}}\\ &- \left(\frac{1}{2v}\tilde{b}_3^2\right)^{\frac{h_2+1}{2}} - b_1 2^{\frac{h_1+1}{2}}\left(\frac{1}{2}s_a^2\right)^{\frac{h_1+1}{2}}\\ &- b_2 2^{\frac{h_2+1}{2}}\left(\frac{1}{2}s_a^2\right)^{\frac{h_2+1}{2}}\end{aligned} \tag{24}$$

Let $c_1 = \min\left\{b_1 2^{\frac{h_1+1}{2}}, 1\right\}$, $c_2 = \min\left\{b_2 2^{\frac{h_2+1}{2}}, 1\right\}$, $\phi = \left(\frac{1}{2v}\tilde{b}_3^2\right)^{\frac{h_1+1}{2}} + \left(\frac{1}{2v}\tilde{b}_3^2\right)^{\frac{h_2+1}{2}}$,$\phi > 0$, then equation (24) can be rewritten as

$$\begin{aligned}\dot{V}_1 \leq{}& -c_1\left(\left(\frac{1}{2v}\tilde{b}_3^2\right)^{\frac{h_1+1}{2}} + \left(\frac{1}{2}s_a^2\right)^{\frac{h_1+1}{2}}\right)\\ &- c_2\left(\left(\frac{1}{2v}\tilde{b}_3^2\right)^{\frac{h_2+1}{2}} + \left(\frac{1}{2}s_a^2\right)^{\frac{h_2+1}{2}}\right) + \phi\end{aligned} \tag{25}$$

According to Lemma 2, it can be known that

$$\begin{aligned}\dot{V}_1 &\leq -c_1 2^{1-\frac{h_1+1}{2}}\left(\frac{1}{2v}\tilde{b}_3^2 + \frac{1}{2}s_a^2\right)^{\frac{h_1+1}{2}}\\ &\quad - c_2\left(\frac{1}{2v}\tilde{b}_3^2 + \frac{1}{2}s_a^2\right)^{\frac{h_2+1}{2}} + \phi\\ &\leq -c_1 2^{1-\frac{h_1+1}{2}} V_1^{\frac{h_1+1}{2}} - c_2 V_1^{\frac{h_2+1}{2}} + \phi\end{aligned} \tag{26}$$

Let $l_1 = c_1 2^{1-\frac{h_1+1}{2}}$, $l_2 = c_2$, according to Lemma 3, V_1 will converge in a fixed time range

$$V_1 \leq \min\left\{\left(\frac{\phi}{(1-\kappa)l_1}\right)^{\frac{2}{h_1+1}}, \left(\frac{\phi}{(1-\kappa)l_2}\right)^{\frac{2}{h_2+1}}\right\} \tag{27}$$

where $0 < \kappa < 1$.

The convergence time of a disturbance observer satisfies:

$$T \leq \frac{1}{l_1}\left(\frac{1}{\kappa(\frac{h_1+1}{2} - 1)}\right) + \frac{1}{l_2}\left(\frac{1}{\kappa(1 - \frac{h_2+1}{2})}\right) \tag{28}$$

The proof is complete.

Remark 2: The AFSMDO designed in this paper can assure the observer is steady in fixed time and reckon the lumped disturbance value simultaneously.

Remark 3: The AFSMDO has a fixed upper bound on the convergence time, and the parameter settings of the observer have nothing to do with the specific value of the disturbance, which makes its application range wider.

Combined with the AFSMDO, the control law of the manipulator system is updated as

$$\begin{aligned}\tau &= M_0(q)\ddot{q}_d + M_0(q)(\hat{k}_1 |s|^{\frac{1}{2}} \operatorname{sign}(s) \\ &\quad + \int_0^t k_2 \operatorname{sign}(s)\mathrm{d}t + \hat{k}_3 s + k_4 \operatorname{sign}(s) \\ &\quad + \frac{1}{\lambda_2 m_2} |\dot{e}|^{2-\lambda_2} \operatorname{sign}(\dot{e})(D^\alpha e + m_1\lambda_1 |e|^{\lambda_1 - 1}) - \hat{d}) \\ &\quad + C_0(q,\dot{q})\dot{q} + G_0(q) \end{aligned} \tag{29}$$

IV. STABILITY ANALYSIS

The Lyapunov function is chosen as

$$V_2 = \frac{1}{2}s^2 + \frac{1}{2\eta_1}\tilde{k}_1^2 + \frac{1}{2\eta_2}\tilde{k}_3^2 \tag{30}$$

Taking the first-order derivation of equation (30), in the light of $\dot{\tilde{k}}_1 = -\dot{\hat{k}}_1$, $\dot{\tilde{k}}_3 = -\dot{\hat{k}}_3$, we can get

$$\dot{V}_2 = s\dot{s} - \frac{1}{\eta_1}\tilde{k}_1\dot{\hat{k}}_1 - \frac{1}{\eta_2}\tilde{k}_3\dot{\hat{k}}_3 \tag{31}$$

Substitute equation (11) and equation (29) into equation (31), and the following equation is got

$$\begin{aligned}\dot{V}_2 &= -sm_2\lambda_2 |\dot{e}|^{\lambda_2 - 1} (\hat{k}_1 |s|^{\frac{1}{2}} \operatorname{sign}(s) + \int_0^t k_2 \operatorname{sign}(s)\mathrm{d}t \\ &\quad + \hat{k}_3 s + k_4 \operatorname{sign}(s) + d - \hat{d}) - \frac{1}{\eta_1}\tilde{k}_1\dot{\hat{k}}_1 - \frac{1}{\eta_2}\tilde{k}_3\dot{\hat{k}}_3 \end{aligned} \tag{32}$$

Combining equation (13) and equation (14), equation (32) can be written as

$$\begin{aligned}\dot{V}_2 &= -sm_2\lambda_2 |\dot{e}|^{\lambda_2 - 1} (\hat{k}_1 |s|^{\frac{1}{2}} \operatorname{sign}(s) + \int_0^t k_2 \operatorname{sign}(s)\mathrm{d}t + \hat{k}_3 s \\ &\quad + k_4 \operatorname{sign}(s) + d - \hat{d}) - \frac{1}{\eta_1}\left(k_1 - \hat{k}_1\right) \\ &\quad \times \left(\eta_1 m_2 \lambda_2 |\dot{e}|^{\lambda_2 - 1} |s|^{\frac{3}{2}}\right) - \frac{1}{\eta_2}\left(k_3 - \hat{k}_3\right) \\ &\quad \times \left(\eta_2 m_2\lambda_2 |\dot{e}|^{\lambda_2 - 1} |s|^2\right) \\ &= -sm_2\lambda_2 |\dot{e}|^{\lambda_2 - 1} (\int_0^t k_2 \operatorname{sign}(s)\mathrm{d}t + k_4 \operatorname{sign}(s) + \tilde{d}) \\ &\quad - k_1 m_2\lambda_2 |\dot{e}|^{\lambda_2 - 1} |s|^{\frac{3}{2}} - k_3 m_2 \lambda_2 |\dot{e}|^{\lambda_2 - 1} |s|^2 \\ &\le -sm_2\lambda_2 |\dot{e}|^{\lambda_2 - 1} \int_0^t k_2 \operatorname{sign}(s)\mathrm{d}t + m_2\lambda_2 |\dot{e}|^{\lambda_2 - 1} \\ &\quad \times \left(-k_4 |s| + \left|\tilde{d}\right| |s|\right) - k_1 m_2\lambda_2 |\dot{e}|^{\lambda_2 - 1} |s|^{\frac{3}{2}} \\ &\quad - k_3 m_2\lambda_2 |\dot{e}|^{\lambda_2 - 1} |s|^2 \end{aligned} \tag{33}$$

Because $k_4 \ge \left|\tilde{d}\right|$, equation (33) can be written as

$$\begin{aligned}\dot{V}_2 &\le -sm_2\lambda_2 |\dot{e}|^{\lambda_2 - 1} \int_0^t k_2 \operatorname{sign}(s)\mathrm{d}t - k_1 m_2\lambda_2 |\dot{e}|^{\lambda_2 - 1} |s|^{\frac{3}{2}} \\ &\quad - k_3 m_2\lambda_2 |\dot{e}|^{\lambda_2 - 1} |s|^2 \\ &\le 0 \end{aligned} \tag{34}$$

In line with the Lyapunov stability theory, the system is stable.

When $\dot{e} = 0$ and $s \neq 0$, substitute equation (29) into equation (7), we can get

$$\ddot{e} = -\hat{k}_1 |s|^{\frac{1}{2}} \operatorname{sign}(s) - \int_0^t k_2 \operatorname{sign}(s)\mathrm{d}t - \hat{k}_3 s - k_4 \operatorname{sign}(s) + \tilde{d} \tag{35}$$

It can be observed that when $\dot{e} = 0$ and $s \neq 0$, $\ddot{e} = 0$ does not always hold, so $\dot{e} = 0$ does not affect the convergence of the system state, the state of the system can arrive at the sliding mode surface in a finite time.

V. SIMULATION RESULTS

In the cause of verifying the superiority of the controller proposed in this study, a simulation is carried out based on the 2-DOF manipulator, as presented in Figure 1. The dynamic model in [33] is adopted.

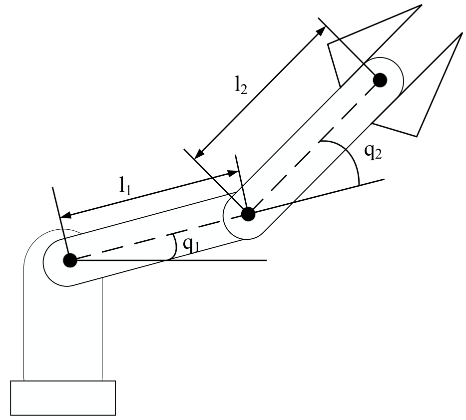

FIGURE 1. The structure of the 2-DOF manipulator.

External disturbance: $\tau_d = \begin{bmatrix} \tau_{d1} \\ \tau_{d2} \end{bmatrix} = \begin{bmatrix} 1 + 0.5\sin(2\pi t) \\ 1 + 0.5\sin(2\pi t) \end{bmatrix}$, model error: $\Delta M(q) = 0.1M(q)$,$\Delta C(q,\dot{q}) = 0.1C(q,\dot{q})$, $\Delta G(q) = 0.1G(q)$. Set the expected trajectories of the two joints as: $q_{d1} = \sin(3t)$, $q_{d2} = \cos(3t)$.

The adaptive FO-NFTSM controller based on AFSMDO presented in this article is denoted as Controller 1. Controller 1 is compared with the FO-NFTSM controller based on AFSMDO and the NTSM controller based on AFSMDO.

Controller 2: the FO-NFTSM controller based on AFSMDO

$$\begin{aligned}\tau &= M_0(q)\ddot{q}_d + M_0(q)(k_1 |s|^{\frac{1}{2}} \operatorname{sign}(s) \\ &\quad + \int_0^t k_2 \operatorname{sign}(s)\mathrm{d}t + k_3 s + k_4 \operatorname{sign}(s) \\ &\quad + \frac{1}{\lambda_2 m_2} |\dot{e}|^{2-\lambda_2} \operatorname{sign}(\dot{e})(D^\alpha e + m_1\lambda_1 |e|^{\lambda_1 - 1}) - \hat{d}) \\ &\quad + C_0(q,\dot{q})\dot{q} + G_0(q) \end{aligned} \tag{36}$$

Controller 3: the NTSM controller based on AFSMDO

$$\tau = M_0(q)\ddot{q}_d + M_0(q)(\hat{k}_1 |s|^{\frac{1}{2}} \operatorname{sign}(s) + \int_0^t k_2 \operatorname{sign}(s) dt + \hat{k}_3 s + k_4 \operatorname{sign}(s) + m_1 \lambda_1 \dot{e} |e|^{\lambda_1 - 1}) - \hat{d}) + C_0(q, \dot{q})\dot{q} + G_0(q) \tag{37}$$

For a reasonable comparison of the performance of the controllers, the parameters of the three controllers are the same. Based on the design requirements and value ranges of the parameters of the controllers, through a large number of simulation experiments, the parameters of the controllers are set as follows. Sliding mode surface parameters: $m_1 = 1/2$, $m_2 = 1/2$, $a = 0.001$, $\lambda_1 = 2$, $\lambda_2 = 4/3$. Adaptive super-twisting reaching law parameters: $r_2 = 0.1$, $r_2 = 0.1$, $\eta_1 = 50$, $\eta_2 = 50$. Observer parameters: $b_1 = 80$, $b_2 = 80$, $v = 0.1$, $h_1 = 1.2$, $h_2 = 0.8$. Lumped disturbance: $F_d = 1 + 0.5\sin(2\pi t) - \Delta M(q)\ddot{q} - \Delta C(q, \dot{q})\dot{q} - \Delta G(q)$. The simulation results are demonstrated in Figure 2-4.

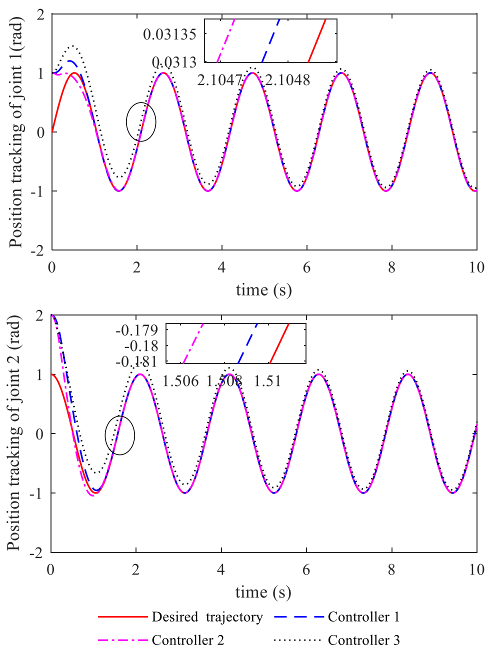

FIGURE 2. Trajectory tracking comparison of joint 1 and joint 2.

Figure 2 manifests the trajectory tracking comparison of the three controllers. It serves to show that the three controllers can finally track the expected trajectory of the joint, but it is obvious that the controller presented in this study has better tracking property.

Figure 3 is a comparison of the position error convergence of the three controllers. Both controller 1 and controller 2 can finally achieve the convergence of the position error, and the convergence effect of the position error of controller 3 is poor. The angular displacement adjustment time and root mean square error (RMSE) of the three controllers are demonstrated in Table 1. It can be obtained from Table 1: the controller raised in this article has a smaller angular

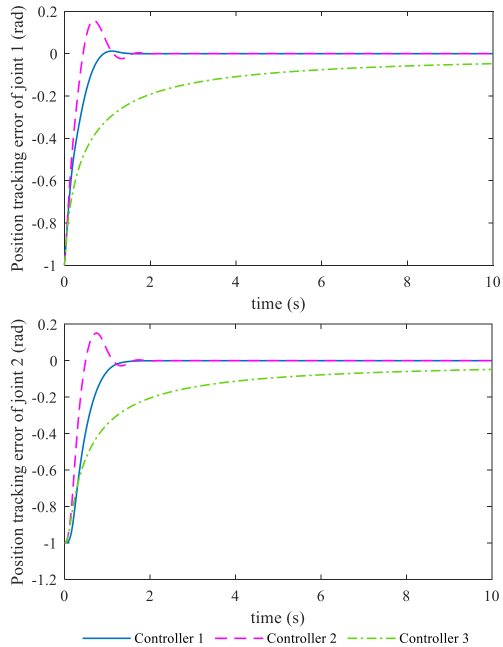

FIGURE 3. Comparison of position tracking error.

TABLE 1. Data comparison of controllers.

	Controller 1	Controller 2	Controller 3
Angular displacement adjustment time of joint 1 (s)	1.20	1.51	>10
Angular displacement adjustment time of joint 2 (s)	1.35	1.52	>10
RMSE of position error of joint 1 (rad)	0.026	0.034	0.106
RMSE of position error of joint 2 (rad)	0.035	0.040	0.113

displacement adjustment time and RMSE value. Comparing controller 1 and controller 2, it indicates that the adaptive reaching law effectively increases the convergence speed. Comparing controller 1 and controller 3, it shows that the sliding mode surface given in this article can improve the control accuracy.

As illustrated in Figure 4, the control input of controller 1 has almost no chattering, and the control input torque is small, which ensures the robustness and feasibility of the controller.

Controller 1 is compared with the adaptive FO-NFTSM controller without disturbance observer (controller 4). The simulation results are shown in Figure 5-7.

As illustrated in Figure 5 and Figure 6, controller 1 can track the desired trajectory more precisely, and the convergence velocity and effect of tracking error are also better than controller 4. Accordingly, the controller designed in this article has a faster convergence speed and better control accuracy.

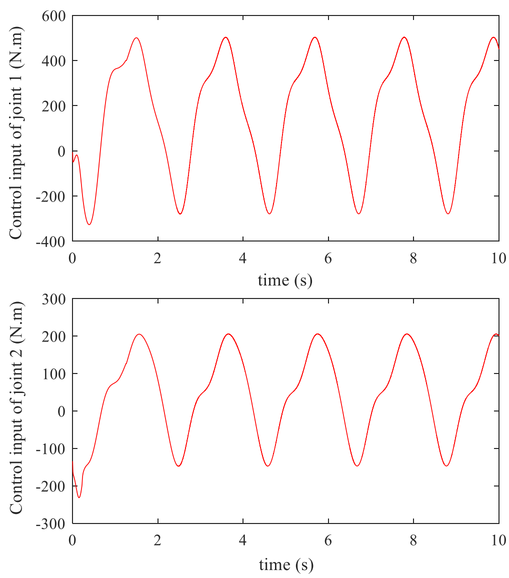

FIGURE 4. Control input of controller 1.

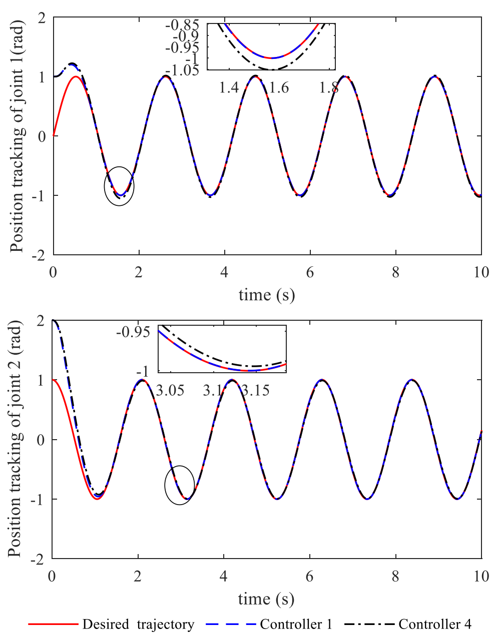

FIGURE 5. Comparison of trajectory tracking.

The introduction of the AFSMDO can well compensate for the control system and raise the performance of the controller.

As you can see from Figure 7, the AFSMDO designed in this article can accurately estimate the lumped disturbance in real-time and ensure the superior property of the controller.

To sum up, the superior performance of the designed adaptive FO-NFTSM controller based on AFSMDO is verified. It provides faster finite-time convergence and high-accuracy tracking and alleviates chattering.

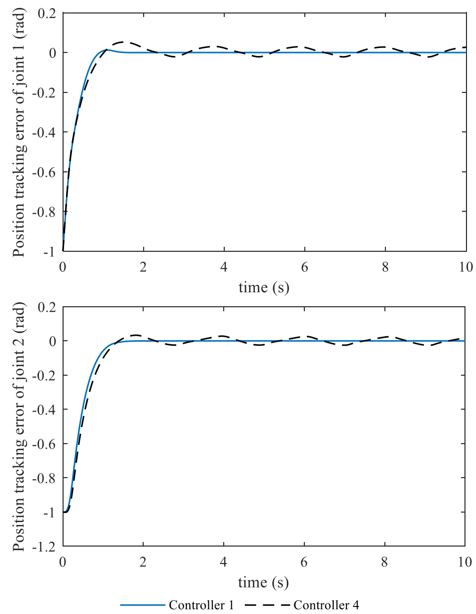

FIGURE 6. Comparison of position tracking error.

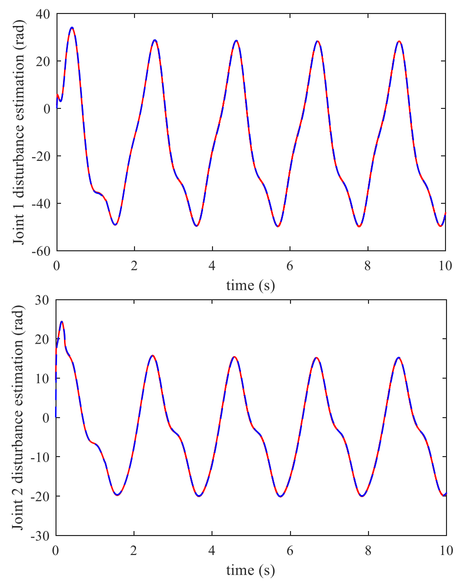

FIGURE 7. Lumped disturbance estimation of joints.

VI. CONCLUSION

For the purpose of realizing the trajectory tracking control of the manipulator with uncertain factors, an adaptive FO-NFTSM controller based on AFSMDO is presented in this article. Firstly, a reaching law is designed using a super-twisting algorithm, which is combined with a FO-NFTSM surface to ensure rapid convergence in the sliding phase and the approaching phase. Secondly, an AFSMDO is designed to gauge the lumped disturbance in the manipulator system so as to compensate for the control system and enhance the control precision of the controller. Then, taking into account the Lyapunov stability theory, the fixed-time convergence of the observer and the stability of the control system are proved. Ultimately, by comparing with other controllers, the simulation results verify that the controller presented in this study can suppress chattering well, and has a faster convergence speed and higher control accuracy.

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