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# **RESEARCH ARTICLE**

# Sensor Fault Tolerant Control for Hybrid Systems With Sinusoidal Disturbance

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**ABSTRACT** This paper aims to design of an active fault-tolerant control AFTC of switched systems in the presence of sensor fault and periodic disturbances. AFTC requires sensor fault estimation. Thus, a databased projection approach is proposed based on performance measures that allow fault estimation without relying on mathematical models and fault assumptions. A new switching control strategy is proposed for fault compensation and recovering the desired performances. These techniques integrate a bank of controllers, corresponding to a set of partial models to design a set of switching control laws, compensating for the fault effect and attenuating the disturbance. A new linear matrix inequality (LMIs) based on Lyapunov stability analysis is proposed to compute control gains. This technique allows finding the optimal values of the control gains matrices, which ensure both sensor fault compensation and disturbance attenuation. A comparative study of the proposed strategy with existing work is carried out to show the effectiveness of the designed fault-tolerant control.

**INDEX TERMS** Switched system, sensor fault, data-based projection approach, disturbance, fault-tolerant control, LMIs.

### I. INTRODUCTION

Over the past few decades, switched hybrid systems received much attention because of their generalized nature and their applications in several real processes, namely robotics, power electronics, chemical processes, network systems, aircraft, DC/DC converters and so on [1], [2]. They are defined by a finite state automaton and a finite number of dynamical subsystems (modes). These modes are represented by differential or difference equations and a switching rule which controls switching between them. The switching rule defines which mode is active at each instant [3]. Above all, the hybrid dynamical systems may be affected by various faults occurring in actuators or sensors. In fact, when a fault occurs, the characteristics of the entire system can cause changes, degrading its performances and even enduring instability [4], [5], [6]. Therefore, fault diagnosis (FD) and fault-tolerant control (FTC) are more and more recommended as they strengthen system safety and maintain

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desirable performances [7]–[9], [10]. In survey papers, two types of FTC are mentioned [11], [12]: passive (PFTC) and active (AFTC) approaches. The former are based on a control design that considers the fault occurrence as a system perturbation [13]. They are designed to be robust against certain faults without requiring information from fault detection and isolation (FDI). Nevertheless, the latter react actively even with the appearance of a fault. Therefore, they need fault identification and an updated control mechanism [14]. Two types of fault identification are considered: a data-driven method and model-based method [15], [16]. In contrast to the data-driven approach (neural networks, fuzzy logic, classification), the model-based method uses the model representation that describes the studied system to find a fault indicator (observer-based approach, parity space approach, parameter estimation techniques) [17]-[20]. In [21], an AFTC design of controllers based on model predictive control is embedded within the hybrid system framework considering the nature of the hybrid FTC law. In [22], the fault-tolerant  $H_{\infty}$  filter against sensor fault is developed based on the Lyapunov function technique for a class of switched systems. It is designed

to guarantee the filtering error is asymptotically stable by ensuring  $H_{\infty}$  norm bound. In [23], an augmented descriptor observer approach is designed to deal with FTC sensor for time delay switched hybrid system under an arbitrary switching signal. In previous works [24]-[26], we dealt with the FTC of switched systems described by the state space and did not take into account the presence of disturbances that can degrade the system performances. To date, few works have treated the problem of switched systems with external disturbances. Most these works are designed under a particular type (constant) and bounded disturbances [27]. Of many control techniques for periodic disturbances attenuation, the predictive control has received widespread attention over the last two decades. In [28], a model predictive control for a class of hybrid system with repetitive disturbance is proposed. It uses past process data and weighted predicted information obtained through solving two separate optimization problems. In [29], FTC algorithms are derived without requiring analytically estimation bound on actuator failure variables. In [30], a controller is designed to guarantee stochastic stability and a disturbance attenuation level. The main contribution of the present work, is to resolve the problem of fault-tolerant control of the switched system with external disturbance to ensure strong robustness and keep invariant properties. Sensor fault estimation is required to design the proposed FTC strategy. The main idea behind fault estimation is to develop the data-based projection approach. Indeed, this approach is based on the inputs-outputs measurements to generate the inputs-outputs databases used to find accurate and rapid fault estimation. Switched systems structures are well known for their flexibility in modeling complex systems with a variable structure. These introduce partial models adapted to each operating zone. For switching systems, a logical rule defines which mode of a set of partial models is active at each instant. Thanks to the features of the switched system, a bank of controllers associated with each local model are designed to synthesize a set of switching control laws to account for the faults and disturbances. To find control gains, an augmented state is constructed. Based on the Lyapunov approach, a stability analysis is achieved, and sufficient conditions for control gains are established. This insures system stability, fault compensation, and disturbance attenuation.

Most of the recent research are designed under the assumption that hybrid systems are affected only by habitual disturbances (constant, ramp, white/gaussian noise signals), [31]–[33]. However, in several nonlinear industrial processes (power distribution, robotics, etc.), disturbances can be harmonic signals. As an indicator, in electrical power distribution, nonlinear loads such as power converters and diode rectifier may be considered as a generators of non-desired harmonic currents [34]. Therefore, an important challenges is the sensitivity of switched hybrid systems to periodic disturbances. In [4], [35], the FTC problem of hybrid systems subject to faults was studied. However, the effect of an unknown disturbances is not addressed. To our knowledge, few results have addressed the attitude tracking control issue of switched hybrid with faults and external sinusoidal disturbances, which motivates our current study. In [36], [37], the problem of faults compensation is considered using the additive FTC law. Indeed, the control gains are computed using the pole placement, LQ optimization, Lyapunov redesign principle, etc. However, in this paper a novel LMIs is developed to compute the control gain matrices that ensures not only the tolerance against the fault but also contribute additional robustness against the sinusoidal disturbance. In a previous work [24], we develop the data-based approach method to estimate the sensor fault of deterministic switched system. In this paper, a novel approach is proposed to deal with sensor fault estimation of disturbed switched system. The novel estimation approach uses only the input-output measured data. The proposed fault estimation approach is certainly intrinsically robust to parameters values and simple to apply. Indeed, no prior knowledge about the dynamical evolution of the fault and single, multiple and simultaneous sensor faults can be considered.

In other words, the key feature of this work can be recapitulated as: first, a new LMIs is developed to compute the control gains for simultaneously perfectly tracking performances and sinusoidal disturbances attenuation. Then, the diagnosis module consists of residual generation using only the inputs and outputs data is proposed. The fault estimation module starts the computation of the additive control law which is able to tolerate the fault effect on the system once the sensor fault is detected. The proposed fault estimation approach is reached as soon as possible to avoid huge losses in system performances.

This paper is organized as follows: Problem statement is presented in section 1. In section 2, we present the additive FTC proposed in a previous work. In section 3, we develop a new solution to the problems related to the FTC of switched systems subject to sensor fault and periodic disturbances. Section 4, is devoted to fault estimation using the data-based projection method. In section 5, a discussion about the work is given. A comparative study between the proposed FTC and the additive FTC is outlined in section 6, which shows the efficiency of the proposed control.

#### **II. PROBLEM STATEMENT**

Switched hybrid system is a dynamical system that consists of a finite number of dynamical subsystems called operating modes and a logical rule that governs switching between these subsystems described by a collection of differential or difference equations. The switching law (logical rule) preconises which mode is active at each time instant. Indeed, switched system is classified based on the dynamics of their subsystems, for instance continuous-time or discrete-time [38]. In this work, we consider the dynamical switched system with linear discrete-time modes described by [39]:

$$\begin{cases} x(k+1) = A_i x(k) + B_i u(k) \\ y(k) = C_i x(k) + f_c(k) + E_i w(k) \end{cases}$$
(1)

where, the vectors  $u(k) \in \mathbb{R}^m$ ,  $x(k) \in \mathbb{R}^n$ ,  $y(k) \in \mathbb{R}^p$  are respectively input, state and output signals at time-instant *kTe* (*Te* being the sampling period).  $f_c(k)$  and w(k) are respectively sensor fault and periodic disturbance.  $i \in \mu = \{1, 2, ..., M\}$ is the mode index and *M* is the number of modes.  $A_i \in \mathbb{R}^{n \times n}$ ,  $B_i \in \mathbb{R}^{n \times m}$ ,  $C_i \in \mathbb{R}^{p \times n}$ ,  $E_i \in \mathbb{R}^{n \times s}$  are constant matrices associated with the stable mode indexed by *i*.

- A.1: The switching signal is known.
- A.2:  $E_i$  is a identity matrix.
- A.3: The disturbance is periodic with a fixed period T.

Given the system described by (1), this work proposes a new active FTC based on the design of nominal control law, sensor fault estimation, and modification of the control law to allow fault compensation and disturbance attenuation. Fault estimation is proposed using a data-based approach. A bank of controllers, corresponding to a set of partial models to design a set of switching control laws is built. Based on the Lyapunov function, a new adequate criterion is deduced by means of linear matrix inequalities (LMIs). The established LMIs are then solved for obtaining the controller gain matrices to compensate for the fault effect and ensure disturbance attenuation.

#### **III. ADDITIVE FTC**

In a previous works [24], we proposed an additive FTC to treat the problem of sensor FTC of a deterministic switched system with linear discrete-time modes described by the model (1) where w(k) = 0. It is performed by combining the state feedback control with integral action with an additive control law. Indeed, before handling sensor fault that can occurs on the system, the goal is to synthesize a nominal tracking control where outputs y(k) are required to track reference signal  $y_r(k)$ . To achieve this task, a controller and integrator vector  $Z_e$  is used to satisfy the following expression:

$$Z_e(k+1) = Z_e(k) + T_e(y_r(k) - y(k))$$
(2)

where,  $y_r(k)$  and  $T_e$  are respectively reference signal, and sampling interval. Then, the expression of the state feedback control with integral action is given by:

$$u(k) = K_i x(k) + G_i Z_e(k)$$
(3)

where,  $K_i$  and  $G_i$  are respectively gains matrices of the state x(k) and gains matrices of the controller  $Z_e$ . Indeed, in most practical systems, controllers are designed, and neglecting that faults can occur. The state feedback control with integral action is updated according to the fault occurrence. For this reason, it is significant to take into account the fact that direct sensor fault accommodation must be considered to keep the system stability and allow trajectory tracking. Consequently, when a sensor fault occurs, an additive control law is computed and added to the nominal one. Then, the additive FTC is computed as:

$$u(k) = K_{i}x(k) + G_{i}Z_{e}(k) + u_{adc}(k)$$
(4)

Since the occurrence of a sensor fault, the partial measured output and the integrator are modified and their expressions are given as following:

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$$w(k) = C_i x(k) = C_i x_0(k) + f_c(k)$$
(5)

$$Z_e(k) = Z_0(k) + \tilde{f}(k) \tag{6}$$

$$\tilde{f}(k) = \tilde{f}(k-1) - T_e f_c(k-1)$$
 (7)

where,  $\tilde{f}$  and  $T_e$  are respectively integral of the sensor fault (integral of  $-f_c(k)$ ) and sampling interval. So, the expression of the additive FTC is given by [24]:

$$u(k) = K_i x_0(k) + G_i Z_0(k) + K_i C_i^+ f_c(k) + G_i \tilde{f}(k) + u_{adc}(k)$$
(8)

Therefore, since the sensor fault is estimated, the fault effect is compensated by computing the additive control  $u_{adc}$  such that:

$$u_{adc} = -K_i C_i^+ \hat{f}_c(k) - G_i \tilde{f}(k)$$
(9)

Indeed, the gain matrices  $K_i$  and  $G_i$  of each mode are computed by the following LMIs developed:

*Theorem 1:* If there is symmetric definite positive matrix P, matrices  $F_i$  and  $H_i$  solution of following LMIs:

$$\begin{cases} P > 0 \\ Q & 0 & QA_i^T + F_iB_i^T & 0 \\ 0 & Q & H_iB_i^T & Q \\ A_iQ^T + B_iF_i^T & B_iH_i^T & Q & 0 \\ 0 & Q^T & 0 & Q \end{bmatrix} > 0 \quad (10)$$

where,  $P^{-1} = \begin{bmatrix} Q & 0 \\ 0 & Q \end{bmatrix}$  then, the switched system is asymptotically stable and the gain matrices  $K_i$  and  $G_i$  of the state feedback control with integral action of each mode are given by:

$$K_i = (Q_i^{-1}F_i)^T$$
$$G_i = (Q_i^{-1}H_i)^T$$

In a previous works, we proposed an additive FTC which showed satisfactory results for the closed loop performance in terms of trajectory tracking and fault tolerance. However, it did not take into account the presence of disturbances. This is not always the case; in fact, most industrial processes, such as robots and converters, are affected by periodic disturbances. However, in the presence of periodic disturbances, the additive FTC cannot ensures disturbance attenuation, which can degrades the closed-loop performances and even provide instability. Therefore, it is necessary to modify the additive FTC algorithm, taking into account periodic disturbances.

#### **IV. PROPOSED FTC**

The general concept of the proposed FTC is summarized by Figure.1. The proposed FTC law is composed of two parts: state feedback integral control and additive control. Firstly, the unknown input observers UIO is used to estimate the unavailable state. Then, we focused on the design of a nominal tracking controller. Thus, a state feedback integral control is developed for simultaneously perfectly tracking



FIGURE 1. Structure of the designed scheme.

performances and sinusoidal disturbances attenuation. The diagnosis module consists of residual generation using only the inputs and outputs data and finally the decision as to which sensor is faulty. The fault estimation module starts the computing of the additive control law that is able to tolerate the fault effect on the system once the sensor fault is detected. Obviously, the fault estimation should be reached as soon as possible to avoid huge losses in system performances.

#### A. UNKNOWN INPUT OBSERVERS

The application of the proposed FTC strategies is based on using a state representation that requires the availability and the knowledge of the state variables. However, for most real processes, there is some state that cannot be measured. Therefore, to estimate the unmeasured state, researchers propose the design of the observers [40], [41]. In [41], the unknown input hybrid observers of the system (1) is as given by:

$$\begin{cases} z(k+1) = F_i z(k) + T_i B_i u(k) + L_i y(k) \\ \hat{x}(k) = z(k) + H_i y(k) \end{cases}$$
(11)

where,  $\hat{x}(k)$  and z(k) are the estimated state and the state of the unknown input observers respectively.  $F_i, T_i, L_i, H_i$  are matrices of each mode which should be determined. Denotes  $A_i^* = T_i A_i$ , in [41],  $\forall (i, j) \in \mu \times \mu$ , if there is  $P_i = P_i^T > 0$  solution of following LMIs:

$$\begin{pmatrix} -P_j & P_j A_i^* - W_{ji} C_i \\ A_i^{*T} P_j - W_{ji}^T & -P_i \end{pmatrix} < 0$$
(12)

then, the gain matrices  $K_i^*$  are given by:

$$K_i^* = P_i^{-1} W_{ji} \tag{13}$$

where:

$$H_i = E_i \Big[ (C_i E_i)^T (C_i E_i) \Big]^{-1} (C_i E_i)^T$$
(14)

$$T_i = I_d - H_i C_i \tag{15}$$

$$F_{i} = A_{i}^{*} - K_{i}^{*}C_{i} \tag{16}$$

$$L_i = K_i^* + F_i H_i \tag{17}$$

#### B. STATE FEEDBACK CONTROL WITH INTEGRAL ACTION

A state feedback control with integral action is developed to ensure the trajectory tracking and disturbance attenuation. First, the following lemma, that will be used next, is considered:

*Lemma:* With *X*, *Y* and  $P = P^T > 0$  matrices of appropriate dimension the following inequality holds [42]:

$$X^T Y + Y^T X \le X^T P X + Y^T P^{-1} Y$$

Consider the classical feedback control law:

$$u(k) = K_i x(k) \tag{18}$$

where,  $K_i$  are the gain matrices of each mode. The goal is to design a partial controller to make the system outputs follow the reference signal as close as possible. Therefore, a comparator integrator  $Z_e(k)$  is added to (18) according to the following relation:

$$Z_e(k+1) = Z_e(k) + T_e(y_r(k) - y(k))$$
(19)

where,  $y_r(k)$  and  $T_e$  are the reference input and the sampling interval respectively. Hence, the feedback control law that guarantees both stability and dynamic behavior of the closedloop system is modified as following:

$$u(k) = \tilde{K}_i \tilde{x}(k) \tag{20}$$

where,  $\tilde{x}(k)$  denotes the new feedback gain calculated based on an augmented partial model.

$$\tilde{K}_i = \begin{bmatrix} K_i & G_i \end{bmatrix}$$
$$\tilde{x}(k) = \begin{bmatrix} x(k) \\ Z_e(k) \end{bmatrix}$$

 $G_i$  is the gain matrix of  $Z_e(k)$  of each mode. The stability of the mode indexed by *i* is studied with the Lyapunov approach in terms of linear matrix inequalities.  $K_i$  and  $G_i$  are computed if a symmetric definite positive matrix  $P = P^T > 0$  and scalar  $\bar{\gamma} > 0$  exist satisfying the following theorem.

Theorem 2:  $\forall i \in \mu$  under assumption (A.3), the system (1) is stabilized by (20) and ensure the attenuation of the disturbance effect, if there exist symmetric definite positive matrix  $P = P^T > 0$ , scalar  $\bar{\gamma} > 0$  and matrices X,  $K_i$  and  $G_i$  of appropriate dimensions solution of the following LMIs in (21), as shown at the bottom of the next page, where,  $\eta = X^T QX - X^T - X$ , H is given matrix,  $\gamma^2 = \bar{\gamma}$  and  $P = \begin{bmatrix} Q & 0 \\ 0 & Q \end{bmatrix}$ .

*Proof:* The novel augmented state model is obtained using (1) and (20):

$$\tilde{x}(k+1) = \begin{bmatrix} A_i + B_i K_i \ B_i G_i \\ -T_e C_i \ I \end{bmatrix} \tilde{x}(k) + \begin{bmatrix} E_i w(k) \\ T_e y_r(k) \end{bmatrix}$$
(22)

The expression (22) is equivalent to (23):

$$\tilde{x}(k+1) = \begin{bmatrix} A_i + B_i K_i \ B_i G_i \\ 0 \ I \end{bmatrix} \tilde{x}(k) + \begin{bmatrix} E_i w(k) \\ T_e(y_r(k) - C_i x(k)) \end{bmatrix}$$
(23)

In fact, when  $k \to +\infty$ , the output of the switched system converges to the reference signal  $(y_r(k) - C_i x(k) \to 0)$ . So, the augmented state model will be:

$$\tilde{x}(k+1) = \tilde{A}_i \tilde{x}(k) + E_i w(k)$$
where,  $\tilde{A}_i = \begin{bmatrix} A_i + B_i K_i \ B_i G_i \\ 0 \ I \end{bmatrix}$ . (24)

VOLUME 10, 2022

The following Lyapunov function is used in order to find the gain matrices  $K_i$  and  $G_i$ :

$$V(\tilde{x}(k), k) = \tilde{x}^{T}(k)P\tilde{x}(k)$$
(25)

The expression of  $\Delta V(\tilde{x})$  is given by:

$$\Delta V(\tilde{x}) = V(\tilde{x}(k+1)) - V(\tilde{x}(k)) \tag{26}$$

Then,  $\Delta V(\tilde{x})$  is as follows:

$$\Delta V(\tilde{x}) = \tilde{x}^T (k+1) P \tilde{x} (k+1) - \tilde{x}^T (k) P \tilde{x} (k)$$
(27)

Replacing (24) into (27):

$$\Delta V(\tilde{x}) = (\tilde{A}_i \tilde{x}(k) + E_i w(k))^T P(\tilde{A}_i \tilde{x}(k) + E_i w(k)) - \tilde{x}^T(k) P \tilde{x}(k)$$
(28)

So,

$$\Delta V(\tilde{x}) = \tilde{x}^{T}(k) \left[ \tilde{A}_{i}^{T} P \tilde{A}_{i} - P \right] \tilde{x}(k) + \tilde{x}^{T}(k) \tilde{A}_{i}^{T} P E_{i} w(k)$$

$$+ w^{T}(k) \left[ E_{i}^{T} P \tilde{A}_{i} \tilde{x}(k) + E_{i}^{T} P E_{i} w(k) \right]$$

$$(29)$$

Then,

$$\Delta V(\tilde{x}) = \psi^T(k)\Omega_i(k)\psi(k) \tag{30}$$

where,  $\psi^{T}(k) = \begin{bmatrix} \tilde{x}^{T}(k) \ w^{T}(k) \end{bmatrix}^{T}$  and  $\Omega_{i}(k) = \begin{bmatrix} \tilde{A}_{i}^{T} P \tilde{A}_{i} - P \ \tilde{A}_{i}^{T} P E_{i} \\ E_{i}^{T} P \tilde{A}_{i} & E_{i}^{T} P E_{i} \end{bmatrix}$ 

The disturbance w must then be taken into account during the synthesis of the gain matrices  $K_i$  and  $G_i$ . For this purpose, an objective signal  $v(k) = H\tilde{x}(k)$  depending only on  $\tilde{x}(k)$  is considered and satisfied the following design objectives:

$$\|v(k)\|_{2}^{2} \le \gamma^{2} \|w(k)\|_{2}^{2}$$

where,  $\gamma$  represents the level of attenuation of the disturbance w(k) and I represents the identity matrix. The matrix H is of appropriate dimension. Then, the system (1) is stabilized and the disturbance effect is minimized, if  $\forall \tilde{x}(k) \neq 0$ , there exist matrix  $P = P^T > 0$ , matrices  $K_i$  and  $G_i$  which verified:

$$\Delta V(\tilde{x}) < -v^T(k)v(k) + \gamma^2 w^T(k)w(k)$$
(31)

Then, using (31) and (30) the following expression is obtained:

$$\psi^{T}(k) \left\{ \Omega_{i}(k) + \begin{bmatrix} H^{T}H & 0\\ 0 & -\gamma^{2}I \end{bmatrix} \right\} \psi(k) < 0 \qquad (32)$$

Then,

$$\begin{bmatrix} \tilde{A}_i^T P \tilde{A}_i - P + H^T H & \tilde{A}_i^T P E_i \\ E_i^T P \tilde{A}_i & E_i^T P E_i - \gamma^2 I \end{bmatrix} < 0$$
(33)

The inequality (33) is equivalent to the following inequality:

$$\begin{bmatrix} -P + H^{T}H & 0\\ 0 & -\gamma^{2}I \end{bmatrix} + \begin{bmatrix} \tilde{A}_{i}^{T}\\ E_{i}^{T} \end{bmatrix} P \begin{bmatrix} \tilde{A}_{i} & E_{i} \end{bmatrix} < 0 \quad (34)$$

78439

Using the Schur Complement the following LMIs is obtained:

$$\begin{bmatrix} -P + H^{T}H & 0 & A_{i}^{T} \\ * & -\gamma^{2}I & E_{i}^{T} \\ * & * & -P^{-1} \end{bmatrix} < 0$$
(35)

(\*) is the transposed element in the symmetric position.

It can be seen from the inequality (35), there exist a nonlinear term which can not be solved using the matlab toolbox, so using the previous lemma and  $P = P^T = \begin{bmatrix} Q & 0 \\ 0 & Q \end{bmatrix} > 0$ , the following LMIs (36), as shown at the bottom of the next page, is obtained which allows to optimize simultaneous the values of the gain matrices  $K_i$  and  $G_i$ . Where,  $\eta = X^T Q X - X^T - X$ and  $\gamma^2 = \overline{\gamma}$ .

Single controller is unable to tolerate fault affecting the nonlinear system modelled by switching state model. Therefore, a bank of controllers are built. Each generated controller generates a local control law.

#### C. COMPUTATION FTC LAW

Fault-tolerant control concepts can be separated into "passive" and "active" methods. To the best of our knowledge, compared with an active FTC, the performance reached by a passive FTC can never be optimal for all design scenarios [43]. Indeed, the overall conservatism increases, if we attempt to design a passive FTC to accommodate an excessive number of faults [44]. Indeed, the active FTC method is generally more efficient in treating with different types of defects [45]. Active FTC methods are mainly classified based on the fault diagnosis unit used in their design (model-based, knowledgebased, and combined model-knowledge-based approaches). The beauty of active FTC architectures arises through their inherent ability to react successfully during a transient period between the fault occurrence and the performance recovery [8], [45]. Motivated by this situation, the novelties of this paper is to design an active FTC of disturbed switched system that provides a good compromise between simplicity of the control system synthesis and the closed loop desired performances. In [8], [46] an additive FTC is proposed for linear system. Indeed, it's design is easy and suitable for the purpose of our paper compared with others existing works. The general concept is to combine the nominal control to an additive control law. Thus, the first part of our proposed FTC law is designed to guarantee the robustness and the stability by an optimal computation of the control gains using a set of novel LMIs generated by Lyapunov function. Then, the objective of the second part is to compensate the fault effect using the additive control  $u_{adc}$  based on the proposed fault estimation approach and adding it to the nominal one. Therefore, the proposed FTC is given by:

$$u(k) = K_i \hat{x}(k) + G_i Z_e(k) + u_{adc}(k)$$
(37)

The additive control  $u_{adc}$  is given by (see proof section additive FTC):

$$u_{adc} = -K_i C_i^+ \hat{f}_c(k) - G_i \tilde{f}(k)$$
(38)

The integrator  $Z_e$  and  $\tilde{f}$  are described by the following expressions:

$$Z_e(k) = Z_0(k) + \tilde{f}(k) \tag{39}$$

$$\tilde{f}(k) = \tilde{f}(k-1) - T_e f_c(k-1)$$
(40)

where,

- $Z_e$  is the integrator (see section B).
- f and  $T_e$  are respectively the integral of  $-f_c(k)$  and the sampling interval (see proof section additive FTC).
- $C_i^+$  is the pseudo inverse of the matrix  $C_i$ .
- *K<sub>i</sub>* and *G<sub>i</sub>* are computed using the novel LMIs developed (theorem 2).
- The estimated state  $\hat{x}$  is computed using the UIO (see section A).

#### **V. DATA BASED PROJECTION METHOD**

When a sensor fault occurs, the closed-loop system behavior is corrupted, and the real output does not converge to the desired signal reference. Fault-tolerant control is designed by the addition of the  $u_{adc}(k)$  control law which relies on the occurrence of fault estimation. Fault estimation is achieved using the data-based projection approach (DPM). It uses only the inputs and outputs of measured data. The main idea is to project an output matrix (generated using output data collected in a time interval T) on the orthogonal of an input matrix (collected using input measurements synthesized in the same time interval T).

#### A. INPUT-OUTPUT DATABASES

The residual is the fault indicator. It is generated using the databases built from the input-output measurements of the system. The expression of the control input database is given by:

$$\bar{U}(k) = \left[ \bar{u}(k - L + 1)_j \ \bar{u}(k - L + 2)_j \ \dots \ \bar{u}(k)_j \right]$$
(41)

$$\begin{cases} P > 0 \\ -Q + H^{T}H & 0 & 0 & 0 & A_{i}^{T} + K_{i}^{T} & 0 \\ 0 & -Q + H^{T}H & 0 & 0 & G_{i}^{T}B_{i}^{T} & I \\ 0 & 0 & -\bar{\gamma}I & 0 & E_{i}^{T} & 0 \\ 0 & 0 & 0 & -\bar{\gamma}I & 0 & E_{i}^{T} \\ * & * & * & * & \eta & 0 \\ * & * & * & * & \eta & 0 \\ * & * & * & * & 0 & \eta \\ \end{cases} < 0$$
(21)

where,  $\bar{U}(k) \in \Re^{m(j+1) \times L}$ 

$$\bar{u}(k)_{j} = \left[ u^{T}(k-j) \ u^{T}(k-j+1) \ \dots \ u^{T}(k) \right]^{T}$$
(42)

The expression of the output database is given by:

$$\bar{Y}(k) = [y(k-L+1) \dots y(k-1) y(k)]$$
 (43)

where, *L* and *j* are two integers which verify: L > m(j + 1)

#### **B.** RESIDUAL

The idea of the DPM is to synthesize mode indicator, called residual, based on the use of the matrix projection techniques and the matrices which are composed of input-output measured data. The residual is computed from the input-output databases already constructed  $(\bar{U}(k), \bar{Y}(k))$  and the projection matrix  $\bar{\Pi}(k)$  to the right of  $\bar{U}(k)$ . The matrix  $\bar{\Pi}(k)$  is given by the following expressions:

$$\bar{\Pi}(k) = I_L - \bar{U}(k)^T \left[ \bar{U}(k)\bar{U}(k)^T \right]^{-1} \bar{U}(k)$$
(44)

Then,

$$\bar{U}(k)\bar{\Pi}(k) = 0 \tag{45}$$

Theorem 3: The residual  $r_c(k)$  is given by the matrix projection:

$$r_c(k) = \bar{Y}(k)\bar{\Pi}(k) \tag{46}$$

*Proof:* We will provide a theoretical analysis on how and when  $\hat{f}_c(k)$  is close to the target signal  $f_c(k)$ . Therefore, the model parameters is used to show how equation (46) can be obtained and why it can be used as residual for sensor fault estimation.

Using assumption 2, and by substituting the state equation (1), the following expression can be obtained:

$$y(k) = C_i A_i^j x(k-j) + H_{i,j} \bar{u}(k)_j + f_c(k) + w(k)$$
(47)

where,

$$H_{i,j} = \left[ C_i A_i^{j-1} B_i | \dots | C_i B_i | D_i \right]$$
(48)

is the Markov parameters matrix of order *j*.

$$\bar{u}(k)_{j} = \begin{bmatrix} u^{T}(k-j) \ u^{T}(k-j+1) \ \dots \ u^{T}(k) \end{bmatrix}^{T}$$
(49)

By concatenating (47) according to the columns over a time interval of size L we obtain:

$$\bar{Y}(k) = C_i A_i^j \left[ x(k-L+1-j) \dots x(k-j) \right] + H_{i,j} \bar{U}(k) + F_c(k) + W_c(k)$$

Where,  $F_c(k)$  and  $W_c(k)$  are constructed in a manner similar to  $\overline{Y}(k)$ .

Then,  $F_c(k) = [f_c(k-L+1) \cdots f_c(k)].$ 

Under stability hypothesis of matrix  $A_i$ ,  $\|A_i^j\| = 0$ , when *j* tends to infinity:

$$\lim_{j \to +\infty} \left\| A_i^j \right\| = 0 \tag{50}$$

Consequently,

$$\bar{Y}(k) \simeq H_{i,j}\bar{U}(k) + F_c(k) + W_c(k)$$
(51)

Let us define  $\Pi(k)$  the orthogonal of the Hankel matrix  $\overline{U}(k)$ . The proposed residual  $r_c(k)$  is obtained by right multiplying equation (51) with  $\Pi(k)$  in order to eliminate the effect of the output and the matrix  $H_{i,j}$  of system parameters.

$$r_c(k) = Y(k)\Pi(k) \simeq \underbrace{H_{i,j}U(k)\Pi(k)}_{=0} + F_c(k)\Pi(k) + W_c(k)\Pi(k)$$
(52)

Then, the computational form of  $r_c(k)$  vector is given by the following expression:

$$r_c(k) = \bar{Y}(k)\bar{\Pi}(k) \tag{53}$$

# C. ESTIMATED ACTUATOR FAULT

The expression of the estimated fault is given by the following theorem.

*Theorem 4: Using the obtained residual (53), the estimated fault is given by:* 

$$\hat{f}_c(k) = r_c(k)Z \tag{54}$$

where,  $Z = (0...01)^T$  is a selective vector. Any column of Z can be chosen as residual.

*Proof:* The main idea to obtain the sensor fault estimation is to impose a special structure of  $\overline{\Pi}(k)$ . The matrix  $\overline{\Pi}(k)$  may be chosen as follows:

$$\bar{\Pi}(k) = \begin{bmatrix} \Pi_c(k) \\ I \end{bmatrix}$$
(55)

where *I* is the identity matrix and  $\Pi_c(k)$  is a matrix to be found such that:

$$\bar{U}(k)\bar{\Pi}(k) = 0 \tag{56}$$

Once the matrix  $\Pi(k)$  is calculated, the proposed residual can be obtained base on a computation form (53). In order to prove that the residual  $r_c(k)$  is able to estimate the sensor

$$\begin{cases} P > 0 \\ -Q + H^{T}H & 0 & 0 & 0 & A_{i}^{T} + K_{i}^{T} & 0 \\ 0 & -Q + H^{T}H & 0 & 0 & G_{i}^{T}B_{i}^{T} & I \\ 0 & 0 & -\bar{\gamma}I & 0 & E_{i}^{T} & 0 \\ 0 & 0 & 0 & -\bar{\gamma}I & 0 & E_{i}^{T} \\ * & * & * & * & \eta & 0 \\ * & * & * & * & \eta & 0 \\ * & * & * & * & 0 & \eta \\ \end{cases} < 0$$
(36)

fault, the evaluation form of the residual is given. Two cases are considered:

\* No fault occurs:

$$r_c(k) = \bar{Y}(k)\bar{\Pi}(k) \simeq H_{i,j}\bar{U}(k)\bar{\Pi}(k) + W_c(k)\bar{\Pi}(k)$$
(57)

Using condition (56), the residual becomes  $(r_c(k) = W_c(k)\overline{\Pi}(k))$ .

\* Sensor fault occurs:

$$r_{c}(k) = \bar{Y}(k)\bar{\Pi}(k)$$
  

$$r_{c}(k) \simeq H_{i,j}\bar{U}(k)\bar{\Pi}(k) + [0|F_{c}(k)]\bar{\Pi}(k) + [0|W_{c}(k)]\bar{\Pi}(k)$$
(58)

Using condition (56), the proposed residual becomes:

$$r_{c}(k) \simeq F_{c}(k)\bar{\Pi}(k) + W_{c}(k)\bar{\Pi}(k) \simeq F_{c}^{*}(k)\bar{\Pi}(k)$$

$$F_{c}^{*}(k) = F_{c}(k) + W_{c}(k)$$
(59)

$$r_c(k) \simeq [0|F_c(k)] \,\bar{\Pi}(k) + [0|W_c(k)] \,\bar{\Pi}(k)$$
 (60)

Replacing the expression of (55) in (60), then the residual is given by:

$$r_{c}(k) = \begin{bmatrix} 0 | F_{c}^{*}(k) \end{bmatrix} \begin{bmatrix} \Pi_{c}(k) \\ I \end{bmatrix}$$
$$r(k) = F_{c}^{*}(k)$$
(61)

From equation (61) and using a selective vector  $Z = (0...01)^T$ , one can deduce that the actuator fault is estimated by:

$$\hat{f}_c(k) = r_c(k)Z \tag{62}$$

*Remark*: From equation (62), it is easy to remark that no model parameter is needed to compute the estimated fault and only the input and output data are used. The obtained estimated fault is used to compute the additive control.

#### **VI. DISCUSSION**

It's well known that the design of the control law is based on the nominal system. However, nonlinear industrial processes are generally complicated and subject to faults and external disturbances that can degrade the system performances. The majority of works dealt with the design of the FTC law of disturbed switched system under a particular type of disturbance (constant, white/gaussian noise) [34]. However, in a variety of industrial systems such as mechanical, power distribution, rotating machine tools, robot manipulators, and converters [47], [48] the disturbances can be periodic signals. So, an important additional difficulties and challenges is the sensitivity of switched system to periodic disturbances. Motivated by this situation, the main challenge of this paper consist in the design of a FTC to compensate the sensor fault and attenuate the sinusoidal disturbance effect in the same time. An effective solution is to develop a novel LMIs using the Lyapunov function to elaborate the necessary conditions to improve the closed-loop performances in terms of trajectory tracking and disturbance attenuating. This technique

### TABLE 1. Interval of activation of modes.

$mode \ i$	1	2
Instants	[0,800[	[800,1600]

provides optimal values of the control gains based on LMIs approach compared with other techniques such as the pole placement. Then, computation of an additive control, based on the estimation fault to ensure fault compensation goals.

## **VII. SIMULATION RESULTS**

In this part, two examples are given to illustrate the effectiveness of the proposed FTC. The first example is given to compare the performance of the proposed FTC with the additive FTC proposed in our previous work [24] (see section 2). The second example is given to evaluate the effectiveness of the proposed strategy with the model predictive control (MPD) proposed in [49].

# A. COMPARISON BETWEEN THE PROPOSED FTC AND THE ADDITIVE FTC

It will be shown that, the proposed FTC control performance is better than the additive FTC. Let us consider a switched system including two partial models [24]:

Partial model 1:

$$A_{1} = \begin{bmatrix} 0.6 & 0.45 \\ -0.2 & 0.3 \end{bmatrix}, B_{1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, C_{1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, E_{1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Partial model 2:

$$A_2 = \begin{bmatrix} 0.4 & -0.3 \\ 0.11 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

The instant of activation of each subsystem are given in the table 1.

The system is then defined by two inputs  $(u_1(k), u_2(k))$ and two outputs  $(y_1(k), y_2(k))$ . Sensor faults can be evoked by several different kinds of problems, are related to wrong readings due to a failure in their components which causes the loss of effectiveness. In fact, they are considered additive signals on the measurements. The evolution of the sensor fault and the disturbance are supposed as follows:

$$w(k) = 0.04sin(0.02k)$$

$$f_c(k) = \begin{cases} 0 \to k \in [0 \dots 100] \\ 0.5 \to k \in ]100 \dots 400] \\ 0 \to k \in ]400 \dots 1000] \\ 0.5 \to k \in ]1000 \dots 1600] \end{cases}$$

The evolution of the outputs using the additive FTC and the FTC proposed in this paper are compared in Fig.2 and Fig.3. The additive sensor fault appears at k = 100 and k=1000. Compared with the additive FTC, the proposed FTC shows its capacity to improve the closed-loop performances in terms of tracking, sensor fault compensation, and disturbance attenuation. As can be seen from Fig. 2 and Fig.3,



**FIGURE 2.** Evolution of the first output  $y_1$ .



**FIGURE 3.** Evolution of the second output  $y_2$ .

the trajectories are stabilized regardless of the occurrence of the faults and sinusoidal disturbance using the proposed FTC. The proposed FTC suppresses the disturbances so that it becomes smooth through the system and has strong antidisturbance capability. However, using the additive FTC, the fault is compensated, but the disturbance persists, which can degrade the closed-loop performances and even lead to instability. The proposed FTC ensures strong robustness and keeps invariant properties compared with additive FTC. Indeed, the control performance relies on the sensor fault estimation quality, that should be rapid and accurate. In the present paper, the proposed data based projection approach is efficient in estimating sensor fault rapidly and accurately. Then, the accurate estimation gives an efficient control law, which allows reference trajectory tracking, fault tolerance, and disturbance attenuation. Fault estimation is achieved by the proposed DPM as illustrated in Fig.4. The proposed DPM performs accurate fault estimation. Thus, allowing the control law to be the most suitable to maintain the trajectory tracking and fault compensation goals. Hence, from these simulation results, it is worth noting that the disturbed switched system subject to sensor fault is stabilized by means of the designed FTC. The evolution of the controls using the proposed FTC are described in Fig.5 and Fig.6. The proposed FTC reacts quickly to cancel for the effect of the sensor fault.

### B. COMPARISON BETWEEN THE PROPOSED FTC AND THE MODEL PREDICTIVE CONTROL (MPD)

To evaluate the accuracy of the obtained FTC, two differents performances criteria are calculated. The first performance index is the mean square error (MSE) and it has the following



FIGURE 4. Sensor fault estimation.



**FIGURE 5.** Evolution of  $u_1$  using the proposed FTC.



**FIGURE 6.** Evolution of  $u_2$  using the proposed FTC.

expression:

$$MSE = \frac{1}{N_H} \sum_{k=1}^{N_H} (y(k) - y_r(k))^2$$
(63)

The second is the variance accounted for (VAF) criterion and it has the following expression:

$$VAF = \max\left\{1 - \frac{var(y(k) - y_r)}{var(y(k))}, 0\right\} 100\%$$
 (64)

where y(k) denotes the measured output of the studied system,  $y_r(k)$  is the reference signal and  $N_H$  is the number of measures. Using the proposed FTC, a good adequation between the system output and the reference signal is obtained and illustrated by the following indices: MSE = 0.0028, VAF = 98.39 percent.

In [49], a passive FTC for the hybrid switched system "AC/DC converter" subject to faults and harmonic disturbance is proposed. The proposed schemes is based on model predictive control (MPC) techniques. To better check the effectiveness of the proposed FTC, we compare the obtained result using the performance criteria VAF with the one obtained using the MPC presented in [49]. Table 2 compares

#### TABLE 2. Error comparison between the proposed FTC and MPC.

	$VAF \ in \ percent$
Proposed FTC MPC	98.39 96

the two approaches. In this case, the difference between the two strategies is clear in the stability and robustness. Hence,the use of proposed FTC gives better results than MPC.

#### **VIII. CONCLUSION**

This paper puts forward an improved active fault tolerant control design for the switched hybrid system with periodic disturbance. A data-projection method is developed for fault estimation. Once, accurate fault estimation is performed, an additive control law is computed aiming, to tolerate the fault effect. A switched control strategy is proposed for fault compensation and disturbance attenuation. Indeed, a bank of controllers corresponding to a set of sub-models is designed. The control gains are computed by solving novel MIs. Compared with existing work, the new control method makes the system more stable and shows strong robustness. In future works, a fault tolerant control for the switched system with an unknown switching mode subject to sensor fault will be designed.

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