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RESEARCH ARTICLE

Output-Feedback Control for Stochastic Feedforward Nonlinear Systems With Markovian Switching

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ABSTRACT This paper studies the output-feedback control for stochastic feedforward nonlinear systems with Markovian switching and unknown measurement sensitivity. By developing a stochastic dual-domination design approach, a state observer and an output feedback control law are designed. By using the generalized Itô formula and Dynkin formula for Markovian switching systems, it is shown that the closed-loop system has a unique solution and the solution of the closed-loop systems is almost surely asymptotically stable. Finally, a simulation example is given to illustrate the effectiveness of the control scheme.

INDEX TERMS Stochastic feedforward nonlinear systems, Markovian switching, output feedback control.

I. INTRODUCTION

Research on the control design of feedforward nonlinear systems (also known as upper-triangular nonlinear systems) has attracted much attention in the past two decades due to their wide practical applications such as planar vertical landing aircraft in [1] and the cart-pendulum system in [2]. For this kind of problems, [3] design control laws by nesting saturation functions for uncertain feedforward nonlinear systems; [4] studies the adaptive controller design for systems with delays of unknown length.

Most researches in the above literature assume that there is no noise in the studied systems. However, real systems are often subject to stochastic noise [5]–[8] in uncertain environments. Therefore, it is necessary and beneficial to study the control of stochastic feedforward nonlinear systems. For the state-feedback control, [9], [10] investigates stochastic feedforward nonlinear systems with time-varying delay; [11] focuses on the decentralized stabilization for large-scale stochastic feedforward nonlinear systems; [12] studies the cooperative control of stochastic feedforward nonlinear multi-agent systems under directed network topology. For the output-feedback control, [13] investigates the output-feedback control of stochastic feedforward nonlinear systems with state time delay; [14] extends the results in [13] to stochastic high-order case; [15] studied the approximate sampled-data observer design for a class of stochastic nonlinear systems with exact output function $y = h(x_1)$; [16] studied the ILC problem for a class of stochastic systems with measurement noise; [17] considers output-feedback control of stochastic feedforward systems with unknown control coefficients and unknown output function; Besides, [18] investigates the problem of output feedback control for a class of stochastic feedforward systems with unknown measurement sensitivity.

It is worth pointing out that, all the output-feedback control schemes provided in [13]–[17], are limited to a special class of systems with strict conditions, where systems are free of sensor sensitivity error and do not consider Markovian switching. And [18] do not consider Markovian switching. However, in practice, there always exists a sensitivity error [19]–[21]. Besides, many physical systems are subject to abrupt variations in their structures, due to random failures or repairs of components and sudden environmental disturbances, which can be effectively described by the differential equations with Markovian switching [22], [23]. Therefore, study of output-feedback control for stochastic feedforward nonlinear systems with Markovian switching and unknown

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measurement sensitivity is of practical significance and thus warranted. To our best knowledge, there is no results in open literature on this topic.

Inspired by [24]–[27] and [31], this paper attempts to solve output-feedback stabilization problem of stochastic feedforward nonlinear systems with Markovian switching and unknown measurement sensitivity. The main features and contributions of this paper are summarized as follows:

(1) This paper is the first result on output-feedback control of stochastic feedforward nonlinear systems with unknown measurement sensitivity. To deal with stochastic noise and the unknown measurement sensitivity simultaneously, stochastic dual-domination approach is developed to construct a state observer and an output-feedback controller. In the stochastic dual-domination design, two gains are designed. One gain is designed to deal with the unknown measurement sensitivity, and the other gain is designed to deal with the feedforward system structure.

(2) Even for the state-feedback control of stochastic feedforward nonlinear systems with Markovian switching, this paper is new since this paper is the first attempt to introduce Markovian switching into stochastic feedforward systems. How to deal with the interconnected term in the infinitesimal generator of Lyapunov function produced by Markovian switching is nontrivial.

The remainder of this paper is organized as follows. Section II is on preliminaries. Section III is for problem formulation. Section IV focuses on controller design and stability analysis. Section V gives a numerical example to show the effectiveness of the theoretical results. Section VI includes some concluding remarks.

II. PRELIMINARY RESULTS AND USEFUL LEMMAS

The following notations will be used throughout this paper. R_+ denotes the set of all nonnegative real numbers, and R^n denotes the real n-dimensional space. For a given vector or matrix X, X^T denotes its transpose, $Tr\{X\}$ denotes its trace when X is square, and $|X|$ is the Euclidean norm of a vector X. Defining $|A| = \left(\sum_{i=1}^n \sum_{j=1}^m a_{ij}^2\right)^{\frac{1}{2}}$ for matrix A. $\mathcal{C}^i$ denotes the set of all functions with continuous ith partial derivatives.

Consider the following stochastic nonlinear system

$$dx(t) = f_\sigma(x(t), t)dt + g_\sigma(x(t), t)d\omega, \tag{1}$$

where $x(t) \in R^n$ is the state of system, the Borel measurable functions $f_\sigma(x(t), t)$ and $g_\sigma(x(t), t)$ are locally Lipschitz in $x \in R^n$ for all $t \geq 0$, and ω is an m-dimensional independent standard Wiener process defined on the complete probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$ with a filtration $\mathcal{F}_t$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all P-null sets). Let $\sigma(t)$ (written as σ for short in this paper) be a right-continuous homogeneous Markov process on the probability space taking values in a finite state space $S = \{1, 2, \ldots, N\}$ with generator $\Gamma = (\gamma_{pq})_{N\times N}$ given by

$$P_{pq}(t) = P\{\sigma(t+s) = q|\sigma(s) = p\} = \begin{cases} \gamma_{pq}t + o(t) & \text{if } p \neq q \\ 1 + \gamma_{pp}t + o(t) & \text{if } p = q \end{cases}$$

for any $s, t \geq 0$. Here $\gamma_{pq} > 0$ is the transition rate from p to q if $p \neq q$ while $\gamma_{pp} = -\sum_{q=1,q\neq p}^N \gamma_{pq}$. for any $s, t \geq 0$. Here $\gamma_{pq} > 0$ is the transition rate from p to q if $p \neq q$ while $\gamma_{pp} = -\sum_{q=1,q\neq p}^N \gamma_{pq}$.

For system (1) and $V \in C^{2,1}(R^n \times R_+ \times S; R_+)$, introduce the infinitesimal generator by

$$\begin{aligned}\mathcal{L}V(x,t,p) &= V_t(x,t,p) + V_x(x,t,p)f(x,t,p) \\ &\quad + \frac{1}{2}\mathrm{Tr}\left[g^T(x,t,p)V_{xx}(x,t,p)g(x,t,p)\right] \\ &\quad + \mathrm{II}V,\end{aligned}$$

where $\mathrm{II}V = \sum_{q=1}^N \gamma_{pq}V(x,t,q)$, $V_t(x,t,p) = \frac{\partial V(x,t,p)}{\partial t}$, $V_x(x,t,p) = (\frac{\partial V(x,t,p)}{\partial x_1}, \ldots, \frac{\partial V(x,t,p)}{\partial x_n})$, $V_{xx}(x,t,p) = (\frac{\partial^2 V(x,t,p)}{\partial x_l \partial x_m})_{n\times n}$.

The following definition and lemmas are useful for the controller design and stability analysis.

Definition 1 [28]: A stochastic process $x(t)$ is said to be bounded in probability if the random variable $|x(t)|$ is bounded in probability uniformly in t; that is

$$\lim_{c\to\infty} \sup_{t>t_0} P\{|x(t)| > c\} = 0.$$

Lemma 1 [24]: For any $l > 0$, define the first exit time η_l as

$$\eta_l = \inf\{t : t \geq t_0,\ |x(t)| \geq l\}.$$

Assume that there exist a positive function $V \in C^{2,1}(R^n \times R_+ \times S; R_+)$, parameters d and $D \geq 0$ such that

$$EV(x, \eta_l \wedge t, \sigma(\eta_l \wedge t)) \leq De^{d(\eta_l\wedge t - t_0)},$$
$$R \to \infty \Longrightarrow V_R = \inf_{t\geq t_0, |x|>R} V(x,t,\sigma(t)) \to \infty.$$

Then for every $x(t_0) = x_0 \in R^n$ and $\sigma(t_0) = i_0 \in S$, there exists a solution $x(t) = x(x_0, i_0; t, \sigma(t))$, unique up to equivalence, of system (1).

Lemma 2 [25]: Let $V \in C^{2,1}(R^n \times R_+ \times S; R_+)$ and τ_1, τ_2 be bounded stopping times such that $0 \leq \tau_1 \leq \tau_2$ *a.s.*. If $V(x,t,\sigma(t))$ and $\mathcal{L}V(x,t,\sigma(t))$ are bounded on $t \in [\tau_1,\ \tau_2]$ a.s., then

$$E[V(x,\tau_2,\sigma(\tau_2)) - V(x,\tau_1,\sigma(\tau_1))] = E\int_{\tau_1}^{\tau_2} \mathcal{L}V(x,t,\sigma(t))dt.$$

Lemma 3 [28]: For $(x, y) \in R^2$, the following inequality holds:

$$xy \leq \frac{\nu^p}{p}|x|^p + \frac{1}{q\nu^q}|y|^q,$$

where $\nu > 0$, the constants $p > 1$ and $p > 1$ satisfy $(p-1)(q-1) = 1$.

Lemma 4 [29]: For $p \in [1, \infty)$ and any $x_i \in R$, $i = 1, \ldots, n$, the following inequality holds:

$$(|x_1| + \cdots + |x_n|)^p \le n^{p-1}(|x_1|^p + \cdots + |x_n|^p).$$

III. PROBLEM FORMULATION

Consider the stochastic feedforward nonlinear systems with Markovian switching and unknown measurement described by

$$\begin{aligned} dx_1 &= x_2dt + f_{\sigma 1}(\tilde{x}_3)dt + g_{\sigma 1}^T(\tilde{x}_3)dw, \\ dx_2 &= x_3dt + f_{\sigma 2}(\tilde{x}_4)dt + g_{\sigma 2}^T(\tilde{x}_4)dw, \\ &\vdots \\ dx_{n-2} &= x_{n-1}dt + f_{\sigma,n-2}(\tilde{x}_n)dt + g_{\sigma,n-2}^T(\tilde{x}_n)dw, \\ dx_{n-1} &= x_ndt, \\ dx_n &= udt, \\ y &= \theta(t)x_1, \end{aligned} \tag{2}$$

where $\tilde{x}_i = (x_i, \cdots, x_n)^T \in R^{n-i+1}$, $u \in R$ and $y \in R$ are the state, the input, and the output of the system, repectively. The functions $f_{\sigma i}$ and $g_{\sigma i}$ are smooth with $f_{\sigma i}(0) = 0$, $g_{\sigma i}(0) = 0$. The sensor sensitivity $\theta(t)$ is a bounded unknown continuous function of $t \in R_+$. The definitions of Wiener process ω and Markov process $\sigma(t)$ (written as σ for short) can be found in system (1). We assume that the Markov process $\sigma(t)$ is independent of the Brownian motion ω.

For system (2), we need the following assumption.

Assumption 1: For $i = 1, \cdots, n-2$, there exist positive constants $b_{\sigma i}$ and $c_{\sigma i}$ such that

$$\begin{aligned} |f_{\sigma i}(\tilde{x}_{i+2})| &\le b_{\sigma i}(|x_{i+2}| + \cdots + |x_n|), \\ |g_{\sigma i}(\tilde{x}_{i+2})| &\le c_{\sigma i}(|x_{i+2}| + \cdots + |x_n|). \end{aligned}$$

Remark 1: What should be emphasized is that, for the output-feedback control of system (2), the existing results [13] and [14] are based on the ideal condition $\theta(t) \equiv 1$. However, in practice, a sensitivity error in $\theta(t)$ often exists for manufacturing reasons, which makes $\theta(t)$ deviate from its real value $\theta_0 = 1$. For instance, as demonstrated by [20], in a magnetic bearing suspension system, there exists $\pm 10\%$ sensitivity error for the displacement sensor. Thus, the output of this sensor may differ from the actual value with $\pm 10\%$ variation. Therefore, with the effect of the sensor sensitivity error, the output-feedback stabilization problem for system (2) is nontrivial.

Remark 2: To the best of our knowledge, all the existing results about stochastic feedforward nonlinear systems, such as [9]–[17], either for state-feedback control or output-feedback control, did not consider Markovian switching. Considering that many physical systems are subject to abrupt variations in their structures, system (2) is more practical model than that considered in [9]–[17].

Remark 3: As shown in [13]–[17], Assumption 1 is a standard assumption for the output-feedback control of stochastic feedforward nonlinear systems, which is frequently used in the observer and output-feedback controller design.

The objective of this paper is to construct a state observer and an output-feedback controller to solve the output-feedback stabilization problem for system (2).

IV. CONTROLLER DESIGN AND STABILITY ANALYSIS

In this section, we aim to solve the output-feedback stabilization problem for system (2) under Assumption 1 via the following four steps:

A) Design a Linear Observer;
B) Construct the output feedback controller;
C) Design the dual domination gains; and
D) Stability of the closed-loop control system is studied.

A. LINEAR OBSERVER DESIGN

Construct the linear observer as

$$\begin{aligned} d\hat{x}_1 &= \hat{x}_2dt - \varepsilon a_1\hat{x}_1dt, \\ d\hat{x}_2 &= \hat{x}_3dt - \varepsilon^2 a_2\hat{x}_1dt, \\ &\vdots \\ d\hat{x}_n &= udt - \varepsilon^n a_n\hat{x}_1dt, \end{aligned} \tag{3}$$

where $0 < \varepsilon < 1$ is a design parameter to be determined later, and $a_i > 0$, $i = 1, \ldots, n$, are coefficients of the Hurwitz polynomial $h_1(s) = s^n + a_1s^{n-1} + \cdots + a_{n-1}s + a_n$.

Define the estimation error

$$e_i = \frac{x_i - \hat{x}_i}{\varepsilon^{i-1}}, \quad i = 1, \ldots, n. \tag{4}$$

Denoting $e = (e_1, \cdots, e_n)^T$, from (2)-(4) we have

$$de = (\varepsilon A_e e + \varepsilon B_e x_1 + F_{\sigma e})dt + G_{\sigma 1}^T dw, \tag{5}$$

where

$$B_e = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}, \quad A_e = \begin{bmatrix} -a_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & \cdots & 1 \\ -a_n & 0 & \cdots & 0 \end{bmatrix},$$

$F_{\sigma e} = [f_{\sigma 1}(\tilde{x}_3), \frac{1}{\varepsilon}f_{\sigma 2}(\tilde{x}_4), \cdots, \frac{1}{\varepsilon^{n-3}}f_{\sigma,n-2}(\tilde{x}_n), 0, 0]^T$, $G_{\sigma 1}^T = [g_{\sigma 1}^T(\tilde{x}_3), \frac{1}{\varepsilon}g_{\sigma 2}^T(\tilde{x}_4), \cdots, \frac{1}{\varepsilon^{n-3}}g_{\sigma,n-2}^T(\tilde{x}_n), 0, 0]^T$. Since A_e is a Hurwitz matrix, there exists a positive-definite matrix P_e satisfying $A_e^TP_e + P_eA_e = -I_n$.

Choosing $V_0(e) = e^TP_ee$, by (5) we get

$$\begin{aligned} \mathcal{L}V_0(e) &= (\varepsilon A_e e + \varepsilon B_e x_1 + F_{\sigma e})^T P_e e + e^T P_e(\varepsilon A_e e \\ &\quad + \varepsilon B_e x_1 + F_{\sigma e}) + \frac{1}{2}Tr\left\{G_{\sigma 1}\frac{\partial^2 V}{\partial e^2}G_{\sigma 1}^T\right\} \\ &\quad + \Pi V_0(e) \\ &= -\varepsilon|e|^2 + 2\varepsilon e^TP_eB_ex_1 + 2e^TP_eF_{\sigma e} \\ &\quad + \frac{1}{2}Tr\left\{G_{\sigma 1}\frac{\partial^2 V}{\partial e^2}G_{\sigma 1}^T\right\} + \Pi V_0(e) \\ &\le -\varepsilon|e|^2 + 2\varepsilon|e||P_e||B_e||x_1| + 2|e||P_e||F_{\sigma e}| \\ &\quad + \frac{1}{2}Tr\left\{G_{\sigma 1}\frac{\partial^2 V}{\partial e^2}G_{\sigma 1}^T\right\} + \Pi V_0(e), \end{aligned} \tag{6}$$

where $\mathrm{II}V_0(e) = \sum_{q=1}^{N} \gamma_{pq} V_0(e)$. From Lemma 3 we have

$$2\varepsilon|e||P_e||B_e||x_1| \leq \frac{1}{2}\varepsilon|e|^2 + 2\varepsilon|P_e|^2|B_e|^2x_1^2. \tag{7}$$

By Assumption 1 we obtain

$$2|e||P_e||F_{\sigma e}| \leq 2b|e||P_e|\left(|x_3| + \cdots + \frac{n-2}{\varepsilon^{n-3}}|x_n|\right), \tag{8}$$

where $b = \sum_{i=1}^{N} \sum_{j=1}^{n-2} b_{ij}$.

By Lemma 3 we get

$$\begin{aligned} |e||x_3| &\leq \varepsilon^2|e|^2 + \frac{1}{4\varepsilon^2}x_3^2, \\ \frac{1}{\varepsilon}|e||x_4| &\leq \varepsilon^2|e|^2 + \frac{1}{4\varepsilon^4}x_4^2, \\ &\vdots \\ \frac{1}{\varepsilon^{n-3}}|e||x_n| &\leq \varepsilon^2|e|^2 + \frac{1}{4\varepsilon^{2n-4}}x_n^2, \end{aligned}$$

which substituting into (8) yields

$$\begin{aligned} &2|e||P_e||F_{\sigma e}| \\ &\leq 2b|P_e|\left(\varepsilon^2|e|^2 + \frac{1}{4\varepsilon^2}x_3^2 + 2\varepsilon^2|e|^2 + 2\frac{1}{4\varepsilon^4}x_4^2 + \cdots \right. \\ &\quad \left. + (n-2)\varepsilon^2|e|^2 + (n-2)\frac{1}{4\varepsilon^{2n-4}}x_n^2\right) \\ &\leq 2b|P_e|\left[\varepsilon^2|e|^2(1+2+\cdots+n-2) \right. \\ &\quad \left. + \frac{1}{4}\varepsilon^2\left(\frac{1}{\varepsilon^4}x_3^2 + \cdots + \frac{n-2}{\varepsilon^{2n-2}}x_n^2\right)\right] \\ &= b\varepsilon^2(n-1)(n-2)|P_e||e|^2 \\ &\quad + \frac{1}{2}b\varepsilon^2|P_e|\left(\frac{1}{\varepsilon^4}x_3^2 + \cdots + \frac{n-2}{\varepsilon^{2n-2}}x_n^2\right) \\ &\leq b\varepsilon^2(n-1)(n-2)|P_e||e|^2 + \frac{1}{2}b\varepsilon^2(n-2)|P_e| \\ &\quad \cdot \left(\frac{1}{\varepsilon^4}x_3^2 + \cdots + \frac{1}{\varepsilon^{2n-2}}x_n^2\right). \end{aligned} \tag{9}$$

By Assumption 1 and Lemma 4 we can get

$$\begin{aligned} &\frac{1}{2}Tr\left\{G_{\sigma 1}\frac{\partial^2 V}{\partial e^2}G_{\sigma 1}^T\right\} \\ &= |P_e|\left(|g_{\sigma 1}|^2 + \cdots + \frac{|g_{\sigma,n-2}|^2}{\varepsilon^{2n-6}}\right) \\ &\leq c^2|P_e|\left((|x_3| + \cdots + |x_n|)^2 + \cdots + \frac{|x_n|^2}{\varepsilon^{2n-6}}\right) \\ &\leq c^2|P_e|\left((n-2)(x_3^2 + \cdots + x_n^2) + \cdots + \frac{x_n^2}{\varepsilon^{2n-6}}\right) \\ &\leq c^2|P_e|\left((n-2)x_3^2 + \cdots + \frac{(n-1)(n-2)}{2\varepsilon^{2n-6}}x_n^2\right) \\ &\leq c^2\varepsilon^4\frac{(n-1)(n-2)}{2}|P_e|\left(\frac{x_3^2}{\varepsilon^4} + \cdots + \frac{x_n^2}{\varepsilon^{2n-2}}\right), \end{aligned} \tag{10}$$

where $c = \sum_{i=1}^{N} \sum_{j=1}^{n-2} c_{ij}$.

Substituting (7), (9), and (10) into (6) yields

$$\begin{aligned} &\mathcal{L}V_0(e) \\ &\leq -\frac{1}{2}\varepsilon|e|^2 + b\varepsilon^2(n-1)(n-2)|P_e||e|^2 \\ &\quad + \frac{1}{2}b\varepsilon^2(n-2)|P_e|\left(\frac{x_3^2}{\varepsilon^4} + \cdots + \frac{x_n^2}{\varepsilon^{2n-2}}\right) \\ &\quad + c^2\varepsilon^4\frac{(n-1)(n-2)}{2}|P_e|\left(\frac{x_3^2}{\varepsilon^4} + \cdots + \frac{x_n^2}{\varepsilon^{2n-2}}\right) \\ &\quad + 2\varepsilon|P_e|^2|B_e|^2x_1^2 + \mathrm{II}V_0(e) \\ &\leq -\frac{1}{2}\varepsilon|e|^2 + b\varepsilon^2(n-1)(n-2)|P_e||e|^2 \\ &\quad + \frac{1}{2}\varepsilon^2[b(n-2) + c^2\varepsilon^2(n-1)(n-2)]|P_e| \\ &\quad \cdot \left(\frac{1}{\varepsilon^4}x_3^2 + \frac{1}{\varepsilon^6}x_4^2 + \cdots + \frac{1}{\varepsilon^{2n-2}}x_n^2\right) \\ &\quad + 2\varepsilon|P_e|^2|B_e|^2x_1^2 + \mathrm{II}V_0(e) \\ &\leq -\frac{1}{2}\varepsilon|e|^2 + b\varepsilon^2(n-1)(n-2)|P_e||e|^2 \\ &\quad + \frac{1}{2}\varepsilon^2[b(n-2) + c^2(n-1)(n-2)]|P_e| \\ &\quad \cdot \left(\frac{1}{\varepsilon^4}x_3^2 + \frac{1}{\varepsilon^6}x_4^2 + \cdots + \frac{1}{\varepsilon^{2n-2}}x_n^2\right) \\ &\quad + 2\varepsilon|P_e|^2|B_e|^2x_1^2 + \mathrm{II}V_0(e). \end{aligned} \tag{11}$$

B. OUTPUT FEEDBACK CONTROLLER DESIGN

Consider the following augmented systems

$$\begin{aligned} dx_1 &= x_2dt + f_{\sigma 1}dt + g_{\sigma 1}^Tdw, \\ d\hat{x}_2 &= \hat{x}_3dt + \varepsilon^2a_2(e_1 - x_1)dt, \\ d\hat{x}_3 &= \hat{x}_4dt + \varepsilon^3a_3(e_1 - x_1)dt, \\ &\vdots \\ d\hat{x}_n &= udt + \varepsilon^na_n(e_1 - x_1)dt. \end{aligned} \tag{12}$$

We design the output feedback controller as

$$v = -b_ny - b_{n-1}z_2 - \cdots - b_2z_{n-1} - b_1z_n, \tag{13}$$

where $b_i > 0, i = 1, \ldots, n$, are coefficients of the Hurwitz polynomial $h_2(s) = s^n + b_1s^{n-1} + \cdots + b_{n-1}s + b_n$ and the change of coordinates are defined by

$$z_1 = x_1, z_i = \frac{\hat{x}_i}{(\varepsilon L)^{i-1}}, v = \frac{u}{(\varepsilon L)^n}, \quad i = 2, \ldots, n, \tag{14}$$

with $L \geq 1$ being a design parameter to be determined later.

From (12) and (14) we have

$$\begin{aligned} dz_1 &= L\varepsilon z_2dt + \varepsilon e_2dt + f_{\sigma 1}dt + g_{\sigma 1}^Tdw, \\ dz_2 &= L\varepsilon z_3dt + \frac{\varepsilon a_2}{L}e_1dt - \frac{\varepsilon a_2}{L}z_1dt, \\ dz_3 &= L\varepsilon z_4dt + \frac{\varepsilon a_3}{L^2}e_1dt - \frac{\varepsilon a_3}{L^2}z_1dt, \\ &\vdots \\ dz_n &= L\varepsilon vdt + \frac{\varepsilon a_n}{L^{n-1}}e_1dt - \frac{\varepsilon a_n}{L^{n-1}}z_1dt. \end{aligned} \tag{15}$$

Substituting (13) into (15) we get

$$dz = \left(\varepsilon LA_z z + \varepsilon LB_z b_n(1-\theta(t))z_1 + \varepsilon D_2 e_2 + \frac{\varepsilon}{L}D_1(e_1 - z_1) + F_{\sigma z}\right)dt + G_{\sigma 2}^T dw, \quad (16)$$

where

$$z = \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix}, \quad B_z = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 \\ a_2 \\ \frac{1}{L}a_3 \\ \vdots \\ \frac{1}{L^{n-2}}a_n \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix}, \quad A_z = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -b_n & -b_{n-1} & \cdots & -b_1 \end{bmatrix},$$

and

$$F_{\sigma z} = \begin{bmatrix} f_{\sigma 1} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad G_{\sigma 2}^T = \begin{bmatrix} g_{\sigma 1}^T \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

Since A_z is a Hurwitz matrix, there exists a positive-definite matrix P_z satisfying $A_z^T P_z + P_z A_z = -I_n$.

Choosing $V_1(z) = z^T P_z z$, from (16)we have

$$\begin{aligned}
\mathcal{L}V_1(z) &= -\varepsilon L|z|^2 + 2\varepsilon L z^T P_z B_z b_n(1-\theta(t))z_1 \\
&\quad + 2z^T P_z\left(\frac{\varepsilon}{L}D_1 e_1 + \varepsilon D_2 e_2 - \frac{\varepsilon}{L}D_1 z_1\right) \\
&\quad + \frac{1}{2}Tr\left\{G_{\sigma 2}\frac{\partial^2 V}{\partial z^2}G_{\sigma 2}^T\right\} + 2z^T P_z F_{\sigma z} + \text{II}V_1(z) \\
&\le -\varepsilon L|z|^2 + 2b_n\varepsilon L|1-\theta(t)||P_z||z|^2 + 2\frac{\varepsilon}{L}|z||P_z| \\
&\quad \cdot |D_1||e| + 2\varepsilon|z||P_z||e| + 2|P_z||F_{\sigma z}||z| \\
&\quad + \frac{1}{2}Tr\left\{G_{\sigma 2}\frac{\partial^2 V}{\partial z^2}G_{\sigma 2}^T\right\} + \text{II}V_1(z), \quad (17)
\end{aligned}$$

where $\text{II}V_1(z) = \sum_{q=1}^N \gamma_{pq}V_1(z)$, $|B_z| = 1$, $|D_2| = 1$.

By Lemma 3 we obtain

$$\begin{aligned}
2\frac{\varepsilon}{L}|z||P_z||D_1||e| &\le \frac{1}{8}\varepsilon|e|^2 + \frac{8\gamma^2\varepsilon}{L^2}|P_z|^2|z|^2, \\
2\varepsilon|z||P_z||e| &\le \frac{1}{8}\varepsilon|e|^2 + 8\varepsilon|P_z|^2|z|^2,
\end{aligned}$$

where $|D_1| \le (\sum_{j=2}^n a_i^2)^{\frac{1}{2}} := \gamma$.

Similarly, from Lemma 3 we have

$$\begin{aligned}
|z||x_3| &\le \varepsilon^2|z|^2 + \frac{1}{4\varepsilon^2}x_3^2, \\
\frac{1}{\varepsilon}|z||x_4| &\le \varepsilon^2|z|^2 + \frac{1}{4\varepsilon^4}x_4^2, \\
&\vdots \\
\frac{1}{\varepsilon^{n-3}}|z||x_n| &\le \varepsilon^2|z|^2 + \frac{1}{4\varepsilon^{2n-4}}x_n^2, \quad (18)
\end{aligned}$$

which together with Assumption 1 and the definition of $F_{\sigma z}$ yields

$$\begin{aligned}
&2|z||P_z||F_{\sigma z}| \\
&\le 2b_{10}|P_z||z|(|x_3| + \cdots + |x_n|) \\
&\le 2b_{10}|P_z||z|\left(|x_3| + \frac{1}{\varepsilon}|x_4| + \cdots + \frac{1}{\varepsilon^{n-3}}|x_n|\right) \\
&\le 2b_{10}|P_z|\varepsilon^2(n-2)|z|^2 \\
&\quad + \frac{1}{2}b_{10}|P_z|\varepsilon^2\left(\frac{1}{\varepsilon^4}x_3^2 + \cdots + \frac{1}{\varepsilon^{2n-2}}x_n^2\right) \\
&= 2b_{10}\varepsilon^2(n-2)|P_z||z|^2 + \frac{1}{2}b_{10}\varepsilon^2|P_z| \\
&\quad \cdot\left(\frac{1}{\varepsilon^4}x_3^2 + \cdots + \frac{1}{\varepsilon^{2n-2}}x_n^2\right), \quad (19)
\end{aligned}$$

where $b_{10} = \Sigma_{i=1}^N b_{i1}$.

From Assumption 1, Lemma 4, and the definition of $G_{\sigma 2}$ we obtain

$$\begin{aligned}
&\frac{1}{2}Tr\left\{G_{\sigma 2}\frac{\partial^2 V}{\partial z^2}G_{\sigma 2}^T\right\} \\
&= |P_z||g_{\sigma 1}|^2 \\
&\le c_{10}^2|P_z|(|x_3| + \cdots + |x_n|)^2 \\
&\le c_{10}^2(n-2)|P_z|(x_3^2 + \cdots + x_n^2) \\
&\le c_{10}^2(n-2)|P_z|\left(\frac{1}{\varepsilon^2}x_3^2 + \cdots + \frac{1}{\varepsilon^{2n-4}}x_n^2\right) \\
&= c_{10}^2\varepsilon^2(n-2)|P_z|\left(\frac{1}{\varepsilon^4}x_3^2 + \cdots + \frac{1}{\varepsilon^{2n-2}}x_n^2\right), \quad (20)
\end{aligned}$$

where $c_{10} = \Sigma_{i=1}^N c_{i1}$.

Substituting (18)-(20) into (17) yields

$$\begin{aligned}
&\mathcal{L}V_1(z) \\
&\le -\varepsilon L(1 - 2b_n|1-\theta(t)||P_z|)|z|^2 + \frac{1}{4}\varepsilon|e|^2 + \frac{8\gamma^2}{L^2} \\
&\quad \cdot\varepsilon|P_z|^2|z|^2 + 8\varepsilon|P_z|^2|z|^2 + \frac{2\gamma\varepsilon}{L}|P_z||z|^2 + 2\varepsilon^2 \\
&\quad \cdot b_{10}(n-2)|P_z||z|^2 + \frac{1}{2}\varepsilon^2(b_{10} + 2(n-2)c_{10}^2) \\
&\quad \cdot|P_z|\left(\frac{1}{\varepsilon^4}x_3^2 + \cdots + \frac{1}{\varepsilon^{2n-2}}x_n^2\right) + \text{II}V_1(z) \\
&= -\varepsilon L(1 - 2b_n|1-\theta(t)||P_z|)|z|^2 + \frac{1}{4}\varepsilon|e|^2 \\
&\quad + \varepsilon L\left(\frac{k_1}{L} + \frac{\varepsilon^2 k_2}{\varepsilon}\right)|z|^2 + \frac{1}{2}\varepsilon^2(b_{10} + 2(n-2) \\
&\quad \cdot c_{10}^2)|P_z|\left(\frac{1}{\varepsilon^4}x_3^2 + \cdots + \frac{1}{\varepsilon^{2n-2}}x_n^2\right) + \text{II}V_1(z), \quad (21)
\end{aligned}$$

where

$$\begin{aligned}
k_1 &= 2\gamma(1 + 4\gamma|P_z|)|P_z| + 8|P_z|^2 > 0, \\
k_2 &= 2b_{10}(n-2)|P_z| > 0.
\end{aligned}$$

C. DUAL DOMINATION GAINS DESIGN

Noting that $x_1 = z_1, x_i = \varepsilon^{i-1}e_i + (\varepsilon L)^{i-1}z_i,\ i = 2, \ldots, n$, from Lemma 4 we get

$$\frac{1}{\varepsilon^{2i-2}}|x_i|^2 \leq 2|e|^2 + 2L^{2i-2}|z|^2, \quad i = 3, \ldots, n,$$

which means that

$$\begin{aligned}
\frac{1}{\varepsilon^4}x_3^2 &\leq 2|e|^2 + 2L^4|z|^2, \\
\frac{1}{\varepsilon^6}x_4^2 &\leq 2|e|^2 + 2L^6|z|^2, \\
&\vdots \\
\frac{1}{\varepsilon^{2n-2}}x_n^2 &\leq 2|e|^2 + 2L^{2n-2}|z|^2.
\end{aligned} \tag{22}$$

From (22) we have

$$\begin{aligned}
&\frac{1}{2}\varepsilon^2[b(n-2) + c^2(n-1)(n-2)]|P_e| \\
&\quad \cdot \left(\frac{1}{\varepsilon^4}x_3^2 + \cdots + \frac{1}{\varepsilon^{2n-2}}x_n^2\right) \\
&\leq \frac{1}{2}\varepsilon^2[b(n-2) + c^2(n-1)(n-2)]|P_e|[2(n-2)|e|^2 \\
&\quad + 2L(L^3 + L^5 + \cdots + L^{2n-3})|z|^2],
\end{aligned}$$

which substitutings into (11) results in

$$\begin{aligned}
&\mathcal{L}V_0(e) \\
&\leq -\frac{1}{2}\varepsilon|e|^2 + b\varepsilon^2(n-1)(n-2)|P_e||e|^2 \\
&\quad + 2\varepsilon|P_e|^2|B_e|^2x_1^2 + \frac{1}{2}\varepsilon^2[b(n-2) \\
&\quad + c^2(n-1)(n-2)]|P_e|[2(n-2)|e|^2 \\
&\quad + 2L(L^3 + L^5 + \cdots + L^{2n-3})|z|^2] + \Pi V_0(e) \\
&\leq -\frac{1}{2}\varepsilon|e|^2 + \varepsilon^2[b(n-1)(n-2)|P_e| + (b(n-2) \\
&\quad + c^2(n-1)(n-2))(n-2)|P_e|]|e|^2 + [2\varepsilon|P_e|^2 \\
&\quad \cdot |B_e|^2 + L\varepsilon^2[b(n-2) + c^2(n-1)(n-2)] \\
&\quad \cdot (L^3 + L^5 + \cdots + L^{2n-3})|P_e|]|z|^2 + \Pi V_0(e) \\
&= -\frac{1}{2}\varepsilon|e|^2 + (2\varepsilon|P_e|^2|B_e|^2 + \varepsilon^2L\hat{k}_2)|z|^2 \\
&\quad + \varepsilon^2\hat{k}_1|e|^2 + \Pi V_0(e),
\end{aligned} \tag{23}$$

where

$$\begin{aligned}
\hat{k}_1 &= b(n-1)(n-2)|P_e| + [b(n-2) \\
&\quad + c^2(n-1)(n-2)](n-2)|P_e| > 0, \\
\hat{k}_2 &= [b(n-2) + c^2(n-1)(n-2)](L^3 + L^5 \\
&\quad + \cdots + L^{2n-3})|P_e| > 0.
\end{aligned}$$

Similar to (23), substituting (22) into (21) we obtain

$$\begin{aligned}
&\mathcal{L}V_1(z) \\
&\leq -\varepsilon L(1 - 2b_n|1-\theta(t)||P_z|)|z|^2 \\
&\quad + \frac{1}{4}\varepsilon|e|^2 + \varepsilon L\left(\frac{k_1}{L} + \frac{\varepsilon^2 k_2}{\varepsilon}\right)|z|^2 \\
&\quad + \frac{1}{2}\varepsilon^2(b_{10} + 2c_{10}^2(n-2))|P_z|[2(n-2)|e|^2 \\
&\quad + 2L(L^3 + L^5 + \cdots + L^{2n-3})|z|^2] + \Pi V_1(z) \\
&= -\varepsilon L(1 - 2b_n|1-\theta(t)||P_z|)|z|^2 + \frac{1}{4}\varepsilon|e|^2 + \varepsilon L \\
&\quad \cdot \left(\frac{k_1}{L} + \frac{\varepsilon^2 k_2}{\varepsilon}\right)|z|^2 + \varepsilon^2(b_{10} + 2c_{10}^2(n-2)) \\
&\quad \cdot (n-2)|P_z||e|^2 + L\varepsilon^2(b_{10} + 2c_{10}^2(n-2))(L^3 \\
&\quad + L^5 + \cdots + L^{2n-3}) \cdot |P_z||z|^2 + \Pi V_1(z) \\
&= -\varepsilon L(1 - 2b_n|1-\theta(t)||P_z|)|z|^2 + \frac{1}{4}\varepsilon|e|^2 + \varepsilon L \\
&\quad \cdot \left(\frac{k_1}{L} + \frac{\varepsilon^2 k_2}{\varepsilon}\right)|z|^2 + \varepsilon^2(b_{10} + 2c_{10}^2(n-2)) \\
&\quad \cdot (n-2)|P_z||e|^2 + L\varepsilon^2\hat{k}_3|z|^2 + \Pi V_1(z),
\end{aligned} \tag{24}$$

where $\hat{k}_3 = (b_{10}+2c_{10}^2(n-2))(L^3+L^5+\cdots+L^{2n-3})|P_z| \geq 0$.

Choosing Lyapunov function $V = V_0(e)+V_1(z)$, from (23) and (24) we have

$$\begin{aligned}
\mathcal{L}V &= \mathcal{L}V_0(e) + \mathcal{L}V_1(z) \\
&\leq -\frac{1}{2}\varepsilon|e|^2 + (2\varepsilon|P_e|^2|B_e|^2 + \varepsilon^2L\hat{k}_2)|z|^2 \\
&\quad + \varepsilon^2\hat{k}_1|e|^2 - \varepsilon L(1 - 2b_n|1-\theta(t)||P_z|)|z|^2 \\
&\quad + \frac{1}{4}\varepsilon|e|^2 + \varepsilon L\left(\frac{k_1}{L} + \frac{\varepsilon^2 k_2}{\varepsilon}\right)|z|^2 \\
&\quad + \varepsilon^2(b_{10} + 2c_{10}^2(n-2))(n-2)|P_z||e|^2 \\
&\quad + L\varepsilon^2\hat{k}_3|z|^2 + \Pi V,
\end{aligned} \tag{25}$$

where

$$\begin{aligned}
\Pi V &= \Pi V_0(e) + \Pi V_1(z) \\
&= \sum_{q=1}^{N}\gamma_{pq}V_0(e) + \sum_{q=1}^{N}\gamma_{pq}V_1(z).
\end{aligned}$$

Now, we choose the allowable sensitive error $\bar{\theta}$ as

$$\bar{\theta} < \theta^* = \frac{1}{2b_n|P_z|}, \tag{26}$$

where θ^* is the upper-bound of the allowable sensitivity error.

From (26) and $1 - \bar{\theta} \leq \theta(t) \leq 1 + \bar{\theta}$ we get

$$1 - 2b_n|1-\theta(t)||P_z| \geq \rho, \tag{27}$$

where $0 < \rho = 1 - 2b_n\bar{\theta}|P_z| < 1$.

Substituting (27) into (25) we obtain

$$\begin{aligned}
\mathcal{L}V &= \mathcal{L}V_0(e) + \mathcal{L}V_1(z) \\
&\leq -\varepsilon L\left(\rho - \frac{2|P_e|^2|B_e|^2 + k_1}{L} - \varepsilon(k_2 + \hat{k}_2\right. \\
&\quad \left. + \hat{k}_3)\right)|z|^2 - \varepsilon\left(\frac{1}{4} - \varepsilon[(b_{10} + 2c_{10}^2(n-2))\right. \\
&\quad \left. \cdot (n-2)|P_z| + \hat{k}_1]\right)|e|^2 + \Pi V \\
&= -\varepsilon L\left(\rho - \frac{\tilde{k}_2}{L} - \varepsilon\tilde{k}_3\right)|z|^2 - \varepsilon\left(\frac{1}{4} - \varepsilon\tilde{k}_1\right)|e|^2 \\
&\quad + \Pi V,
\end{aligned} \tag{28}$$

where

$$\tilde{k}_1 = (b_{10} + 2c_{10}^2(n-2))(n-2)|P_z| + \hat{k}_1 > 0,$$
$$\tilde{k}_2 = 2|P_e|^2|B_e|^2 + k_1 > 0,$$
$$\tilde{k}_3 = k_2 + \hat{k}_2 + \hat{k}_3 > 0.$$

We choose the parameters ε and L in the following order:

$$L \geq \max\left\{1, \frac{4\tilde{k}_2}{3\rho}\right\},$$
$$0 < \varepsilon \leq \min\left\{1, \frac{\rho}{8\tilde{k}_3}, \frac{2-\rho}{8\tilde{k}_1}\right\}, \tag{29}$$

which with (28) yields

$$\mathcal{L}V \leq -\frac{\rho}{8}|e|^2 - \frac{\rho}{8}|z|^2 + \text{II}V. \tag{30}$$

D. STABILITY ANALYSIS

Now, we are in a position to state the main results of this paper.

Theorem 1: If Assumption 1 holds for system (2), with the observer (3) and output feedback control law (13), then we have

(1) for every $x(t_0) = x_0 \in R^n$ and $\sigma(t_0) = i_0 \in S$, the closed-loop system composed of (2), (3), (14), (13), and (29) has a solution, unique up to equivalence.

(2) for any $x_0 \in R^n$ and $i_0 \in S$, the solution of the closed-loop systems is almost surely asymptotically stable.

Proof: Denote $\chi(t) = \left(e^T(t), z^T(T)\right)^T$, from the definition of V we can conclude that

$$V_R = \inf_{t\geq t_0, |\chi(t)|>R} V(\chi(t)) \to \infty \iff R \to \infty. \tag{31}$$

Since V is independent of Markov nodes, we obtain that $\text{II}V = 0$.

For any $l > 0$, define the first exit time

$$\eta_l = \inf\{t : t \geq t_0, \ |\chi(t)| \geq l\}. \tag{32}$$

Let $t_l = \eta_l \wedge t$ for any $t \geq t_0$. Since $|\chi(t)|$ is bounded in the interval $[t_0, \ t_l]$ a.s., which implies that $V(\chi)$ is bounded on $[t_0, \ t_l]$ a.s.. From (30), it can be obtained that $\mathcal{L}V$ is also bounded on $[t_0, \ t_l]$ a.s..

From (30) and Lemma 2 we have

$$EV(\chi(t_l)) \leq EV(\chi(t_0)). \tag{33}$$

By (31), (33), and Lemma 1, we can get conclusion (1).

From (2), (30), and the definition of V, by using Theorem 2.1 in [30], conclusion (2) holds.

Remark 4: In this section, A stochastic dual-domination design technique is developed for the output-feedback control for stochastic feedforward nonlinear systems, in which a high-gain $L > 1$ and a low-gain $0 < \varepsilon < 1$ are introduced. From sections A-C, we can see that this method can effectively deal with the unknown measurement sensitivity, stochastic noise and Markovian switching simultaneously.

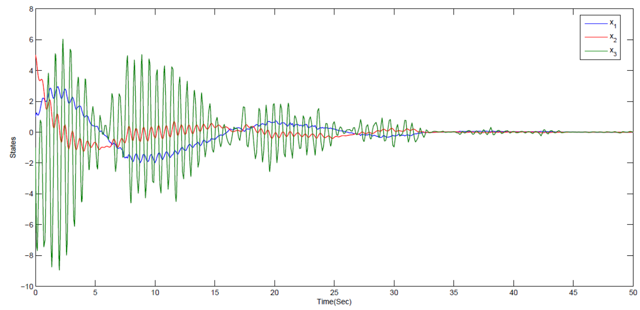

FIGURE 1. The responses of states $x(t)$ for the closed-loop system (34)–(37).

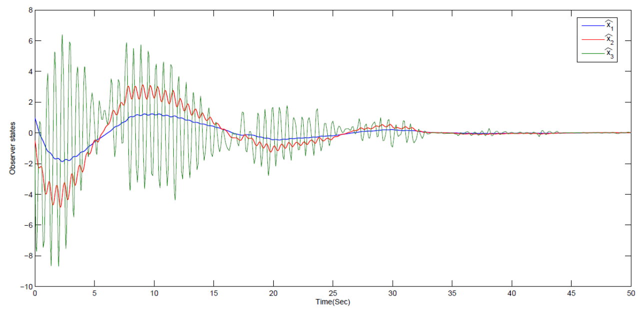

FIGURE 2. The responses of estimations $\hat{x}(t)$ for the closed-loop system (34)–(37).

V. A SIMULATION EXAMPLE

Consider system (2) with two modes. The Markov process $\sigma(t)$ belongs to the space $S = \{1, 2\}$ with generator $\Gamma = (\gamma_{pq})_{2\times 2}$ given by $\gamma_{11} = -4, \gamma_{12} = 4, \gamma_{21} = 3$ and $\gamma_{22} = -3$. We can get $\pi_1 = \frac{3}{7}, \pi_2 = \frac{4}{7}$.

When $\sigma(t) = 1$, the systems is described by

$$\begin{aligned} dx_1 &= x_2dt + \frac{1}{2}x_3dt + \sin x_3 d\omega, \\ dx_2 &= x_3dt, \\ dx_3 &= udt, \\ y &= \theta(t)x_1. \end{aligned} \tag{34}$$

When $\sigma(t) = 2$, the systems can be written as

$$\begin{aligned} dx_1 &= x_2dt + \sin(x_3)dt + \frac{1}{4}x_3d\omega, \\ dx_2 &= x_3dt, \\ dx_3 &= udt, \\ y &= \theta(t)x_1. \end{aligned} \tag{35}$$

In (34)-(35), we choose $\theta(t) = 1 + 0.25\sin(|10t|)$, which means that $\bar{\theta} = 0.25 < \theta^*$ where $\theta^* = 0.2667$.

By following the design procedure developed in section IV, we can design the observer as

$$\begin{aligned} d\hat{x}_1 &= \hat{x}_2dt - \varepsilon a_1\hat{x}_1dt, \\ d\hat{x}_2 &= \hat{x}_3dt - \varepsilon^2 a_2\hat{x}_1dt, \\ d\hat{x}_3 &= udt - \varepsilon^3 a_3\hat{x}_1dt, \end{aligned} \tag{36}$$

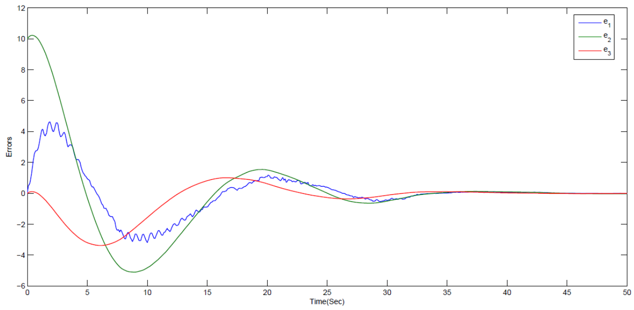

FIGURE 3. The responses of errors e for the closed-loop system (34)–(37).

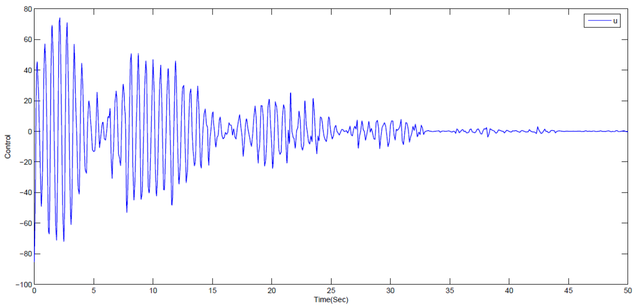

FIGURE 4. The responses of control u for the closed-loop system (34)–(37).

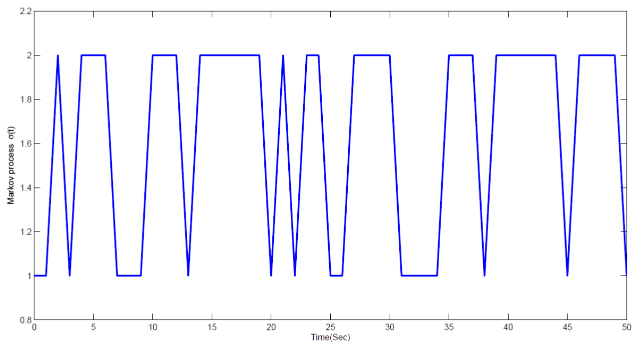

FIGURE 5. The response of Markov process $\sigma(t)$.

and the controller as

$$u = -140.3789y - 85.0781\hat{x}_2 - 12.3750\hat{x}_3. \qquad (37)$$

In the practical simulation, we choose $a_1 = 4, a_2 = 2, a_3 = 1, b_1 = 1.5, b_2 = 1.25, b_3 = 0.25$, the parameters $L = 15, \varepsilon = 0.55$, and the initial states $x_1(0) = 1$, $x_2(0) = 5$, $x_3(0) = -1$, $\hat{x}_1(0) = 1$, $\hat{x}_2(0) = -0.5, \hat{x}_3(0) = -1$. Fig.1-Fig.4 give the responses of closed-loop system (34)-(37). From Fig.1-Fig.4, we can see that $\lim_{t\to+\infty} x_i = \lim_{t\to+\infty} \hat{x}_i = \lim_{t\to+\infty} e_i = \lim_{t\to+\infty} u = 0$ a.s., which verifies the conclusions in Theorem 1, $i = 1, 2, 3$. Fig.5 gives the response of the Markov process $\sigma(t)$.

VI. CONCLUSION

The output-feedback control for stochastic feedforward nonlinear systems with Markovian switching and unknown measurement sensitivity is investigated. A stochastic dual-domination design technique is developed, by which a state observer and an output-feedback controller are designed to guarantee that the closed-loop system has a unique solution and the solution of the closed-loop systems is almost surely asymptotically stable.

For the output-feedback control of stochastic feedforward nonlinear systems, many important issues are still open and worth investigating, such as the generalization of the results in this paper to more general systems [32]–[38].

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