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RESEARCH ARTICLE

Tracking a Ground Target Utilizing Doppler-Only Measurements of a Single Passive Sonar Sensor Assisted by Straight Road Constraints

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ABSTRACT This paper handles tracking a ground target using a single passive sonar sensor measuring doppler shift of the target. We consider a scenario where the frequency of the emitting tone is not known in advance. We further consider a ground target which is constrained to move along a known straight road with a known course. It is assumed that the single sonar sensor is located close to the road, whose shape is known in advance. This article introduces tracking the ground target assisted by road constraints. We address a non-recursive estimation algorithm using a batch filter initialized with samples that are distributed considering the road constraints. As far as we know, our paper is unique in using a single sonar sensor for target localization with doppler-only measurements with unknown emitting frequency. Moreover, our article is unique in target tracking based on doppler-only measurements of a single sonar sensor, assisted by road constraints. The effectiveness of the proposed tracking filter is verified under MATLAB simulations.

INDEX TERMS Doppler-only measurements, road constraints, course constraints, constrained target tracking, passive sonar sensor.

I. INTRODUCTION

We handle tracking a ground target using a single passive sonar sensor measuring doppler shift of the target. The doppler effect became an increasingly important factor in many radar applications. Doppler shift measurements are used in aviation, sounding satellites, meteorology, radiology, and bistatic radar. Doppler shift can separate moving targets from stationary clutter.

For tracking a ground target, the ground moving target indicator (GMTI) radar has been widely used, which can measure range, azimuth, and doppler of the target [1], [2]. However, GMTI works by generating active pulses from a transmitter. GMTI continuously consumes power for generating active pulses, and it cannot operate in a stealthy manner.

In our paper, we use a single passive sonar sensor for tracking a ground target. We consider a passive scenario in [3], where a sonar sensor only measures the doppler shifted sound generated from a non-cooperative target. The authors of [3] considered a scenario where the target approaches a sensor from a far, passes through or at some distance from the sensor, and then departs. Reference [3] assumed that a single moving target emits an acoustic signal (engine noise) with constant but unknown frequency. This assumption is also used in our paper.

A passive sonar sensor works in a stealthy manner, since it only measures the sound of the target in a passive way. Our paper considers a scenario where only one passive sonar sensor is used to estimate the non-cooperative target's emitting frequency as well as the target state (position and velocity).

We consider tracking a ground target which moves with a constant velocity along a known straight road with a known course. Our tracking approach is assisted by road constraints, which are known in advance. We show that if the target's course is not known in advance, then our tracking problem with a known straight road is not observable.

References [4], [5] considered the case where the target moves in a straight line (road segment) at constant

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velocity. Assuming that the frequency of the emitting tone is known, [4] mentioned that three doppler shift measurements of a single sensor are sufficient to determine the target state (position and velocity). Moreover, [5] considered scenarios where multiple sensors are deployed to get doppler shift measurements of the target. Assuming that the frequency of the emitting tone is known, [5] addressed the optimization criterion for sensor placement over the straight road.

Our paper was inspired by [4], [5], since we consider the case where the target moves in a straight line at constant velocity. However, we consider deploying a single sensor, which is distinct from [5]. Moreover, our paper considers the case where the emitting frequency is not known in advance, which is distinct from [4], [5]. As far as we know, our paper is novel in using a single sonar sensor for target localization with doppler-only measurements with unknown emitting frequency.

In our paper, we prove that at least three doppler shift measurements of a single sensor are required to determine the target state (position, velocity, and target's emitting frequency). Assuming that the frequency of the emitting tone is known, [4] proved that three doppler shift measurements of a single sensor are sufficient to determine the target's position and velocity. In our paper, we prove that at least three doppler measurements are required for system observability, even in the case where the frequency of the emitting tone is not known in advance.

Acknowledge that recursive filters, such as Extended Kalman filters (EKF) [6], are useful for tracking a maneuvering target with process noise. However, the initialization of recursive filters is not trivial, since the filter performance heavily depends on accurate track initialization [5]. Note that we consider the case where the initial target state is not known in advance. In doppler-only target tracking, target state initialization is a critical issue as witnessed by the fact that successful doppler-only target tracking has been so far obtained by utilizing particle filters [7], [8]. Thus, we argue that recursive filters are not suitable for our doppler-only target tracking.

We use a batch of multiple doppler measurements in order to estimate the target state (position, velocity, and target's emitting frequency). Since our estimation uses a batch of doppler measurements, we can remove the estimation bias, which may occur as we use recursive filters. Moreover, an estimation error at one time-step does not propagate to the error at the next time-step, since we use non-recursive batch algorithms.

Note that once we estimate the target state using a batch of doppler measurements, we can use the estimated target state for initialization of recursive filters, such as EKF [6]. In other words, a batch of doppler measurements can be used for initialization of recursive filters.

Our paper uses the Gauss-Newton (GN) algorithm to estimate the target state from a batch of doppler measurements. The GN method requires a rather accurate initial estimate for its convergence. For initialization of the GN method, we use distributed samples, so that each sample can randomly generate its associated solution considering the road constraints. In the case where a sample yields a small measurement residual, the sample gets a larger importance weight compared to other samples. At every time-step, we calculate importance weights for every sample, and we can determine the range of re-sampling from the weighted estimate and its variance.

Our contributions are summarized as follows. As far as we know, our paper is novel in using a single sonar sensor for target localization with doppler-only measurements with unknown emitting frequency. Our article is novel in target tracking based on doppler-only measurements of a single sensor, assisted by straight road constraints. Furthermore, our paper is unique in showing that if the target's course is not known in advance, our tracking problem with a known straight road is not observable. The effectiveness of the proposed tracking filter is verified under MATLAB simulations.

Section II presents the literature review related to our paper. Section III discusses the preliminary information of our paper. Section IV presents the tracking filter using a batch of multiple doppler measurements. Section V presents MATLAB simulations. Section VI provides Conclusions.

II. LITERATURE REVIEW

This section presents the literature review related to our paper. It is well known that the target state from doppler-only measurements has poor observability. In doppler-only tracking, many papers [7]–[11] assumed that the frequency of the emitting tone is known. In doppler-only target tracking, target state initialization is a critical issue as witnessed by the fact that successful doppler-only target tracking has been so far obtained by utilizing particle filters [7], [8]. The authors of [9] studied the observability of a constant velocity target from a single doppler sensor, assuming that the frequency of the emitting tone is known. The authors of [10] utilized the Probability Hypothesis Density (PHD) filter to tackle the problem of multi-target detection and tracking over a network of separately located doppler-shift measuring sensors. Reference [11] studied the problem of joint detection and tracking of a target using multi-static doppler-only measurements. The assumption of [11] was that in the surveillance region, a single transmitter of known frequency is active with multiple spatially distributed sensors collecting and reporting doppler-shift frequencies to the data fusion center.

In order to estimate the target position based on doppler-only measurements, several papers [3], [12], [13] considered a sensor network with two or more synchronized nodes with known positions. Considering multiple stationary sensors capable of collecting noiseless doppler shift measurements from a constant velocity target, [14] studied the minimum number of sensors, such that there is a finite number of solutions for target state (position and velocity). The authors of [12] mentioned that at least four sensors are required to yield one candidate target trajectory in 3-D environments.

The assumption of known frequency tone is not feasible, considering a passive scenario where a sensor only measures the signal generated from a non-cooperative target. In our paper, we consider a passive measurement scenario, such that only one sonar sensor measures the sound of a non-cooperative target.

References [1], [2] considered the case where the GMTI radar is used, which can measure range, azimuth, and doppler of the target. References [1], [2] considered tracking a ground target assisted by road constraints, which are known in advance. Based on the assumption that the target is following the road network, [1] enabled the target estimation on the road by using projection approaches. Note that our paper is distinct from [1], [2], since we do not use GMTI radar, which can measure range, azimuth, and doppler of the target.

III. PRELIMINARY INFORMATION

This section presents the preliminary information of this manuscript.

A. DOPPLER-ONLY TRACKING PROBLEM STATEMENT

This manuscript handles how to calculate the target's position under doppler-only measurements. This manuscript considers discrete-time systems. Let dt present the sampling interval in discrete-time systems.

Assume that the target moves along a straight road with a constant velocity. We assume that the road information is known in advance and that the target's course is known a priori. In other words, we consider a ground target which is constrained to move along a known straight road with a known course.

Let $f^e(k)$ denote the emitting frequency of the target at time-step k. We assume that $f^e(k)$ is a constant for all $k > 0$. Thus, we use f^e instead of $f^e(k)$. This assumption is commonly used in the literature [3], [12], [13].

Let $V(k)$ denote the target's speed at time-step k. We consider a constant speed target, which is commonly used in the literature on doppler-only tracking [4], [5]. We assume that $V(k)$ is a constant for all $k > 0$. Thus, we use V instead of $V(k)$.

Let V_{max} denote the maximum speed of the target, which is assumed to be known in advance. Suppose we have a priori information on V_{max}.

Suppose we have a priori information on the maximum sensing range. Let r_{max} denote the maximum sensing range. The upper bound for signal strength of the target can be used to set r_{max}.

Let *road segment* denote a line segment of the road, such that each point in this segment is within r_{max} distance from the sensor. Considering the maximum sensing range, the target must exist on the road segment as the sensor measures the signal of the target.

We consider a single sensor, which is located at the origin. We further set the coordinate system such that the road segment is oriented in x-direction. In this coordinate system, the road segment is parallel to the x-axis and its y-coordinate is y_d, which is known in advance. Here, y_d is the relative distance between the road and the sensor at the origin.

The state vector $X(k) = [x(k), V, f^e]^T$ denotes the target's x-coordinate $x(k)$, target speed V, and emitting frequency f^e at time-step k. Since the target moves along a straight road with a constant velocity, we use

$$X(k) = F_k X(0). \tag{1}$$

where F_k is

$$F_k = \begin{pmatrix} 1 & dt * k & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2}$$

Using (1), the target state $X(k)$ can be derived from $X(0)$. Also, let $\hat{X}(0)$ denote the *initial target estimate*. $\hat{X}(0)$ presents the estimation of $X(0)$.

Let $r(k) = \sqrt{x(k)^2 + y_d^2}$ for convenience. The doppler-only measurement at time-step k is

$$f(k) = f^e * (1 - \frac{\dot{r}(k)}{C}) + e(k). \tag{3}$$

$f(k)$ is the doppler shift of the target measured at time-step k. In (3), C is the signal speed, and $e(k)$ presents the measurement noise and has a Gaussian distribution with $e(k) \sim N(0, \sigma_f^2)$. In (3), $\dot{r}(k)$ is derived as

$$\dot{r}(k) = \frac{x(k) * V}{\sqrt{x(k)^2 + y_d^2}}. \tag{4}$$

(3) and (4) imply that $f(k)$ is a function of $X(k)$. In other words, (3) can be rewritten as the following measurement equation.

$$M(X(k)) = f^e * (1 - \frac{\dot{r}(k)}{C}) + e(k). \tag{5}$$

Furthermore, using (1), $M(X(k))$ in (5) is a function of $X(0)$.

Recall that $\hat{X}(0)$ denotes the initial target estimate. In this paper, we address how to estimate $X(0)$ using a batch of doppler measurements. Suppose that we derived $\hat{X}(0)$ using a batch of doppler measurements. Then, at each time-step k, the target state $X(k)$ can be estimated using (1).

1) OBSERVABILITY ANALYSIS

We assume that the target's course is accessible. We show that if this assumption is not used, the target state is not observable. In Figure 1, arrows indicate the maneuver of the target along a known road. The numbers on the arrows show the time step. The upper sub-figure shows the case where the target moves from left to right, such that the target is to the left of the sensor at time-step 0. Also, the lower sub-figure shows the case where the target moves from right to left, such that the target is to the left of the sensor at time-step 0. Doppler measurements of these two cases are identical, since the range rates are identical. If the target's course is not known in advance, the sensor cannot observe whether the target moves from left to right, or from right to left. Thus, in order to make the target state observable, the target's course must be accessible.

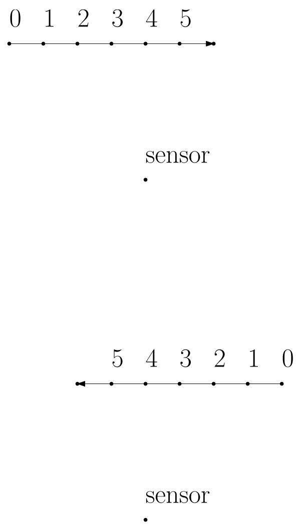

FIGURE 1. **Arrows indicate the maneuver of the target along a known road. The numbers on the arrows show the time step. The upper sub-figure shows the case where the target moves from left to right, such that the target is to the left of the sensor at time-step 0. Also, the lower sub-figure shows the case where the target moves from right to left, such that the target is to the left of the sensor at time-step 0. Doppler measurements of these two cases are identical.**

We further show the conditions where the observability of the doppler-only tracking does not hold. Consider two cases where the emitting frequency are f_1^e and $f_2^e = A * f_1^e$ respectively. Here, $A < 1$ is a positive constant. In the first case, the target moves with range rate $\dot{r}_1$, such that the emitting frequency is f_1^e. In the second case, the target moves with range rate $\dot{r}_2$, such that the emitting frequency is f_2^e. Using (3), these two cases generate the identical frequency measurements when

$$f_1^e * (1 - \frac{\dot{r}_1}{C}) = A * f_1^e * (1 - \frac{\dot{r}_2}{C}) \quad (6)$$

hold. (6) leads to

$$\dot{r}_1 = C - A * C + A * \dot{r}_2. \quad (7)$$

Thus, if (7) holds, then the system is not observable. Since $A < 1$ is a positive constant, $C - A * C$ in (7) is also positive.

In order to check the observability of the doppler-only tracking problem rigorously, we use the observability analysis as presented in [15]. Consider the case where we use a batch of doppler measurements in order to estimate $X(0)$. A batch of measurements s_m has following M doppler measurements.

$$s_m = [f(1), f(2), \ldots, f(M)]^T. \quad (8)$$

Recall that $f(k)$ in (3) is a function of $X(0)$. Thus, in the absence of measurement noise, s_m in (8) is equal to a known function $h(X(0))$. Considering measurement noise, we have

$$s_m = h(X(0)) + n. \quad (9)$$

Here, n presents a Gaussian noise with mean 0 and covariance matrix N.

The observability of $X(0)$ from the measurements s_m is that the following observability matrix has full rank.

$$O = \frac{\partial h(X(0))}{\partial X(0)} \quad (10)$$

Since $X(0)$ has three elements, the observability matrix O must have rank 3.

In this paper, we consider the case where the initial state vector $X(0)$ satisfies that the observability matrix O has rank 3. If this observability condition is not met, then it is impossible to derive $X(0)$ from the measurement set s_m. In order to satisfy that the observability matrix O has rank 3, we require at least $M \geq 3$ measurements.

Assuming that the frequency of the emitting tone is known, [4] proved that three doppler shift measurements of a single sensor are sufficient to determine the target state (position and velocity). We proved that at least three doppler measurements are required for system observability, even in the case where the frequency of the emitting tone is not known in advance.

B. GAUSS-NEWTON (GN) ALGORITHM

Suppose that the doppler-only tracking system is observable, as presented in Section III-A1. This subsection presents the GN algorithm to derive $\hat{X}(0)$ from a batch of doppler measurements s_m in (8). Suppose that we derived $\hat{X}(0)$ using a batch of doppler measurements. Then, at each time-step k, the target's current state $X(k)$ can be estimated using (1).

In this subsection, $\hat{X}_0$ is used instead of $\hat{X}(0)$ for notation simplicity. Also, X_0 is used instead of $X(0)$ for notation simplicity.

The GN algorithm iteratively updates $\hat{X}_0$ to minimize

$$Q(\hat{X}_0) = (s_m - h(\hat{X}_0))^T N^{-1} (s_m - h(\hat{X}_0)). \quad (11)$$

Since the GN algorithm is an iterative algorithm, $\hat{X}_{0,j}$ is utilized to represent $\hat{X}_0$ at j-th iteration. Let G be defined as $G = \frac{\partial h(\hat{X}_0)}{\partial \hat{X}_0}$ at $\hat{X}_0 = \hat{X}_{0,j}$. See that G is equal to the observability matrix O in (10), as we consider the case where $\hat{X}_0$ is identical to the true state X_0. Since the system is observable, G has full rank when $\hat{X}_0 = X_0$.

Let δ define the perturbation on $\hat{X}_{0,j}$. Utilizing (11), we derive

$$Q(\hat{X}_{0,j} + \delta) = D^T N^{-1} D. \quad (12)$$

Here, $D = s_m - h(\hat{X}_{0,j}) - \delta G$. In (12), the Taylor expansion is utilized as follows.

$$h(\hat{X}_{0,j} + \delta) = h(\hat{X}_{0,j}) + \delta G. \quad (13)$$

We differentiate $Q(\hat{X}_{0,j} + \delta)$ with respect to δ, and set it equal to zero.

$$\frac{\partial Q(\hat{X}_{0,j} + \delta)}{\partial \delta} = (-G)^T N^{-1} (s_m - h(\hat{X}_{0,j}) - \delta G) = 0. \quad (14)$$

Then, we get

$$\delta = (G^T N^{-1} G)^{-1} G^T N^{-1} (s_m - h(\hat{X}_{0,j})). \quad (15)$$

Hence, at each iteration of the GN algorithm, we update the estimation using

$$\hat{X}_{0,j+1} = \hat{X}_{0,j} + \delta. \tag{16}$$

Let $V(\hat{X}_0) = E[(\hat{X}_0 - E[\hat{X}_0])(\hat{X}_0 - E[\hat{X}_0])^T]$ denote the covariance of $\hat{X}_0$. Utilizing (15), we can get

$$V(\hat{X}_0) = (G^T N^{-1} G)^{-1}. \tag{17}$$

Considering the GN method, the authors of [16] addressed how to derive the covariance matrix (17) in detail.

The GN method requires a rather accurate initial estimate $\hat{X}_{0,0}$ for its convergence. For initialization of the GN method, we use distributed samples, so that each sample can randomly generate its associated $\hat{X}_{0,0}$ considering the road constraints. In the case where a sample yields a small measurement residual, the sample gets a larger importance weight compared to other samples. At every time-step, we calculate importance weights for every sample, and we can determine the range of re-sampling from the weighted estimate and its variance.

IV. TRACKING FILTER USING A BATCH OF MULTIPLE DOPPLER MEASUREMENTS

For initialization of the GN method, we distribute samples, so that each sample can randomly generate its associated $\hat{X}_{0,0}$. In this paper, samples are distributed considering the road constraints. Since samples are deployed along the road segment, we can decrease the number of samples required for target estimation.

Suppose that we use N samples in total. These samples are used to derive the initial target estimate $\hat{X}(0)$. Once we derived $\hat{X}(0)$, then at each time-step k, the target state $X(k)$ can be estimated using (1).

Let $X^{(j)} = [x^{(j)}, V^{(j)}, f^{(j)}]^T$ denote the state vector associated to the j-th sample at time-step k. In $X^{(j)}$, $[x^{(j)}]$ denotes the x-coordinate of the j-th sample. Also, $V^{(j)}$ denotes the speed of the j-th sample. $f^{(j)}$ is the emitter frequency f^e associated to the j-th sample.

From time-step 0 to time-step k, we can accumulate a batch of doppler measurements composed of k measurements. In other words, (8) leads to the following measurement set.

$$s_m = [f(1), f(2), \ldots, f(k)]^T. \tag{18}$$

The GN method in Section III-B requires that we have sufficiently many doppler shift measurements in the set s_m. We apply the GN method in Section III-B, only in the case where $\|s_m\| \geq M_{min}$. Here, $M_{min} > 0$ is a positive tuning constant. In other words, samples are deployed only when $k \geq M_{min}$. As $k \geq M_{min}$, we can satisfy that $\|s_m\| \geq M_{min}$.

We address how to set our tuning constant M_{min}. Considering the observability condition in Section III-A1, we require that the initial state vector X_0 satisfies that the observability matrix O has rank 3. If this observability condition is not met, then it is impossible to derive X_0 from the measurement set s_m. In order to satisfy that O has rank 3, we require that $M_{min} \geq 3$.

In the optimal case where there is no measurement noise, we can set $M_{min} = 3$. However, considering the doppler measurement noise, $M_{min} = 3$ may lead to an inaccurate solution. Thus, in MATLAB simulations, we set $M_{min} = 40$.

A. SAMPLE INITIALIZATION

At time-step M_{min}, samples are deployed for the first time. At time-step M_{min}, N samples are deployed to estimate the initial state vector $\hat{X}(0)$. Considering the j-th sample ($j \in \{1, 2, \ldots, N\}$), $x^{(j)}_{M_{min}}$ is randomly deployed on the road segment, while satisfying that the relative distance between $x^{(j)}_{M_{min}}$ and the sensor is less than the maximum sensing range r_{max}. Also, $V^{(j)}_{M_{min}}$ is randomly generated using $rand * V_{max}$. Here, $rand$ generates a random number in the interval [0,1]. Also, for all $j \in \{1, 2, \ldots, N\}$, $f^{(j)}$ is set as the doppler shift measurement $f(M_{min})$. The importance weight of each sample at time-step M_{min} is $w^{(j)}_{M_{min}} = 1/N$ where $j \in \{1, 2, \ldots, N\}$.

B. SAMPLE PROPAGATION

At every time-step k ($k \geq M_{min}$), N samples $X^{(j)}$ ($j \in \{1, 2, \ldots, N\}$) are generated to initialize the target solution of the GN method in Section III-B. In other words, considering the j-th sample, $X^{(j)}$ is set as the initial estimate $\hat{X}_{0,0}$ in the GN method.

Let $\hat{E}^{(j)}$ denote the target solution, which is generated by the GN method initialized with the j-th sample $X^{(j)}$ ($j \in \{1, 2, \ldots, N\}$). At each time-step k ($k \geq M_{min}$), the likelihood of $\hat{E}^{(j)}$ ($j \in \{1, 2, \ldots, N\}$) is set as

$$l^{(j)} = \frac{1}{\sqrt{det(N)}} exp(-0.5 q_s^T (N)^{-1} q_s). \tag{19}$$

Here, $q_s = s_m - h(\hat{E}^{(j)})$, and N is the covariance of n in (10). Then, at each time-step k ($k \geq M_{min}$), the importance weight of $\hat{E}^{(j)}$ ($j \in \{1, 2, \ldots, N\}$) is normalized using

$$w^{(j)} = \frac{l^{(j)}}{\sum_{j=1}^{N} l^{(j)}}. \tag{20}$$

The authors of [17] addressed how to combine the estimates and the covariances of multiple filter banks. Inspired by [17], the combined estimate, which is derived at time-step k ($k \geq M_{min}$), is

$$\hat{E} = \sum_{j=1}^{N} w^{(j)} \hat{E}^{(j)}. \tag{21}$$

Here, $\hat{E}$ is the combined target estimation, which is derived at time-step k. Once we derived $\hat{E}$, then at each time-step k, the target state $X(k)$ can be estimated using

$$X(k) = F_k \hat{E} \tag{22}$$

where (1) is used.

Recall that $V(\hat{E})$ in (17) presents the covariance of the solution $\hat{E}$. Inspired by [17], the combined covariance is derived as

$$V(\hat{E}) = \sum_{j=1}^{N} w^{(j)}[V(\hat{E}^{(j)}) + (\hat{E}^{(j)} - \hat{E})(\hat{E}^{(j)} - \hat{E})^T]. \quad (23)$$

Here, $V(\hat{E})$ is the covariance of $\hat{E}$.

C. RE-SAMPLING

At time-step M_{min}, samples are deployed for the first time. Thereafter, at each time-step k ($k > M_{min}$), samples are re-sampled considering the weighted estimate and its variance ($\hat{E}$ and $V(\hat{E})$). Let std_p be defined as

$$std_p = max(rngStdMin, \eta * \sqrt{trace(V(\hat{E}))}). \quad (24)$$

In (24), std_p determines the distribution range of samples when samples are re-sampled. As std_p increases, samples are distributed in a wider space. In (24), $max(a, b)$ returns a bigger value between a and b. It is not desirable that all samples are concentrated in a small space. Thus, in (24), $rngStdMin > 0$ is a tuning parameter, which is used as a lower bound for std_p. Also, $\eta \geq 1$ is a tuning parameter. As η increases, samples are distributed in a wider space.

Recall that $X^{(j)} = [x^{(j)}, V^{(j)}, f^{(j)}]^T$ denotes the state vector associated to the j-th sample at time-step k. Considering the j-th sample, $x^{(j)}$ is re-sampled as follows.

$$x^{(j)} = \hat{E}[1] + rand_{std}. \quad (25)$$

Here, $\hat{E}[n]$ denotes the n-th element in a vector $\hat{E}$. Also, we use $rand_{std} = (rand * 2 * std_p - std_p)$, which implies that $rand_{std}$ is randomly generated in the interval $[-std_p, std_p]$, since $rand$ generates a random number in the interval [0, 1]. In this way, $x^{(j)}$ is generated along the road segment, such that each sample is randomly located inside the circle with center $\hat{E}[1]$ and radius std_p. (25) shows that std_p determines the distribution range of samples when samples are re-sampled.

For all $j \in \{1, 2, \ldots, N\}$, $V^{(j)} \in X^{(j)}$ is randomly re-sampled using $rand * V_{max}$. This implies that $V^{(j)}$ is randomly generated in the interval [0, V_{max}]. Also, $f^{(j)} \in X^{(j)}$ is re-sampled using

$$f^{(j)} = \hat{E}[3]. \quad (26)$$

V. MATLAB SIMULATIONS

MATLAB simulations are utilized to present the effectiveness of our tracking filter using a batch of multiple doppler measurements. According to [3], we consider a scenario where the target approaches a sensor from a far, passes through or at some distance from the sensor, and then departs.

We consider a single passive sonar sensor, which is located at the origin. We further set the coordinate system such that the road segment is oriented in x-direction. In this coordinate system, the road segment is parallel to the x-axis and its y-coordinate is y_d, which is known in advance. In simulations, the single sensor at the origin measures the doppler of sound emitted from the constant velocity target.

The parameter settings are as follows. The sampling interval is $dt = 1$ second. Also, $f^e = 1000$ Hz. We set $r_{max} = 1000$ meters. $N = 300$ samples are used. Furthermore, the sound speed is $C = 350$ m/s. The doppler shift measurement noise σ_f is set as 0.1 Hz. We utilize $V_{max} = 20$ meters per second. For re-sampling, we use $\eta = 3$ and $rngStdMin = 300$ in std_p (24).

The GN method in Section III-B requires that we have sufficiently many doppler shift measurements in the set s_m. Section IV mentions that in the optimal case where there is no measurement noise, we can set $M_{min} = 3$. However, considering the measurement noise, $M_{min} = 3$ may lead to an inaccurate solution. Thus, we set $M_{min} = 40$. We apply the GN method in Section III-B, only in the case where $\|s_m\| \geq M_{min}$. This implies that in the case where $\|s_m\| \geq M_{min}$ is not met, the target estimation is not generated at all.

We run $M_c = 100$ Monte-Carlo (MC) simulations. $\hat{p}^j(k)$ where $j \in \{1, 2, , , M_c\}$ is the target's 2-D position at time-step k utilizing the j-th MC simulation. Let $p(k)$ denote the target's 2-D position at time-step k.

$\|\hat{p}^j(k) - p(k)\|$ is the error of the target estimation at time-step k. The following RMSE is calculated:

$$RMSE_k = \sqrt{\frac{\sum_{j=1}^{M_c} \|\hat{p}^j(k) - p(k)\|^2}{M_c}}. \quad (27)$$

A. POSTERIOR CRAMER-RAO LOWER BOUND (PCRLB)

We use the Posterior Cramer-Rao Lower Bound (PCRLB) as the lowest estimation error of any unbiased estimator [18]. Let $B_{k|k}$ define the error covariance (uncertainty) of the state vector X_k.

At every time-step k, we utilize the EKF in [6] to update $B_{k|k}$. According to [6], we get the recursive form of the PCRLB by merging the prediction and the update step for the error covariance of the EKF.

Since the target moves along a straight road with a constant velocity, (1) leads to

$$X(k + 1) = F_1 X(k). \quad (28)$$

where F_1 is (2) with $k = 1$. Then, the error covariance (uncertainty) of the state vector X_{k+1} is predicted as

$$B_{k+1|k} = F_1 B_{k|k} F_1^T. \quad (29)$$

Using (29), the recursive form of the PCRLB is

$$(B_{k+1|k+1})^{-1} = (F_1 B_{k|k} F_1^T)^{-1} + M^T (R)^{-1} M. \quad (30)$$

Here, M is the Jacobian matrix of $M(X(k))$ in (5) given by $M = \frac{\partial M(X)}{\partial X}|_{X=X(k)}$.

We use the PCRLB in (30) as the lowest estimation error of any unbiased estimator [18]. In (30), the initial iteration uses

$$B_{0|0} = 10^{-6} * diag(r_{max}^2, V_{max}^2, (\delta f)^2). \quad (31)$$

In (31), $diag(a, b, c, \ldots)$ denotes a diagonal matrix with diagonal elements $a, b, c, \ldots$ in this order. Based on (5), (31) uses $\delta f = \frac{f^e * V_{max}}{C}$.

Let $B_{k|k}[j,j]$ denote the j-th diagonal element in $B_{k|k}$. $\sqrt{B_{k|k}[1,1]}$ provides the lower bound for estimation error for the state x_k in X_k. Thus, the PCRLB at time-step k is

$$PCRLB_k = \sqrt{B_{k|k}[1,1]}. \tag{32}$$

B. SCENARIO 1

The sensor is at (0,0), and the initial target is at (-200,200) in meters. The target's velocity vector is (3, 0) in m/s.

Figure 2 depicts the scenario of one MC simulation. The sensor position is marked with a blue asterisk. Samples deployed along the straight road segment are plotted with yellow circles. In this figure, the target's position at every 5 seconds is marked with red diamond. The target estimate at every 5 seconds is plotted with black cross. The target estimate is close to the true target position.

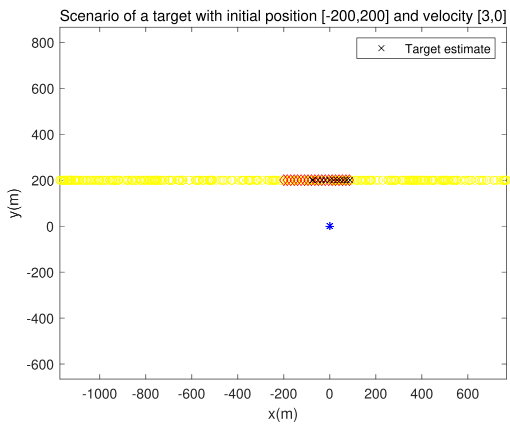

FIGURE 2. **Scenario 1. The target's position at every 5 seconds is plotted with red diamond. The target estimation at every 5 seconds is plotted with black cross. The target estimation is close to the true target position.**

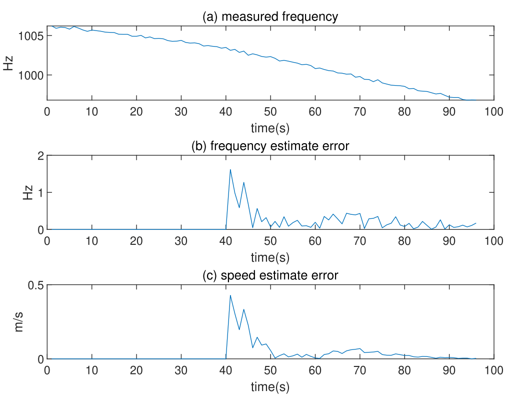

FIGURE 3. **(a) Doppler shift measurements. (b) frequency estimation error. (c) target speed estimation error (Scenario 1).**

Considering the scenario in Figure 2, the subfigure (a) of Figure 3 shows the doppler shift measurements as time goes on. The subfigure (b) shows the difference between the true frequency measurement and the estimated frequency measurement (frequency measurement conjectured using the target estimate) as time goes on. The subfigure (c) shows the difference between the true target speed and the estimated target speed as time goes on. We derive the target estimation using the GN method in Section III-B, only in the case where $\|s_m\| \geq M_{min}$. See that as time goes on, the estimates converges to the true values.

1) RMSE OF MC SIMULATIONS

Considering the scenario in Figure 2, Figure 4 presents $RMSE_k$ with respect to k. The lower sub-figure (b) is the enlarged figure of the upper sub-figure (a). See that the RMSE decreases as time goes on.

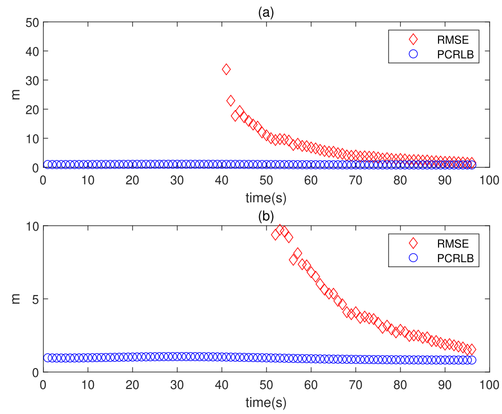

FIGURE 4. **RMSE plot (Scenario 1). The lower sub-figure (b) is the enlarged figure of the upper sub-figure (a). In the case where $\|s_m\| \geq M_{min}$ is not met, the target estimation is not generated at all. Thus, we set $RMSE_k = \infty$ when $k < M_{min}$. As time goes on, the RMSE reduces gradually.**

In the case where $\|s_m\| \geq M_{min}$ is not met, the target estimation is not generated at all. Thus, we set $RMSE_k = \infty$ when $k < M_{min}$. As time goes on, the RMSE reduces gradually.

It takes almost 15 seconds to run one MC simulation using MATLAB. Since the entire scenario runs for 96 seconds (see the x-axis of Figure 4), the proposed doppler-only tracking is suitable for real-time target tracking.

C. SCENARIO 2

The sensor is at (0,0), and the initial target is at (-200,400) in meters. The target's velocity vector is (10, 0) in m/s.

Figure 5 depicts the scenario of one MC simulation. The sensor position is plotted with a blue asterisk. The target's position at every 5 seconds is plotted with red diamond. See that the target moves away from the sensor at the origin. Samples deployed along the straight road segment are plotted with yellow circles. The target estimation at every 5 seconds is plotted with black cross. The target estimation is generated, only in the case where $\|s_m\| \geq M_{min}$.

Considering the scenario in Figure 5, the subfigure (a) of Figure 6 shows the doppler shift measurements as time goes on. The subfigure (b) plots the difference between the true frequency measurement and the estimated frequency measurement as time goes on. The subfigure (c) plots the difference between the true target speed and the estimated target speed

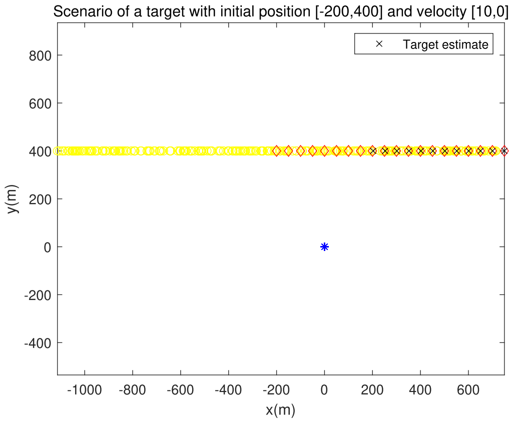

FIGURE 5. Scenario 2. The target's position at every 5 seconds is plotted with red diamond. Samples deployed along the straight road segment are plotted with yellow circles. The target estimation at every 5 seconds is plotted with black cross. The target estimation is close to the true target position.

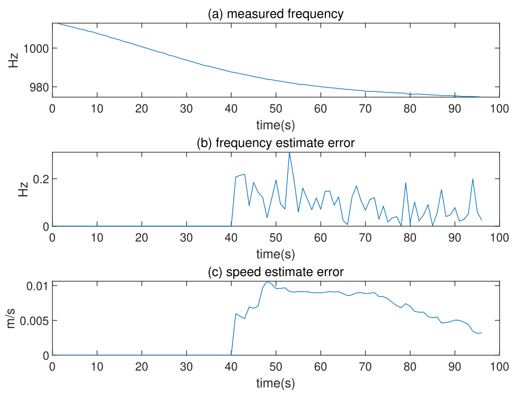

FIGURE 6. (a) Doppler shift measurements. (b) frequency estimation error. (c) target speed estimation error (Scenario 2).

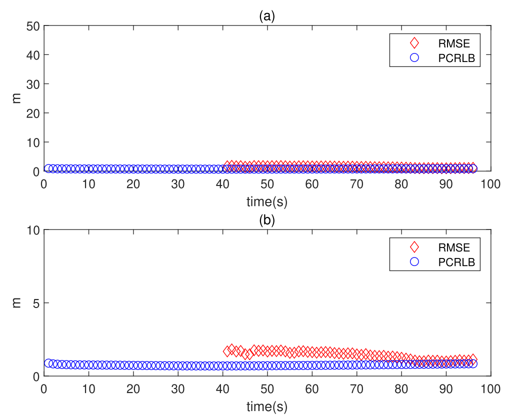

FIGURE 7. RMSE plot (Scenario 2). The lower sub-figure (b) is the enlarged figure of the upper sub-figure (a). The lower subfigure (b) shows that the RMSE decreases as time goes on. In the case where $\|s_m\| \geq M_{min}$ is not met, the target estimation is not generated at all. Thus, we set $RMSE_k = \infty$ when $k < M_{min}$.

as time goes on. In the case where $\|s_m\| \geq M_{min}$ is not met, the target estimation is not generated at all. As time goes on, the estimates converges to the true values.

1) RMSE OF MC SIMULATIONS

Considering the scenario in Figure 5, Figure 7 presents $RMSE_k$ with respect to k. The lower sub-figure (b) is the enlarged figure of the upper sub-figure (a). The lower sub-figure (b) shows that the RMSE decreases as time goes on. In the case where $\|s_m\| \geq M_{min}$ is not met, the target estimation is not generated at all. Thus, we set $RMSE_k = \infty$ when $k < M_{min}$.

It takes almost 15 seconds to run one MC simulation using MATLAB. Since the entire scenario runs for 96 seconds (see the x-axis of Figure 7), the proposed doppler-only tracking is suitable for real-time target tracking.

VI. CONCLUSION

In this paper, we consider tracking a ground target which is constrained to move along a known straight road. It is assumed that a single passive sonar sensor, which is located close to the road, can measure the doppler shift of the signal emitted from the target. We consider a scenario where the frequency of the emitting tone is not known in advance. Also, we assume that the road shape is known to the sensor and that the target's course is known in advance.

We address a non-recursive estimation algorithm using a batch filter based on distributed sampling process. We use the GN algorithm to calculate the target solution from a batch of doppler measurements. The GN method requires a rather accurate initial estimate for its convergence. We thus distribute samples, so that each sample can randomly generate its associated solution considering the road constraints. In the case where a sample yields a small measurement residual, the sample gets a larger importance weight compared to other samples. At every time-step, we calculate importance weights for every sample, and we determine the range of re-sampling from the weighted estimate and its variance.

As far as we know, our paper is novel in using road constraints for target tracking based on doppler-only measurements. The effectiveness of the proposed tracking filter is demonstrated using MATLAB simulations.

The proposed doppler-only tracking can be used for tracking various targets, such as ships or airplanes, whose routes are known in advance. For instance, consider a scenario where a target operates a routine patrol mission. If there is a single sonar sensor close to the route of a target, then the sensor can track the target using the proposed tracking approach.

In the future, we will do experiments with real sonar sensors, in order to verify the effectiveness of the proposed tracking approach. Also, we will tackle doppler-only tracking using multiple heterogeneous passive sonar sensors.

This paper considers a passive scenario in [3], where a sonar sensor only measures the doppler shifted sound generated from a non-cooperative target. Reference [3] considered a scenario where the target approaches a sensor from a far, passes through or at some distance from the sensor, and then departs. In the future, we will tackle doppler-only tracking of multiple non-cooperative targets, using multiple heterogeneous passive sonar sensors.

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